Testing for Malfunction

Let X_t be the monthly produced energy and μ_t is the value estimated by sonnendach.ch for each month. We assume that X_t is normally distributed with variance σ^2 not depending on t. We model the expectation of X_t as the sum of a systematic part γ and the temporal part μ_t, concretely

$$X_t \sim N(\mu_t + \gamma, \sigma^2).$$

Thus we have for $Z_t := X_t - \mu_t$

$$Z_t \sim N(\gamma, \sigma^2)$$

and we assume that the Z_t are independent. This can now be used to construct a statistical test at level α, here $\alpha = 0.05$. Let γ_0 be a real number, in practice γ_0 will be zero. We test the following hypothesis:

$$\begin{aligned} H_0 : &\qquad \gamma \geq \gamma_0 \\ H_1 : &\qquad \gamma < \gamma_0 \end{aligned}$$

If the hypothesis H_0 is rejected, the systematic difference γ is below the level where it is assumed to be working, thus indicating a malfunction. For testing we use the following statistic

$$P = \sqrt{n}\frac{\overline{Z} - \gamma_0}{S_n}$$

where n is the number of months, $\overline{Z}$ is the mean of the Z_t and S_n^2 is its unbiased sample variance. Note that P is t-distributed of degree $n-1$. Therefore, we consider the PV as malfunctioning if

$$P < t_{n-1}^{-1}(\alpha)$$

with t_{n-1}^{-1} the quantile function of the t-distribution.