Damping in the Winter's Model

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Using the terminology of Kraig Winters we need to define a reasonable coefficient for damping.

Given that,

$$\frac{\partial u}{\partial t} = \nu \nabla^{2n} u \tag{1}$$

is spectrally,

$$\frac{\partial u}{\partial t} = \nu (ik2\pi)^{2n} u \tag{2}$$

which has the solution

$$u = e^{\nu (ik2\pi)^{2n} t}. \tag{3}$$

We want to convert this into an e-fold time, so we want

$$e^{-t/T} = e^{\nu (ik2\pi)^{2n} t}. \tag{4}$$

Using that $k = \frac{1}{2\Delta}$ where Δ is the sample interval and solving for ν in terms of the other variables,

$$\nu = \frac{(-1)^{n+1}}{T} \left(\frac{\Delta}{\pi} \right)^{2n} \tag{5}$$

Now add some forcing,

$$\frac{\partial \hat{u}}{\partial t} = \hat{F} + \nu (ik2\pi)^{2n} \hat{u} \tag{6}$$

solution

$$\hat{u} = u_0 e^{\nu (-1)^n (k2\pi)^{2n} t} - (-1)^n \frac{F}{\nu (k2\pi)^{2n}} \tag{7}$$

Then, in steady state,

$$\hat{u}\hat{u}^* = \frac{F^2}{\nu^2} (2\pi k)^{-4n} \tag{8}$$

- We can compute the wavenumber at which damping drops the amplitude more than 50 percent during the length of the simulation.

- We can compute, given U, the cfl criteria, and then ask that the Reynolds number be one at the grid scale.

How does this hyperviscous ν compare to the usual ν_0?

$$\begin{aligned} \nu(k2\pi)^{2n}u =& \nu_0(k2\pi)^2 u && (9) \\ \nu_0 = \nu(k2\pi)^{2n-2} & && (10) \\ \nu_0 = \frac{1}{T}\left(\frac{\Delta}{\pi}\right)^2 & && (11) \end{aligned}$$

$$\begin{aligned} \frac{U\Delta}{\nu_0} =& 1 && (12) \\ \frac{(-1)^{n+1}}{U\Delta}\left(\frac{\Delta}{\pi}\right)^2 =& T && (13) \end{aligned}$$

For one simulation, we have that $U = 0.0365$ m/s and $\Delta = 6750$ m with $n = 3$. This suggests a damping time scale of $T = 18000$s. The actually simulation used twice that.

The linear wave mode propagation speed seems to be the maximum velocity in these simluations.