

Damping in the Winter's Model

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Using the terminology of Kraig Winters we need to define a reasonable coefficient for damping.

Given that,

$$\frac{\partial u}{\partial t} = \nu \nabla^{2n} u \quad (1)$$

is spectrally,

$$\frac{\partial u}{\partial t} = \nu (ik2\pi)^{2n} u \quad (2)$$

which has the solution

$$u = e^{\nu(ik2\pi)^{2n}t}. \quad (3)$$

We want to convert this into an e-fold time, so we want

$$e^{-t/T} = e^{\nu(ik2\pi)^{2n}t}. \quad (4)$$

Using that $k = \frac{1}{2\Delta}$ where Δ is the sample interval and solving for ν in terms of the other variables,

$$\nu = \frac{(-1)^{n+1}}{T} \left(\frac{\Delta}{\pi} \right)^{2n} \quad (5)$$

Now add some forcing,

$$\frac{\partial \hat{u}}{\partial t} = \hat{F} + \nu(ik2\pi)^{2n} \hat{u} \quad (6)$$

solution

$$\hat{u} = u_0 e^{\nu(-1)^n(k2\pi)^{2n}t} - (-1)^n \frac{F}{\nu(k2\pi)^{2n}} \quad (7)$$

Then, in steady state,

$$\hat{u}\hat{u}^* = \frac{F^2}{\nu^2} (2\pi k)^{-4n} \quad (8)$$

- We can compute the wavenumber at which damping drops the amplitude more than 50 percent during the length of the simulation.
- We can compute, given U, the cfl criteria, and then ask that the Reynolds number be one at the grid scale.