

# Energy Storage and Transport

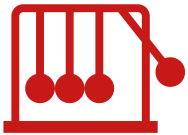
Modeling example

# The model development cycle



## Modelling objective

Analyze the context and define an objective for modeling.  
**Which purpose can a model serve** in the project?



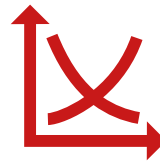
## Physical phenomena

Perform research to understand the physical phenomena that **play a role in the project**. How do they relate to the modelling objective?



## Model selection

Select which phenomena are essential to include and which may be neglected. Which **simplifications and assumptions** can be made?



## Mathematical elaboration

Define and combine the mathematical relations that describe the physical phenomena and apply the selected simplifications.



## Implementation

Create the model which uses the mathematical relations to predict (an) outcome(s) based on given inputs.



## Validation & verification

Validate: does the model solve the right equations?  
Verify: does the model solve the equations correctly?

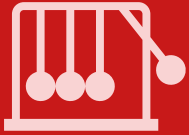


## Reflection

Is the modelling objective met? What are the uses and limitations of the model? How can it be improved?  
**Iterate steps when appropriate**



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



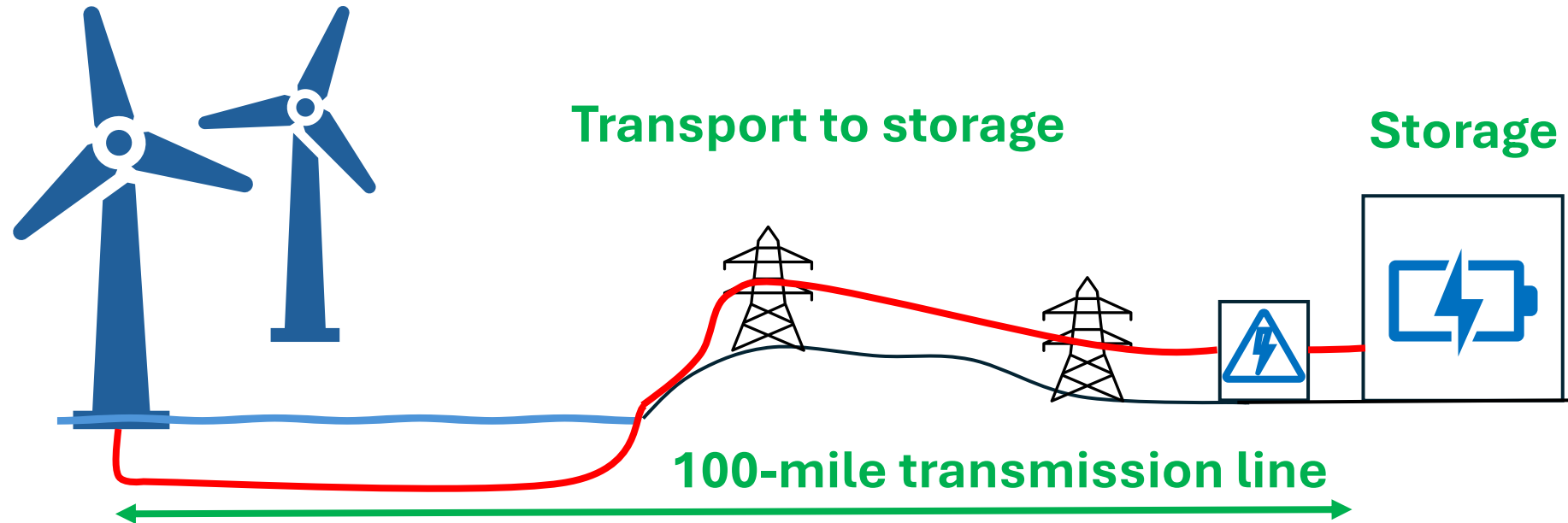
Validation &  
verification



Reflection

# Energy Storage and Transport

Supply: 100 x 10 MW



- Large off-shore wind farm, transporting electric energy to a hydroelectricity dam using high-voltage transmission lines.
- Topic of this lecture:  
**Model development for the transmission lines**



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

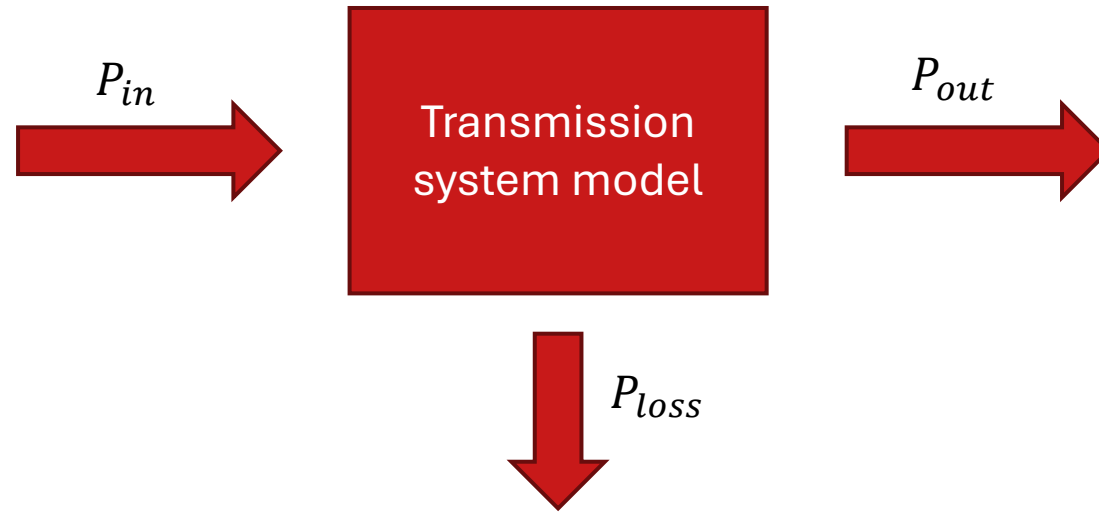


Validation &  
verification



Reflection

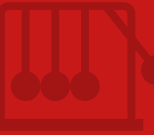
# The model as a black box



- The transmission line model should provide a relation between:
  - **Input:** power supplied by the turbines
  - **Output:** power delivered to the storage



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# Modeling questions

- How much energy is lost in the transmission lines?
- How do these losses depend on the transport system parameters?
  - Length of the transmission lines
  - Electricity transport voltage and power
  - Type of transmission cables
  - Type of transformers
  - Etc.



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

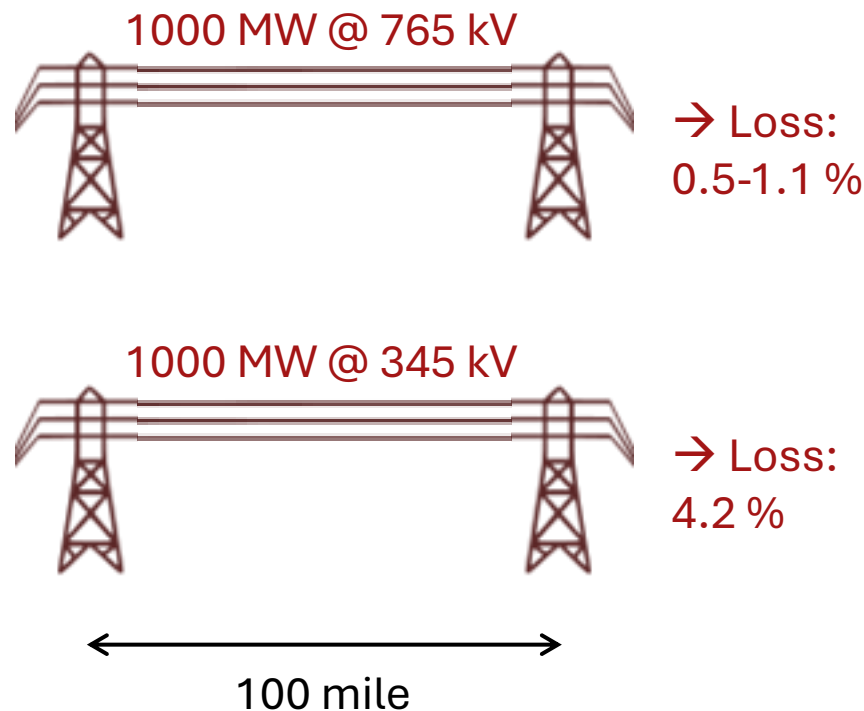


Validation &  
verification



Reflection

# What are typical losses?

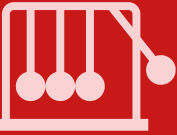


Example from [literature](#):

- A 100 mi (160 km) span at 765 kV carrying 1000 MW of power can have losses of 0.5% to 1.1%.
- A 345 kV line carrying the same load across the same distance has losses of 4.2%.



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

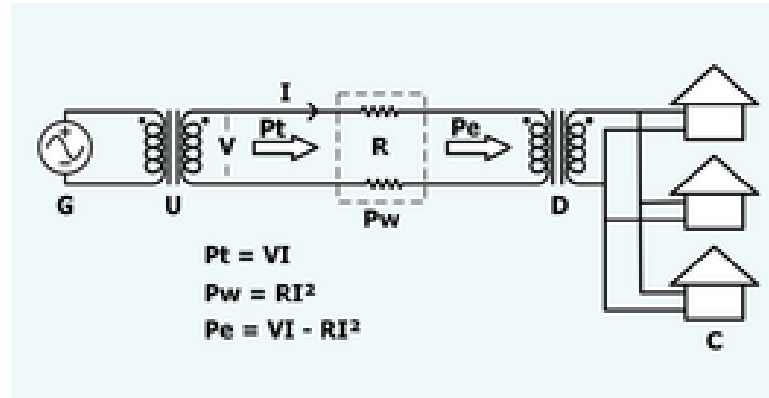


Validation &  
verification



Reflection

# Why is a higher voltage better?



- Transported power = voltage [V] x current (A):
  - $P = VI$
- Power losses = resistance [Ohm] x current (A)<sup>2</sup>
  - $P_{loss} = RI^2$

- Best to transport energy at a low current
- Given a fixed transport power, the current can be lowered by increasing the voltage



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# What causes the losses?

## Joule heating

*resistance of wires*

Dependent on  
properties of cables and  
power current

→ Assumption: **main  
source of loss**

## Inductance

*caused by magnetic field*

Only relevant for  
alternating current

→ Assumption: **direct  
current is used**

## Corona losses

*air ionization*

Only relevant when  
voltage exceeds the  
corona threshold

Dependent on  
weather conditions  
and temperature

→ Assumption: **can  
be neglected**

## Leakage

*to ground*

Minimal in a correctly  
designed system

→ Assumption: **can be  
neglected**

## Transformer losses

*efficiency of transformer*

Expected to be small,  
but significant

→ Assumption: **take  
into account**





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

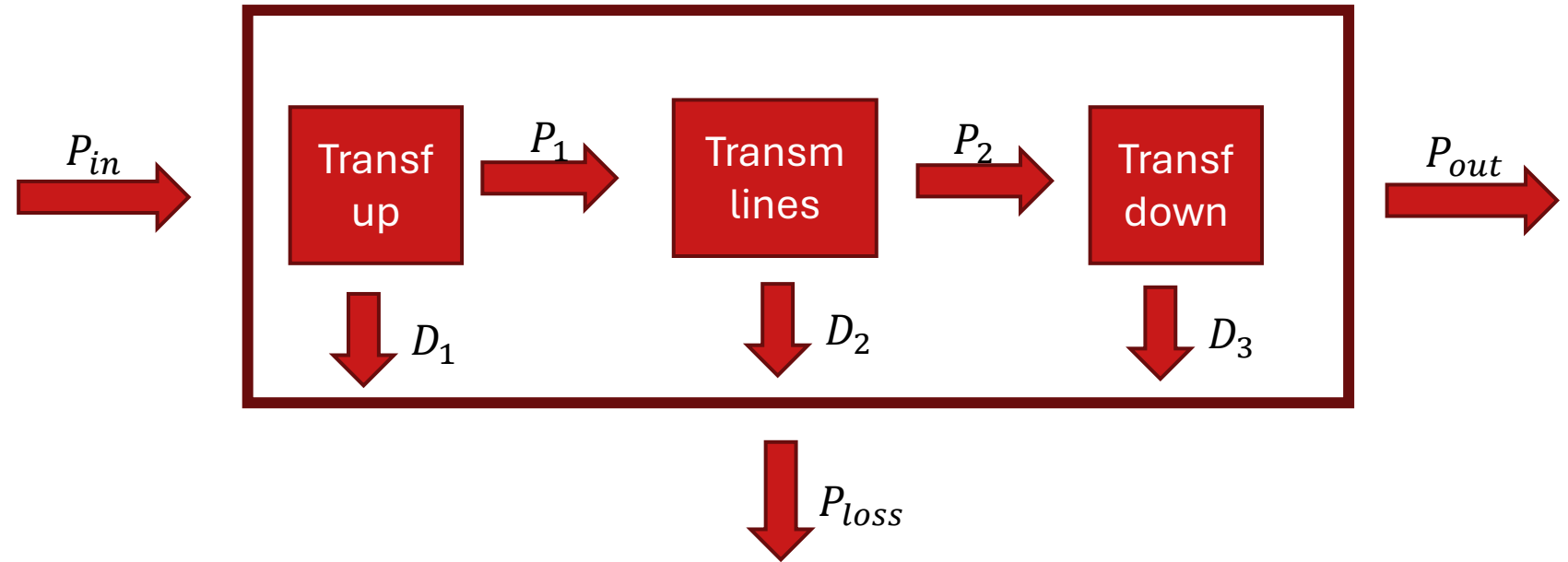


Validation &  
verification



Reflection

# The energy flow diagram



- The structure of your model:
  - How do the sub-components connect ?
  - Where are the losses?



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# The transmission lines

- Joule heating losses in Watts [W]:

$$P_{loss} = R \times I^2 = R \times \left(\frac{P_1}{V}\right)^2$$

- $R$ : resistance in Ohm [ $\Omega$ ]
- $I$ : electric current in Amperes [A]

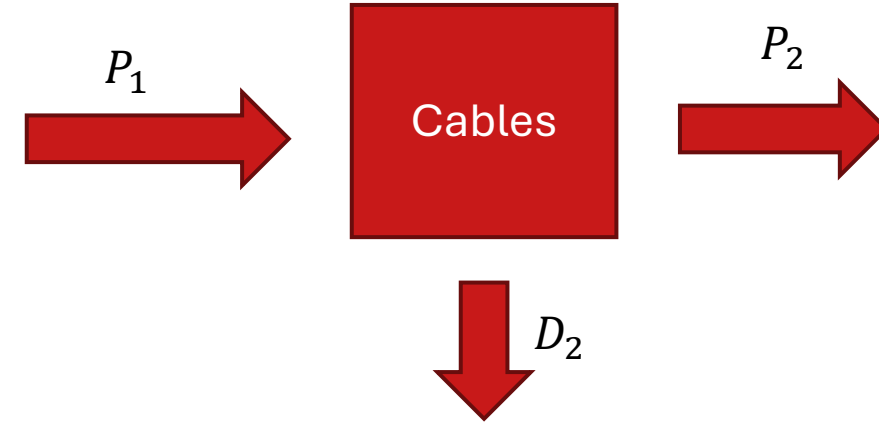
- Wire resistance:

$$R = L \times R'$$

- $L$ : Length of cable
- $R'$ : Resistance per unit length

- Power balance:

$$P_1 - P_2 = L \times R' \times \left(\frac{P_1}{V}\right)^2$$





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
relations



Implementation

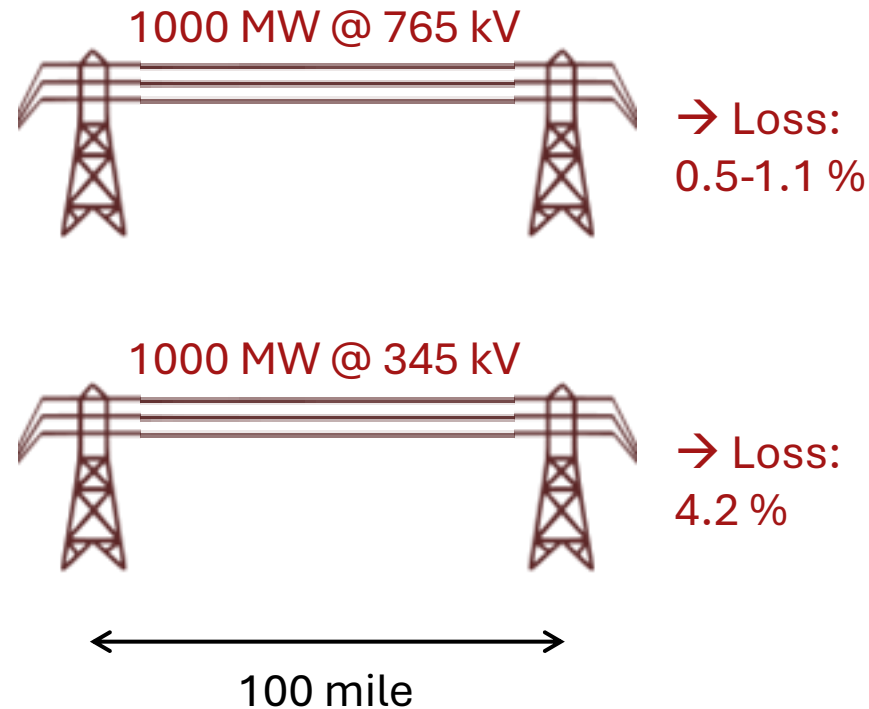


Validation &  
verification




Reflection


# Does it make sense?



[ACSR cable spec sheet:](#)

$$R' = 0.03 \text{ } [\Omega/\text{km}]$$

$$\frac{P_1 - P_2}{P_1} = L \times R' \times \frac{P_1}{V^2}$$
$$= 160 \times 0.03 \times \frac{1000}{765^2} = 0.82\%$$


$$\frac{P_1 - P_2}{P_1} = L \times R' \times \frac{P_1}{V^2}$$
$$= 160 \times 0.03 \times \frac{1000}{345^2} = 4.0\%$$




Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# The transformers

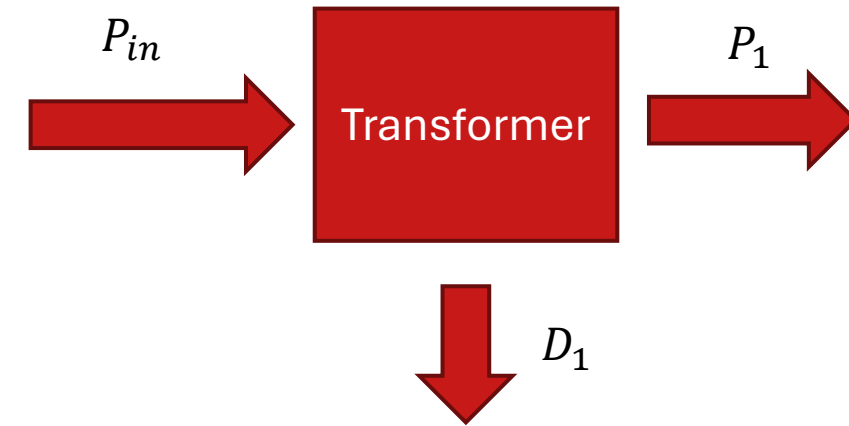
- Heat losses scale with the input power [W]:
  - $D_1 = a \times P_{in}$
  - $a$ : loss coefficient [—]
- Loss coefficient is related to efficiency:

$$\eta = \frac{P_1}{P_{in}} = \frac{P_{in} - D_1}{P_{in}} = 1 - a$$

- Power balance:

$$P_{in} - P_1 = (1 - \eta)P_{in}$$

$$P_2 - P_{out} = (1 - \eta)P_2$$





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

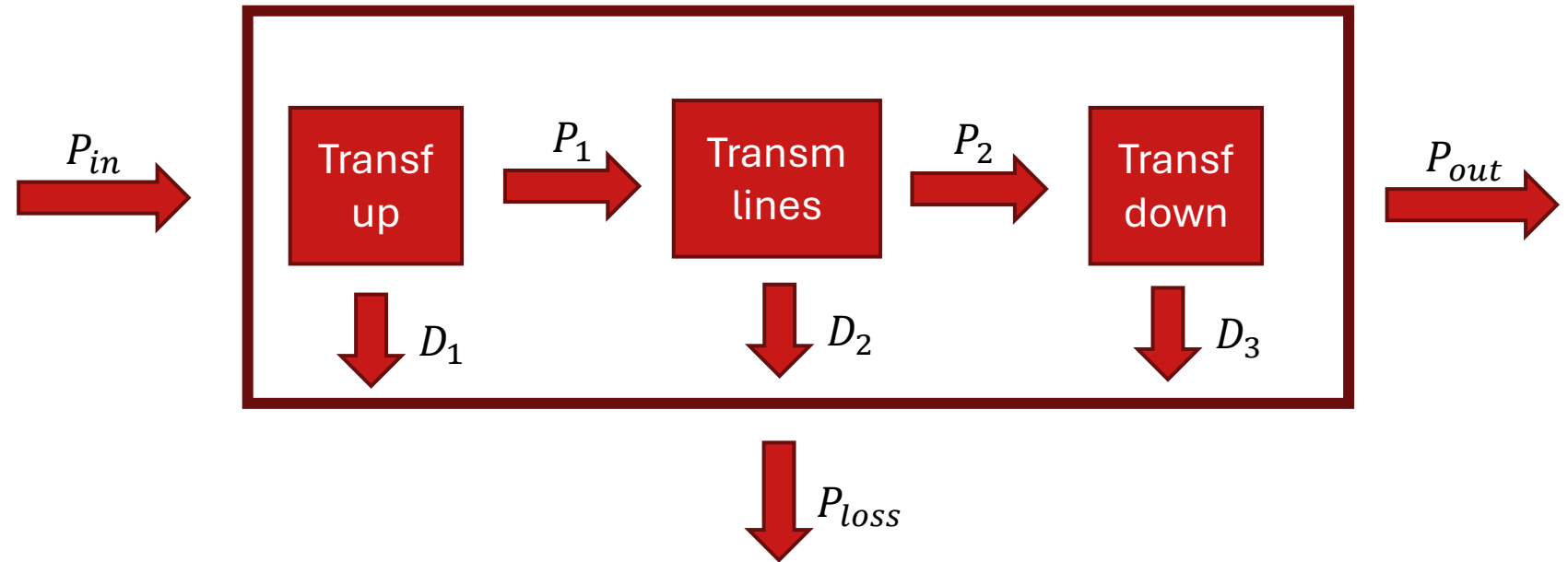


Validation &  
verification



Reflection

# Putting the model together



$$P_1 = \eta P_{in}$$

$$P_2 = P_1 - L \times R' \times \left(\frac{P_1}{V}\right)^2 = \eta P_{in} - L \times R' \times \left(\frac{\eta P_{in}}{V}\right)^2$$

$$P_{out} = \eta P_2 = \eta^2 P_{in} - \eta^3 L \times R' \times \left(\frac{P_{in}}{V}\right)^2$$

$$P_{out} = \eta^2 P_{in} - \eta^3 L \times R' \times \left(\frac{P_{in}}{V}\right)^2$$



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
relations



Implementation

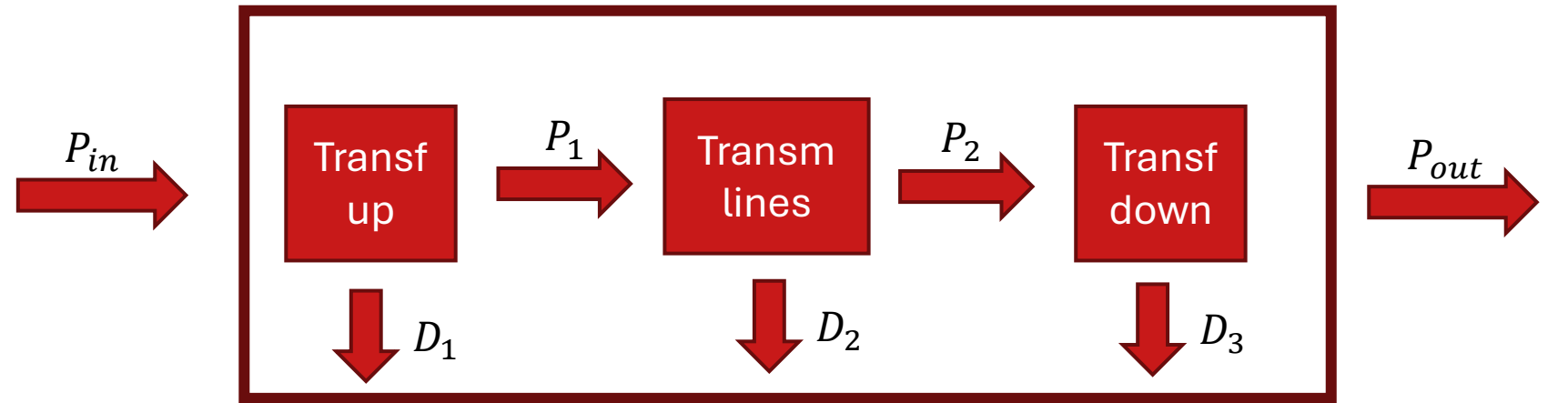


Validation &  
verification



Reflection

# Checking the result



$$D_1 = (1 - \eta)P_{in}$$

$$D_2 = L \times R' \times \left(\frac{\eta P_{in}}{V}\right)^2$$

$$D_3 = (1 - \eta) \left( \eta P_{in} - L \times R' \times \left(\frac{\eta P_{in}}{V}\right)^2 \right)$$

---


$$P_{loss} = (1 - \eta^2)P_{in} + \eta^3 L \times R' \times \left(\frac{P_{in}}{V}\right)^2$$

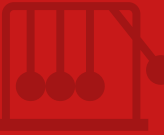


$$P_{loss} = P_{in} - P_{out}$$





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# Does the result make sense?

- Estimating parameters from spec sheets:
  - High-voltage transformer:

$$\eta = 99\%$$

- ACSR cable:

$$L = 160 \text{ [km]}$$
$$R' = 0.03 \text{ [\Omega/km]}$$

- Losses at 1000 MW and 765 kV:

$$P_{loss} = (1 - 0.99^2) \times 1000 + 0.99^3 \times 160 \times 0.03 \times \left(\frac{1000}{765}\right)^2 = 27.9 \text{ [MW]} = 2.79\%$$





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

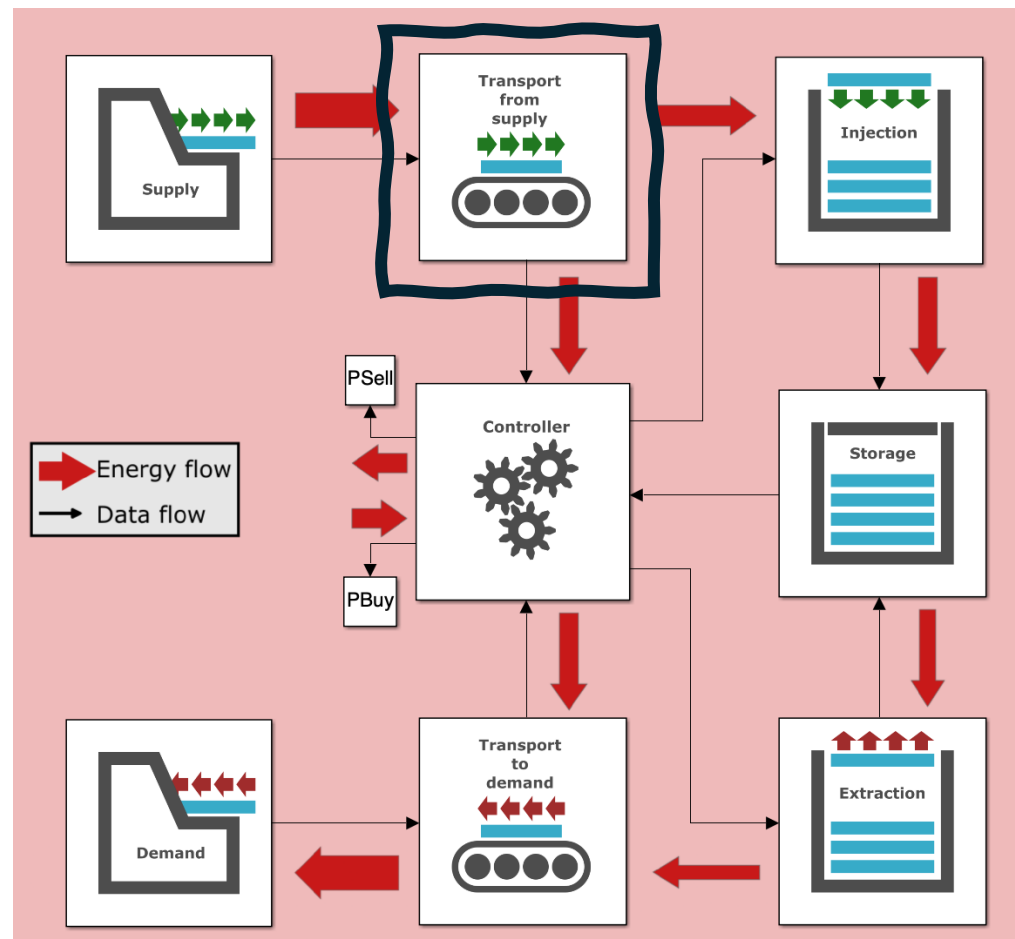


Validation &  
verification



Reflection

# The transport function block







Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



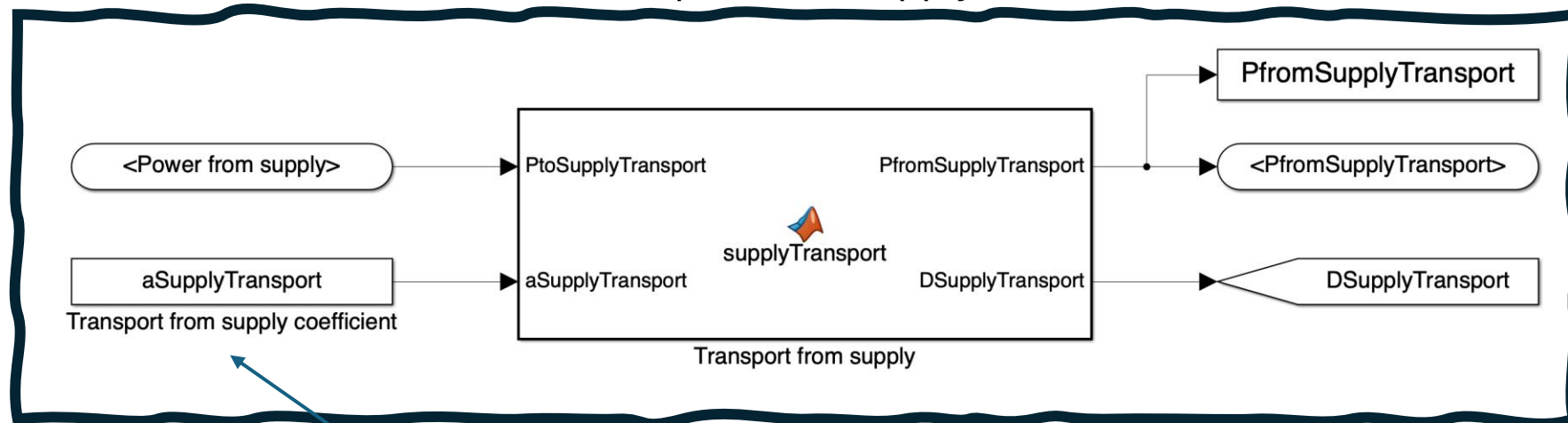
Validation &  
verification



Reflection

# The transport function block

Inside the "Transport from supply" block:



Effective loss  
parameter in baseline  
implementation

```
%% System parameters
preprocessing.m

% transport from supply
aSupplyTransport = 0.01; % Dissipation coefficient

% injection system
aInjection = 0.1; % Dissipation coefficient

% storage system
EStorageMax = 10.*unit("kWh"); % Maximum energy
EStorageMin = 0.0*unit("kWh"); % Minimum energy
EStorageInitial = 2.0*unit("kWh"); % Initial energy
bStorage = 1e-6/unit("s"); % Storage dissipation coefficient

% extraction system
aExtraction = 0.1; % Dissipation coefficient
```



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# The implementation of the model

Inside the "supplyTransport" function block:

```
1 function [PfromSupplyTransport, DSupplyTransport] = supplyTransport(PtoSupplyTransport, aSupplyTransport)
2     DSupplyTransport = aSupplyTransport * PtoSupplyTransport;
3     PfromSupplyTransport = PtoSupplyTransport - DSupplyTransport;
```



$$P_{out} = \eta^2 P_{in} - \eta^3 L \times R' \times \left(\frac{P_{in}}{V}\right)^2$$

Not of the form same form due to  
quadratic dependence on the input  
power.

Requires new implementation of the  
Matlab function.



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification

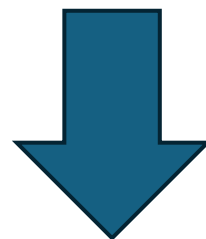


Reflection

# The implementation of the model

Inside the "supplyTransport" function block:

```
1 function [PfromSupplyTransport, DSupplyTransport] = supplyTransport(PtoSupplyTransport, aSupplyTransport)
2     DSupplyTransport = aSupplyTransport * PtoSupplyTransport;
3     PfromSupplyTransport = PtoSupplyTransport - DSupplyTransport;
```

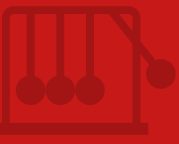


$$P_{out} = \eta^2 P_{in} - \eta^3 L \times R' \times \left(\frac{P_{in}}{V}\right)^2$$

```
1 function [PfromSupplyTransport, DSupplyTransport] = supplyTransport(PtoSupplyTransport, etaTransformer, L, Rprime, V)
2
3     % Model for transformers and transmission line
4     PfromSupplyTransport = (etaTransformer^2)*PtoSupplyTransport - (etaTransformer^3)*L*Rprime*(PtoSupplyTransport/V)^2;
5     DSupplyTransport = PtoSupplyTransport - PfromSupplyTransport;
```



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



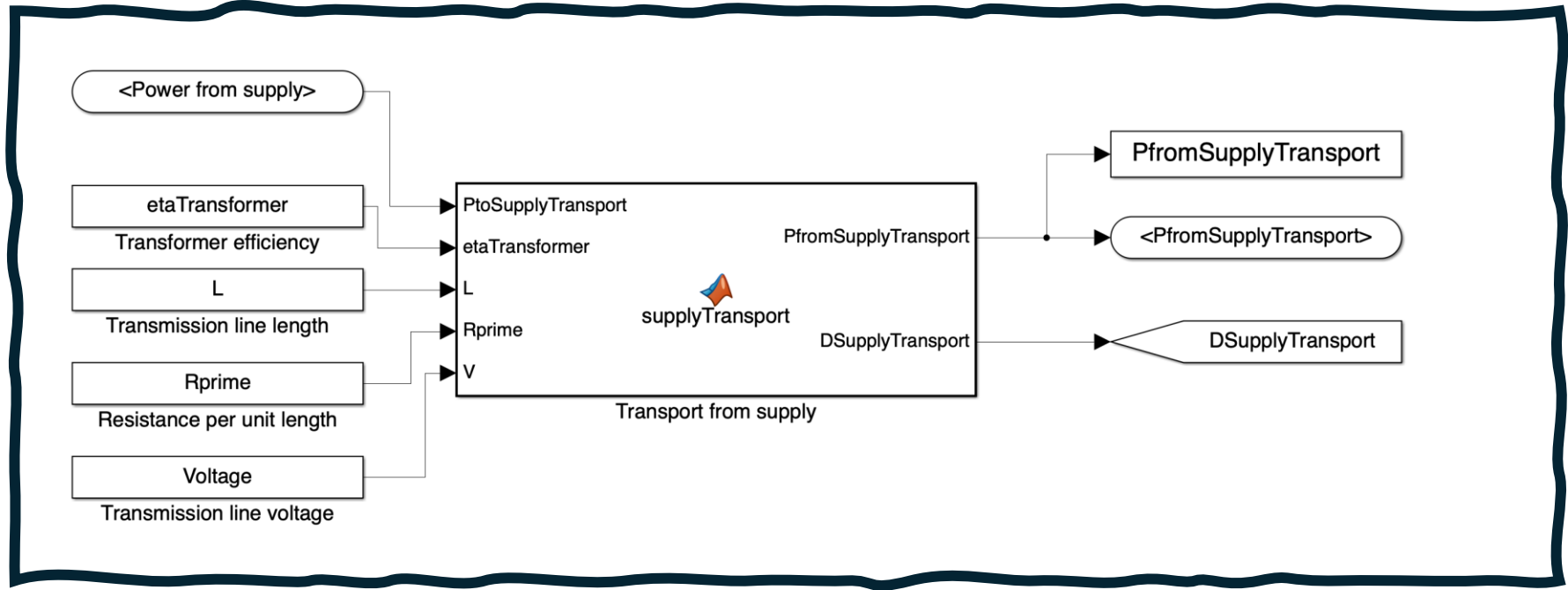
Validation &  
verification



Reflection

# The transport function block

Inside the "Transport from supply" block:





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# The physical input parameters

```
%% System parameters
```

preprocessing.m

```
% transmission line transport from supply
```

```
etaTransformer = 0.99; % Transformer efficiency [-]
```

```
L = 160e3; % Transmission line length [m]
```

```
Rprime = 3e-5; % Resistance per unit length [Ohm/m]
```

```
V = 765e3; % Transmission line voltage [V]
```

```
% injection system
```

```
aInjection = 0.1; % Dissipation coefficient
```

```
% storage system
```

```
EStorageMax = 10.*unit("kWh"); % Maximum energy
```

```
EStorageMin = 0.0*unit("kWh"); % Minimum energy
```

```
EStorageInitial = 2.0*unit("kWh"); % Initial energy
```

```
bStorage = 1e-6/unit("s"); % Storage dissipation coefficient
```

```
% extraction system
```

```
aExtraction = 0.1; % Dissipation coefficient
```

```
% transport to demand
```

```
aDemandTransport = 0.01; % Dissipation coefficient
```



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

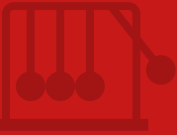
# Debugging

```
Invalid setting in 'EST/Transport from supply/Transmission line voltage' for parameter 'Value'.  
Caused by:  
• Error evaluating parameter 'Value' in 'EST/Transport from supply/Transmission line voltage'  
  • Unrecognized function or variable 'Voltage'.  
    • Variable 'Voltage' does not exist.
```

- Study the error message to understand what is wrong.
- If you cannot find the bug, revert to the version that worked, and then make step-by-step changes.
- Create test cases to track the bug.



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# The physical input parameters

```
%% System parameters
```

preprocessing.m

```
% transmission line transport from supply
```

```
etaTransformer = 0.99; % Transformer efficiency [-]
```

```
L = 160e3; % Transmission line length [m]
```

```
Rprime = 3e-5; % Resistance per unit length [Ohm/m]
```

```
Voltage = 765e3; % Transmission line voltage [V]
```

```
% injection system
```

```
aInjection = 0.1; % Dissipation coefficient
```

```
% storage system
```

```
EStorageMax = 10.*unit("kWh"); % Maximum energy
```

```
EStorageMin = 0.0*unit("kWh"); % Minimum energy
```

```
EStorageInitial = 2.0*unit("kWh"); % Initial energy
```

```
bStorage = 1e-6/unit("s"); % Storage dissipation coefficient
```

```
% extraction system
```

```
aExtraction = 0.1; % Dissipation coefficient
```

```
% transport to demand
```

```
aDemandTransport = 0.01; % Dissipation coefficient
```



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

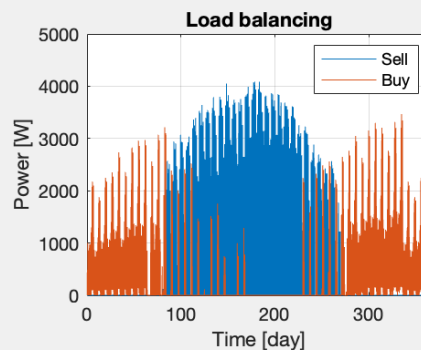
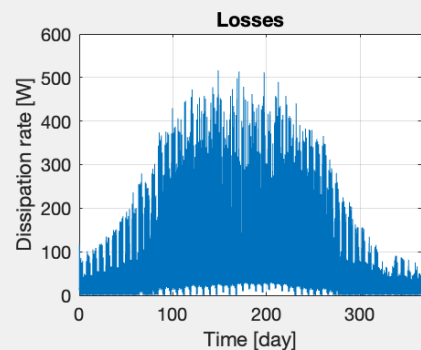
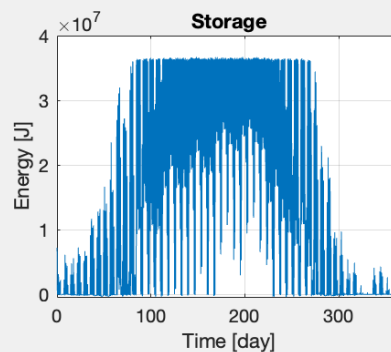
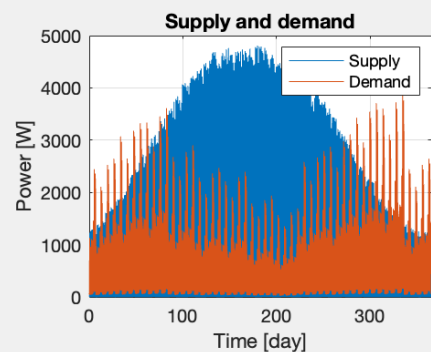


Validation &  
verification

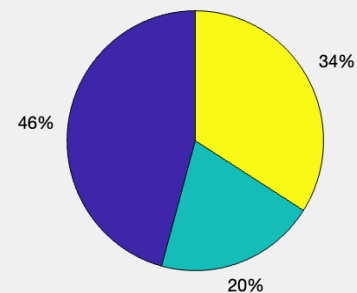


Reflection

# Do the results make sense?

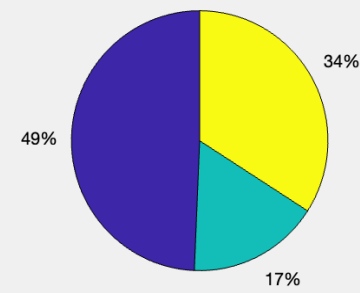


Received energy  $3.09 \times 10^{10}$  [J]



Direct to demand  
To storage  
Sold

Delivered energy  $2.87 \times 10^{10}$  [J]



Direct from supply  
From storage  
Bought

- Standard output does not properly visualize the change.
- Add additional output to assess model change.





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation

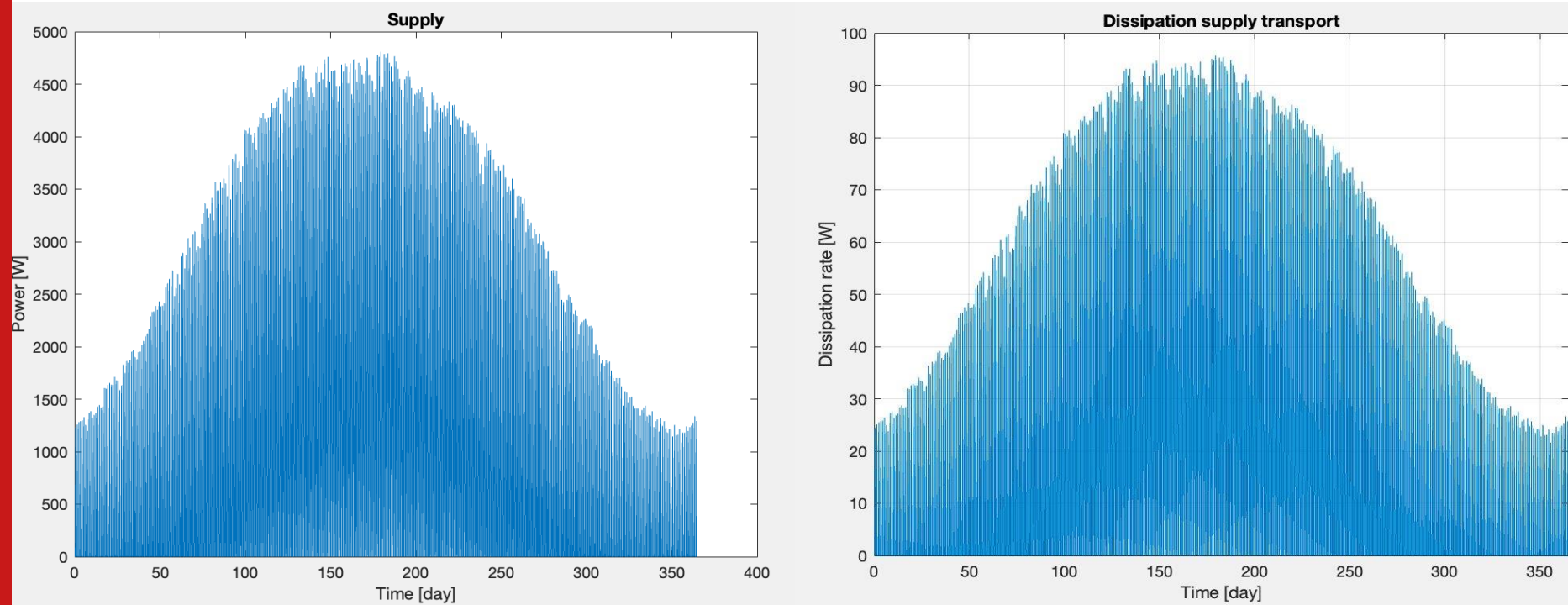


Validation &  
verification



Reflection

# Do the results make sense?



- Losses around 2%
- Dominated by transformers because of low power
- Additional testing at high powers needed





Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



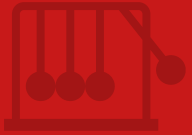
Reflection

# What can the model do?

- Provide estimate of power losses in DC transmission lines
- Model allows variation of:
  - Length of transmission line
  - Resistance of lines (material property)
  - Voltage in transmission line
  - Transformer efficiencies
- Results correspond well with literature observations



Modelling  
objective



Physical  
phenomena



Model selection



Mathematical  
elaboration



Implementation



Validation &  
verification



Reflection

# What can be improved?

- Further validation by considering higher powers
- Extend to AC transmission lines by including inductance in the model
- Add an improved model for the transformers

# Energy Storage and Transport

Modeling example