4,6) a) $T(s) = \frac{2}{s+2}$ Poles: $s = -2$

No zeros

Exp decay

b) $T(s) = \frac{5}{(s+3)(s+6)}$ Poles: $s = -3, s = -6$

No zeros

Overdamped

c) $T(s) = \frac{10(s+1)}{(s+10)(s+20)}$ Poles: $s = -10, s = -20$)

Zeros: $z = -7$

Overdamped

d) $T(s) = \frac{20}{s^2 + 6s + 144}$ Poles: $s = -5 \pm j\ 11.62$

No zeros

Undamped

e) $T(s) = \frac{s+2}{s^2+9}$ Poles: $s = \pm j3$

Zeros: $s = -2$

Undamped

f) $T(s) = \frac{(s+5)}{(s+10)^2}$ Poles $= s = -10, s = 10$

Zeros: $z = -5$

Critically damped

w_n $\zeta = \dfrac{coef\cos S}{2w_n}$

6.11)

d) $T(s) = \dfrac{20}{s^2 + 6s + 144}$

$s^2 + 6s + 144 \rightarrow s^2 + 2\zeta w_n s + w_n^2$

$w_n = 12$ $\qquad \zeta = \dfrac{6}{2(12)} = 0.25$

e) $\dfrac{s+2}{s+9}$

$s^2 + 9 \rightarrow 2\zeta w_n s + w_n^2$

$w_n = 3$ $\qquad \zeta = 0$

6.12) $K = 1$

$\zeta = 0.25$ $\qquad T(s) = \dfrac{K w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$

$w_n = 30$

$R(s) = \dfrac{1}{s}$ $\qquad = \dfrac{900}{s^2 + 15s + 900}$

$Y(s) = T(s)R(s) = \dfrac{900}{s(s^2 + 15s + 900)}$

$S = -7.5 \pm j\,29.04$

$w_d = w_n\sqrt{1 - \zeta^2} = 29.04$

$\phi = \tan^{-1}\left(\dfrac{w_d}{\zeta w_n}\right) = \tan^{-1}\left(\dfrac{29.04}{7.5}\right)$

$y(t) = 1 - e^{-7.5t}\left(\cos(29.04t) + \dfrac{7.5}{29.04}\sin(29.04t)\right)$

6.13) a) $T(s) = \dfrac{16}{s^2 + 3s + 16}$

$w_n = \sqrt{16} = 4$ $2\zeta w_n = 3$ $\zeta = \dfrac{3}{2(4)} = 0.375$

$T_s = \dfrac{4}{(0.375)(4)} = 2.67 \text{ sec}$

$T_p = \dfrac{\pi}{4\sqrt{1-0.375^2}} = 0.85 \text{ sec}$

$T_r \approx \dfrac{1.8}{4} = 0.45 \text{ sec}$

$\%OS = 85.5\%$

b) $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

$2\zeta w_n = 0.02$ $w_n = 0.2$ $\zeta = \dfrac{0.02}{2(0.2)} = 0.05$

$T_s = 400 \text{ sec}$

$T_p = 15.77 \text{ sec}$

$T_r = 9.0 \text{ sec}$

$\%OS = 85.5\%$

c) $T(s) = \dfrac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$

$2\zeta w_n = 1.6 \times 10^3$ $w_n = 3241.1$ $\zeta = \dfrac{1.6 \times 10^3}{2(3241.1)} = 0.247$

$T_s = 0.0049 \text{ sec}$

$T_p = 0.001 \text{ sec}$

$T_r = 0.00056 \text{ sec}$

$\%OS = 43.7\%$

$T(s) = \dfrac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$

$w_n = \sqrt{\text{constant term}}$

$2\zeta w_n = \text{coeff of } s$

$T_s \approx \dfrac{4}{\zeta w_n}$

$T_p = \dfrac{\pi}{w_d} = \dfrac{\pi}{w_n \sqrt{1-\zeta^2}}$

$T_r \approx \dfrac{1.8}{w_n}$

$\%OS = e^{\dfrac{-\zeta \pi}{\sqrt{1-\zeta^2}}} \times 100$