Direct loading

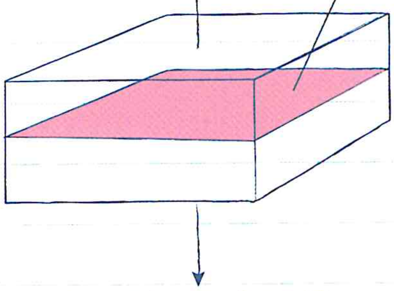
Direct loading includes tensile forces, which pull and stretch a component, and compressive forces, which push and squeeze a component. Direct loading gives rise to direct stress and direct strain.

Direct stress (σ)

Direct stress is a measure of the direct load distribution within a material.

$$\text{Direct stress } (\sigma) = \frac{\text{Normal force } (F)}{\text{Area } (A_\sigma)}$$

Stress has units N/m^2 or Pa (N/m^2 = 1 Pa).

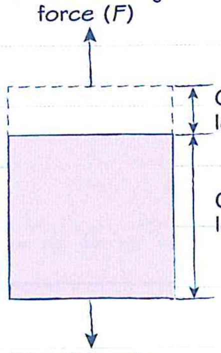


Direct strain (ε)

Direct strain is a measure of the deformation caused by an applied direct stress.

$$\text{Direct strain } (\varepsilon) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

Strain is a dimensionless quantity and has no units.



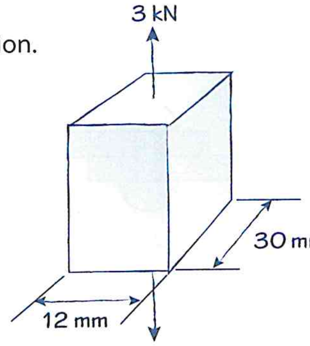
Also known as Young's modulus, the Modulus of elasticity (E) expresses the linear relationship between direct stress and direct strain.

$$\text{Modulus of elasticity } (E) = \frac{\text{Direct stress } (\sigma)}{\text{Direct strain } (\varepsilon)}$$

Modulus of elasticity has units N/m^2 or Pa.

Now try this

The sketch shows part of a structural beam that is loaded in tension. Calculate the direct stress in the beam.



Had a look ☐ Nearly there ☐ Nailed it! ☐

Shear loading

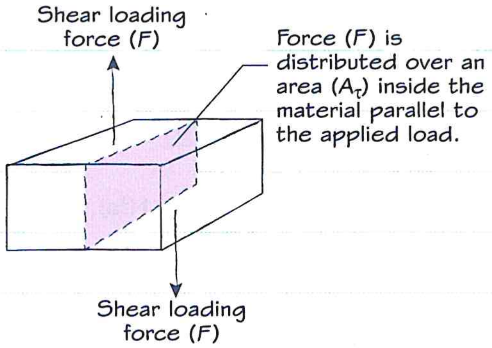
Shear loading is caused by forces that cut across a component and tend to shear or cut it apart. Shear loading gives rise to **shear stress** and **shear strain**.

Shear stress (τ)

Shear stress is a measure of the shear load distribution within a material.

$$\text{Shear stress } (\tau) = \frac{\text{Shear force } (F)}{\text{Shear area } (A_\tau)}$$

Stress has units N/m^2 or Pa (N/m^2 = 1 Pa).

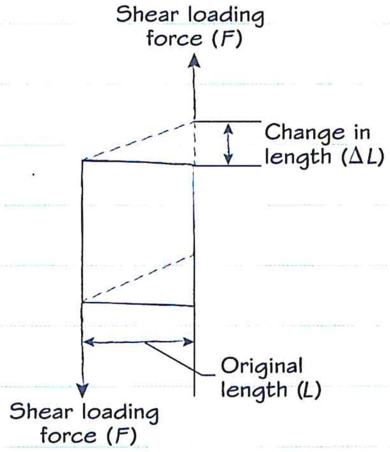


Shear strain (γ)

Shear strain is a measure of the deformation caused by an applied shear stress.

$$\text{Shear strain } (\gamma) = \frac{\text{Change in length } (\Delta L)}{\text{Original length } (L)}$$

Strain is a dimensionless quantity and has no units.



Rigidity

The modulus of rigidity (G) expresses the linear relationship between shear stress and shear strain.

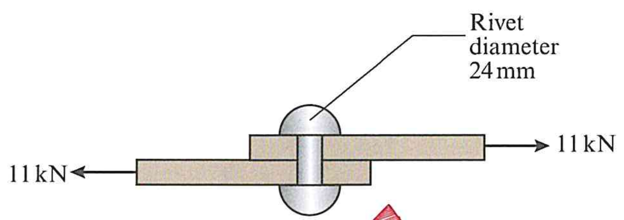
$$\text{Modulus of rigidity } (G) = \frac{\text{Shear stress } (\tau)}{\text{Shear strain } (\gamma)}$$

Modulus of rigidity has units N/m^2 or Pa.

Now try this

Rivets are often used to join two parallel metal plates. A cross-section of such an arrangement is shown in the diagram.

Calculate the shear stress in the rivet.



Don't forget to change the values given in the question to standard units before performing any calculations.

Find the cross-sectional area of the rivet.

Velocity, displacement and acceleration

Engineers use the SUVAT equations to analyse the motion of objects travelling in straight lines with constant acceleration. Although they will be on the formulae sheet in your exam, you need to be confident in using and manipulating them.

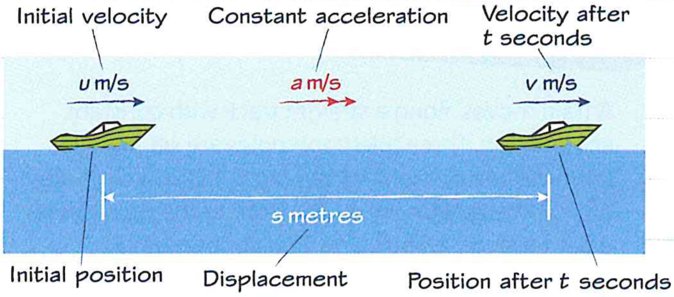
Constant acceleration formulae

Here are the first two formulae you need to use for motion in a straight line with constant acceleration:

1. $v = u + at$
2. $s = \dfrac{(u+v)}{2}t$

Displacement (s)
Initial velocity (u)
Final velocity (v)
Acceleration (a)
Time (t)

Look at the diagram on the right to see what each letter represents.



Using SUVAT

The constant acceleration formulae are sometimes called the SUVAT formulae. In the exam you should write down all five letters.

- Write in any values you KNOW.
- Put a QUESTION MARK next to the value you want to find.
- CROSS OUT any values you don't need for that question.

This will help you choose which formula to use.

Read the question carefully. The distance AB is 1.2 km, but the aircraft does not take off at B.

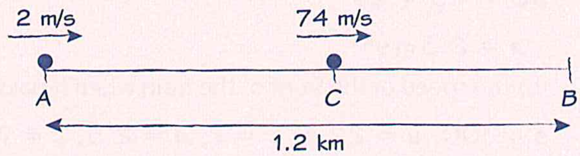
If you have to solve a constant acceleration question involving three points like this one, it's a good idea to draw a quick sketch to help you see what is going on. In part (b), s is the distance AC, in metres. You need to subtract it from 1200 m to find the distance CB.

Worked example

When taking off, an aircraft moves on a straight runway, AB, of length 1.2 km. The aircraft moves from A with initial speed 2 m/s. It moves with constant acceleration and 20 s later it leaves the runway at C with speed 74 m/s. Find:

(a) the acceleration of the aircraft

$s = ?,\ u = 2,\ v = 74,\ a = ?,\ t = 20$



$v = u + at$

$74 = 2 + a \times 20$

$20a = 72$

$a = 3.6\ \text{m/s}^2$

(b) the distance CB.

$s = ?,\ u = 2,\ v = 74,\ a = 3.6,\ t = 20$

$s = \dfrac{(u+v)}{2}t$

$AC = \dfrac{(2+74)}{2} \times 20$

$= 760\ \text{m}$

$CB = 1200\ \text{m} - 760\ \text{m} = 440\ \text{m}$

Now try this

A car moves along a straight stretch of road AB. The car moves with initial speed 2 m/s at point A. It accelerates constantly for 12 seconds, reaching a speed of 23 m/s at point B. Find:

(a) the acceleration of the car

(b) the distance AB.

You can answer both parts of this question using the formulae given on this page. But you can use any of the other SUVAT formulae if you are confident with them.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Applying the SUVAT equations

If you know three of s, u, v, a and t, you can calculate the missing values.

1 $v^2 = u^2 + 2as$

2 $s = ut + \frac{1}{2}at^2$

3 $s = vt - \frac{1}{2}at^2$

For an understanding of what each letter represents, look at the blue box.

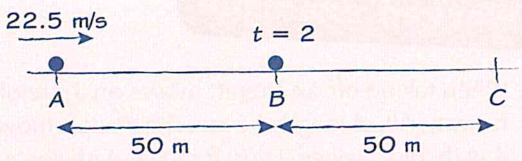
Worked example

A train moves along a straight track with constant acceleration. Three telegraph poles are set at equal intervals beside the track at points A, B and C, where $AB = 50$ m and $BC = 50$ m. The front of the train passes A with speed 22.5 m/s, and 2 s later it passes B.

Find:

(a) the acceleration of the train

$s = 50$, $u = 22.5$, $v = ?$, $a = ?$, $t = 2$



$$s = ut + \frac{1}{2}at^2$$

$$50 = 22.5 \times 2 + \tfrac{1}{2} \times a \times 2^2$$

$$50 = 45 + 2a$$

$$a = 2.5\text{ m/s}^2$$

(b) the speed of the front of the train when it passes C.

$s = 100$, $u = 22.5$, $v = ?$, $a = 2.5$, $t = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 22.5^2 + 2 \times 2.5 \times 100$$

$$= 1006.25$$

$$v = 31.721\ldots = 31.72\text{ m/s (to 2 d.p.)}$$

(b) This formula involves v^2, so there are two possible values of v. You have been asked to find the speed of the front of the train. Speed is the magnitude of the velocity, so you need to give a positive answer.

Units

You need to make sure that your measurements are in the correct units.

- t (time) is measured in seconds.
- s (displacement) is measured in metres.
- u, v (velocity) is measured in m/s.

You will sometimes see ms^{-1} written as m/s. They both mean the same thing: metres per second.

- a (acceleration) is measured in m/s^2.

You will sometimes see m/s^2 written as ms^{-2}. They both mean the same thing: metres per second squared, or metres per second per second.

Now try this

A boat travels in a straight line with constant deceleration, between two buoys, A and B, 300 m apart. The boat passes buoy A with initial speed 16 m/s, and passes buoy B 30 seconds later. Find:

(a) the deceleration of the boat

(b) the speed of the boat as it passes B.

Deceleration is represented in the SUVAT equations by 'a' but remember to make it a **negative** value by putting a '−' in front of it.

Force, friction and torque

You need to understand what is meant by force, friction and torque and be able to calculate their values in different contexts.

Force

A force is a push or a pull acting on an object.
It is measured in newtons (N).

Friction

Friction can be considered as two distinct cases:

- Static frictional force: opposes a static object from starting to move when a force is applied
- Kinetic frictional force: opposes a moving object's motion.

Calculating frictional forces

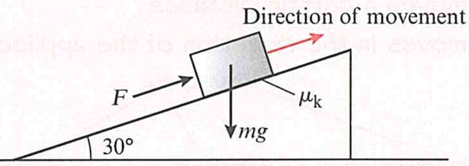
To calculate frictional forces, you need a value for the resistance to movement provided by different surfaces:

Coefficient of static friction: $F_s = \mu_s N$, where F_s is the limiting frictional force (in newtons) and N is the normal reaction between the surfaces (mass × g, in newtons)

Coefficient of kinetic friction: $F_k = \mu_k N$, where F_k is the kinetic frictional force (in newtons) and N is the normal reaction between the surfaces (mass × g, in newtons).

Kinetic friction is often referred to as dynamic or sliding friction.

Worked example



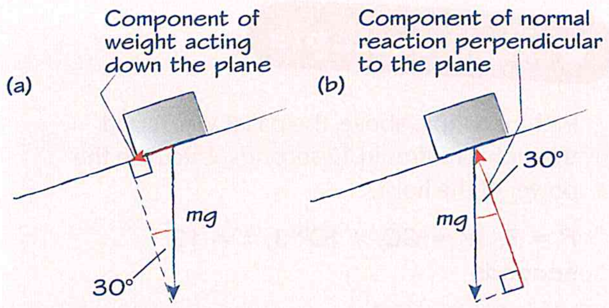
A 30° inclined plane is used to reduce the force required to raise a load of 12 kg, where the coefficient of kinetic friction is 0.4.

(a) Calculate the component of the weight acting down the plane.

(b) Calculate the component of normal reaction acting perpendicular to the plane.

(c) Calculate the kinetic frictional resistance.

(d) Calculate the total force acting down the plane.



(a) $mg \sin 30 = 12 \times 9.81 \times 0.5 = 58.8$ N

(b) $mg \cos 30 = 12 \times 9.81 \times 0.87 = 101.84$ N

(c) $F_k = \mu_k N = 0.4 \times 101.84 = 40.74$ N

(d) $F = 58.8 + 40.74 = 99.54$ N

Worked example

Determine the torque acting on a bolt when a force of 150 N is applied perpendicularly to a spanner with length 30 cm.

$\tau = ?$, $F = 150$ N, $r = 0.30$ m

$\tau = Fr$

$= 150 \times 0.30 = 45$ N m

Remember that torque is the force applied multiplied by the distance from the centre of the object.

Torque

The turning moment of a couple is called torque, represented by the Greek letter τ (pronounced tau).

It can be considered as a force (F) applied tangentially to a shaft or wheel of radius (r), where F is in newtons and r in metres.

Torque is the product of the force and the radius, and is measured in newton metres (N m): $\tau = Fr$.

Now try this

1 A 20° inclined plane is used to reduce the force required to raise a load of 50 kg, where the coefficient of kinetic friction is 0.35. Calculate the total force acting down the plane.

2 A 50 mm diameter bar is being turned on a lathe. The force on the cutting tool, which is tangential to the surface of the bar, is 0.7 kN. Calculate the applied torque.

Work and power

Work is force × distance and **power** is energy transferred in time taken.

Mechanical work

Work is done when force is exerted on an object and

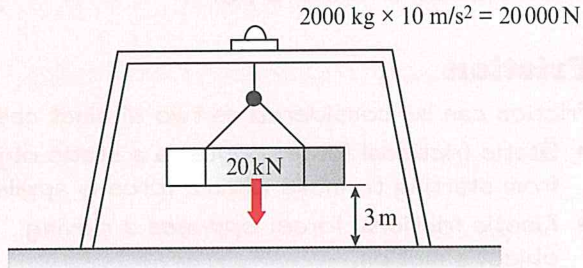
- it moves a certain distance
- it moves in the direction of the applied force,

$W = Fs$

Work in joules (J), where 1 J is 1 N m. Force, in newtons (N) Distance, in metres (m)

- Take care with units: if the force is in kN then the work will be in kJ, not J.
- $F = ma$, or in this special case weight $= mg$.
- The direction of movement is in the direction of the applied force; vertically upwards.

Worked example



A gantry hoist is used to lift a pallet with a mass of 2000 kg through a distance of 3 m. Taking $g = 10\,\text{m/s}^2$, calculate the work done.

$W = ?\ \text{J},\ F = 2000 \times 10\,\text{N},\ s = 3\,\text{m}$

$W = Fs = 2000 \times 10 \times 3$

$= 60\,000\,\text{J}\ (60\,\text{kJ})$

Worked example

In the example above, the pallet was raised through 3 metres in 12 seconds. Calculate the power of the hoist.

$P = ?,\ W = 60 \times 10^3\ \text{J},\ t = 12$ seconds

$$\frac{W}{t} = \frac{60 \times 10^3}{12} = 5 \times 10^3\ \text{W} = 5\,\text{kW}$$

A selector ram pushes a component horizontally at an average velocity of 5.2 m/s with a force of 2450 N. Calculate the power output of the ram.

$W = ?,\ F = 2450\,\text{N},\ s = 5.2\,\text{m},\ t = 1\,\text{s}$

$W = Fs = 2450 \times 5.2$

$= 12\,740\ \text{J} = 12.74\,\text{kJ}$

$$P = \frac{W}{t} = \frac{12.74\,\text{kJ}}{1\ \text{second}}$$

$= 12.74\,\text{kW}$

Power

Energy transferred may be in different forms; for example, heat transfer or lifting a weight.

$$\text{Power} = \frac{\text{Energy transferred}}{\text{Time taken}} = \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t}$$

The unit of power is the watt (W), 1 watt = 1 joule of energy transferred in 1 second.

Also, $P = F \times v$

v is the average velocity (i.e. $\frac{u+v}{2}$), in m/s.

F is the force, in N.

Average and instantaneous power

✓ If the energy transfer is *averaged*; for example, average speed of an object pushed by a force or temperature change of an object averaged over a period of time, then the power will be an 'averaged' value.

✓ The instantaneous power is the average power as the time factor approaches zero.

Now try this

1. A spring is extended by 5.0 cm using 100 J of work. Calculate the average force applied.
2. A car travels 105 metres along a straight road in 10 seconds at a constant velocity. Frictional forces are constant at 500 N. Calculate the power output of the engine, assuming 100% transmission system efficiency.

First find the amount of work done, then calculate the power using the time taken.

Energy

Energy can be thought of as being involved in doing work, making objects move and heating something up. In doing work the energy is not used up, it transfers from one energy store, such as gravitational potential energy, to others, such as kinetic energy, sound and heat.

Gravitational potential energy (GPE)

GPE depends *only* on the **mass** of the object, the **acceleration due to gravity** and the **height** of the object.

It doesn't matter whether the object is moving or resting on another object.

$$\boxed{E_p = mgh}$$

GPE, in joules

Height, in metres

Mass, in kilograms

Acceleration due to gravity (9.81 m/s²)

Remember that $m \times g$ is the same as the weight of an object, which is a force, measured in newtons (N).

In calculations, convert to SI units and make sure you can manipulate very small as well as very large values.

Worked example

In a spray dry production facility, synthetic detergent granules are manufactured by ejecting liquid droplets at high speed from a rotating atomiser disc in the presence of hot air. The individual droplets have a mass of 1.30 milligrams and a GPE at point of ejection from the rotating disc of 51 µJ. Calculate the height of the atomiser disc above the base of the unit.

$E_p = 5.10 \times 10^{-5}$ J, $m = 1.30 \times 10^{-6}$ kg, $g = 9.81$ m/s², $h = ?$

$E_p = mgh$

$5.10 \times 10^{-5} = 1.30 \times 10^{-6} \times 9.81 \times h$

$h = 4.00$ m (to 2 d.p.)

Kinetic energy (KE)

Energy associated with motion such as falling, rotating or moving in a straight line.

$$\boxed{E_k = \tfrac{1}{2} mv^2}$$

v is the velocity, in m/s.

m is the mass, in kg

GPE decreases due to the reducing height.
KE increases due to acceleration under gravity.

Links Read more about the conservation of energy on page 26.

You may be given an energy transfer question in which it is assumed that all the GPE is converted to KE (see question 2 below). That is not the case here because the gondolas are almost perfectly balanced and are driven around using a small motor.

Worked example

Each 600 tonne gondola on the Falkirk Wheel contains 500 000 litres of water and takes 5.5 minutes to complete the 53 metre journey from the upper canal to the lower one. Assuming constant velocity throughout the travel, calculate the kinetic energy of one of the gondolas. Assume the mass of water in one gondola to be 500 000 kg.

$E_k = ?$ J, $v = ?$, $s = 53$ m, $t = (5.5 \times 60)$ s, $m = (600 + 500)$ tonnes $= 1.1 \times 10^6$ kg

$E_k = \frac{1}{2} mv^2$

$v = \frac{d}{t} = \frac{53}{330} = 0.16$ m/s (2 d.p.)

$E_k = \frac{1}{2}(1.1 \times 10^6 \times 0.16^2)$

$= 14080$ J or 14.08 kJ

Make sure that the units are correct; e.g. convert tonnes to kg.

Now try this

1 Using the data above, calculate the GPE of one of the gondolas on the Falkirk Wheel at its highest position (35 metres) above ground level.

2 A 200 kg pile driver, used to drive in reinforced concrete foundation piles, falls 7.5 metres under gravity. Ignoring the effects of air resistance, calculate its velocity at the point of impact with the concrete pile.

Newton's laws of motion, momentum and energy

Real-world engineering deals with objects in motion. You need to understand how to analyse these dynamic systems and the forces involved.

Newton's laws of motion

1. **Inertia:** objects continue in their state of rest, or of uniform velocity, as long as no net force acts. For rotating objects the moment of inertia (I) is related to the mass (m) and the radius of gyration (r): $I = mr^2$.
2. **Force:** the acceleration of an object is proportional to the size of an applied force and takes place in the direction of that force.
3. To every action there is an equal and opposite reaction.

If at rest, a force is needed to start movement. If moving, a force is needed to stop it.

Unbalanced forces lead to acceleration of an object in the direction of the resultant force. At constant mass: $F = ma$.

Provides a sense of symmetry when considering the action of forces.

Momentum

Remember that momentum (p) is a vector quantity and is the product of mass and velocity, or $p = mv$ with units kg m/s.

Conservation of momentum

- Derives from Newton's third law
- Applies to 'isolated systems'
- Usually a collision or impact involved
- The sum of the momentum before the collision equals the sum of momentum after it.

Note that change in momentum is called **impulse** which is also equal to force × time with units of newton seconds (N s).

Worked example

A 400 g piston in a diesel engine has an instantaneous momentum at mid-stroke of 2 kg m/s. Calculate its velocity.

$p = 2\,\text{kg m/s}$, $m = 0.4\,\text{kg}$, $v = ?\,\text{m/s}$

$$p = mv,\ \frac{p}{m} = \frac{2}{0.4} = 5\,\text{m/s}$$

A hammer of mass 0.75 kg with a velocity of 15 m/s hits a nail of mass 20 g. Assuming no rebound, what is the velocity of the hammer and nail immediately after the impact?

$m_h = 0.75\,\text{kg}$, $v_h = 15\,\text{m/s}$, $m_n = 20\,\text{g}$, $v_{h+n} = ?$

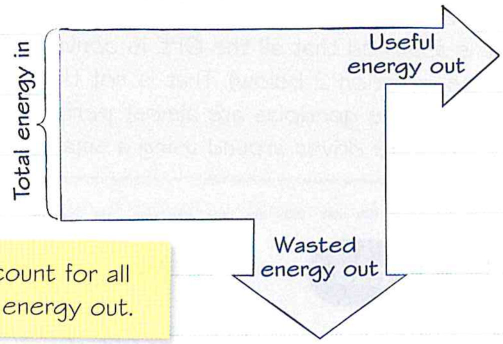
$$m_h \times v_h = m_{h+n} \times v_{h+n}$$

$$0.75 \times 15 = (0.75 + 0.02) \times v_{h+n}$$

$$v_{h+n} = \frac{11.25}{0.77} = 14.61\,\text{m/s (to 2 d.p.)}$$

Conservation of energy

- Energy is not lost when work is done.
- Energy can be considered to change from one energy 'store' to another, or from one form of energy to a different form. 'Energy can be transferred usefully, stored or dissipated. It cannot be created or destroyed.'



Use a Sankey diagram to help you account for all energy changes. Energy in *must* equal energy out.

Now try this

1. A mass of 130 000 kg moves along a straight, level track with a velocity of 15 m/s and collides with a second mass of 50 000 kg travelling in the same direction with a velocity of 10 m/s. After the collision the masses remain locked together. Calculate the velocity of the combined mass.
2. A car of mass 1250 kg stands on an incline of 7°. If the hand brake is released, calculate the velocity of the car after travelling 50 m down the incline. Assume the resistances to motion total 50 N.

Angular parameters

Engineering in the 'real world' often involves rotating objects, such as shafts, or objects moving along a curved path, for example, a F1 car cornering. Basic understanding of this motion and the power/energy calculations involved requires use of the formulae covered below.

Angular and linear velocity

Angular velocity (ω) is the same for all points on a rotating object; but **linear** (tangential) velocity (v) will depend on the distance from the centre (r): $v = r\omega$, where v is in m/s, r in metres and ω in rad/s.

Links You can refresh your memory on radian measurement on page 9.

Worked example

A conveyer belt on a production line travels at 5.0 m/s and is driven by a pulley wheel of diameter 300 mm. Calculate the angular velocity.

$v = 5$ m/s, $r = 0.3$ m, $\omega = ?$ rad/s

$v = r\omega$

$5 = 0.3\omega$

$\omega = 16.67$ rad/s

Centripetal acceleration

A point at distance r from the centre of a body rotating at constant angular velocity (ω) experiences an acceleration even though its speed is not changing. Its direction changes due to the acceleration a towards the centre of rotation: $a = \omega^2 r$ or $a = \dfrac{v^2}{r}$.

We are exploring only **centripetal acceleration** here, not the forces that would result from this acceleration being applied to a mass.

Worked example

The pilot of a Naval F35 carries out a 1.5 km radius turn at a constant speed of 173 m/s.

(a) Determine the centripetal acceleration.

(b) Estimate the additional 'g' acceleration the pilot would experience as a result of the turn.

(a) $a = ?$ m/s², $v = 173$ m/s, $r = 1500$ m

$$a = \frac{v^2}{r} = \frac{173^2}{1500} = 19.95 \text{ m/s}^2$$

(b) Acceleration due to gravity $= 9.81$ m/s², therefore 19.95 m/s² represents an additional acceleration of approximately $2g$.

Power (P)

$$\boxed{P = \tau\omega}$$

Power, the rate of work done (W)

Torque (N m)

Angular velocity (rad/s)

Kinetic energy (E_k)

$$\boxed{E_k = \tfrac{1}{2} I\omega^2}$$

Energy of a rotating body (J)

Moment of inertia (kg m²)

Angular velocity (rad/s)

Remember that $I = mr^2$, so this can be substituted in the formula because the mass and the radius of the rotating mass are given, whereas the moment of inertia is not.

Worked example

A constant torque of 70 N m keeps a pump rotating at 20 rev/s. Neglecting losses, find the power input to the pump.

$P = \tau\omega$

$P = 70 \times (20 \times 2\pi) = 8796$ W $= 8.80$ kW (2 d.p.)

Calculate the angular kinetic energy of a navigational gyroscope rotor, with mass 100 g and radius 40 mm, rotating at 100 rev/s.

$E_k = \frac{1}{2} I\omega^2 = \frac{1}{2}(mr^2) \times (100 \times 2\pi)^2$

$= \frac{1}{2}(0.1 \times 0.04^2) \times (394784.18)$

$= 31.58$ J

Now try this

1 A jet engine applies a constant torque of 850 N m to its turbofan at 100 rev/s. Calculate the power input to the turbofan.

2 A flywheel has a moment of inertia of 4 kg m² about its axis of rotation. Calculate the angular kinetic energy stored in the wheel when it is rotating at 8 rev/s.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Mechanical power transmission

Machines use design features such as levers, gears and screw threads, to make it easier to do work. Engineering projects would be impossible without using machines.

Mechanical advantage (MA)

Make sure you don't confuse load and effort:

1. **Load** is used for the Output force (F_l), measured in newtons (N).
2. **Effort** is used for the Input force (F_e), also measured in newtons.

$$MA = \frac{\text{Load}}{\text{Effort}}$$

- For a pulley system
  MA = the number of pulleys (providing the mass of the pulleys are ignored)
- For a lever

$$MA = \frac{\text{Load}}{\text{Effort}} = \frac{\text{Effort to fulcrum distance}}{\text{Load to fulcrum distance}}$$

Velocity ratio (VR)

This is the ratio of the distances moved by the effort and the load.

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}}$$

or

$$VR = \frac{\text{Velocity with which effort moves}}{\text{Velocity with which load moves}}$$

For a pulley system, VR = the number of ropes supporting the load.

Efficiency

Efficiency (η) (pronounced 'eta') $\eta = \frac{WD_l}{WD_e} \times 100\%$ Useful work output; Work input

Efficiency also can be stated in terms of MA and VR:

$$\eta = \frac{MA}{VR} \times 100\%$$

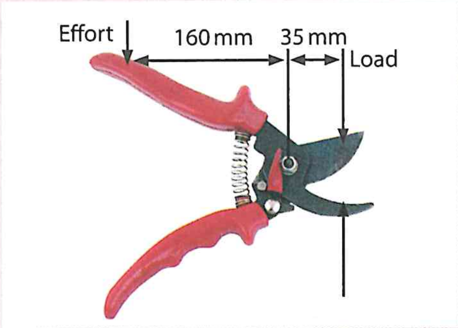
Now try this

1 A machine requires 7 kW of input power and the output power is 5.9 kW. Calculate the efficiency of the machine.

2 A machine lifts a load of 0.5 tonnes through a distance of 75 mm when the effort of 300 N moves through a distance of 2.3 m. Determine the VR, the MA and the efficiency.

Worked example

The force required to cut (shear) a branch with these pruning shears is 0.9 kN. Find the effort needed.



Consider the distances of the load and effort from the fulcrum:

$$MA = \frac{160}{35} = 4.57$$

Now consider the load and effort forces:

$$\text{Effort} = \frac{\text{Load}}{MA} = \frac{900}{4.57} = 197\,\text{N}$$

Worked example

A 180 kg load is lifted through 240 mm when the effort moves through 2.4 m. What is the velocity ratio?

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by load}} = \frac{2.4}{240 \times 10^{-3}}$$

$$VR = 10$$

Worked example

A machine has an efficiency of 65%. If an effort of 160 N raises a load of 1 tonne, find the velocity ratio.

$$\eta = \frac{MA}{VR} \times 100\%$$

$$MA = \frac{\text{Load}}{\text{Effort}} = \frac{1 \times 10^3 \times 9.81}{160} = 61.31$$

$$\eta = \frac{MA}{VR} \times 100 \text{ therefore:}$$

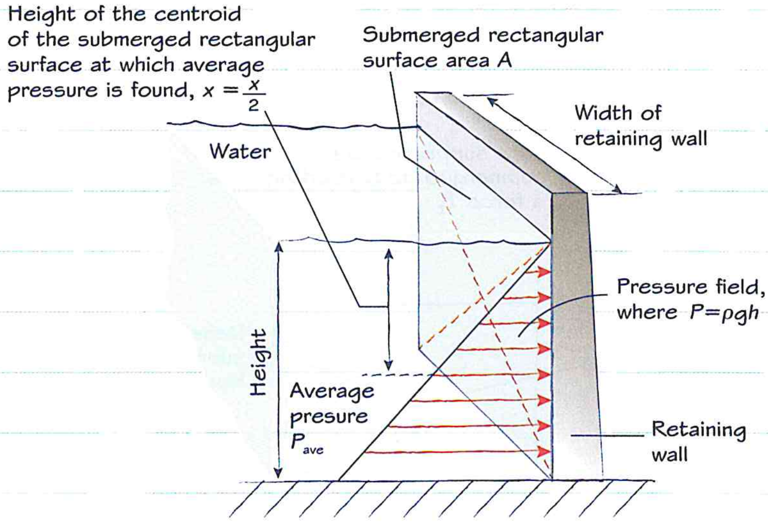
$$65 = \frac{61.31}{VR} \times 100$$

$$VR = 94.3$$

Submerged surfaces

In applications such as dams and storage tanks, forces act on the submerged surfaces that retain the fluids. You need to understand these forces to ensure that the structures are built with sufficient strength to contain the fluids safely.

The pressure exerted on a submerged rectangular surface increases linearly as the submerged height, h, increases. (h is always measured from the surface downwards.)



Hydrostatic pressure

$$P = \rho g h$$

Pressure at any point beneath the surface of the fluid (Pa)
Height (m)
Density (kg/m³)
Gravitational constant (9.81 N/kg)

Hydrostatic thrust

Hydrostatic thrust F_T is the total force acting on the submerged plane surface and can be calculated as a function of average pressure and the area over which it is applied:

$F_T = \rho g A x$

Average hydrostatic pressure

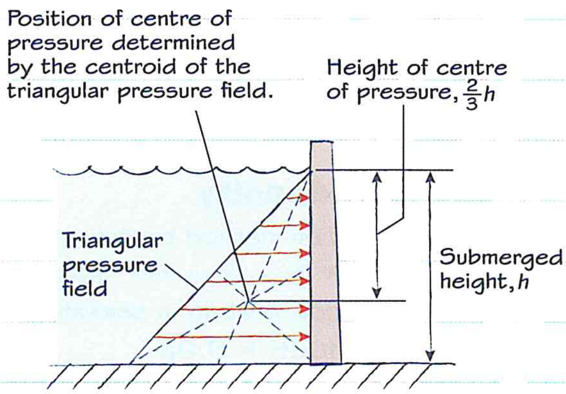
The average pressure on a submerged rectangular plane surface acts at the height of its centroid:

$$P_{ave} = \rho g \frac{h}{2} = \rho g x$$

Centre of pressure

The hydrostatic thrust can be thought of as a single point force acting at the centre of pressure. The height of the centre of pressure below the surface, hp, is given by the centroid of the triangular pressure field described as h increases.

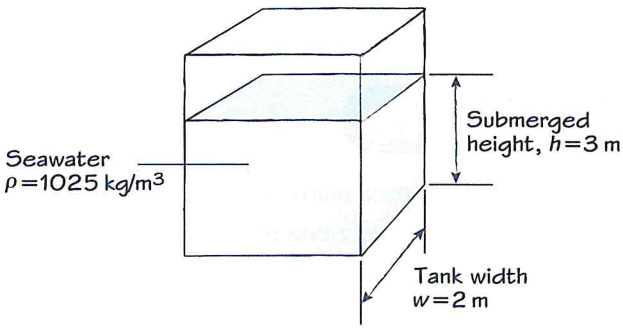
$$hp = \frac{2}{3} h$$



Now try this

A tank is used to store seawater. Calculate:

1 the hydrostatic thrust acting on the tank wall, illustrated in the diagram
2 the height of the centre of pressure.

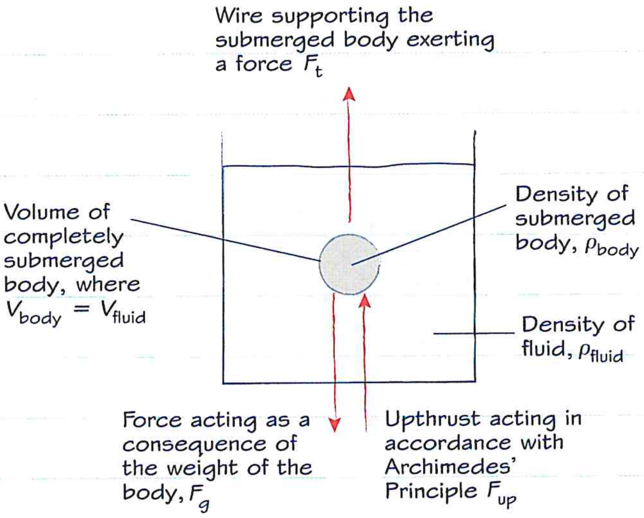


Had a look ☐ Nearly there ☐ Nailed it! ☐

Immersed bodies

Archimedes' principle states that, 'A body totally or partially submerged in a fluid displaces a volume of fluid that weighs the same as the apparent loss in weight of the body.' A floating body experiences an upwards force equal in magnitude to its weight. A body that sinks experiences an apparent weight loss equal to the weight of the fluid it displaces.

Suspended body submerged in a fluid



Assuming that the object is in static equilibrium then the forces acting on the body are:

$F_t = F_g - F_{up}$

$F_g = \rho_{body} V_{body} g$

$F_{up} = \rho_{fluid} V_{fluid} g$

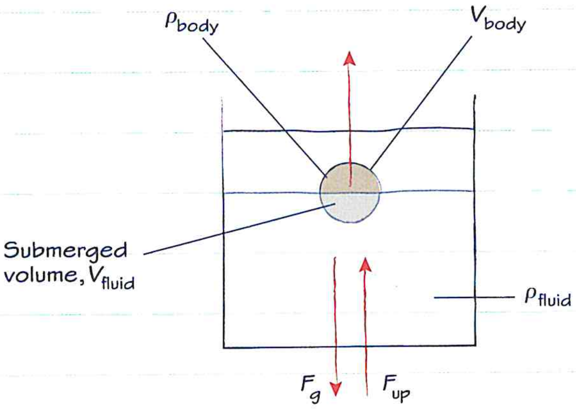
Floating bodies

A partially submerged floating body does not need to be suspended to maintain equilibrium.

When a body is floating and $F_t = 0$ then:

$F_g = F_{up}$

$\rho_{fluid} V_{fluid} = \rho_{body} V_{body}$



Relative density

Relative density (d) is defined as the density of a substance compared to the density of pure water:

$$d_{substance} = \frac{\rho_{substance}}{\rho_{water}}$$

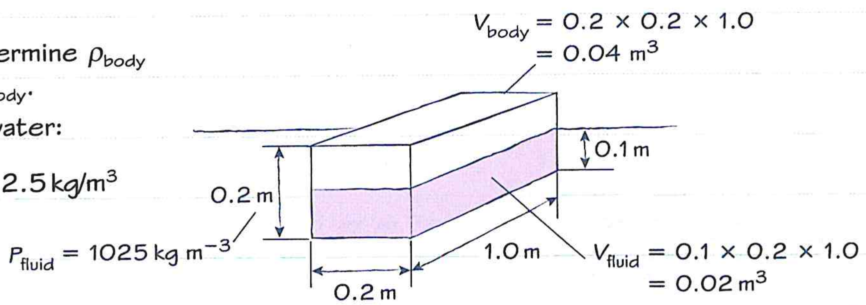
where $\rho_{substance}$ and ρ_{water} are absolute densities.

Determining density

You can use a flotation method to determine ρ_{body} as long as you know ρ_{fluid}, V_{fluid} and V_{body}.

For this block of wood floating in seawater:

$$\rho_{body} = \frac{\rho_{fluid} V_{fluid}}{V_{body}} = \frac{1025 \times 0.02}{0.04} = 512.5\,\text{kg/m}^3$$



Now try this

A partially submerged plank of oak, with height 10 cm, width 25 cm and length 2.4 m, is floating in fresh water.

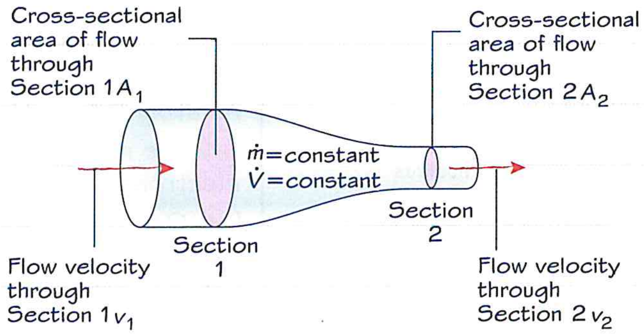
7.4 cm of the height is submerged under the water.

Fresh water has a density of 1000 kg/m³.

Calculate the density of the oak.

Fluid flow in tapering pipes

Tapering pipes can be used to alter the flow velocity of fluids travelling through them. You only need to revise **incompressible** fluids where density remains constant throughout the system.



Volumetric flow rate ($\dot{V}$)

The volume (V) of fluid to pass a given point in time (t). Units m^3/s:

$$\dot{V} = \frac{V}{t}$$

This can also be expressed in terms of flow velocity (v) and the cross-sectional area of the pipe (A):

$$\dot{V} = Av$$

Mass flow rate ($\dot{m}$)

The mass (m) of fluid to pass a given point in time (t). Units kg/s:

$$\dot{m} = \frac{m}{t}$$

This can also be expressed in terms of the density of the fluid (ρ) and the volumetric flow rate ($\dot{V}$):

$$\dot{m} = \rho\dot{V}$$

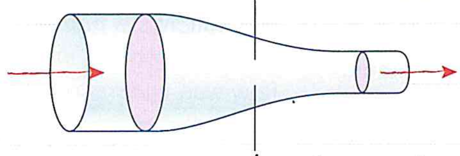
or

$$\dot{m} = \rho Av$$

Equations describing the continuity of flow

Since you are only dealing with incompressible fluids that maintain constant density throughout, then the volumetric and mass flow rates will remain constant throughout.

$$\dot{V} = A_1v_1 = A_2v_2$$



$$\dot{m} = \rho A_1v_1 = \rho A_2v_2$$

Always draw a labelled diagram when solving these types of problems.

Now try this

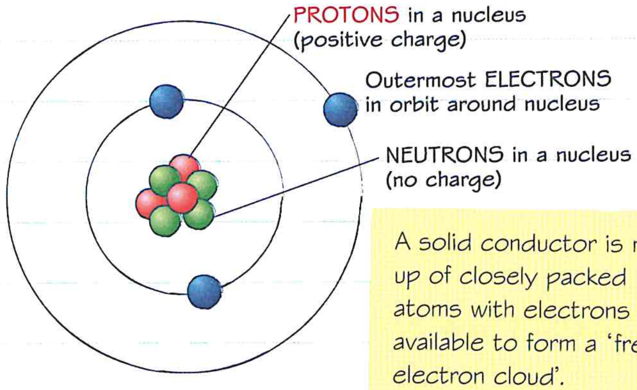
A hose has a gradually tapering nozzle that reduces the flow diameter from 25 mm to 10 mm. The flow velocity in the 25 mm diameter section is 5 m/s. Calculate the flow velocity after the nozzle.

Had a look ☐ Nearly there ☐ Nailed it! ☐

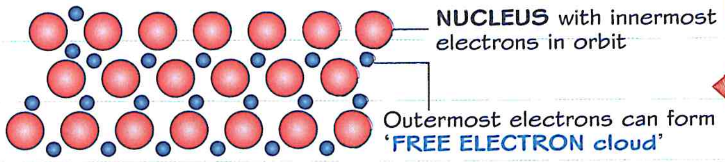
Current flow

Current flow is fundamental to understanding how electricity is used by engineered components such as motors and generators. You will find the following concepts used to explain topics such as capacitors on pages 38 to 39, and DC power sources on page 48.

Atomic structure



A solid conductor is made up of closely packed atoms with electrons available to form a 'free electron cloud'.



Static electricity

An atom consists of charged particles:

- The nucleus is positive and holds electrons in orbit around it.
- Only electrons are transferred when friction is used to charge objects.
- A **positively** charged object has **lost** electrons.
- The outer electrons are less tightly bound because they are farthest from the nucleus.
- In a conductor the outer electrons can drift away from the nucleus to form a 'free electron cloud'.

Metals are good **conductors** because they have many **free electrons**.

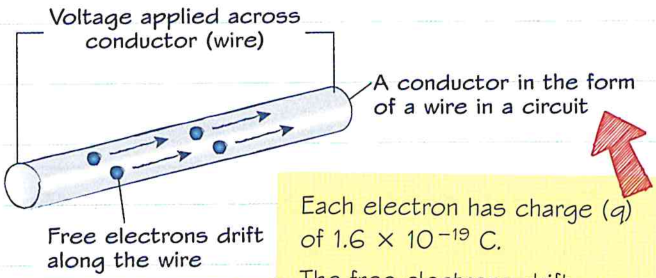
Insulators have few or no free electrons.

Current flow

Electric current, measured in amps (A), is the flow of electric charge (q) in a specified time.

The more free electrons that pass a point per second, the greater the current.

Current (I) is defined as the rate of flow of charge: $I = \frac{q}{t}$, where q is charge, in coulombs (C), and t is time measured in seconds.



Each electron has charge (q) of 1.6×10^{-19} C.

The free electrons drift along a conductor, such as copper wire in a circuit, when a voltage is applied across the ends of the conductor.

The direction of conventional current was decided by early experimenters before the role of electrons in current flow was understood.

Worked example

A current of 2 A flows through a point in a circuit. How much charge passes the point in 3 minutes?

$I = \frac{q}{t}$

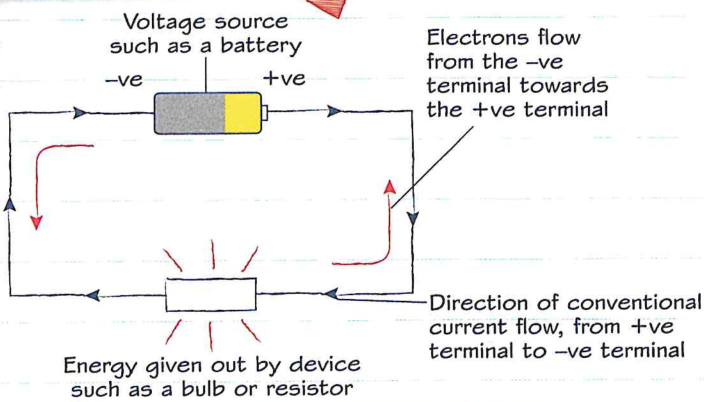
Substitute: $I = 2$ A, $t = 3 \times 60$ s

$2 = \frac{q}{180}$

therefore $q = 2 \times 180$ C $= 360$ C

Conventional current flow

Although electrons drift from the negative to the positive terminal, the conventional **direction of current** is taken as being **from the +ve terminal towards the –ve terminal.**



Now try this

1. A charge of 3 μC flows through an LED in 2 ms. Calculate the current.
2. A total charge of 4320 C passes a point in a circuit in which a current of 25 mA flows. Find the number of days this takes.

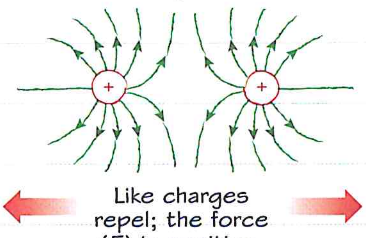
You need to be careful with the units given in question 1.

For question 2, remember that time is usually calculated in seconds, so you will need to convert this to days.

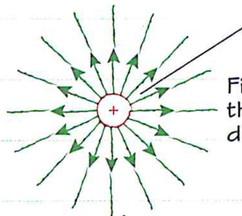
Coulomb's law and electrostatic force

You can use Coulomb's law to calculate the force between two small charged bodies.

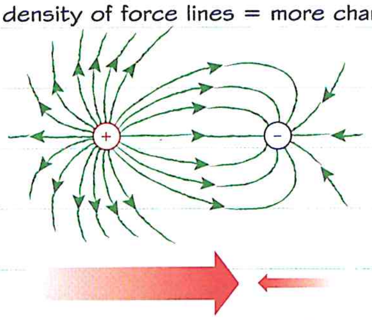
Two equal and like charges



A single charge (q)



Two unequal and opposite charges



Worked example

A positive charge of 1.68 µC and a negative charge of 4.1 µC are separated by 0.99 m. Calculate the force between the two charges.

Sample response extract

$q_1 = +1.68 \times 10^{-6}$ C, $q_2 = -4.1 \times 10^{-6}$ C,
$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $r = 0.99$ m, $F = ?$ N

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$F = \frac{(1.68\times10^{-6})(-4.1\times10^{-6})}{4\pi(8.85\times10^{-12})\times0.99^2}$$

$$= \frac{-6.888\times10^{-12}}{1.112\times10^{-10}\times0.9801}$$

$F = 63.193 \times 10^{-3} = 63.19 \times 10^{-3}$ N (to 2 d.p.)

Don't round values during the intermediate steps of the calculation.

$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$ and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m are both on the formulae sheet, so there is no need to learn them.

Coulomb's law

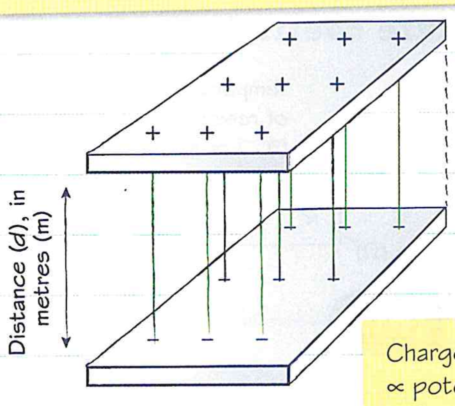
1 $F = \frac{q_1 q_2}{r^2}$ — Charge, in coulombs (C); Force F, in newtons (N), between small charges; Distance (m) between the charges

2 The constant of proportionality is Coulomb's constant (k) $= \frac{1}{4\pi\varepsilon_0}$, where ε_0 is the permittivity of free space.

Permittivity of free space – uniform fields

Two parallel plates at different charge potentials have a potential voltage gradient between them.

Links Energy is needed to create a flow of charge between two points in a circuit. This is called potential difference or voltage. You will find further details about voltage and field strength on page 36.



Charge on plates ∝ potential voltage difference.

Now try this

1 A negative charge of -3.0×10^{-5} C and a positive charge of 8.0×10^{-5} C are separated by 0.20 m. Calculate the force between the two charges.

2 The force between two identical charges separated by 20 mm is equal to 30 N. Find the magnitude of the two charges.

Hint: watch out for the signs of the charge.

Hint: 'identical charges' means that $q_1 = q_2$.

Resistance, conductance and temperature

Resistors are widely used in electronics and you will find details on page 35. Here we look at the factors affecting the resistance of materials and fluids.

Conductivity

- often used in relation to liquids
- constant for a material
- depends on how many free electrons are available to move
- depends on how easily the free electrons can move
- symbol sigma (σ)
- units siemens per metre (S/m)
- equal to 1/resistivity or $\sigma = \frac{1}{\rho}$
- the higher the conductivity, the lower the resistance

Resistivity

- often used in relation to solids
- constant for a material
- depends on how many free electrons are available to move
- depends on how easily the free electrons can move
- symbol rho (ρ)
- units ohm metres (Ωm).

Resistance

Depends on its **dimensions** and the **material** it is made of.

- has symbol R
- units ohms (Ω).

Resistivity, in Ωm

Resistance, in Ω — $\boxed{R = \frac{\rho l}{A}}$ — Length, in m

Cross-sectional area, in m^2

Conductance

The reciprocal of resistance is conductance (G), measured in siemens (S), i.e. $G = \frac{1}{R}$.

Temperature coefficient of resistance

Change in resistance (Ω)

Temperature coefficient of resistance (°C^{-1} or K^{-1})

$\boxed{\frac{\Delta R}{R_o} = \alpha \Delta T}$

Original resistance (Ω)

Change in temperature, in °C or K

Worked example

The resistivity of aluminium is 2.6×10^{-8} Ωm. Find the length of aluminium wire, of diameter 2.5 mm, that has resistance of 0.3 Ω.

Sample response extract

$R = 0.3\ \Omega$, $\rho = 2.6 \times 10^{-8}\ \Omega$m, $A = \pi\left(\frac{2.5}{2}\right)^2$ mm^2

or $\pi\left(\frac{0.0025}{2}\right)^2$ m^2

$R = \frac{\rho l}{A}$

Substituting values:

$$0.3 = \frac{2.6 \times 10^{-8} \times l}{\pi\left(\frac{0.0025}{2}\right)^2}$$

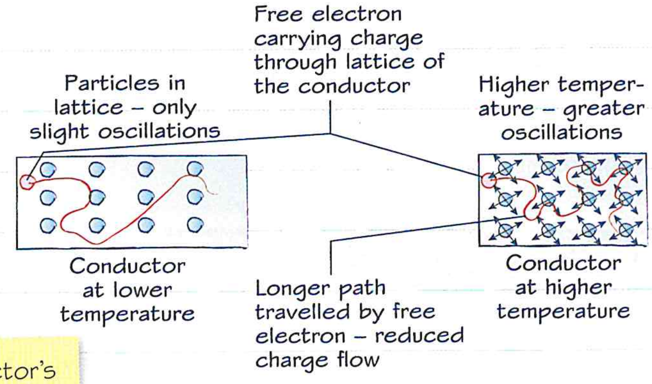
$$l = \frac{\pi\left(\frac{0.0025}{2}\right)^2 \times 0.3}{2.6 \times 10^{-8}}$$

$= 57$ m (to 2 s.f.)

Now try this

1 A coil has a 50 Ω resistance at 50 °C and a 55 Ω resistance at 80 °C. Find its resistance temperature coefficient.

2 What is the conductance of a resistor having a resistance of 100 Ω?



The increased oscillations of particles in a heated conductor's lattice reduce the flow of free electrons by increasing the collisions. This increases the resistance.

Types of resistor

You need to understand the characteristics of different resistors in order to use them effectively.

Uses of fixed resistors

1. Prevent damage to electrical components
2. Control time delays in a circuit when paired with a capacitor
3. Split voltage around a circuit

Variable resistors

✓ Potentiometers: may be used either as a voltage divider, or to vary current flow.

✓ Presets: often included in circuits where the exact resistance is not known until the circuit is constructed.

Make sure that you know how a potentiometer would be wired, both as a voltage divider and as a variable resistance.

Special types

✓ Thermistors: change resistance as their temperature changes. They are used in temperature detecting circuits such as electronic thermometers.

✓ Light-dependent resistors (LDRs) change their resistance as light levels change and are used in light detection circuits such as controlling street lights.

The resistance of a thermistor usually falls as the temperature increases. The resistance of an LDR falls as light levels increase.

Worked example

A radio set contains both pre-set resistors and a potentiometer.

(a) State one use of each in a radio.

(b) Explain how often and why each would be adjusted.

Sample response extract

(a) The radio receiver circuits use a pre-set resistor. The volume is controlled with a potentiometer.

(b) The pre-set resistor is adjusted once, in the factory, to make the radio receiver work. The potentiometer is adjusted by the user many times.

Potentiometers also come in the form of a straight track called **sliders**. They are found, for example, on audio mixing decks to control volume.

Preferred resistor values

Resistors are manufactured in specific ranges to reduce the number of different values required.

The value of a resistor is shown in one of two ways:

- by colour bands; usually four or five bands
- by 'multiplication factor', where R = ×1, K = ×1000 and M = ×1000000.

The **resistor tolerance** is the amount by which the resistance of a resistor may vary from its stated value. If coloured bands are used, the tolerance is the last band, separated from the other bands by a space.

If the multiplication factor is used, then tolerance is given in the form of a letter; e.g. 'J' indicates a tolerance of ± 5%.

Now try this

1 Explain why a carbon composition resistor is unsuitable for use in a high specification audio system.

2 What is the resistance value and tolerance of a resistor specified in a circuit diagram as 5 MΩJ?

Had a look ☐ Nearly there ☐ Nailed it! ☐

Field strength

On page 33 you used Coulomb's law to calculate the force between two small charged bodies. Here you will calculate the force on a small charged body in an electric field. There are practical aspects of this on page 58 (Faraday's laws – induced emf).

Field strength

- The field strength increases closer to the source charge. (You can see that the equipotential lines are closer together nearer to the source charge.)

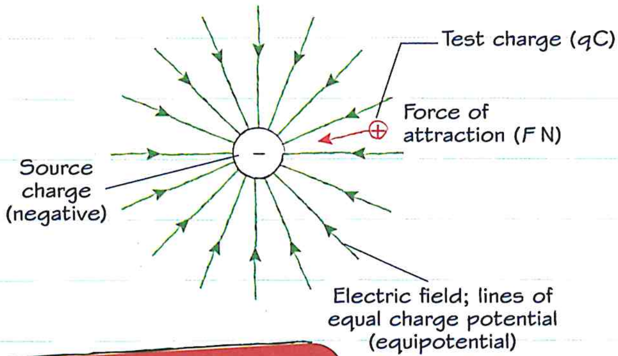
$E \propto \frac{1}{d^2}$

- If the distance of the test charge from the source charge is doubled, then the field strength will decrease by a factor of 4 (2^2).
- The field strength is independent of the size of the test charge.
- Field strength is a vector; it has magnitude and direction.
- The test charge carries a quantity of charge (q).
- The +ve test charge will experience a force (F) of attraction towards the –ve source charge.
- The electric field strength (E) is defined as the force per charge on the test charge or:

$E = \frac{F}{q}$

Non-uniform electric field

Here is a (-ve) source charge with a (+ve) test charge placed at some distance from it to measure the field strength at that point.



Worked example

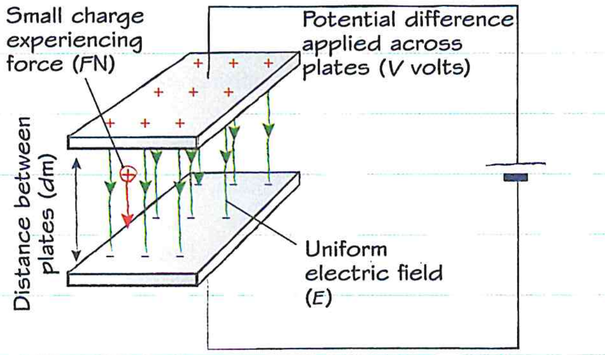
A test charge of 1×10^{-6} C experiences a repulsion force of 0.40 N when placed within the electric field of a positively charged source of 2×10^{-4} C. Find the electric field strength at that point.

Sample response extract

$E = \frac{F}{q} = \frac{0.4}{1 \times 10^{-6}} = 4.0 \times 10^5$ N/C

Uniform electric field

This is a special case when considering electric field strength. Instead of an inverse square relationship, the field strength is constant between the two plates (but not at the edges!).



The field strength is constant, therefore the force on a small charged object is constant.

Field strength (V/m) $E = \frac{V}{d}$ Potential difference applied to the plates (V)

Distance between the plates (m)

By convention, the direction of the field is from the positive to the negative plate.

Worked example

A food preservation process uses pulsed electric fields (PEF) between parallel conducting plates. The uniform field strength is 40 kV/cm. Calculate the voltage applied to the plates if they are 0.3 m apart.

Sample response extract

$E = \frac{V}{d}$

40 kV/cm $= 40 \times 10^3 \times 100$

$= 4.0 \times 10^6$ V/m

Substituting: $4 \times 10^6 = \frac{V}{0.3}$

$V = 4 \times 10^6 \times 0.3$ V

$= 1.2 \times 10^6$ V or 1.2 MV

Now try this

1. A test charge of 2×10^{-5} C experiences a repulsion force of 0.20 N when placed within the electric field of a positively charged source of 3×10^{-4} C. Determine the electric field strength at that point.
2. Find the electric field strength between a pair of parallel conducting plates, positioned 4 cm apart, when the potential difference between them is 300 V.

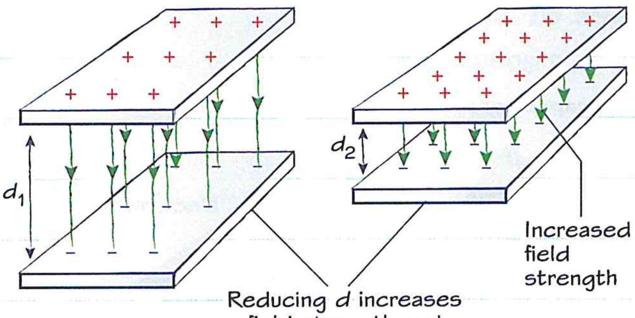
Capacitance

There are three factors that effect the amount of charge stored on parallel plates. You will find details of how these are used in electrical capacitors on pages 38 and 39.

Charge on parallel plates

Amount of charge stored depends on:

- distance between the plates
- area of the plates
- material between the plates.



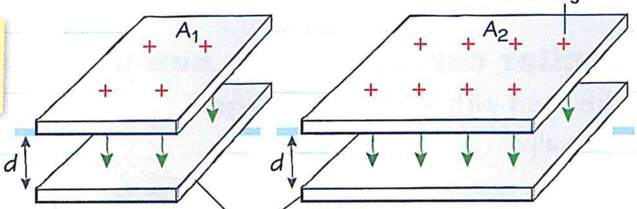
Reducing d increases field strength and therefore capacitance. $C \propto \frac{1}{d}$

Units of capacitance

1 F capacitor, charged by 1 V carries a charge of 1 C.

This is a large amount of charge; usually capacitors have values in the microfarad (10^{-6}) or picofarad (10^{-12}) range.

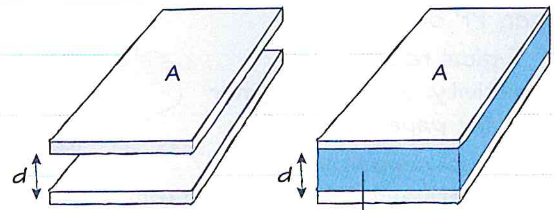
Note that flux density (quantity of charge per unit area ($D = Q/A$)) remains the same.



Increasing A increases charge on the plates and therefore capacitance. $C \propto A$

Permittivity

- Symbol for absolute permittivity is ε (Greek letter epsilon).
- Units of permittivity are farads/metre or F/m.
- The permittivity depends on the material between the plates, called the **dielectric**.
- ε is usually written $\varepsilon_r \varepsilon_0$
- ε_r is the relative permittivity of the dielectric used. The values are relative to a vacuum, which has a value of 1.
- ε_0 is the permittivity of free space (8.85×10^{-12} F/m).
- Typical values of ε_r are 2.5 for paper, 6 for mica and 80 for water. Air has a value very close to 1.



Dielectric (an insulator) between the plates increases capacitance. $C \propto \varepsilon$ where ε is the permittivity of the dielectric.

Calculating capacitance

Absolute permittivity ε is the constant of proportionality in the relationship $C \propto \frac{A}{d}$ such that $C = \varepsilon \frac{A}{d}$. This is more usually written as $C = \varepsilon_r \varepsilon_0 \frac{A}{d}$.

- The value of ε_0 (8.85×10^{-12} F/m) is provided on the formulae and constants sheet.
- The formula will be on the formula sheet in the form $C = \varepsilon \frac{A}{d}$. Remember to substitute ε with $\varepsilon_r \varepsilon_0$ so that you use the form $C = \varepsilon_r \varepsilon_0 \frac{A}{d}$.

Worked example

A parallel plate capacitor in a circuit has plates of area 0.005 m² and 1 mm apart, with a polymer dielectric of relative permittivity 3.5. Calculate the capacitance.

Sample response extract

$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$ F/m, substituting:

$$C = \frac{3.5 \times 8.85 \times 10^{-12} \times 0.005}{1 \times 10^{-3}} = 1.54875 \times 10^{-10}\,\text{F}$$

$$= 154.87 \times 10^{-12}\,\text{F} \text{ or } 154.87\,\text{pF}$$

Now try this

A parallel plate capacitor in a circuit has plates of area 0.01 m² and 1 mm apart, with an electrolytic-based dielectric of relative permittivity $\varepsilon_r = 2$. Calculate the capacitance.

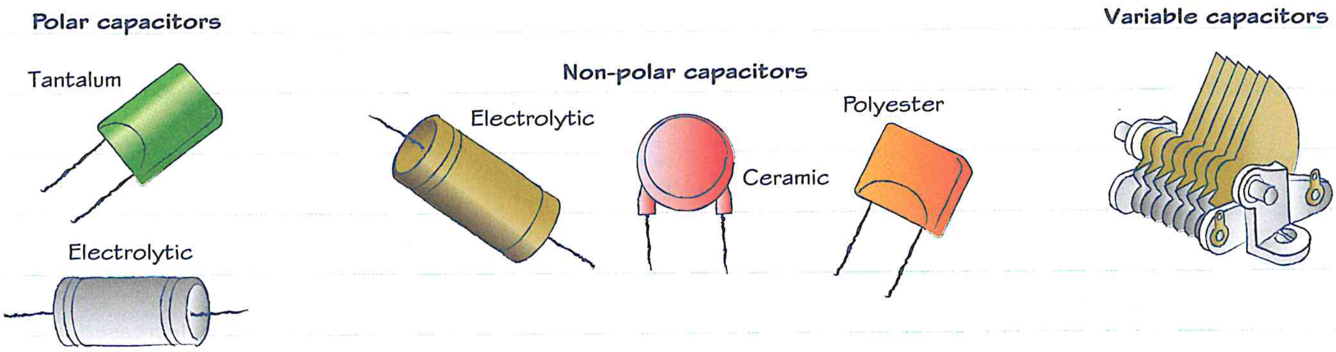
Had a look ☐ Nearly there ☐ Nailed it! ☐

Capacitors – non-polarised

Capacitors find many uses in electronic circuits and some of these are covered on pages 44 and 45. Here and on page 39 you will explore the three main types of capacitor and their construction.

Types of capacitor

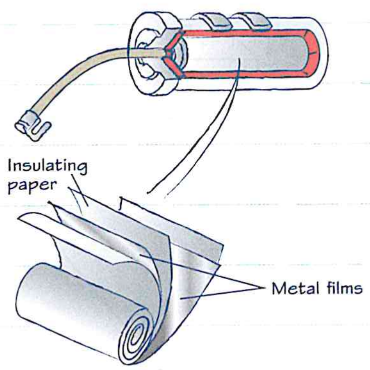
Three main groups:



Capacitor construction – non-polarised

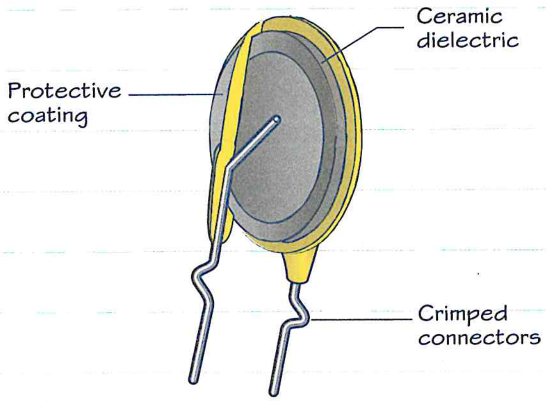
May be used with AC or DC voltages

- fixed value
- film.



- ✓ Waxed paper or polymer dielectric between metal foil.
- ✓ Named after their dielectric, for example, paper, PP or PTFE.
- ✓ Typical relative permittivity: 2.5 for polymer and 4 for paper.
- ✓ Typical capacitance values: between 0.5 μF and 50 pF for polymer, and 10 μF and 10 nF for paper.
- ✓ Working voltages are up to about 600 V for paper and 400 V for polymer.

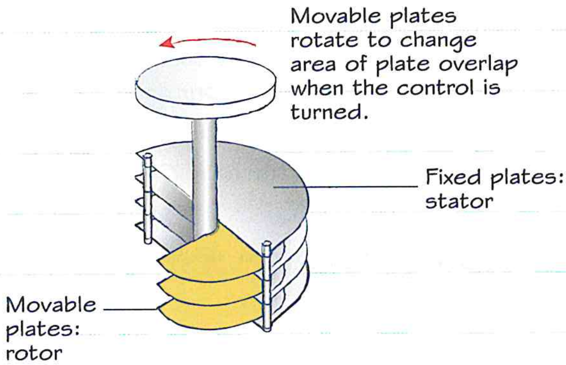
Ceramic



- ✓ Two types: class 1 and class 2, each with different characteristics.
- ✓ Simplest construction is a plate of ceramic coated with silver on each side. Often with crimped wires to keep it clear of PCB-induced movement or stress.
- ✓ Typical relative permittivity: Class 1: 20 to 40, class 2: 200–14 000.
- ✓ Typical capacitance values: 1 μF to 5 pF.
- ✓ Rated voltages up to 500 V.

Variable value

- ✓ The simplest form has an air dielectric with movable plates.
- ✓ Relative permittivity of 1.
- ✓ Typical capacitance values are 1 nF to 100 pF.
- ✓ Working voltages 10 V to 2 kV.



Now try this

Explain why the dielectric material in a capacitor must be an insulator.

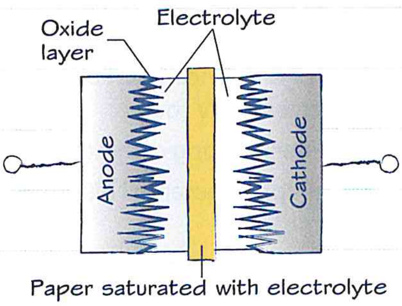
Capacitors – polarised

Capacitors find many uses in electronic circuits and some of these are covered on pages 44 and 45. Following on from the construction of non-polarised capacitors on page 38, here you will explore the construction of polarised capacitors and the calculation of dielectric field strength.

Capacitor construction – polarised

DC voltage only; correct terminal connection is required to avoid breaking down the insulating oxide layer.

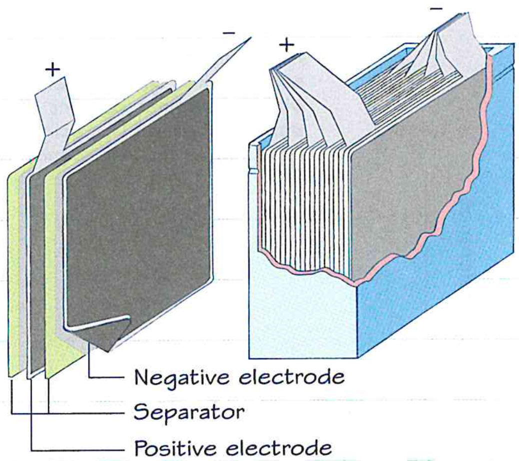
Electrolytic



- Relies on chemical action for operation; oxide forms on +ve plate when voltage applied. This acts as dielectric.
- Historically named after cathode material, for example, aluminium, tantalum or niobium.
- May be radial or axial.
- Relative permittivity: 10 to 40.
- Typical capacitance values: 1 F to 1 μF.
- Working voltages 10–600 V.

Supercapacitors

Supercapacitors bridge the gap between electrolytic capacitor and rechargeable battery.



- Very high capacitance values but lower voltage limits than other capacitors.
- Used where multiple charge/recharge required, such as regenerative braking or CPU cache memory back-up in computers.

Dielectric strength

The electric field strength is defined as $E = \frac{V}{d}$, where E is the electric field, V is the potential difference (in volts) and d is the distance between the plates (in metres).

An applied potential difference greater than rated capacitor voltage can strip atoms of electrons and make the dielectric conduct. Charge is no longer maintained on the plates.

Typical dielectric strengths:

- dry air 3×10^6 V/m
- ceramics 10×10^6 V/m
- polymers 20×10^6 V/m
- mica 40×10^6 V/m

Worked example

The dielectric strength (E) of a polymer is 20×10^6 V/m. Calculate the maximum potential difference that can be applied to a parallel plate capacitor with polymer dielectric thickness 0.2 mm.

Sample response extract

$E = \frac{V}{d}$

$V_{max} = E_{max}d$

Substitute in $V_{max} = 20 \times 10^6 \times 2 \times 10^{-4}$ V/m

$= 4000$ V

$= 4$ kV

Now try this

1 Explain why the terminals on polarised capacitors must have polarity markings. Provide an example of how this is shown.

2 An electrolytic capacitor has a tantalum oxide dielectric of thickness 1 μm and dielectric strength 5×10^6 V/m. What is the maximum voltage that can be used with this capacitor?

Had a look ☐ Nearly there ☐ Nailed it! ☐

Ohm's law, power and efficiency 1

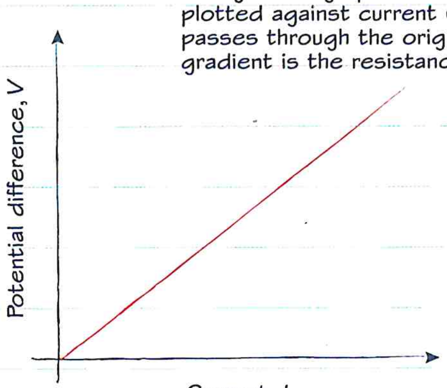
Ohm's law applies to OHMIC CONDUCTORS, whether in a DC or AC circuit. Providing the temperature remains constant, their resistance doesn't change over a wide range of currents.
If you are given two of the values, you can calculate the unknown third value.

1 $V = IR$ 2 $I = \frac{V}{R}$ 3 $R = \frac{V}{I}$

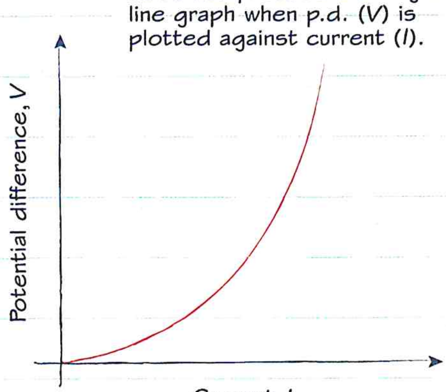
Where V = potential difference measured in volts (V), I = current measured in amps (A), and R = resistance measured in ohms (Ω).

In graphical form

An ohmic resistance produces a straight-line graph when p.d. (V) is plotted against current (I). The line passes through the origin and its gradient is the resistance (R).



A non-ohmic resistance, such as a filament bulb, does not produce a straight-line graph when p.d. (V) is plotted against current (I).



Worked example

A thermistor of resistance 10 kΩ at room temperature is connected across a 12 V source.

(a) Calculate the resulting current.

(b) Sketch a graph of potential difference against current for a thermistor.

(c) Explain whether thermistors obey Ohm's law.

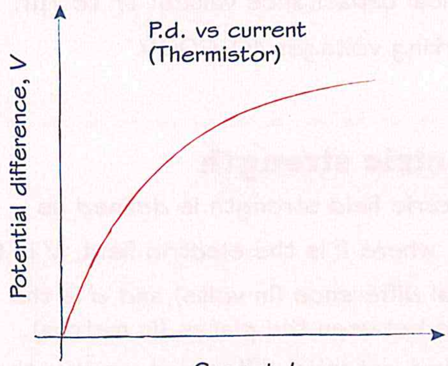
Sample response extract

(a) $V = 12\,\text{V}$, $I = ?$, $R = 10 \times 10^3\ \Omega$

$V = IR \qquad 12 = 10 \times 10^3 I$

$$I = \frac{12}{10 \times 10^3} = 1.2 \times 10^{-3}\ \text{A or } 1.2\ \text{mA}$$

(b)



(c) Although it goes through the origin, the resistance (gradient: $\frac{V}{I}$) of the thermistor shown in the sketch of p.d. against current varies as the current changes: $R \neq \frac{V}{I}$, therefore it does not obey Ohm's law.

Now try this

1 Sketch the graph of current against voltage for a diode and use it to explain why a diode is a non-ohmic device. Your revision on page 47 may help with this.

2 A power tool is supplied with 110 V and has a power rating of 1.5 kW. Calculate the current drawn by the motor.