Ohm's law, power and efficiency 2

Power and energy are very important to engineers when designing electric/electronic products. Whether increasing battery life of a mobile phone by reducing power demands, or reducing the noise from a computer by reducing the need for cooling air, the power equations and efficiency calculations are useful in optimising engineering solutions at the design stage.

Variations on the power equation

Energy transferred to a circuit component is the product of the number of coulombs per second (current in amps, A) and the energy per coulomb (potential difference in volts, V). i.e. Power in watts = VI. But Ohm's law tells us $I = \frac{V}{R}$ so:

✓ $P = V \times \frac{V}{R}$ or $P = \frac{V^2}{R}$

Similarly $V = IR$ so:

✓ $P = IR \times I$ or $P = I^2R$

Engineers refer to wasted energy lost in power cables as I^2R losses.

By transmitting power at higher voltage through the same resistance, the current is correspondingly lower (think Ohm's law: $R = \frac{V}{I}$). Therefore I is greatly reduced, which reduces the wasted energy.

Worked example

A 300 Ω resistor has a power rating of 4 W. Find the maximum current that can be drawn through the resistor without exceeding the power rating.

$R = 300\ \Omega$, $P = 4$ W, $I = ?$ A; $P = I^2R$

Substituting: $4 = I^2 \times 300$, $I^2 = \frac{4}{300}$

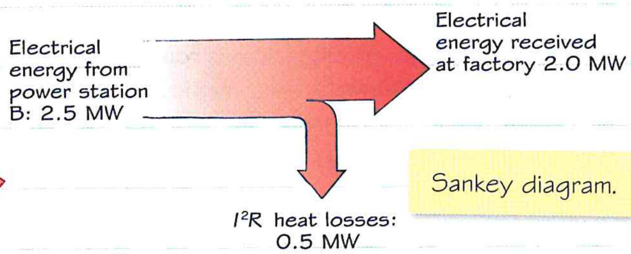
$I = \sqrt{\frac{4}{300}} = 0.116$ A or 116 mA

Efficiency

Usually expressed as a percentage:

$$E = \left(\frac{\text{Power out}}{\text{Power in}}\right) \times 100\%$$

Here you are looking at power, which is the energy consumed per unit time and is measured in joules per second or watts (W).



Sankey diagram.

Worked example

Two power stations transmit 2.5 MW of electrical power at 25 kV for two factories 100 km away. Power station A uses a step-up transformer to raise the voltage to 400 kV before transmission, but power station B does not. In each case the resistance of the transmission line is 50 Ω.

Calculate:

(a) the power lost in transmission lines for each power station

(b) the percentage efficiency of each.

Sample response extract

Current generated by station A: $P = IV$,

$2.5 \times 10^6 = I \times 400 \times 10^3$, $I = \frac{2.5 \times 10^6}{400 \times 10^3} = 6.25$ A

a) Power lost due to heating power line $= I^2R$

$= 6.25^2 \times 50 = 1.95$ kW

b) Efficiency $= \frac{2.5 \times 10^6 - 1.95 \times 10^3}{2.5 \times 10^6} \times 100$

$= 99.92\%$ (to 2 d.p.)

Current generated by station B: $P = IV$,

$2.5 \times 10^6 = I \times 25 \times 10^3$, $I = \frac{2.5 \times 10^6}{25 \times 10^3} = 100$ A

(a) Power lost due to heating power line $= I^2R$

$= 100^2 \times 50 = 0.50$ MW

(b) Efficiency $= ((2.5 - 0.5)/2.5) \times 100 = 80\%$

Now try this

1 If the voltage across a circuit is quadrupled, explain how the current through the circuit changes. Assume the power remains constant.

2 Calculate the efficiency of a power supply that produces a 0.6 W output from a 5 V supply and draws 0.2 A.

Kirchoff's voltage and current laws

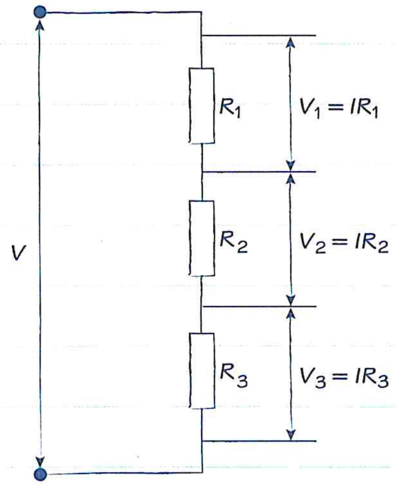
You can use Ohm's law to analyse a simple circuit but you also need Kirchoff's laws for networks of resistors.

Kirchoff's voltage law

When resistors are connected in series the sum of the voltage drop (or potential differences PD) across each one is equal to the total supply voltage.

So where three resistors are in series then:

$V = V_1 + V_2 + V_3$



Combining Kirchoff's and Ohm's law

When you combine Kirchoff's voltage law with Ohm's law you get:

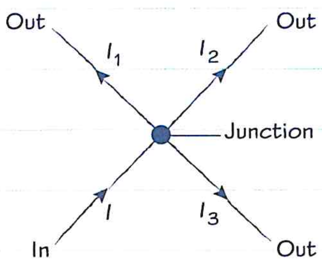
$V = V_1 + V_2 + V_3 = 1R_1 + 1R_2 + 1R_3$

This is sometimes written in terms of the sum of the potential differences (or voltage drops) across each resistor: $\sum PD = \sum 1R$.

Kirchoff's current law

At any junction in an electrical circuit, the total current flowing towards the junction is equal to the total current flowing away from it.

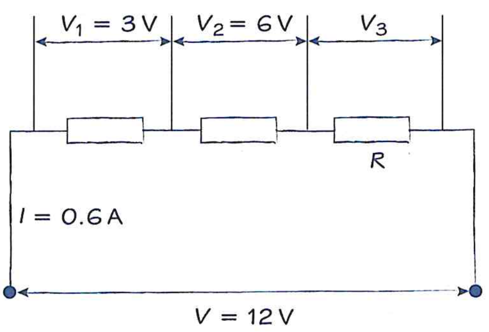
When I flows towards the junction and I_1, I_2 and I_3 flow away from it then $I = I_1 + I_2 + I_3$.



Now try this

The diagram shows a simple series circuit containing three resistors.

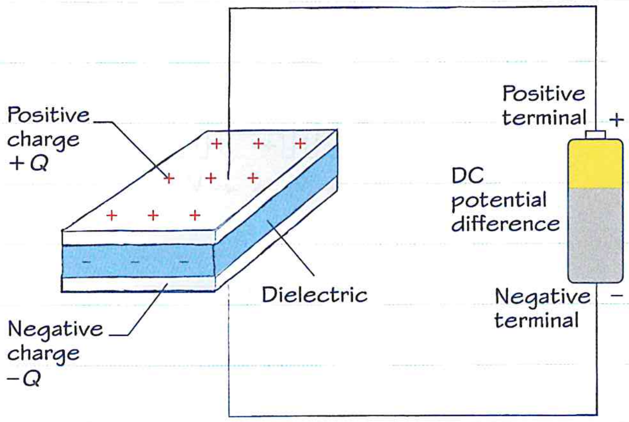
Calculate the value of resistor R.



Capacitors – charging and energy

Capacitors may be found in almost every electrical product. Just a few of their uses are to smooth power supplies, filter wanted or unwanted signals or provide energy storage. Here you will calculate the charge and the energy stored by a capacitor. On pages 44 and 45 you will revise how they can be used in circuits.

Charging a capacitor



Amount of charge (C) — $Q = CV$ — Potential difference (V)

Capacitance (F)

Links See page 37 to revise more about capacitance.

Worked example

A DC 24V circuit contains a 3 μF capacitor. Calculate the charge stored by the capacitor.

Sample response extract

$Q = ?,\ C = 3 \times 10^{-6}$ C, $V = 24$ V

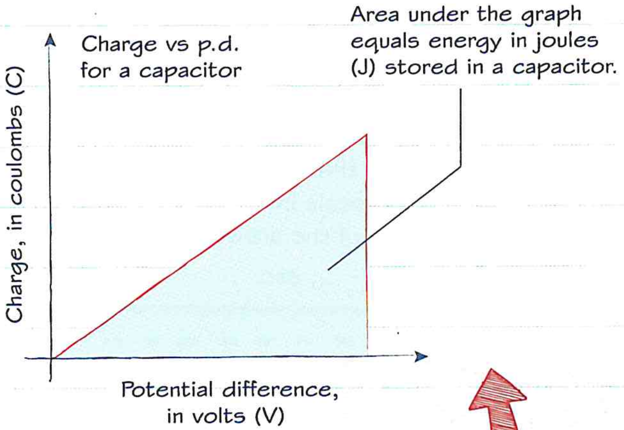
$Q = CV = 3 \times 10^{-6} \times 24$

$= 72 \times 10^{-6}$ C or 72 μC

The symbol for coulomb is an upright C, whereas the variable capacitance is an italic *C*. The algebraic sum of charge on the capacitor plates is 0 because one plate is +*Q* and the other is −*Q*. *Q* refers to the **magnitude** of charge stored on each plate. The fully charged 3 μF capacitor will have +72 μC on the positive plate and −72 μC on the negative plate.

Energy stored in a capacitor

- The energy stored $E = \frac{1}{2}CV^2$
- The energy, in joules, stored in a capacitor is equal to the work done in charging the capacitor from 0 V.



A capacitor is never connected directly to a voltage source. There will always be a resistor to control the rate at which the capacitor is charged – this protects both the capacitor and the voltage source.

Worked example

6.25×10^{-3} J of energy needs to be stored on a capacitor that is supplied with 200V. Calculate the capacitance value required.

Sample response extract

$E = 6.25 \times 10^{-3}$ J, $C = ?$, $V = 200$V

$$E = \frac{1}{2}CV^2$$

$$6.25 \times 10^{-3} = \frac{1}{2} \times C \times 200^2$$

$$C = \frac{2 \times 6.25 \times 10^{-3}}{200^2}$$

$$= 0.31\,\mu\text{F}$$

Links Revise more about this on page 46.

Now try this

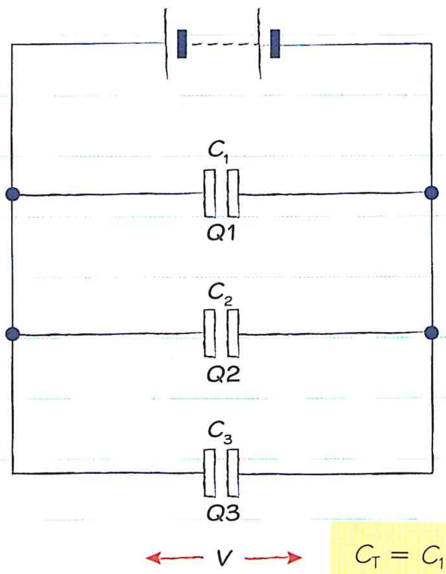
A 3 mF capacitor is connected to a 6V DC power supply. How much charge can be stored by the capacitor?

Capacitors – networks

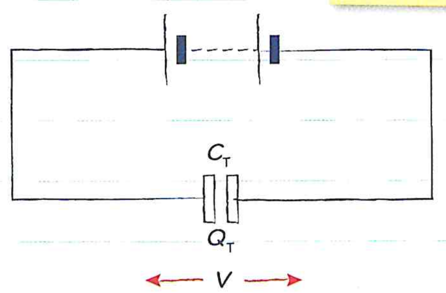
Here you will revise the different results from connecting capacitors in series and in parallel.

Capacitors in parallel and series

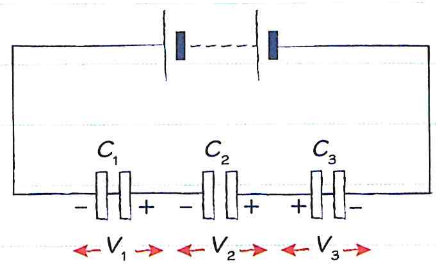
Parallel

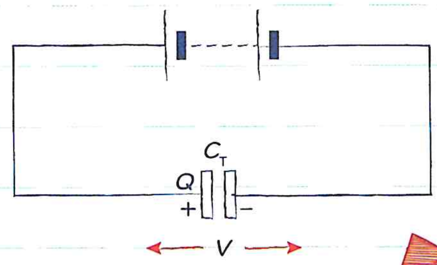


$C_T = C_1 + C_2 + C_3$



Series



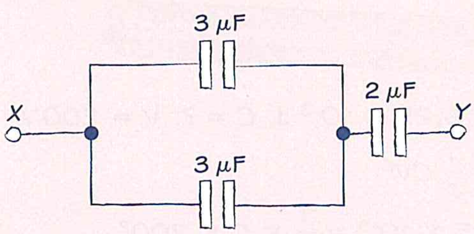


$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

The potential difference applied will equal the sum of the potential differences across each capacitor: $V = V_1 + V_2 + V_3$

Worked example

Find the total capacitance between points X and Y in the circuit shown.



First consider the two 3 μF capacitors; they are in parallel and so can be replaced by one capacitor of value 6 μF ($C_T = C_1 + C_2$).

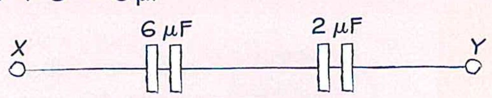
Next, consider the two capacitors in series: 6 μF and 2 μF: $C_T = \frac{C_1C_2}{C_1 + C_2}$.

Remember 'product over sum' works only for two capacitors. For three or more, you need to calculate the reciprocals individually and then take the reciprocal of the answer to obtain C_T:

i.e. $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ … etc.

Sample response extract

For parallel capacitors: $C_T = C_1 + C_2$
$= 3 + 3 = 6\,\mu F$



The two capacitors are in series: $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$

$C_T = \frac{C_1C_2}{C_1 + C_2} = \frac{6 \times 2}{6 + 2} = \frac{12}{8} = 1.50\,\mu F$ (to 2 d.p.)

Now try this

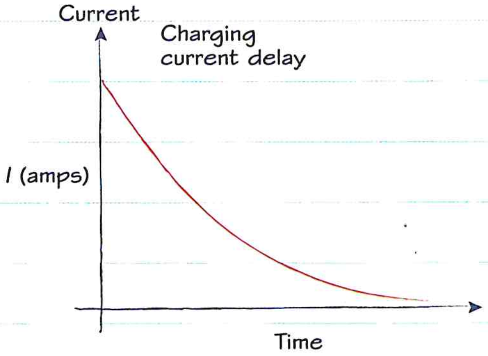
1. When using identical capacitors, state which has the largest capacitance.
   (a) three capacitors in series
   (b) three capacitors in parallel
2. Two capacitors with values of 35 μF and 76 μF are connected in series. What is the total capacitance of this arrangement?

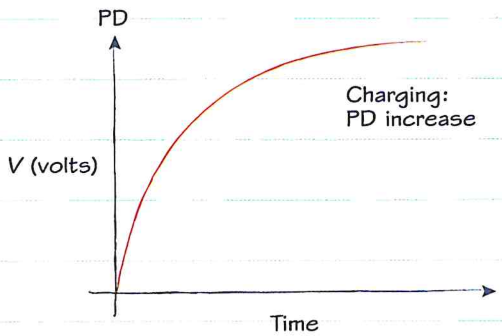
Capacitors in circuits – the time constant

Capacitors are charged and discharged through a resistor, which controls the flow of current and therefore the time taken for the capacitor to charge and discharge. On the next page you will revise how to analyse the RC transients during the exponential-based charging and discharge cycle.

Capacitor charging

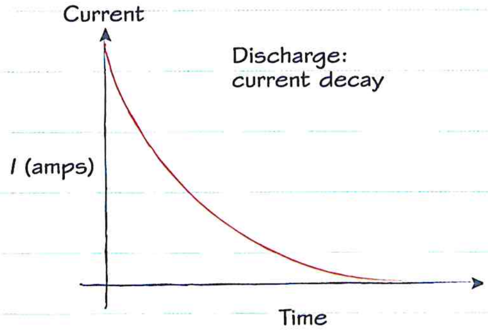
The current flow decays. The potential difference across the plates increases as the charge accumulates.

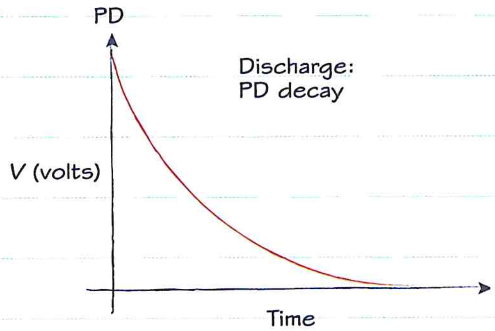




Capacitor discharging

The current flow, potential difference across the plates and the charge all follow the same type of decay curve during the discharge.





RC transient period

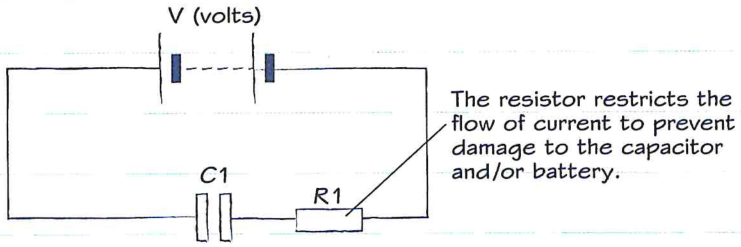
The generally accepted time for a capacitor to be fully charged/discharged is taken as 5 time constants. The RC **transient period** is taken to be where $0 < t < (5 \times \text{time constant})$.

RC transients

The time constant (Greek letter 'tau' τ), in seconds, for the capacitor to charge/discharge depends on the capacitance of the capacitor (C, in farads) and the resistance of the resistor (R, in ohms).

It is given by the product of the two values:

$\tau = RC$



Now try this

A 2200 µF capacitor is charged to 12 V through a 100 kΩ resistor. Calculate

(a) the maximum charge stored by the capacitor

(b) the time constant of the circuit.

Had a look ☐ Nearly there ☐ Nailed it! ☐

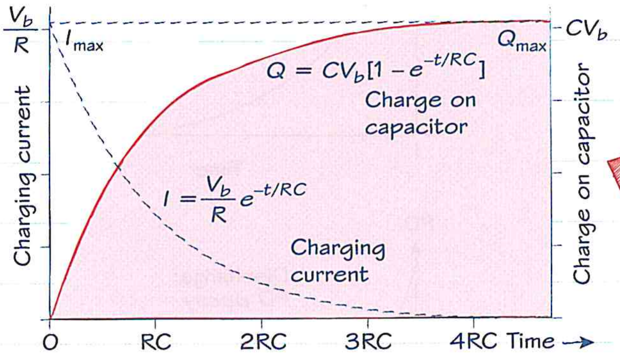
Capacitors in circuits – RC transients

You need to know how to analyse the *RC* transients during the exponential-based charging and discharge cycles of capacitors.

Voltage change on capacitor charge/discharge

The value of the voltage across the capacitor (V_c) at any instant in time during the charge period is given by $V_c = V_s(1 - e^{\frac{-t}{\tau}})$.

The equivalent discharge voltage is given by $V_c = V_s e^{\frac{-t}{\tau}}$, where V_s is the supply voltage and t is the elapsed time since the application/removal, respectively, of the supply voltage.



Worked example

An extrusion machine in a plastic bottle manufacturing factory includes an *RC* network with a time constant of 0.56 s. The value of the capacitor is 3.9 μF. Calculate the value of the resistor.

Sample response extract

$\tau = 0.56$, $R = ?$, $C = 3.9\,\mu F$

$\tau = RC$

$0.56 = 3.9 \times 10^{-6} R$

$R = \frac{0.56}{3.9 \times 10^{-6}} = 144\,k\Omega$

If these expressions are required for the exam, they will be provided on the sheet. There is no need to learn them.

Make sure you know the laws of logs and indices, which can be revised on pages 1 and 2.

Worked example

A single-phase squirrel cage motor requires a secondary winding, in series with a 'start capacitor', to generate sufficient torque to start the motor turning. A typical start capacitor has a capacitance of 100 μF and a supply voltage of 250 V. What is the discharge voltage after 2 time constants?

Use the 'ln' button on your calculator, together with the 'shift' key, to access the e^x function.

$V_c = ?V$, $V_s = 250V$, $t = 2\tau$, $\tau = ?$

Ignore the capacitance because you don't need to calculate the time constant, τ.

$V_c = V_s e\ -t/\tau$

$V_c = 250e^{-2\tau/\tau}$

$V_c = 250e^{-2} = 250 \times 0.1353$

$V_c = 33.83\,V$ (to 2 d.p.)

Don't forget to change the sign of the power 2 to a minus.

Now try this

1 An *RC* timing circuit has a time constant of 0.56 s. The value of the resistor is 15 kΩ. Calculate the value of the capacitor.

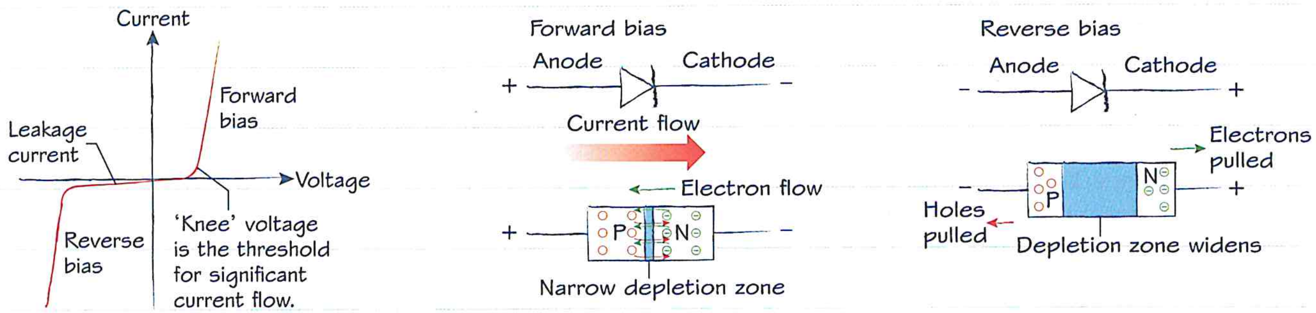
2 An *RC* circuit has a time constant of 5 seconds. How long should be allowed for the capacitor to be considered fully charged?

Diodes – bias and applications

Here you will explore how diodes work and the many uses they have in electronic engineering.

Bias characteristics

Simple diodes allow current to flow in one direction only. Modern diodes are based on a semiconductor pn junction. The voltage/current characteristics can be amended by the addition of doping material at manufacture.



Remember: **forward** bias is **positive** terminal to **p** side of the junction. **Reverse** bias is **negative** terminal to **p** side of the junction. P (Anode) side of junction has holes. N (Cathode) side of junction has electrons.

Forward bias applications

Remember that before a diode will allow significant current to pass there must be an applied voltage to collapse the depletion region. For germanium diodes the forward voltage is 0.3 V and for silicon it is 0.7 V.

You should be able to sketch a suitable diagram to support your description of uses for a forward-biased diode.

Worked example

Explain how a diode can provide:

(a) rectification (b) clamping function (c) circuit protection.

Sample response extract

(a) One diode can provide half wave rectification of an AC current, but full wave rectification requires four diodes in a bridge circuit.

(b) A diode connected with reverse bias in parallel to a component protects the circuit from back EMF when the component is switched off.

(c) A series-connected diode protects the circuit if the battery is connected the wrong way round.

Reverse bias

In reverse bias:

- a small leakage current, although there is a high resistance
- avalanche breakdown or Zener effect, application of sufficient (precise) voltage causes a significant current to flow.

Typical uses for Zener reverse-biased diodes include:

1. voltage regulator
2. power dissipation
3. reference voltage generation.

Worked example

Describe how a Zener diode can be used to regulate voltage in a circuit.

Sample response

A Zener diode provides a controlled breakdown when reverse biased. It will try to limit the voltage drop across it to the breakdown voltage even if the current changes. The voltage drop is very nearly constant across a wide range of reverse currents.

Now try this

Describe, with the aid of a sketch, how you could achieve full wave rectification using a bridge rectifier containing four diodes.

Had a look ☐ Nearly there ☐ Nailed it! ☐

DC power sources

Direct current (DC) power sources are able to provide the voltage and current required to power a range of electrical and electronic devices.

Cells

Cells are electrochemical devices able to generate an emf and are used as a source of electrical current.

Batteries

Batteries contain multiple cells connected together to provide higher voltage or current.

Internal resistance

The internal structure of a cell or battery has some resistance to the flow of current. This is known as internal resistance and is the reason why batteries tend to get hot when discharged rapidly.

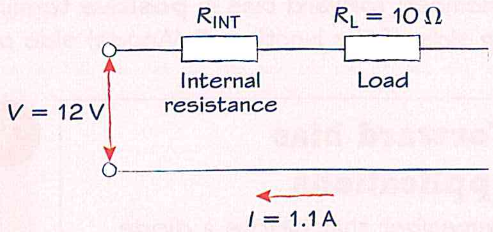
You can calculate the internal resistance of a battery by considering it as an additional series resistor in a circuit.

You might have to remind yourself about Ohm's law and resistors in series to follow this worked example.

Worked example

The circuit contains a battery supplying an emf of 12V. A current of 1.1 A flows through an external load of 10Ω.

Calculate the internal resistance of the battery.



Sample response extract

Calculate total resistance, R_T, using Ohm's law:

$R_T = \frac{V}{I} = \frac{12}{1.1} = 10.909\,\Omega$

For resistors connected in series:

$R_T = R_{INT} + R_L$

Rearrange to make R_{INT} the subject:

$R_{INT} = R_T - R_L$

$R_{INT} = 10.909 - 10 = 0.909\,\Omega$

Stabilised power supply

- mains powered
- produces an accurately regulated pre-set DC output
- more versatile, controllable and safer than batteries
- found in science labs and electronics workshops.

Photo-voltaic cells

- semiconductor devices that generate an emf when exposed to light
- generate small voltages
- usually connected together into large arrays to provide higher voltages
- can be mounted on roofs for domestic use.

Now try this

A battery with an emf of 12V and internal resistance 0.5Ω is connected to a load with resistance 9Ω. Calculate the voltage across the battery terminals.

Resistors in series or parallel

You need to know how to analyse circuits that contain resistors connected together in series or parallel.

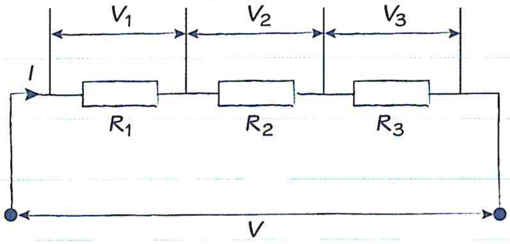
Resistors in series

When resistors are connected in **series** the **same current** flows throughout the circuit and the current flowing through each resistor is the same.

The total resistance (R_T) for a number (n) of resistors in series is simply:

$$R_T = R_1 + R_2 \ldots + R_n$$

Resistors in **series** where current (I) is constant.



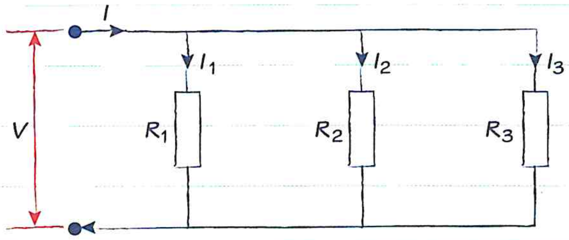
Resistors in parallel

When resistors are connected in **parallel** there is the **same voltage** across each resistor.

The total resistance (R_T) for a number (n) of resistors in parallel is:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \ldots + \frac{1}{R_n}$$

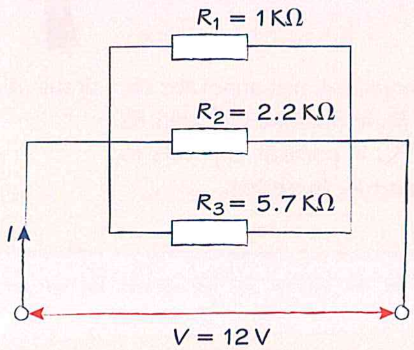
Resistors in **parallel** where voltage (V) is constant.



Worked example

A simple circuit contains three resistors connected in parallel, where $R_1 = 1\,\text{k}\Omega$, $R_2 = 2.2\,\text{k}\Omega$ and $R_3 = 5.7\,\text{k}\Omega$. The circuit supply voltage $V = 12\,\text{V}$.

Calculate the circuit current, I.



Sample response extract

For resistors in parallel:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2.2} + \frac{1}{5.7}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1} + \frac{1}{2.2} + \frac{1}{5.7}$$

$$R_T = \frac{1}{1.981} = 0.614\,\text{k}\Omega$$

Apply Ohm's law:

$$I = \frac{V}{R} = \frac{12}{0.614 \times 10^3} = 0.0195\,\text{A (to 3 s.f.)}$$

Follow these steps:

1. Annotate the circuit diagram.
2. Find the total resistance of the parallel resistors.
3. Use Ohm's law to find the current.

Now try this

A simple circuit contains three resistors connected in parallel. The circuit supply current $V = 12\,\text{V}$ and current $I = 25\,\text{mA}$. Two of the resistors have known values, where $R_1 = 2.2\,\text{k}\Omega$ and $R_2 = 5.3\,\text{k}\Omega$.

Calculate the value of resistor R_3.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Resistors in series and parallel combinations

It is common for electrical and electronic circuits to contain resistors connected together in series and parallel combinations. It's important that you understand the voltage and current characteristics of these networks.

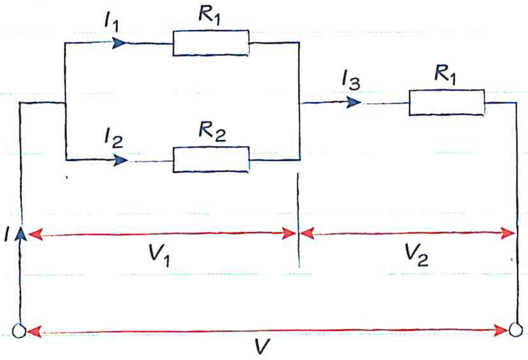
Series and parallel combination

First calculate the equivalent resistance of the parallel element, using

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

Then use:

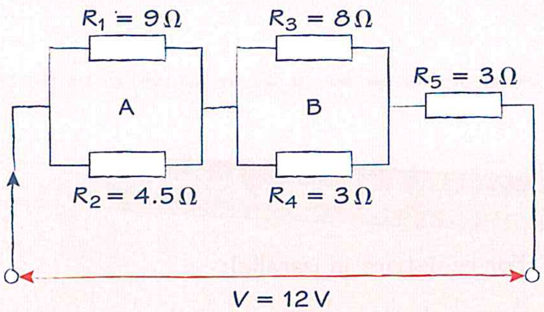
$R_T = R_p + R_3$ to find the total resistance of the combination.



Resistors connected in a **series** and **parallel** combination.

Worked example

In the circuit shown, calculate current *I*.



You may need to remind yourself about Ohm's law before working through this example.

Sample response extract

Simplifying parallel combination A:

$$\frac{1}{R_A} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{9} + \frac{1}{4.5}$$

$R_A = 3\,\Omega$

Simplifying parallel combination B:

$$\frac{1}{R_B} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{9} + \frac{1}{3}$$

$R_B = 2.18\,\Omega$

Combining series elements of simplified circuit:

$R_T = R_A + R_B + R_5 = 3 + 2.18 + 3$

$R_T = 0.18\,\Omega$

Apply Ohm's law:

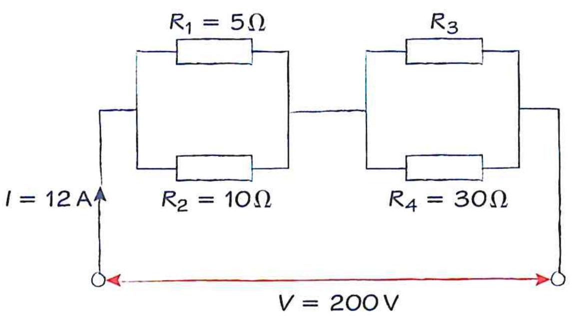
$I = \frac{V}{R_T} = \frac{12}{8.18} = 4.47\,A$ (to 3 s.f.)

Follow these steps:

1 Identify the parts of the circuit that can be simplified and annotate the circuit diagram.
2 Find the combined total resistance of R_1 and R_2 in parallel. Call this R_A.
3 Find the combined total resistance of R_3 and R_4 in parallel. Call this R_B.
4 Find the combined total resistance of R_A, R_B and R_5 in series.
5 Apply Ohm's law to find *I*.

Now try this

In the circuit shown calculate the value of R_3.



Use Ohm's law to find the equivalent total resistance. Next find the equivalent total resistances of the parallel combinations. Substitute in $\frac{1}{R_p} = \frac{1}{R_3} + \frac{1}{R_4}$ to find the unknown value R_3.

Resistors and diodes in series

It is common for electrical and electronic circuits to contain diodes connected together in series with resistors. It's important that you understand the voltage and current characteristics of these simple circuits.

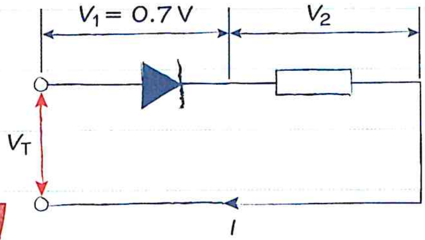
Resistors and diodes in series

When a forward biased diode is connected in series with a resistor there is a voltage drop across the diode.

For silicon diodes the voltage drop is typically 0.7 V.

From Kirchoff's voltage law, the total voltage (V_T) for a silicon diode and resistor in series is:

$V_T = 0.7 + V_2$

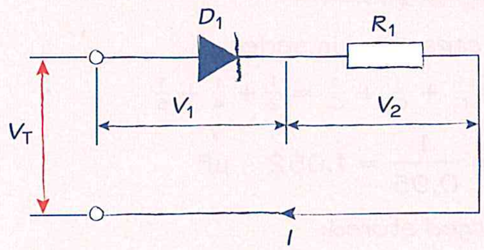


Resistor connected in **series** with a forward biased silicon diode.

Worked example

A simple circuit contains a typical silicon diode and resistor connected in series. The voltage drop across the diode is 0.7 V. The supply voltage is 12 V DC and $R_1 = 3.3\,k\Omega$.

Find the current flowing in the circuit.



Sample response extract

$V_T = 12\,V$, $V_1 = 0.7\,V$:

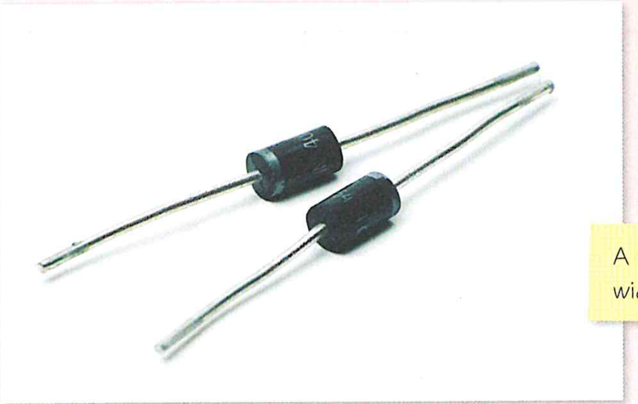
$V_2 = 12 - 0.7 = 11.3\,V$

From Ohm's law:

$$I = \frac{V_2}{R_1} = \frac{11.3}{3.3 \times 10^3} = 0.0034\,A \text{ (to 2 s.f.)}$$

Follow these steps:

1. The voltage drop for a silicon diode is 0.7 V.
2. From Kirchoff's voltage law: $V_T = V_1 + V_2$. Rearrange to make V_2 the subject and substitute known values to find V_2 across known resistor R_1.
3. Apply Ohm's law to find the current flowing through R_1, which will be the same throughout a series circuit.



A typical silicon diode used in a wide range of electronic circuits.

Now try this

A simple circuit contains a silicon diode and resistor connected in series. The supply voltage is 9 V DC and the circuit current is 6 mA. The voltage drop across the diode is 0.7 V.

Find the resistance of the series resistor.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Capacitors in series or parallel

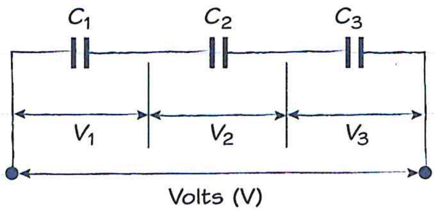
You need to know how to analyse circuits that contain capacitors connected together in series or parallel.

Capacitors in series

When capacitors are connected in **series** the **charge (Q)** stored by each capacitor is the same.

The total capacitance (C_T) for a number (n) of capacitors in series is $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \dots - \frac{1}{C_n}$.

Note that capacitors in series and parallel are treated differently from resistors in series and parallel.



Capacitors in series where charge (Q) is a constant.

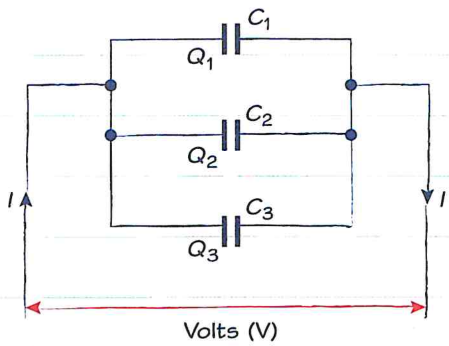
Capacitors in parallel

When capacitors are connected in **parallel** there is the **same voltage** across each capacitor.

The total capacitance (C_T) for a number (n) of capacitors in parallel is:

$C_T = C_1 + C_2 \dots + C_n$

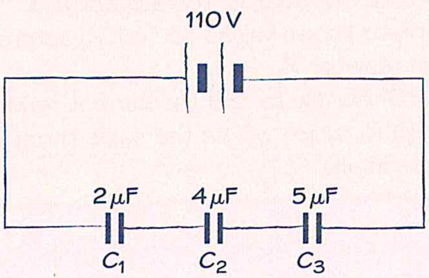
Capacitors in parallel where voltage (V) is constant.



Worked example

A simple circuit contains three capacitors connected in **series**, where $C_1 = 2\,\mu F$, $C_2 = 4\,\mu F$ and $C_3 = 5\,\mu F$. The circuit supply voltage $V = 110\,V$ DC.

Calculate the charge stored in each capacitor.



Sample response extract

For capacitors in series

$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{2} + \frac{1}{4} + \frac{1}{5}$

$C_T = \frac{1}{0.95} = 1.052\dots\,\mu F$

Charged stored:

$Q = CV$

$Q = 1.052\dots \times 10^{-6} \times 110 = 1.16 \times 10^{-4} C$

Each capacitor holds the same charge.

Follow these steps:

1. Annotate the circuit diagram.
2. Find the combined total capacitance of the series capacitors.
3. Use $Q = CV$ to find the charge required.
4. Remember that in a series circuit the charge stored in each capacitor is the same.

Now try this

A simple circuit contains three capacitors connected in **parallel**, where $C_1 = 2\,\mu F$ and $C_2 = 4\,\mu F$, $C_3 = 5\,\mu F$. The circuit supply current $V = 110\,V$ DC. Calculate the charge stored in each capacitor.

Capacitors in series and parallel combinations

It is common for electrical and electronic circuits to contain capacitors connected together in series and parallel combinations. It's important that you understand the characteristics of these networks.

Series and parallel combinations

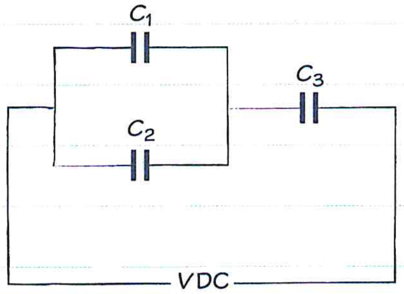
First calculate the equivalent resistance of the parallel element, using:

$C_P = C_1 + C_2$

Then use:

$$\frac{1}{C_T} = \frac{1}{C_P} + \frac{1}{C_3}$$

to find the total capacitance of the combination.

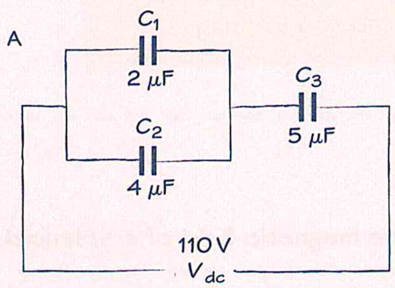


Capacitors connected in a series and parallel combination.

Worked example

A circuit, as shown, contains three capactitors in a series/parallel combination, where $C_1 = 2\,\mu F$, $C_2 = 4\,\mu F$ and $C_3 = 5\,\mu F$. The circuit supply voltage $V = 110\,V$ DC.

Calculate the total stored charge in the circuit.



Sample response extract

Simplifying parallel combination:

$C_A = C_1 + C_2 = 2 + 4 = 6\,\mu F$

Combining series elements of simplified circuit:

$$\frac{1}{C_T} = \frac{1}{C_A} + \frac{1}{C_3} = \frac{1}{6} + \frac{1}{5} = \frac{11}{30}$$

$C_T = 2.72\,\mu F$

Charge stored:

$Q = CV$

$Q = 2.72 \times 10^{-6} \times 110 = 2.99 \times 10^{-4}\,C$

Follow these steps:

1. Identify the parts of the circuit that can be simplified and annotate the circuit diagram.
2. Find the combined total capacitance of C_1 and C_2 in parallel. Call this C_A.
3. Find the combined total capacitance of C_A and C_3 in series.
4. Use $Q = CV$ to find the charge.

Now try this

Continue with the Worked example and find the charge stored in capacitor C_1.

Capacitors in series carry the same charge. Find the voltage across the parallel combination using $V = Q/C$. Find the charge in C_1 using $Q = CV$.

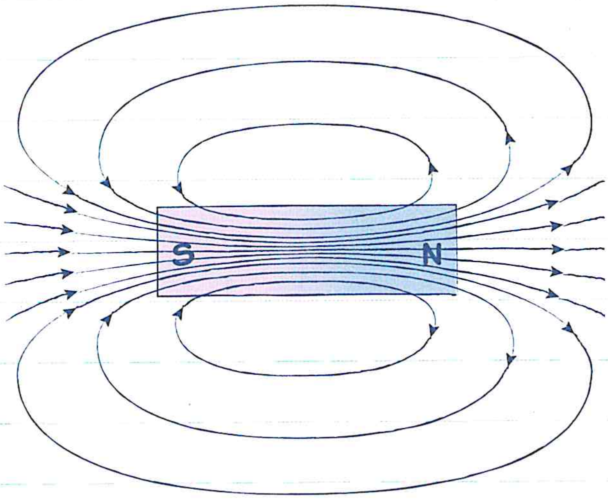
Had a look ☐ Nearly there ☐ Nailed it! ☐

Magnetism and magnetic fields

Magnetism underpins a huge range of important areas of engineering, such as electrical generators, motors, transformers and a host of other electro-mechanical devices.

Magnetic fields

Magnetic flux (Φ) is used to measure the total magnetic field produced by a source of magnetism. Units webers (Wb).



Lines of magnetic flux around a bar magnet.

Magnetic flux density

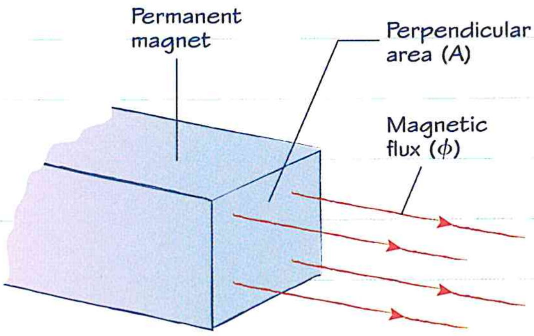
Magnetic flux density (the intensity of a magnetic field), in Wb/m² or tesla (T)

$$B = \frac{\Phi}{A}$$

Magnetic flux, in Wb

Area, in m²

Units Wb/m² or Tesla (T).



Lines of magnetic flux from the end of a bar magnet.

Ferromagnetic materials

- iron and steel
- can be magnetised by a permanent magnet or the magnetic field of a solenoid.

Solenoids

Magneto motive force (F_m)

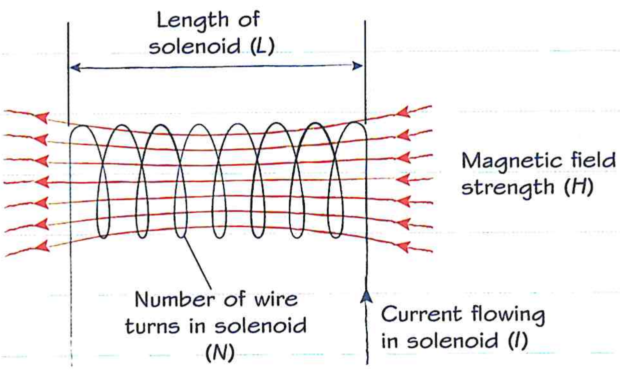
Magneto motive force (A)

$$F_m = NI$$

Number of turns

Electrical current through a solenoid (A)

Units A.



Magnetic field strength (H)

Magnetic field strength (A/m)

$$H = \frac{NI}{L}$$

Number of turns

Electrical current through a solenoid (A)

Length (m)

Units A/m¹.

Now try this

A solenoid is 25 mm in length and has 250 turns. When connected to a power supply, a current of 1.2 A flows through the solenoid.

Calculate the magnetic field strength generated inside the solenoid.

Permeability

Permeability is a measure of the degree of magnetisation a material undergoes when it is exposed to a magnetic field.

Permeability (μ)

The ratio of the magnetic flux density (*B*) generated inside a material and the external magnetic field strength (*H*) that causes the magnetising effect.

$$\boxed{\mu = \frac{B}{H}}$$

Permeability (H/m) — μ; Magnetic flux density (T) — B; Magnetic field strength (A/m) — H

Units H/m.

Permeability of free space (μ_o)

The theoretical ratio of magnetic flux density (*B*) and magnetising field strength (*H*) in a vacuum. This is a fixed constant and used as a benchmark to which other materials are compared.

$\mu_0 = 4\pi \times 10^{-7}$ H/m

The unit is the henry per metre.

Relative permeability (μ_r)

This compares the permeability (μ) observed in a given material to the permeability of free space (μ_0).

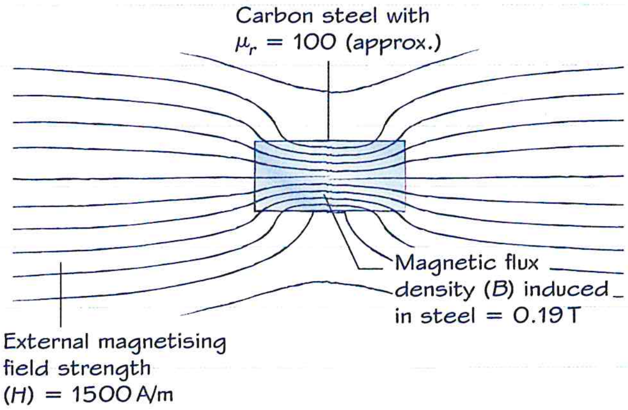
$$\mu_r = \frac{\mu}{\mu_0}$$

This relationship can also be expressed as:

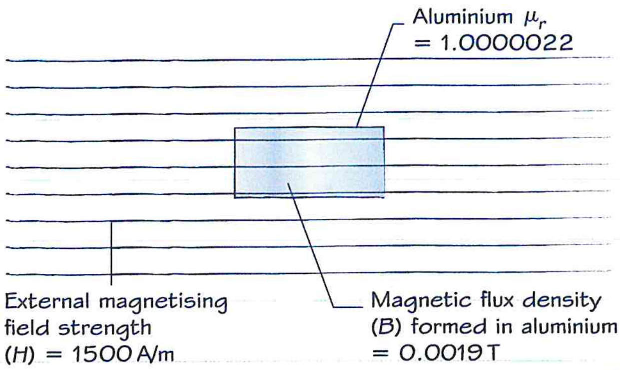
$$\frac{B}{H} = \mu_0 \mu_r$$

Values of the relative permeability (μ_r) of some common materials.

Medium	Relative permeability (μ_r)
Vacuum	1
Air	1.000 000 37
Wood	1.000 000 43
Aluminium	1.000 022
Carbon steel	100
Electrical steel	4000
Iron	5000
Permalloy	8000



In carbon steel the magnetising field *H* = 1500 A/m causes a magnetic flux density *B* = 0.19 T.



The same magnetising field in aluminium causes a much lower magnetic flux density *B* = 0.0019 T, which is little more than exists in the surrounding air.

Now try this

A bar of electrical steel with $\mu_r = 4000$ is placed inside a solenoid and exposed to a magnetic field strength which is $H = 2200$ A/m.

The permeability of free space, μ_0, is $4\pi \times 10^{-7}$ H/m.

Calculate the magnetic flux density (*B*) formed in the electrical steel.

Had a look ☐ Nearly there ☐ Nailed it! ☐

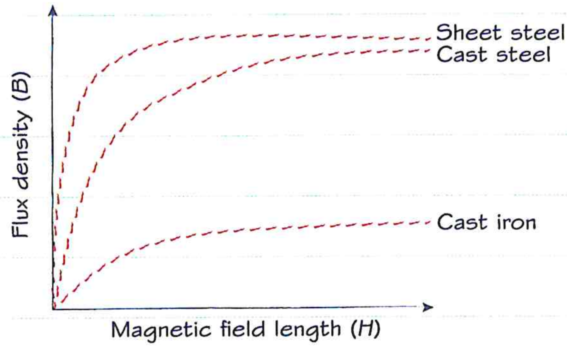
B/H curves, loops and hysteresis

It is important that you are aware of the relationship between the magnetic field strength (H) and magnetic flux density (B) in ferromagnetic materials where permeability is not a constant.

B/H curves in ferromagnetic materials

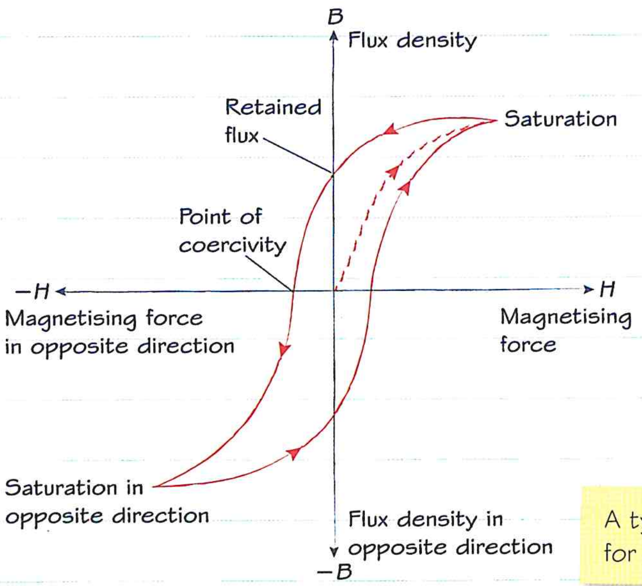
A B/H curve is simply a graph of the magnetic flux density (B) and applied magnetic field strength (H) for a given material.

These are not straight lines as you might expect. They are curves because permeability (μ) decreases as the applied magnetic field (H) increases.



Typical B/H curves for a range of ferromagnetic materials.

At low values of H where μ is high, a small increase in H leads to a large increase in B.
At high values of H, where the material approaches magnetic saturation, μ tends to 0 and an increase in H leads to no further increase in B.



1. A B/H loop shows the full magnetisation of a ferromagnetic material along the dotted line.
2. When the magnetising field H is reduced to zero, some of the flux B is retained.
3. When the magnetising field H is reversed, the retained flux B reduces to zero at the point of coercivity.
4. As the reversed magnetic field H increases, saturation in the opposite direction is reached.
5. Repeating this cycle causes a closed hysteresis loop to be formed. The area inside this loop is proportional to the energy lost as heat during the magnetisation cycle.

A typical B/H hysteresis loop for a ferromagnetic material.

Now try this

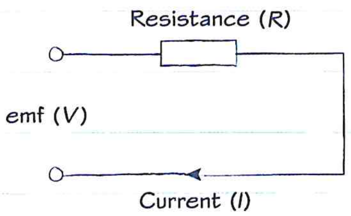
In many applications, ferromagnetic materials undergo the magnetisation cycle many times a second.
Explain the importance of specifying a ferromagnetic material with a narrow hysteresis loop when designing a transformer core.

Reluctance and magnetic screening

Some materials oppose the flow of magnetic flux. Reluctance (S) is used to describe this magnetic resistance. Units are 1/H.

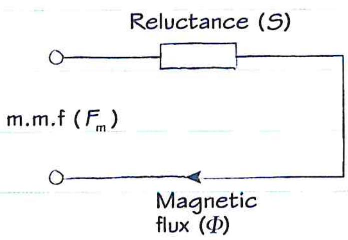
Analogy of reluctance and resistance

You can think of reluctance as being similar to electrical resistance:



$R = \frac{V}{I}$

Electrical resistance.



$S = \frac{F_m}{\Phi}$

Magnetic resistance.

Reluctance (S) can be expressed as the ratio of magneto motive force (F_m) to magnetic flux (Φ):

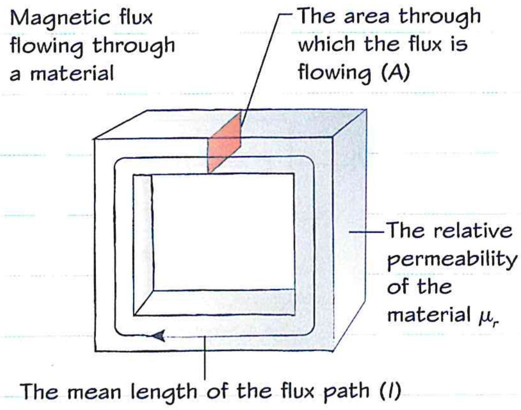
$$S = \frac{F_m}{\Phi}$$

Reluctance (S)

You can also express reluctance in terms of the characteristics of the material used in a magnetic circuit.

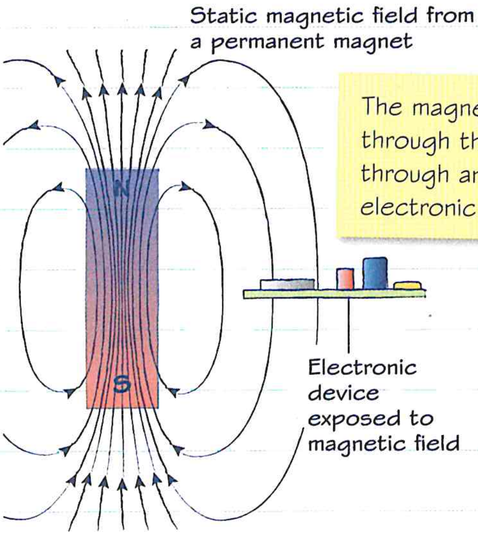
$$S = \frac{l}{\mu_0 \mu_r A}$$

μ_0 = permeability of free space



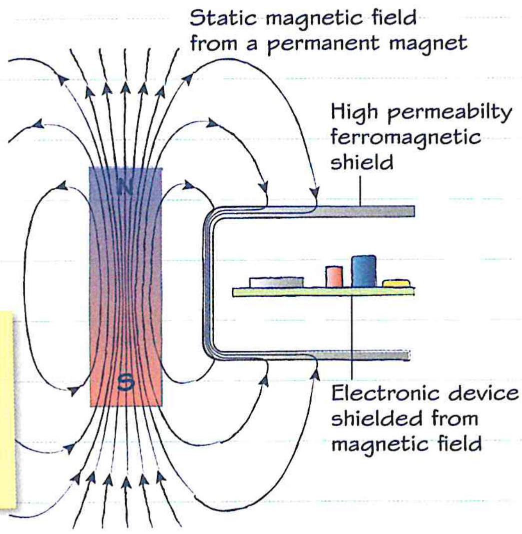
Magnetic screening

Often it is necessary to screen sensitive electronic components or devices from magnetic fields to prevent any unwanted effects. Effective screening can be achieved using ferromagnetic materials with low reluctance. These provide a pathway for lines of magnetic flux around the components being protected.



The magnetic field passes through the air and then through an unshielded electronic device.

A magnetic field finds it easier to pass through the low reluctance shield rather than high reluctance air.



Now try this

A steel circular core with relative permeability 190 has a mean length of 0.2 m and a cross-sectional area of 0.01 m^2. The permeability of free space is $4\pi \times 10^{-7}$ H/m.

Calculate the magnetic reluctance of the circuit.

Had a look ☐ Nearly there ☐ Nailed it! ☐

Electromagnetic induction

Electromagnetic induction is the key process in the generation of electricity.

Faraday's law of electromagnetic induction

The magnitude of the emf generated is given by Faraday's law:

'When a magnetic flux through a coil is made to vary, an emf is induced. The magnitude of this emf is proportional to the rate of change of flux.'

Lenz's law of electromagnetic induction

The direction of the induced current is given by Lenz's law:

'An induced current always acts in such a direction so as to oppose the change in flux producing the current.'

The emf induced in a coil

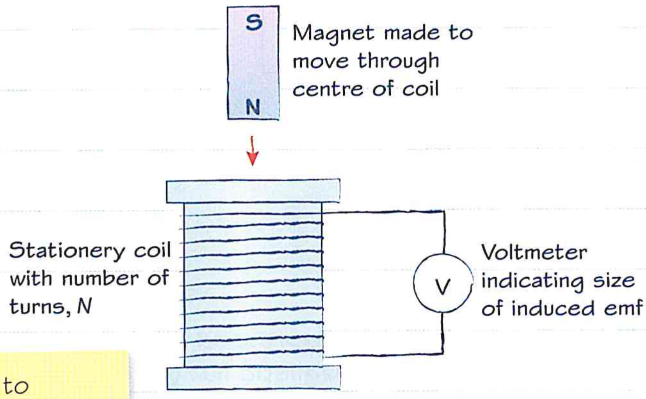
An emf is generated in a stationary coil by the movement of a permanent magnet, which provides the changing magnetic flux.

$$\boxed{E = -N\frac{d\Phi}{dt}}$$

emf induced (V)

Number of turns

Rate of change of magnetic flux (the –ve sign is a consequence of Lenz's law)



A simple experiment can be carried out to demonstrate electromagnetic induction in a coil.

The emf induced in a moving conductor

An emf is generated in a conductor moving through a stationary magnetic field at right angles to the lines of flux.

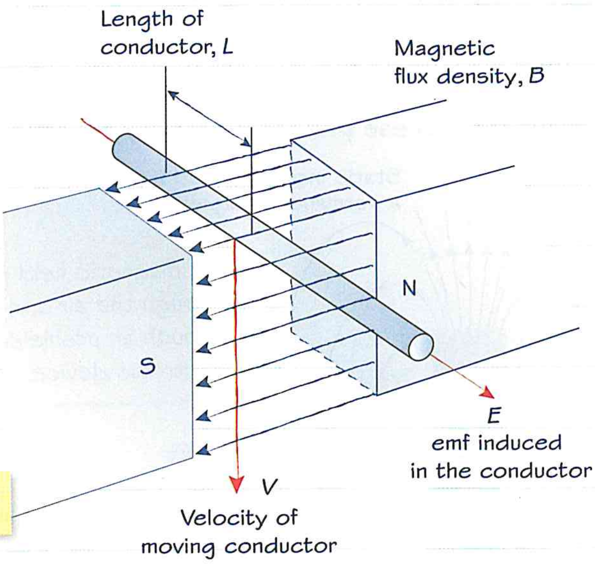
$$\boxed{E = BLv}$$

emf induced (V)

Velocity (m/s)

Length (m)

Flux density (T)



emf induced in a wire moving in a magnetic field.

Now try this

A conductor moving at right angles to a magnetic field with a flux density of 0.04 T at a velocity of 0.5 m/s generates an emf of 2.4×10^{-3} V.

Calculate the length of the conductor moving within the field.