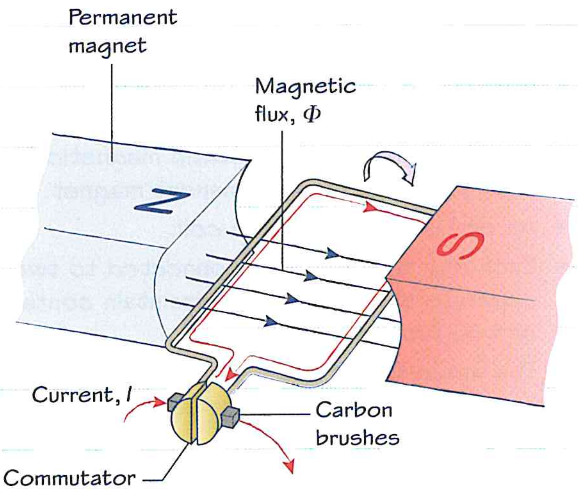
# DC motors

Motors convert electrical energy to mechanical rotation able to do work.

## The basic operation of a DC motor

- A permanent magnet provides a static magnetic field in which a coil is free to rotate.
- When current flows in the coil, the magnetic field generated opposes the static field from the magnets and the coil rotates until the fields are aligned.
- As this happens, the commutator reverses the current in the coil.
- This, in turn, reverses its magnetic field, causing the rotation to continue.

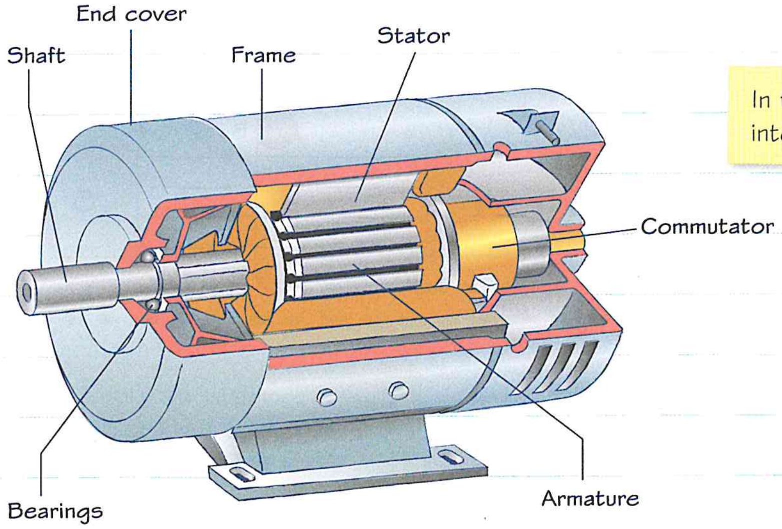



The DC motor. Current is fed through carbon brushes to a split ring, commutator, which reverses the current every half-turn.

## Industrial DC motors

In practice, motors are more refined. Some features of industrial motors include:

1. The rotating section, including the coils (or windings), commutator and drive shaft, is called an **armature** and is supported in bearings to allow free rotation.
2. The commutator is split into several segments, feeding current to several coils or windings in turn to provide smooth operation.
3. The stationary part of the motor, or **stator**, uses electromagnets to provide the static magnetic field, because permanent magnets have a limit to the size of magnetic field they can produce.




In this cut away you can see the internal parts of an industrial DC motor.

## Now try this

Explain the function of the commutator and the armature in a DC electric motor.

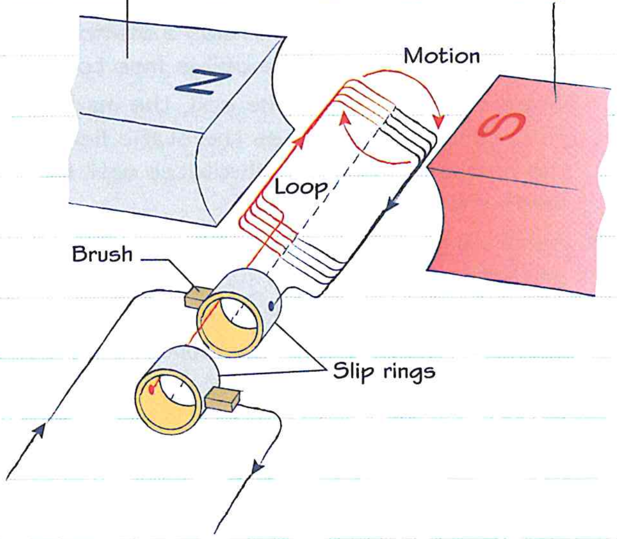
Had a look ☐ Nearly there ☐ Nailed it! ☐

# Electric generators

Electric generators are used to convert mechanical power, usually provided by a rotating turbine or engine, to electricity.

## Operation of an electric generator

- Generators rely on the principles of electromagnetic induction.
- A coil rotates inside a static magnetic field, here provided by a permanent magnet.
- An emf is induced in the coil.
- Each end of the coil is connected to two separate slip rings, which maintain contact with carbon brushes.
- The induced emf causes an electric current to flow between the brushes as the coil rotates.




This sketch illustrates the basic construction of an electric generator.

## Factors effecting induced emf

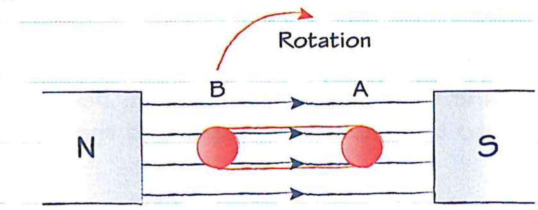
The magnitude of the emf generated is proportional to:

- the number of turns in the coil
- the speed of rotation
- the strength of the magnetic field in which the coil rotates
- the angle of the coil relative to the magnetic field in which it is rotating.

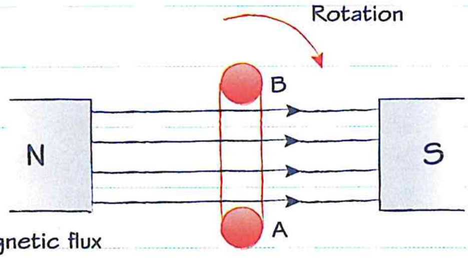
Max emf induced when coil is cutting through perpendicular lines of magnetic flux.

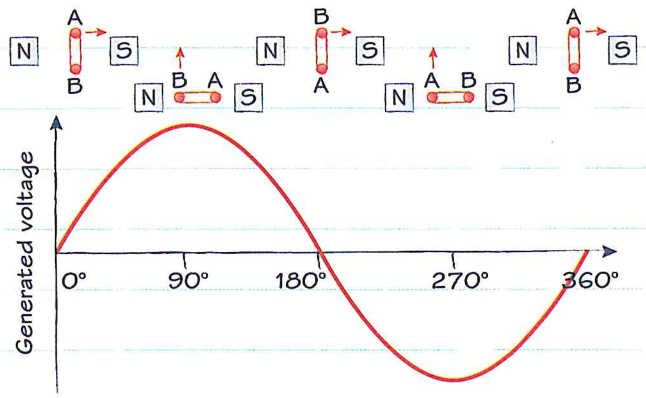
## Sinusoidal output

As the induced emf at any time depends on the angle at which the coil is cutting through the lines of magnetic flux, the output from this type of generator is sinusoidal.




Min emf (V = 0) when coil is moving parallel to lines of magnetic flux.




Over a complete rotation of the coil a single cycle of a sinusoidal waveform is generated. This waveform is better known as an alternating current (AC) and for this reason generators of this type are often called **alternators**.

## Now try this

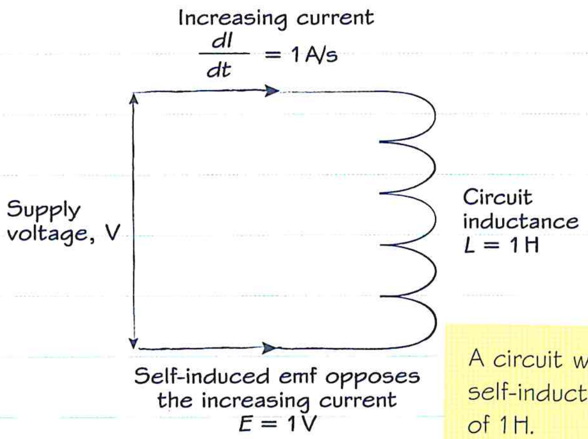
State three factors that increase the emf generated by an alternator.

# Inductors and self-inductance

Self-inductance occurs when the varying magnetic field generated by a change in the current flowing in a circuit induces an emf in the same circuit.

## Inductance ($L$)

Inductance is measured in henry (H), whereby 1 H is the inductance present in a circuit where a changing current of 1 A/s induces an emf of 1 V. Units H.




A circuit with self-inductance of 1 H.

## Electromotive force or emf ($E$)

$$\boxed{E = -L\frac{dI}{dt}}$$

emf (V); Inductance (H); Rate of change of current (A/s)

Units V.

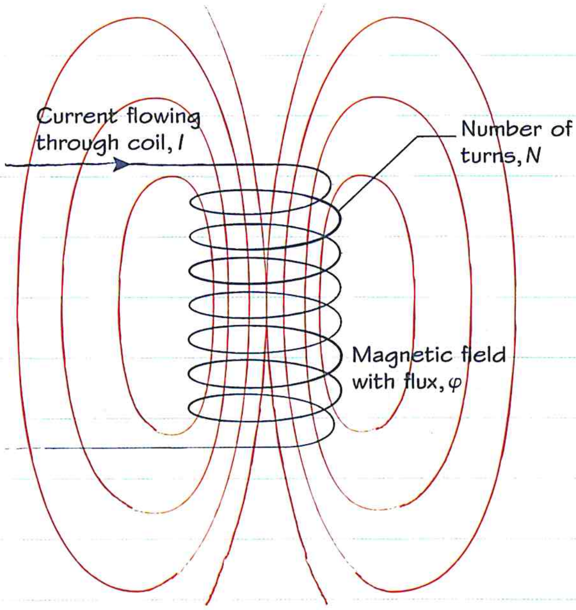
## Self-inductance in a coil ($L$)

$$\boxed{L = \frac{N\Phi}{I}}$$

Self-inductance (H); Number of turns; Magnetic flux generated (Wb); Current (A)

Units H.

A coil with self inductance $L$.



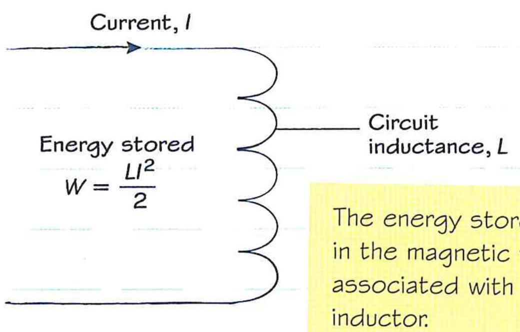

## Energy stored in an inductor ($W$)

When a flow of current is established in a conductor a magnetic field is also generated. The work done in creating this field is stored within it until the current stops flowing. As the current flow ceases the collapse of the magnetic field returns the stored energy as an induced emf.

$$\boxed{W = \frac{LI^2}{2}}$$

Energy stored (J); Inductance (H); Current (A)

Units J.




The energy stored in the magnetic field associated with an inductor.

### Now try this

An inductor with inductance 2.2 H passes a current of 0.3 A.
Calculate the energy stored in the inductor.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# Transformers and mutual inductance

Transformers use the principles of mutual inductance to change the voltage and current characteristics of an AC supply.

## Mutual inductance (*M*)

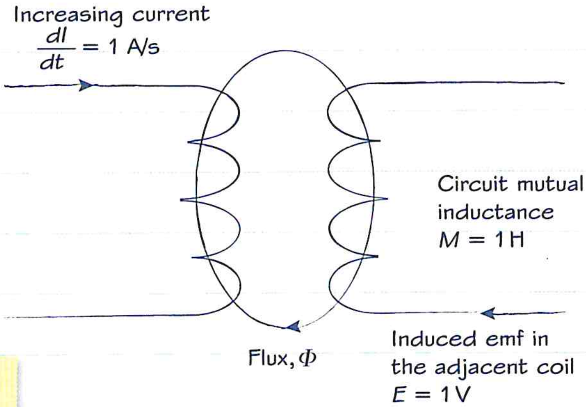
Mutual inductance occurs when the varying magnetic field generated by a change in the current flowing in one circuit induces an emf in an adjacent circuit.

$$\boxed{E = -M\frac{dI}{dt}}$$

emf induced (V)

Mutual inductance (H)

Rate of change of current (A/s)

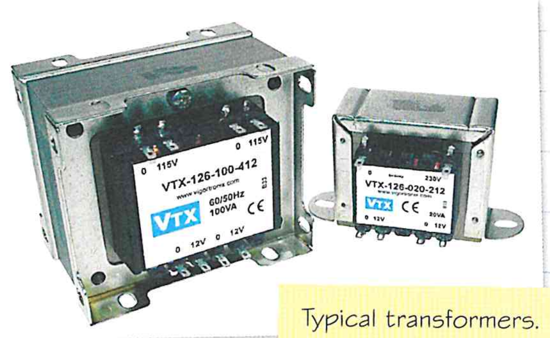



A circuit with mutual inductance of 1 H.

## Transformers

A transformer

- has two separate coils or windings
- laminated iron core reduces losses through eddy currents
- iron core minimises hysteresis losses.




Typical transformers.

## Important transformer calculations

For a primary coil with turns $N_1$ and voltage $V_1$ and a secondary coil with turns $N_2$ and voltage $V_2$, the transformer voltage ratio is given by:

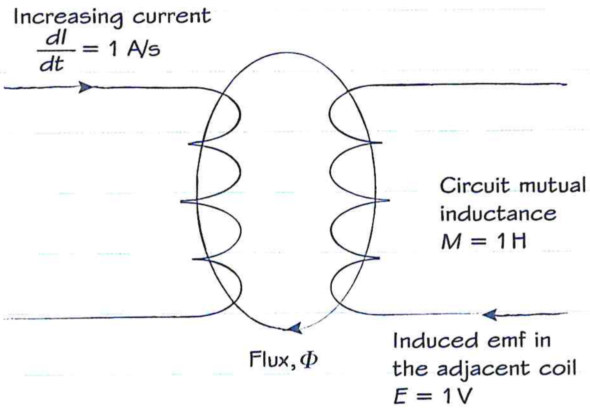
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

The transformer current ratio is given by:

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

A step-up transformer increases the supply voltage.

A step-down transformer reduces the supply voltage.




A schematic diagram of a transformer.

## Now try this

A transformer with a primary supply voltage of 230 V AC has a primary winding with 1200 turns and a secondary winding with 500 turns.

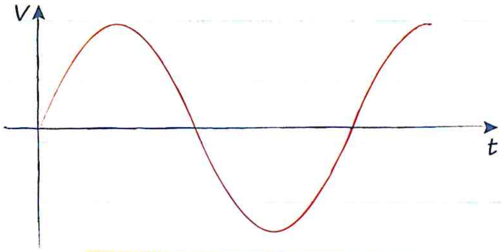
Calculate the voltage output from the secondary winding.

# AC waveforms

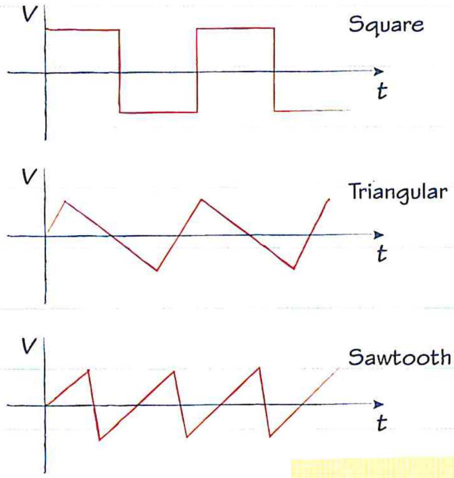
An alternating current can be analysed by looking at how voltage and current vary over time. The resulting graphs are known as waveforms.

## Sinusoidal waveforms

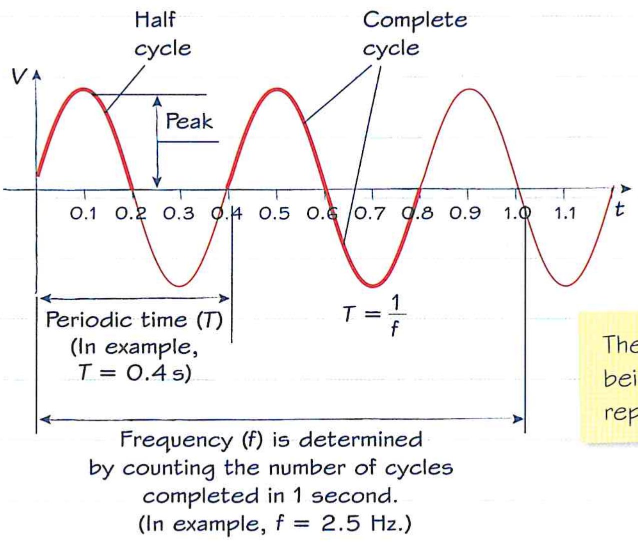
- the output from electrical alternators (generators)
- all mains electricity.




The sinusoidal waveform.




Non-sinusoidal waveforms.




The sinusoidal waveform is periodic, being made up of a series of repeating cycles.

## Now try this

A sinusoidal waveform has a periodic time of 0.016 s.
Calculate the frequency of the waveform.

# Single phase AC parameters

Different parameters are used to define the characteristics of a single phase AC supply.

## AC parameters

- **Peak voltage ($V_{peak}$)**

The maximum value of voltage or current reached in a positive or negative half cycle.

(For the example shown below $V_{peak}$ = 325 V.)

- **Peak-to-peak voltage**

The difference between the positive peak and the negative peak voltage in a full cycle.

(For the example shown below peak to peak voltage = 650 V.)

- **Root mean square (rms) voltage ($V_{rms}$)**

The equivalent DC voltage and how AC voltages are usually quoted. For a sine wave:

$$V_{rms} = \frac{1}{\sqrt{2}} V_{peak}$$

(For the example shown below $V_{rms}$ = 230 V.)

- **Average voltage ($V_{ave}$)**

The average of all the instantaneous measurements in one half cycle. For a sine wave:

$$V_{ave} = \frac{2}{\pi} V_{peak}$$

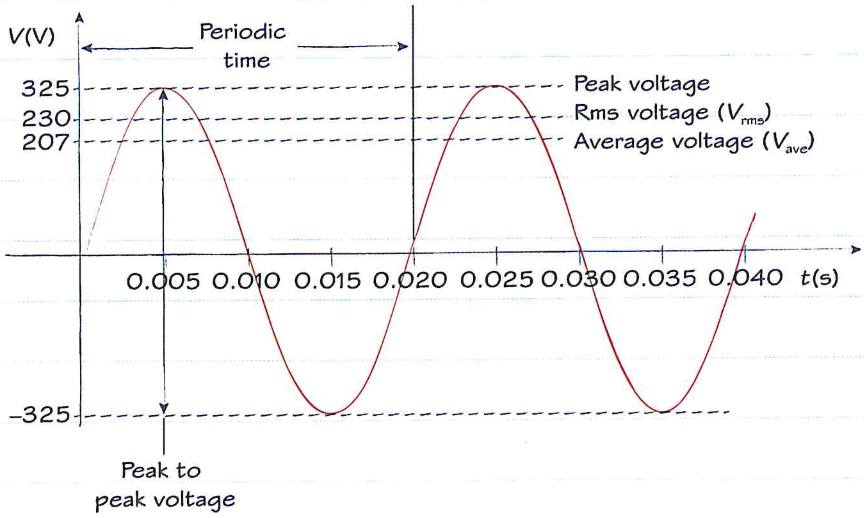
(For the example shown below $V_{ave}$ = 207 V.)

- **Form factor**

$$\text{Form factor} = \frac{V_{rms}}{V_{ave}}$$

For a sinusoidal waveform the form factor is a constant 1.11.

Note: These relationships are also true when considering current instead of voltage.




Example AC waveform illustrating 230 V 50 Hz supply used in UK mains.

## Now try this

Building sites use 110 V AC to power portable equipment, as this is less hazardous than the standard 230 V AC mains supply.

Calculate the peak voltage for a 110 V AC supply.

# Analysing AC voltages using phasors

You can represent a sinusoidal AC voltage as a sine wave on a graph. Another useful tool is to think of this wave in terms of a rotating vector or phasor.

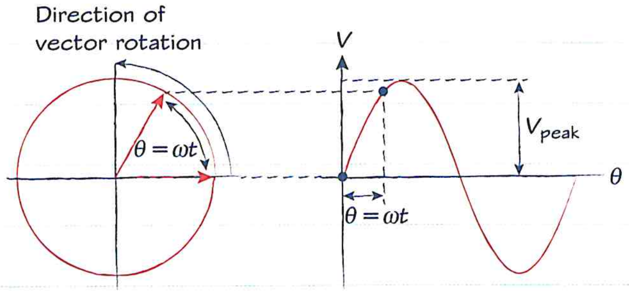
As a phasor rotates, its vertical height at any instant corresponds to the voltage at that point on the corresponding sine wave.

$$\boxed{V = V_{peak}\sin(\omega t + \phi)}$$

Voltage (in V) at time $t$ (s)

Phase angle (rad)

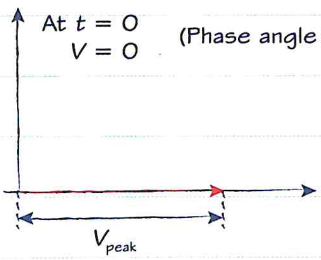
Angular velocity (rad/s) $\boxed{\omega = 2\pi f}$ Frequency (Hz)



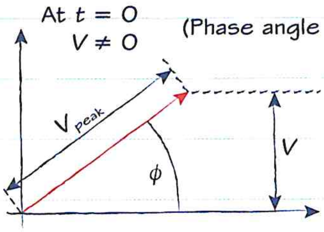

Rotating phasor with angular velocity $\omega$ rad/s

Corresponding sinusoidal waveform, where $V = V_{peak}\sin(\omega t + \phi)$

Using phasors to represent a sinusoidal waveform.



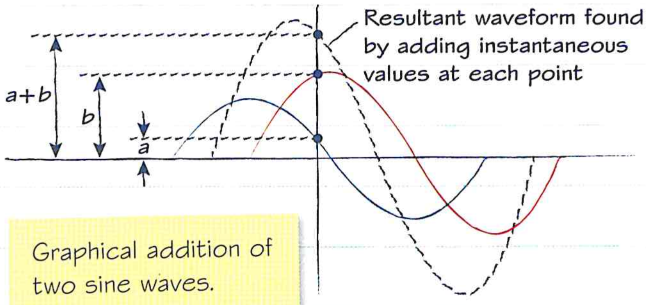

Phasor representing standard sinusoidal waveform, where $V = V_{peak}\sin(\phi t + \phi)$ and $\phi = 0$.




Phasor representing a sinusoidal waveform with a leading phase angle, where $V = V_{peak}\sin(\omega t + \phi)$ and $\phi \neq 0$.

## Graphical addition

Sinusoidal waveforms can be added together graphically. However, this is laborious and time consuming.

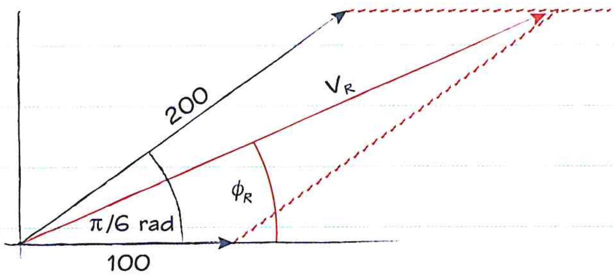



Graphical addition of two sine waves.

## Phasor addition

Sinusoidal waveforms of the same frequency can be added together using the vector addition of phasors.

For example, adding $v_1 = 100\sin(100\omega t)$ and $V_2 = 200\sin(100\omega t + \frac{\pi}{6})$ can be represented as:




The resultant peak voltage $V_R$ and phase angle $\phi_R$ can be determined using basic trigonometric methods.

**Links** Trigonometric methods for carrying out vector addition can be found on page 13.

### Now try this

Two sinusoidal voltages of the same frequency are represented by the equations $V_1 = 100\sin(100\omega t)$ and $V_2 = 200\sin(100\omega t + \frac{\pi}{6})$.

If these are combined, determine the equation for the resultant waveform.

**Links** For an example of how to solve phasor addition problems graphically, see page 76.

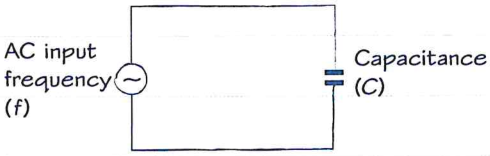
Had a look ☐ Nearly there ☐ Nailed it! ☐

# Reactance and impedance

In AC circuits the equivalent to resistance is impedance ($Z$), which is a combination of conventional resistance ($R$), capacitative reactance ($X_C$) and inductive reactance ($X_L$).

## Capacitative reactance ($X_C$)

**Capacitative** reactance is the opposition to the flow of alternating current exhibited by a capacitor.




Capacitative reactance (Ω)

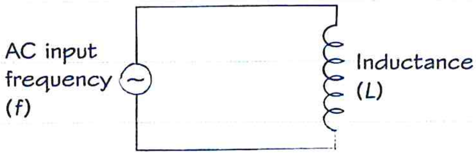
$$X_C = \frac{1}{2\pi f C}$$

Capacitance (F)

Frequency of AC (Hz)

## Inductive reactance ($X_L$)

**Inductive** reactance is the opposition to the flow of alternating current exhibited by an inductor.




Inductive reactance (Ω)

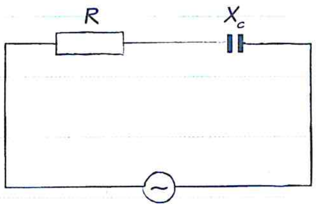
$$X_L = 2\pi f L$$

Inductance (H)

Frequency of AC (Hz)

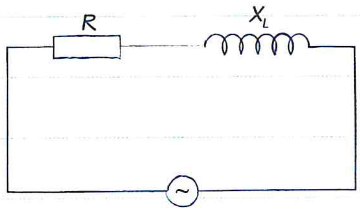
## Resistor/capacitor series circuit

In circuits containing a capacitor and a resistor in series then the total impedance ($Z$) is made of the resistance ($R$) and capacitative reactance ($X_C$).



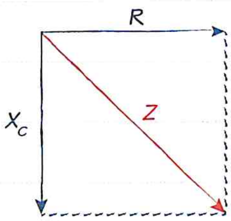

## Resistor/inductor series circuit

In circuits containing an inductor and a resistor in series then the total impedance ($Z$) is made up of the resistance ($R$) and the inductive reactance ($X_L$).




## Total impedance of a resistor/capacitor series circuit

In AC circuits resistance ($R$) and capacitative reactance ($X_C$) are vector quantities.

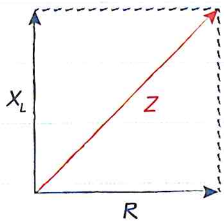



Total impedance ($Z$) is given by:

$$Z = \sqrt{X_C^2 + R^2}$$

## Total impedance of a resistor/inductor series circuit

In AC circuits resistance ($R$) and inductive reactance ($X_L$) are vector quantities.




Total impedance ($Z$) is given by:

$$Z = \sqrt{X_L^2 + R^2}$$

## Now try this

An engineer is testing an inductor with an inductance of 1.2 H and resistance 2.5 Ω. It is connected to a 230 V, 50 Hz AC supply.

(a) Calculate the impedance of the circuit.

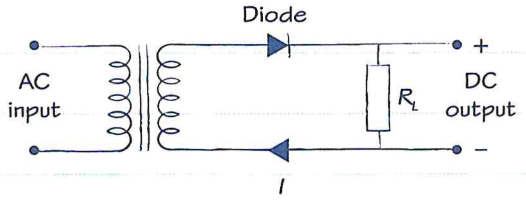
(b) Calculate the current drawn from the supply.

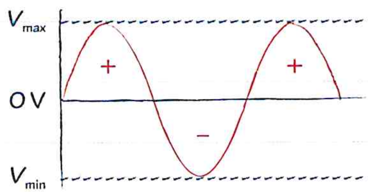
# Rectification

An important application of diodes is in the rectification of AC to DC. Rectifier circuits are commonly used to convert the low voltage AC output of a mains transformer to a DC voltage suitable for running electronic devices.

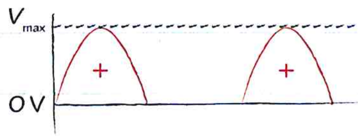
## Simple half wave rectifier

A simple circuit connected to the output of a transformer uses a single diode to provide half wave rectification by eliminating the -ve half cycle.



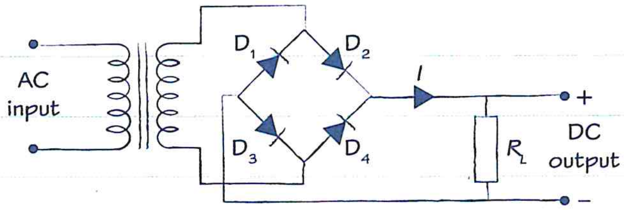

AC input.

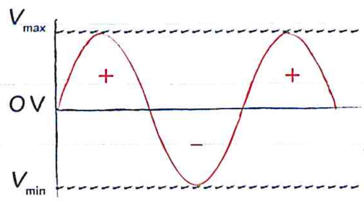



DC output with –ve half cycle eliminated.

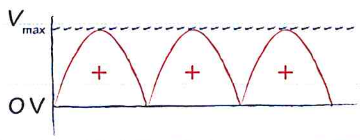
## Full wave bridge rectifier

A circuit connected to the output of a transformer uses a 4-diode bridge to provide full wave rectification making both half cycles +ve.



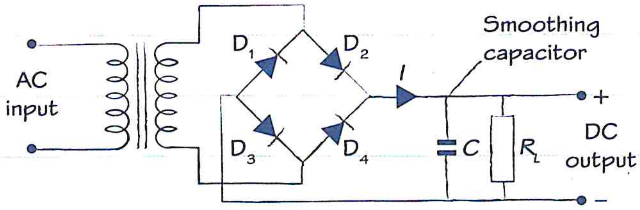

AC input.

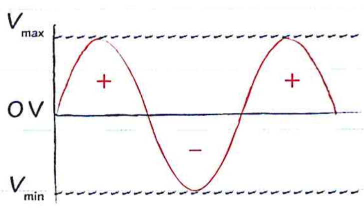



DC output with –ve half cycle inverted to become +ve.

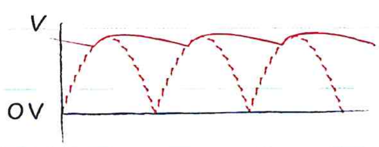
## Smoothed full wave bridge rectifier

The addition of a smoothing capacitor to the output of the rectifier reduces voltage ripple and provides a smoother DC output.




AC input.




Smoothed DC output.

## Now try this

Electricity distribution systems use high voltage AC (alternating current) to carry electricity over long distances between power stations and homes and businesses. However, many electronic devices require a much lower voltage DC (direct current) electrical supply in order to function.

Use notes and diagrams to explain how 240V AC mains electricity could be converted to a 5V DC supply suitable for use in a phone charger.

# Your Unit 1 exam

Your Unit 1 exam will be set by Pearson and could cover any of the essential content in the unit. You can revise the unit content in this Revision Guide. This skills section is designed to revise skills that might be needed in your exam.

## Exam checklist

Before your exam, make sure you:

- ✓ Have double-checked the time and date of your exam.
- ✓ Have a black pen you like and at least one spare.
- ✓ Have checked what you need to take into the exam; e.g. a ruler, protractor, pencil, scientific calculator that must not be programmable and that meets the requirements stated.
- ✓ Are prepared to show all your working using the appropriate units in your answers, to the appropriate degree of accuracy.
- ✓ Get a good night's sleep.

## Check the Pearson website

The questions provided in this section are designed to demonstrate the skills that might be needed in your exam. The details of the actual exam may change so always make sure you are up to date. Check the Pearson website for the most up-to-date **Sample Assessment Material** to understand the structure of your paper and how much time you are allowed.

**Exam skills**

- 'State' and 'describe' questions See example on page 70
- 'Explain' questions See an example on page 71
- 'Find' questions See an example on page 72
- 'Calculate' questions See examples on pages 73 and 74
- 'Solve' questions See an example on page 75
- 'Draw' questions Revise this skill on page 76
- 'Label' questions Revise this skill on page 77
- 'Name' and 'identify' questions See an example on page 78
- 'Convert' questions See an example on page 79
- Using the formulae booklet See page 80
- Showing your working See page 69

## Now try this

Visit the Pearson website and find the page containing the course materials for BTEC National Engineering. Look at the latest Unit 1 Sample Assessment Material (SAM) to get an indication of:

- the number of papers you have to take
- whether a paper is in parts
- how much time is allowed and how many marks are allocated
- what types of questions appear on the paper.

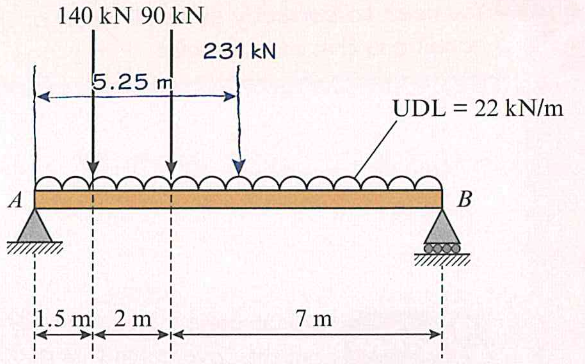
Your tutor or instructor may already have provided you with a copy of the Sample Assessment Material. You can use these as a 'mock' exam to practise before taking your actual exam.

# Showing your working

Here are some examples of the skills involved when showing your workings and methods in your mathematical calculations.

## Worked example

The supports for the loaded beams of a bridge must be able to support the load safely.




Calculate the reaction force at point $A$. **6 marks**

### Sample response extract

Find equivalent point force for UDL.

Magnitude of force:

$(1.5 + 2 + 7) \times 22 = 231$ kN

Position of force:

$$\frac{(1.5 + 2 + 7)}{2} = 5.25 \text{ m from point } A$$

Find support reaction at point $A$.

Take moments about $B$:

$R_A \times 10.5 = (140 \times 9) + (90 \times 7) + (231 \times 5.25)$

$R_A \times 10.5 = 1260 + 630 + \underline{\underline{1112.75}}$ Numerical error

$$R_A = \frac{3002.75}{10.5}$$

$R_A \times 285.98$ kN

If answering, for example, a simply supported beam question that includes a UDL (uniformly distributed load), add the equivalent point load to the sketch provided.

You should set out your working clearly and, where necessary, explain in words what you are doing at each stage.

The response to this question clearly shows the method followed. However, in this case, the final solution is incorrect because of a numerical error made in one of the calculations.

Despite the fact that the final answer is inaccurate, the correct method is clearly shown. So this would be reflected in the marking, as shown below.

## Final answers, methods and marks

In your exam, marks may be given for:

✓ your final answer ✓ the method used to reach the final answer. For example:

| Working | Answer | Notes |
|---|---|---|
| Taking moments about $B$:<br>$R_A \times 10.5 = (140 \times 9) + (90.7 \times 7) + (281 \times 5.25)$<br>$R_A \times 10.5 = 1260 + 630 + 1212.75$<br>$R_A = \frac{3102.75}{10.5}$<br>$R_A = 295.5$ kN<br>Up to 5 marks for method | $R_A = 295.5$ kN<br>1 mark for answer | M1 for recognising F=Fd<br>M1 for converting UDL to point load<br>M1 for balancing clockwise and anti-clockwise moments<br>M1 for correct substitution<br>M1 for rearranging to make Ra the subject.<br>A1 for value of $R_A$. |

## Now try this

Continue with the Worked example to calculate the reaction force at point $B$.

Links You could revise page 18 to help you.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# 'State' and 'describe' questions

Here are some examples of skills involved when answering '**state**' and '**describe**' questions.

## Worked example

A body in static equilibrium will remain at rest or in motion with constant velocity.

State all three conditions that must be met for a system of coplanar non-concurrent forces to be in static equilibrium.

3 marks

If answering a 'state' question, recall the specific facts required and state them clearly in your answer.

You need to correctly state all three of the conditions that must be met.

### Sample response extract

The sum of all the vertical components of the forces acting in the system is zero.

The sum of all the horizontal components acting in the system is zero.

The sum of all the moments acting about any point in the system is zero.

**Links** Look at page 16 to revise the content covered in this question.

## Worked example

A maintenance engineer strips down a DC electric motor in order to find and rectify a fault that is causing it to malfunction. They have traced the fault to worn brushes, which will need replacing.

Describe the function of the brushes in a DC electric motor.

2 marks

The first part of the question establishes the **engineering context** of the question so that you will know the subject area being considered.

### Sample response extract

A pair of stationary carbon brushes allow electrical current to be supplied to the coils in the rotating armature of the motor. They are spring-loaded and stay in contact with the commutator, which feeds current to each coil in turn as the motor spins.

Brushes are usually made of carbon and wear down gradually over the lifetime of the motor. Worn brushes are a common fault.

When answering a 'describe' question, you need to give a clear description of a particular component, feature or process. Here, the answer is structured in sentences and provides a good description of the function of the brushes.

Whenever possible you should use appropriate technical language in your answers. This example correctly uses the terms **armature** and **commutator** in connection with the function of brushes.

The second paragraph of this answer gives an indication of a good understanding of the practical aspects of using electric motors. However, you are unlikely to have time to add in extra detail in this way, so concentrate your efforts and write concise answers.

## Now try this

Friction is the main cause of energy losses in mechanical systems.

State one factor that can help reduce friction in a mechanical system.

**Links** You can revise friction on page 23.

Electrical generators or alternators are widely used to generate electricity in a wide range of applications.

Describe the function of the slip rings in a simple electric generator.

# 'Explain' questions

Here are some examples of skills involved when answering **explain** questions.

## Worked example

Electric motors are used widely to convert electrical energy into mechnical energy to run everying from extractor fans to trains. They are clean, reliable and able to operate at efficiencies of up to 90%.

Explain one form of mechanical loss that limits the efficiency of an electric motor. **2 marks**

When answering an 'explain' question you need to give a clear explanation relating to a particular component, feature or process.

## Sample response extract

Mechanical losses in electric motors are mainly due to friction. This occurs wherever surfaces move over one another, such as in the bearings that allow the armature of the motor to rotate. Friction losses take the form of waste heat.

The answer is structured in two useful parts.

1. The first sentence identifies a source of mechanical loss.
2. The rest of the answer expands on this to explain its cause, characteristics and effects.

Always write as neatly as possible and make sure your answers **make sense** when you read them back to yourself. They should be easy to read and understand.

Remember to use appropriate **technical language** in your answers wherever it is appropriate.

### Answering the question

'**Explain**' questions may require answers that are fairly short, as in this example, or a bit longer.

For example, you may be asked to explain two components, features or processes, instead of just one. If so, you can explain each one using the same approach shown here.

## Now try this

Transformers are used to change the voltage in an AC circuit.

Explain one cause of losses which effect the efficiency of transformers.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# 'Find' questions

Here are some examples of skills involved when answering **find** questions.

When answering a '**find**' question, determine the solution to a problem given in the information provided in the question. This might involve applying the particular technique or mathematical method mentioned in the question.

## Worked example

The radius, $r$, of an enclosed cylindrical tank with height of 0.5 m and total surface area of 5 $m^2$ is given by the equation:

$5 = 2\pi r^2 + \pi r$

A question may have a short scenario giving the **engineering context** for the question.

Use the quadratic formula to find the radius of the tank. **4 marks**

In this example, the equation has been recognised as a **quadratic equation**. The first step in finding the solution is to rearrange the equation into its standard form, which can be found in the formula booklet.

### Sample response extract

$0 = 2\pi r^2 + \pi r - 5$

$y = ax^2 + bx + c$

$a = 2\pi \qquad x = r$

$b = \pi$

$c = -5$

Writing down the values of coefficients $a$, $b$ and $c$ helps to **make the working clear**. This is helpful when giving an answer.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In this example, a reminder of the quadratic formula would be found in the **formula booklet** and copied into the solution, then substituting in the values for $a$, $b$ and $c$.

$$x = \frac{-\pi \pm \sqrt{\pi^2 + 40\pi}}{4\pi}$$

$$x = \frac{-\pi \pm 11.639}{4\pi}$$

$x = 0.676$ or $-1.176$

The quadratic formula will generate two possible solutions. You would need to decide whether one or both of these are relevant in the context of the question. In this case, it is not possible to have a negative radius and so this possible solution would be ignored.

Negative answers are not possible in this scenario.

Required radius = 0.676 m

You need to state your **final answer** clearly to an appropriate level of accuracy. This will usually be to 2 d.p. or 3 s.f.

If time allows, always **check your solution** by substituting it back into the equation given in the question.

## Now try this

The radius, $r$, of an enclosed cylindrical tank with height of 1 m and total surface area of 16 $m^2$ is given by the equation:

$16 = 2\pi r^2 + 2\pi r$

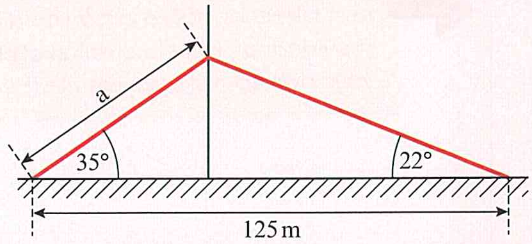
Use the quadratic formula to find the radius of the tank.

# 'Calculate' questions 1

Here are some examples of skills involved if answering '**calculate**' questions where you need to find the number or amount of something based on information given in the question.

## Worked example

The diagram shows a radio mast supported by a triangular system of cables. An engineer has made a ground level survey of the mast supports and measured the angles and distances shown in the diagram.




Calculate the length of cable *a*. **4 marks**

The question has a short scenario giving the **engineering context** for the calculation, and you need to focus on the application of a mathematical method or formula.

You may find a small **diagram** is included to illustrate what is required in the question.

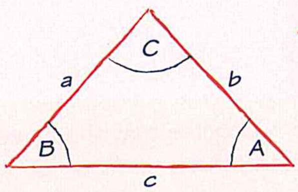
## Sample response extract

In this case:

$B = 35°$

$A = 22°$

$C = 125\,\text{m}$




You might find it useful to add your own labels or notes to a small **sketch** or the diagram itself as you work. In this example, labels have been added to help identify the sides and angles involved.

Angle $C = 180 - 35 - 22$

$C = 123°$

Using the sine rule:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

This question involves a non right-angled triangle and you must recognise that the sine rule can be applied to find *a* if the missing angle *C* can be determined. You will find a reminder of the sine rule in the **formula booklet**.

Rearrange:

$$a = \frac{\sin A \times c}{\text{Sin}C}$$

Substitute known values:

$$a = \frac{\sin 22 \times 125}{\text{Sin}123}$$

You should clearly organise your answer including a short **explanation** at each stage so that the solution **is easy to follow**.

$a = 55.83$ (to 2 d.p)

The length of cable *a* is 55.83 m.

You should state your final answer **clearly**, to an appropriate level of **accuracy** and including the **correct units**.

## Now try this

Continue with the Worked example. Calculate the length of the second cable supporting the radio mast.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# 'Calculate' questions 2

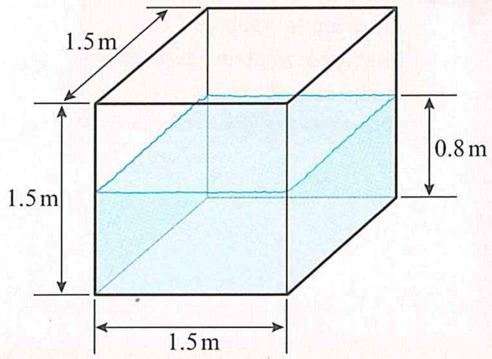
Here are some examples of skills involved if answering '**calculate**' questions where you need to find the number or amount of something based on information given in the question and, in addition to applying a mathematical method or formula, you also use additional subject knowledge. Some questions may require longer answers than others. The example below is of a fairly short answer.

## Worked example

A steel tank has dimensions 1.5 m x 1.5 m x 1.5 m. The tank is used to store oil in a vehicle maintenance workshop.

A depth gauge indicates an oil level of 0.8 m inside the tank.

Assume the density of the oil is 860 kg/m$^3$.




Calculate the hydrostatic thrust on one wall of the tank. **6 marks**

The question has a scenario giving the **engineering context** for the question.

You might find it useful to add your own labels or notes onto the diagram provided, or make a small **sketch** of your own to make things clear.

### Sample response extract

From the formula booklet

$F = \rho gAx$

$\rho = 860\ \text{kg/m}^3$

$g = 9.81\ \text{m/s}^2$

$A = 0.8 \times 1.5 = 1.2\ \text{m}^2$

$x =$ the height where the average pressure is found $= \dfrac{h}{2}$

$F = 860 \times 9.81 \times 1.2 \times \dfrac{0.8}{2}$

$F = 4049.568\ldots$

$F = 4050$ N

In this example, you must recognise that this question concerns the forces acting on an immersed plane surface. You will find the applicable formula in the Fluid principles section of the **formula booklet**.

Your answer should **list** the variables used in the formula, **identify** each one and **establish** its value from the information given in the question. This will make your next steps far more straightforward.

Once you have established values for each variable, these should be substituted into the formula. By **writing this out carefully** you will find it far easier to go back and spot any mistakes that you might have made at this stage.

You should state your answer **clearly** using an appropriate level of accuracy and units. As a rule of thumb numerical answers are usually given to 2 d.p. or 3 s.f.

## Now try this

A steel tank has dimensions 1.2 m x 1.2 m x 1.2 m. The tank is used to store kerosene to run a heating system. A depth gauge indicates the tank is half full. Assume the density of the kerosene is 810 kg/m$^3$.

Calculate the hydrostatic thrust on one wall of the tank.

# 'Solve' questions

Here are some examples of skills involved when answering '**solve**' questions.

## Worked example

The tension in a drive belt at either side of a powered rotating pulley can be represented by the following equation:

$300 = 100e^{\pi\mu}$

where $\mu$ is the unknown coefficient of friction between the belt and pulley materials.

Solve the equation to find $\mu$. Show evidence of the use of logarithms in your answer.

5 marks

Although you are given an engineering context, this type of question is about your ability to apply **mathematical methods** to **solve** the given equation.

The question asks you to refer to a **particular mathematical method** so you must ensure you show evidence of this. Use the information in the engineering context and question to inform your approach to the question.

## Sample response extract

$300 = 100e^{\pi\mu}$

Take natural logs of both sides:

$\ln 300 = \ln 100e^{\pi\mu}$

Use the rule $\ln AB = \ln A + \ln B$:

$\ln 300 = \ln 100 + \ln e^{\pi\mu}$

Use the rule $\ln A^x = x \ln A$:

$\ln 300 = \ln 100 + \pi\mu \ln e$

Since $\ln e = 1$ then:

$\ln 300 = \ln 100 + \pi\mu$

Rearrange:

$$\frac{(\ln 300 - \ln 100)}{\pi} = \mu$$

$\mu = 0.350$

Check answer:

$100e^{(\pi \times 0.350)} = 300$

In this example, the question has been correctly understood to need the use of natural logarithms to solve the equation. The rules for manipulating logarithms are needed, as given in the **formula booklet**.

The **rule** that is being applied at **each stage** of the answer is being **correctly written down**. This makes it absolutely clear that an appropriate method is being followed and helps you to keep track of your calculations.

An important element of this question is that you recall that $\ln e = 1$. This is not in the formula booklet.

The final answer is **stated clearly** to an appropriate level of accuracy. This will usually be 3 s.f. or 2 d.p.

As well as marks for an **accurate solution**, marks are also available for applying the **correct method**. These can be awarded only if you **show your working clearly**.

If time allows, always **check your answer** by substituting your final solution back into the equation given in the question.

## Now try this

The tension in a drive belt at either side of a powered rotating pulley can be represented by the following equation:

$350 = 100e^{2\pi\mu}$

where $\mu$ is the unknown coefficient of friction between the belt and pulley materials.

Solve the equation to find $\mu$. Show evidence of the use of logarithms in your answer.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# 'Draw' questions

Here are some examples of skills involved when answering '**draw**' questions.

## Worked example

An electrical engineer is working on a system that combines two AC voltages with the same frequency into a combined output. The two AC voltage waveforms can be represented by:

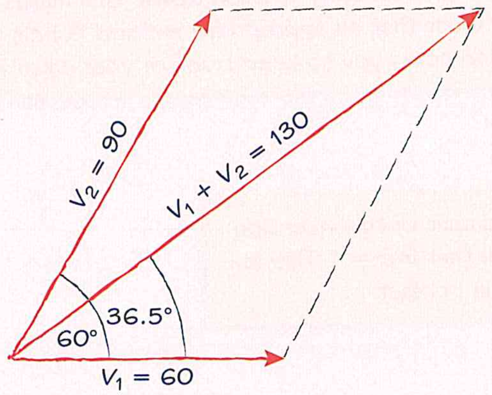
$V_1 = 60 \sin(80\omega t)$

$V_2 = 90 \sin(80\omega t + \frac{\pi}{3})$

Draw a phasor diagram to represent $V_1 + V_2$ and find the magnitude and phase angle of the resultant phasor. 6 marks

## Sample response extract

$V_2$ phase angle $\frac{\pi}{3}$ is equivalent to 60°.
Scale used: 1 cm to 10 V




By measurement from diagram:

Magnitude of resultant phasor = 130 V

Phase angle (above horizontal) = 36.5°

### Diagrams and graphs

In response to 'draw' questions, you might need to create a graph or diagram. This should be done accurately and to the appropriate scale. You might also need to label the diagram and provide clear annotations.

You can solve questions involving vectors or phasors **analytically** using trigonometry or **graphically** using an accurately drawn diagram. Here, the command verb '**draw**' tells you that the **graphical** approach is required.

To allow measurements to be taken from the diagram an **appropriate scale** needs to be chosen. This allows the drawing to fit comfortably in the space provided. In this example, a scale has been used of 1 cm to 10 V.

It is vital you use a **ruler** and **protractor** when drawing diagrams and that you work as accurately as possible. This example allows the use of a standard protractor by converting angles given in radians to degrees.

Take any measurements from the diagram carefully and accurately. You need to convert your measurements to the **units** required in the solution according to the scale you have chosen.

Although the question may not specifically ask that the diagram be labelled, if you correctly provide appropriate labels and annotation, it will help to make your **working clear**.

## Now try this

An electrical engineer is working on a system that combines two AC voltages with the same frequency into a combined output.

The two AC voltage waveforms can be represented by:

$V_1 = 90 \sin(120\omega t)$

$V_2 = 80 \sin(120\omega t - \frac{\pi}{3})$

Draw a phasor diagram to represent $V_1 + V_2$ and find the magnitude and phase angle of the resultant phasor.

# 'Label' questions

Here are some examples of skills involved when answering **'label'** questions.

## Worked example

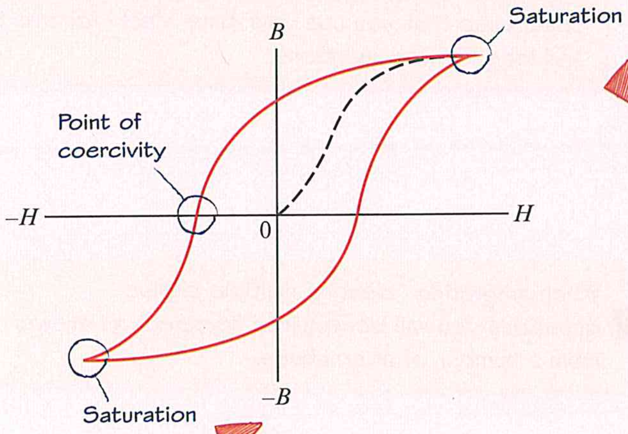
An electrical engineer is working on the designs for a transformer and is studying the *B*/*H* loop for a ferromagnetic material used in the transformer core.

On the graph, label the points at which the material reaches magnetic saturation and the point of coercivity. **2 marks**

When answering 'label' questions, you need to mark a particular feature or component on a diagram or sketch.

## Sample response extract

*B*/*H* diagram for ferromagnetic material



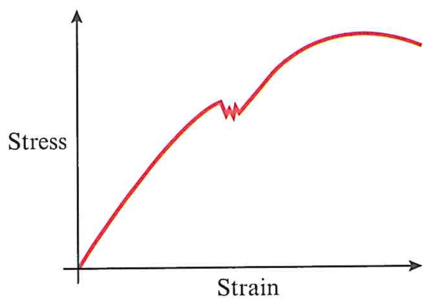

You need to label all the characteristics asked for in the question.

You need to clearly identify points or components with a leader line to the label.

## Now try this

An engineer is working on the designs for a structural tie bar and is studying the results of a tensile test on the component.

On the graph, label the point of failure, UTS and limit of proportionality.



# 'Name' and 'identify' questions

Here are some examples of skills involved when answering '**name**' and '**identify**' questions.

## Worked example

A well-designed electrical transformer can achieve an efficiency of between 95% and 98.5%.

Name two types of core losses that limit transformer efficiency. **2 marks**

### Sample response extract

Circulating eddy currents.

Hysteresis losses.

The first part of a question may establish the **engineering scenario** around which it is based.

Here, the question is testing your knowledge of transformer losses. Note that the question asks you to consider **core losses only**. You do not need to mention the ohmic losses caused by the resistance of the copper windings.

When answering 'name' questions, you need to recognise and recall the identity of particular features or processes. Keep your answers concise so that you use your time wisely for short and long answer questions.

---

## Worked example

Identify the term which describes the turning effect of a force about a point of rotation. **1 mark**

- **A** Moment ☑
- **B** Uniformly distributed load ☐
- **C** Thrust ☐
- **D** Angular momentum ☐

When answering 'identify' multiple choice questions you will be required to select an answer from a number of alternatives.

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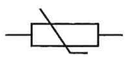
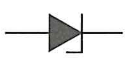
## Now try this

An electrical maintenance engineer is repairing a printed circuit board and is studying the corresponding circuit diagram.

Identify the following components that appear on the circuit diagram.

1 

2 

3 

# 'Convert' questions

Here are some examples of skills involved when answering 'convert' questions.

## Worked example

An engineer wants to use the SUVAT equations to analyse the motion of a train whose velocity is given in km/h.

Convert 178 km/h into m/s. **2 marks**

### Sample response extract

There are 1000 m in 1 km.

So 178 km/h × 1000 = 178 000 m/h

There are 3600 s in 1 hour.

So 178 000 m/h = 178 000/3600 = 49.4 m/s

36 km/h is equivalent to 49.4 m/s.

It is often necessary to convert values into the standard units used in equations. In this example SUVAT equations only work when velocity is stated in m/s.

When converting one unit into another break it down and carry out calculations one step at a time.

## Worked example

A engineer wants to calculate the area of a sector of a circle which subtends an angle of 126° using the formula $A = \frac{1}{2}r^2\theta$

Convert 126° into radians. **2 marks**

### Sample response extract

There are $2\pi$ radians and 360° in a full circle.

So, $\theta$ (radians) = 126/360 × $2\pi$ = 2.20

126° is equivalent to 2.20 radians.

You have to remember that the angles used in many engineering formulae must be stated in radians.

The relationship between degrees and radians is given in the formulae booklet.

Remember that $2\pi$ = 6.28. If you're converting an angle between 0 and 360° your answer will be between 0 and 6.28 radians. If not, then go back and check your calculations.

## Worked example

An engineer wants to calculate the stress in a loaded member with cross-sectional area 250 mm$^2$.

Convert 250 mm$^2$ into m$^2$. **1 mark**

### Sample response extract

In 1 m there are 1000 mm.

This means that 1 m$^2$ (1 m × 1 m) is the equivalent of 1 000 000 mm$^2$ (1000 mm × 1000 mm).

So, area in m$^2$ = 250/1 000 000 = 0.00025 m$^2$

250 mm$^2$ is equivalent to 0.00025 m$^2$.

You have to remember that the lengths, areas and volumes used in engineering formulae must be stated in m, m$^2$ or m$^3$ respectively.

Here an area given in mm$^2$ must be converted to m$^2$. If you can't remember that 1 m$^2$ is the equivalent of 1 000 000 mm$^2$ then you can work it out as shown.

## Now try this

The formula for calculating angular velocity requires all angles to be stated in radians.

Convert 36° into radians.

Had a look ☐ Nearly there ☐ Nailed it! ☐

# Using the formulae booklet

During your exam for Unit 1 you will have access to a booklet containing a list of formulae and physical constants. You must become familiar with this booklet, know what is included and what you will need to remember. This is shown on pages 81–85 and is included in the Sample Assessment Material for Unit 1 on the BTEC National Engineering section of the Pearson website. Always check the booklet in the latest Sample Assessment Materials to **make sure you are up to date**.

## Formulae and constants

These include:

- rules of indices
- rules of logarithms
- trigonometric rules
- volume and area of regular shapes
- quadratic formula
- equations of linear motion with uniform acceleration
- stress and strain
- work, power, energy and forces
- angular parameters
- fluid principles
- static and DC electricity theory
- capacitance
- magnetism and electromagnetism
- single phase alternating current theory.

## Compound unit styles

Fundamental quantities such as mass, length or time have their own units. These are usually stated in their abbreviated form, which would be kg, m and s, respectively. Calculated quantities such as density, velocity or acceleration have what are known as compound units that are stated as a combination of fundamental units. You need to be familiar with two styles of expressing compound units. For example, in your Unit 1 exam and throughout this Revision Guide:

✓ The units for density are given as $kg/m^3$; elsewhere you may see this expressed as $kg\,m^{-3}$.

✓ The units for velocity are given as m/s; elsewhere you may see this expressed as $m\,s^{-1}$.

✓ The units for acceleration are given as $m/s^2$; elsewhere you may see this expressed as $m\,s^{-2}$.

The two styles can be used interchangeably and there is no right or wrong method, although you should be consistent.

## Uses

1. Having the formulae booklet available means you can concentrate on applying your knowledge rather than remembering a large number of new formulae.
2. Use the booklet to check the formula you need. Don't get a question wrong just because you remembered a formula incorrectly.
3. If you're stuck, look through the list of formula; you may find it reminds you of the knowledge that you need.

## Limitations

1. You won't be awarded any marks for remembering a formula when it is given in the booklet.
2. Many of the straightforward relationships, like trigonometric ratios, for example, are not included. You'll be expected to remember these.
3. When learning a topic do this alongside the formula booklet. Make a note of any formulae that are not included. You'll need to learn these.
4. Make sure you are familiar with the letters and symbols used in each of the formulae. The booklet does not define any of these terms. To be able to use the formulae you must learn how they are applied.

## Now try this

Use the Formulae and constant booklet (pages 81–85) to find the formula used to calculate the following. Define all the terms used.

1 Voltage decay on capacitor discharge $V_C$.

2 The force between two electrostatically charged particles, *F*. (Hint: Coulomb's law.)

3 Hydrostatic thrust on an immersed plane surface, *F*.

# Formulae and Constants

## Maths

### Rules of Indices

$a^m \times a^n = a^{(m+n)}$

$a^m \div a^n = a^{(m-n)}$

$(a^m)^n = a^{mn}$

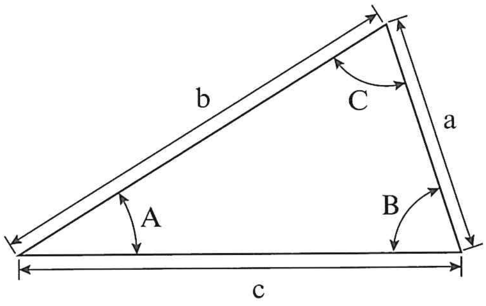
### Rules of Logarithms

$log\ AB = logA + logB$

$log\ \frac{A}{B} = logA - logB$

$logA^x = xlogA$

### Trigonometric rules




### Sine rule

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC} \text{ or } \frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

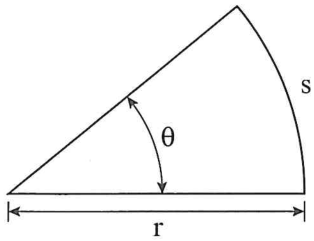
### Cosine rule

$a^2 = b^2 + c^2 - 2bc \cos A$

**Formulae and constants**

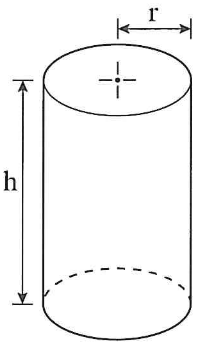
The formulae and constants booklet (pages 81–85) is in the Unit 1 Sample Assessment Material on the BTEC Nationals Engineering page of the Pearson website. Always check the website to ensure you are up to date.

**Volume and area of regular shapes**



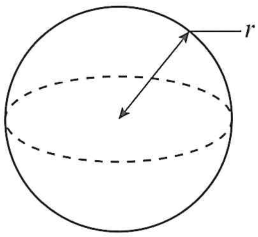

length of an arc of a circle $s = r\theta$ (where θ is expressed in radians)

area of a sector of a circle $A = \frac{1}{2}r^2\theta$ (where θ is expressed in radians)



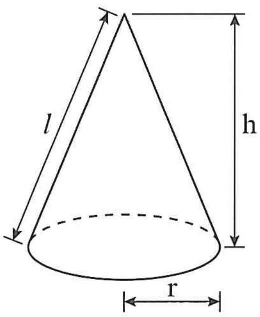

volume of a cylinder $v = \pi r^2 h$

total surface area of a cylinder $TSA = 2\pi rh + 2\pi r^2$




volume of sphere $v = \frac{4}{3}\pi r^3$

surface area of a sphere $SA = 4\pi r^2$




volume of a cone $v = \frac{1}{3}\pi r^2 h$

curved surface area of cone $CSA = \pi r l$

**Quadratic Formula**

To solve $ax^2 + bx + c = 0, a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Physical constants

| | |
|---|---|
| Acceleration due to gravity | $g = 9.81\ m/s^2$ |
| Permittivity of free space | $\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$ |
| Permeability of free space | $\mu_0 = 4\pi \times 10^{-7}\ H/m$ |

### Equations of linear motion with uniform acceleration

v = final velocity, u = initial velocity, a = acceleration, t = time and s = distance

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

### Stress and strain

| | |
|---|---|
| Direct stress | $\sigma = F/A$ |
| Direct strain | $\varepsilon = \Delta L/L$ |
| Shear stress | $\tau = F/A$ |
| Shear strain | $\gamma = a/b$ |
| Young's Modulus (modulus of elasticity) | $E = \sigma/\varepsilon$ |
| Modulus of rigidity | $G = \tau/\gamma$ |

### Work, Power, Energy and Forces

| | |
|---|---|
| Force | $F = ma$ |
| Components of forces | $F_x = F\cos\theta, F_y = F\sin\theta$ where $\theta$ is measured from the horizontal |
| Mechanical work | $W = Fs$ |
| Mechanical power | $P = Fv, P = W/t$ |
| Mechanical Efficiency | Efficiency $(\eta) = \frac{P_{out}}{P_{in}}$ |
| Force to overcome limiting friction | $F = \mu N$ (where N is the normal force) |
| Gravitational potential energy | $PE = mgh$ |
| Kinetic energy | $KE = \frac{1}{2}mv^2$ |

### Angular parameters

| | |
|---|---|
| Centripetal acceleration | $a = \omega^2 r$ or $a = v^2/r$ |
| Power | $P = T\omega$ |
| Rotational Inertia | $I = kmr^2$ |
| Rotational Kinetic energy | $KE = \frac{1}{2}I\omega^2$ |
| Angular frequency | $\omega = 2\pi f$ |
| Frequency | $f = \frac{1}{\text{time period}}$ |
| Radians to degrees conversion | $\theta_{(degrees)} = \frac{360\ \theta_{(radians)}}{2\pi}$ where $2\pi$ radians = 360° |
| Degrees to radians conversion | $\theta_{(radians)} = \frac{2\pi\ \theta_{(degrees)}}{360}$ |

## Fluid Principles

| | |
|---|---|
| Continuity of volumetric flow | $A_1v_1 = A_2v_2$ |
| Continuity of mass flow | $\rho A_1v_1 = \rho A_2v_2$ |
| Hydrostatic thrust on an immersed plane surface | $F = \rho gAx$ |
| Density | $\rho = m/V$ |

## Static and DC electricity theory

| | |
|---|---|
| Current/electron flow | $I = \frac{q}{t}$ |
| Coulomb's law | $F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$ |
| Resistance | $R = \frac{\rho l}{A}$ |
| Resistance: temperature coefficient | $\frac{\Delta R}{R_0} = \alpha\Delta T$ |
| Ohm's Law DC circuits | $I = \frac{V}{R}$ |
| Total for resistors in series | $R_T = R_1 + R_2 + R_{3\ldots}$ |
| Total for resistors in parallel | $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$ |
| Power | $P = IV,\ P = I^2R,\ P = \frac{V^2}{R}$ |
| Electrical Efficiency | Efficiency $(\eta) = \frac{P_{out}}{P_{in}}$ |
| Kirchoff's Current Law | $I = I_1 + I_2 + I_3 \ldots$ |
| Kirchoff's Voltage Law | $V = V_1 + V_2 + V_{3\ldots}$ or $\Sigma PD = \Sigma IR$ |

## Capacitance

| | |
|---|---|
| Electric Field Strength | $E = \frac{F}{q}$ or $E = \frac{V}{d}$ for uniform electric fields |
| Capacitance | $C = \frac{\varepsilon A}{d}$ |
| Time constant | $\tau = RC$ |
| Charge stored | $Q = CV$ |
| Energy stored in a Capacitor | $W = \frac{1}{2}CV^2$ |
| Capacitors in series | $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$ |
| Capacitors in parallel | $C_T = C_1 + C_2 + C_3 \ldots$ |
| Voltage decay on Capacitor discharge | $v_c = Ve^{\left(\frac{-t}{\tau}\right)}$ |

## Magnetism and Electromagnetism

| | |
|---|---|
| Magnetic Flux Density | $B = \frac{\phi}{A}$ |
| Magneto Motive Force | $F_m = NI$ |
| Magnetic Field Strength or Magnetising force | $H = \frac{NI}{l}$ |
| Permeability | $\frac{B}{H} = \mu_0\mu_r$ |
| Reluctance | $S = \frac{F_m}{\phi}$ |
| Induced EMF | $E = Blv,\ E = -N\frac{d\phi}{dt} = -L\frac{dI}{dt}$ |
| Energy stored in an inductor | $W = \frac{1}{2}LI^2$ |
| Inductance of a coil | $L = N\frac{\phi}{I}$ |
| Transformer equation | $\frac{V_1}{V_2} = \frac{N_1}{N_2}$ |

## Single Phase Alternating Current Theory

| | |
|---|---|
| Time Period | $T = \frac{1}{f}$ |
| Capacitive reactance | $X_C = \frac{1}{2\pi fC}$ |
| Inductive reactance | $X_L = 2\pi fL$ |
| Ohm's Law AC circuits | $I = \frac{V}{Z}$ (when voltage and current are in phase) |
| Root mean square voltage | $r.m.s.\ voltage = \frac{peak\ voltage}{\sqrt{2}}$ |
| Total impedance of an inductor in series with a resistance | $Z = \sqrt{X_L^2 + R^2}$ |
| Total impedance of a capacitor in series with a resistance | $Z = \sqrt{X_C^2 + R^2}$ |
| Waveform average value | $Average\ value = \frac{2}{\pi} \times maximum\ value$ |
| Form factor of a waveform | $Form\ factor = \frac{r.m.s.\ value}{average\ value}$ |

# Design triggers 1

Design triggers prompt the design of new or improved products or processes. They differ between **market-led** and **technology-led** companies, although in reality companies seldom use a single approach.

## Market pull

Market-driven companies are constantly researching their **customers' needs** and what their **competitors** are offering. Their aim is to follow market trends and develop **new** or **improved products** to fill an identified need.

The design and operation of online supermarket delivery services was triggered by market pull for a service.

## Technology push

Technology-driven companies concentrate on **research** and **development** to **push the boundaries** of what is possible in consumer products. They tend not to have an established market and so are not guaranteed success. However, when they get it right it can be revolutionary.

The inclusion of the first digital camera in a mobile phone was triggered by technology push in a product.

## Demand

Demand is the number of products that could be sold into a particular market.

- **Market-led** companies try to measure demand through research.
- **Technology-led** companies hope to stimulate demand by launching innovative new products.

## Profitability

Profitability is basically the **difference** between the **cost** of making a product and the **price** it can then be sold for. To maintain or increase profitability designers are often asked to redesign a product or service to reduce manufacturing or running costs.

## Innovation

Innovation is the **key to the success** of both **technology-** and **market-led** companies. It is required to ensure that products remain **competitive**. The use of emerging technology or just approaching a problem in an unconventional way can transform the performance of an existing product or lead to new products with better performance for less cost.


A series of innovations in bicycle design helped maintain the dominance of Great Britain's cycling squad at Rio 2016.

## Now try this

Explain whether the following rely mostly on market pull or technology push to trigger design activities:

1 high street clothes retailer
2 Formula 1 team
3 mobile phone manufacturer.