Q.4. ([illegible] points) A 200-V 2-pole separately excited dc motor drives a load with a <u>constant</u> torque demand of 191 Nm. The field winding has $N_f = 1000$ turns.

The magnetization characteristic of the machine is obtained at 1000 rpm, and is approximated by a straight line, $E_a = 200I_f$ Volts.

Armature resistance R_a is 0.2 Ω, friction and windage torque is negligibly small, and the field excitation is kept constant at $I_f = 1$ A throughout the problem.

(a) Show that the shaft speed versus torque characteristic of the separately excited dc motor operating at steady-state is given by: $\omega = \dfrac{V_t}{K_a\phi} - \dfrac{R_a}{(K_a\phi)^2}T$,
where ω is the angular shaft speed, V_t is the applied motor voltage, φ is the field flux per pole, K_a is the motor constant, and T is the torque.

(b) Calculate the no-load shaft-speed in rpm for $V_t = 200$ V.

(c) With the same terminal voltage, if, now, the constant-torque load of 191 Nm is applied to the shaft, what will the new shaft speed be? Neglect armature reaction. Comment on the result.

(d) If the speed is to be reduced to 450 rpm by armature voltage control, what should the new value of the applied motor voltage be? Briefly compare speed control techniques by varying the armature voltage and field current.

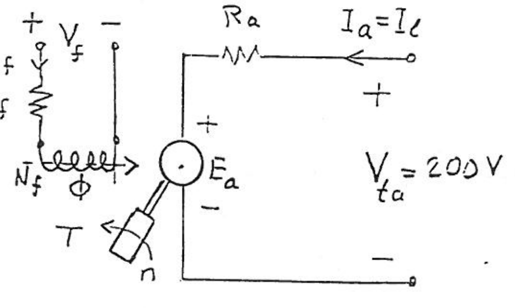
(e) Suppose now that the speed is to be increased to the no-load value found in part (b) while the machine is operating as in part (c) (V_t = 200 V, T_L = 191 Nm), by equipping the motor with a series field winding of negligible resistance. Determine the type of compounding and the number of turns of the series field winding to be inserted.

(f) Find the shaft-speed for the operating condition in part (c) (V_t = 200 V, T_L = 191 Nm) by assuming that the demagnetizing effect of the armature reaction is AR = $2I_a$ Ampere-turns. Briefly discuss the effect of armature reaction on the induced armature emf.

(g) Assume now that the applied motor voltage is suddenly reduced to 100 V, while the motor is operating as in part (c) (T_L = 191 Nm) with armature reaction neglected. Determine the directions and magnitudes of armature current, electromechanical torque and electromechanical power just after this operation by assuming that the speed remains momentarily constant. Also identify the mode of operation of the machine in this case.

<u>SOLUTION</u>

$R_a = 0.2\,\Omega$, $I_f = 1\,A = $ const, $E_a = 200 I_f$ at $n = 1000$,



a) $V_{ta} = R_a I_a + E_a$

$E_a = K_a \phi \omega$

$T = K_a \phi I_a \rightarrow I_a = \dfrac{T}{K_a\phi}$

$V_{ta} = R_a \dfrac{T}{K_a\phi} + K_a\phi\omega \rightarrow V_{ta} - R_a\dfrac{T}{K_a\phi} = K_a\phi\omega \rightarrow \omega = \dfrac{V_{ta}}{K_a\phi} - R_a\dfrac{V_{ta}}{(K_a\phi)^2}$

b) For $I_f = 1$ A, $n = 1000$ rpm $\rightarrow E_a = 200 \times 1 = 200$ V $= V_{ta}$ at no-load

Thus; $n_o = 1000$ rpm

c) At $n_o = 1000$ rpm $\Rightarrow \omega_o = 2\pi\dfrac{1000}{60} = 104.72$ and $I_f = 1$ A, $\mathcal{F}_f = N_f I_f = 1000$

$E_a = 200 \times 1 = K_a\phi_f\omega_o$, $K_a\phi_f = \dfrac{200}{104.72} = 1.91$, $T = K_a\phi_f I_a = 191$ Nm

$I_a = \dfrac{191}{1.91} = 100$ A, $E_a = K_a\phi_f\omega_o$, $V_{ta} = E_a + R_a I_a$, $E_a = 200 - 0.2 \times 100 = 180$ V

$I_f = 1\,A = \text{const}$, $E_a = 200\,V$ at $n = 1000$ rpm
$\quad$ " $\qquad\qquad\quad E_a = 180\,V$ at $n = ?$ $\Big]$ $n = 1000\,\dfrac{180}{200} = 900$ rpm

speed regulation $= \dfrac{1000-900}{900} = 11.1\%$. satisfactory.

d) $I_f = 1\,A$, $n = 1000$ rpm $\rightarrow E_a = 200\,V$
$I_f = 1\,A$, $n = 450$ rpm $\rightarrow E_a = ?$ $\Big\}$ $E_a = \dfrac{450}{1000} * 1000 = 90\,V$

$T = 191$ Nm $\longrightarrow I_a = 100\,A$ (part c) , $V_{ta} = E_a + R_a I_a = 90 + 0.2*100 = 110\,V$

e) $n = n_0 = 1000$ rpm , $V_{ta} = 200\,V$,

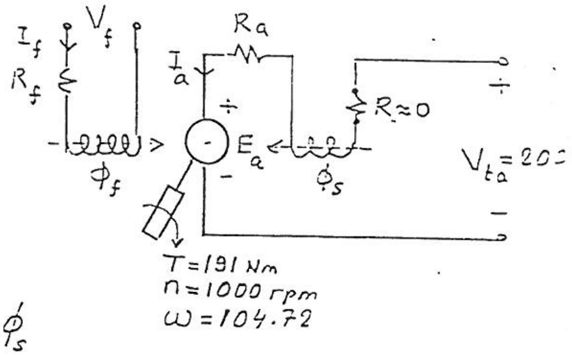
$I_a = 100\,A$, $T = 191$ Nm , $I_f = 1\,A$

$E_a = V_{ta} - R_a I_a = 200 - 0.2*100 = 180\,V$

$E_a = K_a \phi' \omega$, $\phi' = \phi_f \mp \phi_s$

$(K_a \phi') = \dfrac{180}{104.72} = 1.719$

$\delta(K_a \phi) = 1.91 - 1.719 = -0.191 \rightarrow \phi = \phi_f - \phi_s$



(subtractive). Reduction in the generated voltage $= \delta E = 200 - 180 = 20$

At $n = 1000$ rpm $I_f = 1$, $N_f I_f = \mathcal{F}_f = 1000$ At $\rightarrow E_a = 200\,V$
$\quad$ " $\qquad\qquad\qquad \delta\mathcal{F} \qquad\qquad \rightarrow \delta E_a = 20\,V$ $\Big]$ $\delta\mathcal{F} = \dfrac{20}{200} \times 1000 = 10$

$\delta\mathcal{F} = N_f I_f = N_s I_a = 100$ At , $N_s = \dfrac{100}{100} = 1$ turn

f) $V_{ta} = 200\,V$, $I_a = 100\,A$, $T = 191$ Nm , $I_f = 1\,A$, $E_a = 180\,V$
$n = ?$, $K_a\phi = ?$, $E_a = ?$
$T = K_a \phi I_a = K \mathcal{F} I_a$, $E_a = K_a \phi \omega = K \mathcal{F} \omega$, $\mathcal{F} \rightarrow$ mmf
For $I_f = 1$, $N_f = 1000$, $\mathcal{F} = 1000$, $T = 191 = K \times 10^3 * 10^2$, $K = 191 * 10^{-5}$
mmf including armature reaction $= N_f I_f - 2 I_a = 10^3 - 2 I_a$

$191 = 191 \times 10^{-5} (10^3 - 2 I_a) I_a \rightarrow 1 = 10^{-5} (10^3 - 2 I_a) I_a \rightarrow$

$2 I_a^2 - 10^3 I_a + 10^5 = 0 \quad I_a = \dfrac{10^3 \mp (10^6 - 8 \times 10^5)^{1/2}}{4} = \dfrac{10^3 \mp 447.2}{4}$ $\begin{matrix} \nearrow I_a = 138.2\,A \\ \searrow I_a = 361.8\,A \end{matrix}$

$I_a = 138.2\,A$, $E_a = V_{ta} - R_a I_a = 200 - 138.2*0.2 = 172.36\,V$

$\mathcal{F} = (10^3 - 2 * I_a) = (10^3 - 2 * 138.2) = 723.6$

$\omega = \dfrac{E_a}{K\mathcal{F}} = \dfrac{172.36}{191 \times 10^{-5} * 723.6} = 124.71 \rightarrow n = \dfrac{60\,\omega}{2\pi} = \dfrac{60 * 124.71}{2 * \pi} = 1191$ rpm

g) $n = 1000$ rpm , $E_a = 180\,V$, $K_a\phi = \text{const} = 1.91$
$V_t = 100\,V < E_a$, I_a and T changes direction
$E_a = V_t + R_a I_a$, $I_a = (180 - 100)/0.2 = 400\,A$, $P_{out} = 100 * 400 = 40$ kW
$T = K_a \phi I_a = 1.91 * 400 = 764$ Nm re-generative braking (recuperation
(generator operation).

Q.3. (35 points) A 2-pole shunt dc generator has an approximate magnetization characteristic measured at 1500 rpm as given below.

$E_a = 20$ Volts if $0 \le I_f \le 0.1$ A
$E_a = 200 I_f$ Volts if $0.1 \le I_f \le 0.5$ A
$E_a = 100 I_f + 50$ Volts if $0.5\text{ A} \le I_f$,

where I_f is the shunt field current.

The armature has a resistance of 0.2 Ω, and the field circuit has a resistance of 129.15 Ω.

Throughout the problem the generator is driven at a constant speed of 1500 rpm by its prime mover.

Neglect armature reaction for your calculations.

(a) Assuming that the machine is operating as a self-excited shunt generator at no-load, sketch the magnetization curve and calculate the armature terminal voltage V_t.

(b) The self-excited generator is now loaded by connecting a resistive load to its armature terminals. Calculate the armature terminal voltage V_t by assuming that the resistive load carries a current of 50 A. Compute also the load resistance.

(c) The self-excited generator above is now equipped with a series field winding in order to raise the armature terminal voltage in part (b) to 220 V. The series field winding is connected in series with the load so that the machine is converted into a short-compound generator. Calculate the number of series-turns of the series field winding by assuming that the shunt field winding has 1000 turns, and the series field winding resistance is negligibly small. Define also the type of compounding used.

(d) Compute the efficiency of the self-excited generator in part (b) by assuming that friction and windage losses are 150 W.

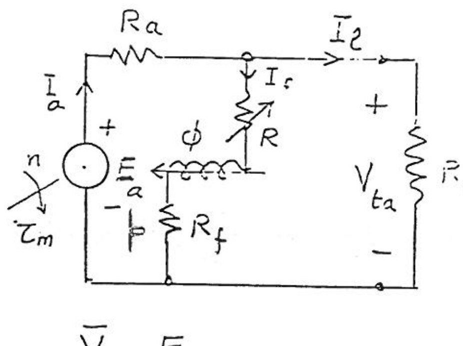
SOLUTION

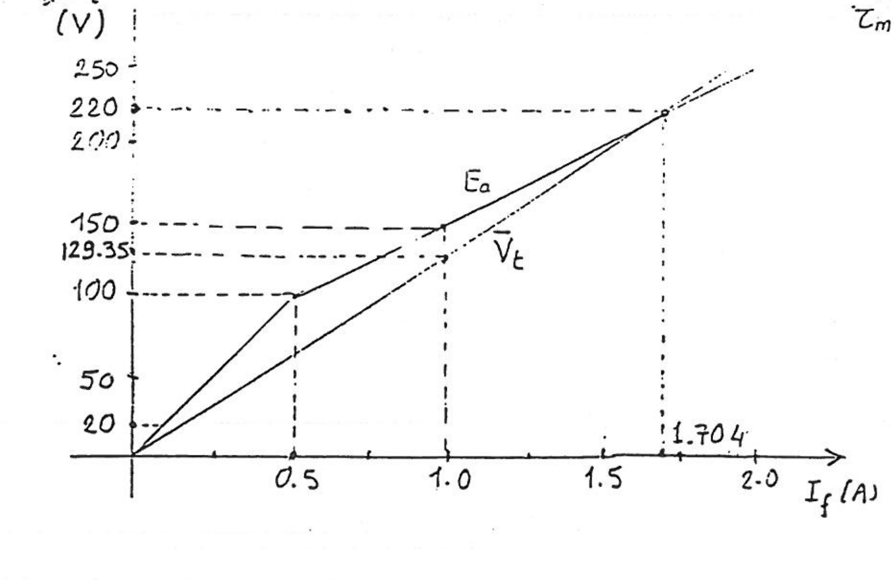
a) $R_a = 0.2\,\Omega$, $R_t = R_f + R = 129.15\,\Omega$

At no-load: $I_l = 0$; $I_f = I_a$

$E_a = (R_a + R_t)\, I_f = \bar{R} I_f = \bar{V}_t$; $\bar{R} = R_a + R_t = 129.35\,\Omega$

$I_f = 1\text{ A}$, $\bar{V}_t = 129.35\text{ V}$





$\bar{V}_t = E_a$

$129.35 I_f = 50 + 100 I_f$

$29.35 I_f = 50$

$I_f = 1.706$ A

$V_{ta} = R_t I_f = 129.15 \times 1.706 = 220$ V b)

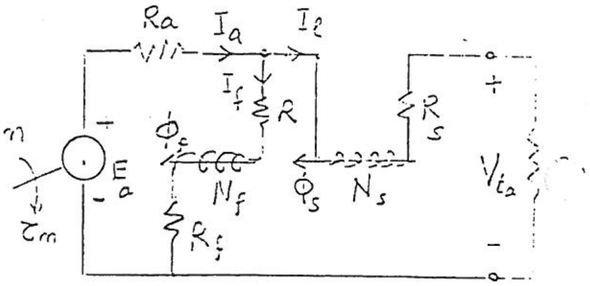
$$\begin{cases} I_a = I_l + I_f = 50 + I_f, \quad I_l = 50\ A \\ V_{ta} = R_t I_f = 129.15\, I_f \\ V_{ta} = E_a - R_a I_a = (50+100 I_f) - 0.2 I_a = 50 + 100 I_f - 0.2(50+I_f) = 99.8 I_f + 40 \end{cases}$$

$129.15\, I_f = 99.8\, I_f + 40 \rightarrow \quad I_f = \dfrac{40}{29.35} = 1.363\,A \quad , I_a = 51.363\,A$

$V_{ta} = I_f * R_t = 176\ V \quad , \quad R_l = \dfrac{V_{ta}}{I_l} = 3.52\ \Omega$

c) $N_f = 1000$ trn, $R_s \approx 0$, $V_{ta} = 220\ V$

Additive: $\phi = \phi_f + \phi_s$



$R_l = 3.52\ \Omega \quad , \quad I_l = \dfrac{220}{3.52} = 62.5\ A$

$I_f = \dfrac{220}{129.15} = 1.703\ A$

$I_a = I_l + I_f = 64.20\ A$

$E_a = V_{ta} + R_a I_a = 220 + 0.2 * 64.2 = 220 + 12.84 = 232.84\ V$

The required total equivalent shunt current $\bar{I}_f$

$232.84 = 100\, \bar{I}_f + 50, \quad \bar{I}_f = \dfrac{232.84 - 50}{100} \cong 1.83\ A.$

The corresponding required total shunt mmf is

$N_f \bar{I}_f = 1000 * 1.83 = 1830$ At

The shunt field provides $1000 * 1.703 = 1703$ At

The difference $\delta F = 1830 - 1703 = 127$ At should additionally be provided by the series field winding.

$N_s I_l = 127, \quad N_s \cong \dfrac{127}{62.5} = 2$ turns

d) $P_{out} = V_{ta} * I_l = 176 * 50 = 8800\ W$

$P_f = 129.15 * 1.363^2 = 239.9\ W \quad , \quad P_a = 0.2 * 51.363^2 = 527.63\ W$

$P_{in} = P_{out} + P_f + P_r + P_{fw} = 8800 + 239.9 + 527.63 + 150 = 9717.53\ W$

Efficiency $= \dfrac{8800}{9717.53} = 90.6\,\%$

Q5 (20 pts.) The (constant speed) magnetization curve for a 20-kW, 250-V DC machine at the rated speed of 1100 rpm is given in the table.

The armature winding resistance is 0.125 Ω and the field winding resistance is 30 Ω. The rotational loss at 1100 rpm is 2000 W and the armature reaction is neglected.

The machine is operated as separately excited DC generator. When delivering its rated power, at rated voltage and rated speed, find:

a) The rated armature current and field current.
b) The overall efficiency and voltage regulation.

The same machine is now operated as shunt generator, by connecting the field winding in parallel with the armature winding. With the same rated armature current, rated terminal voltage and rated speed. Calculate:

c) The value of the external resistance connected in series with the field winding.
d) Efficiency and voltage regulation.
e) Compare the two generators and comment.

E_a (V)	140	195	240	260	275
I_f (A)	1	1,5	2	2,5	3

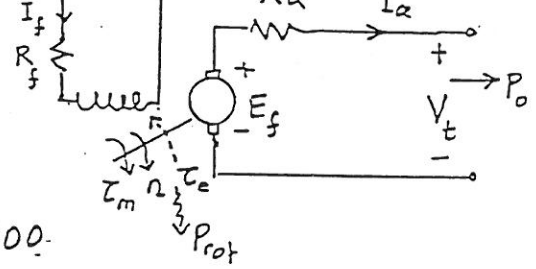
SOLUTION

$\cdot\ P_o = 20\text{ kW},\ V_t = 250\text{ V},\ n = 1100\text{ rpm},\ R_a = 0.125\,\Omega,\ R_f = 30\,\Omega,\ P_{rot} = 2\text{ kW}$

a) $P_o = V_t I_a,\ I_a = \dfrac{20\times10^3}{250} = 80\text{ A}$

$E_a = V_t + R_a I_a = 250 + 0.125 \doteq 260\text{ V}$

$E_a = 260\text{ V} \rightarrow I_f = 2.5\text{ A}$



b) $P_{loss} = R_f I_f^2 + R_a I_a^2 + P_{rot} = 30 * 2.5^2 + 0.125 * 80^2 + 2000$

$P_{loss} = 2987.5\text{ W}$

$$\eta = \frac{P_o}{P_o + P_{loss}} = \frac{20000}{20000 + 2987.5} = 87\,\%$$

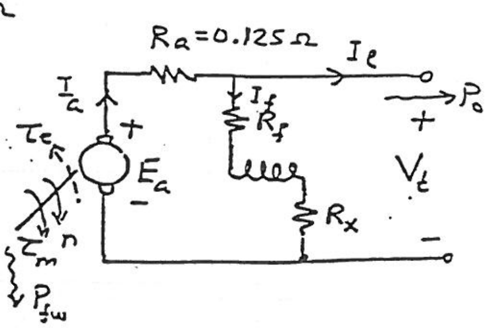
$$\text{Regulation} = \frac{V_{tn\text{-}ld} - V_t}{V_{trt}} = \frac{260 - 250}{250} = 4\,\%$$

c) $V_t = 250\text{ V},\ I_a = 80\text{ A},\ n = 1100\text{ rpm},\ R_f = 30\,\Omega$

$E_a = V_t + R_a I_a = 250 + 0.125 * 80 = 260\text{ V}$

$E_a = 260\text{ V} \rightarrow I_f = 2.5\text{ A}.$

$R = R_f + R_x = \dfrac{250}{2.5} = 100\,\Omega,\ R_x = 70\,\Omega$



d) $I_l = I_a - I_f = 80 - 2.5 = 77.5\ A$

$P_o = 250 * 77.5 = 19375\ w$

$P_{loss} = P_{fw} + R_a I_a^2 + R I_f^2 = 2000 + 0.125 * 80^2 + 100 * 2.5^2 = 3425\ w$

$$\eta = \frac{19375}{\underbrace{19375 + 3425}_{22800}} = 84.98\ \% \approx 85\%$$

Voltage regulation $= \dfrac{V_{nld} - V_{ld}}{V_{rt}} = VR$

$V_{ld} = V_{rt} = V_t = 250V$

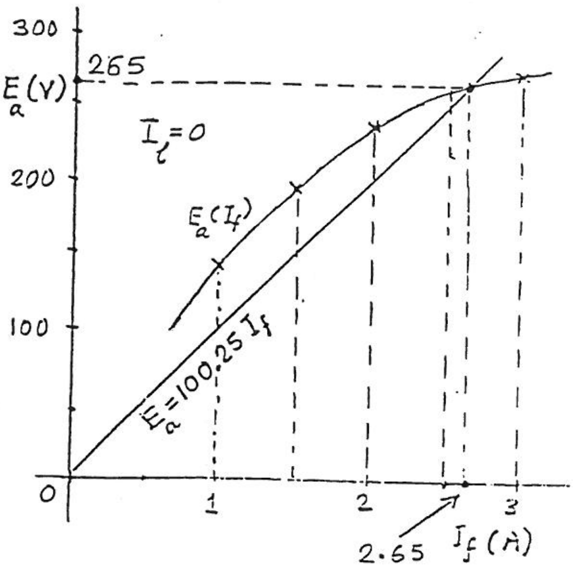
No-load voltage $= V_{nld}$, $I_l = 0$, $I_a = I_f$

$E_a = (R_a + R_f) I_f = 100.125 \times I_f$

The intersection point gives: $I_f = 2.65A$

$E_a = 265V$, $V_{nld} = R_f I_f = 100 * 2.65 = 265V$.

$$VR = \frac{265 - 250}{250} = 6\%$$



e) <u>Comment:</u>

The two generators show almost the same performance. A separate DC source is not needed for shunt-generator operation. This is an important advantage (gain). Thus, shunt-generator operation is freferable.

Q.3 (35 points) A 10-kW dc shunt motor is connected to a constant 200 V dc supply and is driving a load with the following power-speed characteristic (n: speed in rpm):

$$P_L = Kn^2 = 3.9\times10^{-3}n^2$$

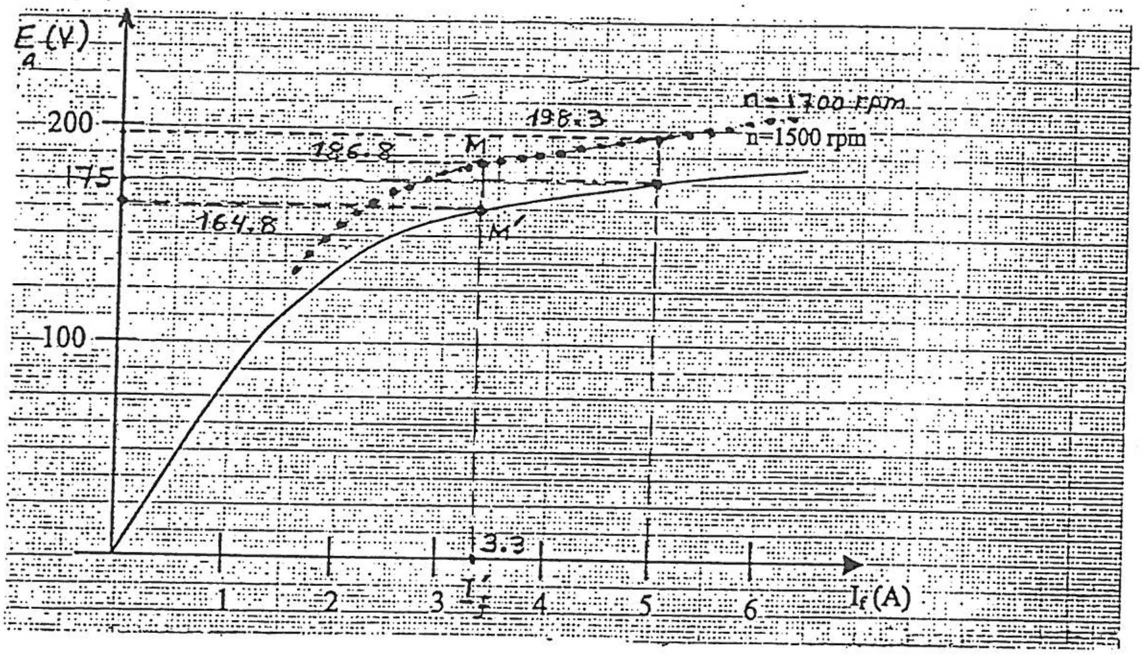
The field winding resistance is 40 Ω and the friction and windage losses of the motor are constant and given as 250 W.

a) Calculate the motor speed and the load torque if the motor is supplying rated power to the load.

b) At the rated load conditions determine the armature and field current of the motor.

c) Calculate the current drawn from the electrical terminals for the condition in part (b). Determine the efficiency of the motor.

d) The shaft speed of the motor is measured as 1700 rpm while the armature current is as in part (c). Calculate the armature reaction mmf magnitude for this condition (Number of field winding turns = 1000).



SOLUTION:

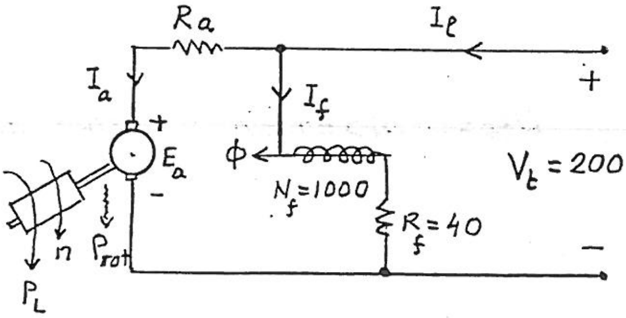
a) $P_L = Kn^2 = 3.9*10^{-3}n^2$

$P_{out} = P_L = P_{rt} = 10\,000$ W

$10000 = 3.9*10^{-3}n^2$; $n = 1601.3$ rpm

$\omega_m = 2\pi \frac{1601.3}{60} = 167.68$ 1/rad

$T_{ld} = \frac{P_o}{\omega_m} = 59.64$ Nm



b) $I_f = \frac{200}{40} = 5$ A

$P_e = P_{out} + P_{rot} = 10\,000 + 250 = 10250$ W at $n = 1601.3$

At $n = 1500$ rpm $\rightarrow I_f = 5$ A $\rightarrow E_a' = 175$ V

$n = 1601.3$ rpm $\rightarrow I_f = 5$ A $\rightarrow E_a = 175\,\frac{1601.3}{1500} = 186.8$ V

$P_e = P_{out} + P_{rt} = 10000 + 250 = 10250$, $P_e = E_a I_a$

$I_a = \frac{10250}{186.8} = 54.87$ A , $R_a = \frac{V_t - E_a}{I_a} = \frac{200 - 186.8}{54.87} = 0.24\ \Omega$

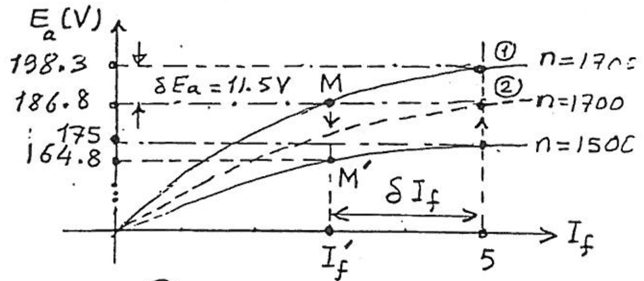
c) $I_l = I_a + I_f = 59.87$ A. $P_{in} = 200 * 59.87 = 11974$ W

d) $n = 1700$ rpm , $I_a = 54.87$ A , $E_a = V_t - R_a I_a = 200 - 0.24 * 54.87 = 186.8$ V

At $n = 1700$ rpm , $I_f = 5$ A $E_a' = ?$
$n = 1500$ " , $I_f = 5$ A $E_a = 175$

$$E_a' = 175 \frac{1700}{1500} = 198.3 \text{ V}$$ (without armature reaction

At $n = 1700$ rpm, $I_f = 5$ A $\delta E_a = 198.3 - 186.8 = 11.5$ V induced voltage loss due to reduction in flux caused by armature reaction. $E_a = 186.8$ V could be induced at I_f', if there were no armature reaction, i.e δI_f is excitation loss due to armature reaction. The E_a-I_f characteristic for $n=1700$ (①) can be obtained from the E_a-I_f



① - No-armature reaction
② - With " " at constant $I_a = 54.87$ A

characteristic at $n=1500$, by increasing voltage values by a factor of 1700/1500 at a constant I_f. At a selected I_f, a point of (E_a-I_f) curve for $n=1700$, the corresponding point on (E_a-I_f) for $n=1500$ is found by decreasing the voltage value by a factor of 1500/1700. Thus, at $I = I_f'$, the corresponding voltage of 186.8 V on $n=1500$ curve is $186.8 * \frac{1500}{1700} = 164.8$ V. From the given E_a-I_a characteristic, for $E_a = 164.8 \rightarrow I_f' \cong 3.3$ A is found. Therefore, the E_a-I_a for $n=1700$ may not be drawn.

$\delta I_f = 5 - 3.3 \cong 1.7$ A , $\delta(\mathcal{F}) = N_f \, \delta I_f = 1000 * 1.7 \cong 1700$ At.