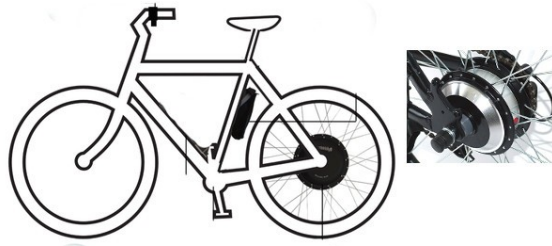


Q4) (25pts + 5 pts bonus) You bought an electric bike with a permanent-magnet DC motor (which can be considered as a separately excited DC motor with constant field current) connected to the rear wheel directly as shown in the figure below.



As you don't know the specifications of the motor, you make a few tests to estimate the equivalent circuit parameters:

- You apply 48 V to the motor at no-load and it rotates at 200 rpm (neglecting the losses).
- When the motor is stationary ( $\omega_r=0$ ), you measure the armature current as 12 A and the output torque as 27.5 Nm if you apply 6 V to the motor terminals.

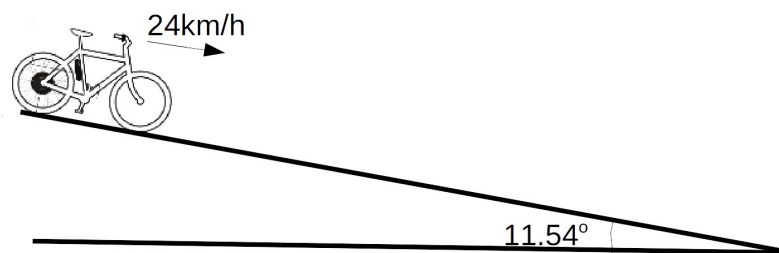
a) (5 pts) Draw the equivalent circuit of the motor, find out the armature resistance and write down induced voltage and torque expressions.

b) (10 pts) You power the motor from a 48 V battery and start cycling on a flat road. Calculate the starting torque, starting current, and the current, speed, and efficiency of the motor when it reaches to the steady-state speed. Assume the friction on the road is reflected to the motor as a constant torque of 5.5 Nm. Also sketch the waveforms for torque and speed as a function of time.

c) (10 pts) You want to eliminate the mechanical brakes in the bike, thus you disconnect the motor from the battery and connect to an external resistor when going downhill (i.e. dynamic breaking mode).

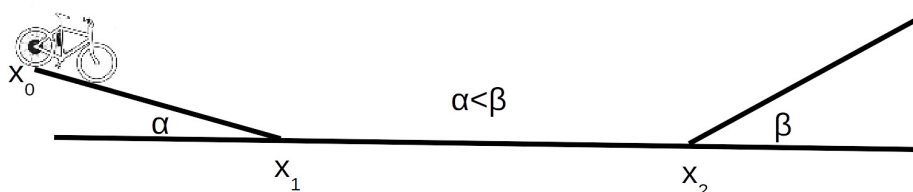
i) Draw the armature circuit and mark polarity of the induced voltage and direction of current.

ii) Find out the value of the external resistor, if you want to keep the speed constant at 24 km/h on a road with an decline of  $11.54^\circ$  degrees as shown in the figure, (Total mass: 80 kg,  $g=10\text{m/s}^2$ , Wheel diameter=600 mm, **Neglect friction**)

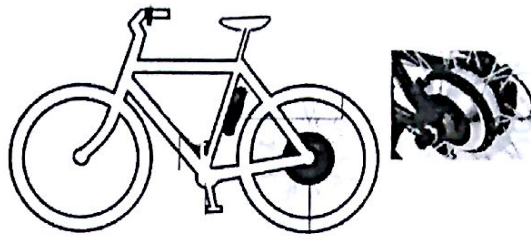


d) (5 pts bonus) The resistor is not an efficient way of breaking as you dissipate the mechanical energy, so you decide to remove the resistor and connect a capacitor to the motor terminals without the battery (i.e. regenerative breaking mode).

Then, you take a test-drive on a road with a profile as shown in the figure. Sketch the waveform for, the capacitor voltage, armature current, and speed. You don't have to calculate any numerical values, but describe each mode of operation in detail and give equations for full-credit (Capacitor initially uncharged, **do not neglect friction** on the road.)



Q4) (25pts + 5 pts bonus) You bought an electric bike with a permanent-magnet DC motor (which can be considered as a separately excited DC motor with constant field current) connected to its rear wheel directly as shown in the figure.



As you don't know the specifications of the motor, you make a few tests to estimate the equivalent circuit parameters:

- You apply 48 V to the motor at no-load and find out it rotates at 200 rpm (neglect the losses).
- When the motor is stationary ( $\omega_r=0$ ), you apply 6V and measure the armature current as 12 A, and measure the output torque as 27.5 Nm.

a) (5 pts) Draw the equivalent circuit of the motor, find out the armature resistance and write down induced voltage and torque expressions

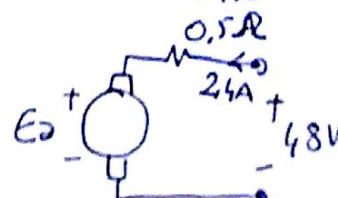
$E_a \rightarrow 48V \rightarrow 200rpm$   
 $I_a \rightarrow 12A \rightarrow 27,5Nm$   
 $\omega_r=0 \rightarrow E_a=0$   
 $R_a = \frac{V_t - E_a}{I_a}$   
 $R_a = \frac{6-0}{12} = 0,5\Omega$

$E_a = K_a \Phi_{pp} \cdot \omega$   
 $T = K_a \Phi_{pp} \cdot I_a$

b) (10 pts) You start driving the bike, powered from a 48 V battery, on a flat road. Calculate the starting torque, starting current, final current, final speed, and efficiency of the motor at the steady-state speed. Assume the friction on the road is reflected to the motor as a constant torque of 5.5 Nm. Also sketch the waveforms for torque and speed as a function of time.

Starting  $\omega_r=0$   
 $V_t=48V$   
 $E_a=0$   
 $I_a = \frac{48-0}{0,5} = 96A$   
 $T = 27,5 \cdot \frac{96}{12} = 220Nm$

Steady-state  $T_e = T_m$   
 $T_e = 5,5Nm$   
 $I_a = \frac{5,5}{27,5} \cdot 12 = 2,4A$

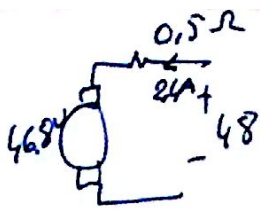


$E_a = 48 - 2,4 \cdot 0,5$   
 $E_a = 46,8V$

$E_a \rightarrow 200rpm$

$E_a = 48V$  at 200rpm

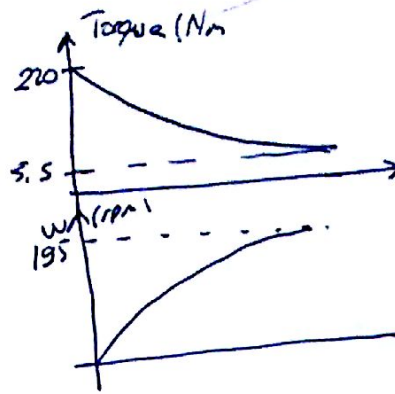
thus  $\omega_{final} = \frac{46,8}{48} \cdot 200 = 195rpm$



$$P_{\text{tot}} = 48 \cdot 2.4 = 115.2 \text{ W}$$

$$P_{\text{out}} = 46.8 \cdot 2.4 = 112.32 \text{ W}$$

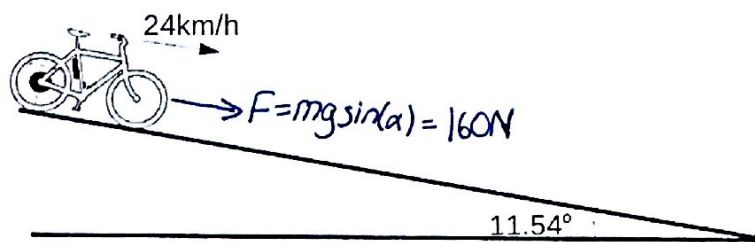
$$\text{Eff} = \frac{112.32}{115.2} = \%97.5$$



c) (10 pts) You want to eliminate the mechanical brakes in the bike, thus you disconnect the motor from the battery and connect to an external resistor when going downhill (i.e. dynamic breaking mode of operation).

i) Draw the armature circuit and mark polarity of the induced voltage and direction of current.

ii) Find out the value of the external resistor, if you want to keep the speed constant at 24 km/h on a road with an decline of  $11.54^\circ$  degrees as shown in the figure, (Total mass: 80 kg,  $g=10\text{m/s}^2$ , Wheel diameter=600 mm, Neglect friction)



$$T = Fr$$

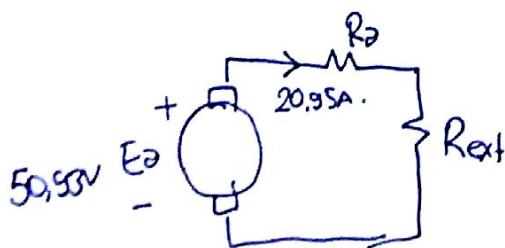
$$T = 160 \cdot 0.3 = 48 \text{ Nm}$$

$$24 \text{ km/h} \Rightarrow 400 \text{ m/min} \Rightarrow \pi \cdot D \cdot N = 400$$

$$\pi \cdot 0.6 \cdot N = 400$$

$$N_{\text{rpm}} = 212.2 \text{ rpm}$$

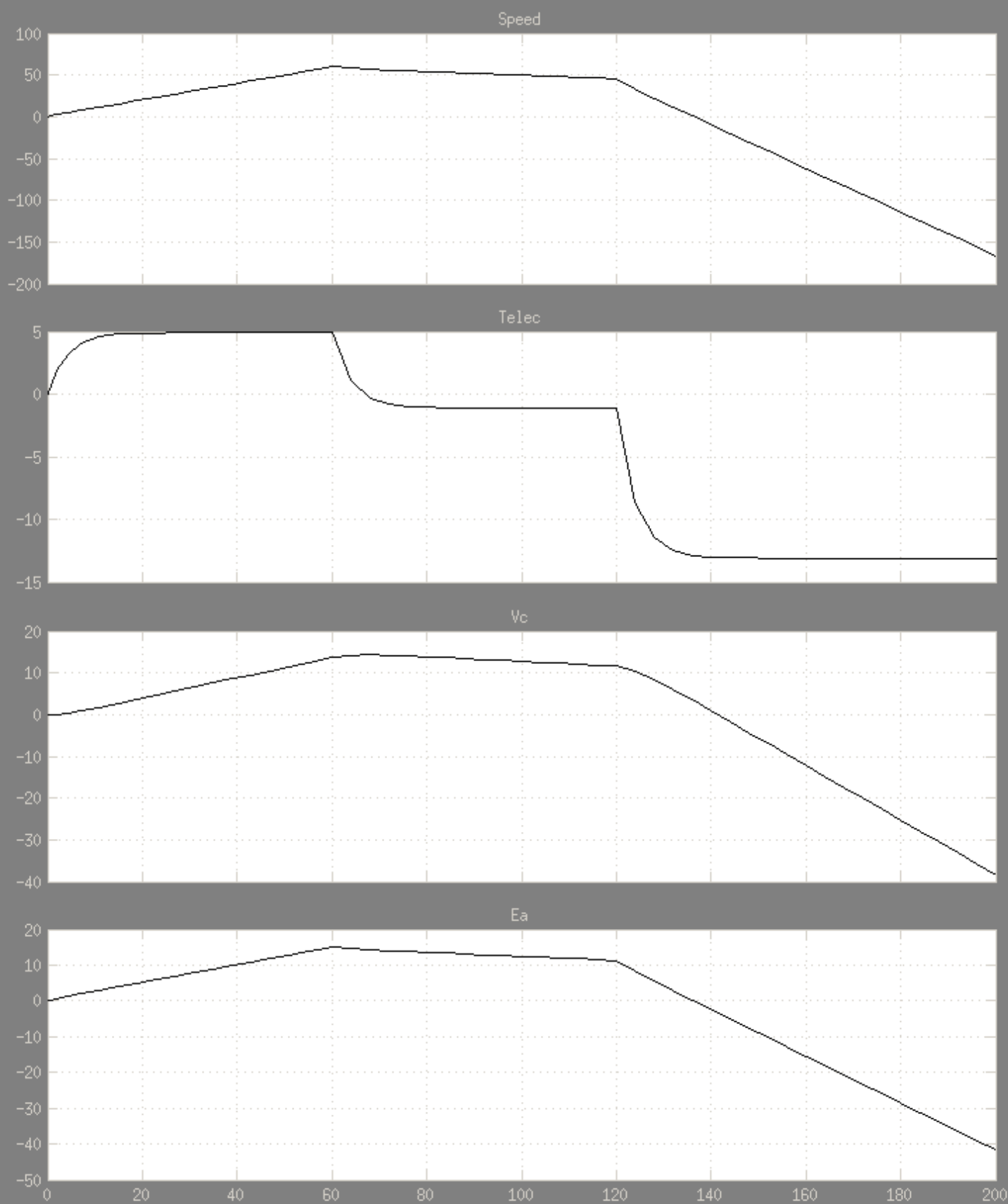
$$E_a = 48 \cdot \frac{212.2}{200} = 50.93 \text{ V}$$



$$I_a = \frac{12.48}{27.5} = 20.95 \text{ A}$$

$$R_{\text{total}} = \frac{E_a}{I_a} = \frac{50.93}{20.95} = 2.43 \Omega$$

$$R_{\text{ext}} = 2.43 - 0.5 = \boxed{1.93 \Omega}$$



Time offset: 0