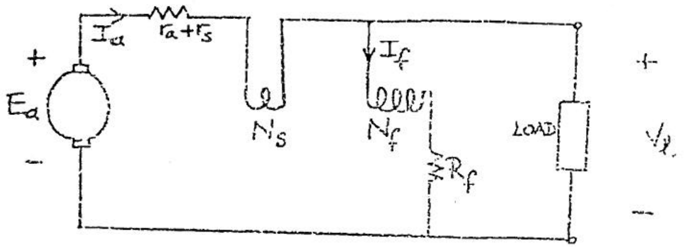
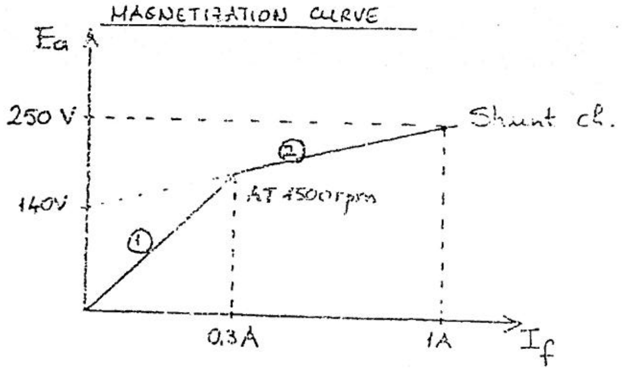
Ex 3 A cumulatively compounded long-shunt connected dc generator delivers 15 A to a 10 Ω load.

$r_a + r_s = 1\,\Omega \quad r_f = 250\,\Omega, \quad \frac{N_f}{N_s} = 52$

The magnetization curve is as shown below. Find the shaft speed in rpm and the electrical torque produced.





① : $E_a \cong 567\, I_f$

② : $E_a = 140 + 100 I_f$

Soln

Terminal voltage under load: $15 \times 10 = 150\,V$

$I_f = \frac{150}{250} = 0.6\,A \Rightarrow I_a = I_f + I_L = 15.6\,A$

$E_a = V_t + I_a(r_a + r_s) = 150 + 15.6 = 165.6\,V$

Total mmf applied = Shunt field mmf + Series field mmf

$= N_f I_f + N_s I_a$

Equivalent shunt field amp: $I_f' = I_f + \frac{I_a N_s}{N_f} = 0.9\,A$

$E_a' = 140 + 100 I_f' = 230\,V$ at 1500 rpm

$$\frac{E_{a_{1500}}}{E_{a_N}} = \frac{1500}{N} \Rightarrow N = \frac{1500 \times 165.6}{230}$$

$$N \cong 1075 \text{ rpm} \;//$$

$$\text{Electromagnetic torque} = \frac{E_a I_a}{\omega}$$

$$T = \frac{165.6 \times 15.6 \times 60}{2\pi \times 1075} = 23 \text{ Nm} \;//$$

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Solved Problems

Ex2

A 200 V dc series motor has the following O.C.C. measured at 1000 rpm (O.C.C. is obtained by separately exciting the field winding).

Field Amps, I_f	5	10	15	20	25	30
Open-circuit voltage, E_a Volts	80	160	202	222	236	244

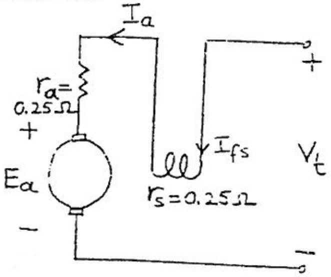
The armature resistance is 0.25 Ω and the series field resistance is 0.25 Ω.

Find the speed of the m/c WHEN,

a) the armature current is 22.5 A

b) the gross torque is 36 Nm
($T_e = T_l + T_{f\&w}$)

Solution



The induced armature emf E_a at the unknown speed is:

$E_a = V_t - I_a(r_a + r_s)$

$= 200 - 22.5 \times 0.5 = 188.75\text{ V}$

Now, assume that the motor is running at 1000 rpm and supplying the same armature current $I_a = I_{fs} = 22.5$ A.

From the magnetization curve,

When $I_{fs} = 22.5$ A, $E_a = 230$ V

Since $E_a = K I_a \omega$ & I_a remains unchanged for both cases,

$$\left.\frac{E_{a_{1000}}}{E_{a_N}}\right|_{I_a = 22.5\text{ A}} = \frac{1000}{N} \Rightarrow N = 819 \text{ rpm}$$

b) When T = 36 Nm, Find N

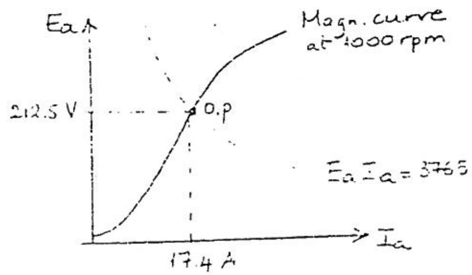
For a series motor $T = K I_a^2$ and when T is constant I_a is also constant & independent of shaft speed.

$T = \left(\frac{E_a}{\omega}\right) I_a$ ← (E_a/ω ratio is constant)

Assume the machine is running at 1000 rpm:

$36\left(1000 \times \frac{2\pi}{60}\right) = E_a I_a$

from which $E_a I_a = 3765$



For the unknown speed N:

$E_a = V_t - I_a(r_a + r_s)$

$= 200 - 17.4 \times 0.5$

$= 191.3\text{ V}$ ← actual value

$$\left.\frac{212.5}{191.3}\right|_{I_f = I_a = 17.4\text{A}} = \frac{1000}{N} \Rightarrow N = 902 \text{ rpm}//$$

A, B

Q.4. (22 points) A 15-kW 200-V separately excited dc motor has a field resistance of 200 Ω and an armature resistance of 0.2 Ω. The magnetization characteristic of the motor measured at 1500 rpm is given as:

I_f, A	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
E_a, V	45	90	134	176	210	240	260	270

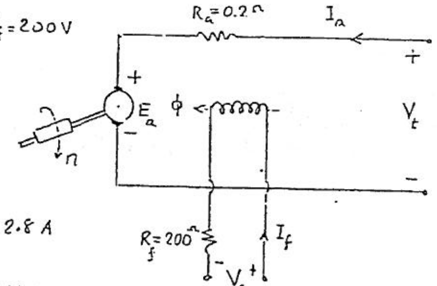
The speed of the motor is controlled by independently adjusting the armature and field circuits' terminal voltages. For this purpose, both the armature and field circuits are fed with separate variable voltages whose maximum value is 200 V for both circuits.

Armature reaction and rotational losses are to be neglected.

a) Calculate the shaft speed and load torque when the motor is supplying 10 kW to its load with maximum voltage applied to both field and armature circuits.
b) Now the shaft speed is to be reduced to 1100 rpm while the motor is supplying the <u>same power</u> as in part (a). State how you can achieve this. Assuming that the uncontrolled variable is kept at its maximum value, calculate the new value of the control parameter to satisfy this condition.
c) This time the shaft speed is to be increased to 1600 rpm while the motor is supplying the <u>same torque</u> as in part (a). State how you can achieve this. Assuming that the uncontrolled variable is kept at its maximum value, calculate the new value of the control parameter to satisfy this condition.

Solution:

a) $P_o = 10000 = E_a I_a = P_e$, $V_t = 200\,V$, $V_f = 200\,V$



$I_f = 200/200 = 1\,A$, $n, \tau \to ?$

$\begin{cases} E_a I_a = 10\,000 \\ E_a = V_t - R_a I_a = 200 - 0.2 I_a \end{cases}$

$(200 - 0.2 I_a) I_a = 10000$

$0.2 I_a^2 - 200 I_a + 10000 = 0 \to I_a = 52.8\,A$

$E_a = 200 - 0.2 \times 52.8 = 189.4\,V$

$\left.\begin{array}{l} I_f = 1 \to n = 1500 \to E_a = 210\,V \\ I_f = 1 \to n = ? \to E_a = 189.4 \end{array}\right\} n = 1500 \dfrac{189.4}{210} = 1353\,rpm$

$\omega_m = 2\pi \dfrac{1353}{60} = 141.68\,rad/sec$, $\tau = \dfrac{P_e}{\omega_m} = \dfrac{10000}{141.68} = 70.57\,Nm$

b) $n = 1100\,rpm$, $P_e = 10\,000\,w$. Take $V_f = 200\,V$ and $I_f = 1\,A$

$\left.\begin{array}{l} I_f = 1 \to n = 1500 \to E_a = 210 \\ I_f = 1 \to n = 1100 \to E_a = ? \end{array}\right\} E_a = 210 \dfrac{1100}{1500} = 154\,V$

$P_e = E_a I_a$, $I_a = \dfrac{10000}{154} = 64.9\,A$, $V_t = E_a + R_a I_a = 154 + 0.2 \times 64.9 = 167\,V$

The speed is reduced by the terminal voltage, keeping flux constan[t]

c) $n = 1600\,rpm$, $\tau = 70.57\,Nm$; $P_e = \tau \cdot \omega_m$, $\omega_m = 2\pi \dfrac{1600}{60} = 167.55\,rad/sec$.

$P_e = \tau \times \omega_m = 70.57 \times 167.55 = 11824.1\,w$. Take $V_t = 200\,V$

$E_a I_a = P_e = (V_t - R_a I_a) I_a \to 0.2 I_a^2 - 200 I_a + 11824.1 = 0 \to I_a = 63.1\,A$

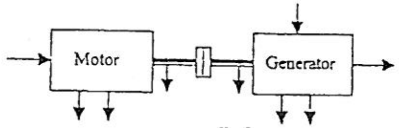
$E_a = 200 - 0.2 \times 63.1 = 187.4\,V$, at $n = 1600\,rpm$

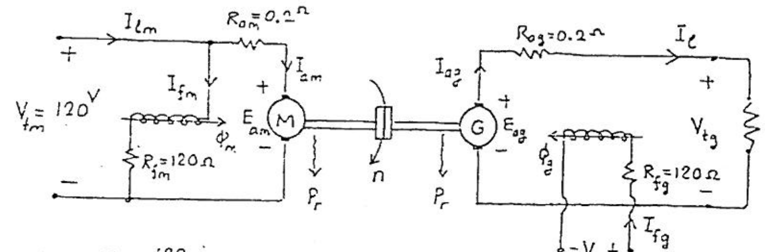
At $n = 1500$ $E_a = 187.4 \dfrac{1500}{1600} = 176\,V \longrightarrow I_f = 0.8\,A \to V_f = 160\,V$

The field voltage is reduced keeping the terminal voltage at 200 V

... Consider two identical dc machines having an armature resistance $R_a = 0.2\,\Omega$ and a field winding resistance $R_f = 120\,\Omega$. One of the machines is used as a shunt motor to drive the other to operate it as a separately excited generator to give power to a load. The shunt motor draws 6 kW from a constant voltage source of 120 V. The excitation currents of both machines are <u>equal</u> and the rotational loss of each machine is constant and is 200 W per machine.

a) Draw the complete circuit diagram indicating voltages, currents and winding resistances. Use subscript "m" and "g" for motor and generator quantities, respectively.
b) Calculate the induced voltage of the motor and of the generator.
c) Find the armature current and output power of the generator.
d) Complete the power flow diagram of the system given below, by finding the corresponding numerical values for each arrow. Calculate the overall efficiency of the system.





b) $V_{tm} = 120\,V$, $I_{fm} = \dfrac{120}{120} = 1\,A = I_{fg}$, $R_{fm} = R_{fg} = 120\,\Omega$, $V_{cg} = 1 \times 120 = 120\,V$

$P_{im} = 6000 = V_{tm} \times I_{lm}$, $I_{lm} = \dfrac{6000}{120} = 50\,A$, $I_{am} = 49\,A$

$E_{am} = V_{tm} - R_{am} I_{am} = 120 - 49 \times 0.2 = 110.2\,V$

$P_{em} = E_{am} I_{am} = 110.2 \times 49 = 5399.8$, $P_{om} = P_{em} - P_r = 5399.8 - 200 = 5199.8\,W$

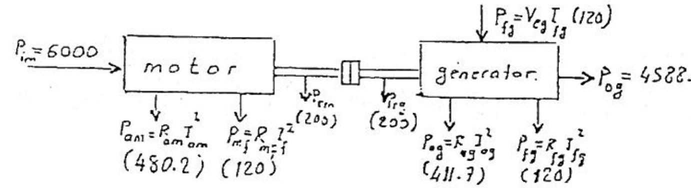
$E_{ag} = E_{am} = 110.2\,V$ as $\phi_m = \phi_g = \phi$, $K_{am} = K_{ag} = K_a$, $\omega_{mm} = \omega_{mg} = \omega_m$, $E_a = K_a \phi \omega_m$

c) $E_{ag} = 110.2\,V$: $P_{om} = P_{ig} = 5199.8\,W$, $P_{eg} = P_{ig} - P_r = 5199.8 - 200 = 4999.8\,w$

$I_{ag} = \dfrac{P_{eg}}{E_{ag}} = \dfrac{4999.8}{110.2} = 45.37\,A = I_l$, $V_{tg} = E_{ag} - R_{ag} I_{ag} = 110.2 - 0.2 \times 45.37 = 101.13\,V$

$P_{og} = V_{tg} \times I_{lg} = 101.13 \times 45.37 = 4588.27\,w$

d)



$\sum P_{in} = 6000 + 120 = 6120\,w$, $\sum P_{loss} = 480.2 + 120 + 2 \times 200 + 411.7 + 120 = 1531.9\,w$

$\eta = \dfrac{\sum(P_{in} - P_{loss})}{\sum P_{in}} = \dfrac{6120 - 1531.9}{6120} = 74.97\,\%$

EE 361 Electromechanical Energy Conversion I
Final Examination
Date: 12.01.2004

(A,B) **Q.1. (22 points)** Consider the cylindrical relay shown below. The diameter of the plunger is 2 cm and the initial length of the air gap is 4 mm. The exciting coil has 3141 turns.

The relay is excited by a sinusoidal voltage $v(t) = 310\cos 100\pi t$ V.

a) Neglecting coil resistance and leakage flux, calculate the maximum flux density in the air gap.

b) Comment on the input current shape if the core is infinitely permeable.

c) What happens to the current waveform if the core has a nonlinear B-H characteristic?

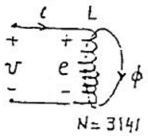
Now, assume that the core material of the relay is infinitely permeable.

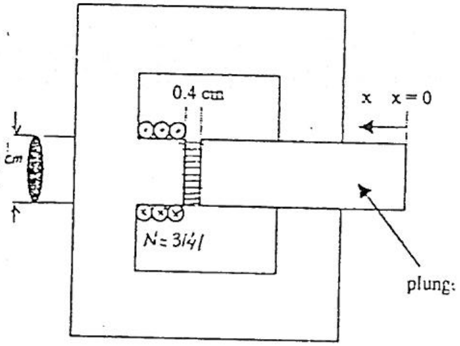
d) Find an expression for the inductance of the core as a function of plunger position x ($\mu_0 = 4\pi.10^{-7}$ H/m).

e) Find an expression for the instantaneous force acting on the plunger as a function of x and the current in the coil.

f) Calculate the initial magnetic force acting on the plunger if the coil current is I = 1 A.

Solution:





a) $v = 310\cos 100\pi t$

$v = e = N\cdot\frac{d\phi}{dt}$

$\phi = \frac{1}{N}\int v\,dt = \frac{310}{N\omega}\sin\omega t$

$\phi_{mx} = \frac{310}{N\omega}$, $B_{mx} = \frac{\phi_{mx}}{A}$, $A = \pi r^2 = \pi * 0.01^2$

$B_{mx} = \frac{310}{3141*100\pi*0.01^2*\pi} \cong 1$ T

b) If $\mu\to\infty$, the system is linear and the current shape will be the same as input voltage, i.e. sinusoidal

c) In case, the core has a non-linear B-H characteristic, the current shape will be distorted if peak flux pushes the core into saturation. Otherwise, the current shape will remain sinusoidal.

d) $L = N^2/\mathcal{R}$, $\mathcal{R} = \frac{g}{\mu_0 A} = \frac{(0.004-x)}{4\pi*10^{-7}*\pi*0.01^2} = 25.33*10^{8}*(0.004-x)$, $L = \frac{N^2}{\mathcal{R}} = \frac{3.89*10^{-3}}{(0.004-x)}$

e) $F(x) = \frac{1}{2}i^2\frac{dL}{dx} = \frac{i^2}{2}\,\frac{3.89*10^{-3}}{(0.004-x)^2}$

07 Let $i = 1$ A , $x_0 = 0$, $F = \frac{1}{2}\,\frac{3.89\times10^{-3}}{0.004^2} = 121.6$ N

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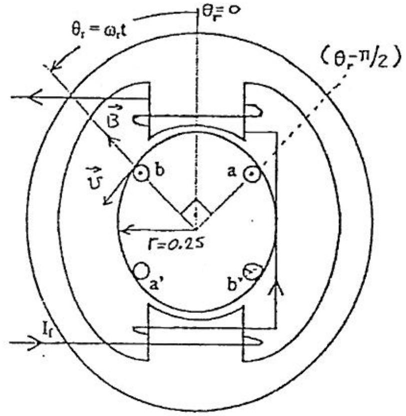
(B) **Q.2. (22 points)** Consider the simple electric brake shown below. The rotor winding consists of two identical shorted loops, each having a resistance of 0.005 Ω. The stator winding produces a flux density distribution in the air gap given by $B(\theta_r) = 1.0\cos\theta_r$ T. The axial length and diameter of the rotor are both 0.5 m. The core material is infinitely permeable. The rotor is rotating at a speed $\omega_r = 154$ rad/sec, so that $\theta_r = \omega_r t$.

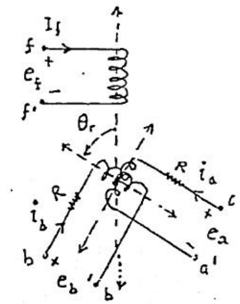


a) Derive expressions for the emf and for the currents in each of the shorted loops as a function of time. Hint: $e = l\cdot(\vec{v}\times\vec{B})$

b) Show that the total power dissipated in the two loops is constant over each revolution, provided that ω_r and I_f are constant.

c) Given that the field winding self-inductance is L_f = 100 mH, the self-inductance of armature coil a-a' is $L_a = 0.10 + 0.05\cos 2\theta_r$ mH and the mutual inductance between field and armature coil a-a' is $M_{fa} = 0.2\cos\theta_r$ mH, find the self and mutual inductances of coil b-b'. Derive an expression for the electromagnetic torque acting on the rotor as a function of time.

Solution: $L_f = 100$ mH , $L_a = 0.10+0.05\cos 2\theta_r$ mH



$M_{fa} = 0.2\cos\theta_r$, $B(\theta_r) = 1\cos\theta_r$ T , $\ell = 0.5$ m , $D_r = 0.5$ m

$\theta_r = \omega_r t$, $\omega_r = 154$ rad/sec , $R = 0.005\,\Omega = 5*10^{-3}\,\Omega$

a) $v = r\omega_r = 0.25*154 = 38.5$ m/s

$e_b = 2*v\ell B = 2*38.5*0.5*1\cos\theta_r = 38.5\cos\theta_r$

$e_a = 2*v\ell B = 2*38.5*0.5*\cos(\theta_r - \pi/2) = 38.5\sin\theta_r$

$i_b = \frac{38.5}{R}\cos\theta_r = 7.7*10^3\cos\theta_r$ A , $i_a = 7.7*10^3\sin\theta_r$

b) Instantaneous power $= p(t) = R(i_a^2 + i_b^2) = 5*10^{-3}(7.7*10^3)^2(\sin^2\theta_r + \cos^2\theta_r)$

$P = 296450 =$ constant , $\int_{\theta_r=0}^{2\pi} p\,d\theta_r = p*2\pi =$ constant.

c) If $L_a = 0.1+0.05\cos 2\theta_r$ mH, 90° angle between axes of aa' and bb' coils.

$L_b = 0.1+0.05\cos 2(\theta_r - 90°)$ mH , $M_{fb} = 0.2\cos(\theta_r + \pi/2)$ mH and $M_{ab} = 0$

$= -0.2\sin\theta_r$.

$M_{af} = 0.2\cos\theta_r$

Q.3. (35 points) A 2-pole shunt dc generator has an approximate magnetization characteristic measured at 1500 rpm as given below.

$E_a = 20$ Volts	if $0 \le I_f \le 0.1$ A
$E_a = 200 I_f$ Volts	if $0.1 \le I_f \le 0.5$ A
$E_a = 100 I_f + 50$ Volts	if $0.5\ \text{A} \le I_f$,

where I_f is the shunt field current.

The armature has a resistance of 0.2 Ω, and the field circuit has a resistance of 129.15 Ω.

Throughout the problem the generator is driven at a constant speed of 1500 rpm by its prime mover.

Neglect armature reaction for your calculations.

(a) Assuming that the machine is operating as a self-excited shunt generator at no-load, sketch the magnetization curve and calculate the armature terminal voltage V_t.

(b) The self-excited generator is now loaded by connecting a resistive load to its armature terminals. Calculate the armature terminal voltage V_t by assuming that the resistive load carries a current of 50 A. Compute also the load resistance.

(c) The self-excited generator above is now equipped with a series field winding in order to raise the armature terminal voltage in part (b) to 220 V. The series field winding is connected in series with the load so that the machine is converted into a short-compound generator. Calculate the number of series-turns of the series field winding by assuming that the shunt field winding has 1000 turns, and the series field winding resistance is negligibly small. Define also the type of compounding used.

(d) Compute the efficiency of the self-excited generator in part (b) by assuming that friction and windage losses are 150 W.

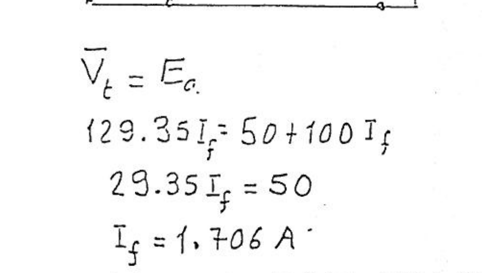
SOLUTION

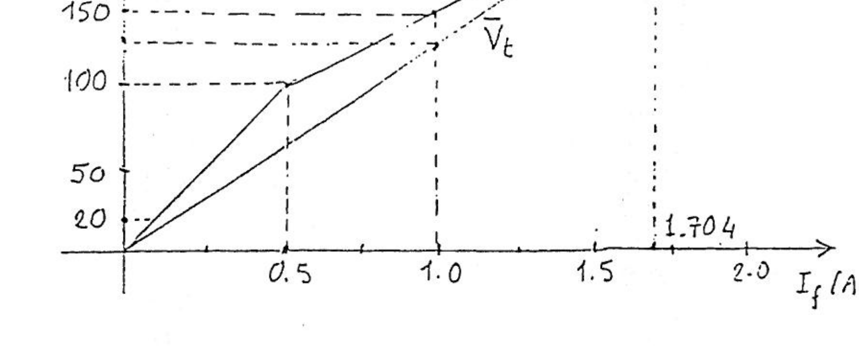
a) $R_a = 0.2\,\Omega$, $R_t = R_f + R = 129.15\,\Omega$

At no-load: $I_\ell = 0$, $I_f = I_a$

$E_a = (R_a + R_t) I_f = \bar{R} I_f = \bar{V}_t$; $\bar{R} = R_a + R_t = 129.35\,\Omega$

$I_f = 1$ A , $\bar{V}_t = 129.35$ V





$\bar{V}_t = E_a$

$129.35 I_f = 50 + 100 I_f$

$29.35 I_f = 50$

$I_f = 1.706$ A

$V_{ta} = R_t I_f = 129.15 \times 1.706 = 220$

Q.3. (35 points) A 2-pole shunt dc generator has an approximate magnetization characteristic measured at 1500 rpm as given below.

$E_a = 20$ Volts if $0 \le I_f \le 0.1$ A
$E_a = 200 I_f$ Volts if $0.1 \le I_f \le 0.5$ A
$E_a = 100 I_f + 50$ Volts if $0.5 \text{ A} \le I_f$,

where I_f is the shunt field current.

The armature has a resistance of 0.2 Ω, and the field circuit has a resistance of 129.15 Ω.

Throughout the problem the generator is driven at a constant speed of 1500 rpm by its prime mover.

Neglect armature reaction for your calculations.

(a) Assuming that the machine is operating as a self-excited shunt generator at no-load, sketch the magnetization curve and calculate the armature terminal voltage V_t.

(b) The self-excited generator is now loaded by connecting a resistive load to its armature terminals. Calculate the armature terminal voltage V_t by assuming that the resistive load carries a current of 50 A. Compute also the load resistance.

(c) The self-excited generator above is now equipped with a series field winding in order to raise the armature terminal voltage in part (b) to 220 V. The series field winding is connected in series with the load so that the machine is converted into a short-compound generator. Calculate the number of series-turns of the series field winding by assuming that the shunt field winding has 1000 turns, and the series field winding resistance is negligibly small. Define also the type of compounding used.

(d) Compute the efficiency of the self-excited generator in part (b) by assuming that friction and windage losses are 150 W.

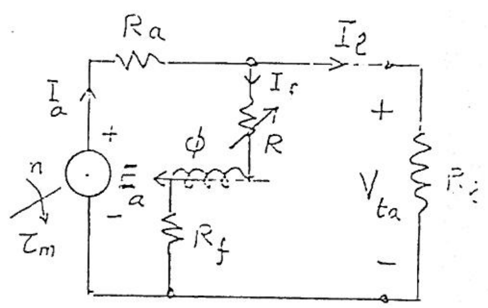
SOLUTION

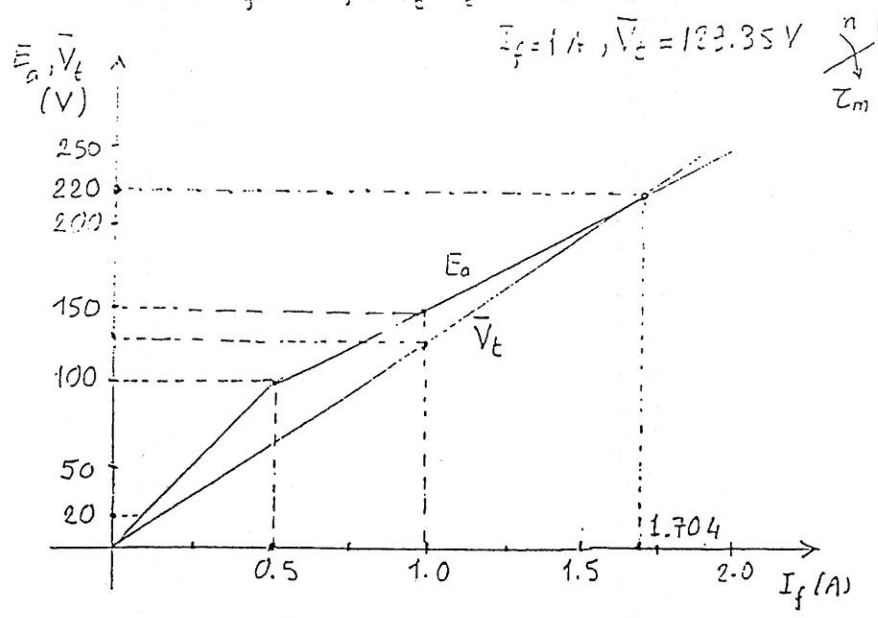
a) $R_a = 0.2\ \Omega$, $R_t = R_f + R = 129.15\ \Omega$

At no-load: $I_L = 0$, $I_f = I_a$

$E_a = (R_a + R_t) I_f = \bar{R} I_f = \bar{V}_t$; $\bar{R} = R_a + R_t = 129.35\ \Omega$

$I_f = 1\text{ A}$, $\bar{V}_t = 129.35\text{ V}$





$\bar{V}_t = E_a$

$129.35 I_f = 50 + 100 I_f$

$29.35 I_f = 50$

$I_f = 1.706\text{ A}$

$V_{ta} = R_t I_f = 129.15 \times 1.706 = 220$

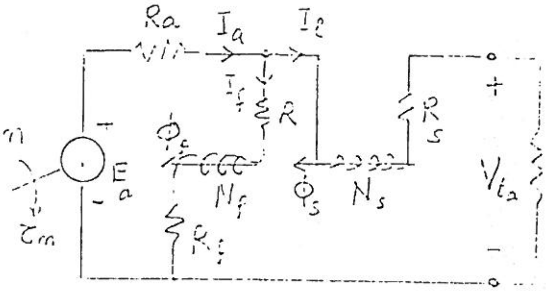
b)
$$\begin{cases} I_a = I_\ell + I_f = 50 + I_f \quad , \ I_\ell = 50\text{ A} \\ V_{ta} = R_t I_f = 129.15\, I_f \\ V_{ta} = E_a - R_a I_a = (50 + 100 I_f) - 0.2 I_a = 50 + 100 I_f - 0.2(50 + I_f) = 99.8 I_f + 40 \end{cases}$$

$129.15\, I_f = 99.8\, I_f + 40 \rightarrow \quad I_f = \dfrac{40}{29.35} = 1.363\text{ A} \quad , \ I_a = 51.363\text{ A}$

$V_{ta} = I_f * R_t = 176\text{ V} \quad , \quad R_\ell = \dfrac{V_{ta}}{I_\ell} = 3.52\ \Omega$

c) $N_f = 1000$ trn, $R_s \simeq 0$, $V_{ta} = 220$ V

Additive: $\phi : \phi_f + \phi_s$



$R_\ell = 3.52\ \Omega \quad , \ I_\ell = \dfrac{220}{3.52} = 62.5\text{ A}$

$I_f = \dfrac{220}{129.15} = 1.703\text{ A}$

$I_a = I_\ell + I_f = 64.20\text{ A}$

$E_a = V_{ta} + R_a I_a = 220 + 0.2 * 64.2 = 220 + 12.84 = 232.84\text{ V}$

The required total equivalent shunt current $\bar{I}_f$

$232.84 = 100\, \bar{I}_f + 50, \quad \bar{I}_f = \dfrac{232.84 - 50}{100} \cong 1.83\text{ A}.$

The corresponding required total shunt mmf is:

$N_f \bar{I}_f = 1000 * 1.83 = 1830\text{ At}$

The shunt field provides $1000 * 1.703 = 1703$ At

The difference $\delta F = 1830 - 1703 = 127$ At should additionally be provided by the series field winding.

$N_s I_\ell = 127, \quad N_s \cong \dfrac{127}{62.5} = 2$ turns

d) $P_{out} = V_{ta} * I_\ell = 176 * 50 = 8800\text{ W}$

$P_f = 129.15 * 1.363^2 = 239.9\text{ W} \quad , \ P_a = 0.2 * 51.363^2 = 527.63\text{ W}$

$P_{in} = P_{out} + P_f + P_r + P_{fw} = 8800 + 239.9 + 527.63 + 150 = 9717.53\text{ W}$

$\text{Efficiency} = \dfrac{8800}{9717.53} = 90.6\%$

Q.4. (60 points) A 200-V 2-pole separately excited dc motor drives a load with a constant torque demand of 191 Nm. The field winding has $N_f = 1000$ turns.

The magnetization characteristic of the machine is obtained at 1000 rpm, and is approximated by a straight line, $E_a = 200 I_f$ Volts.

Armature resistance R_a is 0.2 Ω, friction and windage torque is negligibly small, and the field excitation is kept constant at $I_f = 1$ A throughout the problem.

(a) Show that the shaft speed versus torque characteristic of the separately excited dc motor operating at steady-state is given by: $\omega = \dfrac{V_t}{K_a\phi} - \dfrac{R_a}{(K_a\phi)^2}T$,

where ω is the angular shaft speed, V_t is the applied motor voltage, ϕ is the field flux per pole, K_a is the motor constant, and T is the torque.

(b) Calculate the no-load shaft-speed in rpm for $V_t = 200$ V.

(c) With the same terminal voltage, if, now, the constant-torque load of 191 Nm is applied to the shaft, what will the new shaft speed be? Neglect armature reaction. Comment on the result.

(d) If the speed is to be reduced to 450 rpm by armature voltage control, what should the new value of the applied motor voltage be? Briefly compare speed control techniques by varying the armature voltage and field current.

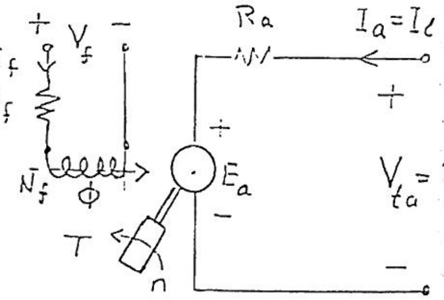
(e) Suppose now that the speed is to be increased to the no-load value found in part (b) while the machine is operating as in part (c) (V_t = 200 V, T_L = 191 Nm), by equipping the motor with a series field winding of negligible resistance. Determine the type of compounding and the number of turns of the series field winding to be inserted.

(f) Find the shaft-speed for the operating condition in part (c) (V_t = 200 V, T_L = 191 Nm) by assuming that the demagnetizing effect of the armature reaction is AR = $2I_a$ Ampere-turns. Briefly discuss the effect of armature reaction on the induced armature emf.

(g) Assume now that the applied motor voltage is suddenly reduced to 100 V, while the motor is operating as in part (c) (T_L = 191 Nm) with armature reaction neglected. Determine the directions and magnitudes of armature current, electromechanical torque and electromechanical power just after this operation by assuming that the speed remains momentarily constant. Also identify the mode of operation of the machine in this case.

SOLUTION

$R_a = 0.2\,\Omega$, $I_f = 1\,A = \text{const}$, $E_a = 200 I_f$ at $n = 1000$,



a) $V_{ta} = R_a I_a + E_a$

$E_a = K_a \phi \omega$

$T = K_a \phi I_a \longrightarrow I_a = \dfrac{T}{K_a\phi}$

$$V_{ta} = R_a \frac{T}{K_a\phi} + K_a\phi\omega \rightarrow V_{ta} - R_a\frac{T}{K_a\phi} = K_a\phi\omega \rightarrow \omega = \frac{V_{ta}}{K_a\phi} - R_a\frac{V_{ta}}{(K_a\phi)^2}$$

b) For $I_f = 1\,A$, $n = 1000$ rpm $\rightarrow E_a = 200 \times 1 = 200\,V = V_{ta}$ at no-load

Thus; $n_o = 1000$ rpm

c) At $n_o = 1000$ rpm $\Rightarrow \omega_o = 2\pi \dfrac{1000}{60} = 104.72$ and $I_f = 1\,A$, $\mathcal{F}_f = N_f I_f = 1000$

$E_a = 200 \times 1 = K_a \phi_f \omega_o$, $K_a\phi_f = \dfrac{200}{104.72} = 1.91$, $T = K_a \phi_f I_a = 191.$ Nm

$I = \dfrac{191}{\ldots} = 100\,A$. $E_a = K_a \phi_f \omega$. $V_{ta} = E_a + R_a I_a$. $E = 200 - 0.2 \times 100 = 180\,V$

$I_f = 1\,A = \text{const}$, $E_a = 200\,V$ at $n = 1000$ rpm
" $E_a = 180\,V$ at $n = ?$ $\Big]$ $n = 1000 \, \dfrac{180}{200} = 900$ rpm

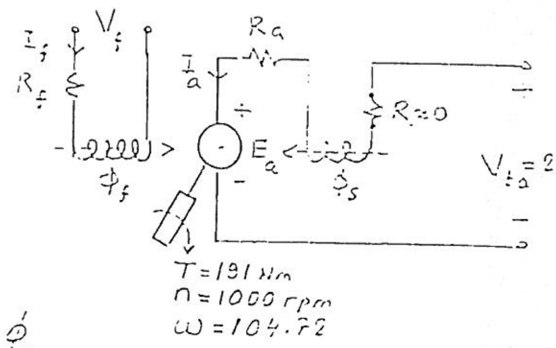
speed regulation $= \dfrac{1000 - 900}{900} = 11.1\,\%$. satisfactory.

d) $I_f = 1\,A$, $n = 1000$ rpm $\rightarrow E_a = 200\,V$
$I_f = 1\,A$, $n = 450$ rpm $\rightarrow E_a = ?$ $\Big\}$ $E_a = \dfrac{450}{1000} * 1000 = 90\,V$

$T = 191$ Nm $\longrightarrow I_a = 100\,A$ (part c) , $V_{ta} = E_a + R_a I_a = 90 + 0.2 * 100 = 110\,V$

e) $n = n_o = 1000$ rpm , $V_{ta} = 200\,V$,

$I_a = 100\,A$, $T = 191$ Nm , $I_f = 1\,A$



$E_a = V_{ta} - R_a I_a = 200 - 0.2 * 100 = 180\,V$

$E_a = K_a \phi' \omega$, $\phi' = \phi_f \mp \phi_s$

$(K_a \phi') = \dfrac{180}{104.72} = 1.719$

$\delta(K_a \phi) = 1.91 - 1.719 = -0.191 \rightarrow \phi' = \phi_f - \phi_s$

(subtractive). Reduction in the generated voltage $= \delta E_r = 200 - 180 = 20$

At $n = 1000$ rpm $I_f = 1$, $N_f I_f = \mathcal{F} = 1000$ At $\rightarrow E_a = 200\,V$
" $\delta\mathcal{F}$ $\rightarrow \delta E_a = 20\,V$ $\Big]$ $\delta\mathcal{F} = \dfrac{20}{200} \times 1000 = 100$

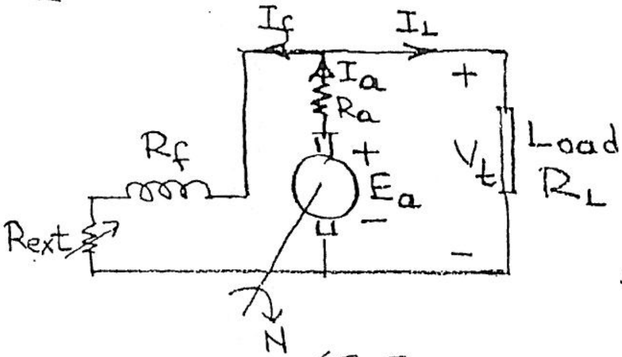
$\delta\mathcal{F} = N_f I_f = N_s I_a = 100$ At , $N_s = \dfrac{100}{100} = 1$ turn

f) $V_{ta} = 200\,V$, $I_a = 100\,A$, $T = 191$ Nm , $I_f = 1\,A$, $E_a = 180\,V$
$n = ?$, $K_a \phi = ?$, $E_a = ?$
$T = K_a \phi I_a = K \mathcal{F} I_a$, $E_a = K_a \phi \omega = K \mathcal{F} \omega$, $\mathcal{F} \rightarrow$ mmf
For $I_f = 1$, $N_f = 1000$, $\mathcal{F} = 1000$, $T = 191 = K \times 10^3 * 10^2$, $K = 191 * 10^{-5}$
mmf including armature reaction $= N_f I_f - 2 I_a = 10^3 - 2 I_a$
$191 = 191 \times 10^{-5} (10^3 - 2 I_a) I_a \Rightarrow 1 = 10^{-5} (10^3 - 2 I_a) I_a \rightarrow$
$2 I_a^2 - 10^3 I_a + 10^5 = 0$ $\quad I_a = \dfrac{10^3 \mp (10^6 - 8 \times 10^5)^{1/2}}{4} = \dfrac{10^3 \mp 447.2}{4} = \begin{cases} I_a = 138.2\,A \\ I_a = 361.8\,A \end{cases}$

$I_a = 138.2\,A$, $E_a = V_{ta} - R_a I_a = 200 - 138.2 * 0.2 = 172.36\,V$
$\mathcal{F} = (10^3 - 2 * I_a) = (10^3 - 2 * 138.2) = 723.6$
$\omega = \dfrac{E_a}{K \mathcal{F}} = \dfrac{172.36}{191 \times 10^{-5} * 723.6} = 124.71 \rightarrow n = \dfrac{60\,\omega}{2\pi} = \dfrac{60 * 124.71}{2 * \pi} = 1191$ rpm

g) $n = 1000$ rpm , $E_a = 180\,V$, $K_a \phi = \text{const} = 1.91$
$V_t = 100\,V < E_a$, I_a and T changes direction
$E_a = V_t + R_a I_a$, $I_a = (180 - 100)/0.2 = 400\,A$, $P_{out} = 100 * 400 = 40$ kW
$T = K_a \phi I_a = 1.91 * 400 = 764$ Nm re-generative braking (recuperation)
(generator operation).

Ex. Self-excited shunt dc generator



Assumptions: R_a, R_f, R_{ext} known

I_L, E_a vs I_f Ch. given

Find E_a, I_f and V_t

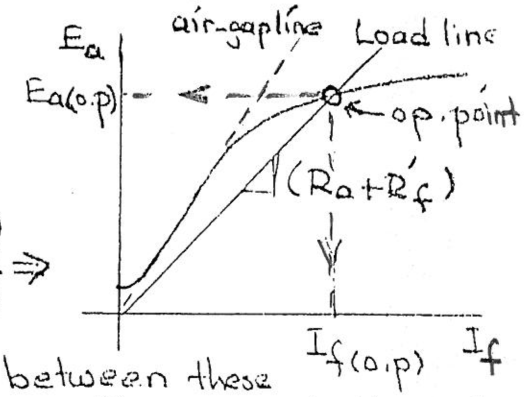
At no-load: $I_L = 0$ and hence

Net output power $= V_t I_L = 0$

$(E_a I_a = (R_f + R_{ext}) I_f^2)$

$\Rightarrow I_a = I_f$

$$\left.\begin{aligned} V_t &= E_a - R_a I_a \\ &= E_a - R_a I_f \\ V_t &= (R_f + R_{ext}) I_f \\ &= R_f' I_f \end{aligned}\right\} \Rightarrow \left.\begin{aligned} & R_f' I_f = E_a - R_a I_f \\ & E_a = (R_a + R_f') I_f \\ & E_a \text{ vs } I_f \text{ given} \end{aligned}\right\} \Rightarrow$$

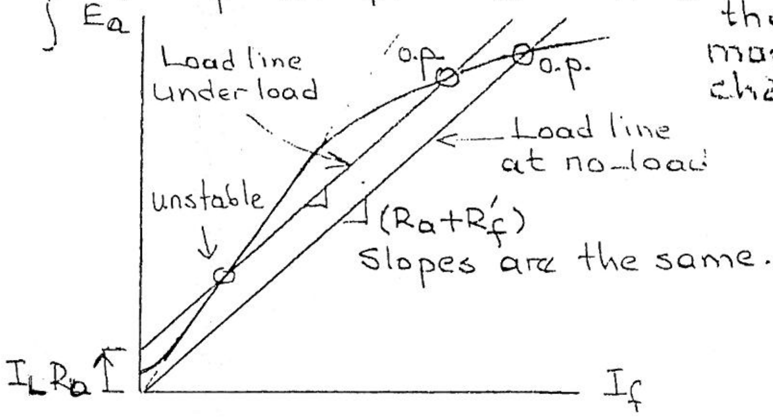


Intersection point between these two chs is the operation point in the s.s

Under load: $I_L > 0 \Rightarrow V_t I_L > 0$, $I_a = I_L + I_f$

$$\left.\begin{aligned} V_t &= E_a - R_a (I_f + I_L) \\ V_t &= I_f R_f' \end{aligned}\right\} \Rightarrow I_f (R_a + R_f') = E_a - R_a I_L$$

Plot it on the given magnetization characteristic



Therefore,

$V_t = E_a - I_a R_a$

Comment on the result: Load connected to the armature terminals causes a drop in terminal voltage.

Ex. If R_L is known instead of I_L, compute E_a, I_f and V_t.

$$\begin{aligned} V_t &= E_a - R_a I_a \\ &= E_a - R_a (I_f + I_L) \end{aligned}$$

Since $I_f = V_t / R_f'$ and $I_L = V_t / R_L$

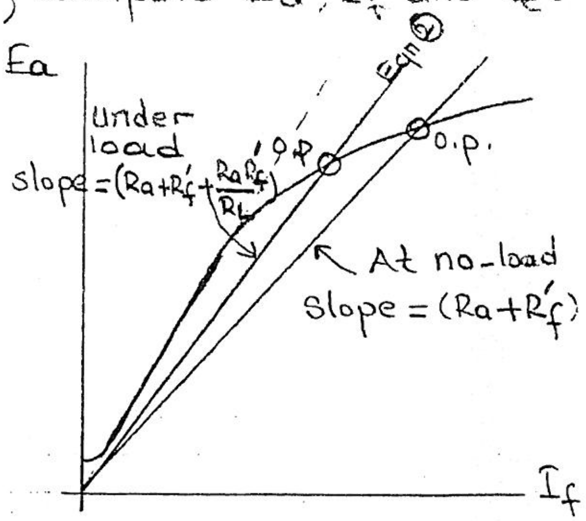
then $V_t = E_a - R_a \left(\frac{V_t}{R_f'} + \frac{V_t}{R_L} \right)$

by rearranging $V_t = E_a \left[\frac{R_f' R_L}{(R_f' R_L + R_a R_L + R_f' R_a)} \right] \ldots (1)$

Replace V_t in Eqn (1) by $V_t = R_f' I_f$

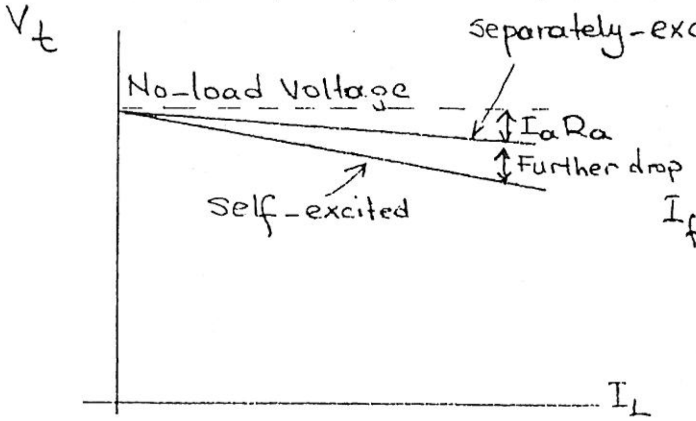
This substitution yields:

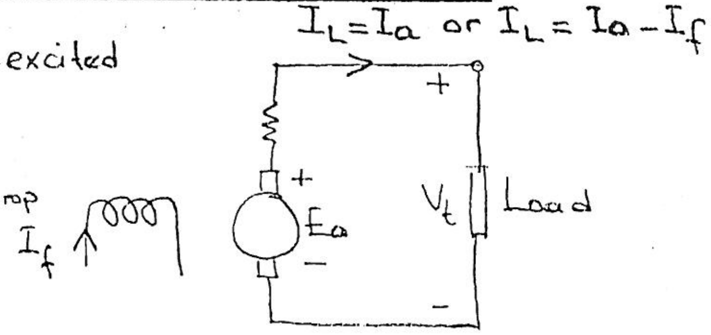
$$E_a = \left[R_f' + R_a + \frac{R_f' R_a}{R_L} \right] I_f \ldots (2)$$



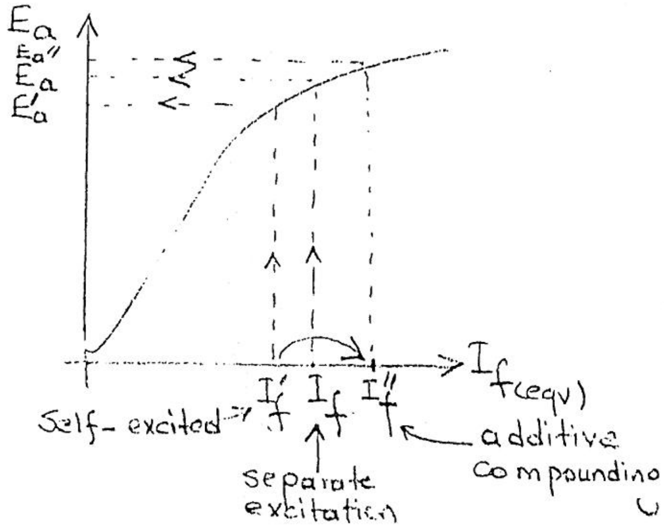
Find the intersection point between E_a vs I_f and Eqn (2).

A Comparison of dc generator terminal characteristics

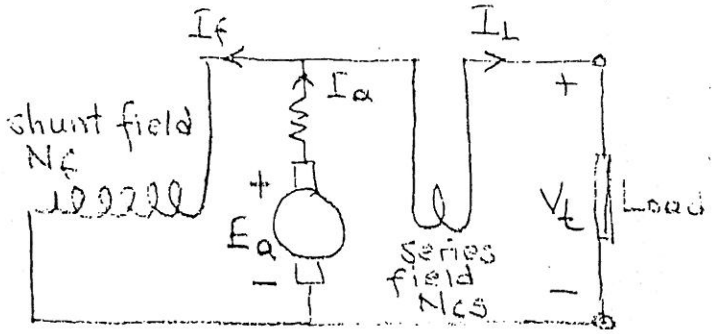




For a self-excited dc generator as I_L is increased V_t falls further because as V_t decreases $I_f = V_t/R_f$ and hence E_a decreases



To compensate for the fall in terminal voltage as Load is increased, additive (cumulative) Compounding can be used.



Separate excitation: $V_t = E_a - I_aR_a$

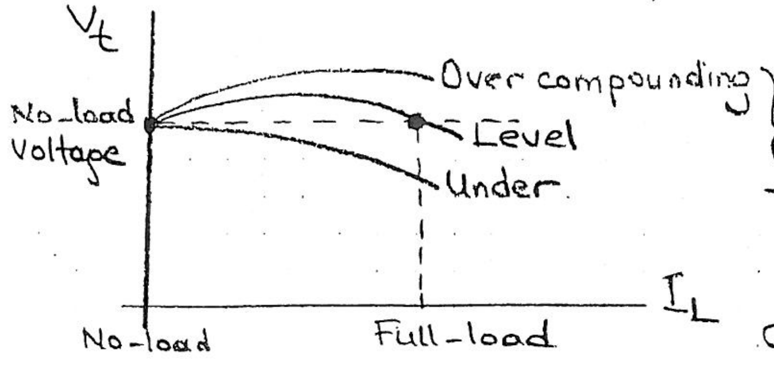
Self-excitation: $V_t' = E_a' - I_aR_a$

GROSS MMF $\simeq N_fI_f + N_{cs}I_a$ ($I_L \simeq I_a$)

Equivalent shunt field amps, $I_{f(equiv)} \simeq I_f + \frac{N_{cs}}{N_f} I_a$

Cumulative Comp: $V_t'' = E_a'' - I_aR_a$

By using cumulative compounding the terminal characteristic of a self-excited generator can be approximated to that of an ideal direct voltage source

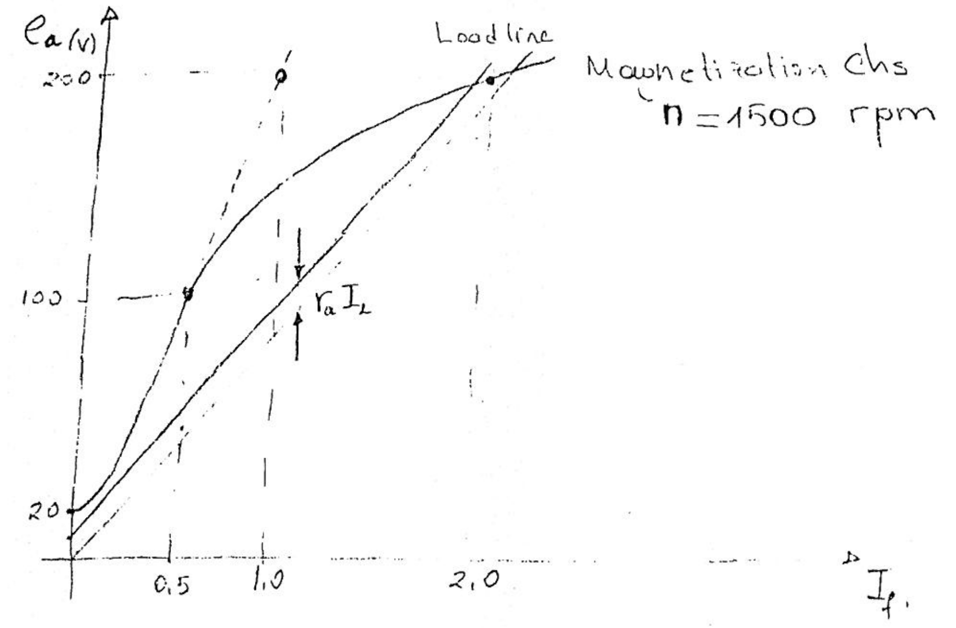


Depending on the strength of the series field wdg relative to shunt field wdg.

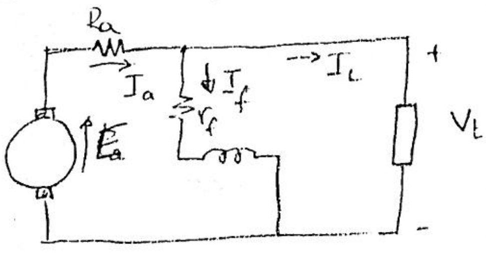
Since the fall in V_t is drastic as I_L is increased, differential Compounding is never used.

Example

A shunt generator has a magnetization characteristic at n=1500r, as shown in the figure below. The armature has a resistance of 0.2 Ω, and the field has a total resistance of 100Ω. a) Find the terminal voltage and the field current of the generator, when it delivers 50 A to a resistive load. b) Find the terminal voltage and the field current when the load is disconnected.



Solution



Terminal voltage eqn. $\quad E_a - r_a I_a = V_t \qquad (1)$

Also

$$r_f I_f = V_t \qquad (2)$$

$$I_a = I_f + I_L \qquad (3)$$

From eqns. (1), (2) and (3)

$$E_a = r_a (I_f + I_L) + r_f I_f$$

$$\boxed{E_a = (r_a + r_f) I_f + r_a I_L}$$

$$= 100.2\, I_f + 10 \qquad (4)$$

eqn (4) is the load line. Solutions for e_a and I_f are at the intersection point of the magnetization characteristic with the load-line.

$$e_a \simeq 200\ V \qquad I_f \simeq 1.9\ A$$

$$I_a = 51.9\ A \Rightarrow V_t = 200 - (0.2)\times 51.9 \simeq 190\ V.$$

b) When the load is disconnected, the load line becomes

$$e_a = 100.2\ I_f \qquad (E_a = (R_a + R_f) I_f)$$

$$\Rightarrow I_f \simeq 2.1\ A \Rightarrow e_a = 210\ V$$

$$V_t = e_a - I_f . r_a \simeq 210\ V$$

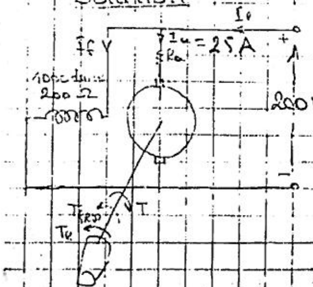
One important consequence of armature reaction is a reduction in the field flux ϕ_d. Since E_a is directly proportional to $K_a \phi_d \omega_m$, a reduced flux will cause the induced armature emf E_a to decrease. The effect of armature reaction then is approximately the same as a demagnetizing mmf AR acting on the main field axis i.e. d-axis.

The net direct-axis mmf is then assumed to be

$$\text{Net mmf} = \text{Gross mmf} - AR$$
$$= N_f I_f \pm N_s I_s - AR$$

Example. A 200 V 7.5 HP d.c. motor drives a no-load at 1500 RPM. At this speed the no-load rotational losses are 275 Watts and armature current = 25 Amp. Demagnetizing armature reaction mmf acting on d-axis is given by $AR = 4 \times I_a$ AT. 1000 turn field wdg has a resistance of 200 Ohms. You are given the magnetization curve (E_a vs F) of the m/c. Find E_a and net output power in hp.

Solution



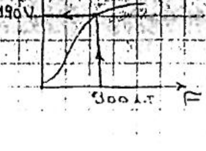
$$I_f = \frac{200\ V}{200\ \Omega} = 1\ \text{Amp}$$

$$\text{Gross mmf} = N_f I_f = 1000 \times 1 = 1000\ AT$$

$$\text{Net mmf} = \text{Gross mmf} - AR$$
$$= N_f I_f - AR$$
$$= 1000 - 4 \times 25$$
$$= 900\ AT$$

From the magnetization curve

When Net mmf = 900 AT, $E_a = 190$ Volts.



Gross output power $= E_a I_a = 190 \times 25$

$= 4750$ Watts (Energy converted to mechanical form)

Net output power $= 4750 -$ No-load rotational losses $= 4750 - 275$

$= 4475$ Watts $= \frac{4475}{746} = 6$ H.P.