DERIVATION OF MOSFET I_{DS} VS. V_{DS} + V_{GS}

Derive the current expressions in the MOSFET:

Linear Region: $I_D = \mu C_{ox} \frac{W}{L}[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}]$

Saturation Region: $I_D = \mu C_{ox} \frac{W}{2L}(V_{GS} - V_{TH})^2$

1. Linear Region

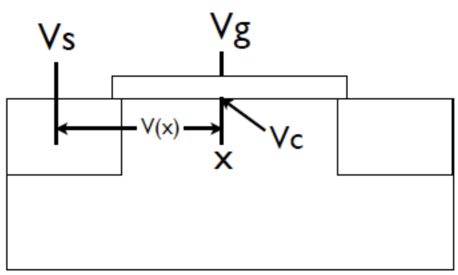

Figure 1. Concentration Contours in Linear Region. A uniform narrow channel exists.

KVL:

$$V_G - V_S = V_G - V_C + V_C - V_S$$

$$V_G - V_S = V_{GS}$$

$$V_G - V_C = V_{GC}$$

$$V_C - V_S = V(x)$$

$V_{GS} = V_{GC} + V(x)$ or $V_{GS} - V(x) = V_{GC}$

Total charge density at x on capacitor(C_{OX}) is $Q_T(x)$:

$Q_T(x) = V_{GC}C_{OX} = (V_{GS} - V(x))C_{OX}$

$Q_T(x) = Q(x)_{mobile} + Q(x)_{depletion}$

$Q(x)_{mobile}$=mobile electron charge in channel at x

$Q(x)_{mobile} = [V_{GS} - V(x) - V_{TH}]C_{OX}$

Use mobile charge to get current:

$J_n = q\mu nE + qD_n\frac{dn}{dx} = q\mu nE$ (no diffusion current in the channel)

$qn(x) = Q(x)_{mobile} = Q_m(x)$

$J_n = Q_m(x)\mu E$, but $E = -\frac{dV}{dx}$

$J_n = -Q_m(x)\mu\frac{dV}{dx}$, substitute for $Q_m(x)$

$J_n = \mu C_{ox}(V_{GS} - V(x) - V_{TH})\frac{dV}{dx}$, separate variables and neglect (-) sign. Consider only the magnitude.

$J_n dx = \mu C_{ox}[(V_{GS} - V_{TH}) - V(x)]dV$

Due to continuity, $Jn = constant$ (no hole current or no generation, recombination). Integrating from source to drain or from x=0 to x=L, where L=gate length:

$J_n \int_0^L dx = \mu C_{ox} \int_{V(0)}^{V(L)} [(V_{GS} - V_{TH}) - V(x)]dV$

V(L) = VDS , V(0)=0

$J_n \int_0^L dx = \mu C_{ox} \int_0^{V_{DS}} [(V_{GS} - V_{TH}) - V(x)]dV$

$J_n L = \mu C_{ox}[(V_{GS} - V_{TH})V - \frac{V^2}{2}]_0^{V_{DS}}$

$J_n = \frac{\mu C_{ox}}{L}[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}]$

$I_D = J_n W$ (W=Device Width)

J_n for channel is Amp/cm since $Q_m = Charge/cm^2$

I_D for Linear Region: $I_D = \mu C_{ox}\frac{W}{L}[(V_{GS} - V_{TH})V_{DS} - \frac{V_{DS}^2}{2}]$

2. Saturation Region

When $V_{DS} \geq (V_{GS} - V_{TH})$ channel pinches off. This means that the channel current near the drain spreads out and the channel near drain can be approximated as the depletion region. After this occurs, at $V_{DS} = (V_{GS} - V_{TH})$, if you make V_{DS} larger, the current I_D does not change (to zero approximation). This is because any additional V_{DS} you add will get dropped across the depletion region and won't change the current I_D.

So for $V_{DS} \geq (V_{GS} - V_{TH})$ we find I_D by setting $V_{DS} = (V_{GS} - V_{TH})$ substituting into the linear equation.

$I_D = \mu C_{ox}\frac{W}{L}[(V_{GS} - V_{TH})(V_{GS} - V_{TH}) - \frac{(V_{GS}-V_{TH})^2}{2}]$

I_D for Saturation Region:

$I_D = \mu C_{ox}\frac{W}{2L}(V_{GS} - V_{TH})^2$

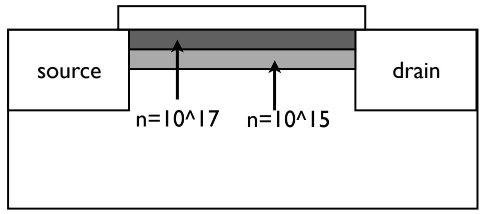

FIGURE 2. Concentration Contours in Linear Region. A uniform narrow channel exists.

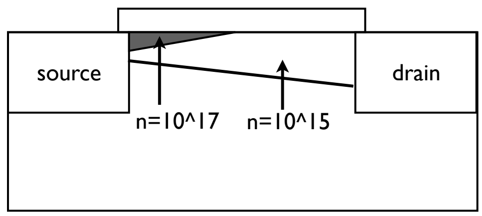

FIGURE 3. Concentration Contours in Saturation Region. Channel narrow near source and spreads out and widens near drain, said to be "pinched off".