Bipolar transistors

- **Two port representation of the bipolar transistor**
- **Models of bipolar transistors**

Objectives:

Systems of r, g and h parameters of BJTs

Meanings of r, g and h parameters

Finding (calculation) of h and other parameters

Methodology of composing of the T-type equivalent circuit for a BJT in its CB configuration

Calculation of parameters of thr equivalent circuit and application of the circuit

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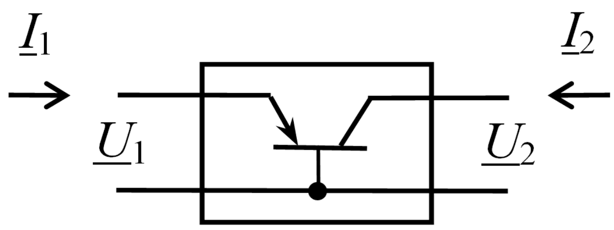
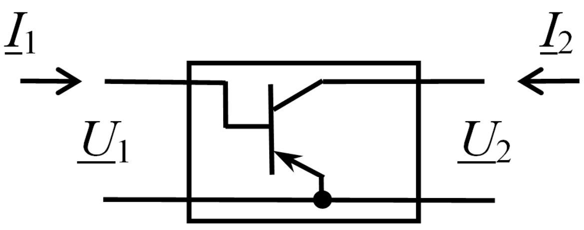
Two port representation of a BJT

1. *I-U* characteristics of BJTs are not linear. The powerfull tools of linear circuit analysis cannot be used.
2. When transistor is used for processing of small signals, only small parts of non-linear characteristics are used. Then the linear approximations of the characteristics at the quiescent operating points are possible.
3. **At processing of small signals, the linear models of BJTs can be used.**



(a)



(b)

Sets of equations, describing BJTs

$$\underline{U}_1 = \underline{Z}_{11}\underline{I}_1 + \underline{Z}_{12}\underline{I}_2 \qquad \underline{U}_1 = r_{11}\underline{I}_1 + r_{12}\underline{I}_2$$

$$\underline{U}_2 = \underline{Z}_{21}\underline{I}_1 + \underline{Z}_{22}\underline{I}_2 \qquad \underline{U}_2 = r_{21}\underline{I}_1 + r_{22}\underline{I}_2$$

$\underline{Z}$ – impedance;
r – resistance

$$\underline{I}_1 = \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \qquad \underline{I}_1 = g_{11}\underline{U}_1 + g_{12}\underline{U}_2$$

$$\underline{I}_2 = \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \qquad \underline{I}_2 = g_{21}\underline{U}_1 + g_{22}\underline{U}_2$$

$\underline{Y}$ – admittance;
g – conductance

$$\underline{U}_1 = \underline{H}_{11}\underline{I}_1 + \underline{H}_{12}\underline{U}_2 \qquad \underline{U}_1 = h_{11}\underline{I}_1 + h_{12}\underline{U}_2$$

$$\underline{I}_2 = \underline{H}_{21}\underline{I}_1 + \underline{H}_{22}\underline{U}_2 \qquad \underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{U}_2$$

$\underline{H}$, h – hybrid parameters

r parameters

$$\underline{U}_1 = r_{11}\underline{I}_1 + r_{12}\underline{I}_2$$

$$\underline{U}_2 = r_{21}\underline{I}_1 + r_{22}\underline{I}_2$$

$$r_{11} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{I}_2 = 0}$$ – the input resistance when output is open

$$r_{12} = \left.\frac{\underline{U}_1}{\underline{I}_2}\right|_{\underline{I}_1 = 0}$$ – the reverse transfer resistance when the input is open

$$r_{21} = \left.\frac{\underline{U}_2}{\underline{I}_1}\right|_{\underline{I}_2 = 0}$$ – the forward transfer resistance when the output is open

$$r_{22} = \left.\frac{\underline{U}_2}{\underline{I}_2}\right|_{\underline{I}_1 = 0}$$ – the output resistance when the input is open

1. The open circuit conditions (for alternating current) must be arranged.
2. Difficulties arise when resistances r_{11} and r_{21} are measured.

g parameters

$$\underline{I}_1 = g_{11}\underline{U}_1 + g_{12}\underline{U}_2$$
$$\underline{I}_2 = g_{21}\underline{U}_1 + g_{22}\underline{U}_2$$

$$g_{11} = \left.\frac{\underline{I}_1}{\underline{U}_1}\right|_{\underline{U}_2 = 0}$$ – the input conductance when the output is shortened

$$g_{12} = \left.\frac{\underline{I}_1}{\underline{U}_2}\right|_{\underline{U}_1 = 0}$$ – the reverse transfer conductance when the input is shortened

$$g_{21} = \left.\frac{\underline{I}_2}{\underline{U}_1}\right|_{\underline{U}_2 = 0}$$ – the forward transfer conductance when the output is shortened

$$g_{22} = \left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{U}_1 = 0}$$ – the output conductance when the input is shortened

1. The short circuit regimes (for alternating voltage) must be arranged.
2. Difficulties arise when conductances g_{12} and g_{22} are measured.

h parameters

$$\underline{U}_1 = h_{11}\underline{I}_1 + h_{12}\underline{U}_2$$

$$\underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{U}_2$$

$$h_{11} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{U}_2 = 0}$$ – input resistance with shortened output

$$h_{12} = \left.\frac{\underline{U}_1}{\underline{U}_2}\right|_{\underline{I}_1 = 0}$$ – open circuit reverse voltage gain

$$h_{21} = \left.\frac{\underline{I}_2}{\underline{I}_1}\right|_{\underline{U}_2 = 0}$$ – short circuit forward current gain

$$h_{22} = \left.\frac{\underline{I}_2}{\underline{U}_2}\right|_{\underline{I}_1 = 0}$$ – output conductance with input open

1. There are no difficulties to arrange output shortened and input open conditions and measure *h* parameters.
2. The set of *h* parameters includes information about current gain:
   $h_{21\mathrm{B}} = -\alpha$, $h_{21\mathrm{E}} = \beta$

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Parameters of BJTs

A BJT as a linear network can be represented by three sets of parameters:

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \qquad \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \qquad \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

h parameters of a BJT are known. Derive formulas for calculation of its *r* parameters.

Parameters of BJTs

$$\underline{U}_1 = h_{11}\underline{I}_1 + h_{12}\underline{U}_2$$

$$\underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{U}_2$$

$$\underline{U}_2 = -\frac{h_{21}}{h_{22}}\underline{I}_1 + \frac{1}{h_{22}}\underline{I}_2$$

$$\underline{U}_1 = (h_{11} - \frac{h_{12}h_{21}}{h_{22}})\underline{I}_1 + \frac{h_{12}}{h_{22}}\underline{I}_2$$

$$\underline{U}_1 = r_{11}\underline{I}_1 + r_{12}\underline{I}_2$$

$$\underline{U}_2 = r_{21}\underline{I}_1 + r_{22}\underline{I}_2$$

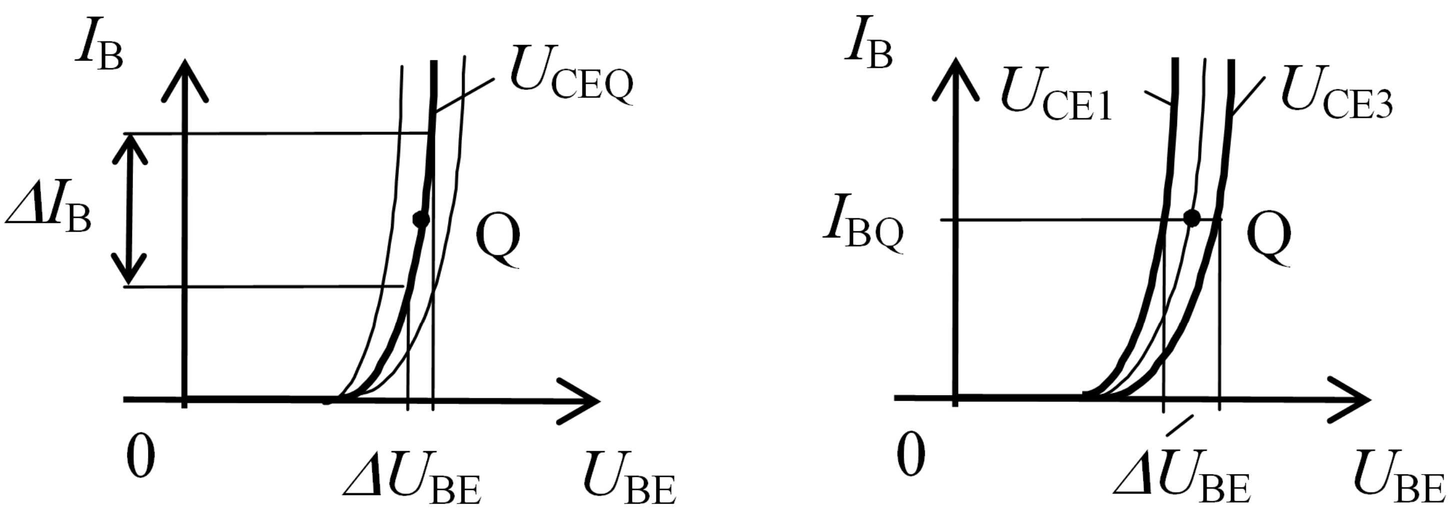
$$r_{11} = h_{11} - \frac{h_{12}h_{21}}{h_{22}}, \quad r_{12} = \frac{h_{12}}{h_{22}},$$

$$r_{21} = -\frac{h_{21}}{h_{22}}, \quad r_{12} = \frac{1}{h_{22}}.$$

Evaluation of *h* parameters

$$h_{11E} = \left.\frac{\underline{U}_1}{\underline{I}_1}\right|_{\underline{U}_2 = 0} = \left.\frac{\Delta U_{BE}}{\Delta I_B}\right|_{U_{CE} = \mathrm{const} = U_{CEQ}}$$

$$h_{12E} = \left.\frac{\underline{U}_1}{\underline{U}_2}\right|_{\underline{I}_1 = 0} = \left.\frac{\Delta U_{BE}}{\Delta U_{CE}}\right|_{I_B = \mathrm{const}} = \left.\frac{\Delta U_{BE}}{U_{CE3} - U_{CE1}}\right|_{I_B = I_{BQ}}$$



(a) (b)

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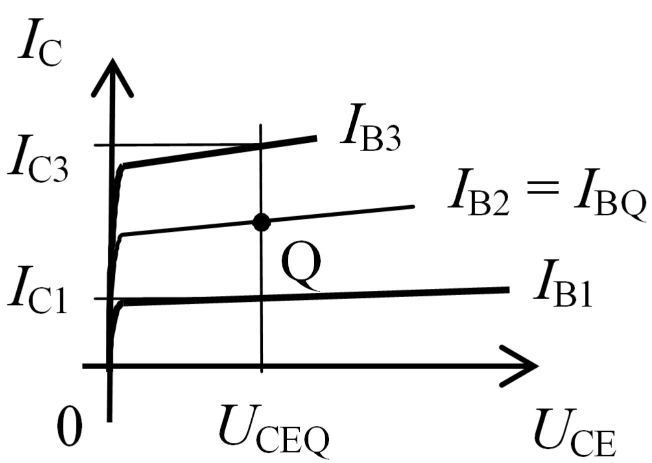
Evaluation of *h* parameters

$$h_{21E} = \frac{\underline{I}_2}{\underline{I}_1}\bigg|_{\underline{U}_2=0} = \frac{\Delta I_C}{\Delta I_B}\bigg|_{U_{CE}=\text{const}} = \frac{I_{C3}-I_{C1}}{I_{B3}-I_{B1}}\bigg|_{U_{CE}=U_{CEQ}}$$

$$h_{22E} = \frac{\underline{I}_2}{\underline{U}_2}\bigg|_{\underline{I}_1=0} = \frac{\Delta I_C}{\Delta U_{CE}}\bigg|_{I_B=\text{const}=I_{BQ}}$$



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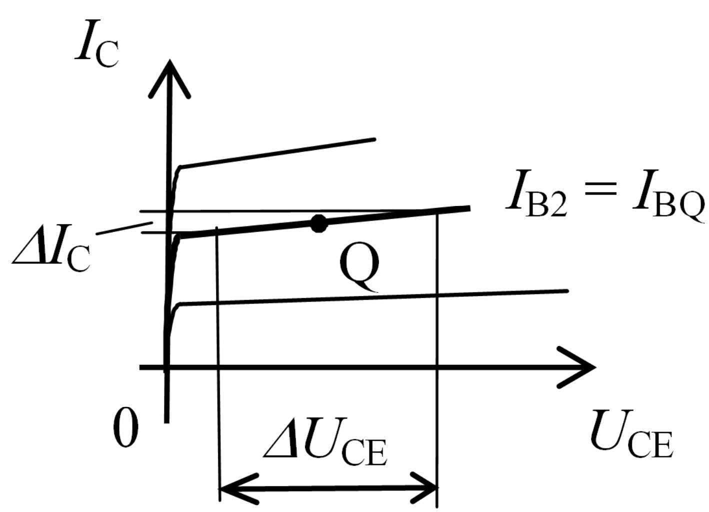
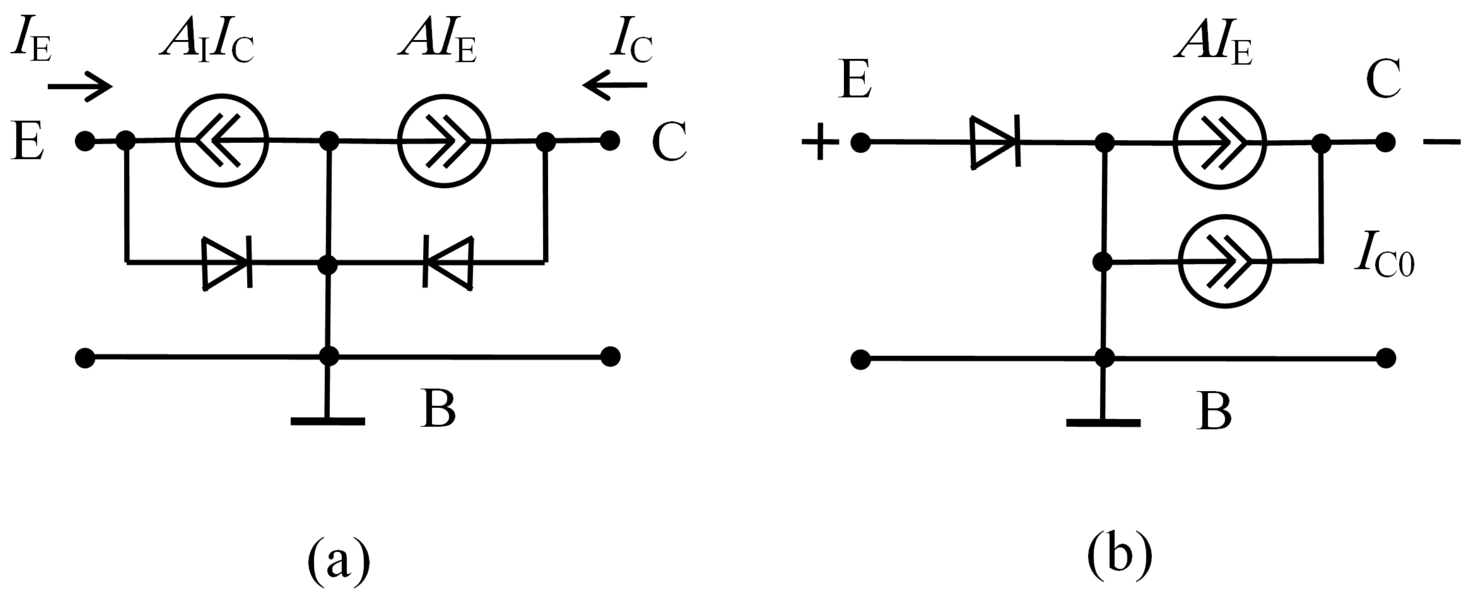


(b)

Transistor models

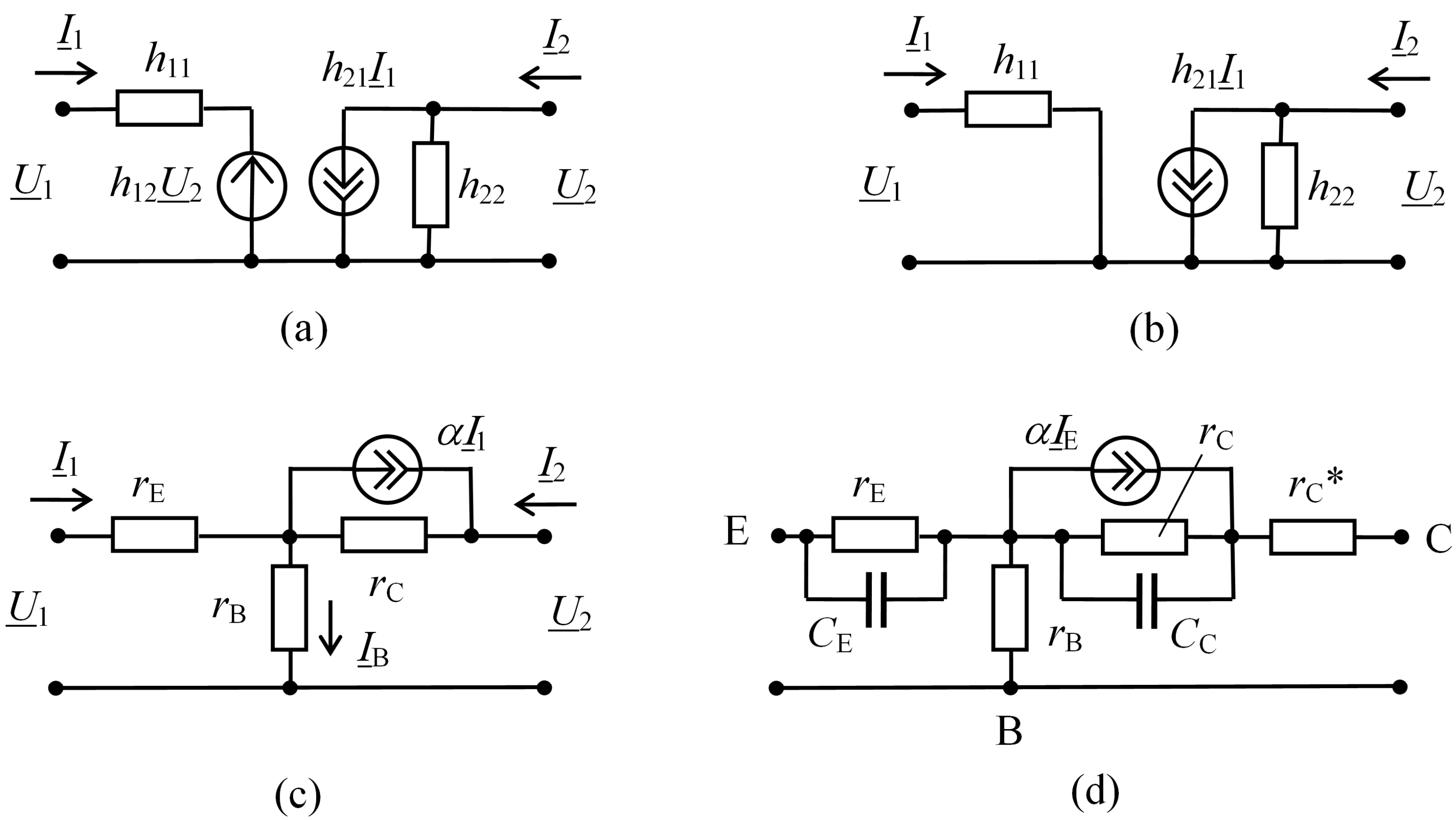


Equivalent circuits of a pnp transistor corresponding to the Ebers-Moll equations

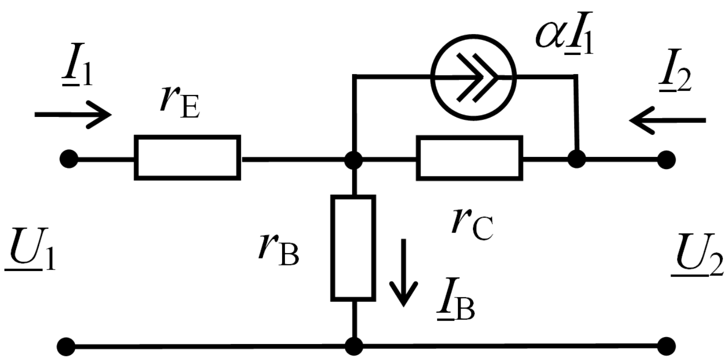
The T-type model of a BJT in its CB configuration

$$\underline{U}_1 = h_{11}\underline{I}_1 + h_{12}\underline{U}_2$$

$$\underline{I}_2 = h_{21}\underline{I}_1 + h_{22}\underline{U}_2$$



The T-type model of a BJT in its CB configuration



$$\underline{U}_1 = r_E \underline{I}_E + r_B \left(\underline{I}_E + \underline{I}_C\right)$$
$$\underline{U}_2 = r_B \left(\underline{I}_E + \underline{I}_C\right) + \underline{U}_C$$
$$\underline{U}_C = r_C \left(\underline{I}_C + \alpha \underline{I}_E\right)$$

$$\underline{U}_1 = \left(r_B + r_E\right)\underline{I}_E + r_B \underline{I}_C$$
$$\underline{U}_2 = \left(r_E + \alpha\, r_K\right)\underline{I}_B + \underline{I}_C \left(r_B + r_C\right)$$

$$\underline{U}_1 = h_{11B}\underline{I}_1 + h_{12B}\underline{U}_2$$
$$\underline{I}_2 = h_{21B}\underline{I}_1 + h_{22B}\underline{U}_2$$

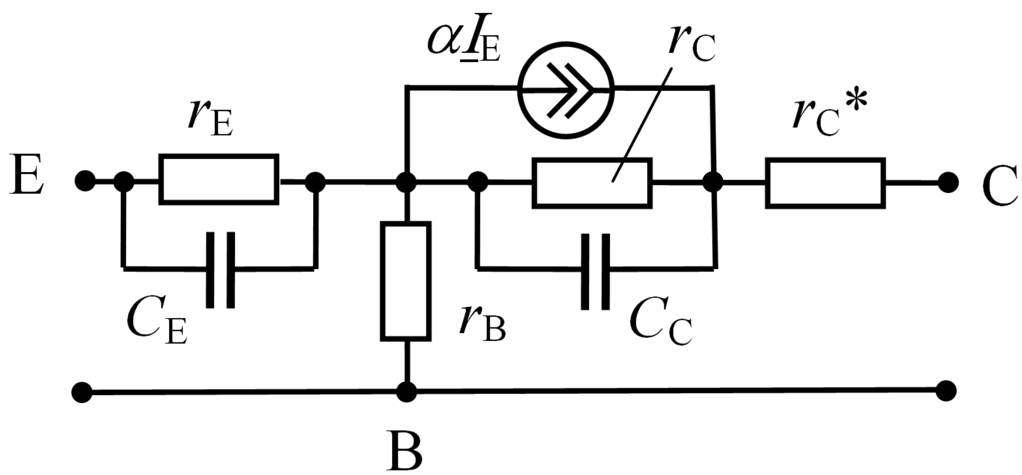
$$r_B = h_{12B} / h_{22B}$$
$$r_B + r_E = \ldots$$
$$h_{11B} = r_B(1-\alpha) + r_E$$
$$r_B + r_K = 1/h_{22B}$$

$$\underline{U}_1 = \left(h_{11B} - \frac{h_{12B}h_{21B}}{h_{22B}}\right)\underline{I}_E + \frac{h_{12B}}{h_{22B}}\underline{I}_C$$

$$\underline{U}_2 = -\frac{h_{21B}}{h_{22B}}\underline{I}_E + \frac{1}{h_{22B}}\underline{I}_C$$

$$r_E \cong \frac{kT/q}{I_{EQ}} \cong \frac{0{,}025}{I_{EQ}}$$
$$r_B \cong \beta\left(h_{11B} - r_E\right)$$
$$r_C \cong 1/h_{22B}$$

The T-type model of a BJT in its CB configuration



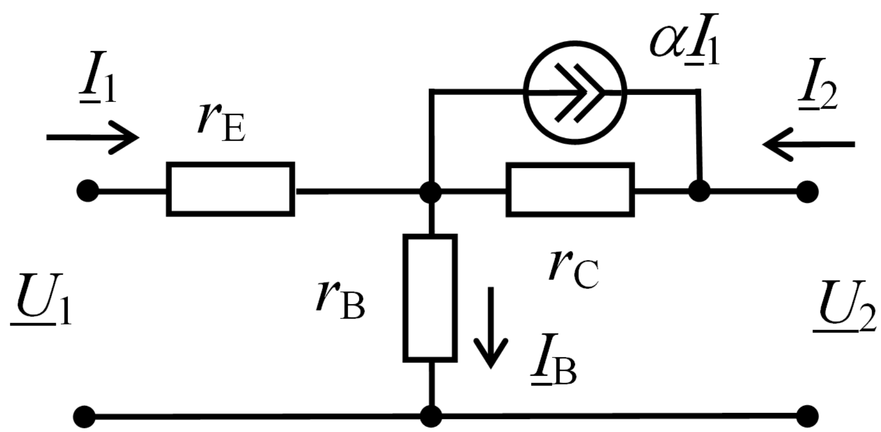
$$r_E \cong \frac{kT}{q}\frac{1}{I_{CQ}} \cong \frac{0{,}025}{I_{CQ}} \qquad r_B \cong \beta\left(h_{11B} - r_E\right) \qquad r_C \cong 1/h_{22B} \cong \beta\, r_{CE}$$

$$\tau_\alpha = \frac{1}{2\pi f_T} \qquad \tau_B \cong \tau_\alpha - \tau_C \qquad \tau_C = r_B C_C \qquad C_E = \tau_B / r_E$$

The T-type model of a BJT in its CB configuration

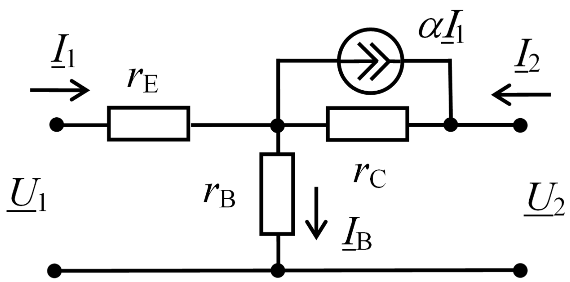
A BJT is in its CB configuration. According to *I-U* characteristics *h* parameters were found at the given Q point (emitter current – 15 mA, collector-base voltage – 5 V): input resistance 4 Ω, emitter current gain – 0,99, output conductance – 50 μS. Sketch the T-type model of the transistor and find parameters of equivalent circuit elements.

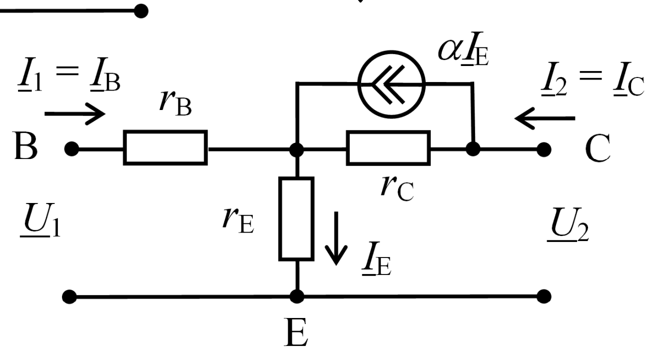
The T-type model of a BJT in its CB configuration



$$r_E \cong \frac{kT}{q}\frac{1}{I_{CQ}} \cong \frac{0{,}025}{I_{CQ}} \qquad r_B \cong \beta\left(h_{11B} - r_E\right) \qquad r_C \cong 1/h_{22B} \cong \beta\, r_{CE}$$

The T-type model of a BJT in its CE configuration





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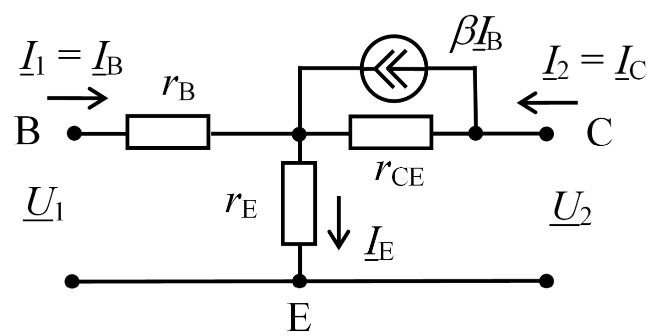


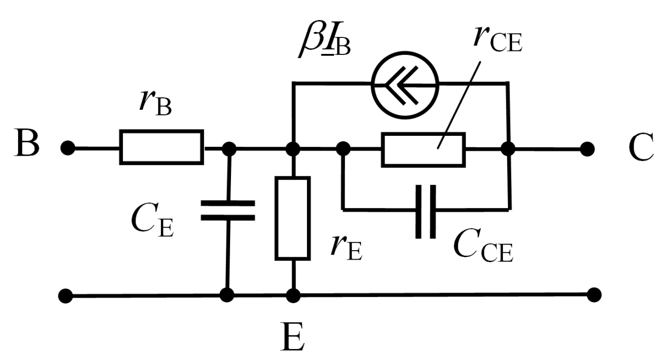
(b)

$$\underline{U}_K = r_K\left(\underline{I}_K - \alpha \underline{I}_E\right)$$

$$\underline{U}_K = r_K(1-\alpha)\left(\underline{I}_K - \frac{\alpha}{1-\alpha}\underline{I}_B\right) =$$

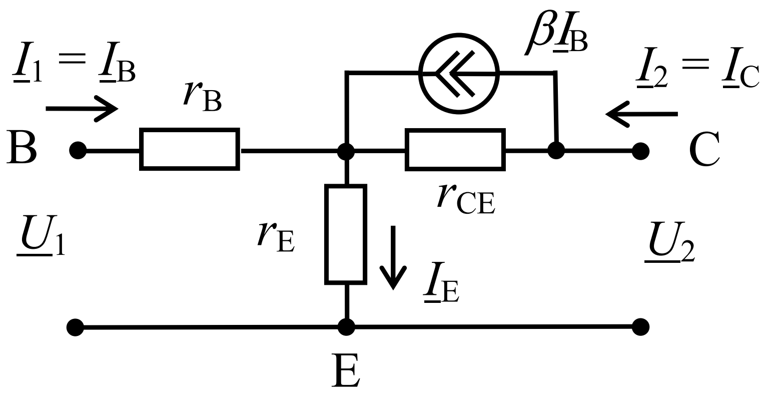
$$= r_{KE}\left(\underline{I}_K - \beta \underline{I}_B\right)$$

$$r_{KE} = r_K(1-\alpha)$$



(c)

The T-type model of a BJT in its CE configuration



$$\underline{U}_1 = r_B \underline{I}_B + r_E (\underline{I}_B + \underline{I}_C)$$
$$\underline{U}_2 = r_E (\underline{I}_B + \underline{I}_C) + \underline{U}_C$$
$$\underline{U}_C = r_{CE} (\underline{I}_C - \beta \underline{I}_B)$$

$$\underline{U}_1 = (r_B + r_E)\underline{I}_B + r_E \underline{I}_C$$
$$\underline{U}_2 = (r_E - \beta r_{CE})\underline{I}_B + \underline{I}_C (r_E + r_{CE})$$

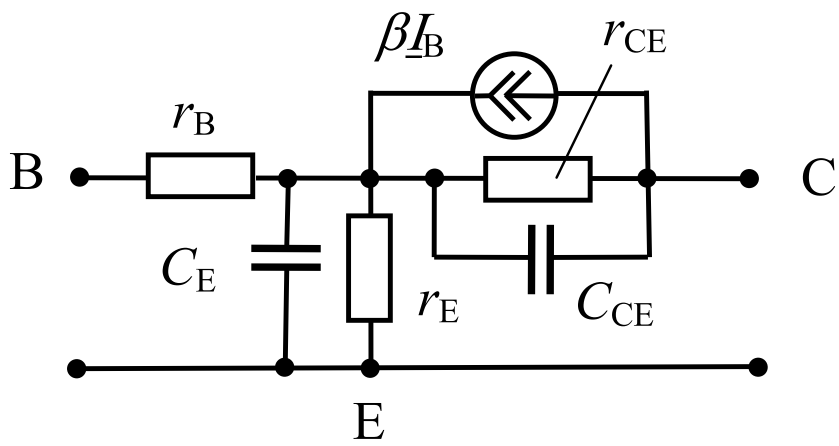
$$\underline{U}_1 = h_{11E} \underline{I}_1 + h_{12E} \underline{U}_2$$
$$\underline{I}_2 = h_{21E} \underline{I}_1 + h_{22E} \underline{U}_2$$

$$\underline{U}_1 = \left(h_{11E} - \frac{h_{12E} h_{21E}}{h_{22E}} \right) \underline{I}_B + \frac{h_{12E}}{h_{22E}} \underline{I}_C$$
$$\underline{U}_2 = -\frac{h_{21E}}{h_{22E}} \underline{I}_B + \frac{1}{h_{22E}} \underline{I}_C$$

$$\Rightarrow$$

$$r_E = \frac{h_{12E}}{h_{22E}} = \dots \cong \frac{kT/q}{I_{EQ}}$$
$$r_B = h_{11E} - (1 + \beta) r_E$$
$$r_{CE} = \frac{1}{h_{22E}} - r_E \cong \frac{1}{h_{22E}}$$

The T-type model of a BJT in its CE configuration



$$r_{E} \cong \frac{kT}{q}\frac{1}{I_{CQ}} \cong \frac{0{,}025}{I_{CQ}} \qquad r_{B} = h_{11E} - r_{E}(1+\beta) \qquad r_{CE} \cong 1/h_{22E}$$

$$\beta \Rightarrow \underline{\beta} = \frac{\beta_{0}}{1+j\omega/\omega_{\beta}} \qquad f_{\beta} \cong f_{T}/\beta_{0}$$

$$C_{CE} \cong \beta C_{C} \qquad C_{E}, C_{CE}(K)$$

Composing and application of the T-type equivalent circuit of a BJT in its CE configuration

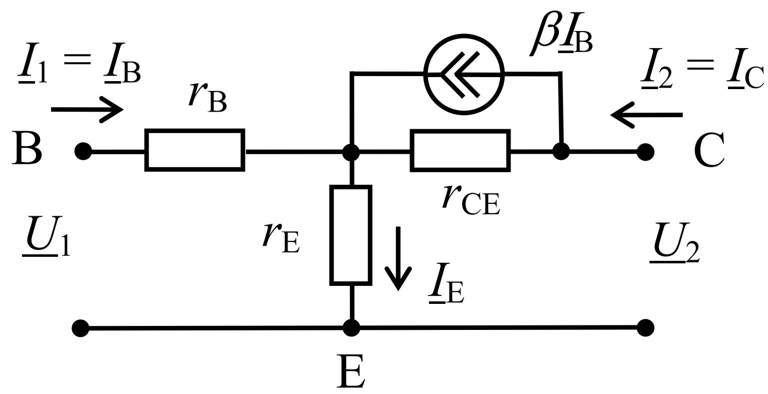
A BJT is in its CE configuration. At the given Q-point (base current 0.1 mA, CE voltage 5 V) the input resistance is 300 Ω, the base current gain is 100, and the output conductance is 200 μS.

Sketch the equivalent circuit of the BJT.

Find the parameters of the circuit.

Find the voltage gain at input voltage of 10 mV and load resistance of 300 Ω.

Composing and application of the T-type equivalent circuit of a BJT in its CE configuration



$$r_E \cong \frac{kT}{q}\frac{1}{I_{CQ}} \cong \frac{0{,}025}{I_{CQ}} \qquad r_B \cong h_{11B} - (\beta+1) r_E \qquad r_C \cong 1/h_{22B} \cong \beta\, r_{CE}$$