NONLINEAR TWO-PORT NETWORK DC MODEL FOR FET

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***Abstract* - On base of the decomposition of function from many varyables on simple in dimensions functions a classification of nonlinear models is entered for a family of the formal models of nonlinear devices in class of Volterra-Wiener systems. DC models for FET are built as nonlinear two-port networks in classes A, AB, B and C operations. Bernstein polynoms are used for the monotone approximation of measured output and transfer characteristics, which can be used in further identification processes.**

***Keywords* - Volterra-Wiener polynoms, classification, DC models, FET, classes A, AB, B, C operations.**

I. INTRODUCTION

It is well known that linear two-port network models are characterized measured sets of parameters (Z, Y, H, ABCD, S and others) which are basis for analysis, modeling and designing of linear microwave devices.

It is possible to define the sets of similar measered parameters for nonlinear two-port networks [1]. In this case nonlinear input/output mappings described by Volterra-Wiener polynoms

$$y(t) = \sum_{n=1}^{N} H_n \circ x^n(t), \qquad (1)$$

where operators

$$H_n \circ x^n(t) = \int_0^\infty \ldots \int_0^\infty h_n(\tau_1,\ldots,\tau_n) \prod_{i=1}^{n} x(t-\tau_i) d\tau_i \ .$$

The functions h_n(.) are defined as a multidimensional transfer functions (MPF), and their family defines the corresponding to parameters of nonlinear two-port networks. However, at present, measurements of MPF is complexity problem of engineering.

Meantime, the comparatively simple models of nonlinear devices are used in practical designing. These models have a simple procedures for its building. However the question about connection of these methods of the presentation was not considered earlier related to more general presentations.

In this paper, on base of the decomposition for functions from many variables on simple in dimensions functions [2-3] it is presented classification for formal models of nonlinear devices. In the following part of the paper are built DC models for the FET as nonlinear two-port networks in classes A, AB, B and C operations. For presentation of the measured characteristics of the FET used aproximation by Bernstein polynoms, saving monotonous.

II. CLASSIFICATION OF THE NONLINEAR MODELS GENERATED BY PRESENTATIONS ON VOLTERRA-WIENER POLYNOMS

We shall consider the presentation of the nonlinear transformations by Volterra-Wiener polynoms (1) as general. Obviously, that these polynoms generate others, more simple presentations of the input/output mappings. We shall conduct their systematic classification on base of the decomposition for functions of many variables on simple in dimensions functions ([3]). For our development we use Laplace transfom for MTF

$$H(s_1,...,s_n) = \int_0^\infty ... \int_0^\infty h_n(\tau_1,...,\tau_n)\exp(-s_1\tau_1 - ... - s_n\tau_n)d\tau_1...d\tau_n \text{ ,}$$

but record input/output functions shall do as before – in time area. Then record of Volterra-Wiener polynom can be expressed in the symbolic form

$$y(t) = \sum_{n=1}^{N} H_n(s_1,...,s_n) \circ x^n(t) \qquad (2)$$

The decomposition function of many variables on simple in dimensions functions has a following sense. Each function of many variables $H_n(\ .)$ define the family of the functions of smaller number variables – its narrowings in zero point: $H_n(0,\dots,0)$, $\{H_n(s_1,0,\dots,0), H_n(0,s_2,0,\dots,0), , H_n(0,\dots,0,s_n)\}$, $\{ H_n(s_1,s_2, 0,\dots,0), H_n(s_1,0,s_3,0,\dots,0), H_n(0,\dots,0,s_{n-1}, s_n) \}$ and so on before family of the narrowings to dimension *n-1*. From the narrowings of function $H_n(s_1,s_2,\dots s_n)$ there are formed simple in dimensions functions from one, two, before *n* variables. Algebraic building of simple functions is reduced to subtraction from narrowings of dimension r of all narrowings of smaller dimensions ([3]). Each simple function from many variables, on building, equal to zero, when any one variable of simple function is a zero. In mathematical and technical applications turn out to be useful following two properties of the simple in dimension functions.

Property 1. The functions of n variables are approximated exactly by sum of simple in dimensions functions ([3]).

Property 2. The shortcut decompositions functions of many variables on simple in dimensions functions, under determined condition to smoothness, can be considered as aproximations of source function by sum of simple in dimensions functions to smaller dimensions in even metrics ([2]).

E x a m p l e 1. For MPF $H_2(s_1,s_2)$ the decomposition on simple in dimensions functions in point $(s_1,s_2)=(0,0)$ has a following presentation by narrowings MPF $H_2(s_1,s_2)$

$$\begin{aligned} H_2(s_1,s_2) &= H_2(0,0) + \{H_2(s_1,0) - H_2(0,0)\} + \{H_2(0,s_2) - H_2(0,0)\} + \\ &\{H_2(s_1,s_2) - H_2(0,0) - H_2(s_1,0) - H_2(0,s_2) - H_2(0,0) - H_2(0,0)\}, \end{aligned}$$

where simple on dimensionality of the functions are comprised in figured parentheses.

The presentation for MPF $H_2(s_1,s_2)$ it is possible to write as sum of simple on dimensionality functions

$$H_2(s_1,s_2) = H_2(0,0) + \hat{H}_2(s_1,0) + \hat{H}_2(0,s_2) + \hat{H}_2(s_1,s_2) \text{ ,}$$

or else

$$H_2(s_1,s_2) = H_2(s^{(0)}) + \hat{H}_2(s^{(1)}) + \hat{H}_2(s^{(2)}),$$

where $s^{(0)}=(0,0)$; $s^{(1)}=(s_r,0)$, $r=(1,2)$; $s^{(2)}=(s_1,s_2)$.

Thus we can write Volterra-Wiener's representation (2) as the sum, in which the kernels (MTF) are considered to be the sum of the simple in dimensions functions in frequency area:

$$
\begin{aligned}
y(t) = \sum_{m=1}^{n} H_m(s^{(0)}) \circ x^m(t) + \\
\sum_{m=1}^{n} \hat{H}_m(s^{(1)}) \circ x^m(t) + \\
\sum_{m=2}^{n} \hat{H}_m(s^{(2)}) \circ x^m(t) + \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \qquad (3) \\
\sum_{m=n-1}^{n} \hat{H}_m(s^{(n-1)}) \circ x^m(t) + \\
\hat{H}_n(s^{(n)}) \circ x^n(t)
\end{aligned}
$$

Observe from eq. (3) that the decomposition of Volterra-Wiener kernels (MTF) on simple in dimensions functions generate the scale of omplexity for polynomial models in class of Volterra-Wiener models and have a simple technical sense.

The minimum level in complexity model (in zero dimension) give line 1 in formula (3), which presents statical (DC) model of nonlinear device. For building of the statical (DC) model, as it is well known, it is enough to measure DC characteristics under test signal $x(t)=x_0$, x_0 e $[x_{min}, x_{max}]$.

The first level in complexity model in one dimension give the lines 1 and 2 in formula (3) as a result we have Volterra-Wienera model in one dimension. For building of the models in one dimension in class of the Volterra-Wienera models it is enough to realize the small-signal (the linear measurements) of amplitude-frequency and phase-frequency characteristics on set of biases for x_0e $[x_{min},x_{max}]$.

The r-level in complexity model in r dimensions give the lines from 1 to r+1 in formula (3) as result we have Volterra-Wiener model in r dimensions. For building r-measured models in class of the Volterra-Wiener models in r dimensions, as it is possible show, it is enough to realize the measurements of DC characteristics, Volterra-Wiener model in one dimensions and all Volterra-Wiener models in before r dimensions. Volterra-Wiener models in before r dimensions can be determined from measurements of amplitude-frequency and phase-frequency characteristics in corresponding dimensions on set of biases for x_0e $[x_{min},x_{max}]$ under test signal $x(t) = x_0 + \sum_{i=1}^{r} m \cos(w_i t)$, x_0e$[x_{min},x_{max}]$.

III. VOLTERRA-WIENER MODEL IN ZERO DIMENSION FOR FET- DC MODELS FOR FET.

We determine the model in zero dimension for the transistor "Pirat-24" from DC measurements to frame presented above classification. The basic concept used in analytical representation DC characteristics consists in using a monotone approximation. From the known methods we use Bernstein polynoms providing the monotone approximation [5]. The intent of this representation is to verify to what extent the resistive model can be used modeling, analysis. Other goal is using of the model in zero dimension in further identification processes .

Building of family I-V characteristics for transistor "Pirat-24" for class A operation is executed on measured dates: three output (v_g={0V, -2.1V, -3V}) and two tranfer (v_d=2V and v_d=3V) characteristics. Measured transfer characteristics of transistor "Pirat-24" are replaced averaged one, which is presented by polynom of third degree

$$i_d(v_g,3)=0.309999v_g+0.45v_g^2-0.45v_g^3 \qquad (5).$$

Then we approximate measured the output “middle” I-V characteristic for v_g=-2.1V which is presented by Bernstein polynoms

$$i_d(-2.1,v_d) = 2.1v_d - 7.518v_d^2 + 14.840v_d^3 - 17.29v_d^4 + 12.09v_d^5 - 4.627v_d^6 + 0.707v_d^7 \qquad (6)$$

characteristics build the family of rate setting for class A operation on and the transfer characteristic. For building of family output characteristics we duplicate it on rule, which is given the transfer characteristic. It is possible show that for building family output features it is necessary to rate setting the transfer characteristic concerning “middle” I-V characteristic for v_g=-2.1V so that it was equal about unit. As a result we get function-multiplier

$$fn := y \rightarrow 1.24623\ y + 1.809\ y^2 - 0.603\ y^3 \qquad (7).$$

in which y is v_g.

Now it is possible to build a family of the output I-V characteristics, which coincide with measured output I-V characteristics ((v_g={0V, -2.1V, -3V})) and which the average transfer characteristic (5). The rule of generating for a family of the output I-V characteristics consists in multiplication function (7) on I-V characteristic (6). As a result we get equation for a family of I-V output characteristics (Fig.1) for the transistor “Pirat-24”

$$z := (x, y) \rightarrow (2.100\ x - 7.518\ x^2 + 14.840\ x^3 - 17.290\ x^4 + 12.096\ x^5 - 4.627\ x^6 + 0.707\ x^7)\,(1.24623\ y + 1.809\ y^2 - 0.603\ y^3)$$

in which x is v_d, and y is v_g .

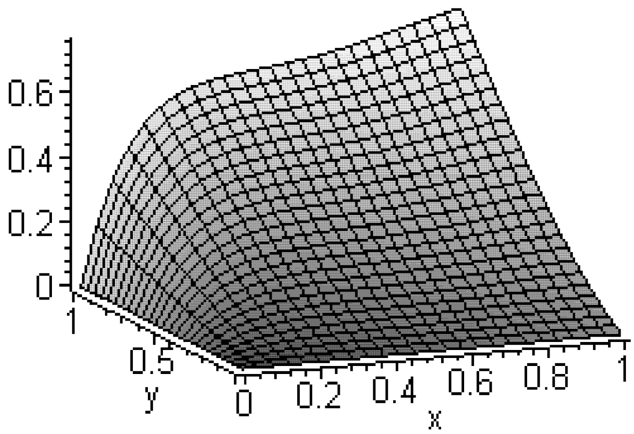

Fig 1 . Measured and modeled $i_d(v_g,v_d)$=z(x,y) of the “Pirat-24” FET for classis A and AB operation (i_{dmax}=1000mA, v_{dmax}=7V, v_{gmax}=-3V).

The building of family I-V characteristics for transistor Pirat-24 for classes AB, B and C is conducted on the same manner, as building I-V characteristics for class A but only a region of the change the gate voltage and the drain current prolonged up to cut-off. Naturally it is necessary to use the approximation by Bernstein more high degree. For this case the equation for output current $i_d(v_d,v_g)=z(x,y)$ (Fig. 2) of the transistor "Pirat-24" has a following type

$$
\begin{aligned}
z(x,y)= & \\
& 8.\,y + 84.0\,x^8 y^2 - 28.\,y^2 + 56.\,y^3 - 70.\,y^4 + 56.\,y^5 - 28.\,y^6 + 8.\,y^7 - 1.\,y^8 - 32.0\,x\,y \\
& + 224.0\,x^3 y - 560.0\,x^4 y + 672.0\,x^5 y - 448.0\,x^6 y + 160.0\,x^7 y - 24.0\,x^8 y \\
& + 112.0\,x\,y^2 - 784.0\,x^3 y^2 + 1960.0\,x^4 y^2 - 2352.0\,x^5 y^2 + 1568.0\,x^6 y^2 \\
& - 560.0\,x^7 y^2 + 1568.0\,x^6 y^6 - 224.0\,x\,y^3 + 1568.0\,x^3 y^3 - 3920.0\,x^4 y^3 \\
& + 4704.0\,x^5 y^3 - 3136.0\,x^6 y^3 + 1120.0\,x^7 y^3 - 168.0\,x^8 y^3 + 280.0\,x\,y^4 \\
& - 1960.0\,x^3 y^4 + 4900.0\,x^4 y^4 - 5880.0\,x^5 y^4 + 3920.0\,x^6 y^4 - 1400.0\,x^7 y^4 \\
& + 210.0\,x^8 y^4 - 224.0\,x\,y^5 + 1568.0\,x^3 y^5 - 3920.0\,x^4 y^5 + 4704.0\,x^5 y^5 \\
& - 3136.0\,x^6 y^5 + 1120.0\,x^7 y^5 - 168.0\,x^8 y^5 + 112.0\,x\,y^6 - 784.0\,x^3 y^6 \\
& + 1960.0x^4 y^6 - 2352.0x^5 y^6 - 560.0x^7 y^6 + 84.0x^8 y^6 - 32.0x\,y^7 \\
& + 224.0x^3 y^7 - 560.0x^4 y^7 + 672.0x^5 y^7 - 448.0x^6 y^7 + 160.0x^7 y^7 \\
& - 24.0x^8 y^7 + 4.0x\,y^8 - 28.0x^3 y^8 + 70.0x^4 y^8 - 84.0x^5 y^8 + 56.0x^6 y^8 \\
& - 20.0x^7 y^8 + 3.0x^8 y^8
\end{aligned}
$$

in which x is v_d, and y is v_g .

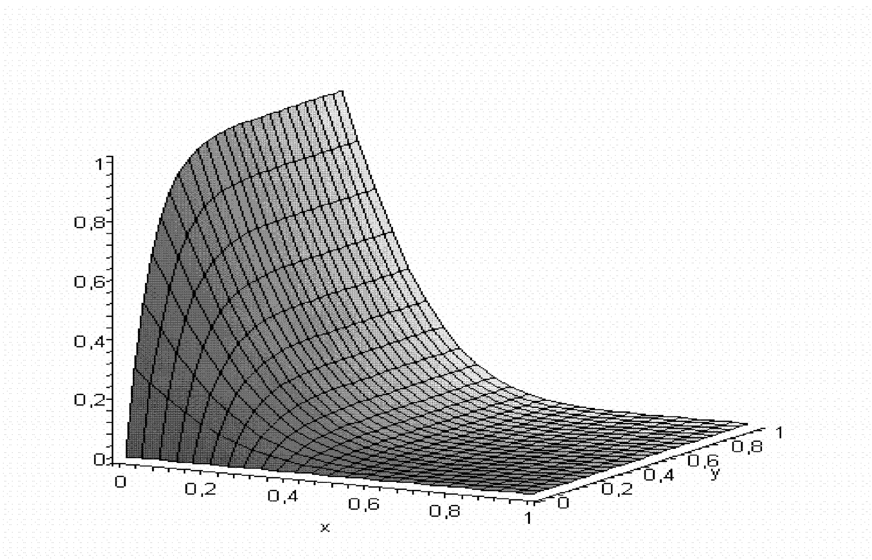

Fig. 2 . Measured and modeled $i_d(v_g,v_d)=z(x,y)$ of the “Pirat-24” FET for classis A AB,B и C operation (i_{dmax}=1000mA, v_{dmax}=7V, v_{gmax}=-6)V).

Thus, Volterra-Wiener models in zero dimension is constructed for output features of the transistor “Pirat-24” for classes A AB, B and C operations, which present itself models for the transistor in the manner of nonlinear two-port networks. It must noted that the constructed model for class A is equivalent of cubic model Curtice [4]. From that fact follow interesting, on our point, conclusion. A bilding formal models is not connected with physical phenomenas and technological process. In Curtice model (and other known models [5]) involved the parameters of the physical processes, which are connected with technology of the fabrication transistor. But since now relationship between physical model Curtice (and with other similar model) is installed, that, consequently, it is possible carry in formal model physical and technological parameters.

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