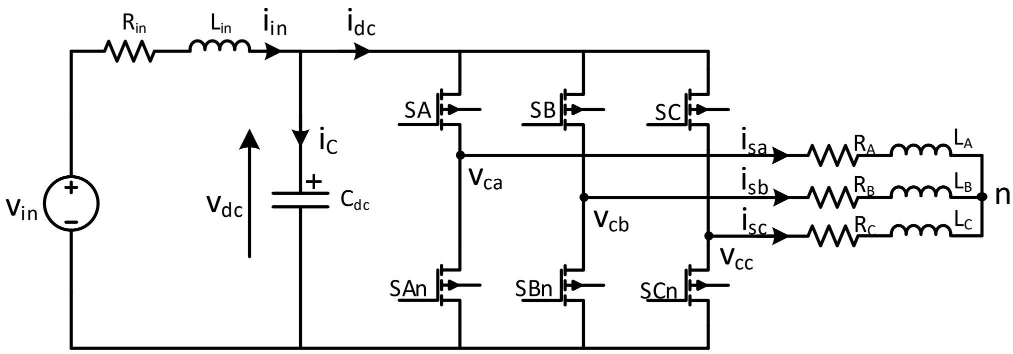
In this study, a state-space representation of a motor drive inverter is obtained. the circuit diagram is as follows:



The model has a three-phase RL load, an inverter with a single DC bus capacitor and without parasitics, and a simple DC bus input model with a controllable DC supply. the parameter classification is as follows:

Parameter	**Type**	**Description**
V_{in}	Input	It is a controlled variable. Harmonics (such as 300 Hz may be injected).
I_{in}	State	This parameter is treated as a state since the harmonic content to the input side is unknown at this point.
R_{in}, L_{in}	Constant	These are modeling the parasitics between the source and inverter. Values of these parameters are unknown.
V_{dc}	State	This is obviously a state parameter, ripple content of which is to be kept at acceptable levels.
I_{dc}	Mid-state	This parameter can be represented in terms of phase currents (which are states). Therefore, it is treated as a mid-state variable.
C_{dc}	Constant	Parameter is known.
I_c	Mid-state	This parameter can be represented in terms of DC bus voltage (which is state). Therefore, it is treated as a mid-state variable.
SA, SB, SC	Constant	These are the switching states. They are controlled by the user, and can be treated as constants. n's are inverter switching states.
V_{ca}, V_{cb}, V_{cc}	Mid-state	These parameters can be represented in terms of DC bus voltage (which is state). Therefore, they are treated as a mid-state variable. They represent the voltage between each phase and neutral.
I_{sa}, I_{sb}, I_{sc}	State	Phase currents are state parameters.
R_A, R_B, R_C, L_A, L_B, L_C	Constant	Parameters are known. They can be used for unbalanced cases.

SA, SB and SC depend on the switching method. A model without switching harmonics shown in (1) - (3), for Sinusoidal PWM, where m_a is the modulation index, ω_0 is the fundamental frequency and δ is the load angle.

$$SA = \frac{1}{2}[1 + m_a \sin(\omega_0 t + \delta)] \tag{1}$$

$$SB = \frac{1}{2}\left[1 + m_a \sin\left(\omega_0 t + \delta - {}^{2\pi}/_{3}\right)\right] \tag{2}$$

$$SC = \frac{1}{2}\left[1 + m_a \sin\left(\omega_0 t + \delta + {}^{2\pi}/_{3}\right)\right] \tag{3}$$

Since this model does not include switching harmonics, and it is time consuming to obtain such a model, switching states will be obtained using a carrier signal for the time being.

V_{in} has the characteristics shown in (4) to represent a constant DC voltage source (V_{dcin}) with a rectifier, where V_{6th} is the peak voltage of sixth harmonic, ω_{r0} is the fundamental frequency and φ_{r0} is the fundamental phase of rectifier.

$$V_{in} = V_{dcin} + V_{6th} \sin(6\omega_{r0} t + \varphi_{r0}) \tag{4}$$

We have the inverter output voltages in terms of DC bus voltage and phase currents as in (5) - (7).

$$V_{ca} = V_{dc}\, SA = R_A\, I_{sa} + L_A\, \dot{I_{sa}} \tag{5}$$

$$V_{cb} = V_{dc}\, SB = R_B\, I_{sb} + L_B\, \dot{I_{sb}} \tag{6}$$

$$V_{cc} = V_{dc}\, SC = R_C\, I_{sc} + L_C\, \dot{I_{sc}} \tag{7}$$

The DC bus current can also be represented by phase currents as in (8). On the DC bus, the input current flows through capacitor and inverter as shown in (9). Finally, the DC bus voltage can be related to capacitor current as in (10).

$$I_{dc} = SA\, I_{sa} + SB\, I_{sb} + SC\, I_{sc} \tag{8}$$

$$I_{in} = I_C + I_{dc} \tag{9}$$

$$I_C = C\, \dot{V_{dc}} \tag{10}$$

On the input side, the voltage drop equation is shown in (11) to relate the input current and voltage.

$$V_{in} - V_{dc} = R_{in}\, I_{in} + L_{in}\, \dot{I_{in}} \tag{11}$$

Now, from (5) – (7), we have the state equations of phase currents, by eliminating V_{ca}, V_{cb} *and* V_{cc}, as in (12) – (14).

$$\dot{I_{sa}} = \frac{SA}{L_A} V_{dc} - \frac{R_A}{L_A} I_{sa} \tag{12}$$

$$\dot{I_{sb}} = \frac{SB}{L_B} V_{dc} - \frac{R_B}{L_B} I_{sb} \tag{13}$$

$$\dot{I_{sc}} = \frac{SC}{L_C} V_{dc} - \frac{R_C}{L_C} I_{sc} \tag{14}$$

From (8) – (10), we have the state equation of DC bus voltage, by eliminating I_{dc} *and* I_c, as in (15).

$$\dot{V_{dc}} = \frac{1}{C_{dc}} I_{in} - \frac{SA}{C_{dc}} I_{sa} - \frac{SB}{C_{dc}} I_{sb} - \frac{SC}{C_{dc}} I_{sc} \tag{15}$$

And from (11), the input current state equation can be rearranged as in (16).

$$\dot{I_{in}} = \frac{1}{L_{in}} V_{in} - \frac{1}{L_{in}} V_{dc} - \frac{R_{in}}{L_{in}} I_{in} \tag{16}$$

The state space representation of this model in the form of $\dot{x} = Ax + Bu$ is shown in (17).

$$\begin{bmatrix} \dot{I_{sa}} \\ \dot{I_{sb}} \\ \dot{I_{sc}} \\ \dot{V_{dc}} \\ \dot{I_{in}} \end{bmatrix} = \begin{bmatrix} -{R_A}/{L_A} & 0 & 0 & {SA}/{L_A} & 0 \\ 0 & -{R_B}/{L_B} & 0 & {SB}/{L_B} & 0 \\ 0 & 0 & -{R_C}/{L_C} & {SC}/{L_C} & 0 \\ -{SA}/{C_{dc}} & -{SB}/{C_{dc}} & -{SC}/{C_{dc}} & 0 & {1}/{C_{dc}} \\ 0 & 0 & 0 & -{1}/{L_{in}} & -{R_{in}}/{L_{in}} \end{bmatrix} \begin{bmatrix} I_{sa} \\ I_{sb} \\ I_{sc} \\ V_{dc} \\ I_{in} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ {1}/{L_{in}} \end{bmatrix} [V_{in}] \tag{17}$$