

Harmonic Analysis of Sinusoidal Pulse Width Modulation

Sonal Arvind Barge & S. R. Jagtap

E & Tc. Dept., R.I.T.

E-mail : sonalbarge_7@rediffmail.com, satyawar.jagtap@ritindia.edu

Abstract - This paper discuss about inverters that transfers power from a DC source to an AC load. Inverters are widely used in motor drives, uninterruptible power supplies (ups), etc. Output voltage from an inverter can also be adjusted by exercising a control within the inverter itself. The most efficient method of doing this is by pulse width modulation control used within an inverter. This paper presents study of performance of Sinusoidal Pulse Width Modulation (SPWM) technique for voltage source inverter. SPWM scheme is modeled in Pspice. The model is used to predict harmonics under various conditions of modulation index. The performance of technique is presented in terms of the Total Harmonic Distortion (THD) and output RMS voltage. Basically, Total Harmonic Distortion describes the quality of the output waveform. Effects of harmonics on output of sinusoidal pulsewidth modulation (PWM inverter) is presented.

Keywords - Harmonics, Inverter, Sinusoidal PWM.

I. INTRODUCTION

In many industrial applications, it is often required to vary the output voltage of inverter due to following reasons:

- 1) To compensate for the variations in the input voltage.
- 2) To compensate for the regulation of inverters.
- 3) To supply some special loads which need variations of voltage with frequency, such as an induction motor.

There are various methods for the control of the output voltage of inverters are as under:

- a) External control of a.c. output voltage.
- b) External control of d.c. output voltage.
- c) Internal control of inverter.

The first two methods require use of peripheral components, whereas the third method require no peripheral components. Internal control is to adjust the

gain of inverter to control the output voltage. The two possible ways of doing this are:

- a) Series inverter control
- b) Pulse-width modulation control

The most efficient method of controlling output voltage is to incorporate PWM control within inverters[3]. In this method, a fixed d.c. voltage is supplied to inverter and a controlled a.c. output voltage is obtained by adjusting on-off period of inverter devices.

The basic PWM techniques are:

1. Single Pulse Width Modulation
 2. Multi Pulse Width Modulation
 3. Sinusoidal Pulse Width Modulation (SPWM)
- Pulse Width Modulation variable speed drives are

increasingly applied in many new industrial applications that require superior performance. Recently, developments in power electronics and semiconductor technology have lead improvements in power electronic systems. Hence, different circuit configurations namely PWM inverters have become popular and considerable interest by researcher are given on them. A number of Pulse width modulation (PWM) schemes are used to obtain variable voltage and frequency supply. The most widely used PWM scheme for voltage source inverters is sinusoidal PWM.

II. DEVELOPMENT OF INVERTER WITH SPWM.

SPWM is commonly used in industrial application. In this scheme the width of each pulse is varied in proportion to the amplitude of a sine wave evaluated at the center of same pulse. The gating signals are generated by comparing a sinusoidal reference signal with a triangular carrier wave of frequency f_c . The frequency of reference signal f_r determines the inverter

output frequency f_o ; and its peak amplitude A_r controls the modulation index M , and then in turn the RMS output voltage V_o . The number of pulses per half-cycle depends on the carrier frequency. Within the constraint that two transistors of same arm cannot conduct at the same time, the instantaneous output voltage is shown in Fig. 1.[3].

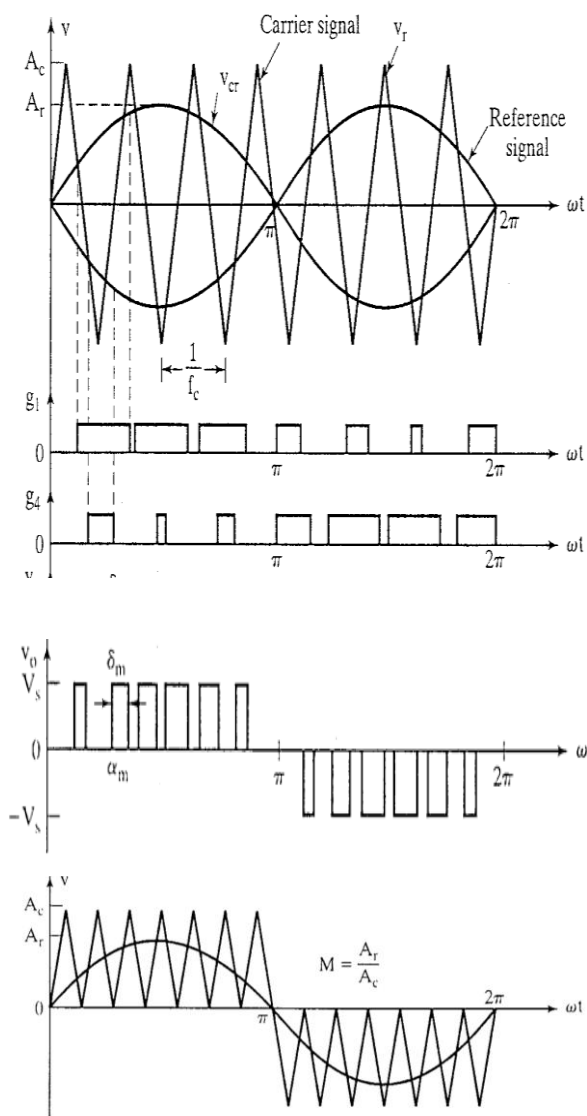


Fig. 1: Sinusoidal PWM.

It can be observed that the area of each pulse corresponds approximately to the area under the sine wave between the adjacent midpoints of off periods on the gating signals. The same gating signals can be generated by using unidirectional triangular carrier wave as shown in Fig.1.[3].

III. DEVELOPMENT OF SPICE MODEL FOR SPWM INVERTER

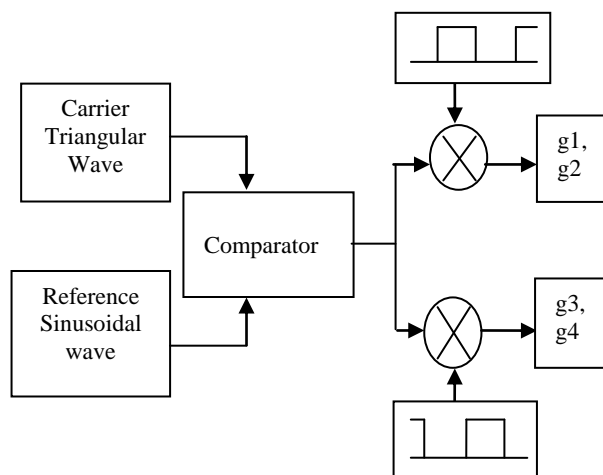


Fig. 2 : SPICE A/D model of PWM inverter.

In proposed SPICE model PWM inverter, unidirectional triangular carrier wave is compared with absolute value of reference sinusoidal wave. This output after comparison is then multiplied to 50% duty cycle signal, having amplitude 1 volt, such that g1 and g4 can not be released at same time. Where g1, g2, g3, g4 are switches of inverter. Pulses per half cycle is denoted as 'p'.

IV. OUTPUT OF PWM INVERTER

After applying SPWM gating signal to inverter switches, with modulation index $M = 0.83$, we get Pspice simulated output as shown in Fig.3.

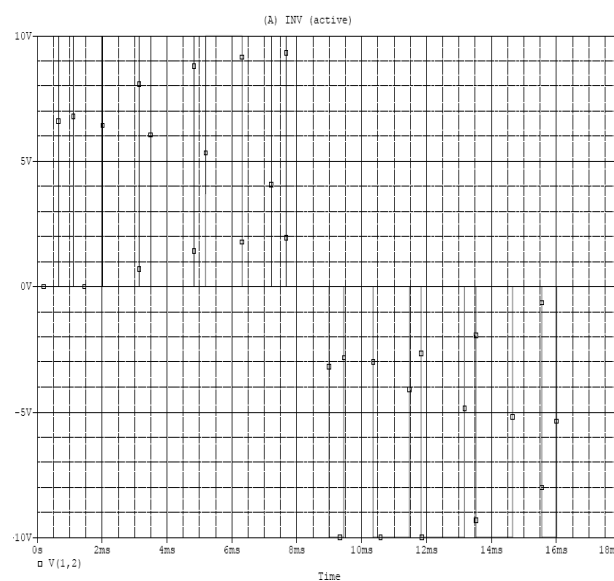


Fig. 3 : Output waveform of PWM inverter.

In Pspice, we can perform fourier analysis of result[4], which is shown in Fig.4. This harmonics obtained by simulation is verified theoretically with the equation given below[3]. If the positive pulse of m th pair starts at $\omega t = \alpha_m$ and ends at $\omega t = \alpha_m + \delta_m$, the Fourier coefficient for a pair of pulses is

$$b_n = \frac{2Vs}{\pi} \left\{ \int_{\alpha_m + \frac{\delta_m}{2}}^{\alpha_m + \frac{\delta_m}{2}} \sin(n\omega t) d\omega t - \int_{\pi + \alpha_m}^{\pi + \alpha_m + \frac{\delta_m}{2}} \sin(n\omega t) d\omega t \right\}$$

Where $n=1,3,5\dots$

The coefficient B_n can be found by adding the effects of all pulses[3]

$$B_n = \sum_{m=1}^{2p} \frac{4Vs}{n\pi} \sin \frac{n\delta_m}{4} \left[\sin n \left(\alpha_m + \frac{3\delta_m}{4} \right) - \sin n \left(\pi + \alpha_m + \frac{\delta_m}{4} \right) \right]$$

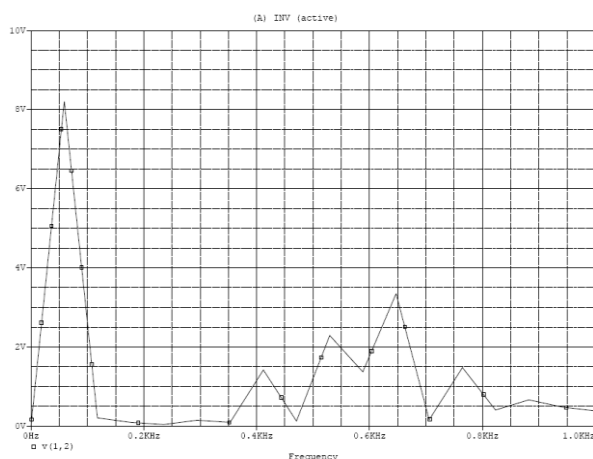


Fig.4: Harmonic profile at $M=0.83$ & $p=5$

Observed harmonics are verified by theoretically calculating as shown in I. given below,

Harmonics									
Calculated					Observed				
1	3	5	7	9	1	3	5	7	9
8.29	3E-3	0.143	1.51	2.99	8.29	2.92E-3	1.49E-1	1.50	2.97

I. Calculated & observed harmonics at $M=0.83$

The RMS output voltage can be varied by varying modulation index M . It can be observed that the area of each pulse corresponds approximately to the area under the sine wave between the adjacent midpoints of off periods on the gating signals[3]. If δ_m is width of m th pulse, the rms output voltage is

$$V_o = V_s \left(\sum_{m=1}^{2p} \frac{\delta_m}{\pi} \right)$$

Fig.5 shows RMS output voltage of inverter for $M=0.83$. By varying modulation index RMS output voltage is varied as shown in II.

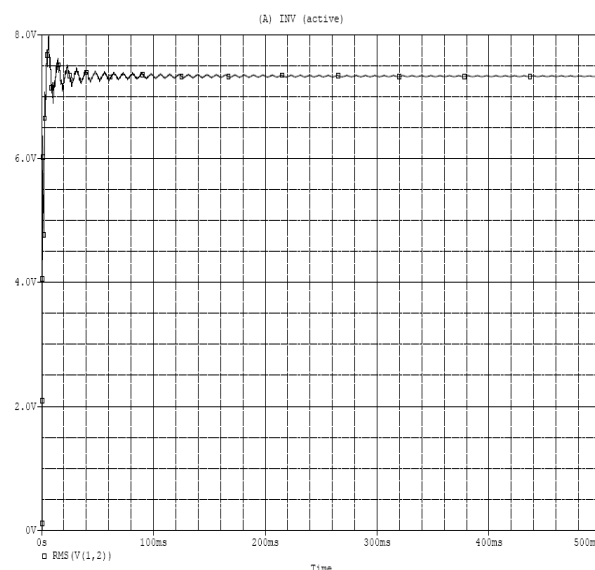


Fig. 5: Output RMS voltage of PWM inverter.

II. Effect of modulation index on Total harmonic distortion.

M	Output RMS		Harmonics					THD
	Calc.	Obs.	1	3	5	7	9	
0.1	2.55	2.55	0.99	1.05E-3	4.1E-3	2.7E-3	9.861E-1	98%
0.2	3.6	3.56	1.99	3.8E-3	8.2E-4	3.3E-2	1.90	95%
0.3	4.4	4.4	2.99	1.32E-3	1.3E-3	1.0E-1	2.67	89%
0.4	5.079	5.08	3.99	2.1E-3	6.5E-3	2.4E-1	3.25	81%
0.5	5.715	5.7	4.99	1.12E-3	1.5E-2	4.4E-1	3.60	72%
0.6	6.24	6.24	5.99	1.68E-3	3.4E-2	7.1E-1	3.69	62%
0.7	6.743	6.75	6.99	2.6E-3	7.1E-2	1.032	3.53	52%
0.8	7.2	7.2	7.99	6.2E-3	1.2E-1	1.39	3.14	43%
0.9	7.64	7.64	8.99	1.2E-2	2.1E-1	1.764	2.54	34%
1	8.04	8.05	9.99	2.2E-2	3.3E-1	2.12	1.80	28%

Here, it can be observed that as modulation index increases total harmonic distortion THD reduces.

V. CONCLUSION

In this paper it is shown theoretically and experimentally that when modulation index is increased, output RMS voltage get increased and harmonic distortion get reduced. It is observed that for modulation index 0.2 to 0.5, 3rd harmonics has variation. 5th and 7th harmonics goes on increasing while 9th harmonics goes on increasing up to 0.5 and goes on decreasing onwards. We can say that the output voltage control with this method can be obtained without any additional components. With the method, lower order harmonics can be eliminated or minimized along with its output voltage control. As higher order harmonics can be filtered easily, the filtering requirements are minimized.

VI. REFERENCES

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