State-Space Model

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1 Introduction

In this report, the state-space model of 3-phase voltage inverter with rectifier is investigated.State variable selections and mathematical idea of the model are argued. Inverter and rectifier are split in two different state space model and the combination of two state space is shown. Also, results are compared with simulink model of 3-phase inverter and the reliability of the state space model is discussed.

2 State Space

State Space is a representation of a physical system as mathematical model by using differential equations. State variables changes with respect to time and input variables and output can be calculated by using the state variables. In addition, two state space model can be merged by adjusting inputs and outputs. Output of one model can be taken as input of other one and vice verse.

State space representation can be shown in the form of matrix as:

Changes of states are calculated by using past value of states and present value of inputs.

3 3-phase Voltage Inverter

Figure **??** shows the circuit schematic of 3- phase voltage inverter. It includes three half-bridge legs and each output of leg is one-phase of load.Also,in addition to voltage source, a capacitor is required to keep the input voltage constant at switching time and to filters the high frequency ripples.Each transistor at half-bridge are driven by using Sinosoidal-PWM.

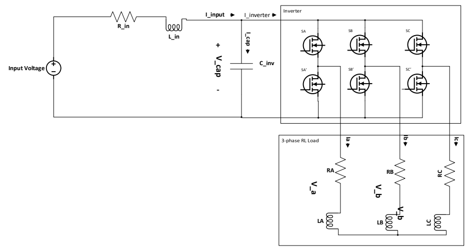

Figure 1: Three Phase Voltage Inverter

4 State Space Representation of 3-Phase Voltage Inverter

First of all, states and input of the model should be selected. Figure **??** shows the circuit. The circuit includes switching elements and it makes the circuit behave non-linear. To represent the switching elements, the harmonics of switching functions can be used. Switching functions for top switches are written as :

$$SwitchingFunction = \frac{1}{2} + \frac{M}{2}\cos(w_o + \theta_o)$$

$$+(\frac{2}{\pi})\sum_{m=1}^{inf} J_o(m\pi\frac{M}{2}))sin(m\frac{\pi}{2})cos(m(w_c + \theta_c))$$

$$+(\frac{2}{pi})\sum_{m=1}^{inf}\sum_{n=-inf}^{inf}(\frac{1}{m})J_n(\frac{mM\pi}{2}))sin(\frac{(m+n)\pi}{2})\ cos(m(w_c + \theta_c) + n(\theta_o + w_o))$$

The function is very hard to implement the state space model including all harmonics. However, the switches can be thought as zero resistive (or R_on resistance) and infinite resistive element with respect to input of switches. If the switches are in conduction, input of switch is '1' and in off mod, input of switch is '0'. In short, there are 3 parameters to change to voltage or current with respect to time. The parameters are boolean and it can be found with respect to time by using Sinosoidal-PWM. . Also, there is an input that is given by voltage source.

In addition, states are chosen by rule of thumb. Capacitor voltage or inductor current creates the states. DC-Link capacitor of the inverter is chosen as one state of the circuit. Phase current of load can be thougth as inductor current and they can be chosen as state. However, system is Y-connected 3- wire 3-phase system. The sum of phase currents is zero. Thus, two phase current are chosen

states to avoid redundancy. In addition, the input current which depends on the switch parameters and phase currents is chosen as a state.
To conclude, there are 4 states such as input current, capacitor voltage and two phase current.There is one input as voltage source. In addition, switches are chosen as parameters that changes with respect to time as zero or one. They are listed table 1

Variables	**Type**	**Description**
V_{in}	Input	It can be DC with low-frequency ripple
I_{in}	State	Input current is a state.
I_a	State	2 phase current a state. Other one iş redundant.
I_c	State	2 phase current a state. Other one iş redundant
V_{cap}	State	It is a state.
SA,SB,SC	Parameter as changing variable	It can be thought as variable parameter changing from 1 to 0 and vice verse.
R_a, L_a, R_b,L_b, R_c,L_c	Constant	Load parameters.
R_{in}, L_{in}	Constant	Source parameters.
C_{cap}	Constant	DC-Link parameters

Table 1: All Inputs, States and Parameters of 3-Phase Inverter

4.1 State and Input Matrices

$$\begin{bmatrix} \dot{I_a} \\ \dot{I_c} \\ \dot{V_{cap}} \\ \dot{I_{in}} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} I_a \\ I_c \\ V_{cap} \\ I_{in} \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} [V_{in}]$$

(1)

Equation 1 shows that the input and state matrices. It is required to find elements of matrices with respect to circuit components such as resistor, inductor, capacitor values and switch parameters.

Calculation for input current:

$$V_{in} - I_{in}R_{in} - L_{in}\dot{I_{in}} = V_{cap}$$

(2)

$$\dot{I_{in}} = 0I_a + 0I_c - \frac{1}{L_{in}}V_{cap} - \frac{R_{in}}{L_{in}}I_{in} + \frac{1}{L_{in}}V_{in} \tag{3}$$

Equation 2 is extracted by using KVL and the fourth column of state and input matrix is calculated in Equation 3
Calculation for capacitor voltage:

$$I_{dc} = (SA - SB)I_a + (SC - SB)I_c \tag{4}$$

$$C\dot{(V_{cap})} = I_{in} - I_{dc}\dot{(V_{cap})} = \frac{1}{C}(SA - SB)I_a + \frac{1}{C}(SC - SB)I_c + 0V_{cap} + \frac{1}{C}I_{in}$$

(5)

$$\dot{(V_{cap})} = \frac{1}{C}(SA - SB)I_a + \frac{1}{C}(SC - SB)I_c + 0V_{cap} + \frac{1}{C}I_{in}$$

(6)

Equation 4 and 5 are used to extract the third column of matrices in equation 6

Calculation for phase currents:

$$(SA - SC)V_{cap} = R_aI_a + R_cI_c + L_a\dot{(I_a)} + L_c\dot{(I_c)} \tag{7}$$

$$(SA - SB)V_{cap} = (R_a - R_b)I_a + (L_a - L_b)\dot{(I_a)} - R_bI_c - L_b\dot{(I_c)} \tag{8}$$

$$\dot{I_a} = \frac{-R_bL_c - R_aL_c - R_aL_b}{L_aL_b + L_aL_c - L_bL_c}I_a + \frac{R_cL_b - R_bL_c}{L_aL_b + L_aL_c - L_bL_c}I_b + \frac{SA(L_a + L_c) - SBL_c - SCL_b}{L_aL_b + L_aL_c - L_bL_c}V_{cap} + 0I_{in} + 0V_{in}$$

(9)

$$\dot{I}_c = \frac{R_aL_b - R_bL_a}{L_aL_b + L_aL_c - L_bL_c} I_a + \frac{R_cL_a - R_cL_b}{L_aL_b + L_aL_c - L_bL_c} I_b + \frac{SC(L_a + L_b) - SAL_b - SBL_a}{L_aL_b + L_aL_c - L_bL_c} V_{cap} + 0I_{in} + 0V_{in}$$

(10)

From equation 7 and 8, state and input matrices column 3 and 4 are calculated in 9 and 10

$$\begin{bmatrix} \dot{I}_a \\ \dot{I}_c \\ \dot{V}_{cap} \\ \dot{I}_{in} \end{bmatrix} = \begin{bmatrix} \frac{-R_bL_c - R_aL_c - R_aL_b}{L_aL_b + L_aL_c - L_bL_c} & \frac{R_cL_b - R_bL_c}{L_aL_b + L_aL_c - L_bL_c} & \frac{SA(L_a + L_c) - SBL_c - SCL_b}{L_aL_b + L_aL_c - L_bL_c} & 0 \\ \frac{R_aL_b - R_bL_a}{L_aL_b + L_aL_c - L_bL_c} & \frac{R_cL_a - R_cL_b}{L_aL_b + L_aL_c - L_bL_c} & \frac{SC(L_a + L_b) - SAL_b - SBL_a}{L_aL_b + L_aL_c - L_bL_c} & 0 \\ \frac{1}{C}(SA - SB) & \frac{1}{C}(SC - SB) & 0 & \frac{1}{C} \\ 0 & 0 & -\frac{1}{L_{in}} & \frac{R_{in}}{L_{in}} \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_c \\ V_{cap} \\ I_{in} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{L_{in}} \end{bmatrix} \begin{bmatrix} V_{in} \end{bmatrix}$$

5 Rectifier

6 Summary

6.1 The moral of a Story