Nonlinear Electronics: a Linear Time-Varying Circuit Approach

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Abstract— **Given a nonlinear electronic circuit, an associated linear time-varying small-signal circuit is formally derived by the tableau-method. It has the same topology as the original circuit while each original circuit element is replaced by an incremental one, evaluated along the signal-dependent nonlinear circuit solution. Since the variational circuit is linear in the first place, the designer is now invited to use the results of linear circuit theory. Furthermore, we present time-discrete companion models of linear time-varying elements. By them, the small-signal circuit equations can be efficiently solved numerically.**

Keywords— **Nonlinear electronic circuit design, linear time-varying small-signal circuits.**

I. Introduction

Recently, a linear time-varying (LTV) approach was applied for nonlinear electronic circuit design [1], [2], [3], [4], [5], [6], [7]. Thereby, first the nonlinear state-equations had to be formulated and subsequently solved in order to obtain the signal dependent bias trajectory in state-space. The Jacobian along the solution trajectory constitutes the time-varying system matrix of the associated variational LTV state-equations.

From them, the dynamic small-signal behavior was deduced by using the concept of dynamical eigenvalues [8], [9], [10], [11], [12], [13].

However, such a system-oriented approach completely obscures the underlying circuit topology. Therefore, the electronic designer does not fully exploits his expertise with linear design methodologies.

In this paper a complementary, *circuit*-oriented approach is proposed. To that aim, we derive a LTV variational circuit, associated with the original one. Since the resulting small-signal circuit is *linear* in the first place, the designer is now invited to use the results of linear circuit theory. In particular, the superposition principle holds. As a consequence, one may use for example the Thévenin and Norton theorems for ports, including those ports associated with LTV-dynamic elements [14]. Also, linear two-port theory becomes available, together with a variety of inspection techniques that simplify the circuit topology.

It should be understood however, that there is a clear distinction between the familiar linear time-invariant (LTI) small-signal circuit and the LTV small-signal circuit discussed here. Where the LTI circuit models small departures from a static bias point, now the dynamic solution is the reference for small variations.

However, there is a common design aspect, too. The linear designer first has to choose a bias point in a linear region of operation of a (fundamentally nonlinear) transistor circuit. So, by solving the biasing problem, he essentially deals with nonlinear resistive circuits. On its turn, the nonlinear circuit designer has to choose a nonlinear dynamic solution as a design objective.

In order to cope succesfully with this challenge, the nonlinear designer should have the disposal of nonlinear functional building blocks for which the dynamic behavior is already understood.

It is expected that the LTV small-signal approximation will be instrumental to reach this objective. It seems especially suited for stability analysis and for studying noise propagation (noise dealt with as small-signal) in nonlinear transistor circuits, respectively. Also, nonlinear distortion may be treated in a more analytical fashion [15], in contrast to brute force computer calculations [16], [17].

In the next section, the associated LTV variational circuit is formally derived by using the tableau-method [18]. In this method, Kirchhoff's voltage law (KVL) is effectuated by writing each branch voltage as a difference of two node-to-datum voltages, while Kirchhoff's current law (KCL) is required for any node, except the datum node, respectively. The branch voltages and branch currents, together with the node-to-datum voltages are all taken as unknown variables. Then, the (always linear) Kirchhoff-equations and the (possibly nonlinear) terminal relations of the constituent circuit elements are collected as subsets of equations in the resulting tableau, respectively. Thus, thanks to the introduction of the node-to-datum voltages as additional unknowns, the Kirchhoff-equations and the terminal relations appear as seperated sets of equations. This makes the tableau formalism eminently suited for theoretical purposes.

Now, it becomes completely transparant that the LTV variational circuit has the same topology as the original nonlinear circuit, while each original circuit element is replaced by an incremental one, evaluated along the signal-dependent nonlinear circuit solution. As a matter of fact, our argumentation is just a generalization of the derivation of the LTI variational circuit in [15]. Although the LTV small-signal circuit *equations* were obtained earlier by a different, by far less transparant method, no attempt was made to translate these equations into a real circuit *topology* [19]. By omitting this potential visualization aspect, the designer seems not really tempted to use linear circuit theory. He therefore neither will realize easily that once the variational circuit is obtained, he can proceed further by using for example node-analysis, mesh-analysis or any other solution method, including a variety of inspection techniques.

Finally, in section 3 we present time-discrete companion models for the LTV circuit elements. By them, the LTV circuit problem can be efficiently solved numerically by standard routines.

II. LTV small-signal circuit

Given a nonlinear electronic circuit $\mathcal{C}$ with a directed graph composed of n nodes and b branches. One arbitrarily node is selected as the datum node. The b branch currents and b branch voltages are respectively collected in the b-vectors $\boldsymbol{i}$ and $\boldsymbol{u}$ as follows

$$\boldsymbol{i} = [i_1\, i_2 \ldots i_b]^T \quad \text{and} \quad \boldsymbol{u} = [u_1\, u_2 \ldots u_b]^T\,, \tag{1}$$

where T denotes the transpose, while the $(n-1)$ node-to-datum voltages go into the $(n-1)$-vector $\boldsymbol{v}$, thus

$$\boldsymbol{v} = [v_1\, v_2 \ldots v_{n-1}]^T\,.$$

The Kirchhoff-equations of $\mathcal{C}$ can now be put into the following linear matrix form

$$\mathbf{A}\boldsymbol{i} = \boldsymbol{0} \;\; (KCL) \quad \text{and} \quad \boldsymbol{u} = \mathbf{A}^T\boldsymbol{v} \;\; (KVL)\,, \tag{2}$$

where $\mathbf{A}$ denotes the reduced node-branch incidence matrix of dimension $(n-1)\times b$. Since $\mathbf{A}$ is of full rank $(n-1)$, we have $(n-1)$ and b linear independent KCL's and KVL's, respectively, making a total of $(n-1)+b$ linear independent Kirchhoff-equations.

Next to the Kirchhoff-equations we also have the (nonlinear) terminal relations of the constituent circuit elements of $\mathcal{C}$. If we restrict ourselves for the moment to 2-termimal elements (later on we get rid of this restriction), we distinguish the following types of nonlinear elements: 1. independent voltage sources, 2. independent current sources, 3. current controlled resistors, 4. voltage controlled resistors, 5. voltage controlled capacitors, 6. charge controlled capacitors, 7. current controlled inductors and 8. flux controlled inductors. Except for the sources, we further suppose the nonlinear elements to be time-invariant (although this is not a real constraint).

If we collect same types of elements into a single numbered group, the b branch relations of a general circuit $\mathcal{C}$, composed of 2-terminal elements, are specified by

$$\begin{array}{ll} 1.\, u = e & 5.\; q = \tilde{q}(u) \\ 2.\, i\, = j & 6.\; u = \tilde{u}(q) \\ 3.\, u = \tilde{u}(i) & 7.\; \phi = \tilde{\phi}(i) \\ 4.\, i = \tilde{i}(u) & 8.\;\; i = \tilde{i}(\phi) \end{array} \tag{3}$$

Here, e and j denote the source strengths, while a superscript refers to a nonlinear function description of a constitutive variable. Furthermore, q and ϕ are the electric charge and the magnetic flux, respectively. They are related to i and u by the auxiliary equations

$$q = \int^t i d\tau \quad \Leftrightarrow \quad \dot{q} = i \tag{4a}$$

and

$$\phi = \int^t u d\tau \quad \Leftrightarrow \quad \dot{\phi} = u \tag{4b}$$

where τ denotes a dummy variable, while the dot refers to the Newton notation of a time-derivative. In general, all variables are a function of time t, explicitely denoted as $x = x(t)$ for any variable x.

The collection of the $(n-1)+b$ independent Kirchhoff-equations (2) and the b branch relations (3) constitute the $(n-1)+2b$ independent tableau-equations. Note that the number of equations equals the number of unknowns: $2b$ branch variables (u's and i's) plus $(n-1)$

note-to-datum voltages; the tableau formalism generates a well-posed problem.

We now assume that a dynamic nonlinear solution of the tableau-equations (2) and (3) is known. Next, we consider the same nonlinear electronic circuit $\mathcal{C}$, but with small departures from the known solution, for example caused by small source variations. Then, the value of any variable $x(t)$ in the tableau-equations has to be replaced by a new value $x_{\mathcal{C}}(t) + \hat{x}(t)$, in which $x_{\mathcal{C}}(t)$ and $\hat{x}(t)$ denotes the known solution and a small variation, respectively.

As a consequence, the Kirchhoff-equations of $\mathcal{C}$ now become

$$\mathbf{A}(\boldsymbol{i}_{\mathcal{C}} + \hat{\boldsymbol{i}}) = \boldsymbol{0} \quad (KCL) \tag{5a}$$

and

$$\boldsymbol{u}_{\mathcal{C}} + \hat{\boldsymbol{u}} = \mathbf{A}^T(\boldsymbol{v}_{\mathcal{C}} + \hat{\boldsymbol{v}}) \quad (KVL)\,, \tag{5b}$$

while the branch relations read

$$\begin{array}{ll} 1.\, u_{\mathcal{C}} + \hat{u} = e + \hat{e} & 5.\; q_{\mathcal{C}} + \hat{q} = \tilde{q}(u_{\mathcal{C}} + \hat{u}) \\ 2.\, i_{\mathcal{C}} + \hat{i} \; = j + \hat{j} & 6.\, u_{\mathcal{C}} + \hat{u} = \tilde{u}(q_{\mathcal{C}} + \hat{q}) \\ 3.\, u_{\mathcal{C}} + \hat{u} \; = \tilde{u}(i_{\mathcal{C}} + \hat{i}) & 7.\, \phi_{\mathcal{C}} + \hat{\phi} = \tilde{\phi}(i_{\mathcal{C}} + \hat{i}) \\ 4.\, i_{\mathcal{C}} + \hat{i} \; = \tilde{i}(u_{\mathcal{C}} + \hat{u}) & 8.\; i_{\mathcal{C}} + \hat{i} = \tilde{i}(\phi_{\mathcal{C}} + \hat{\phi}) \end{array} \tag{6}$$

with auxiliary equations

$$q_{\mathcal{C}} + \hat{q} = \int^t (i_{\mathcal{C}} + \hat{i})d\tau \;\Leftrightarrow\; (q_{\mathcal{C}} + \hat{q})^{\cdot} = i_{\mathcal{C}} + \hat{i} \tag{7a}$$

and

$$\phi_{\mathcal{C}} + \hat{\phi} = \int^t (u_{\mathcal{C}} + \hat{u})d\tau \;\Leftrightarrow\; (\phi_{\mathcal{C}} + \hat{\phi})^{\cdot} = u_{\mathcal{C}} + \hat{u}. \tag{7b}$$

Since the variations are supposed to be small, we may neglect higher order terms in the Taylor-expansion for any nonlinear function $y = \tilde{y}(x)$ of a constitutive variable x. Thus

$$y_{\mathcal{C}} + \tilde{y} = \tilde{y}(x_{\mathcal{C}} + \tilde{x}) = \tilde{y}(x_{\mathcal{C}}) + (d\tilde{y}\,/\,dx)_{\mathcal{C}}\hat{x}\,, \tag{8}$$

in which the derivative $(d\tilde{y}\,/\,dx)$ is evaluated at the known solution $x_{\mathcal{C}} = x_{\mathcal{C}}(t)$.

Next, we substract the tableau-equations (2) and (3) line-by-line from the tableau-equations (5) and (6), respectively. Then, in view of the approximation (8), we arrive at the following tableau-equations for the small variations

$$\mathbf{A}\hat{\boldsymbol{i}} = \boldsymbol{0} \;\; (KCL) \quad \text{and} \quad \hat{\boldsymbol{u}} = \mathbf{A}^T\hat{\boldsymbol{v}} \;\; (KVL)\,, \tag{9}$$

and

$$\begin{array}{ll} 1.\, \hat{u} \; = \hat{e} & 5.\, \hat{q} = c\hat{u} \\ 2.\, \hat{i} \; = \hat{j} & 6.\, \hat{u} = s\hat{q} \\ 3.\, \hat{u} = r\hat{i} & 7.\, \hat{\phi} = l\hat{i} \\ 4.\, \hat{i} = g\hat{u} & 8.\; \hat{i} = \gamma\hat{\phi} \end{array} \tag{10}$$

with auxiliary equations

$$\hat{q} = \int^t \hat{i}d\tau \quad \Leftrightarrow \quad \dot{\hat{q}} = \hat{i} \tag{11a}$$

$$\hat{\phi} = \int^t \hat{u}d\tau \quad \Leftrightarrow \quad \dot{\hat{\phi}} = \hat{u}\,. \tag{11b}$$

In (10), the incremental quantities r, g, c, l and γ are called the differential resistance $[\Omega]$, - conductance $[S]$, - capicitance $[F]$, - elastance $[F^{-1}]$, - inductance $[H]$ and - inverse inductance $[H^{-1}]$, respectively. They are given by

$$\begin{array}{ll} r(t) = (d\tilde{u}\,/\,di)_{\mathcal{C}} & s(t) = (d\tilde{u}\,/\,dq)_{\mathcal{C}} \\ g(t) = (d\tilde{i}\,/\,du)_{\mathcal{C}} & l(t) = (d\tilde{\phi}\,/\,di)_{\mathcal{C}} \\ c(t) = (d\tilde{q}\,/\,du)_{\mathcal{C}} & \gamma(t) = (d\tilde{i}\,/\,d\phi)_{\mathcal{C}} \end{array} \tag{12}$$

where the notation underlines that the derivatives are all evaluated at the known solution $x_{\mathcal{C}} = x_{\mathcal{C}}(t)$ of the orginal nonlinear circuit $\mathcal{C}$ and hence are time-dependent. They are subsequently interpreted as the constitutive coefficients of linear time-varying circuit elements.

Finally, in view of the tableau-equations (2) and (3), it is clearly observed that the tableau-equations (9) and (10) define a new, LTV-variational circuit $\hat{\mathcal{C}}$, characterized by 1. the same topology as $\mathcal{C}$ (same circuit structure matrix $\mathbf{A}$) and 2. any elemnt of $\mathcal{C}$ is replaced by an associated LTV circuit element in $\hat{\mathcal{C}}$.

Before generalizing this result to circuits with nonlinear three- and more- terminal elements, we first present some simple examples.

In the nonlinear dynamic circuit $\mathcal{C}$ of figure 1.a the nonlinear elements are given by the terminal relations

$$\begin{array}{lll} R : i = \tilde{i}_1(u) & & \\ L : i = \tilde{i}_2(\phi) & \text{with} & \dot{\phi} = u \\ C : u = \tilde{u}(q) & \text{with} & \dot{q} = i \end{array} \tag{13}$$

We want to study the combined effect of small variations $\hat{j}$ and $\hat{e}$ in the source strength j and e, respectively, upon the nonlinear large signal solution of $\mathcal{C}$. By taking q and ϕ as state-variables, the nonlinear

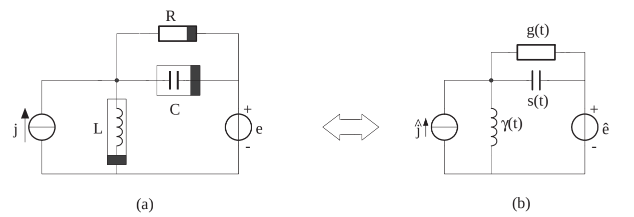

Fig. 1. Nonlinear circuit $\mathcal{C}$ (a) and its LTV small- signal circuit $\hat{\mathcal{C}}$ (b).

state-equations are obtained as (see [18] for writing state-equations)

$$\begin{bmatrix} q \\ \phi \end{bmatrix}^{\cdot} = \begin{bmatrix} -\tilde{i}_1(\tilde{u}(q)) - \tilde{i}_2(\phi) \\ \tilde{u}(q) \end{bmatrix} + \begin{bmatrix} j \\ e \end{bmatrix} , \tag{14}$$

or in shorthand notation

$$\dot{\boldsymbol{x}} = \tilde{\boldsymbol{x}}(\boldsymbol{x}, \boldsymbol{s}) , \tag{15}$$

where $\boldsymbol{x} = [q \quad \phi]^T$ and $\boldsymbol{s} = [j \quad e]^T$ are the state-vector and the source-vector, respectively, while $\tilde{\boldsymbol{x}}$ denotes a nonlinear vectorfunction.

Suppose we have found a solution of (15). Then the associated LTV small-signal circuit $\hat{\mathcal{C}}$ is known immediately ; it is depicted in figure 1.b. In it, the LTV circuit elements are given by (12).

Independently of (14), the LTV state-equations are obtained from figure 1.b as

$$\begin{bmatrix} \hat{q} \\ \hat{\phi} \end{bmatrix}^{\cdot} = \begin{bmatrix} -g(t)s(t) & -\gamma(t) \\ s(t) & 0 \end{bmatrix} \begin{bmatrix} \hat{q} \\ \hat{\phi} \end{bmatrix} + \begin{bmatrix} \hat{j} \\ \hat{e} \end{bmatrix} . \tag{16}$$

Note that the time-varying system matrix equals precisely the Jacobian of $\tilde{\boldsymbol{x}}$ with respect to $\boldsymbol{x}$ in (15). This is always the case.

However, it is not necessary to formulate the nonlinear state-equation (15) first and subsequently solve it, in oder to arrive at (16). For example, by using a numerical integration rule instead, the nonlinear dynamic elements in $\mathcal{C}$ are at each time-step replaced by nonlinear resistors. On their turn, the nonlinear resistors are next replaced by equivalent Newton-Raphson linearized elements [18]. Then, one proceed further by using for example the MNA-method for linear resistive circuits in order to find the solution of $\mathcal{C}$ recursively. This is not a too difficult task for relatively small-sized nonlinear circuits. Once the solution of $\mathcal{C}$ is found, the LTV elements of $\hat{\mathcal{C}}$ can be computed.

In the end, the second order LTV-state-equation (16) has to be solved. This can be performed by a modal expansion, which in turn requires the solution of a scalar Riccati-equation [8], [13].

In the next example, we want to study the influence of a small-signal source $\hat{e}$ upon the nonlinear behavior of the circuit $\mathcal{C}$ in figure 2.a. The terminal relations of the constituent elements are given by

$$\begin{aligned} R_1 &: i \ = G_1 u \\ R_2 &: i = \tilde{i}_2(u) \\ R_3 &: i = \tilde{i}_3(u) \\ C &: u = \tilde{u}(q) \quad \text{with} \quad \dot{q} = i \end{aligned} \tag{17}$$

Given a solution of $\mathcal{C}$ with $\hat{e} = 0$, the associated LTV

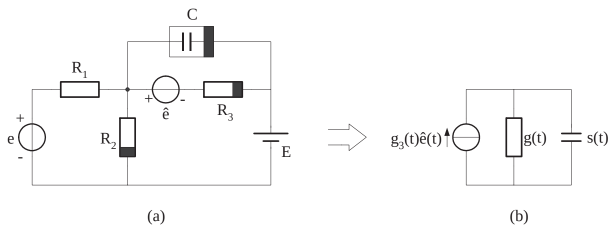

Fig. 2. Original nonlinear circuit $\mathcal{C}$ (a) and reduced LTV-small-signal circuit, with $g = G_1 + g_2 + g_3$ (b).

small-signal circuit $\hat{\mathcal{C}}$ is known, too. Due to its linearity, the topology of $\hat{\mathcal{C}}$ can be drastically reduced by using Norton's theorem together with the rule for parallel connections of linear conductances. The result is shown in figure 2.b.

Taking $\hat{q}$ as state-variable, the LTV state-equation is easily obtained as

$$\dot{\hat{q}} = \lambda(t)\hat{q} + g_3(t)\hat{e}(t) , \tag{18}$$

in which $\lambda(t) = -g(t)s(t)$ denotes the dynamical eigenvalue of (18). Since stability is guaranteed by a negative Lyapunov-exponent, we directly deduce from (18) that the solution of $\mathcal{C}$ is stable iff [10]

$$\lim_{t\to\infty} t^{-1} \int_0^t \lambda(\tau) d\tau < 0 . \tag{19}$$

We leave it to an excercise to solve (18) by the method of variation of constants.

Finally, it is noted that the charge q is preferred to the voltage u as state-variable. By absence of a current impulse through a capacitor (which is always the case in a robust model circuit), the charge is invariable a continuous function of time (see (11a)). This is not necessary true for the voltage across a LTV-capacitor: $c(t)$ or $s(t)$ may be discontinuous functions of time (compare (12)) [14]. A dual argument holds for a LTV inductor.

The derivation of the LTV variational circuit pertaining to circuits containing nonlinear N-terminal elements is essentially the same as before. Taking one terminal as the datum node, a N-terminal element defines $(N-1)$ branches. Each corresponding branch relation interacts with the other ones. For example, a nonlinear 3-terminal resistor generates two coupled branch relations. For them, we have six possible representations (a choice of two out of four variables). If we take the voltage controlled representation (e.g. a resistive Ebers-Moll model for a common base bipolar transistor), the two branch relations

$$i_1 = \tilde{i}_1(u_1, u_2) \quad \text{and} \quad i_2 = \tilde{i}_2(u_1, u_2) \tag{20}$$

give rise to the following LTV small-signal branch relations

$$\hat{i}_1 = g_{11}\hat{u}_1 + g_{12}\hat{u}_2 \quad \text{and} \quad \hat{i}_2 = g_{21}\hat{u}_1 + g_{22}\hat{u}_2 \;, \tag{21}$$

in which the incremental quantities

$$g_{ij}(t) = (\partial\tilde{i}_i \,/\, \partial u_j)_{\mathcal{C}} \qquad (i, j = 1, 2) \tag{22}$$

denote differential conductances, evaluated at the signal-dependent solution of the original nonlinear circuit $\mathcal{C}$. Hence, they define the constitutive coefficients of a LTV voltage controlled 3-terminal resistor.

III. Time-discrete LTV models

In this section it is shown that the LTV small-signal circuit is easily converted into a time-discrete resistive circuit. To that aim, the LTV resistive elements are taken at their value at discrete times t_k, while the dynamic LTV elements are replaced by discrete recursive resistive elements.

To start with, consider a LTV capacitor with voltage controlled branch relation (10.5). Combined with the auxiliary equation (11a) yields the $u-i$-relation

$$c(t)u(t) = \int^t i(\tau)d\tau \;, \tag{23}$$

where we have suppressed the notation for small variations. For two successive times t_k and t_{k+1} it follows

$$c_{k+1}u_{k+1} = c_k u_k + \int_{t_k}^{t_{k+1}} i(\tau)d\tau \;, \tag{24}$$

in which index k refers to time t_k. Next, the integral in (24) is approximated by a suitable numerical integration rule. For example, the backward Euler-integration rule with step size h yields

$$c_{k+1}u_{k+1} = c_k u_k + h i_{k+1} \;, \tag{25}$$

which is subsequently rewritten as

$$i_{k+1} = (h^{-1}c_{k+1})u_{k+1} - (h^{-1}c_k)u_k \;. \tag{26}$$

Finally, this expression is recognized as the $u-i$-relation of a recursive (dynamic) resistive one-port at time t_{k+1}, as shown in figure 3.a. In analogy, for a

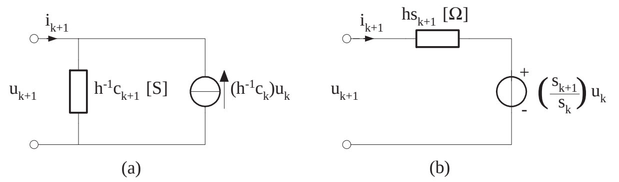

Fig. 3. Time-discrete LTV capacitor (Backward Euler). Voltage controlled (a) and charge controlled (b).

charge controlled LTV capacitor with branch relation (10.6) the recursive resistive port of figure 3.b is obtained. By setting $c_k = s_k^{-1}$, it is easily seen that the one-ports in figure 3 are equivalent. For LTV inductors, dual time-discrete models are obtained.

Finally, by replacing each LTV element in a LTV small-signal circuit by an appropriate time-discrete element, there results a time-discrete resistive circuit that can be solved recursively [20].

IV. Conclusions

For a given nonlinear electronic circuit, an associated linear time-varying small-signal circuit is formally derived by the tableau method. Due to its linearity,linear circuit theory may be applied. It opens the possibility of simplifying the circuit topology. Then, the LTV state-equations are most easily obtained. In general, the solution of the LTV state-equation requires the solution of a Riccati-equation. The LTV solution then follows by modal expansion. Stability analysis is performed by checking the Lyapunov exponents.

Alternatively, the LTV solution can be found recursively by replacing each LTV circuit element by an appropriate time-discrete resistive model.

References

[1] F.C.M. Kuijstermans, A. van Staveren, P. van der Kloet, F.L. Neerhoff and C.J.M. Verhoeven, *Dynamic Exponent based Electronics*, Proc. ProRisc/IEEE Workshop CSSP, Mierlo, the Netherlands, 1997.

[2] F.C.M. Kuijstermans, A. van Staveren, P. van der Kloet, F.L. Neerhoff and C.J.M. Verhoeven, *Dynamic Behavior of Nonlinear Circuits: the Linear Time-Varying Approach*, Proc. NDES'98, Budapest, Hungary, July 1998, pp. 279-285.

[3] F.C.M. Kuijstermans, F.M. Diepstraten, W.A. Serdijn, P. van der Kloet, A. van Staveren, and A.H.M. van Roermund, *The linear time-varying approach applied to a first-order dynamic translinear filter*, Proc. ISCAS, Orlando, USA, June 1999, pp. VI-69-72.

[4] A. van Staveren, T.H.A.J. Cordenier, F.C.M. Kuijstermans, P. van der Kloet, F.L. Neerhoff, C.J.M. Verhoeven and A.H.M. van Roermund, *The linear time-varying approach applied to the design of a negative- feedback class-B output amplifier*, Proc. ISCAS, Orlando, USA, June 1999, pp. II-204-207.

[5] F.M. Diepstraten, F.C.M. Kuijstermans, W.A. Serdijn, P. van der Kloet, A. van Staveren, F.L. Neerhoff, C.J.M. Verhoeven and A.H.M. van Roermund, *Dynamic Behaviour of Dynamic Translinear Circuits: the Linear Time-Varying Approach*, accepted for publ. IEEE Trans. on Circuits and Systems.

[6] T. Falk, F.C.M. Kuijstermans and A. van Staveren, *Investigation of the Stability Properties of a Class of Oscillators using Lyapunov Transformations*, Proc. NDES'99, Ronne, Denmark, July 15-17, 1999, pp. 221-224.

[7] A. van Staveren, F.C.M. Kuijstermans, P. van der Kloet, F.L. Neerhoff, C.J.M. Verhoeven and A.H.M. van Roermund, *Analyzing Nonlinear Circuits using the Linear Time-Varying Approach*, Proc. X. Int. Symp. Theor. Electr. Eng. Magdeburg, Germany, 1999, pp. 337-341.

[8] P. van der Kloet and F.L. Neerhoff, *On Eigenvalues and Poles for Second Order Linear Time-Varying Systems*, Proc. NDES'97, Moscow, Russia, June 26-27, 1997, pp. 300-305.

[9] P. van der Kloet, F.L. Neerhoff, F.C.M. Kuijstermans, A. van Staveren and C.J.M. Verhoeven, *Generalizations for the Eigenvalue and Pole Concept with respect to Linear Time-Varying Systems*, Proc. ProRisc/IEEE Workshop CSSP, Mierlo, the Netherlands, 1997.

[10] P. van der Kloet and F.L. Neerhoff, *Behaviour of Dynamic Eigenpairs in Slowly-Varying Systems*, Proc. NDES'99, Ronne, Denmark, July 15-17, 1999, pp. 9-12.

[11] P. van der Kloet and F.L. Neerhoff, *Iteration Schemes for the Modal Solutions of Linear Time-Varying Systems*, Proc. NDES'99, Ronne, Denmark, July 15-17, 1999, pp. 25-28.

[12] P. van der Kloet and F.L. Neerhoff, *Diagonalization Algorithms for Linear Time-Varying Dynamic Systems*, accepted for publ. Int.Journ. Systems Sci.

[13] P. van der Kloet F.C.M. Kuijstermans, F.L. Neerhoff, A. van Staveren and C.J.M. Verhoeven, *A note on Dynamic Eigenvalues and Slowly-Varying Systems*, Proc. X. Int. Symp. Theor. Electr. Eng. Magdeburg, Germany, 1999, pp. 141-144.

[14] C.A. Desoer and E.S. Kuh, *Basic Circuit Theory*, Mc.Graw-Hill, 1969.

[15] C.A. Desoer and K.K. Wong, *Small-Signal Behavior of Nonlinear Lumped Networks*, Proc. IEEE, Vol. 56, No. 1, 1968, pp. 14-22.

[16] E. Kleihorst, *Frequency domain analysis for nonlinear electronic circuits*, Ph. D-thesis, TU-Delft, 1994.

[17] P. van der Kloet and F.L. Neerhoff, *a Minimum Equation Harmonic Balance Method*, Proc. NDES'96, Seville, Spain, June 27-28, 1996, pp. 139-144.

[18] L.O.Chua, C.A.Desoer and E.S.Kuh, *Linear and Nonlinear Circuits*, McGraw-Hill, 1987.

[19] T.E.Stern, *Theory of Nonlinear Networks and Systems, an Introduction*, Addison-Wesley, 1965.

[20] P. Dewilde and A. van der Veen, *Time-Varying Systems and Computations*, Kluwer Ac. Publ., 1998.