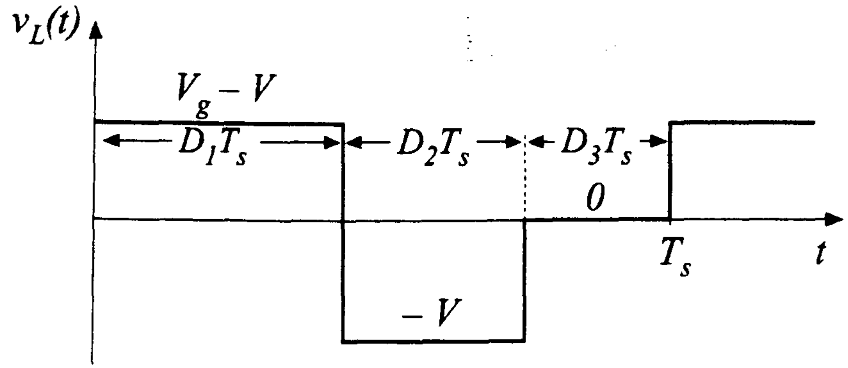
Inductor volt-second balance

(Buck converter)



Volt-second balance:

$$\langle v_L(t) \rangle = D_1(V_g - V) + D_2(-V) + D_3(0) = 0$$

Solve for V:

$$V = V_g \frac{D_1}{D_1 + D_2}$$

note that D_2 is unknown

Capacitor charge balance

node equation:

$$i_L(t) = i_C(t) + V / R$$

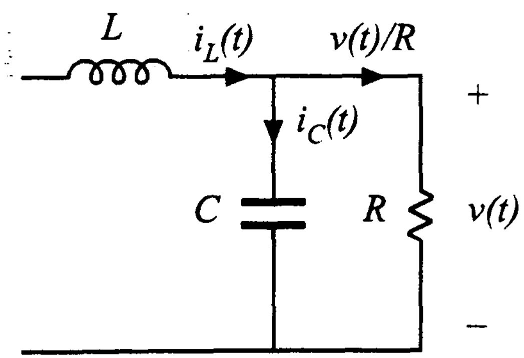
capacitor charge balance:

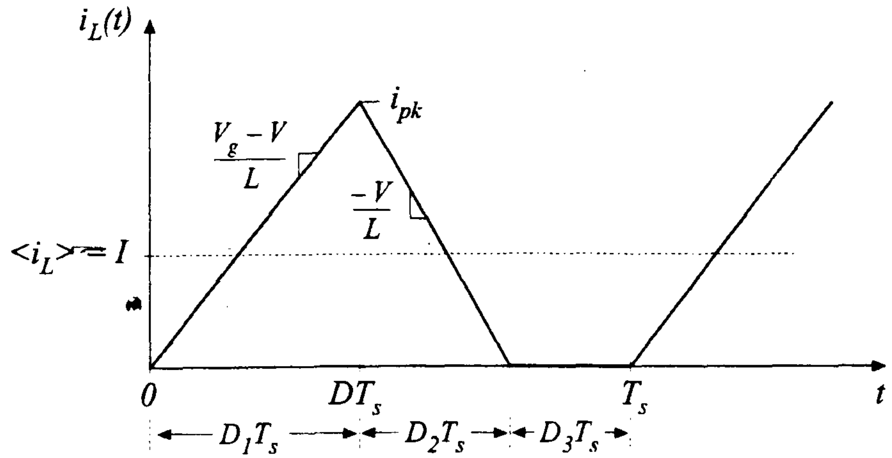
$$\langle i_C \rangle = 0$$

hence

$$\langle i_L \rangle = V / R$$

must compute dc component of inductor current and equate to load current (for this buck converter example)





Inductor current waveform

peak current:

$$i_L(D_1T_s) = i_{pk} = \frac{V_g - V}{L} D_1T_s$$

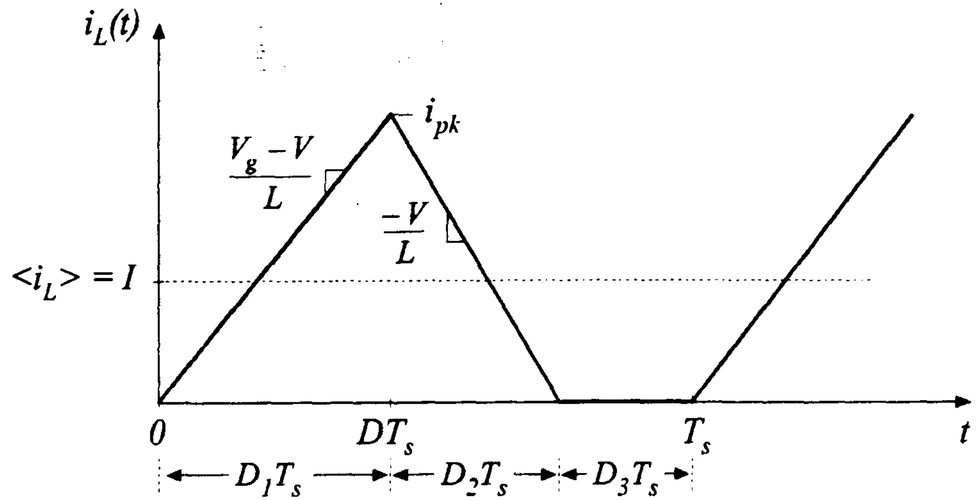
average current:

$$\left\langle i_L \right\rangle = \frac{1}{T_s}\int_0^{T_s} i_L(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_L(t)\, dt = \frac{1}{2}\, i_{pk}\,(D_1 + D_2)T_s$$

$$\left\langle i_L \right\rangle = (V_g - V)\,\frac{D_1T_s}{2L}\,(D_1 + D_2)$$



equate dc component to dc load current:

$$\frac{V}{R} = \frac{D_1T_s}{2L}\,(D_1 + D_2)\,(V_g - V)$$

Solution for V

Two equations and two unknowns (V and D_2):

$$V = V_g \frac{D_1}{D_1 + D_2} \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V}{R} = \frac{D_1 T_s}{2L} (D_1 + D_2)(V_g - V) \qquad \text{(from capacitor charge balance)}$$

Eliminate D_2 , solve for V :

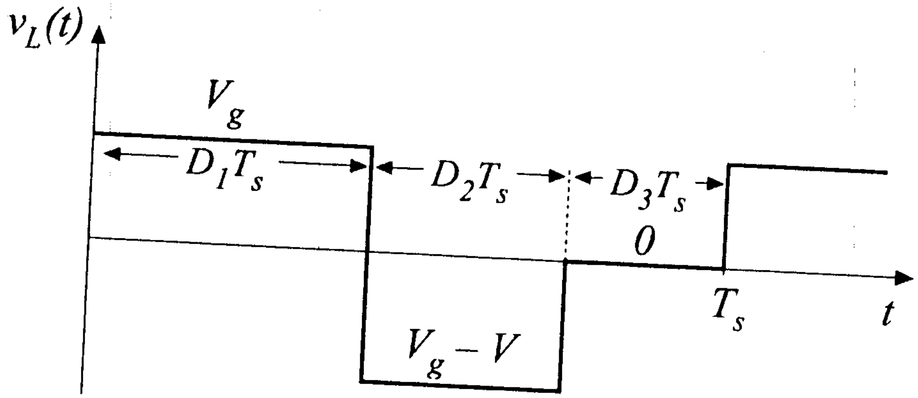
$$\frac{V}{V_g} = \frac{2}{1 + \sqrt{1 + 4K / D_1^2}}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}$

Boost Converter

Inductor volt-second balance



Volt-second balance:

$$D_1 V_g + D_2(V_g - V) + D_3(0) = 0$$

Solve for V:

$$V = \frac{D_1 + D_2}{D_2} V_g$$

note that D_2 is unknown

Capacitor charge balance

node equation:

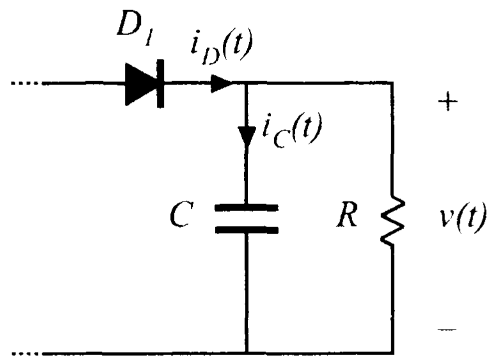
$$i_D(t) = i_C(t) + v(t) / R$$

capacitor charge balance:

$$\langle i_C \rangle = 0$$

hence

$$\langle i_D \rangle = V / R$$



must compute dc component of diode current and equate to load current (for this boost converter example)

Inductor and diode current waveforms

peak current:

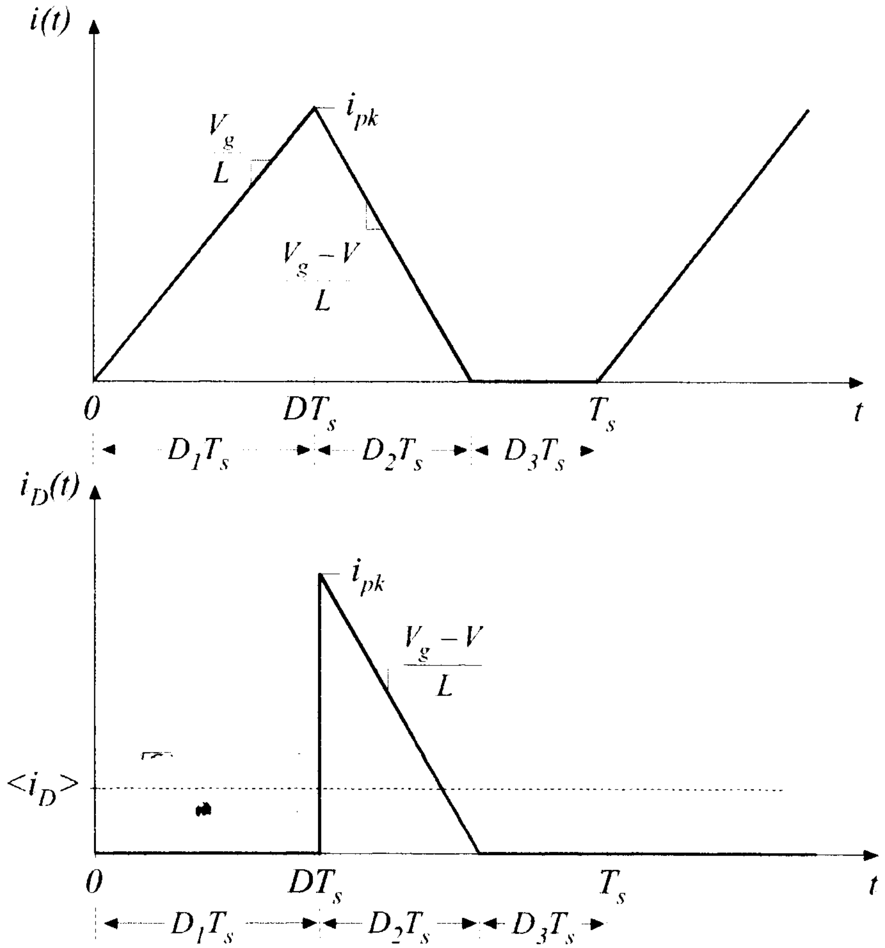
$$i_{pk} = \frac{V_g}{L} D_1 T_s$$

average diode current:

$$\left\langle i_D \right\rangle = \frac{1}{T_s} \int_0^{T_s} i_D(t)\, dt$$

triangle area formula:

$$\int_0^{T_s} i_D(t)\, dt = \frac{1}{2}\, i_{pk}\, D_2 T_s$$



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Equate diode current to load current

ave-
rage diode current:

$$\langle i_D \rangle = \frac{1}{T_s}\left(\frac{1}{2}\, i_{pk}\, D_2 T_s\right) = \frac{V_g D_1 D_2 T_s}{2L}$$

equate
ate to dc load current:

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R}$$

Solution for V

Two equations and two unknowns (V and D_2):

$$V = \frac{D_1 + D_2}{D_2} V_g \qquad \text{(from inductor volt-second balance)}$$

$$\frac{V_g D_1 D_2 T_s}{2L} = \frac{V}{R} \qquad \text{(from capacitor charge balance)}$$

Eliminate D_2 , solve for V. From volt-sec balance eqn:

$$D_2 = D_1 \frac{V_g}{V - V_g}$$

Substitute into charge balance eqn, rearrange terms:

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Solution for V

$$V^2 - VV_g - \frac{V_g^2 D_1^2}{K} = 0$$

Use quadratic formula:

$$\frac{V}{V_g} = \frac{1 \pm \sqrt{1 + 4D_1^2 / K}}{2}$$

Note that one root leads to positive V, while other leads to negative V. Select positive root:

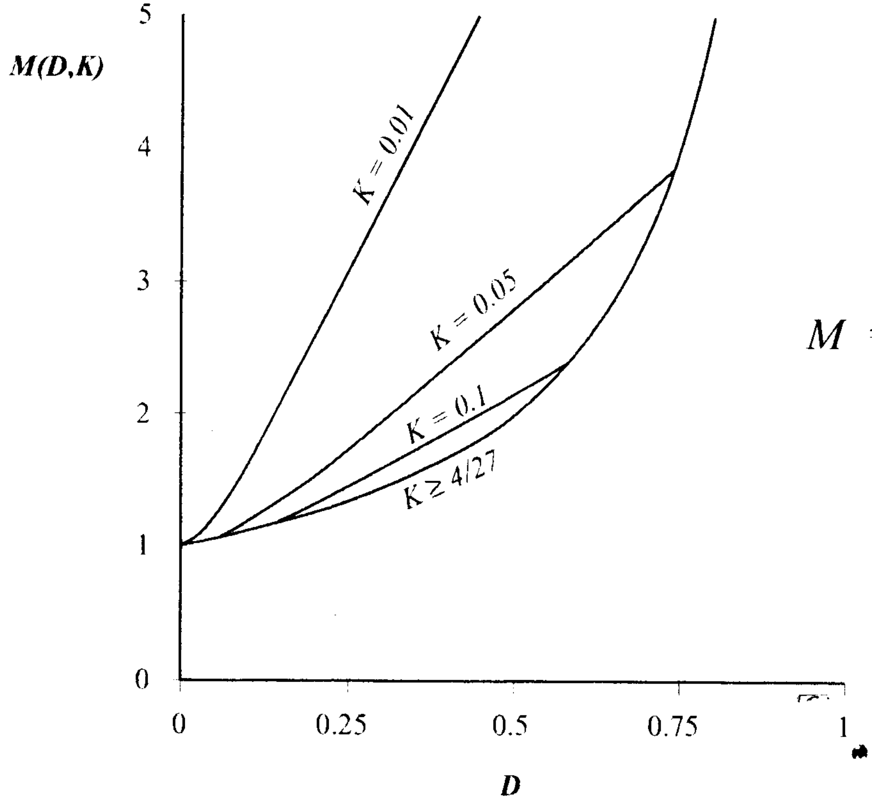
$$\frac{V}{V_g} = M(D_1, K) = \frac{1 + \sqrt{1 + 4D_1^2 / K}}{2}$$

where $K = 2L / RT_s$

valid for $K < K_{crit}(D)$

Transistor duty cycle D = interval 1 duty cycle D_1

Boost converter characteristics



$$M = \begin{cases} \dfrac{1}{1-D} & \text{for } K > K_{crit} \\ \dfrac{1+\sqrt{1+4D^2/K}}{2} & \text{for } K < K_{crit} \end{cases}$$

Approximate M in DCM:

$$M \approx \frac{1}{2} + \frac{D}{\sqrt{K}}$$

Summary of DCM characteristics

Table 5.2. Summary of CCM-DCM characteristics for the buck, boost, and buck-boost converters

Converter	$K_{crit}(D)$	DCM $M(D,K)$	DCM $D_2(D,K)$	CCM $M(D)$
Buck	$(1-D)$	$\dfrac{2}{1+\sqrt{1+4K/D^2}}$	$\dfrac{K}{D}M(D,K)$	D
Boost	$D(1-D)^2$	$\dfrac{1+\sqrt{1+4D^2/K}}{2}$	$\dfrac{K}{D}M(D,K)$	$\dfrac{1}{1-D}$
Buck-boost	$(1-D)^2$	$-\dfrac{D}{\sqrt{K}}$	$\sqrt{K}$	$-\dfrac{D}{1-D}$

with $K = 2L/RT_s$. *DCM occurs for* $K < K_{crit}$.

$\dfrac{2L}{RT_s} < 1-D$ (*)

$\dfrac{2Lf_s}{R_L} < (1-D)^2 D$ (**)

(*) $I_{Lcrit.} < \dfrac{(1-D)V_g\,DT_s}{2L} \Rightarrow \boxed{\dfrac{2L}{RT_s} < 1-D}$

(**) $\dfrac{2Lf_s}{R_L} < (1-D)^2 D$; $I_{Lcrit.} = \dfrac{V_{in}\,D}{2Lf_s}$

$I_{LOAD} = (1-D)I_{Lcrit} = \dfrac{V_{out}(1-D)^2 D}{2Lf_s}$

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