III. MAGNETICS DESIGN

Outline:

3.1. Inductor/Transformer Design Relationships

3.2. Magnetic Cores and Materials, Core Geometries

3.3. Power Dissipation in Magnetic Devices

Low frequency losses, High frequency losses

3.4. Thermal Considerations

3.5. Inductor Design Procedures

3.6. Transformer Design Procedures

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3.1. Inductor/Transformer Design Relationships

❖ *The purpose of a **power transformer** in SMPS is to transfer power efficiently, and instantaneously from the source to the load. It also provides the followings:*

- *The primary-to-secondary turns ratio can be established to efficiently accomodate widely different input/output voltage levels.*

- *Multiple secondaries with different number of turns can be used to achieve multiple outputs at different voltage levels.*

- *Separate primary and secondary windings facilitate high voltage input/output isolation, especially important for safety in off-line applications.*

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- *Ideally, a transformer stores no energy, all energy is transferred instantaneously from input to output.*

- *A practical transformer does store some energy in the mutual (magnetizing) inductance and leakage inductances, which degrade the circuit performance. These inductances are normally considered as undesirable parasitics, whose minimization is one of the important goals of transformer design. ,*

- *Leakage inductance delays the transfer of current between switches and rectifiers during switching transitions. These delays are proportional to load current, are the main cause of regulation problems.*

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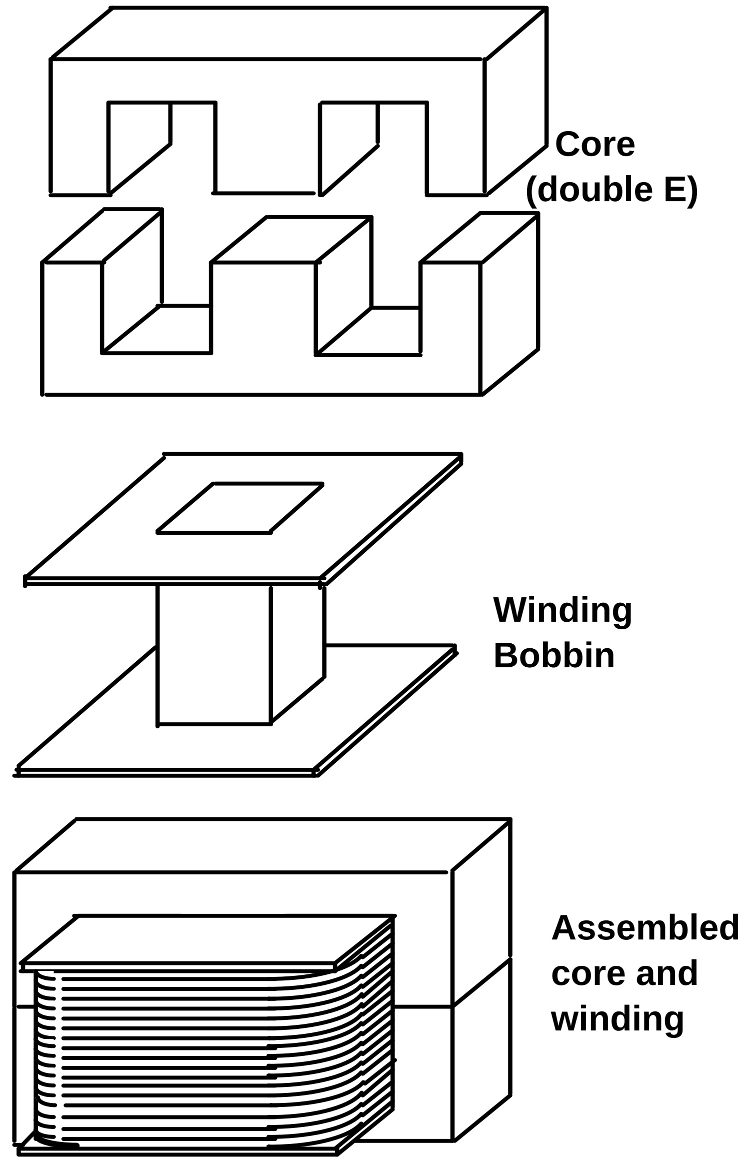
❖ *Filter inductors, boost inductors, and flyback transformers are all members of the 'power inductor' family. They all function by taking energy from the electrical circuit, storing it in a magnetic field, and subsequently returning this energy to the circuit. As discussed previously, a flyback transformer is actually a multi-winding coupled inductor, unlike the true transformer.*

❖ *Parasitic elements inherent in high frequency transformers or inductors cause a variety of circuit problems including high losses, high voltage spikes necessitating snubbers or clamps, poor cross regulation between multiple outputs, noise coupling to input or output, restricted duty cycle range etc...*

Magnetic Component Design Responsibility of Circuit Designer

- Ratings for inductors and transformers in power electronic circuits vary too much for commercial vendors to stock full range of standard parts.
- Instead only magnetic cores are available in a wide range of sizes, geometries, and materials as standard parts.
- Circuit designer must design the inductor/transformer for the particular application.
- Design consists of:

  1. Selecting appropriate core material, geometry, and size
  2. Selecting appropriate copper winding parameters: wire type, size, and number of turns.

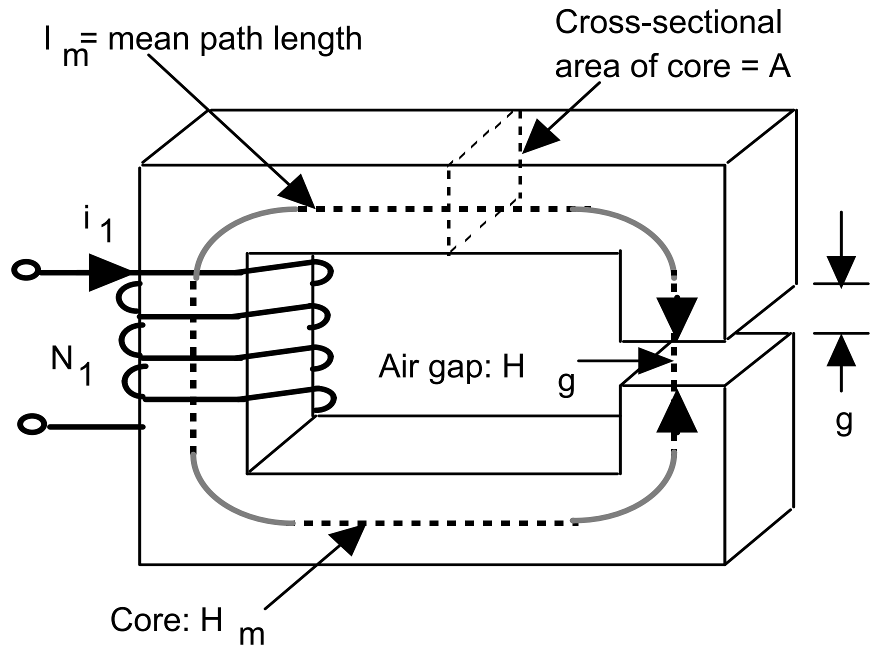


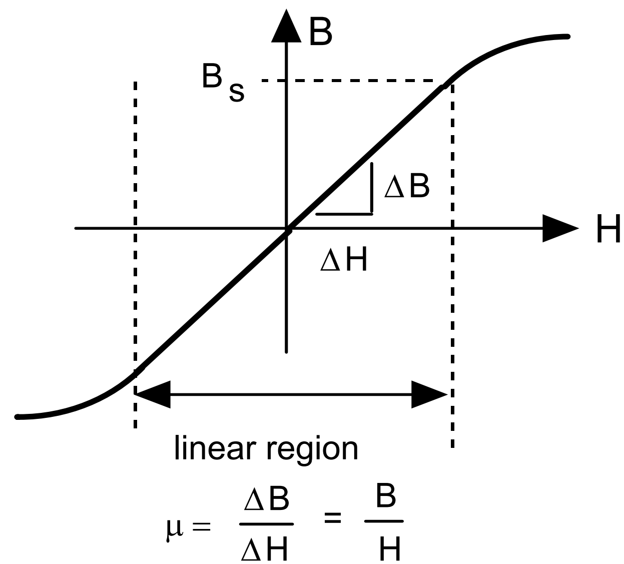
Review of Inductor Fundamentals

- Assumptions
  - No core losses or copper winding losses
  - Linearized B-H curve for core with $\mu_m >> \mu_o$
  - $l_m >> g$ and $A >> g^2$
  - Magnetic circuit approximations (flux uniform over core cross-section, no fringing flux)

- Starting equations
  - $H_m l_m + H_g g = N I$ (Ampere's Law)
  - $B_m A = B_g A = \phi$ (Continuity of flux assuming no leakage flux)
  - $\mu_m H_m = B_m$ (linearized B-H curve) ; $\mu_o H_g = B_g$

- Results
  - $B_s > B_m = B_g = \dfrac{N I}{l_m/\mu_m + g/\mu_o} = \phi/A$
  - $LI = N\phi$; $L = \dfrac{A N^2}{l_m/\mu_m + g/\mu_o}$

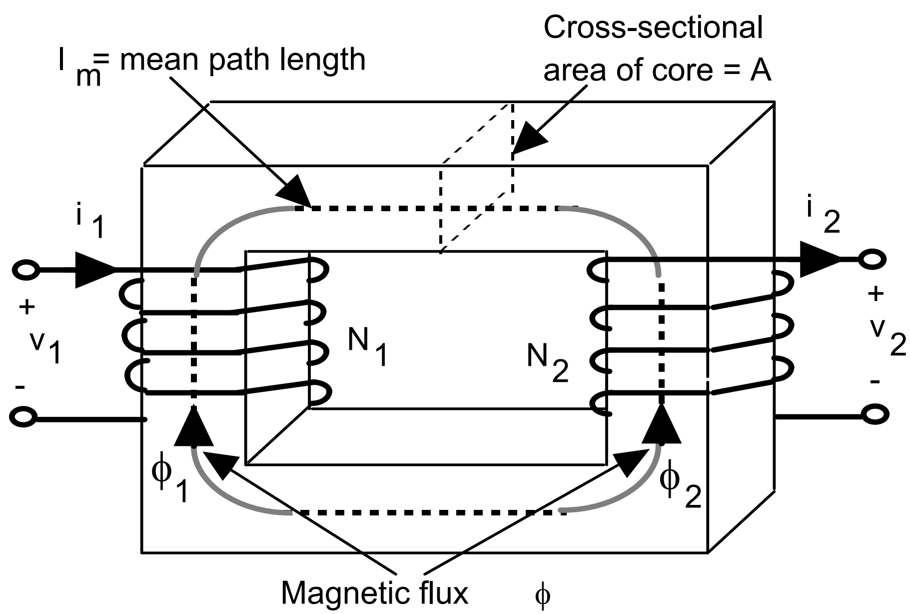




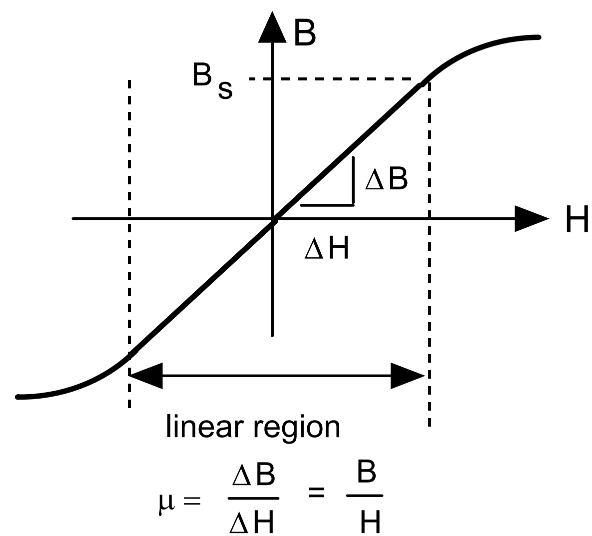
Review of Transformer Fundamentals

- Assumptions same as for inductor

- Starting equations
  - $H_1 L_m = N_1 I_1$; $H_2 L_m = N_2 I_2$ (Ampere's Law)
  - $H_m L_m = (H_1 - H_2) L_m = N_1 I_1 - N_2 I_2$
  - $\mu_m H_m = B_m$ (linearized B-H curve)
  - $v_1 = N_1 \frac{d\phi_1}{dt}$; $v_2 = N_2 \frac{d\phi_2}{dt}$ (Faraday's Law)
  - Net flux $\phi = \phi_1 - \phi_2 = \mu_m H_m A = \frac{\mu_m A (N_1 I_1 - N_2 I_2)}{L_m}$

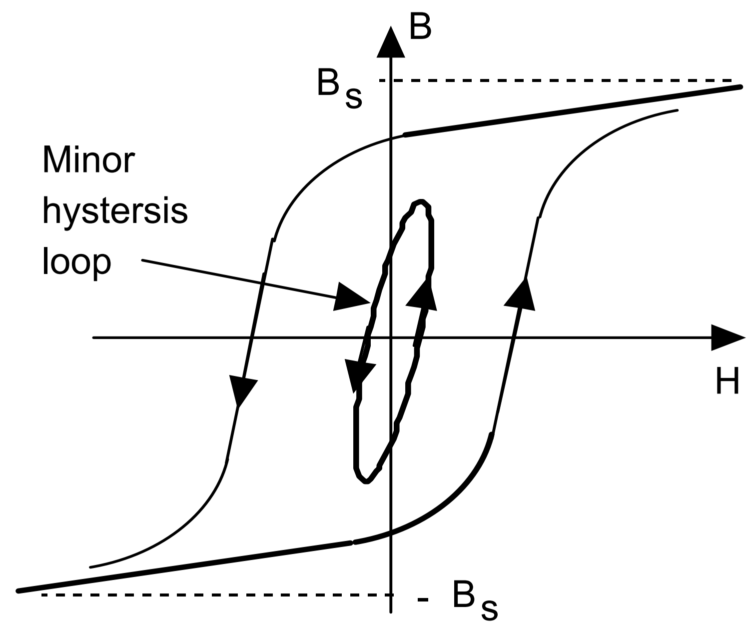


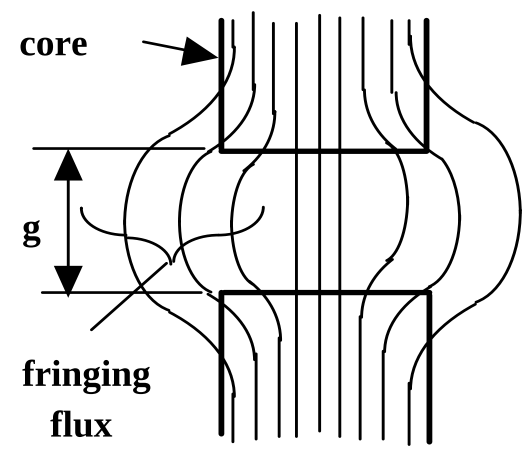
- Results assuming $\mu_m \Rightarrow \infty$, i.e. ideal core or ideal transformer approximation.
  - $\frac{\phi}{\mu_m} = 0$ and thus $N_1 I_1 = N_2 I_2$
  - $\frac{d(\phi_1 - \phi_2)}{dt} = 0 = \frac{v_1}{N_1} - \frac{v_2}{N_2}$; $\frac{v_1}{N_1} = \frac{v_2}{N_2}$



Current/Flux Density Versus Core Size

- Larger electrical ratings require larger current I and larger flux density B.
  - Core losses (hysteresis, eddy currents) increase as B^2 (or greater)
  - Winding (ohmic) losses increase as I^2 and are accentuated at high frequencies (skin effect, proximity effect)
- To control component temperature, surface area of component and thus size of component must be increased to reject increased heat to ambient.
  - At constant winding current density J and core flux density B, heat generation increases with volume V but surface area only increases as $V^{2/3}$.
  - Maximum J and B must be reduced as electrical ratings increase.
- Flux density B must be $< B_s$
  - Higher electrical ratings $\Rightarrow$ larger total flux $\Rightarrow$ larger component size
  - Flux leakage, nonuniform flux distribution complicate design





Magnetic Component Design Problem

- Challenge - conversion of component operating specs in converter circuit into component design parameters.

- Goal - simple, easy-to-use procedure that produces component design specs that result in an acceptable design having a minimum size, weight, and cost.

- Inductor electrical (e.g.converter circuit) specifications.
  - Inductance value L
  - Inductor currents rated peak current I, rated rms current I_{rms} , and rated dc current (if any) I_{dc}
  - Operating frequency f.
  - Allowable power dissipation in inductor or equivalently maximum surface temperature of the inductor T_s and maximum ambient temperature T_a.

- Transformer electrical (converter circuit) specifications.
  - Rated rms primary voltage V_{pri}
  - Rated rms primary current I_{pri}
  - Turns ratio N_{pri}/N_{sec}
  - Operating frequency f
  - Allowable power dissipation in transformer or equivalently maximum temperatures T_s and T_a

- Design procedure outputs.
  - Core geometry and material.
  - Core size (A_{core} , A_w)
  - Number of turns in windings.
  - Conductor type and area A_{cu}.
  - Air gap size (if needed).

- Three impediments to a simple design procedure.

1. Dependence of J_{rms} and B on core size.

.

2. How to chose a core from a wide range of materials and geometries.

3. How to design low loss windings at high operating frequencies.

- Detailed consideration of core losses, winding losses, high frequency effects (skin and proximity effects), heat transfer mechanisms required for good design procedures.

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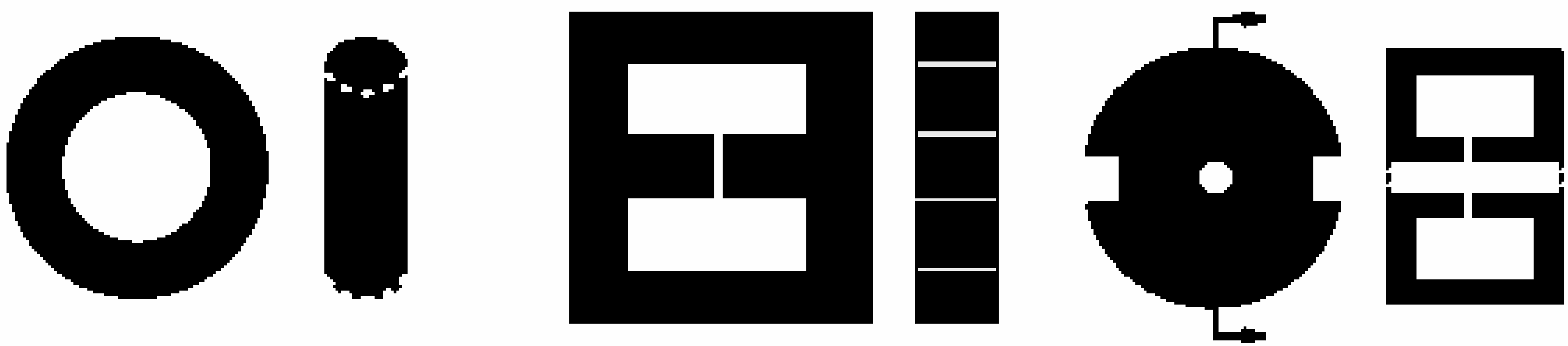
3.2. Magnetic Cores and Materials, Core Geometries

Some Common Core Types:



A. TORROID   B. E-E CORE   C. POT CORE

Core Shapes and Sizes

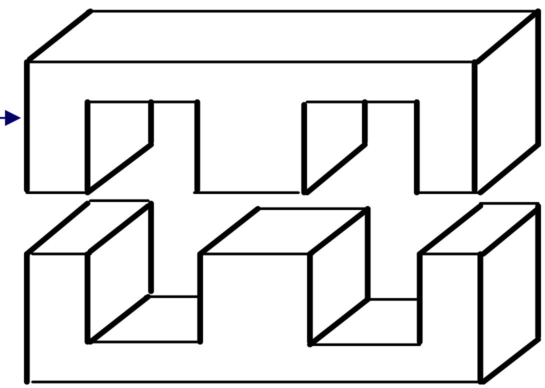
- Magnetic cores available in a wide variety of sizes and shapes.

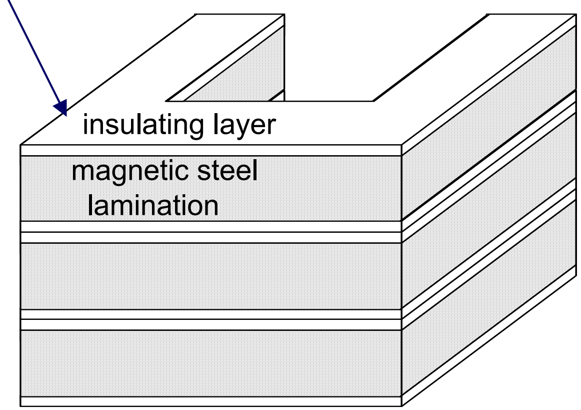
  - Ferrite cores available as U, E, and I shapes as well as pot cores and toroids.
  - Laminated (conducting) materials available in E, U, and I shapes as well as tape wound toroids and C-shapes.
  - Open geometries such as E-core make for easier fabrication but more stray flux and hence potentially more severe EMI problems.
  - Closed geometries such as pot cores make for more difficult fabrication but much less stray flux and hence EMI problems.

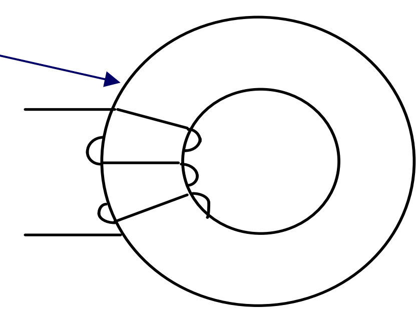
- Bobbin or coil former provided with most cores.

- Dimensions of core are optimized by the manufacturer so that for a given rating (i.e. stored magnetic energy for an inductor or V-I rating for a transformer), the volume or weight of the core plus winding is minimized or the total cost is minimized.

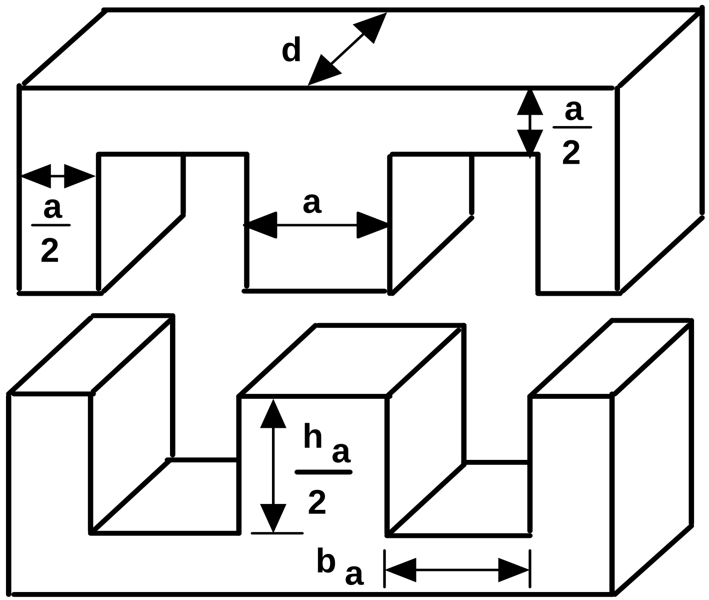
  - Larger ratings require larger cores and windings.
  - Optimization requires experience and computerized optimization algorithm.
  - Vendors usually are in much better position to do the optimization than the core user.



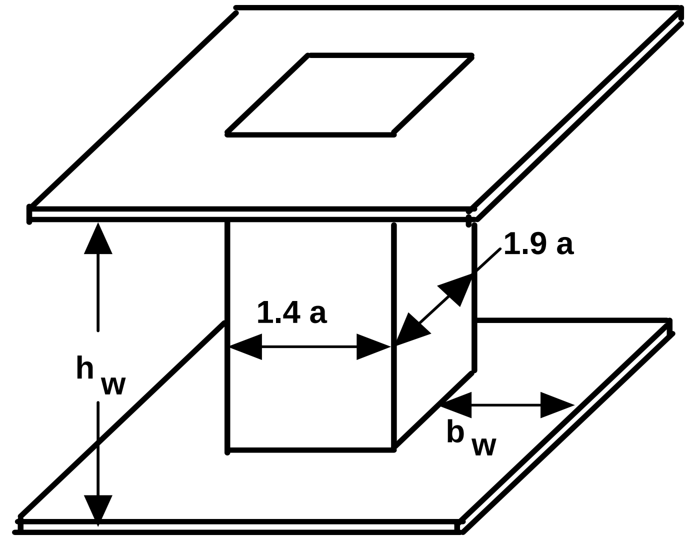




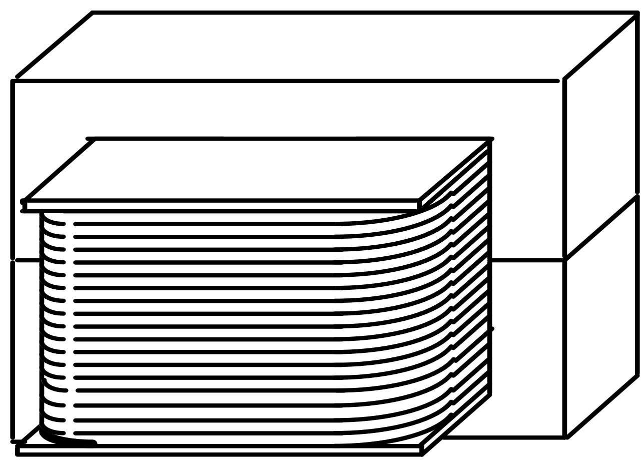
Double-E Core Example



Core



Bobbin



Assembled core and winding

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Core materials

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors
Amorphous Metal	**1.5 – 1.6 T**	**low**	**10 – 100 kHz transformers, Magamps**

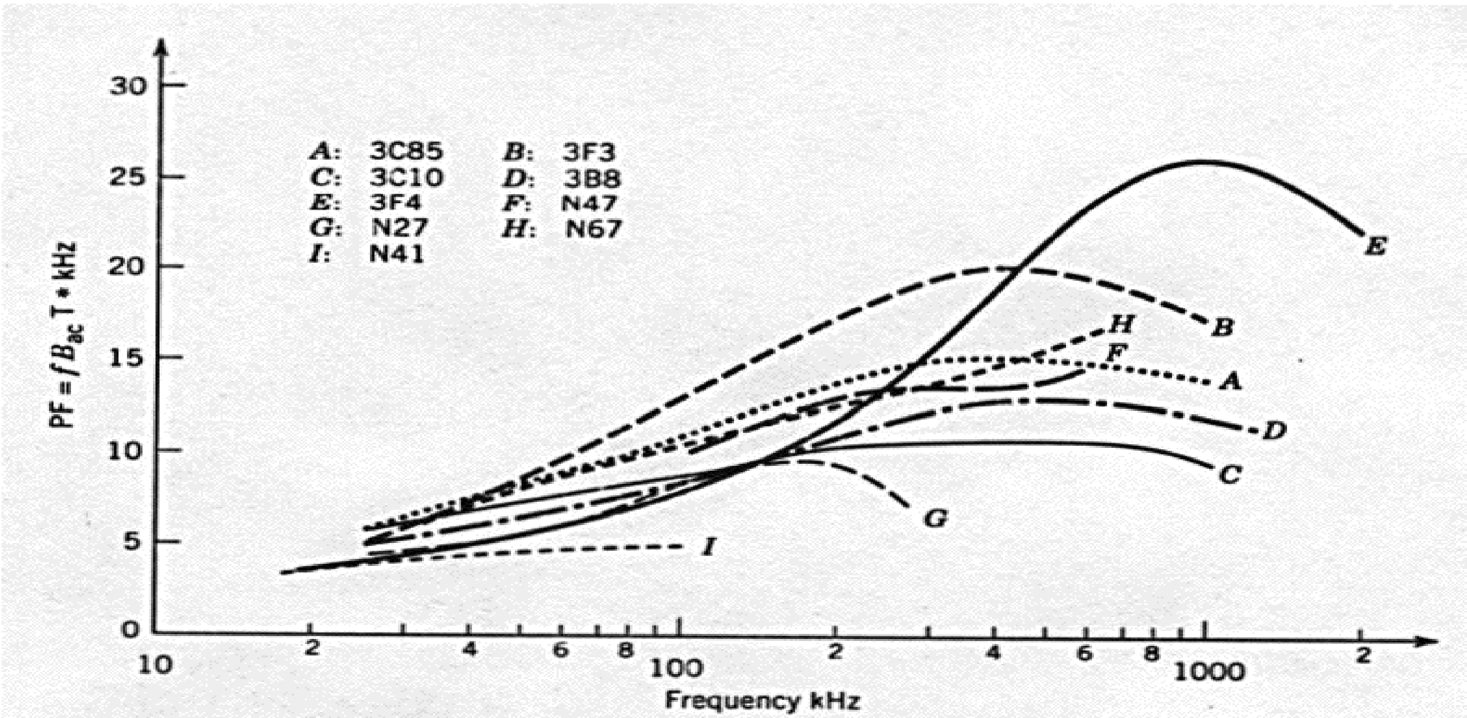
Types of Core Materials

- Iron-based alloys
  - Various compositions
    - Fe-Si (few percent Si)
    - Fe-Cr-Mn
    - METGLASS (Fe-B, Fe-B-Si, plus many other compositions)
  - Important properties
    - Resistivity _ = (10 - 100) ρ_{Cu}
    - B_s = 1 - 1.8 T (T = tesla = 10^4 oe)
  - METGLASS materials available only as tapes of various widths and thickness.
  - Other iron alloys available as laminations of various shapes.
  - Powdered iron can be sintered into various core shapes. Powdered iron cores have larger effective resistivities.

- Ferrite cores
  - Various compositions - iron oxides, Fe-Ni-Mn oxides
  - Important properties
    - Resistivity ρ very large (insulator) - no ohmic losses and hence skin effect problems at high frequencies.
    - B_s = 0.3 T (T = tesla = 10^4 oe)

Core Material Performance Factor

- Volt-amp (V-A) rating of transformers proportional to f B_{ac}
- Core materials have different allowable values of B_{ac} at a specific frequency. B_{ac} limited by allowable $P_{m,sp}$.
- Most desirable material is one with largest B_{ac}.
- Choosing best material aided by defining an emperical performance factor PF = f B_{ac}. Plots of PF versus frequency for a specified value of $P_{m,sp}$ permit rapid selection of best material for an application.
- Plot of PF versus frequency at $P_{m,sp}$ = 100 mW/cm^3 for several different ferrites shown below.



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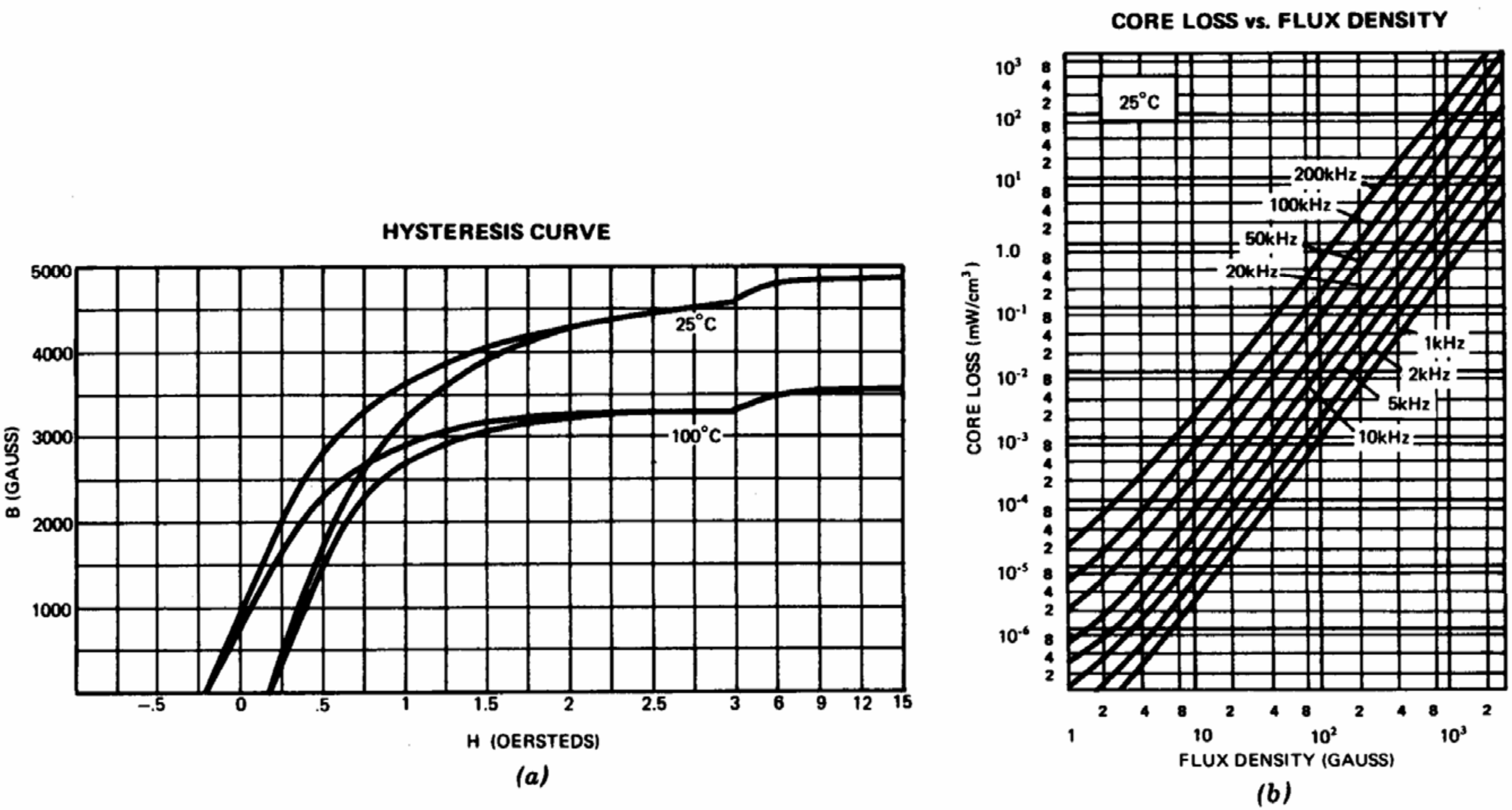
Ferrite core material characteristics



3C8 ferrite characteristic curves: (*a*) B–H loop; (*b*) core loss curves. (*Courtesy of Ferroxcube Division of Amperex*

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3.3. Power Dissipation in Magnetic Devices

Low-frequency losses:

- Dc copper loss
- Core loss: hysteresis loss

High-frequency losses: the skin effect

- Core loss: classical eddy current losses
- Eddy current losses in ferrite cores

High frequency copper loss: the proximity effect

- Proximity effect: high frequency limit
- MMF diagrams, losses in a layer, and losses in basic multilayer windings

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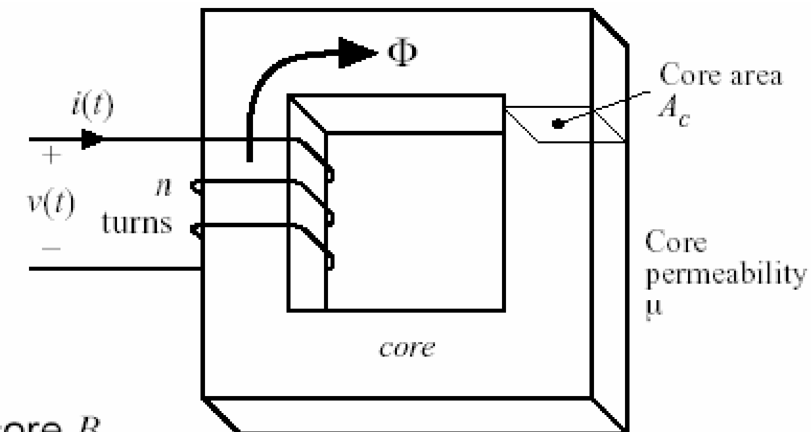
A. CORE LOSS:

Energy per cycle W flowing into n-turn winding of an inductor, excited by periodic waveforms of frequency f:

$$W = \int_{one\ cycle} v(t)i(t)dt$$



Relate winding voltage and current to core B and H via Faraday's law and Ampere's law:

$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)\ell_m = ni(t)$$

Substitute into integral:

$$W = \int_{one\ cycle} \left(nA_c \frac{dB(t)}{dt}\right)\left(\frac{H(t)\ell_m}{n}\right)dt$$

$$= \left(A_c \ell_m\right) \int_{one\ cycle} H\, dB$$

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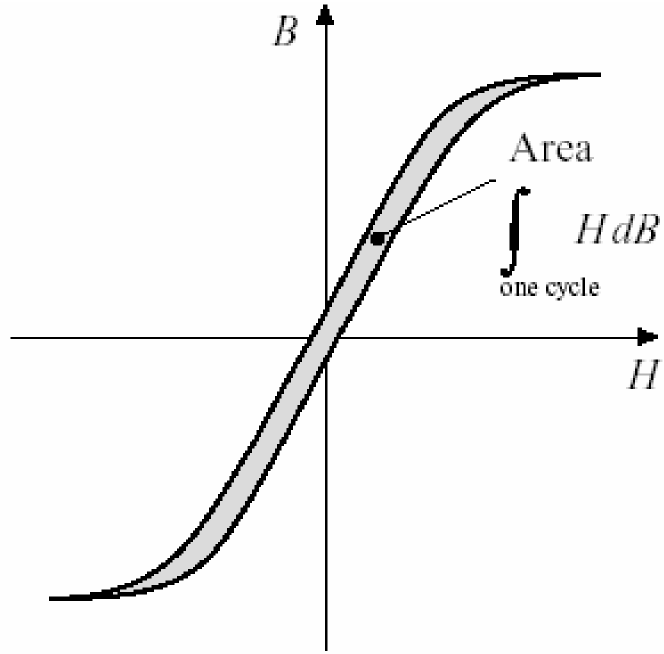
a. Hysteresis Loss:

$$W = \left(A_c \ell_m\right) \int_{one\ cycle} H\, dB$$



The term $A_c \ell_m$ is the volume of the core, while the integral is the area of the B–H loop.

(energy lost per cycle) = (core volume) (area of B–H loop)

$$P_H = \left(f\right)\left(A_c \ell_m\right) \int_{one\ cycle} H\, dB$$

Hysteresis loss is directly proportional to applied frequency

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Hysteresis Loss:

- Hysteresis loss varies directly with applied frequency
- Dependence on maximum flux density: how does area of B–H loop depend on maximum flux density (and on applied waveforms)? Empirical equation (Steinmetz equation):

$$P_H = K_H f B_{\max}^{\alpha}(core\ volume)$$

The parameters K_H and α are determined experimentally.

Dependence of P_H on B_{max} is predicted by the theory of magnetic domains.

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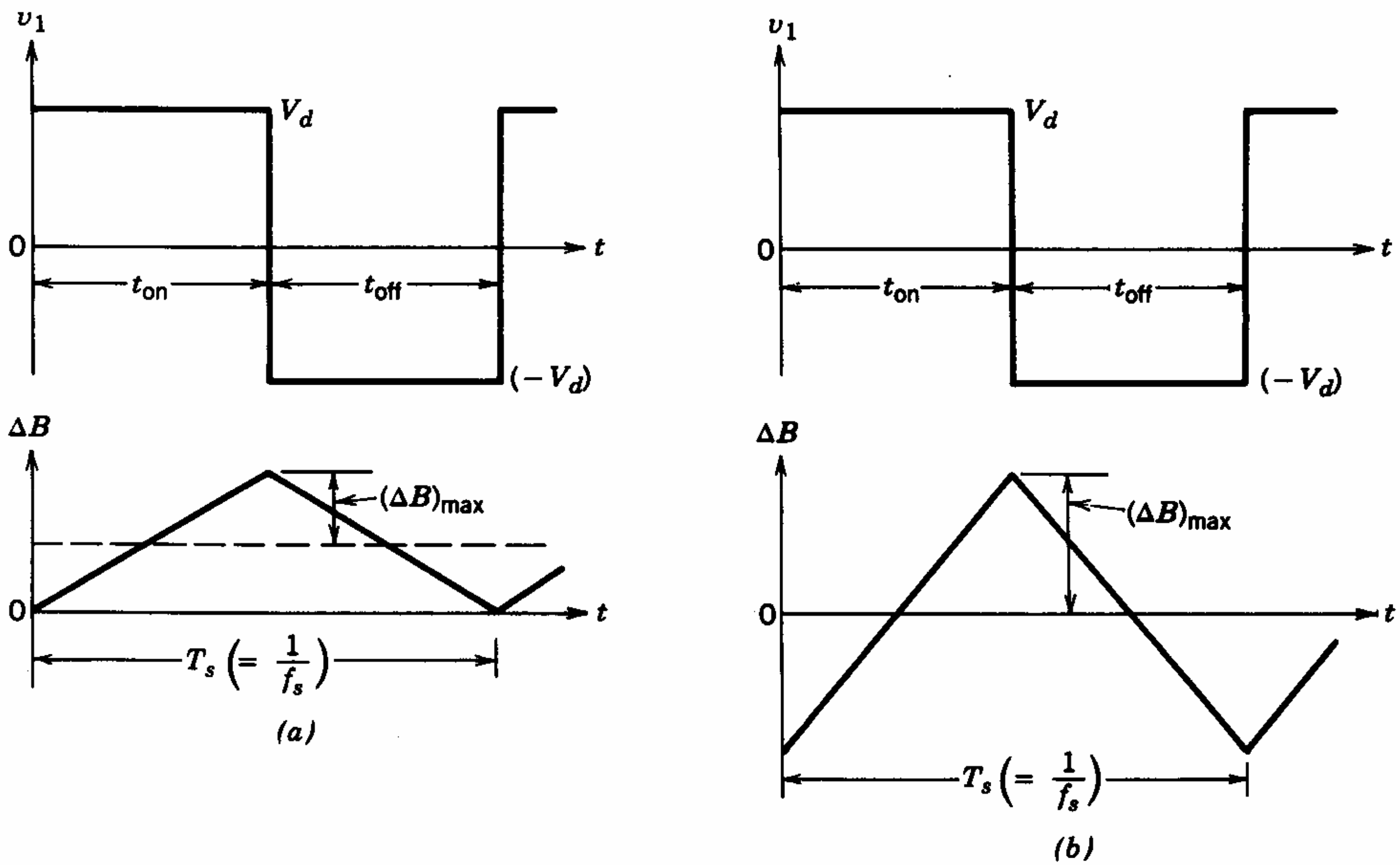
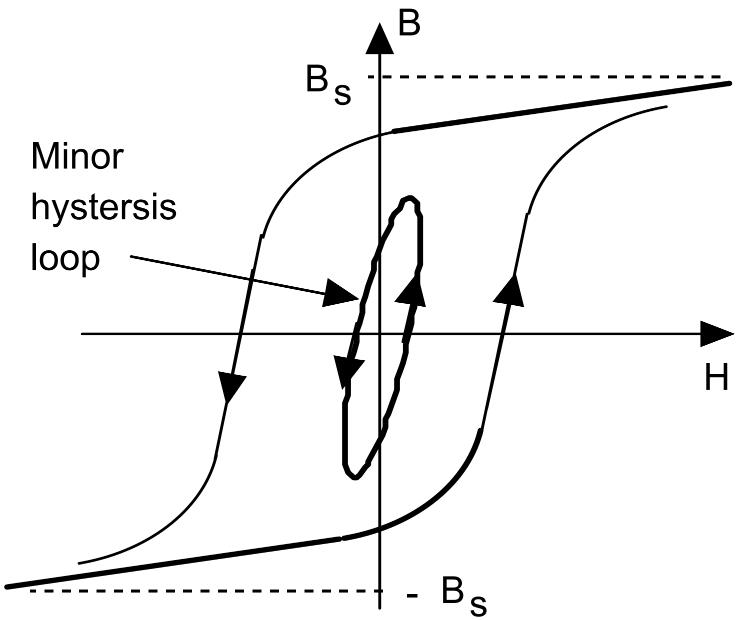
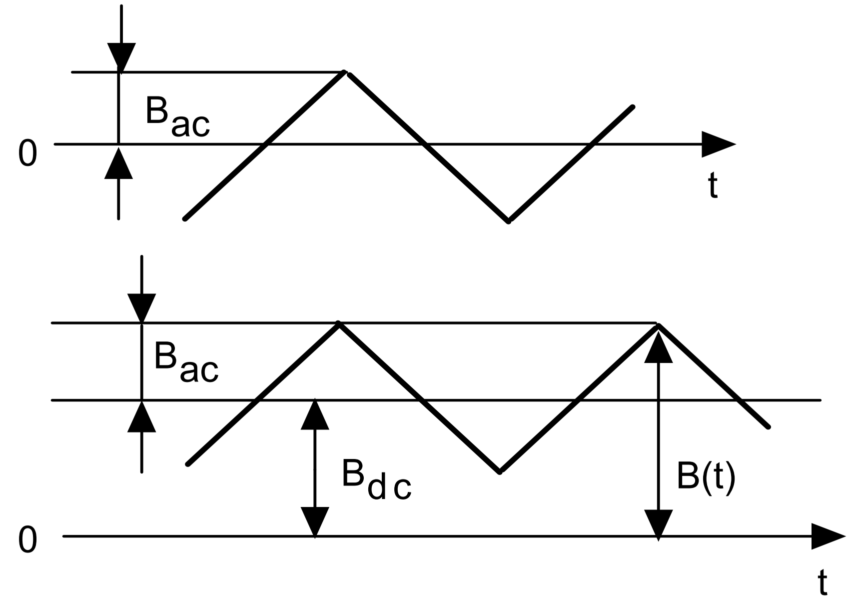
Core utilization in various converter topologies



Core excitation: (a) forward converter, $D = 0.5$; (b) full-bridge converter,

Hysteresis Loss in Magnetic Materials





- Area encompassed by hysteresis loop equals work done on material during one cycle of applied ac magnetic field. Area times frequency equals power dissipated per unit volume.

- Typical waveforms of flux density, B(t) versus time, in an inductor.
- Only B_{ac} contributes to hysteresis loss.

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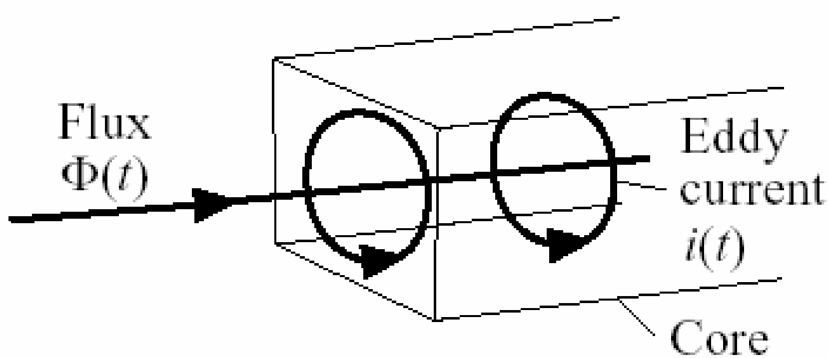
b. Eddy Current Loss:

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux which opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.



Eddy current loss $i^2(t)R$

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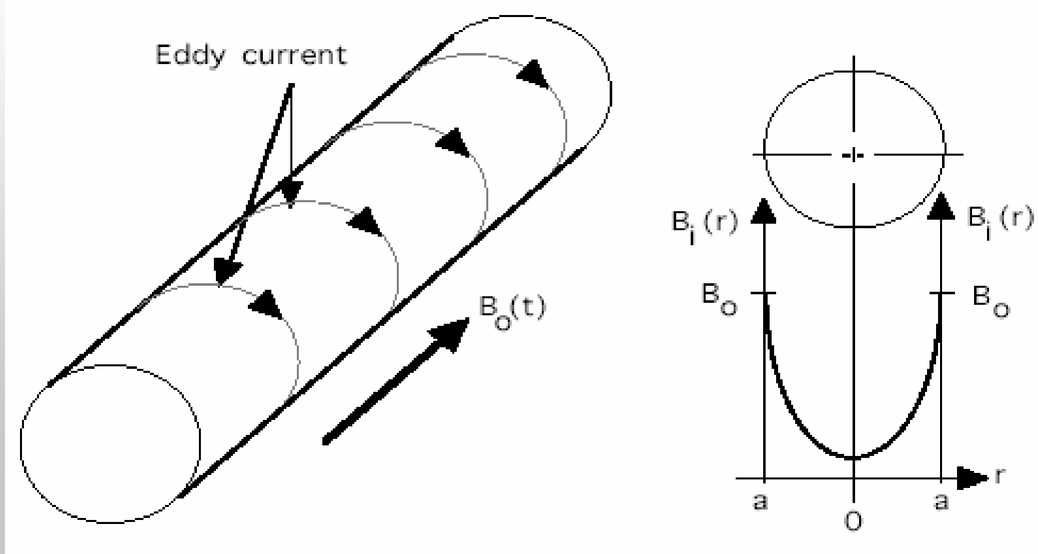
Modeling eddy current loss:

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to derivative of $\Phi(t)$. In consequence, magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to voltage: $i(t) = v(t)/R$. Hence magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with square of excitation frequency f.
- Classical Steinmetz equation for eddy current loss:

$$P_E = K_E f^2 B_{max}^2 (core\ volume)$$

- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as f^4.

Eddy Current Losses in Magnetic Cores



- $\dfrac{B_i(r)}{B_o} = \exp(\{r - a\}/\delta)$
- δ = skin depth = $\sqrt{\dfrac{2}{\omega\mu\sigma}}$
  - $\omega = 2\pi f$, f = frequency
  - μ = magnetic permeability ; $\mu > \mu_o$ for magnetic materials.
  - σ = conductivity of material.

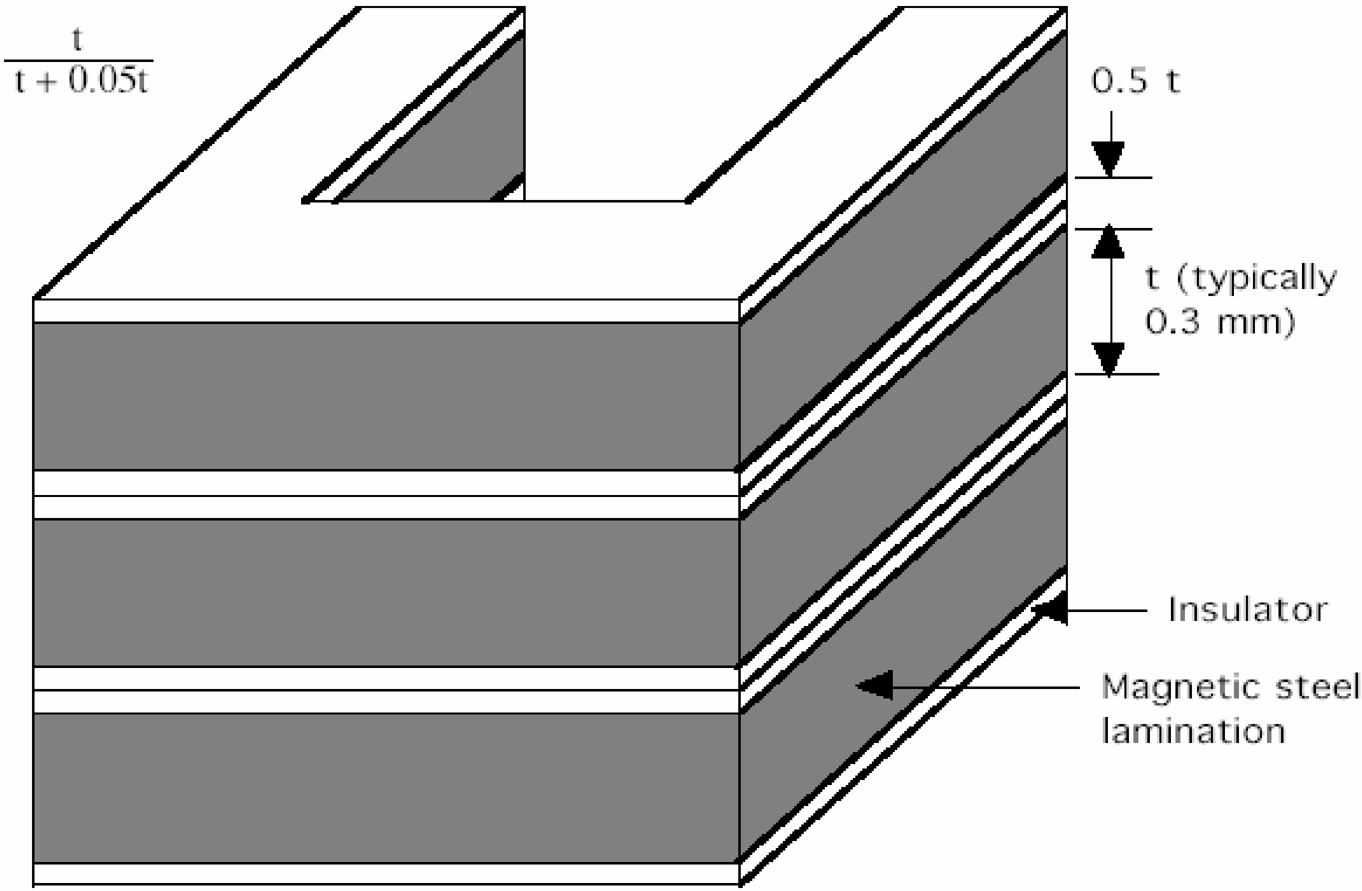
- AC magnetic fields generate eddy currents in conducting magnetic materials.
  - Eddy currents dissipate power.
  - Shield interior of material from magnetic field.

- Numerical example
  - $\sigma = 0.05\ \sigma_{cu}$; $\mu = 10^3\ \mu_o$
    f = 100 Hz
  - δ = 1 mm

Laminated Cores

- Cores made from conductive magnetic materials must be made of many thin laminations. Lamination thickness < skin depth.

- Stacking factor $k_{stack} = \frac{t}{t + 0.05t}$



Eddy Current Losses in Laminated Cores (additional info.)

- Flux $\phi(t)$ intercepted by current loop of area 2xw given by $\phi(t) = 2xw\ B(t)$
- Voltage in current loop $v(t) = 2xw \dfrac{dB(t)}{dt}$

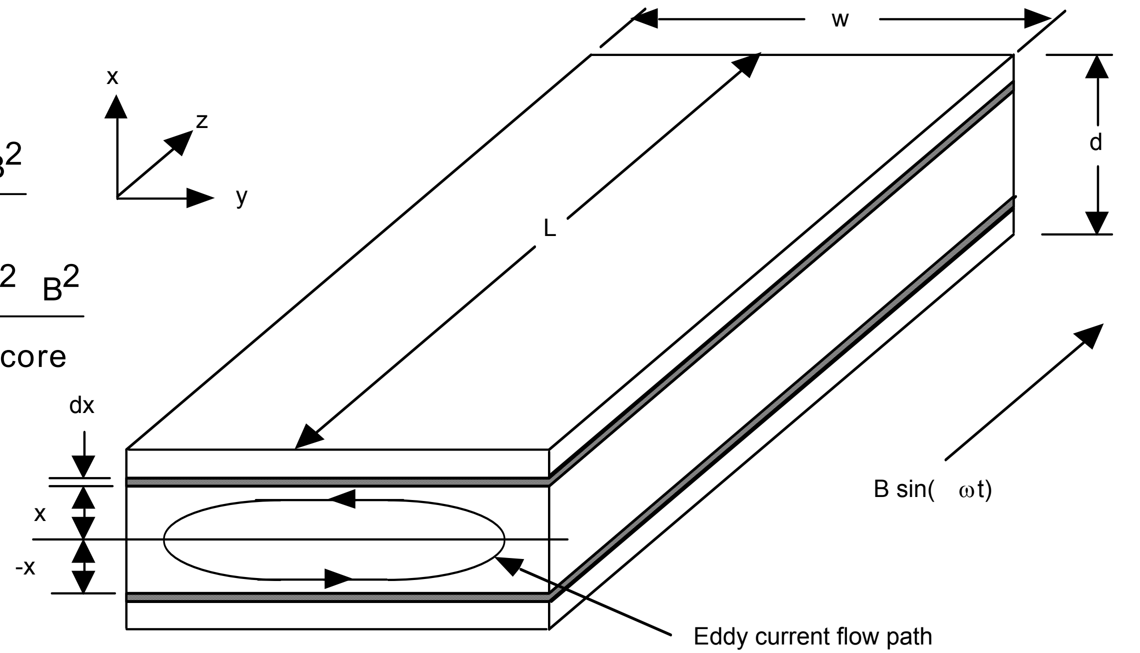
  $= 2wx\omega B\cos(\omega t)$
- Current loop resistance $r = \dfrac{2w\,\rho_{core}}{L\ dx}$; $w >> d$
- Instantaneous power dissipated in thin loop

  $\delta p(t) = \dfrac{[v(t)]^2}{r}$
- Average power P_{ec} dissipated in lamination

  given by $P_{ec} = \langle \int \delta p(t) dV \rangle = \dfrac{w\ L\ d^3\ \omega^2\ B^2}{24\ \rho_{core}}$
- $P_{ec,sp} = \dfrac{P_{ec}}{V} = \dfrac{w\ L\ d^3\ \omega^2\ B^2}{24\ \rho_{core}} \dfrac{1}{dwL} = \dfrac{d^2\ \omega^2\ B^2}{24\ \rho_{core}}$

- Average power P_{ec} dissipated in lamination

  given by $P_{ec} = \langle \int \delta p(t) dV \rangle = \dfrac{w\ L\ d^3\ \omega^2\ B^2}{24\ \rho_{core}}$
- $P_{ec,sp} = \dfrac{P_{ec}}{V} = \dfrac{w\ L\ d^3\ \omega^2\ B^2}{24\ \rho_{core}} \dfrac{1}{dwL} = \dfrac{d^2\ \omega^2\ B^2}{24\ \rho_{core}}$



Pec is specified in manufacturer's data sheets as part of core loss

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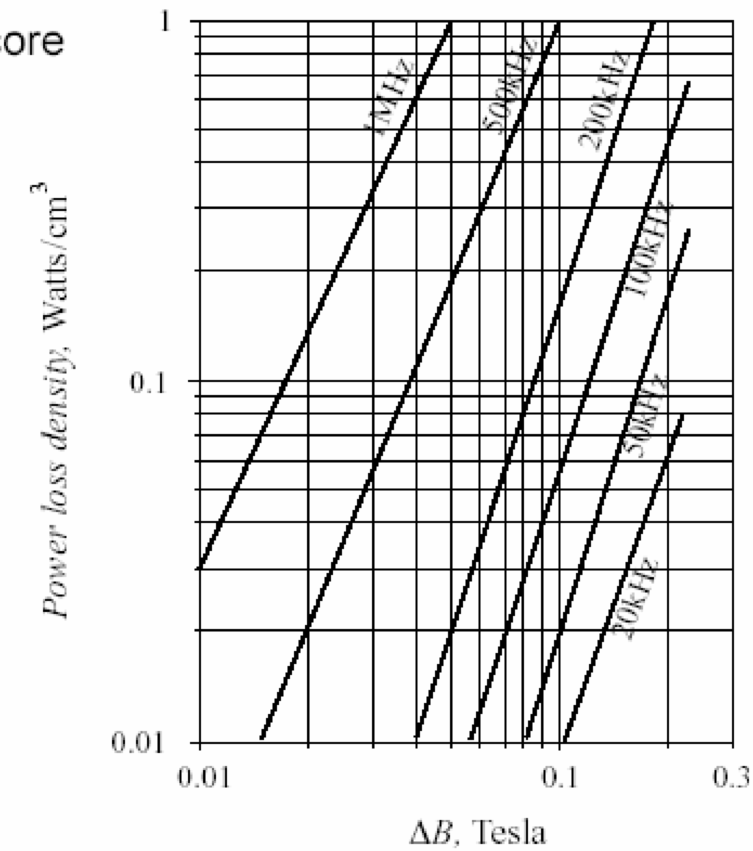
Total core loss: Manufacturer's data

Ferrite core material



Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe}(\Delta B)^{\beta} A_c \ell_m$$

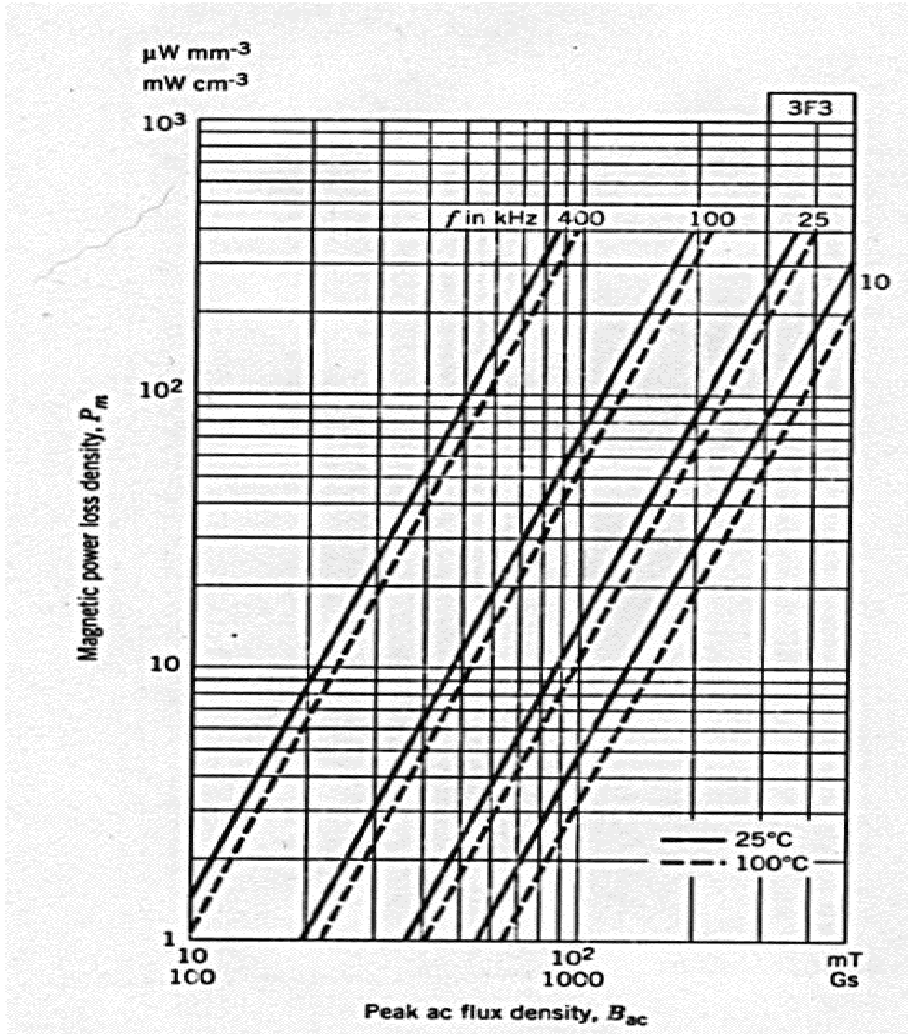
Quantitative Description of Core Losses

- Eddy current loss plus hysteresis loss = core loss.
- Empirical equation - $P_{m,sp} = k f^a [B_{ac}]^d$.

  f = frequency of applied field. B_{ac} = base-to-peak value of applied ac field. k, a, and d are constants which vary from material to material

  - $P_{m,sp} = 1.5 \times 10^{-6} f^{1.3} [B_{ac}]^{2.5}$ mW/cm^3 for 3F3 ferrite. (f in kHz and B in mT)
  - $P_{m,sp} = 3.2 \times 10^{-6} f^{1.8} [B_{ac}]^{2}$ mW/cm^3 METGLAS 2705 M (f in kHz and B in mT)
  - Example: 3F3 ferrite with f = 100 kHz and B_{ac} = 100 mT, $P_{m,sp}$ = 60 mW/cm^3

- 3F3 core losses in graphical form.



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B. Copper Loss (low frequency)

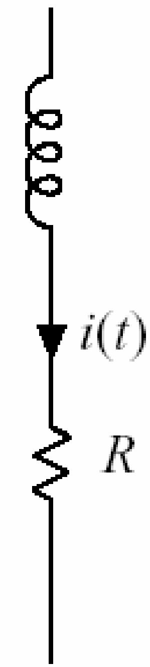
DC resistance of wire

$$R = \rho \frac{\ell_b}{A_w}$$

where A_w is the wire bare cross-sectional area, and ℓ_b is the length of the wire. The resistivity ρ is equal to $1.724 \cdot 10^{-6}\ \Omega\ \text{cm}$ for soft-annealed copper at room temperature. This resistivity increases to $2.3 \cdot 10^{-6}\ \Omega\ \text{cm}$ at 100°C.

The wire resistance leads to a power loss of

$$P_{cu} = I_{rms}^2 R$$



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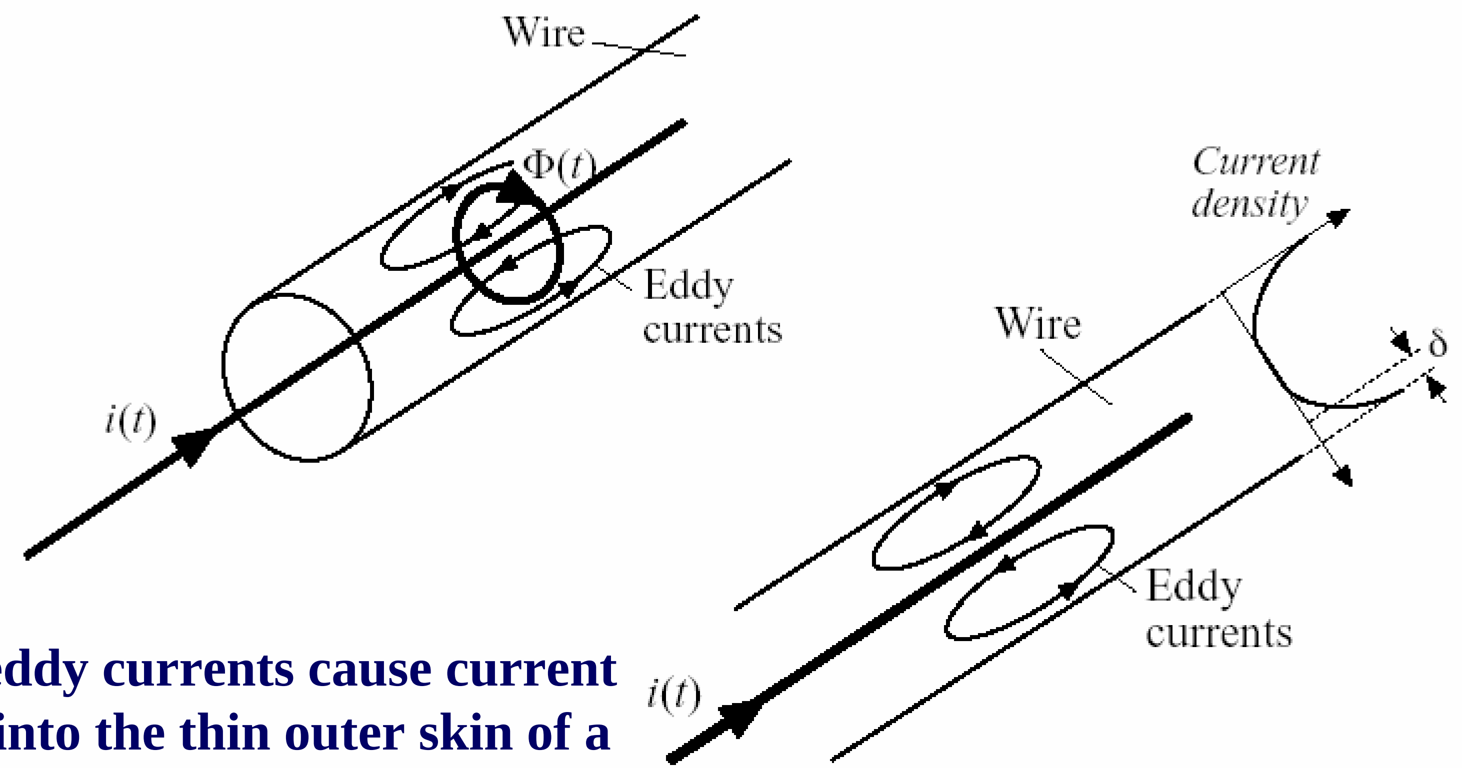
C. Eddy currents in winding conductors: Introduction to Skin and Proximity Effects



Induced eddy currents cause current to crowd into the thin outer skin of a conductor.

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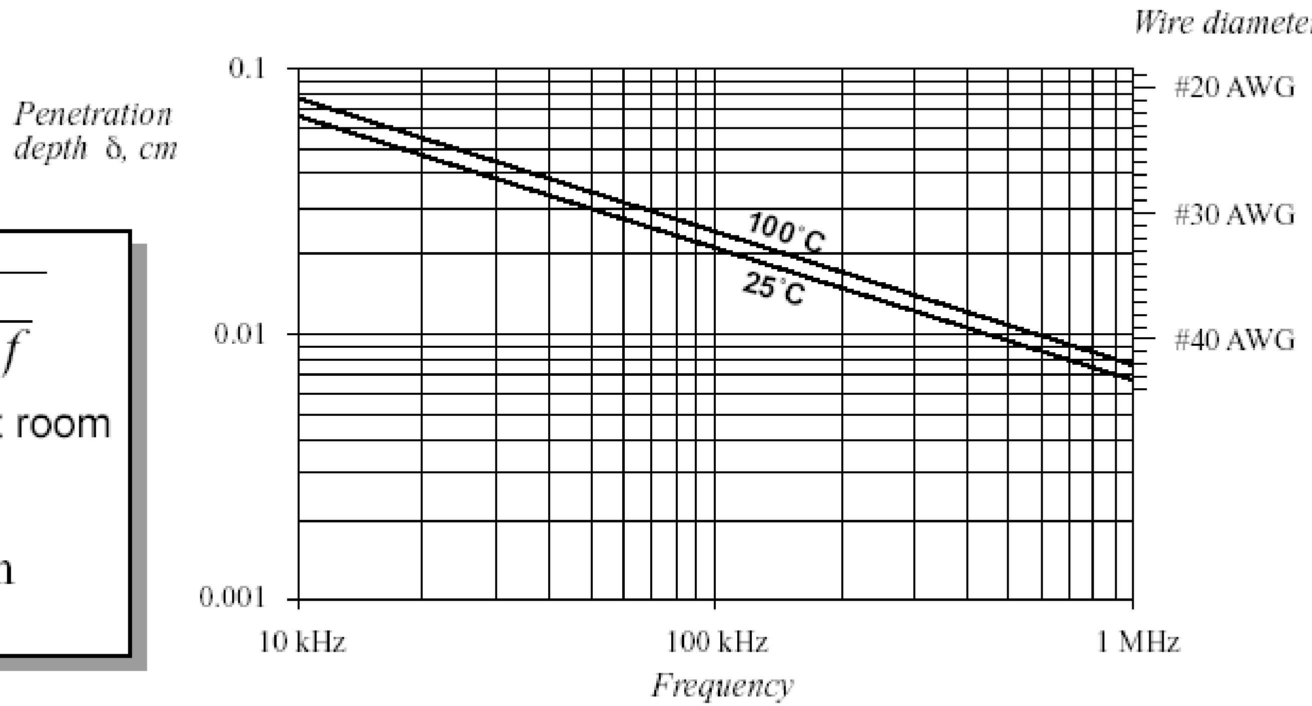
Penetration depth:

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length δ known as the *penetration depth* or *skin depth*.

$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

$$\delta = \frac{7.5}{\sqrt{f}} \text{ cm}$$



Skin depth is the distance below the surface where the current density has fallen to 1/e or 37% of its value at the surface

Power Dissipation in Windings

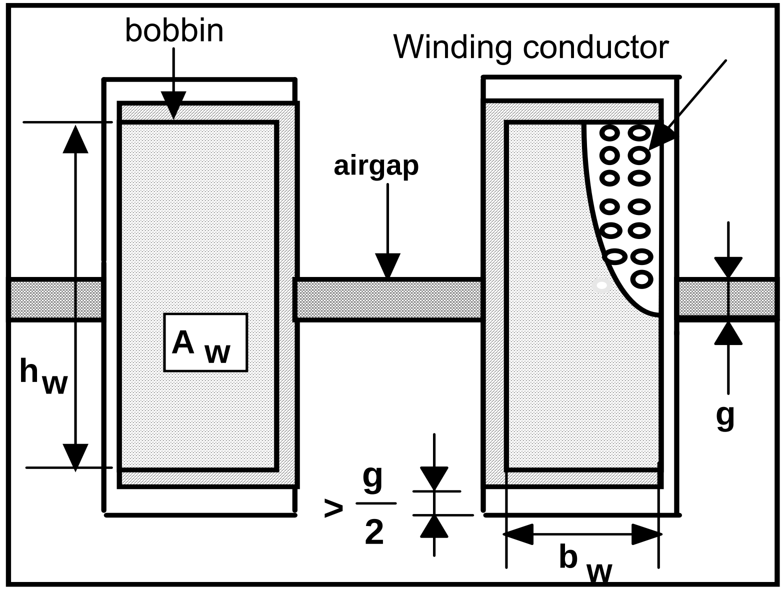
- Average power per unit volume of copper dissipated in copper winding = $P_{cu,sp} = \rho_{cu} (J_{rms})^2$ where $J_{rms} = I_{rms}/A_{cu}$ and ρ_{cu} = copper resistivity.

- Average power dissipated per unit volume of winding = $P_{w,sp} = k_{cu} \; \rho_{cu} (J_{rms})^2$; $V_{cu} = k_{cu} V_w$ where V_{cu} = total volume of copper in the winding and V_w = total volume of the winding.

- Copper fill factor $k_{cu} = \dfrac{N A_{cu}}{A_w} < 1$

  - N = number of turns; A_{cu} = cross-sectional area of copper conductor from which winding is made; $A_w = b_w l_w$ = area of winding window.

  - $k_{cu} = 0.3$ for Leitz wire; $k_{cu} = 0.6$ for round conductors; $k_{cu} \Rightarrow$ 0.7-0.8 for rectangular conductors.



Double-E core example

- $k_{cu} < 1$ because:

  - Insulation on wire to avoid shorting out adjacent turns in winding.

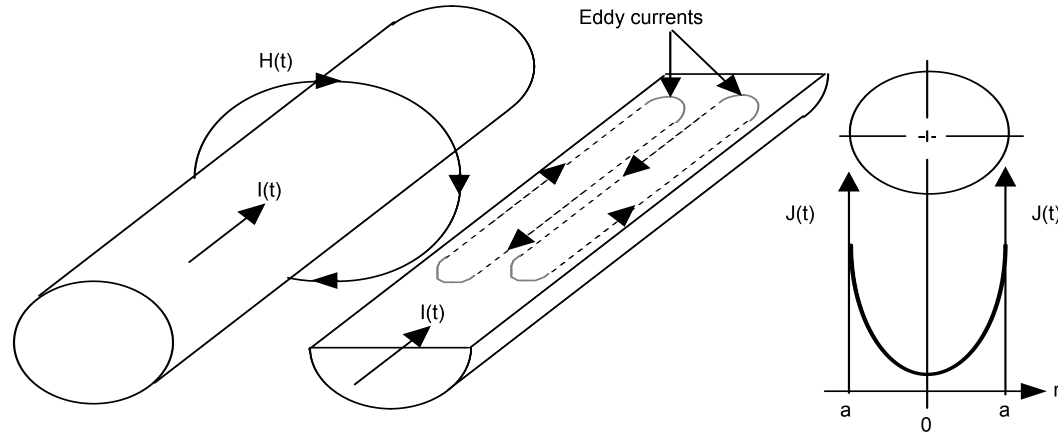
  - Geometric restrictions. (e.g. tight-packed circles cannot cover 100% of a square area.)

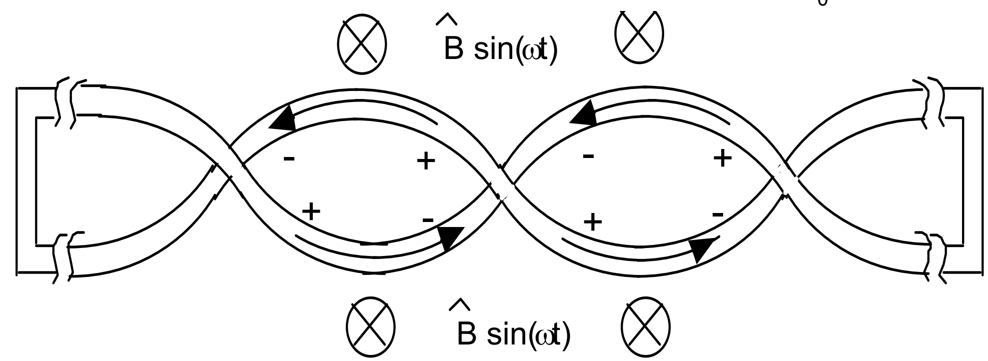
Eddy Currents Increase Winding Losses

- AC currents in conductors generate ac magnetic fields which in turn generate eddy currents that cause a nonuniform current density in the conductor . Effective resistance of conductor increased over dc value.
  - $P_{w,sp} > k_{cu}\, \rho_{cu}\, (J_{rms})^2$ if conductor dimensions greater than a skin depth.
  - $\dfrac{J(r)}{J_o} = \exp(\{r - a\}/\delta)$
  - δ = skin depth = $\sqrt{\dfrac{2}{\omega\mu\sigma}}$
    - $\omega = 2\pi f$; f = frequency of ac current
    - μ = magnetic permeability of conductor; $\mu = \mu_o$ for nonmagnetic conductors.
    - σ = conductivity of conductor material.

- Numerical example using copper at 100 °C

Frequency	50 Hz	5 kHz	20 kHz	500 kHz
Skin Depth	10.6 mm	1.06 mm	0.53 mm	0.106 mm





- Mnimize eddy currents using Leitz wire bundle. Each conductor in bundle has a diameter less than a skin depth.

- Twisting of paralleled wires causes effects of intercepted flux to be canceled out between adjacent twists of the conductors. Hence little if any eddy currents.

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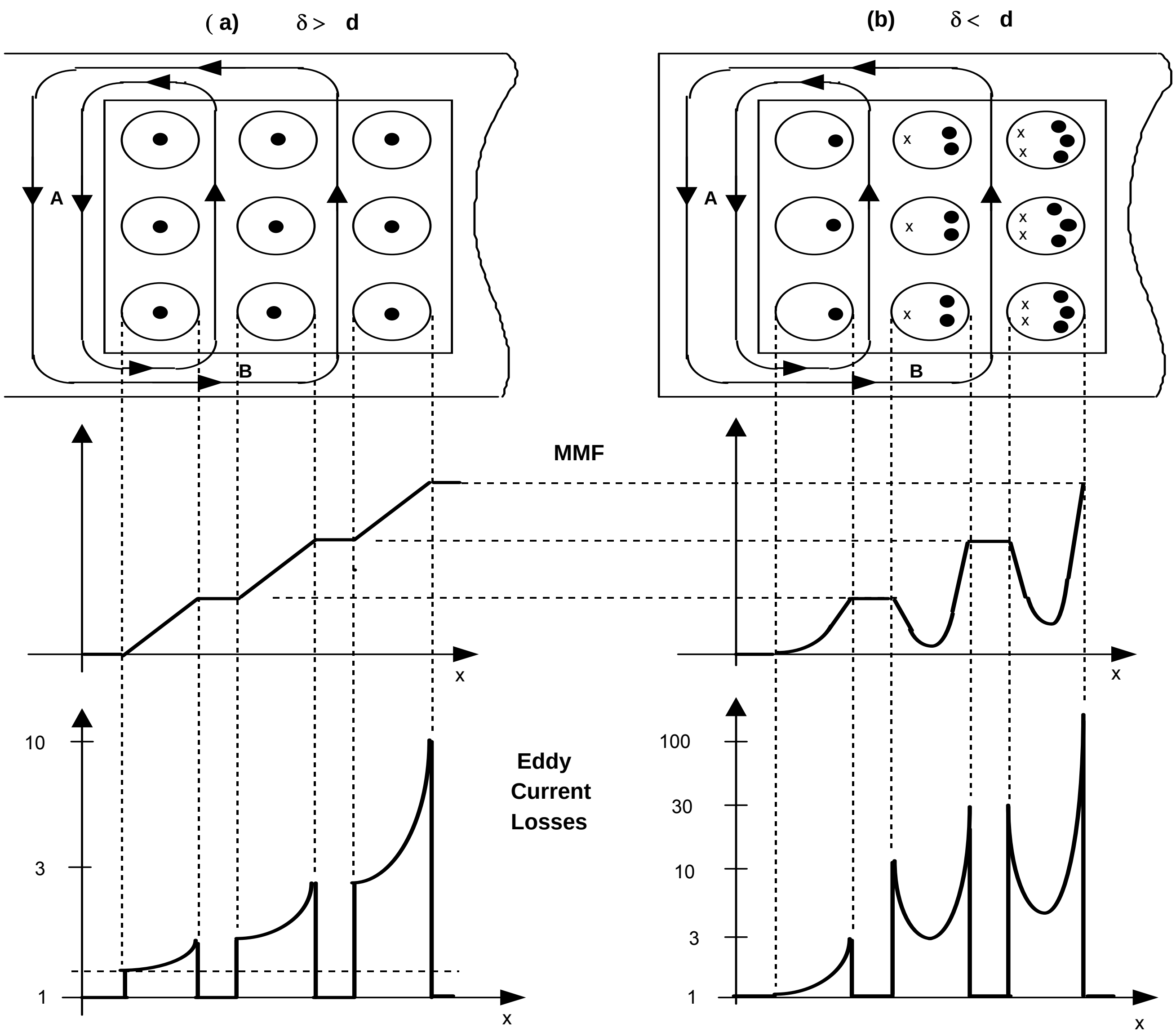
Proximity effect: *is caused by alternating magnetic fields arising from currents in adjacent wires or from currents in adjacent winding layers in a multi-layer coil. It is more serious than skin effect because the latter increases copper losses only, by restricting the conducting area of the wire to a thin skin on its surface, but it does not change the magnitude of currents flowing (only the current density at the wire surfaces).*

In contrast, in proximity effect, eddy currents caused by the magnetic fields of currents in adjacent coil layers increase exponentially in amplitude as the number of coil layers increases.

Proximity Effect Further Increases Winding Losses



- Proximity effect - losses due to eddy current generated by the magnetic field experienced by a particular conductor section but generated by the current flowing in the rest of the winding.

- Design methods for minimizing proximity effect losses discussed later.

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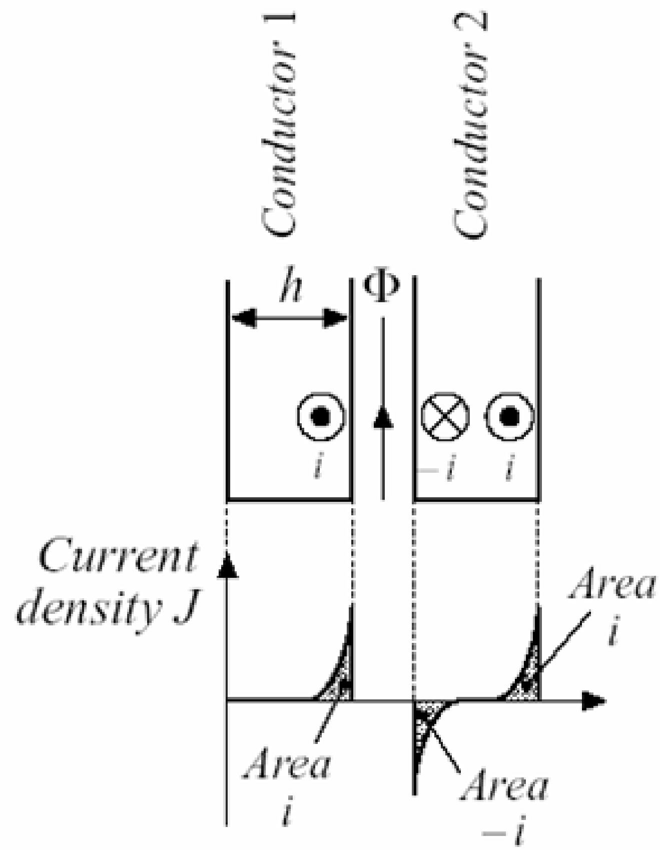
The Proximity Effect

Ac current in a conductor induces eddy currents in adjacent conductors by a process called the *proximity effect*. This causes significant power loss in the windings of high-frequency transformers and ac inductors.

A multi-layer foil winding, with $h \gg \delta$. Each layer carries net current $i(t)$.



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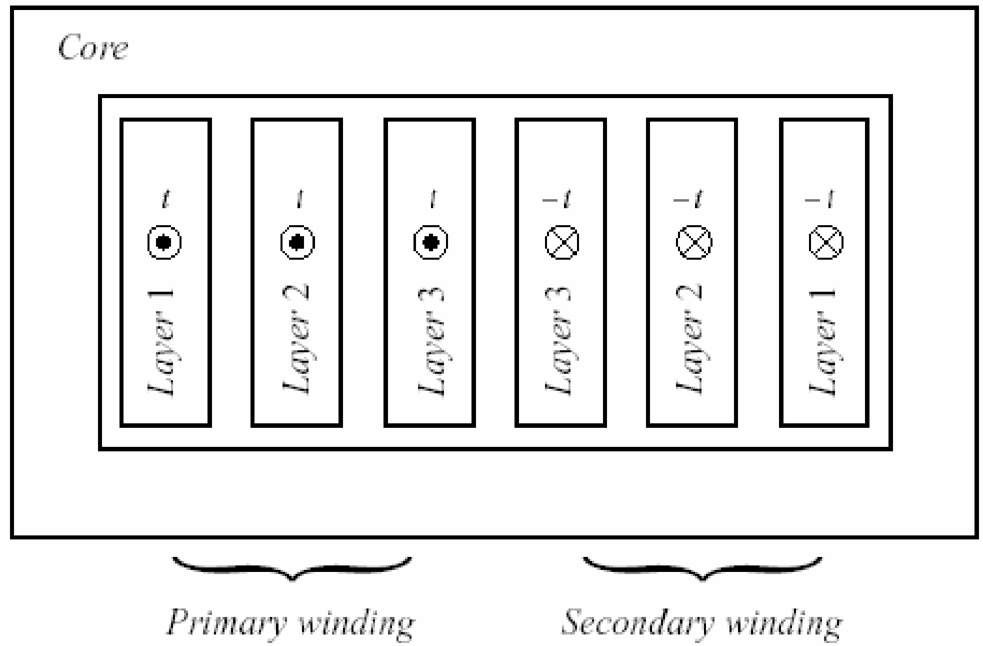
Example of a two-winding transformer

Cross-sectional view of two-winding transformer example. Primary turns are wound in three layers. For this example, let's assume that each layer is one turn of a flat foil conductor. The secondary is a similar three-layer winding. Each layer carries net current $i(t)$. Portions of the windings that lie outside of the core window are not illustrated. Each layer has thickness $h \gg \delta$.



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Distribution of currents on surfaces of conductors:

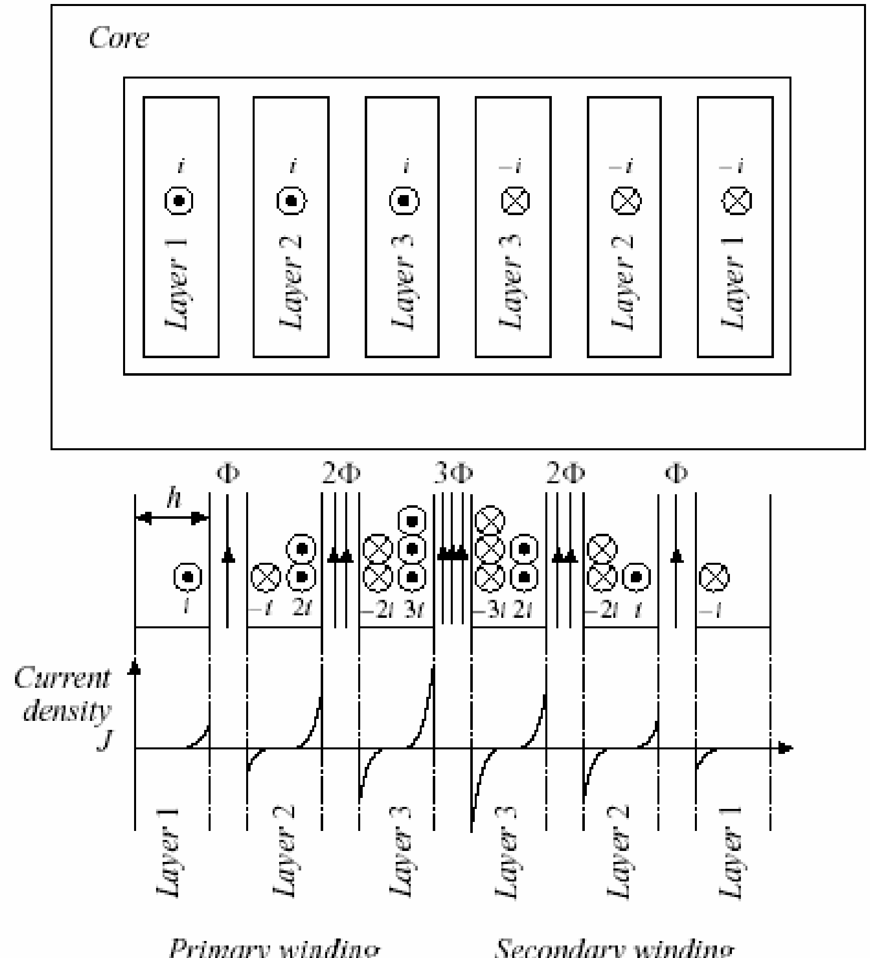
Skin effect causes currents to concentrate on surfaces of conductors

Surface current induces equal and opposite current on adjacent conductor

This induced current returns on opposite side of conductor

Net conductor current is equal to $i(t)$ for each layer, since layers are connected in series

Circulating currents within layers increase with the numbers of layers



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Estimating proximity loss: high-frequency limit

The current $i(t)$ having rms value I is confined to thickness d on the surface of layer 1. Hence the effective "ac" resistance of layer 1 is:

$$R_{ac} = \frac{h}{\delta} R_{dc}$$

This induces copper loss P_1 in layer 1:

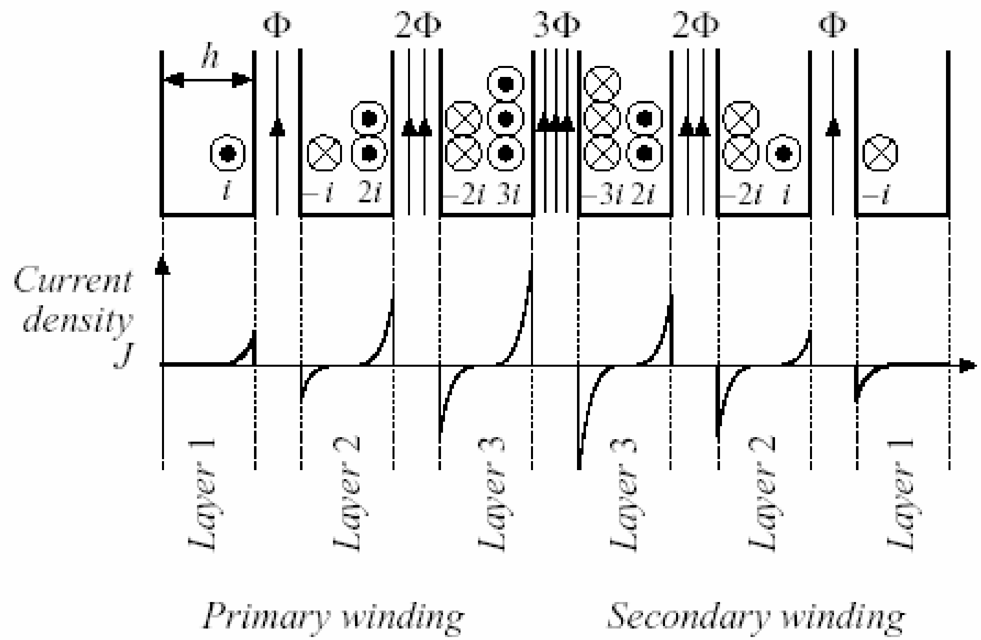
$$P_1 = I^2 R_{ac}$$

Power loss P_2 in layer 2 is:

$$P_2 = P_1 + 4P_1 = 5P_1$$

Power loss P_3 in layer 3 is:

$$P_3 = \left(2^2 + 3^2\right) P_1 = 13 P_1$$



Power loss P_m in layer m is:

$$P_m = I^2 \left[(m-1)^2 + m^2 \right] \left(\frac{h}{\delta} R_{dc} \right)$$

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Total loss in M-layer winding: high-frequency limit

Add up losses in each layer:

$$P = I^2\left(\frac{h}{\delta} R_{dc}\right) \sum_{m=1}^{M} \left[(m-1)^2 + m^2\right]$$
$$= I^2\left(\frac{h}{\delta} R_{dc}\right) \frac{M}{3}\left(2M^2 + 1\right)$$

Compare with dc copper loss:

If foil thickness were $H = \delta$, then at dc each layer would produce copper loss P_1. The copper loss of the M-layer winding would be

$$P_{dc} = I^2 M R_{dc}$$

So the proximity effect increases the copper loss by a factor of

$$F_R = \frac{P}{P_{dc}} = \frac{1}{3}\left(\frac{h}{\delta}\right)\left(2M^2 + 1\right)$$

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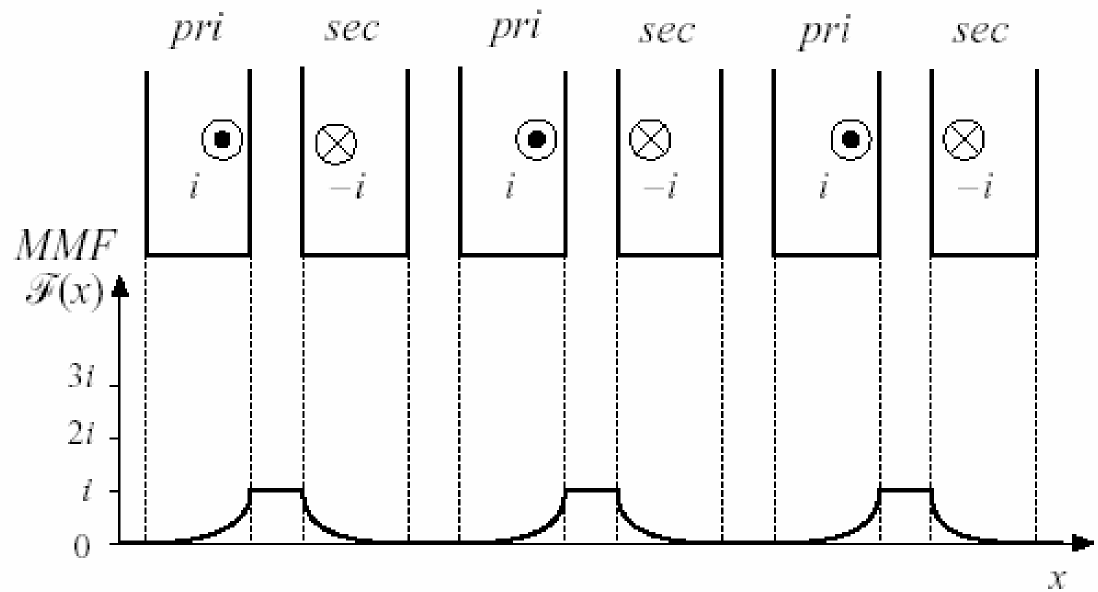
Interleaving the windings

Same transformer example, but with primary and secondary layers alternated

Each layer operates with $\mathscr{F} = 0$ on one side, and $\mathscr{F} = i$ on the other side

Proximity loss of entire winding follows $M = 1$ curve



For $M = 1$: minimum loss occurs with $\varphi = \pi/2$, although copper loss is nearly constant for any $\varphi \geq 1$, and is approximately equal to the dc copper loss obtained when $\varphi = 1$.

Discussion: design of winding geometry to minimize proximity loss

- Interleaving windings can significantly reduce the proximity loss when the winding currents are in phase, such as in the transformers of buck-derived converters or other converters
- In some converters (such as flyback the winding currents are out of phase. Interleaving then does little to reduce the peak MMF and proximity loss. See *Vandelac and Ziogas* [10].
- For sinusoidal winding currents, there is an optimal conductor thickness near $\varphi = 1$ that minimizes copper loss.
- Minimize the number of layers. Use a core geometry that maximizes the width ℓ_w of windings.
- Minimize the amount of copper in vicinity of high MMF portions of the windings

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Litz wire

- A way to increase conductor area while maintaining low proximity losses
- Many strands of small-gauge wire are bundled together and are externally connected in parallel
- Strands are twisted, or transposed, so that each strand passes equally through each position on inside and outside of bundle. This prevents circulation of currents between strands.
- Strand diameter should be sufficiently smaller than skin depth
- The Litz wire bundle itself is composed of multiple layers
- Advantage: when properly sized, can significantly reduce proximity loss
- Disadvantage: increased cost and decreased amount of copper within core window

III. MAGNETICS DESIGN

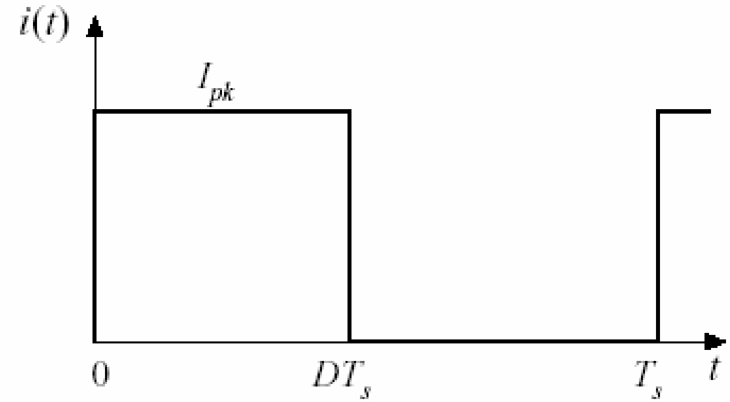
Harmonic Loss Factor F_H

Effect of harmonics: F_H = ratio of total ac copper loss to fundamental copper loss

$$F_H = \frac{\sum_{j=1}^{\infty} P_j}{P_1}$$

The total winding copper loss can then be written

$$P_{cu} = I_0^2 R_{dc} + F_H F_R I_1^2 R_{dc}$$



3.4. Thermal Considerations in Magnetic Components

- Losses (winding and core) raise core temperature. Common design practice to limit maximum interior temperature to 100-125 °C.
  - Core losses (at constant flux density) increase with temperature increases above 100 °C
  - Saturation flux density B_s decreases with temp. Increases
  - Nearby components such as power semi-conductor devices, integrated circuits, capacitors have similar limits.
- Temperature limitations in copper windings
  - Copper resistivity increases with temperature increases. Thus losses, at constant current density increase with temperature.
  - Reliability of insulating materials degrade with temperature increases.

- Surface temperature of component nearly equal to interior temperature. Minimal temperature gradient between interior and exterior surface.
  - Power dissipated uniformly in component volume.
  - Large cross-sectional area and short path lengths to surface of components.
  - Core and winding materials have large thermal conductivity.

.• Thermal resistance (surface to ambient) of magnetic component determines its temperature.

- $P_{sp} = \dfrac{T_s - T_a}{R_{\theta sa}(V_w + V_c)}$; $R_{\theta sa} = \dfrac{h}{A_s}$
  - h = convective heat transfer coefficient = 10 °C-m^2/W
  - A_s = surface area of inductor (core + winding). Estimate using core dimensions and simple geometric considerations.
  - Uncertain accuracy in h and other heat transfer parameters do not justify more accurate thermal modeling of inductor.

Scaling of Core Flux Density and Winding Current Density

- Power per unit volume, P_{sp}, dissipated in magnetic component is $P_{sp} = k_1/a$; k_1 = constant and a = core scaling dimension.

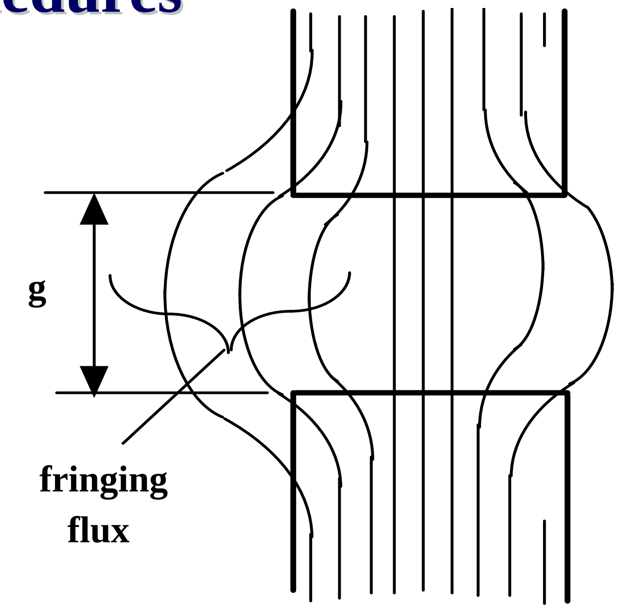
- $$P_{w,sp} V_w + P_{m,sp} V_m = \frac{T_s - T_a}{R_{\theta sa}} :$$
  T_a = ambient temperature and $R_{\theta sa}$ = surface-to-ambient thermal resistance of component.

- For optimal design $P_{w,sp} = P_{c,sp} = P_{sp}$:
  Hence $$P_{sp} = \frac{T_s - T_a}{R_{\theta sa}(V_w + V_c)}$$

- $R_{\theta sa}$ proportional to a^2 and $(V_w + V_c)$ proportional to a^3

3.5. Inductor Design Procedures

Analysis of a Specific Inductor Design

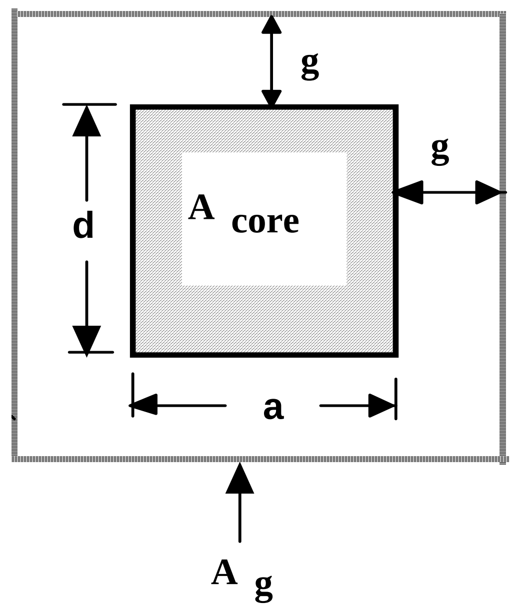


- Inductor specifications
  - Maximum current = 4 amp rms at 100 kHz
  - Double-E core with a = 1 cm using 3F3 ferrite.
  - Distributed air-gap with four gaps, two in series in each leg; total gap length Σg = 3 mm.
  - Winding - 66 turns of Litz wire with $A_{cu} = 0.64\ mm^2$
  - Inductor surface black with emissivity = 0.9
  - $T_{a,max}$ = 40 °C

- Find; inductance L, $T_{s,max}$; effect of a 25% overcurrent on T_s

- Power dissipation in winding, $P_w = V_w\, k_{cu}\, \rho_{cu}\, (J_{rms})^2$ = 3.2 Watts
  - $V_w = 12.3\ cm^3$ (table of core characteristics)
  - k_{cu} = 0.3 (Leitz wire)
  - ρ_{cu} at 100 °C (approx. max. T_s) = 2.2×10^{-8} ohm-m
  - $J_{rms} = 4/(.64) = 6.25\ A/mm^2$

- Power dissipation in 3F3 ferrite core,
  $P_{core} = V_c 1.5 \times 10^{-6} f^{1.3} (B_{ac})^{2.5}$ = 3.3 W
  - B_{ac} ? = 0.18 T; assumes $H_g >> H_{core}$
    - $A_g = (a + g)(d + g) = 1.71\ cm^2$; g = 3mm/4 = .075 mm
    - $A_c = 1.5\ cm^2$ (table of core characteristics)
    - $V_c = 13.5\ cm^3$ (table of core characteristics)
    - f = 100 kHz

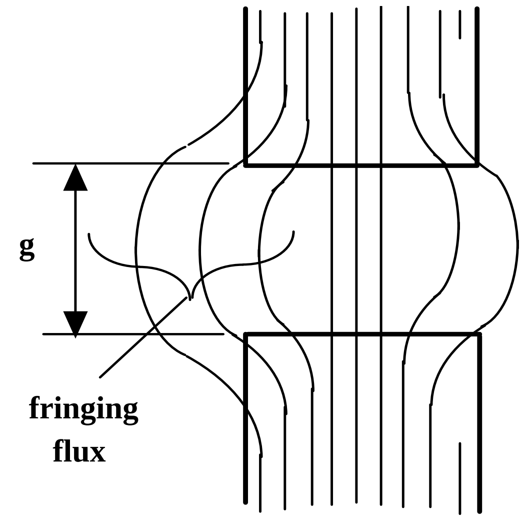


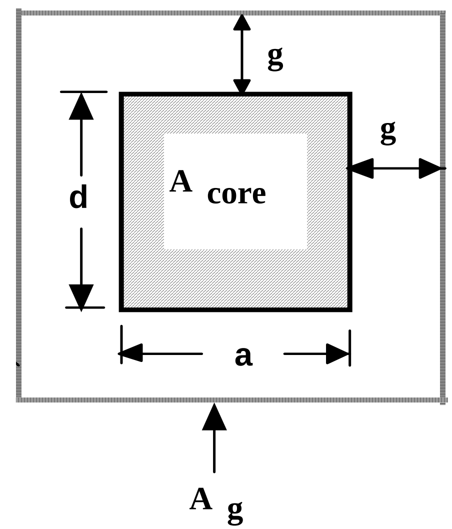
Analysis of a Specific Inductor Design (cont.)

- $L = \frac{N\,\phi}{I} = 310\ \mu H$
  - $\phi = B_{ac}\,A_c = (0.18\ T)(1.5\text{x}10^{-4}\ m^2) = 2.6\text{x}10^{-5}\ Wb$

- Surface temperature $T_s = T_a + R_{\theta sa}\,(P_w + P_{core}) = 104\ °C$
  - $R_{\theta sa} = R_{\theta,rad} \,||\, R_{\theta,conv} = 9.8\ °C/W$

- Overcurrent of 25% (I= 5 amp rms) makes $T_s = 146\ °C$
  - $P_w = (3.2\ W)(1.25)^2 = 5\ W$; $P_{core} = (3.3\ W)(1.25)^{2.5} = 5.8\ W$
  - $T_s = (9.8\ °C/W)(10.8\ W) + 40\ °C = 146\ °C$





Stored Energy Relation - Basis of Inductor Design

- Input specifications for inductor design
  - Inductance value L.
  - Rated peak current I
  - Rated rms current I_{rms}.
  - Rated dc current (if any) I_{dc}.
  - Operating frequency f.
  - Maximum inductor surface temperature T_s and maximum ambient temperature T_a.
- Design consists of the following:
  - Selection of core geometric shape and size
  - Core material
  - Winding conductor geometric shape and size
  - Number of turns in winding
- Design procedure starting point - stored energy relation
  - $[L\ I]\ I_{rms} = [N\ \phi]\ I_{rms}$
  - $N = \dfrac{k_{cu} A_w}{A_{cu}}$
  - $\phi = B\ A_{core}\ ;\ I_{rms} = J_{rms}\ A_{cu}$
  - $L\ I\ I_{rms} = k_{cu}\ J_{rms}\ B\ A_w\ A_{core}$
  - Equation relates input specifications (left-hand side) to needed core and winding parameters (right-hand side)
- A good design procedure will consists of a systematic, single-pass method of selecting k_{cu}, J_{rms}, B, A_w, and A_{core}.

Goal: Minimize inductor size, weight, and cost.

Core Database - Basic Inductor Design Tool

- Interactive core database (spreadsheet-based) key to a single pass inductor design procedure.
  - User enters input specifications from converter design requirements. Type of conductor for windings (round wire, Leitz wire, or rectangular wire or foil) must be made so that copper fill factor k_{cu} is known.
  - Spreadsheet calculates capability of all cores in database and displays smallest size core of each type that meets stored energy specification.
  - Also can be designed to calculate (and display as desired) design output parameters including J_{rms}, B, A_{cu}, N, and air-gap length.
  - Multiple iterations of core material and winding conductor choices can be quickly done to aid in selection of most appropriate inductor design.

- Information on all core types, sizes, and materials must be stored on spreadsheet. Info includes dimensions, A_w, A_{core}, surface area of assembled inductor, and loss data for all materials of interest.

- Pre-stored information combined with user inputs to produce performance data for each core in spreadsheet. Sample of partial output shown below.

Core No.	Material	AP = $A_w A_{core}$	R_θ ΔT=60 °C	P_{sp} @ ΔT=60 °C	J_{rms} @ ΔT=60 °C & P_{sp}	B_{ac} @ ΔT=60 °C & 100 kHz	$k_{cu} J_{rms} \hat{B}$ • $A_w A_{core}$
•	•	•	•	•	•	•	•
8	3F3	2.1 cm^4	9.8 °C/W	237 mW/cm^3	$3.3/\sqrt{k_{cu}}$	170 mT	$.0125\sqrt{k_{cu}}$
•	•	•	•	•	•	•	•

Details of Interactive Inductor Core Database Calculations

- User inputs: L, I, I_{rms}, I_{dc}, f, T_s, T_a, and k_{cu}

- Stored information (static, independent of converter requirements)
  - Core dimensions, A_w, A_{core}, V_c, V_w, surface area, mean turn length, mean magnetic path length, etc.
  - Quantitative core loss formulas for all materials of interest including approximate temperature dependence.

- Calculation of core capabilities (stored energy value)
  1. Compute converter-required stored energy value: $L\, I\, I_{rms}$.
  2. Compute allowable specific power dissipation $P_{sp} = [T_s - T_a] / \{ R_{\theta sa} [V_c + V_w] \}$. $R_{\theta sa} = h/A_s$ or calculated interactively using input temperatures and formulas for convective and radiative heat transfer from Heat Sink chapter.
  3. Compute allowable flux density $P_{sp} = k\, f^b\, [B_{ac}]^d$ and current density $P_{sp} = k_{cu}\, \rho_{cu}\, \{J_{rms}\}^2$.
  4. Compute core capabilities $k_{cu}\, A_w\, A_{core}\, B\, J_{rms}$

- Calculation of inductor design parameters.
  1. Area of winding conductor $A_{cu} = I / J_{rms}$.
  2. Calculate skin depth δ in winding. If $A_{cu} > \delta^2$at the operating frequency, then single round conductor cannot be used for winding.
     - Construct winding using Leitz wire, thin foils, or paralleled small dia. ($\leq \delta$) round wires.

Details of Interactive Core Database Calculations (cont.)

3. Calculate number turns of N in winding: $N = k_{cu} A_w / A_{cu}$.

4. Calculate air-gap length L_g. Air-gap length determined on basis that when inductor current equals peak value I, flux density equals peak value B.
   - Formulas for air-gap length different for different core types. Example for double-E core given in next slide.

5. Calculate maximum inductance L_{max} that core can support. $L_{max} = N A_{core} B_{peak} / I_{peak}$.

   If L_{max} > required L value, reduce L_{max} by removing winding turns.
   - Save on copper costs, weight, and volume.
   - P_w can be kept constant by increasing $P_{w,sp}$
   - Keep flux density B_{peak} constant by adjusting gap length L_g.

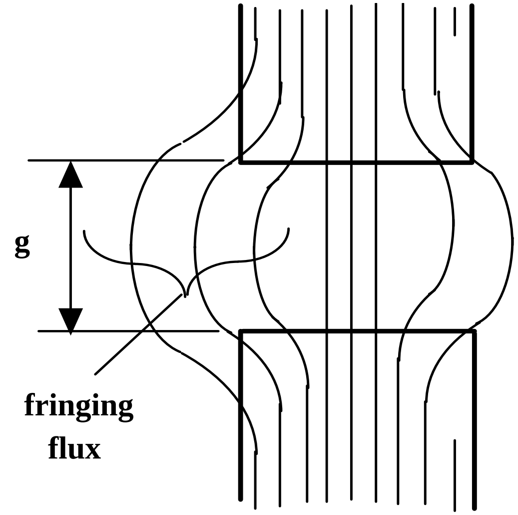
6. Alternative L_{max} reduction procedure, increasing the value of L_g, keeping everything else constant, is a poor approach. Would not reduce copper weight and volume and thus achieve cost savings. Full capability of core would not be utilized.

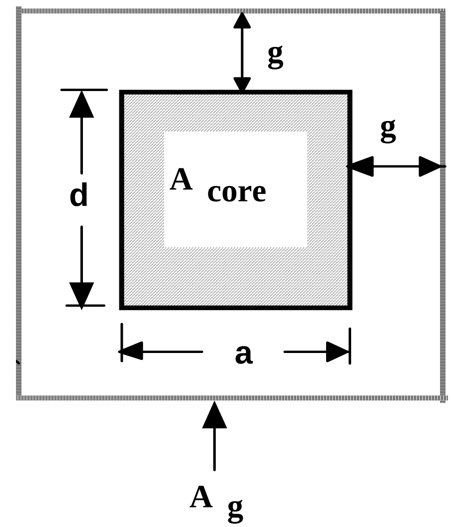
Setting Double-E Core Air-gap Length

- Set total airgap length L_g so that B_{peak} generated at the peak current I_{peak}.
- $L_g = N_g\, g$; N_g = number of distributed gaps each of length g. Distributed gaps used to minimize amount of flux fringing into winding and thus causing additional eddy current losses.
- $R_m = \dfrac{N\, I_{peak}}{A_c\, B_{peak}} = R_{m,core} + R_{m,gap} \approx R_{m,gap} = \dfrac{L_g}{\mu_o A_g}$
- $L_g = \dfrac{N\, I_{peak}\, \mu_o\, A_g}{A_c\, B_{peak}}$
- For a double-E core, $A_g = \left(a + \dfrac{L_g}{N_g}\right)\left(d + \dfrac{L_g}{N_g}\right)$
  - $A_g \approx ad + (a + d)\dfrac{L_g}{N_g}$; $\dfrac{L_g}{N_g} << a$
- Insertion of expression for $A_g(L_g)$ into expression for $L_g(A_g)$ and solving for L_g yields

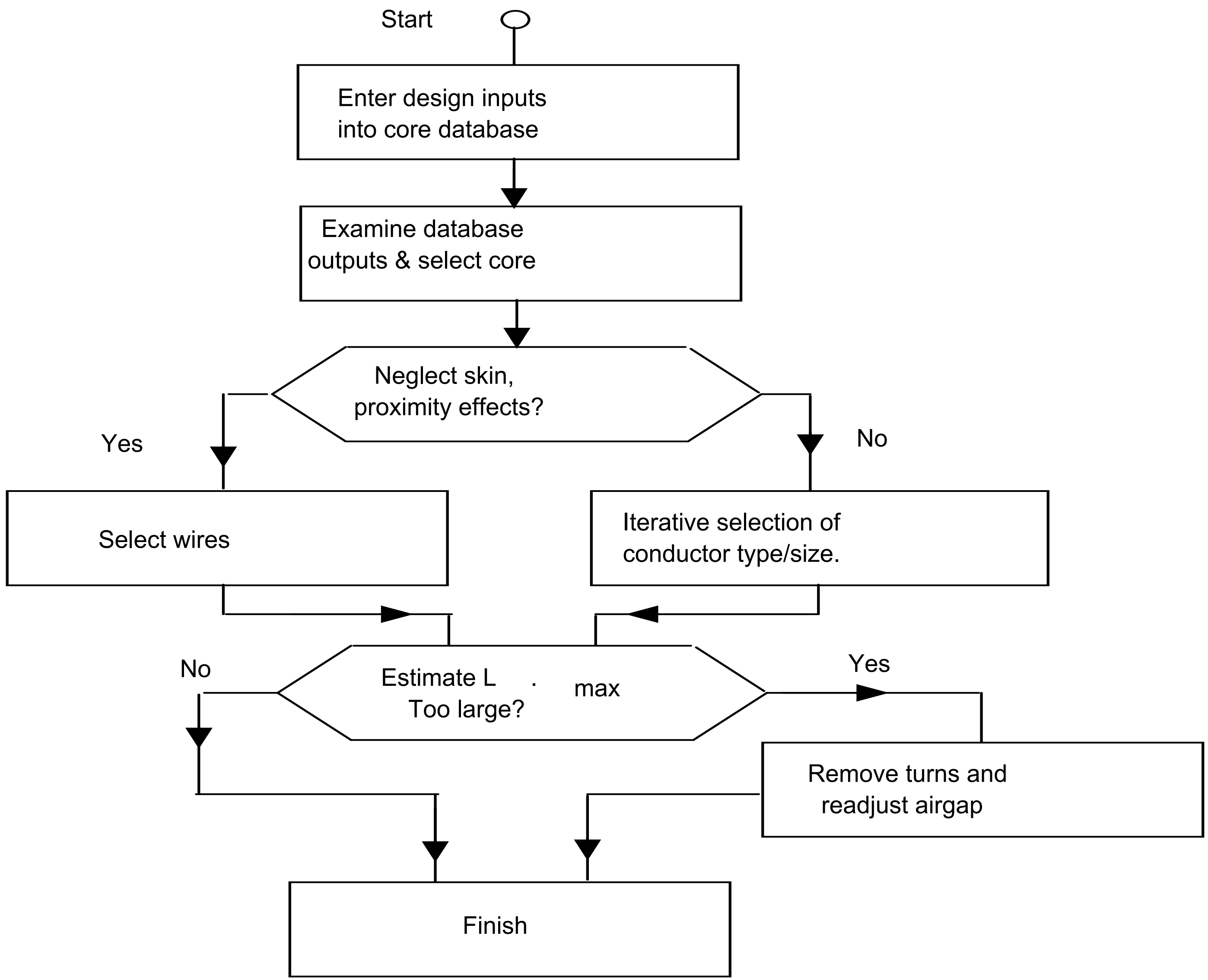
$$L_g = \frac{a}{\dfrac{B_{peak}\, A_c}{d\, \mu_o\, N\, I_{peak}} - \dfrac{a + d}{d\, N_g}}$$

- Above expression for L_g only valid for double-E core, but similar expressions can be developed for other core shapes.





Single Pass Inductor Design Procedure



Inductor Design Example

- Assemble design inputs
  - L = 300 microhenries
  - Peak current = 5.6 A, sinewave current, I_{rms} = 4 A
  - Frequency = 100 kHz
  - T_s = 100 °C ; T_a = 40 °C
- Stored energy $L\,I\,I_{rms} = (3\text{x}10^{-4})(5.6)(4)$

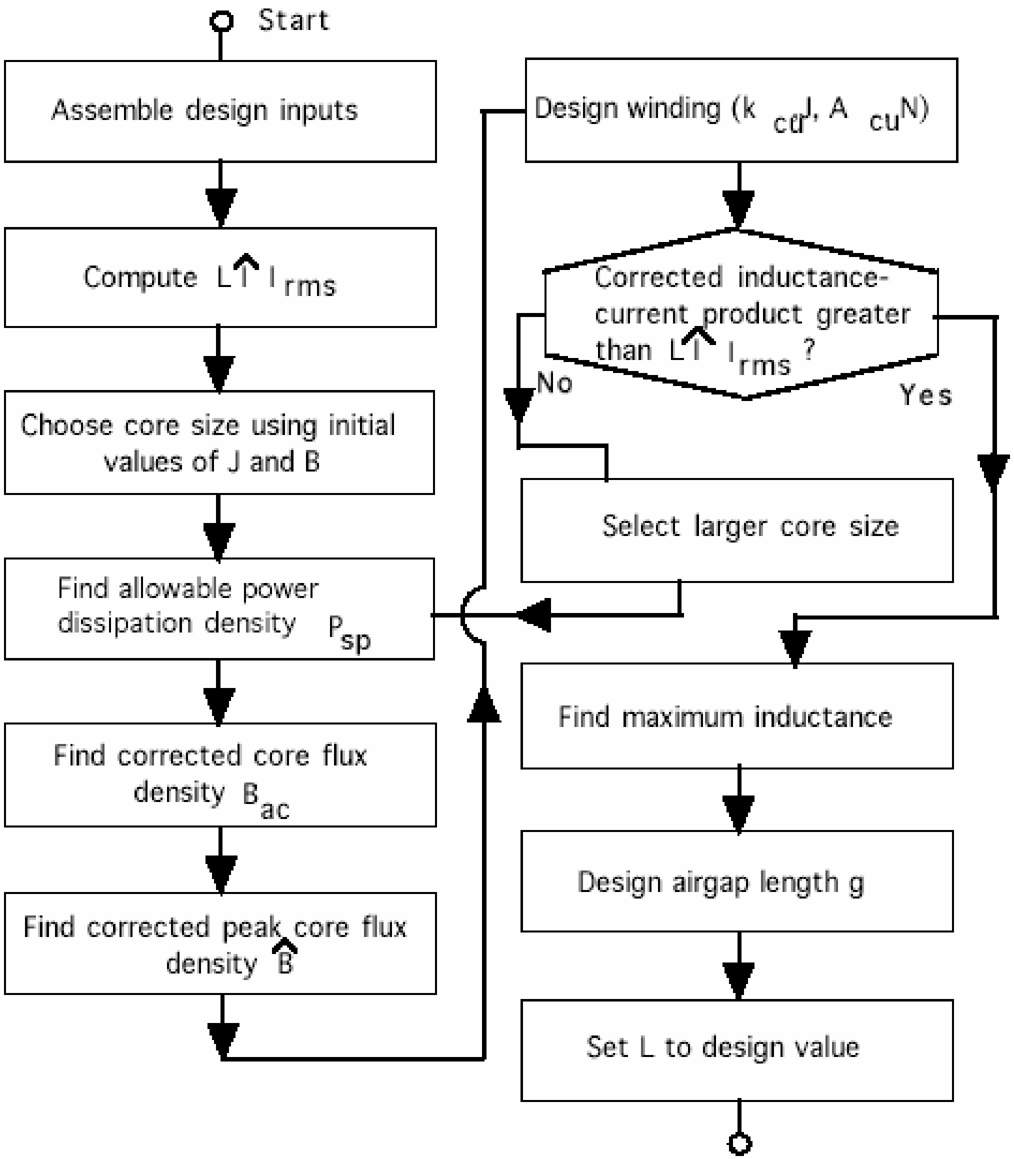
  $= 0.00068$ J-m^{-3}
- Core material and geometric shape
  - High frequency operation dictates ferrite material. 3F3 material has highest performance factor PF at 100 kHz.
  - Double-E core chosen for core shape.
- Double-E core with a = 1 cm meets requirements.

  $k_{cu}\, J_{rms}\, \hat{B} \quad A_w\, A_{core}$ □$0.0125 \sqrt{k_{cu}}\ 0.0068$

  for $k_{cu} > 0.3$
- Database output: R_θ = 9.8 °C/W and

  P_{sp} = 237 mW/cm^3
- Core flux density B =170 mT from database. No I_{dc}, B_{peak} = 170 mT.
- Winding parameters.
  - Litz wire used, so k_{cu} = 0.3. J_{rms} = 6 A/mm^2
  - A_{cu} = (4 A)/(6 A/mm^2) = 0.67 mm^2
  - N = (140 mm^2)((0.3)/(0.67 mm^2) = 63 turns.
- $$L_{max} = \frac{(63)(170\ \text{mT})(1.5\text{x}10^{-4}\ \text{m}^2)}{5.6\ \text{A}}$$
  \- 290 microhenries
- $$L_g = \frac{10^{-2}}{\dfrac{(0.17)\ (1.5\text{x}10^{-4})}{(1.5\text{x}10^{-2})(4\text{š}\text{x}10^{-7})(63)(5.6)} - \dfrac{2.5\text{x}10^{-2}}{(4)(1.5\text{x}10^{-2})}}$$
  L_g - 3 mm
- L_{max} - L so no adjustment of inductance value is needed.

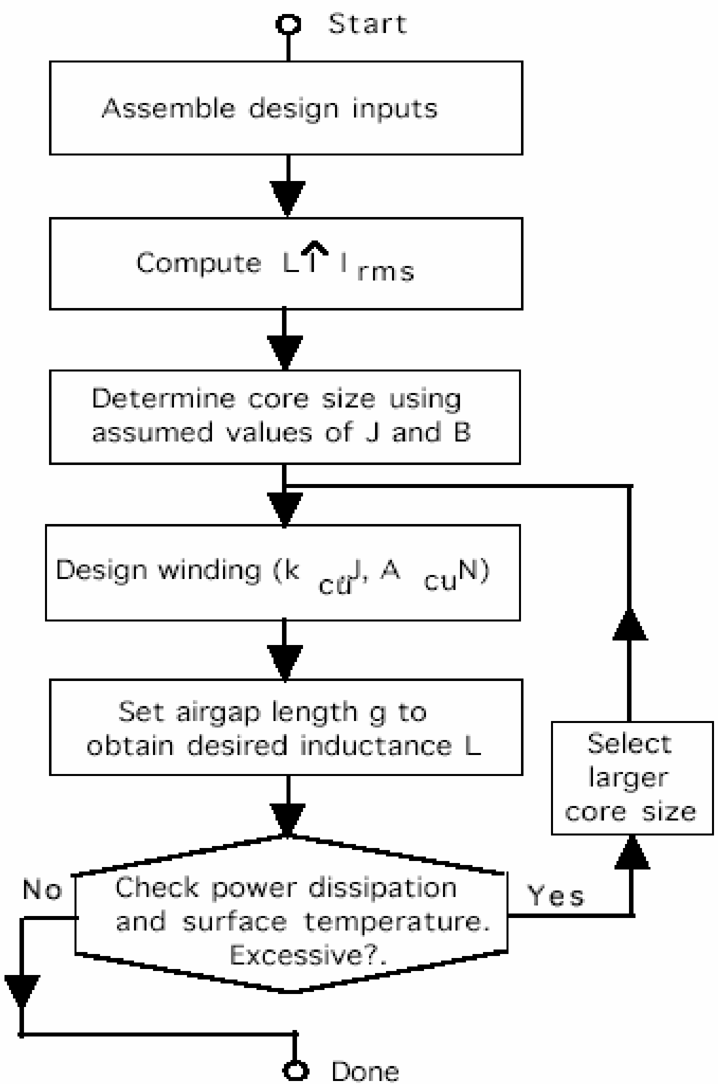
Iterative Inductor Design Procedure



- Iterative design procedure essentially consists of constructing the core database until a suitable core is found.
- Choose core material and shape and conductor type as usual.
- Use stored energy relation to find an initial area product $A_w A_c$ and thus an initial core size.
  - Use initial values of J_{rms} = 2-4 A/mm^2 and B_{ac} = 50-100 mT .
- Use initial core size estimate (value of a in double-E core example) to find corrected values of J_{rms} and B_{ac} and thus corrected value of $k_{cu}\, J_{rms}\, \hat{B}\ \ A_w\, A_{core}$.
- Compare $k_{cu}\, J_{rms}\, \hat{B}\ \ A_w\, A_{core}$ with $L\, I\, I_{rms}$ and iterate as needed into proper size is found.

Simple, Non-optimal Inductor Design Method

(Area Product Method)



- Assemble design inputs and compute required LI I_{rms}
- Choose core geometry and core material based on considerations discussed previously.
- Assume J_{rms} = 2-4 A/mm^2 and B_{ac} = 50-100 mT and use LI $I_{rms} = k_{cu} J_{rms} B_{ac} A_w A_{core}$ to find the required area product $A_w A_{core}$ and thus the core size.
    - Assumed values of J_{rms} and B_{ac} based on experience.
- Complete design of inductor as indicated.
- Check power dissipation and surface temperature using assumed values of J_{rms} and B_{ac}. If dissipation or temperature are excessive, select a larger core size and repeat design steps until dissipation/temperature are acceptable.
- Procedure is so-called area product method. Useful in situations where only one ore two inductors are to be built and size/weight considerations are secondary to rapid construction and testing..

3.6. Transformer Design Procedures

Analysis of Specific Transformer Design

- Transformer specifications
  - Wound on double-E core with a = 1 cm using 3F3 ferrite.
  - I_{pri} = 4 A rms, sinusoidal waveform; V_{pri} = 300 V rms.
  - Frequency = 100 kHz
  - Turns ratio N_{pri}/N_{sec} = 4 and N_{pri} = 32.
  - Winding window split evenly between primary and secondary and wound with Litz wire.
  - Transformer surface black (E = 0.9) and T_a Š 40 °C.
- Find: core flux density, leakage inductance, and maximum surface temperature T_s, and effect of 25% overcurrent on T_s.

- Areas of primary and secondary conductors, $A_{cu,pri}$ and $A_{cu,sec}$.
  - $A_{w,pri} = \dfrac{N_{pri}\, A_{cu,pri}}{k_{cu,pri}}$; $A_{w,sec} = \dfrac{N_{sec}\, A_{cu,sec}}{k_{cu,sec}}$
  - $A_{w,pri} + A_{w,sec} = A_w = \dfrac{N_{pri}\, A_{cu,pri}}{k_{cu}} + \dfrac{N_{sec}\, A_{cu,sec}}{k_{cu}}$

    where $k_{cu,pri} = k_{cu,sec} = k_{cu}$ since we assume primary and secondary are wound with same type of conductor.
- Equal power dissipation density in primary and secondary gives

$$\frac{I_{pri}}{I_{sec}} = \frac{A_{cu,pri}}{A_{cu,sec}} = \frac{N_{sec}}{N_{pri}}$$

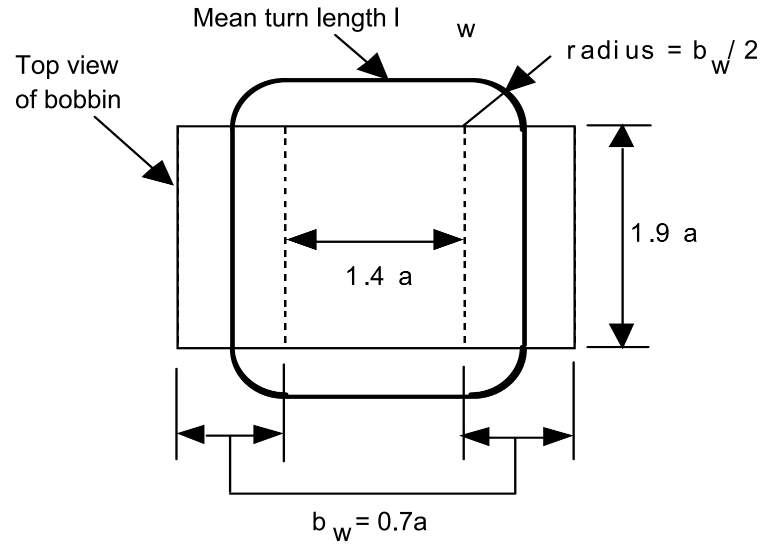
  - Using above equations yields $A_{cu,pri} = \dfrac{k_{cu}\, A_w}{2\, N_{pri}}$ and

    $A_{cu,sec} = \dfrac{k_{cu}\, A_w}{2\, N_{sec}}$
  - Numerical values: $A_{cu,pri} = \dfrac{(0.3)(140\ \mathrm{mm}^2)}{(2)(32)} = 0.64\ \mathrm{mm}^2$

    and $A_{cu,sec} = \dfrac{(0.3)(140\ \mathrm{mm}^2)}{(2)(8)} = 2.6\ \mathrm{mm}^2$

Analysis of Specific Transformer Design (cont.)

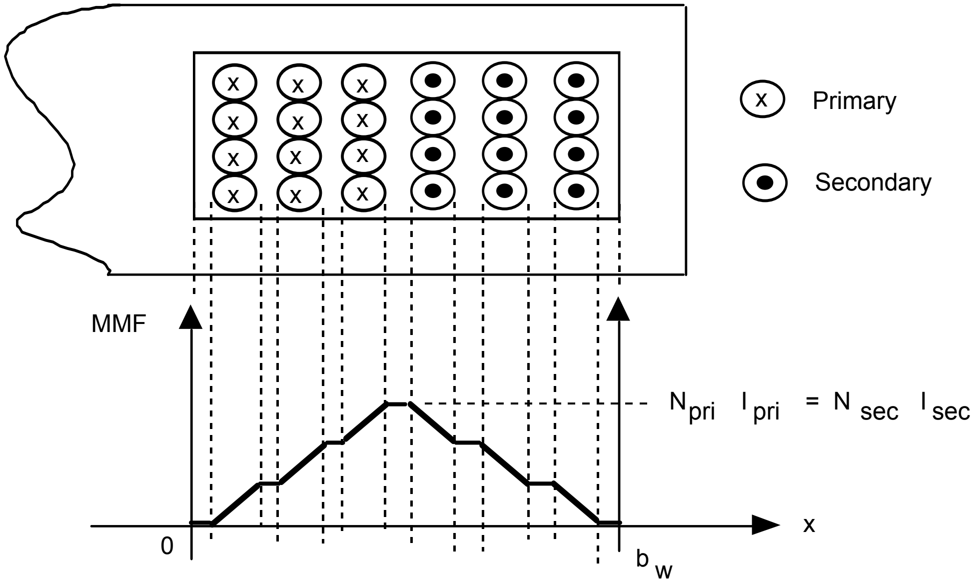
- Power dissipation in winding $P_w = k_{cu}\, \rho_{cu} (J_{rms})^2\, V_w$
  - $J_{rms} = (4\ A)/(0.64\ mm^2) = (16\ A)/(2.6\ mm^2) = 6.2\ A/mm^2$
  - $P_w = (0.3)(2.2x10^{-8}\ ohm\text{-}m)(6.2x10^{6}\ A/m^2)^2 (1.23x10^{-5}\ m^3)$
    $P_w = 3.1\ watts$
- Flux density and core loss
  - $V_{pri,max} = N_{pri}\, A_c\, \omega\, B_{ac} = (1.414)(300\ \ ) = 425\ V$
  - $B_{ac} = \dfrac{425}{(32)\ (1.5\,x10^{-4}\ m^2)(2\, š)(10^5\ Hz)} = 0.140\ T$
  - $P_{core} = (13.5\ cm^3)(1.5x10^{-6})(100\ kHz)^{1.3}(140\ mT)^{2.5} = 1.9\ W$
- Leakage inductance $L_{leak} = \dfrac{\mu_o (N_{pri})^2\, b_w\, l_w}{3\ h_w}$
  - $l_w = 8\ a = 8\ cm$
  - $L_{leak} = \dfrac{(4\,š\,x10^{-7})(32\ )^2(0.7)(10^{-2})(8\ x10^{-2})}{(3)\ (2x10^{-2})}$ - 12 microhenries



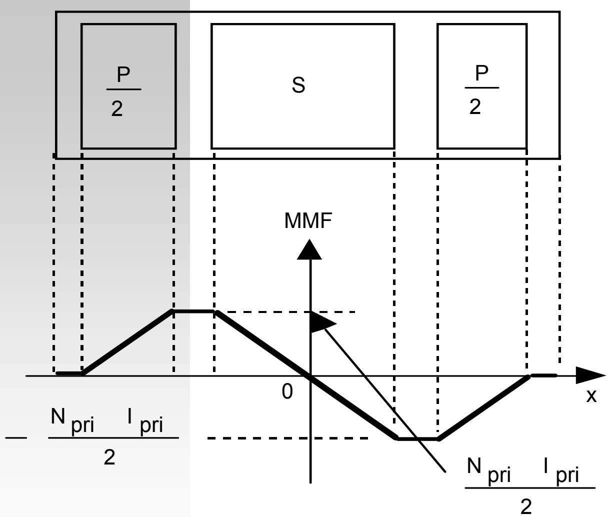
$l_w = (2)(1.4a) + (2)(1.9a) + 2š\,(0.35b\ \ ) = 8\ a_w$

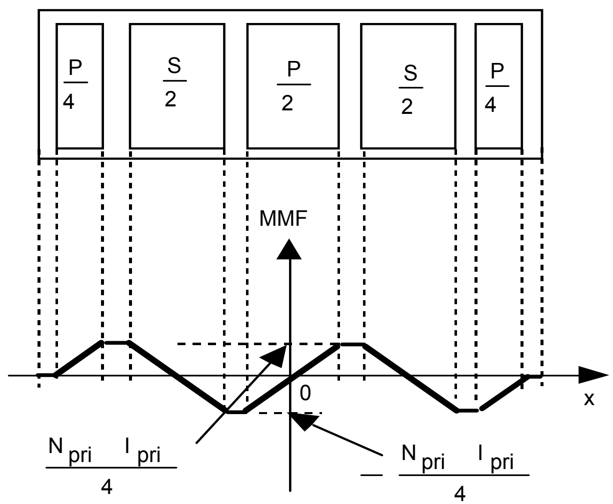
- Surface temperature T_s.
  - Assume $R_{\theta,sa}$ - 9.8 °C/W.
    Same geometry as inductor.
  - $T_s = (9.8)(3.1 + 1.9) + 40 = 89$ °C
- Effect of 25 % overcurrent.
  - No change in core flux density.
    Constant voltage applied to primary keeps flux density constant.
  - $P_w = (3.1)(1.25)^2 = 4.8\ watts$
  - ` $T_s = (9.8)(4.8 + 1.9) + 40 = 106$ °C

Sectioning of Transformer Windings to Reduce Winding Losses



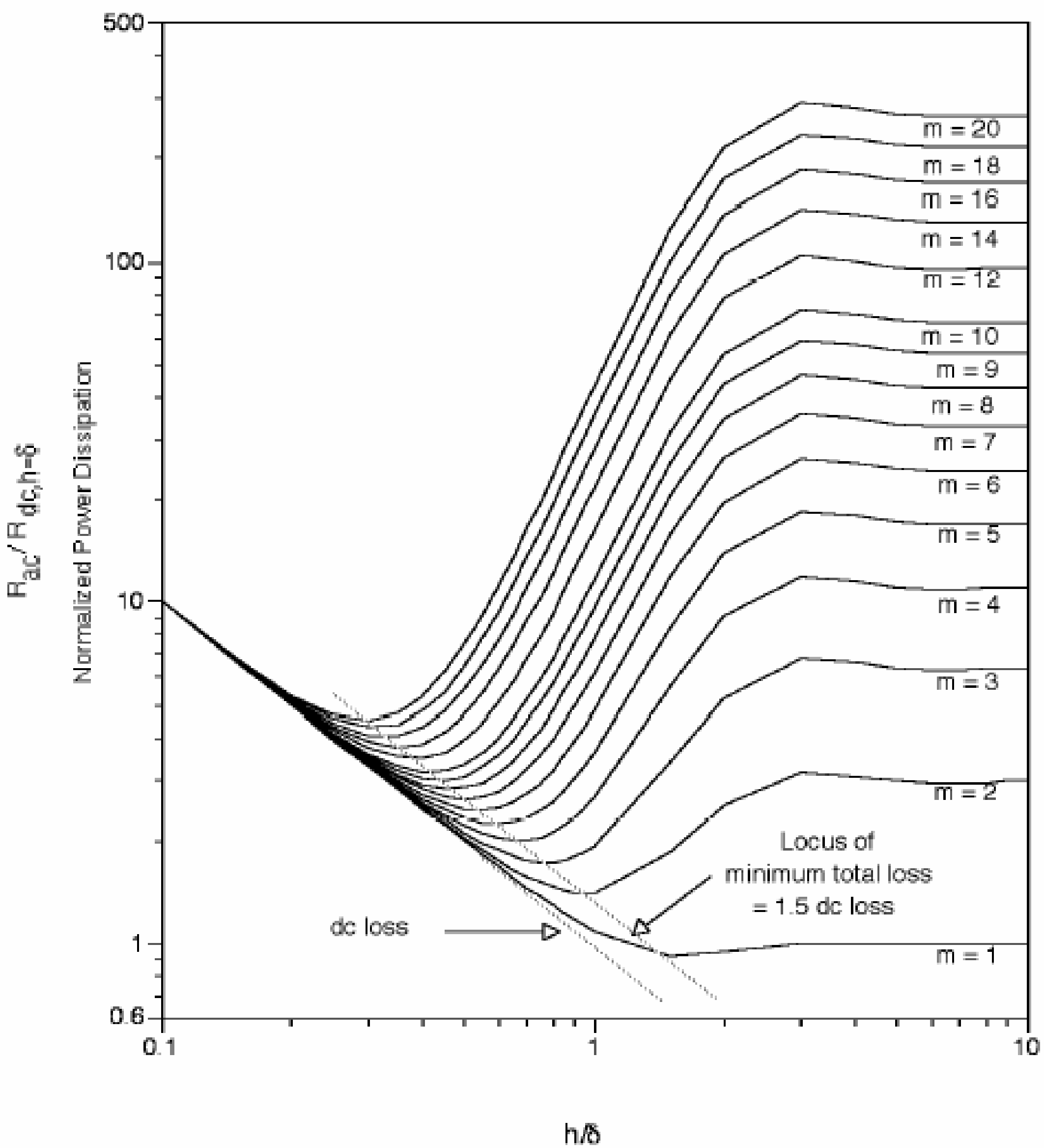
- Reduce winding losses by reducing magnetic field (or equivently the mmf) seen by conductors in winding. Not possible in an inductor.
- Simple two-section transformer winding situation.



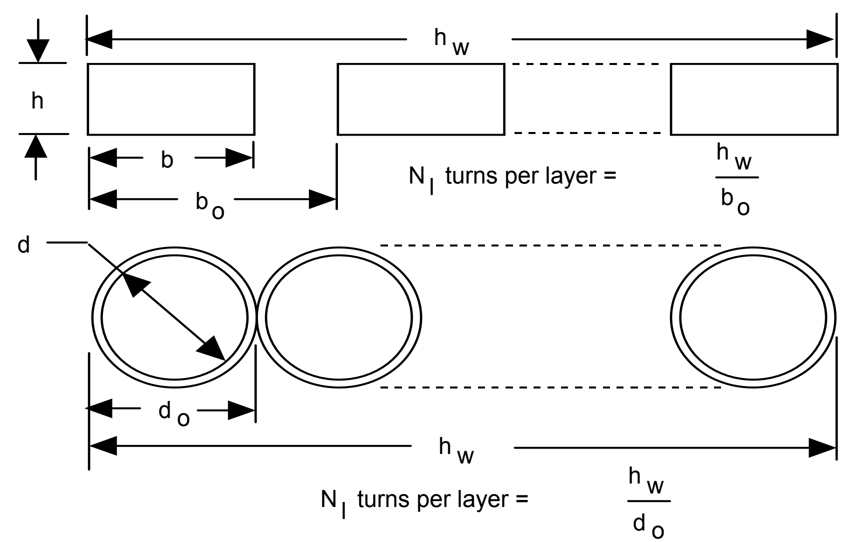


- Division into multiple sections reduces MMF and hence eddy current losses.

Optimization of Solid Conductor Windings

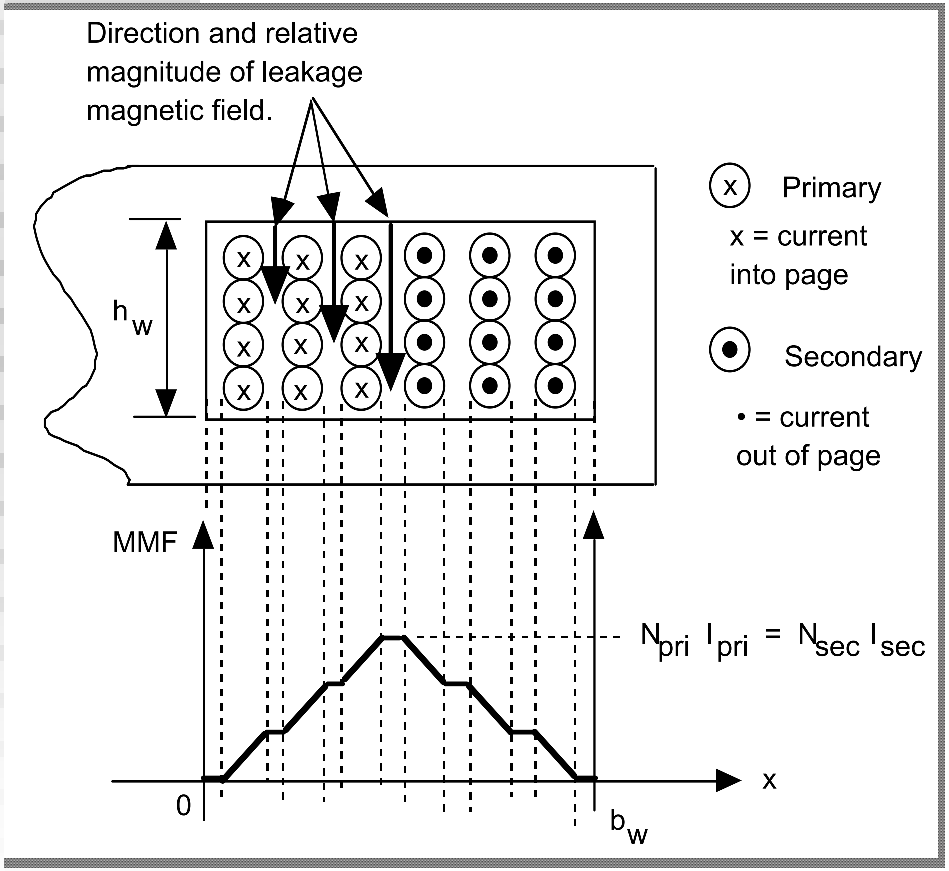


- Nomalized power dissipation = $\dfrac{P_w}{R_{dc,h=\delta}(I_{rms})^2} = \dfrac{F_R R_{dc}}{R_{dc,h=\delta}}$
- Conductor height/diameter $\dfrac{\sqrt{F_l}\,h}{\delta}$
- F_l = copper layer factor
  - $F_l = b/b_o$ for rectangular conductors
  - $F_l = d/d_o$ for round conductors
- h = effective conductor height
  - $h = \sqrt{\dfrac{\pi}{4}}\,d$ for round conductors
- m = number of layers



Transformer Leakage Inductance (additional info.)

- Transformer leakage inductance causes overvoltages across power switches at turn-off.
- Leakage inductance caused by magnetic flux which does not completely link primary and secondary windings.



- Linear variation of mmf in winding window indicates spatial variation of magnetic flux in the window and thus incomplete flux linkage of primary and secondary windings.
- $H_{window} = H_{leak} = \dfrac{2 N_{pri} I_{pri}\, x}{h_w b_w}$; $0 < x < b_w/2$

  $H_{leak} = \dfrac{2 N_{pri} I_{pri}}{h_w}(1 - x/b_w)$; $b_w/2 < x < b_w$
- $\dfrac{L_{leak} (I_{pri})^2}{2} = \dfrac{1}{2} \displaystyle\int_{V_w} \mu_o (H_{leak})^2 dV$
- Volume element $dV = h_w l_w(x)dx$; $l_w(x)$ equals the length of the conductor turn located at position x.
  - Assume a mean turn length l_w ? 8a for double-E core independent of x.
- $\dfrac{L_{leak} (I_{pri})^2}{2} = (2)\dfrac{1}{2} \displaystyle\int_0^{b_w/2} \mu_o \left[\frac{2 N_{pri} I_{pri}\, x}{h_w b_w}\right]^2 h_w l_w dx$
- $L_{leak} = \dfrac{\mu_o (N_{pri})^2 l_w b_w}{3 p^2 h_w}$
- If winding is split into p+1 sections, with p > 1, leakage inductance is greatly reduced.

Volt-Amp (Power) Rating - Basis of Transformer Design

- Input design specifications
  - Rated rms primary voltage V_{pri}
  - Rated rms primary current I_{pri}
  - Turns ratio N_{pri}/N_{sec}
  - Operating frequency f
  - Maximum temperatures T_s and T_a
- Design consists of the following:
  - Selection of core geometric shape and size
  - Core material
  - Winding conductor geometric shape and size
  - Number of turns in primary and secondary windings.

- Design proceedure starting point - transformer V-A rating S
  - $S = V_{pri}\, I_{pri} + V_{sec}\, I_{sec} = 2\, V_{pri}\, I_{pri}$
  - $V_{pri} = N_{pri}\,\frac{d\phi}{dt} = \frac{N_{pri}\, A_{core}\, \omega\, B_{ac}}{\sqrt{2}}$; $I_{pri} = J_{rms}\, A_{cu,pri}$
  - $S = 2\, V_{pri}\, I_{pri} = 2\,\frac{N_{pri}\, A_{core}\, \omega\, B_{ac}}{\sqrt{2}}\, J_{rms}\, A_{cu,pri}$
  - $A_{cu,pri} = \frac{k_{cu}\, A_w}{2\, N_{pri}}$
  - $S = 2\, V_{pri}\, I_{pri} = 2\,\frac{N_{pri}\, A_{core}\, \omega\, B_{ac}}{\sqrt{2}}\, J_{rms}\,\frac{k_{cu}\, A_w}{2\, N_{pri}}$
  - $S = V_{pri}\, I_{pri} = 4.4\, k_{cu}\, f\, A_{core}\, A_w\, J_{rms}\, B_{ac}$
- Equation relates input specifications (left-hand side) to core and winding parameters (right-hand side).
- Desired design procedure will consist of a systematic, single-pass method of selecting k_{cu}, A_{core}, A_w, J_{rms}, and B_{ac}.

Core Database - Basic Transformer Design Tool

- Interactive core database (spreadsheet-based) key to a single pass tramsformer design procedure.
  - User enters input specifications from converter design requirements. Type of conductor for windings (round wire, Litz wire, or rectangular wire or foil) must be made so that copper fill factor k_{cu} is known.
  - Spreadsheet calculates capability of all cores in database and displays smallest size core of each type that meets V- I specification.
  - Also can be designed to calculate (and display as desired) design output parameters including J_{rms}, B, $A_{cu,pri}$, $A_{cu,sec}$, N_{pri}, N_{sec}, and leakage inductance..
  - Multiple iterations of core material and winding conductor choices can be quickly done to aid in selection of most appropriate tranformer design.
- Information on all core types, sizes, and materials must be stored on spreadsheet. Info includes dimensions, A_w, A_{core}, surface area of assembled transformer , and loss data for all materials of interest.
- Pre-stored information combined with user inputs to produce performance data for each core in spreadsheet. Sample of partial output shown below.

Core No.	Material	$AP = A_w A_c$	R_θ $\Delta T = 60$ °C	P_{sp} @ $T_s = 100$ °C	J_{rms} @ $T_s = 100$ °C & P_{sp}	$\hat{B}_{rated}$ @ $T_s = 100$ °C & 100 kHz	$2.22\, k_{cu}\, f\, J_{rms}\, \hat{B}\, AP$ (f = 100kHz)
•	•	•	•	•	•	•	•
8	3F3	2.1 cm^4	9.8 °C/W	237 mW/cm^3	$(3.3/\sqrt{k_{cu}})\cdot\sqrt{\frac{R_{dc}}{R_{ac}}}$ A/mm^2	170 mT	$2.6\times10^3 \cdot \sqrt{\frac{k_{cu} R_{dc}}{R_{ac}}}$ [V-A]
•	•	•	•	•	•	•	•

Details of Interactive Transformer Core Database Calculations

- User inputs: V_{pri}, I_{pri}, turns ratio N_{dc}/ N_{sec}, f, T_s, T_a, and k_{cu}

- Stored information (static, independent of converter requirements)
  - Core dimensions, A_w, A_{core}, V_c, V_w, surface area, mean turn length, mean magnetic path length, etc.
  - Quantitative core loss formulas for all materials of interest including approximate temperature dependence.

- Calculation of core capabilities
  1. Compute converter-required stored energy value: $S = 2\, V_{pri}\, I_{pri}$
  2. Compute allowable specific power dissipation $P_{sp} = [T_s - T_a] / \{ R_{\theta sa} [V_c + V_w]\}$. $R_{\theta sa} = h/A_s$ or calculated interactively using input temperatures and formulas for convective and radiative heat transfer from Heat Sink chapter.
  3. Compute allowable flux density $P_{sp} = k\, f^b\, [B_{ac}]^d$ and current density $P_{sp} = k_{cu}\, \rho_{cu}\, \{J_{rms}\}^2$.
  4. Compute core capabilities $4.4\, f\, k_{cu}\, A_w\, A_{core}\, B_{ac}\, J_{rms}$

- Calculation transformer parameters.
  1. Calculate number of primary turns $N_{pri} = V_{pri} /\{2\pi\, f\, A_{cpre} B_{ac}\}$ and secondary turns $N_{sec} = V_{sec} /\{2\pi\, f\, A_{cpre} B_{ac}\}$
  2. Calculate winding conductor areas assuming low frequencies or use of Leitz wire
     - $A_{cu,pri} = [k_{cu} A_w]/[2\, N_{pri}]$ and $A_{cu,sec} = [k_{cu} A_w]/[2\, N_{sec}]$

Details of Interactive Transformer Core Database Calculations (cont.)

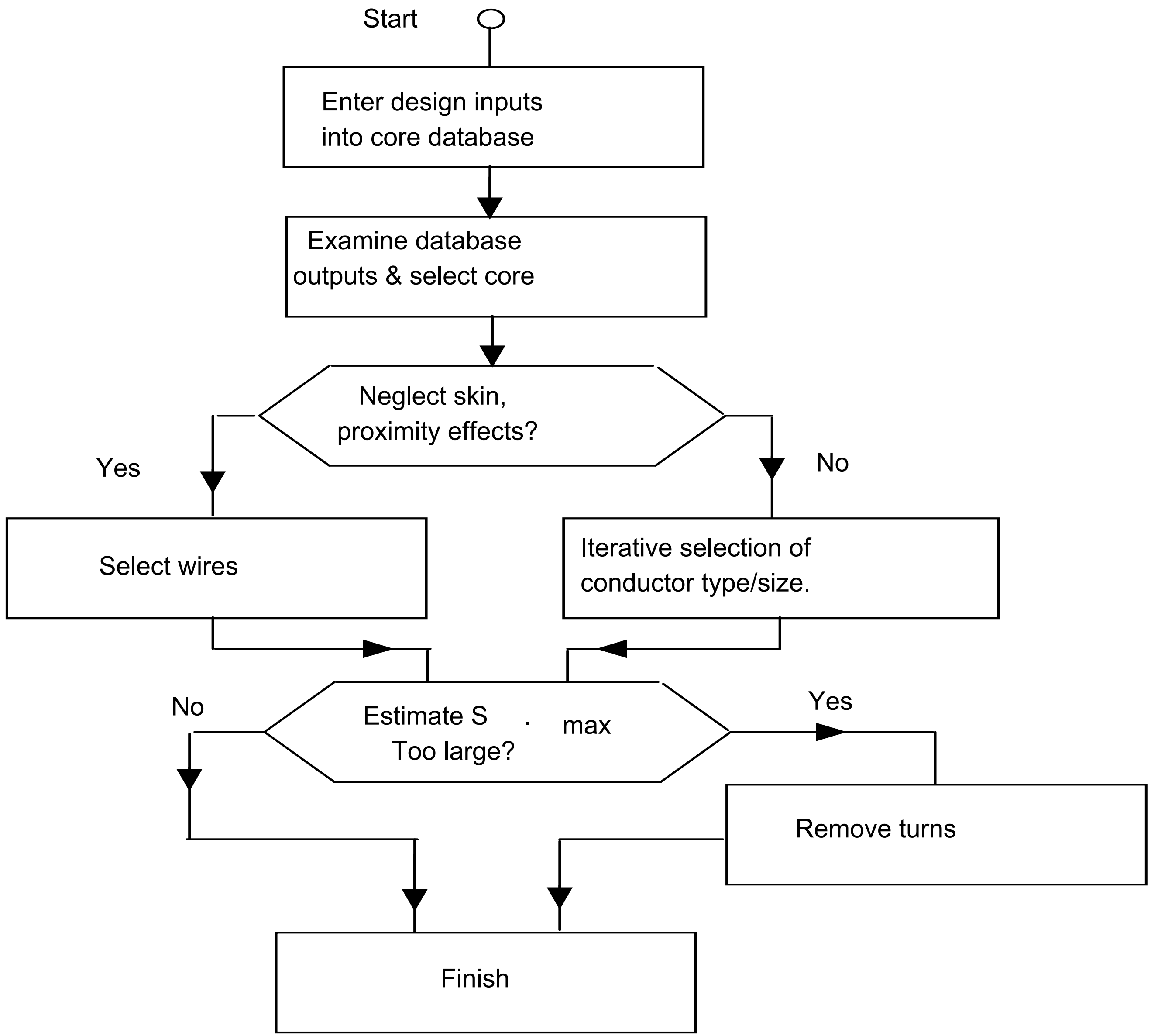
3. Calculate winding areas assuming eddy current/proximity effect is important
   - Only solid conductors, round wires or rectangular wires (foils), used. $J_{rms} = [\{P_{sp}\, R_{dc}\}/\{R_{ac}\, k_{cu}\, r_{cu}\}]^{1/2}$
   - Conductor dimensions must simultaneously satisfy area requirements and requirements of normalized power dissipation versus normalized conductor dimensions.
   - May require change in choice of conductor shape. Most likely will require choice of foils (rectangular shapes).
   - Several iterations may be needed to find proper combinations of dimensions, number of turns per layer, and number of layers and sections.
   - Best illustrated by a specific design example.

4. Estimate leakage inductance $L_{leak} = \{\mu_o \{N_{pri}\}^2\, l_w\, b_w\} / \{3\, p^2\, h_w\}$

5. Estimate $S_{max} = 4.4\, k_{cu}\, f\, A_{core}\, A_w\, J_{rms}\, B_{ac}$

6. If $S_{max} > S = 2\, V_{pri}\, I_{pri}$ reduce S_{max} and save on copper cost, weight, and volume.
   - If $N_{pri}\, w\, A_c\, B_{ac} > V_{pri}$, reduce S_{max} by reducing N_{pri} and N_{sec}.
   - If $J_{rms}\, A_{cu,\,pri} > I_{rms}$, reduce $A_{cu,pri}$ and $A_{cu,\,sec}$.
   - If $S > S_{max}$ by only a moderate amount (10-20%) and smaller than S_{max} of next core size, increase S_{max} of present core size.
   - Increase I_{rms} (and thus winding power dissipation) as needed. Temperature T_s will increase a modest amount above design limit, but may be preferable to going to larger core size.

Single Pass Transformer Design Procedure



Transformer Design Example

- Design inputs
  - $V_{pri} = 300$ V rms ; $I_{rms} = 4$ A rms
  - Turns ratio n = 4
  - Operating frequency f = 100 kHz
  - $T_s = 100$ °C and $T_a = 40$ °C
- V - I rating S = (300 V rms)(4 A rms) = 1200 watts
- Core material, shape, and size.
  - Use 3F3 ferrite because it has largest performance factor at 100 kHz.
  - Use double-E core. Relatively easy to fabricate winding.
- Core volt-amp rating $= 2{,}600 \sqrt{k_{cu}} \sqrt{\frac{R_{dc}}{R_{ac}}}$
  - Use solid rectangular conductor for windings because of high frequency. Thus $k_{cu} = 0.6$ and $R_{ac}/R_{dc} = 1.5$.
  - Core volt-amp capability $= 2{,}600 \sqrt{\frac{0.6}{1.5}}$ = 1644 watts. > 1200 watt transformer rating. Size is adequate.
- Using core database, $R_\theta = 9.8$ °C/W and $P_{sp} = 240$ mW/cm^3.
- Flux density and number of primary and secondary turns.
  - From core database, $B_{ac} = 170$ mT.
  - $N_{pri} = \frac{300\sqrt{2}}{(1.5\text{x}10^{-4}\text{m}^2)(2š)(10^5\text{Hz})(0.17\text{ T})}$ = 26.5 - 24. Rounded down to 24 to increase flexibility in designing sectionalized transformer winding.
  - $N_{sec} = \frac{24}{6} = 6.$
- From core database $J_{rms} = \frac{3.3}{\sqrt{(0.6)(1.5)}}$ = 3.5 A/mm^2.
  - $A_{cu,pri} = \frac{4\text{ A rms}}{3.5\text{ A rms/mm}^2} = 1.15\text{ mm}^2$
  - $A_{cu,sec} = (4)(1.15\text{ mm}^2) = 4.6\text{ mm}^2$

Transformer Design Example (cont.)

- Primary and secondary conductor areas - proximity effect/eddy currents included. Assume rectangular (foil) conductors with $k_{cu} = 0.6$ and layer factor $F_1 = 0.9$.

  - Iterate to find compatible foil thicknesses and number of winding sections.
  - 1st iteration - assume a single primary section and a single secondary section and each section having single turn per layer. Primary has 24 layers and secondary has 6 layers.

- Primary layer height $h_{pri} = \dfrac{A_{cu,pri}}{F_1 \, h_w}$

$$= \frac{1.15 \text{ mm}^2}{(0.9)(20 \text{ mm})} = 0.064 \text{ mm}$$

  - Normalized primary conductor height

$$\phi = \frac{\sqrt{F_1}\, h_{pri}}{d} = \frac{\sqrt{0.9}\,(0.064 \text{ mm})}{(0.24 \text{ mm})} = 0.25 \,;$$

    $\delta = 0.24$ mm in copper at 100 kHz and 100 °C.

  - Optimum normalized primary conductor height $\phi = 0.3$ so primary winding design is satisfactory.

- Secondary layer height $h_{sec} = \dfrac{A_{cu,sec}}{F_1 \, h_w}$

$$= \frac{4.6 \text{ mm}^2}{(0.9)(20 \text{ mm})} - 0.26 \text{ mm.}$$

  - Normalized secondary conductor height

$$\phi = \frac{\sqrt{F_1}\, h_{sec}}{d} = \frac{\sqrt{0.9}\,(0.26 \text{ mm})}{(0.24 \text{ mm})} = 1$$

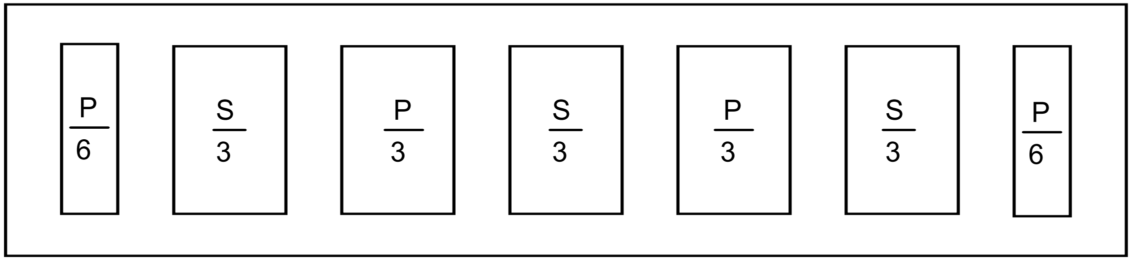
  - However a six layer section has an optimum $\phi = 0.6$. A two layer section has an optimum $\phi = 1$. 2nd iteration needed.

- 2nd iteration - sectionalize the windings.

  - Use a secondary of 3 sections, each having two layers, of height $h_{sec} = 0.26$ mm.

  - Secondary must have single turn per layer. Two turns per layer would require $h_{sec} = 0.52$ mm and thus $\phi = 2$. Examination of normalized power dissipation curves shows no optimum $\phi = 2$.

Transformer Design Example (cont.)



- Three secondary sections requires four primary sections.
  - Two outer primary sections would have 24/6 = 4 turns each and the inner two sections would have 24/3 = 8 turns each.
  - Need to determine number of turns per layer and hence number of layers per section.

Turns/ layer	h_{pri}	No. of Layers	ϕ	Optimum ϕ
1	0.064 mm	8	0.25	0.45
2	0.128 mm	4	0.5	0.6
4	0.26 mm	2	1	1

  - Use four turns per layer. Two interior primary sections have two layers and optimum value of ϕ. Two outer sections have one layer each and ϕ not optimum, but only results in slight increase in loss above the minimum.

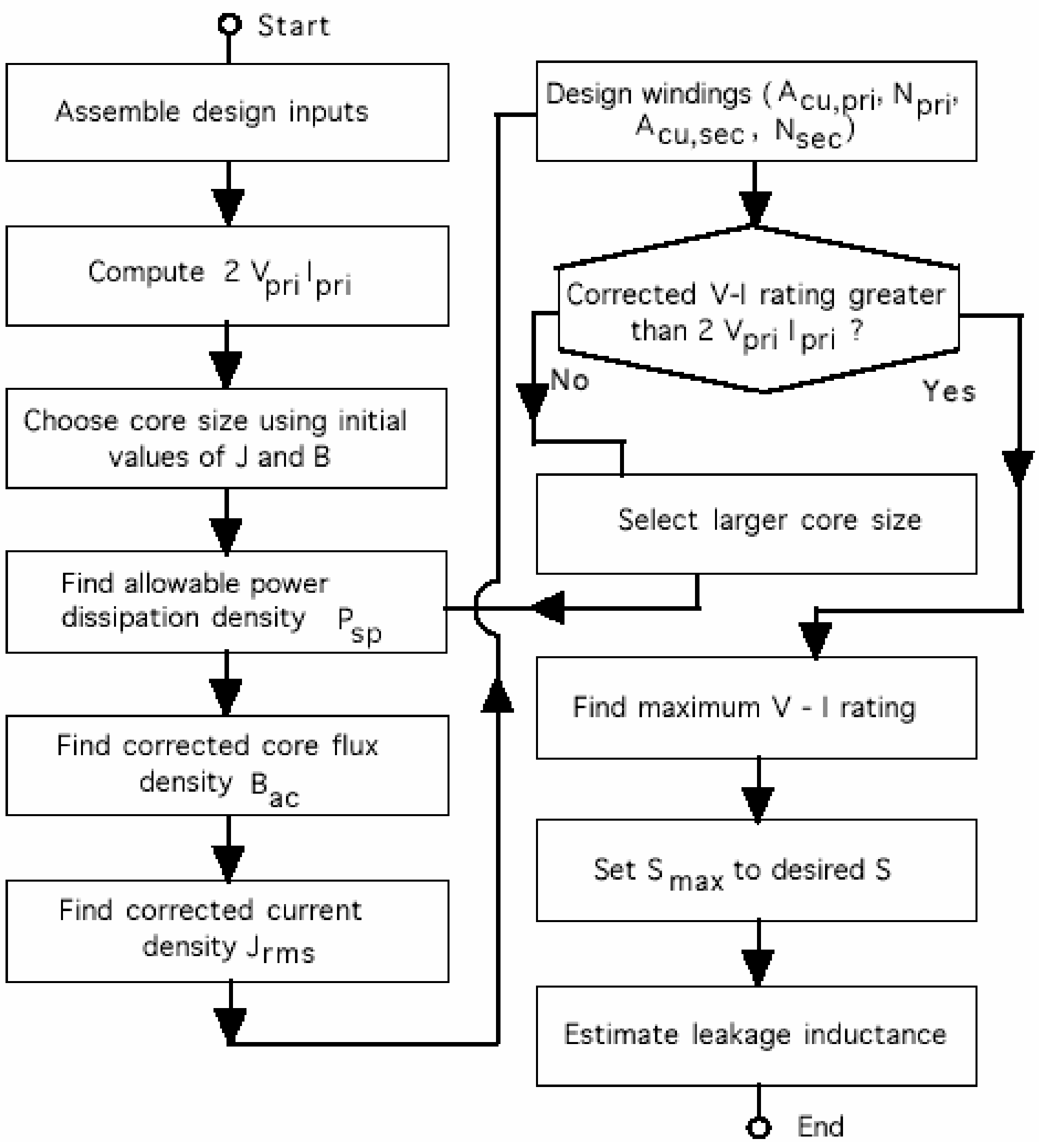
- Leakage inductance L_{leak}

$$= \frac{(4\pi \times 10^{-9})(24)^2(8)(0.7)(1)}{(3)(6)^2(2)} = 0.2\ \mu H$$

  - Sectionalizing increases capacitance between windings and thus lowers the transformer self-resonant frequency.

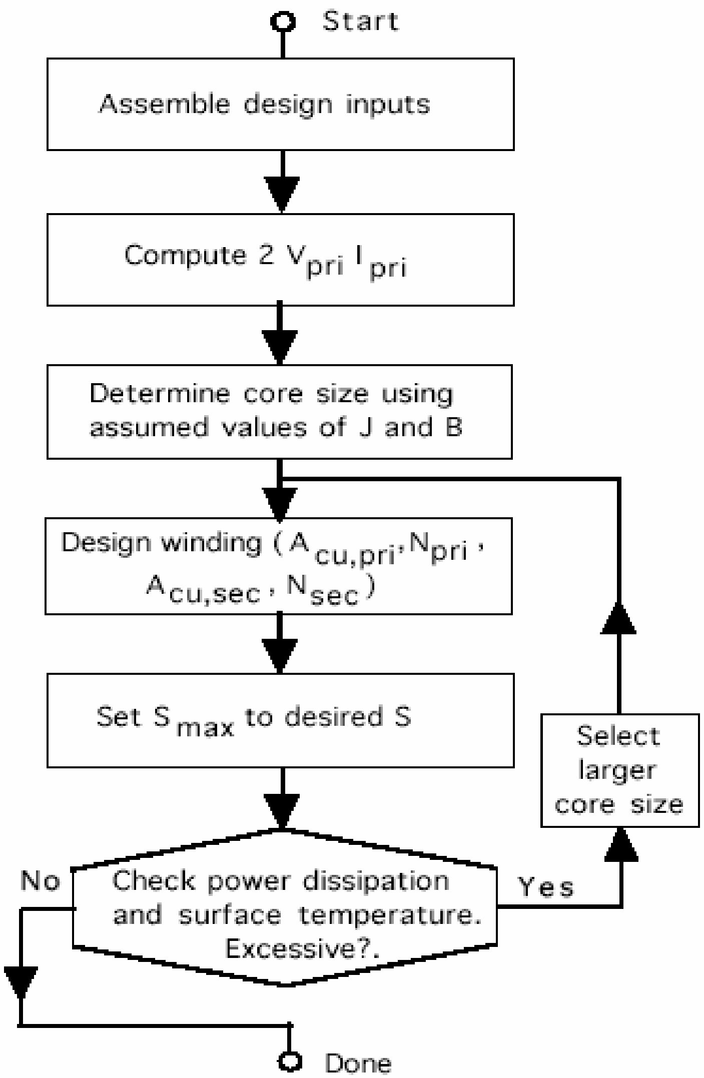
- S_{max} = 1644 watts
  - Rated value of S = 1200 watts only marginally smaller than S_{max}. Little to be gained in reducing S_{max} to S unless a large number of transformer of this design are to be fabricated.

Iterative Transformer Design Procedure



- Iterative design procedure essentially consists of constructing the core database until a suitable core is found.
- Choose core material and shape and conductor type as usual.
- Use V - I rating to find an initial area product $A_w A_c$ and thus an initial core size.
  - Use initial values of $J_{rms} = 2\text{-}4\ \mathrm{A/mm^2}$ and $B_{ac} = 50\text{-}100\ \mathrm{mT}$.
- Use initial core size estimate (value of a in double-E core example) to find corrected values of J_{rms} and B_{ac} and thus corrected value of $4.4\, f\, k_{cu}\, J_{rms}\, \hat{B}\ A_w\, A_{core}$.
- Compare $4.4\, f\, k_{cu}\, J_{rms}\, \hat{B}\ A_w\, A_{core}$ with $2\, V_{pri}\, I_{pri}$ and iterate as needed into proper size is found.

Simple, Non-optimal Transformer Design Method



- Assemble design inputs and compute required 2 V_{pri} I_{pri}
- Choose core geometry and core material based on considerations discussed previously.
- Assume J_{rms}= 2-4 A/mm^2 and B_{ac} = 50-100 mT and use 2 V_{pri} I_{pri} = 4.4 f k_{cu} J_{rms} B_{ac} A_w A_{core} to find the required area product A_w A_{core} and thus the core size.
  - Assumed values of $J_{rms}$and B_{ac} based on experience.
- Complete design of transformer as indicated.
- Check power dissipation and surface temperature using assumed values of $J_{rms}$and B_{ac}. If dissipation or temperature are excessive, select a larger core size and repeat design steps until dissipation/temperature are acceptable.
- Procedure is so-called area product method. Useful in situations where only one ore two transformers are to be built and size/weight considerations are secondary to rapid construction and testing..

III. MAGNETICS DESIGN

Summary

Core Area-Product $A_p = A_{core} A_{window}$

inductor: $$A_p = \frac{L \hat{I} I_{rms}}{k_w J_{\max} B_{\max}}$$

transformer: $$A_p = \frac{k_{conv} \sum V_y I_{y,rms}}{k_w B_{\max} J_{\max} f_s}$$

Design Procedure Based on Area-Product A_p

inductor: $$N = \frac{L \hat{I}}{B_{\max} A_{core}} \qquad L \simeq \frac{N^2}{\Re_g} \qquad \Re_g \simeq \frac{\ell_g}{\mu_o A_{core}} \qquad \ell_g = \frac{N^2 \mu_o A_{core}}{L}$$

transformer: $$N_y = \frac{k_{conv} V_y}{A_{core} f_s B_{\max}}$$

III. MAGNETICS DESIGN

DESIGN EXAMPLE OF AN INDUCTOR

In this example, we will discuss the design of an inductor that has an inductance $L = 100\mu H$. The worst-case current through the inductor is shown in Fig. 9-3, where the average current $I = 5.0A$, and the peak-peak ripple $\Delta I = 0.75A$ at the switching frequency $f_s = 100kHz$. We will assume the following maximum values for the flux density and the current density: $B_{\max} = 0.25T$, and $J_{\max} = 6.0A/mm^2$ (for larger cores, this is typically in a range of 3 to $4A/mm^2$). The window fill factor is assumed to be $k_w = 0.5$.

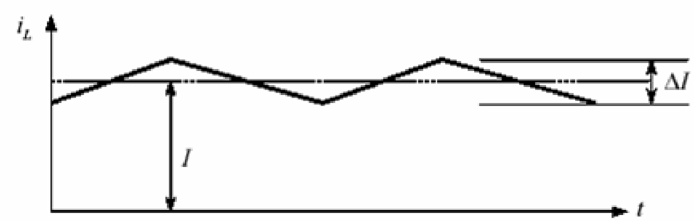


Figure 9-3 Inductor current waveforms.

$$\hat{I} = I + \frac{\Delta I}{2} = 5.375A$$

$$I_{rms} = \sqrt{I^2 + \frac{1}{12}\Delta I^2} \simeq 5.0A$$

$$A_p = \frac{100\times10^{-6}\times5.375\times5}{0.5\times0.25\times6\times10^6}\times10^{12} = 3587\,mm^4$$

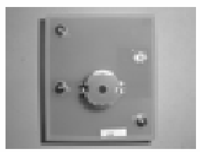
III. MAGNETICS DESIGN

From the Magnetics, Inc. catalog [2], we will select a P-type material, which has the saturation flux density of $0.5T$ and is quite suitable for use at the switching frequency of $100\,kHz$. A pot core 26×16, which is shown in Fig. 9-4 for a laboratory experiment, has the core Area $A_{core} = 93.1\,mm^2$ and the window Area $A_{window} = 39\,mm^2$. Therefore, we will select this core, which has an Area-Product $A_p = 93.1 \times 39 = 3631\,mm^4$.

$$N = \frac{100\mu \times 5.375}{0.25 \times 93.1 \times 10^{-6}} \simeq 23$$

Winding wire cross sectional area $A_{cond} = I_{rms} / J_{\max} = 5.0 / 6.0 = 0.83\,mm^2$. We will use five strands of American Wire Gauge AWG 25 wires [3], each with a cross-sectional area of $0.16\,mm^2$, in parallel.

$$\ell_g = \frac{23^2 \times 4\pi \times 10^{-7} \times 93.1 \times 10^{-6}}{100\mu} \simeq 0.62\,mm$$



III. MAGNETICS DESIGN

DESIGN EXAMPLE OF A TRANSFORMER FOR A FORWARD CONVERTER

The required electrical specifications for the transformer in a Forward converter are as follows: $f_s = 100kHz$ and $V_1 = V_2 = V_3 = 30V$. Assume the rms value of the current in each winding to be $2.5A$. We will choose the following values for this design:

$B_{\max} = 0.25\,\mathrm{T}$ and $J_{\max} = 5\,\mathrm{A/mm^2}$. $k_w = 0.5$ $k_{conv} = 0.5$

$$A_p = \frac{k_{conv}}{k_w f_s B_{\max} J_{\max}} \sum_y \hat{V}_y I_{rms,y} = 1800\ \mathrm{mm}^4$$

For the pot core 22×13 [2], $A_{core} = 63.9\ \mathrm{mm}^2$, $A_{window} = 29.2\ \mathrm{mm}^2$, and therefore $A_p = 1866\ \mathrm{mm}^4$.

$$A_{cond,1} = \frac{I_{1,rms}}{J_{\max}} = \frac{2.5}{5} = 0.5\ \mathrm{mm}^2$$

We will use three strands of AWG 25 wires [3], each with a cross-sectional area of $0.16\,mm^2$, in parallel for each winding.

$$N_1 = \frac{0.5 \times 30}{\left(63.9 \times 10^{-6}\right) \times \left(100 \times 10^3\right) \times 0.25} \simeq 10 \qquad N_1 = N_2 = N_3 = 10$$

III. MAGNETICS DESIGN

Example of Transformer Design:

-The first step in the process is to define the application Parameters that should not change, regardless of subsequent iterations in the selection of a specific core type and size.

Example of a forward converter transformer design:

***Step 1:* Define the power supply parameters:**

Vin range: 100 – 190 V

Output: 5V, 50A

Topology: Forward Converter

Switching frequency: 200 kHz

Max. Loss: 2.5 W

Max Temp. Rise: 40°C

Cooling: Natural Convection

III. MAGNETICS DESIGN

Transformer Design:

- **Step 2:** Define absolute duty cycle limit D_{lim}, D_{max} at low V_{in} (to provide headroom for dynamic response) and normal V_{in} D:

D_{lim}: 0.47
Normal D_{max}: 0.42
V_{in} D: V_{inmin} D_{max}= 42V
V_{inmax} D_{lim}: 89.3V

-**Step 3:** Calculate output voltages + diode and secondary
-IR drops at full load:
V_o: 5.0 + 0.4 = 5.4 V

III. MAGNETICS DESIGN

Transformer Design:

- **Step 4:** Calculate desired turns ratios: P-S1; S1-S2 etc.
Remember that choices with low voltage secondaries will be limited.

$n = N_p / N_{s1} = V_{in} D / V_o = 42 / 5.4 = 7.8$

Possible choices: 8:1; 7:1; 15:2

- **Step 5:** Core Selection: Select core material, shape, and tentative size, using manufacturer's data, and area product formula.

Core material: Ferrite, Magnetics Type P

Core Type: ETD

Core Size: 34 mm ETD 34

III. MAGNETICS DESIGN

Step 6: For the specific core selected note: Effective Core Area, Window Area, Volume, Path, Length (cm)

$A_e = 0.97\ cm^2$; $A_w = 1.89/1.23\ cm^2$; $V_e = 7.64\ cm^3$; $l_e = 7.9\ cm$

Step 7: Define thermal resistance and loss limit

Obtain thermal resistance from data sheets or calculate from window area.

Calculate loss limit based on max. temperature rise:

$P_{lim} = °C\ rise / R_{th}$

In the ex: $R_{th} = 36 / A_w$ for ETD series, $P_{lim} = 40 / 19 = 2.1W$

Compare with loss limit in Step 1.

III. MAGNETICS DESIGN

Step 8: Loss Limited Flux Swing:
Calculate max. Core loss per cm^3.
$P_{clim} / V_e = 1 / 7.64 = 131\ mW / cm^3$
Using this core loss value, enter the core loss curve for P-material.
At the transformer frequency, find the peak flux density, and double it to find the flux swing.
At 131 mW / cm^3 and f = 200 kHz: $\Delta B = 2 \times 800\ G = 0.16\ T$

Step 9: Using Faraday's Law, calculate the number of secondary Turns: $N_s = V_o\, T_s / \Delta B \,.\, A_e = 5.4 \times 5\ 10^{-6} / (0.16 \times 0.97\ 10^{-4})$
$= 1.74$ turns. Round to 2 turns.

Step 10: Recalculate the flux swing and core loss: $\Delta B' = 0.14\ T$
From the core loss curves at 0.14 / 2 T, $P_c = 110\ mW/cm^3$.
Hence $P_{core} = 0.84\ W$.

III. MAGNETICS DESIGN

Step 11: ***Finalize the choice of primary turns. A larger turns ratio results in lower peak current, larger D, and more copper loss. From the possiblities, trial solutions show the best choice to be Np= 15 turns. (7.5 : 1 turns ratio). Recalculate normal V_{in} D and flux swing under worst case $V_{in\ max}$ D_{lim} conditions.***

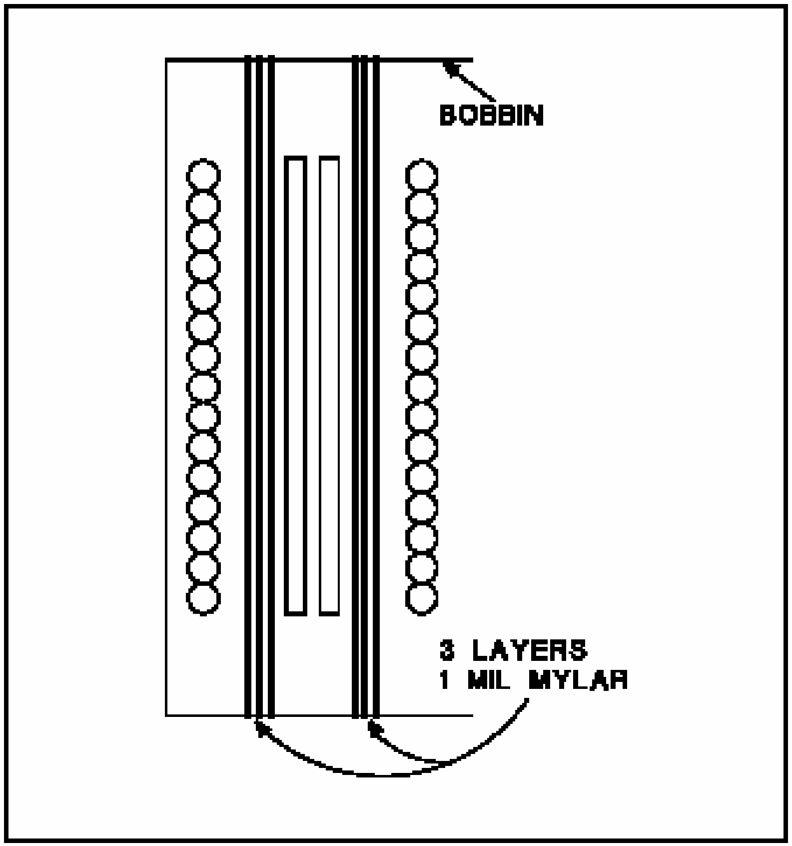
$V_{in}\, D = n\, V_o = 7.5 \times 5.4 = 40.5\ V$

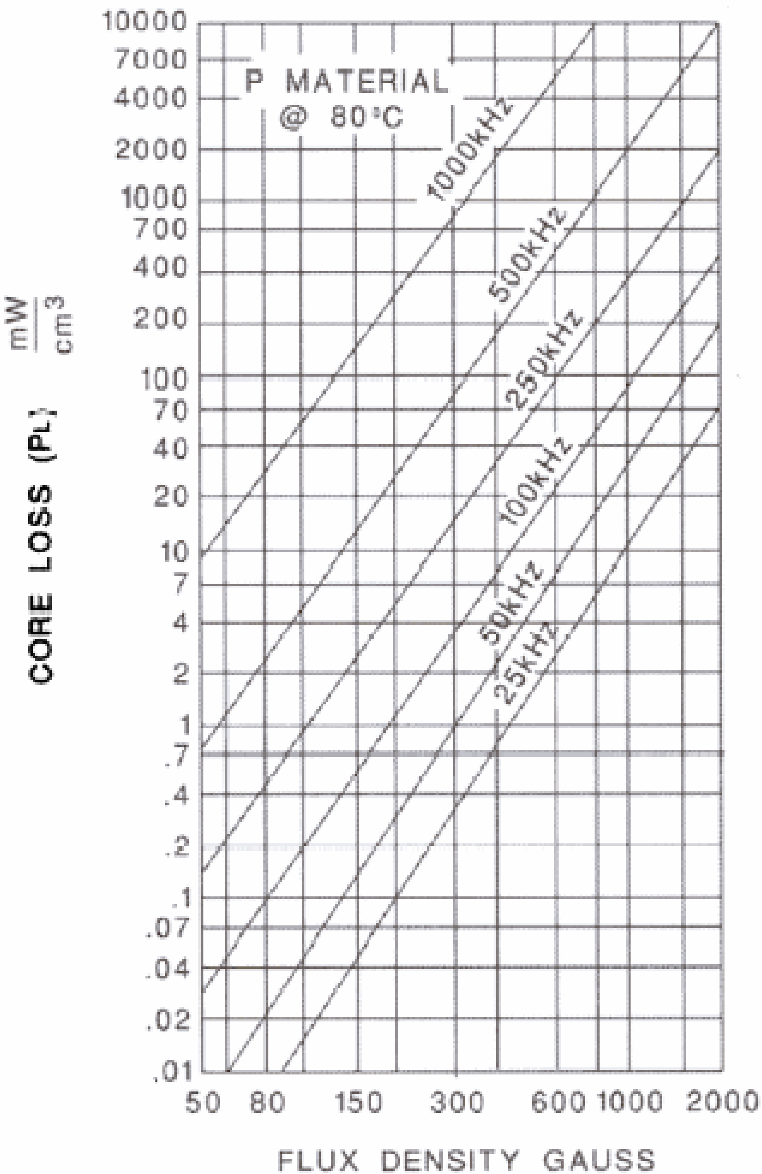
$\Delta B_{lim} = 0.14 \times 89.3 / 40.5 = 0.31\ T$ --- OK!

Step 12: ***Define the winding structure:***

An interleaved structure will minimize leakage inductance and winding losses. This results in 2 wdg sections. Primary wdg of 15 turns in each section are connected in parallel. Secondary wdg of 1 turn copper foil in each section are connected in series resulting in a 2-turn secondary.

III. MAGNETICS DESIGN





III. MAGNETICS DESIGN

Step 13: Calculate dc and rms ac currents in each winding at $V_{in\ min}$ and D_{max}.

$I_{Sdc} = 50\ D_{max} = 50 \times 0.405 = 20.25$

$I_{Sac} = I_{Sdc}\ ((1-D)/D)^{1/2} = 24.5A$

$I_{pdc} = 20.25 / 7.5 = 2.7\ A$

$I_{pac} = 24.5 / 7.5 = 3.27\ A$

Step 14: Define the primary winding: One layer of 15 turns spread across the available wdg breadth of 1.3 cm allows max.
Wire diameter of 0.87 mm. AWG 21 – 0.72 mm copper will be used.

From Dowell's curves R_{ac}/R_{dc} for 1 layer is 3:1. This will result in unacceptable ac loss. So Litz wire will be used.

III. MAGNETICS DESIGN

A Litz wire consisting of 100 strands with a diameter of 0.81 mm and R= 0.545 mohm / cm will be used. The dc resistance of single Layer is 0.55 10^{-3} x (MLT=6.1) x (N_s=15) = 50 mohm.

Total primary dc loss = 2 x 1.35^2 x 50 10^{-3} = 0.18 W.

The diameter of each wire is 0.064 mm, but there are effectively 10 layers of fine wire in a single layer of Litz wire (10x10 array). Q is nearly 1/10th of solid wire, or 0.3, resulting in
R_{ac} / R_{dc} = 1.2.
P_{ac} = 2 x 1.652 x 0.06 = 0.32 W

P_{dc} + P_{ac} = 0.5 W

III. MAGNETICS DESIGN

Step 15: Define the secondary wdg.

The sec. wdg. consists of 2 turns (two layers) of copper strip or foil, 1.3 cm wide, and 0.13 cm. Thick. There is one secondary layer in each of the two sections of the interleaved structure.

Q = Layer thickness / D_{pen} = 0.13 / 0.017 = 7.6
(acceptable due to very low dc resistance)

$R_{dc} = \rho \, MLT \, N_s / (b_w \, h) = 0.068$ W
$P_{ac} = 0.75$ W
Total sec. Loss= 0.82 W. Then total copper loss = 0.82 + 0.5 = 1.32W
Core + Copper Loss = 0.84 + 1.32 = 2.16 W
(below the limit specified in Step 1)

III. MAGNETICS DESIGN

Table I
Magnetic Parameters and Conversion Factors

		SI	CGS	CGS to SI
FLUX DENSITY	B	Tesla	Gauss	10^{-4}
FIELD INTENSITY	H	A-T/m	Oersted	$1000/4\pi$
PERMEABILITY (space)	μ_0	$4\pi\cdot10^{-7}$	1	$4\pi\cdot10^{-7}$
PERMEABILITY (relative)	μ_r			
AREA (Core Window)	A_e, A_w	m	cm^2	10^{-4}
LENGTH (Core, Gap)	ℓ_e, ℓ_g	m	cm	10^{-2}
TOTAL FLUX = $\int B\,dA$	ϕ	Weber	Maxwell	10^{-8}
TOTAL FIELD = $\oint H\,d\ell$	$\mathcal{F}$,mmf	A-T	Gilbert	$10/4\pi$
RELUCTANCE = $\mathcal{F}/\phi$	$\mathcal{R}$	A-T/Wb	Gb/Mx	$10^{9}/4\pi$
PERMEANCE = $1/\mathcal{R}$	$\mathcal{P}$			$4\pi\cdot10^{-9}$
INDUCTANCE = $\mathcal{P}\cdot N^2$ (SI)	L	Henry	(Henry)	
ENERGY	W	Joule	Erg	10^{-7}

III. MAGNETICS DESIGN

A key design decision: the choice of maximum operating flux density B_{max}

- Choose B_{max} to avoid saturation of core, or
- Further reduce B_{max}, to reduce core losses

Different design procedures are employed in the two cases.

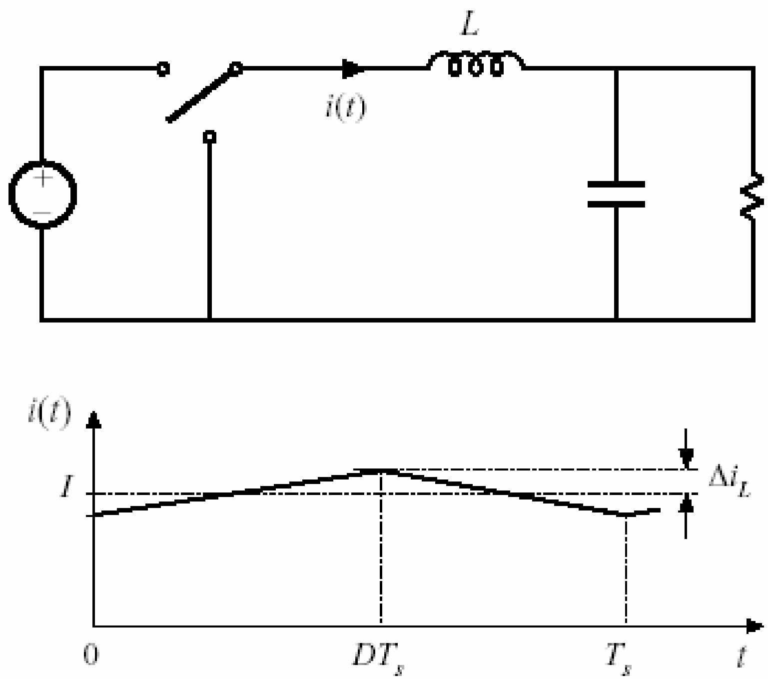
Types of magnetic devices:

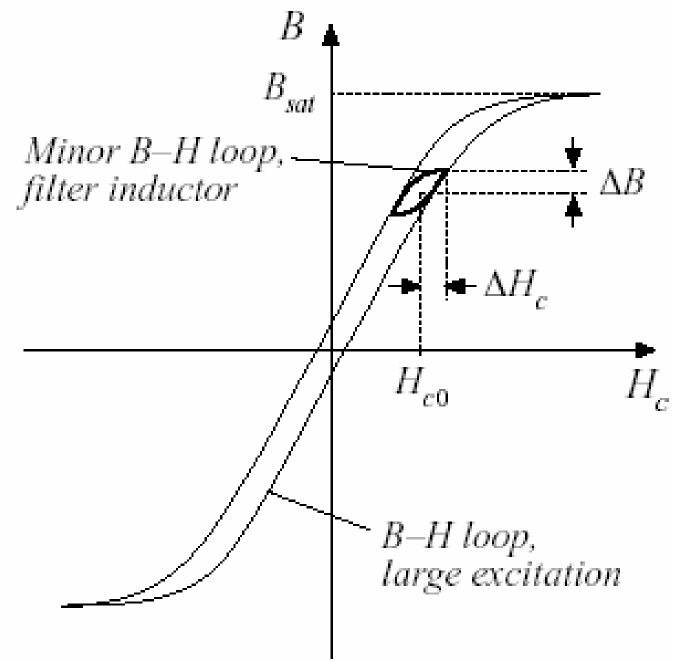
Filter inductor	AC inductor
Conventional transformer	Coupled inductor
Flyback transformer	
Magnetic amplifier	Saturable reactor

III. MAGNETICS DESIGN

Filter Inductor

CCM buck example



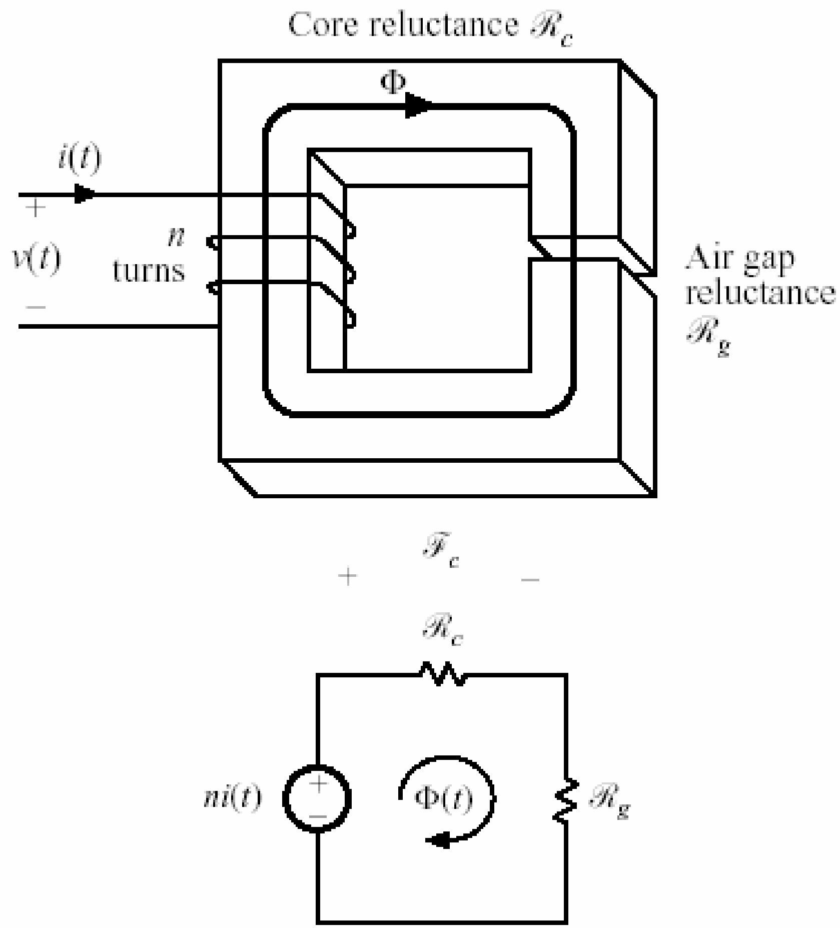


$$H_c(t) = \frac{n i(t)}{\ell_c} \frac{\mathcal{R}_c}{\mathcal{R}_c + \mathcal{R}_g}$$

III. MAGNETICS DESIGN

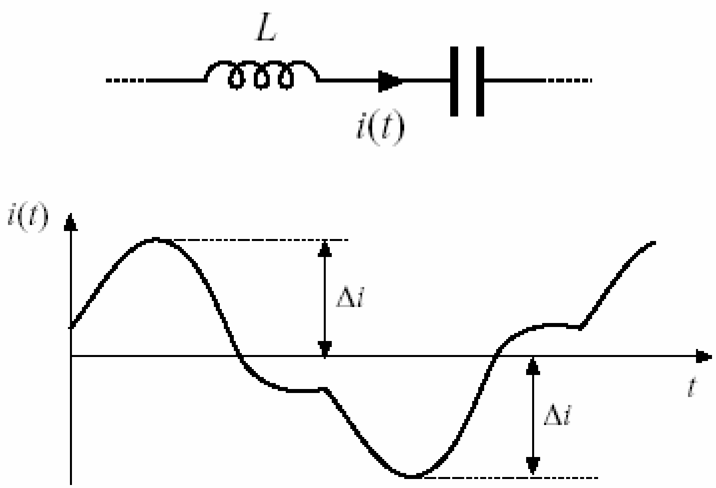
Filter Inductor, cont.

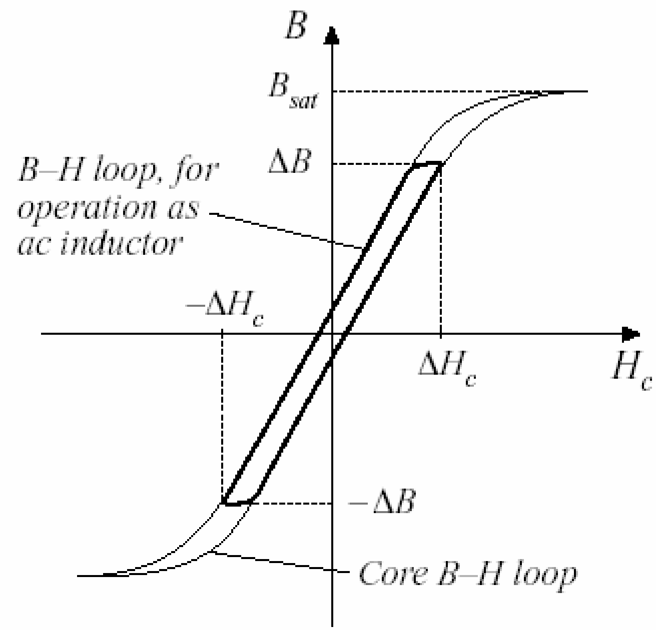
- Negligible core loss, negligible proximity loss
- Loss dominated by dc copper loss
- Flux density chosen simply to avoid saturation
- Air gap is employed
- Could use core materials having high saturation flux density (and relatively high core loss), even though converter switching frequency is high



III. MAGNETICS DESIGN

AC Inductor

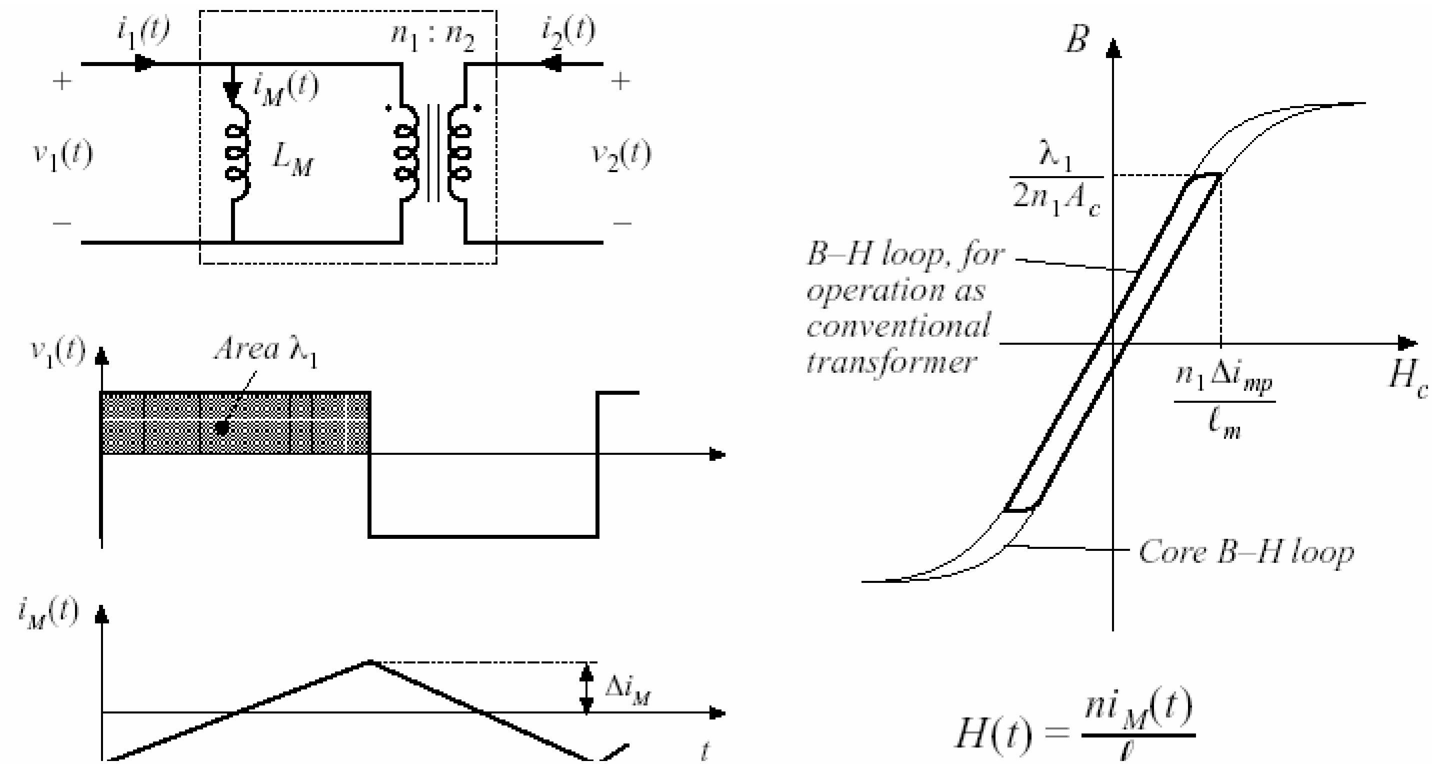




- Core loss, copper loss, proximity loss are all significant
- An air gap is employed
- Flux density is chosen to reduce core loss
- A high-frequency material (ferrite) must be employed

III. MAGNETICS DESIGN

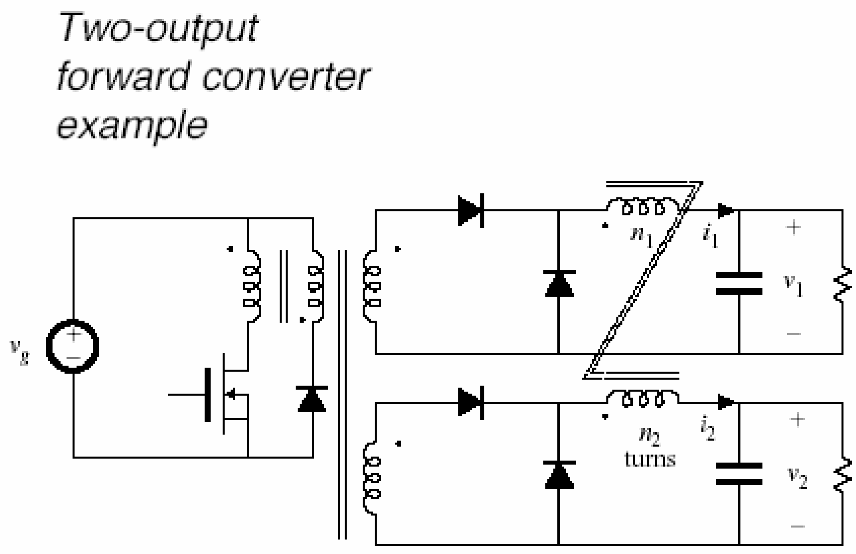
Conventional Transformer

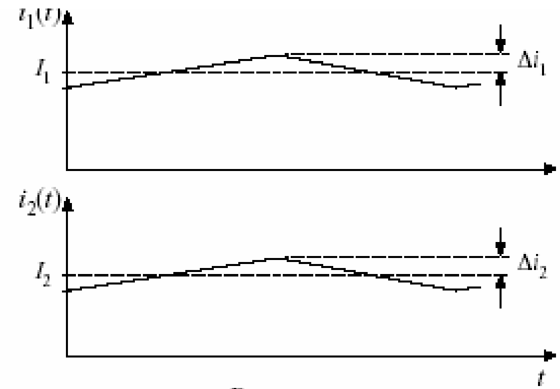


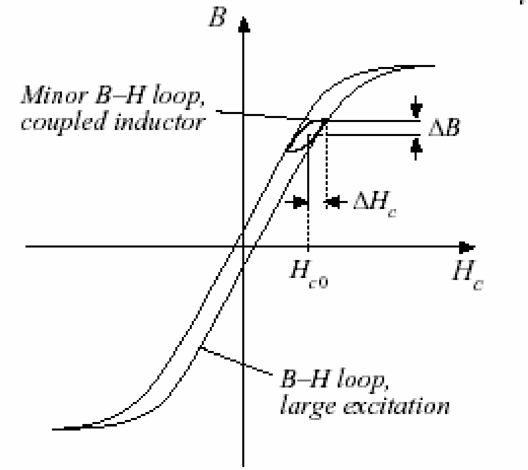
- Core loss, copper loss, and proximity loss are usually significant
- No air gap is employed
- Flux density is chosen to reduce core loss
- A high frequency material (ferrite) must be employed

III. MAGNETICS DESIGN

Coupled Inductor





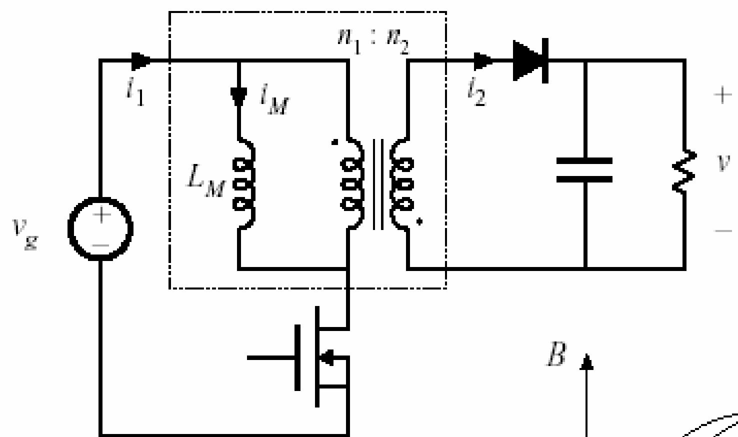


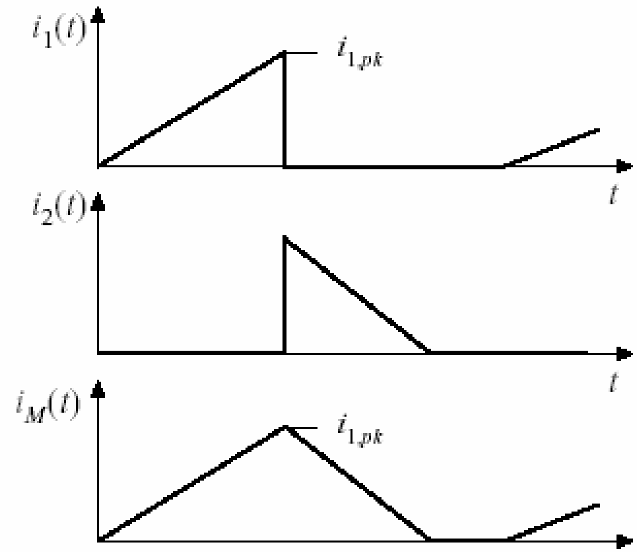
$$H_c(t) = \frac{n_1 i_1(t) + n_2 i_2(t)}{\ell_c} \frac{\mathcal{R}_c}{\mathcal{R}_c + \mathcal{R}_g}$$

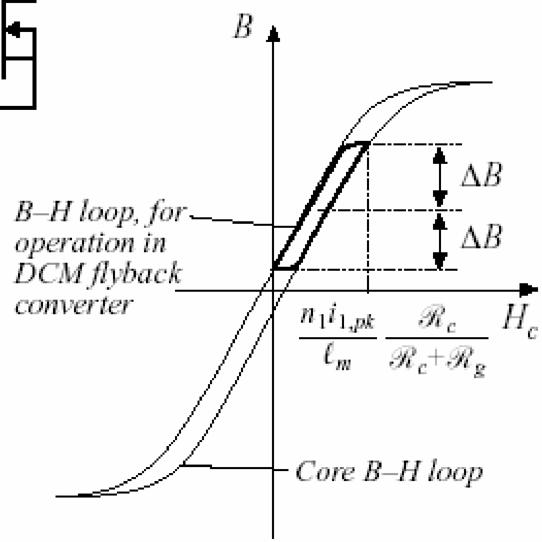
- A filter inductor having multiple windings
- Air gap is employed
- Core loss and proximity loss usually not significant
- Flux density chosen to avoid saturation
- Low-frequency core material can be employed

III. MAGNETICS DESIGN

DCM Flyback Transformer







- Core loss, copper loss, proximity loss are significant
- Flux density is chosen to reduce core loss
- Air gap is employed
- A high-frequency core material (ferrite) must be used

III. MAGNETICS DESIGN

Summary of key points

Air gaps are employed in inductors to prevent saturation when a given maximum current flows in the winding, and to stabilize the value of inductance. The inductor with air gap can be analyzed using a simple magnetic equivalent circuit, containing core and air gap reluctances and a source representing the winding MMF.

Conventional transformers can be modeled using sources representing the MMFs of each winding, and the core MMF. The core reluctance approaches zero in an ideal transformer. Nonzero core reluctance leads to an electrical transformer model containing a magnetizing inductance, effectively in parallel with the ideal transformer. Flux that does not link both windings, or "leakage flux," can be modeled using series inductors.

The conventional transformer saturates when the applied winding volt-seconds are too large. Addition of an air gap has no effect on saturation. Saturation can be prevented by increasing the core cross-sectional area, or by increasing the number of primary turns.

III. MAGNETICS DESIGN

Summary of key points

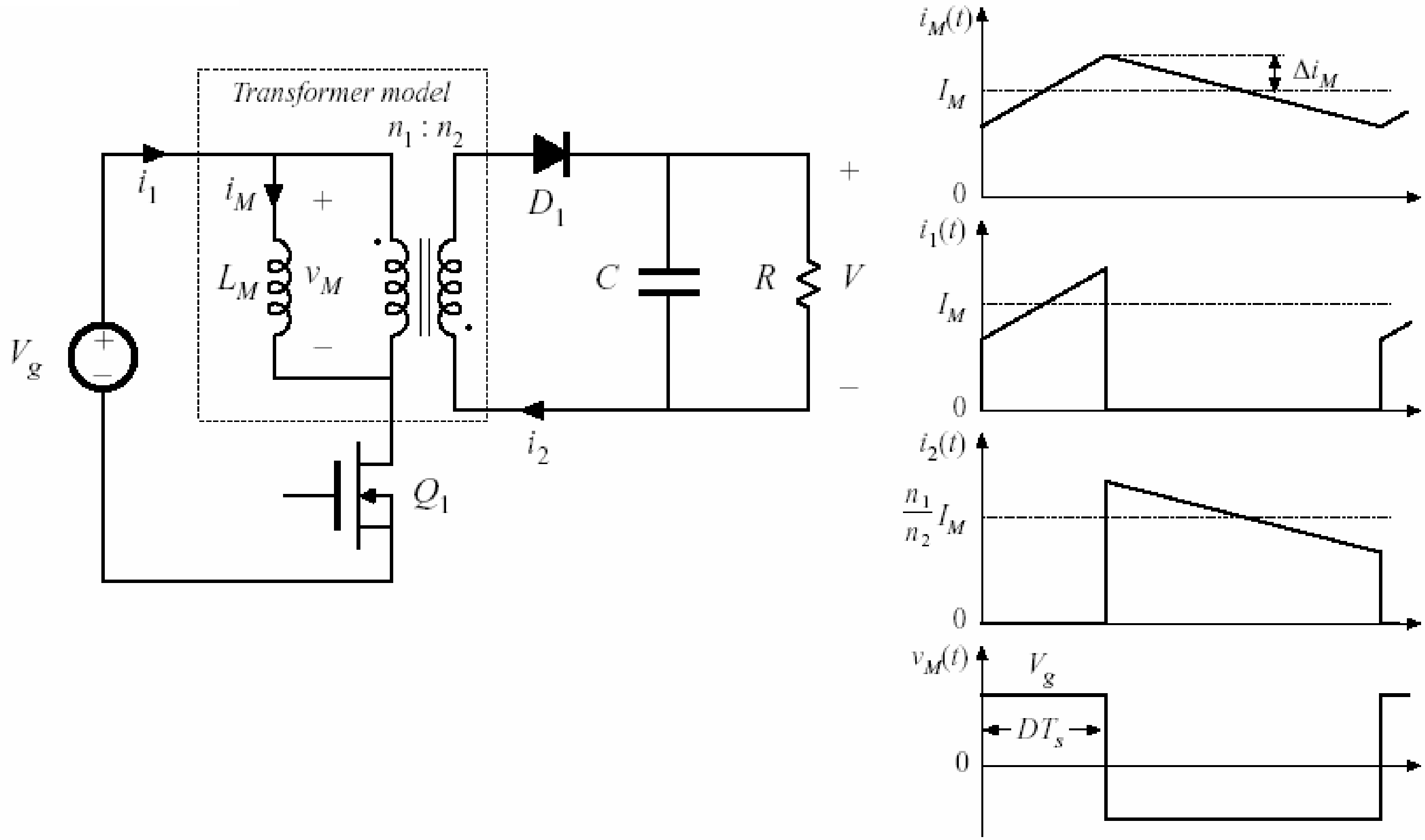
Magnetic materials exhibit core loss, due to hysteresis of the B–H loop and to induced eddy currents flowing in the core material. In available core materials, there is a tradeoff between high saturation flux density B_{sat} and high core loss P_{fe}. Laminated iron alloy cores exhibit the highest B_{sat} but also the highest P_{fe}, while ferrite cores exhibit the lowest P_{fe} but also the lowest B_{sat}. Between these two extremes are powdered iron alloy and amorphous alloy materials.

The skin and proximity effects lead to eddy currents in winding conductors, which increase the copper loss P_{cu} in high-current high-frequency magnetic devices. When a conductor has thickness approaching or larger than the penetration depth δ, magnetic fields in the vicinity of the conductor induce eddy currents in the conductor. According to Lenz's law, these eddy currents flow in paths that tend to oppose the applied magnetic fields.

The magnetic field strengths in the vicinity of the winding conductors can be determined by use of MMF diagrams. These diagrams are constructed by application of Ampere's law, following the closed paths of the magnetic field lines which pass near the winding conductors. Multiple-layer noninterleaved windings can exhibit high maximum MMFs, with resulting high eddy currents and high copper loss.

III. MAGNETICS DESIGN

Example : CCM flyback transformer



III. MAGNETICS DESIGN

Specifications

Input voltage	$V_g = 200\text{V}$
Output (full load)	20 V at 5 A
Switching frequency	150 kHz
Magnetizing current ripple	20% of dc magnetizing current
Duty cycle	$D = 0.4$
Turns ratio	$n_2/n_1 = 0.15$
Copper loss	1.5 W
Fill factor	$K_u = 0.3$
Maximum flux density	$B_{max} = 0.25\ \text{T}$

III. MAGNETICS DESIGN

Basic converter calculations

Components of magnetizing current, referred to primary:

$$I_M = \left(\frac{n_2}{n_1}\right)\frac{1}{D'}\frac{V}{R} = 1.25 \text{ A}$$

$$\Delta i_M = (20\%) I_M = 0.25 \text{ A}$$

$$I_{M,max} = I_M + \Delta i_M = 1.5 \text{ A}$$

Choose magnetizing inductance:

$$L_M = \frac{V_g D T_s}{2\Delta i_M}$$
$$= 1.07 \text{ mH}$$

RMS winding currents:

$$I_1 = I_M \sqrt{D}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 0.796 \text{ A}$$

$$I_2 = \frac{n_1}{n_2} I_M \sqrt{D'}\sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 6.50 \text{ A}$$

$$I_{tot} = I_1 + \frac{n_2}{n_1} I_2 = 1.77 \text{ A}$$

The smallest EE core that satisfies this inequality (Appendix D) is the EE30.

III. MAGNETICS DESIGN

Choose air gap and turns

$$\ell_g = \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4$$
$$= \frac{\left(4\pi \cdot 10^{-7}\text{H/m}\right)\left(1.07 \cdot 10^{-3}\text{ H}\right)\left(1.5\text{ A}\right)^2}{\left(0.25\text{ T}\right)^2\left(1.09\text{ cm}^2\right)} 10^4$$
$$= 0.44\text{ mm}$$

$$n_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4$$
$$= \frac{\left(1.07 \cdot 10^{-3}\text{ H}\right)\left(1.5\text{ A}\right)}{\left(0.25\text{ T}\right)\left(1.09\text{ cm}^2\right)} 10^4$$
$$= 58.7\text{ turns}$$

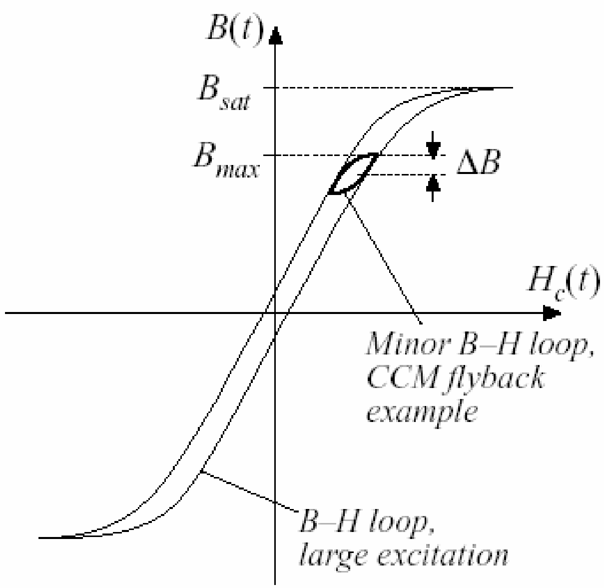
$$n_2 = \left(\frac{n_2}{n_1}\right) n_1$$
$$= \left(0.15\right) 59$$
$$= 8.81$$

Round to $n_1 = 59$ $n_2 = 9$

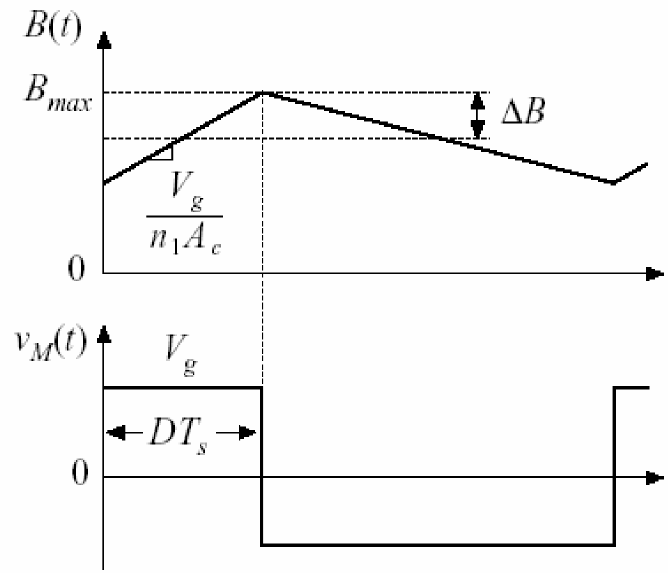
Core loss

CCM flyback example

B-H loop for this application:



The relevant waveforms:



$B(t)$ vs. applied voltage, from Faraday's law:

$$\frac{dB(t)}{dt} = \frac{v_M(t)}{n_1 A_c}$$

For the first subinterval:

$$\frac{dB(t)}{dt} = \frac{V_g}{n_1 A_c}$$

Calculation of ac flux density and core loss

Solve for ΔB:

$$\Delta B = \left(\frac{V_g}{n_1 A_c}\right)\left(DT_s\right)$$

Plug in values for flyback example:

$$\Delta B = \frac{\left(200\ \mathrm{V}\right)\left(0.4\right)\left(6.67\ \mu\mathrm{s}\right)}{2\left(59\right)\left(1.09\ \mathrm{cm}^2\right)}\,10^4$$

$$= 0.041\ \mathrm{T}$$

From manufacturer's plot of core loss (at left), the power loss density is 0.04 W/cm³. Hence core loss is

$$P_{fe} = \left(0.04\ \mathrm{W/cm}^3\right)\left(A_c \ell_m\right)$$

$$= \left(0.04\ \mathrm{W/cm}^3\right)\left(1.09\ \mathrm{cm}^2\right)\left(5.77\ \mathrm{cm}\right)$$

$$= 0.25\ \mathrm{W}$$