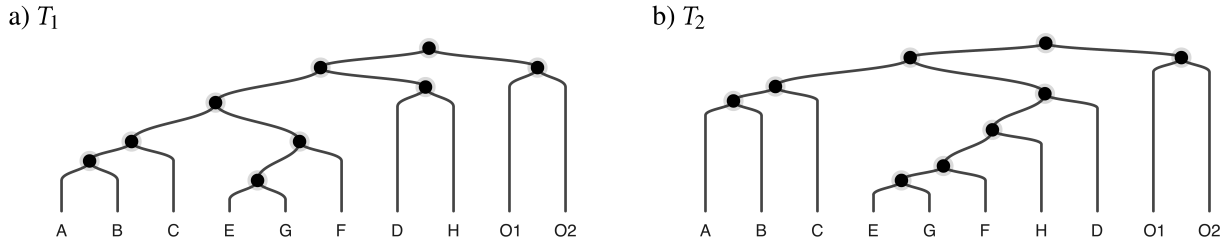
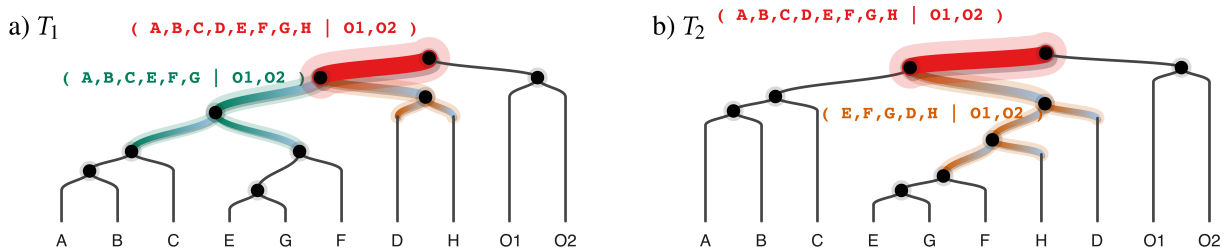


**Figure A: Input Trees for Comparison**



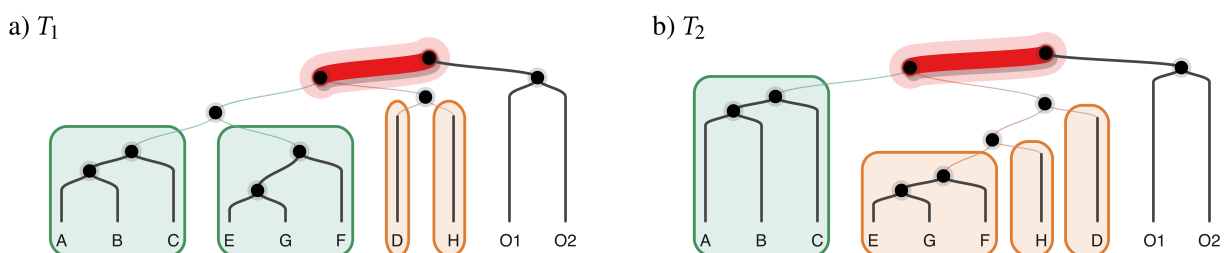
This figure presents the starting point of our tree comparison analysis, showing two phylogenetic trees,  $T_1$  and  $T_2$ , in their standard representation without any special highlighting. Both trees contain the same taxa (labeled A through H, plus outgroups O1 and O2), but with different topological relationships between them. At first glance, identifying the structural differences between these trees is challenging due to their complexity and the rearrangement of multiple branches.

**Figure B: Identifying Lattice Edges**



The thick red branches represent split points that are identical in both trees – these form the common split (A, B, C, D, E, F, G, H | O1, O2) preserved across both phylogenetic trees. Connected to these common splits are branches with gradient coloring (teal/green in  $T_1$  and orange in  $T_2$ ), which represent the unique structural differences between the trees. In  $T_1$ , the unique splits include: The teal/green on the left: (A, B, C, E, F, G | D, H) Further divisions into teal/green (A, B, C) and (E, F, G) groups The teal/green split: (D, H | A, B, C, E, F, G, O1, O2) In  $T_2$ , the unique splits include: The orange partition: (E, F, G, D, H | A, B, C) Further subdivisions into orange (E, F, G, H | D) And the split between orange (E, F, G) | H These connection zones – where common splits interface with unique branches – are the critical regions where tree transformation occurs during interpolation.

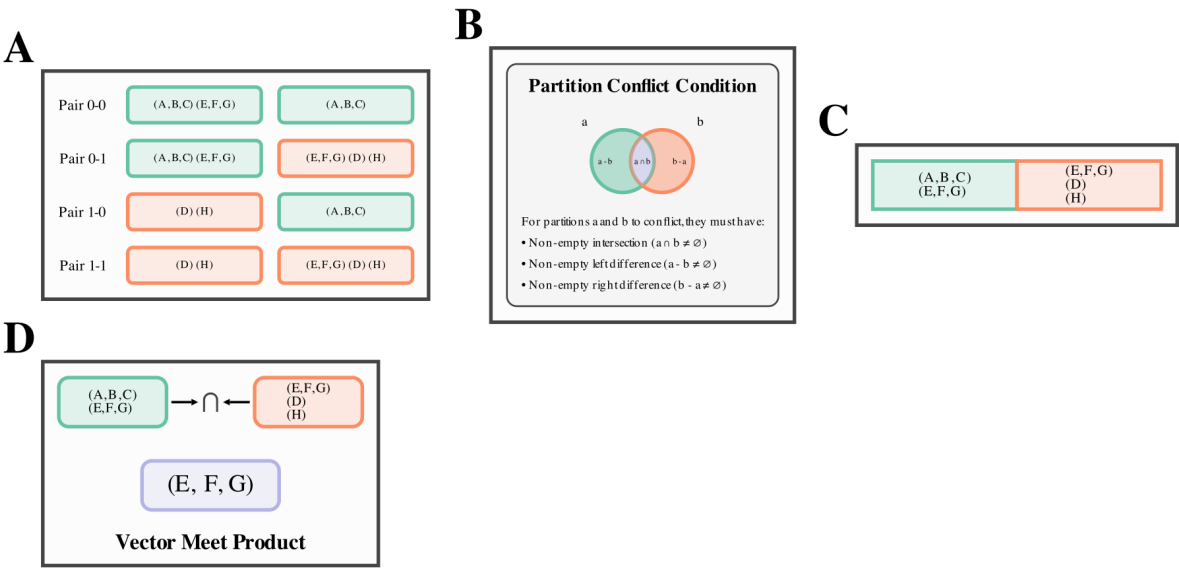
**Figure C: Analyzing Partition Sets (Atoms) within Lattice Edges**



This figure shows how we find the conflicting partition sets. The teal/green enclosures in  $T_1$  and orange enclosures in  $T_2$  represent distinct partition elements that form the minimal separating sets in each structure. Each enclosed region maps to a fundamental mathematical unit in the lattice edge's computational representation. These visual groupings— $\{A, B, C\}$  and  $\{E, F, G\}$  in  $T_1$  versus  $\{E, F, G\}$  and separate single-element partitions  $\{D\}$  and  $\{H\}$  in  $T_2$ —highlight the set-theoretic differences between the graph structures. The algorithm identifies these partition sets through set-theoretic operations that isolate topologically significant element groupings. By highlighting these abstract structural differences, we provide a direct visualization of the mathematical foundation underlying the lattice transformation process between the two structures.

**Figure D: Analyzing Partition Sets (Atoms) within Lattice Edges**

Lattice Edge Processing: Mapping between cover sets in trees  $T_1$  and  $T_2$



Detailed visualization of the computational steps: **(A)** Partition pairs extracted from input structures, **(B)** Mathematical conflict condition formalized as set relationships ( $a \cap b \neq \emptyset, a - b \neq \emptyset, b - a \neq \emptyset$ ), **(C)** Candidate conflict matrix with computational row highlighting, and **(D)** Vector meet product operation demonstrating the set intersection calculation central to determining partition conflicts in the algorithm.