CAUSAL AI IN tnreason

RESEARCH NOTES IN THE ENEXA PROJECT

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1 Intervention Queries

In a Bayesian Network, when intervening on a variable X_v, the corresponding conditional probability gets trivialized. The probability tensor is then captured by

$$\mathbb{P}\left[X_{\tilde{\mathcal{V}}}|\mathrm{do}\left(X_{\bar{\mathcal{V}}}=x_{\bar{\mathcal{V}}}\right)\right]=\left\langle \{\mathbb{P}\left[X_v|X_{\mathrm{Pa}(v)}\right]\,:\,v\notin\bar{\mathcal{V}}\}\cup\{\frac{1}{m_v}\mathbb{I}\left[X_v,X_{\mathrm{Pa}(v)}\right]\,:\,v\in\bar{\mathcal{V}}\}\cup\{e_{x_v}\left[X_v\right]\,:\,v\in\bar{\mathcal{V}}\}\right\rangle\left[X_{\tilde{\mathcal{V}}}\right].$$

Since $\frac{1}{m_v}\mathbb{I}\left[X_v,X_{\mathrm{Pa}(v)}\right]$ is directed with $X_{\mathrm{Pa}(v)}$ incoming and X_v outgoing, the partition function of the network stays 1 and the normation is captured by the contraction.

1.1 Intervention Variables

We modify conditional probability cores in Bayesian Networks to capture interventions, by introducing intervention variable

$$T\left[X_v,X_{\mathrm{Pa}(v)},D_v\right]=\mathbb{P}\left[X_v|X_{\mathrm{Pa}(v)}\right]\otimes e_0\left[D_v\right]+\mathbb{I}\left[X_v,X_{\mathrm{Pa}(v)}\right]\otimes e_1\left[D_v\right]$$

1.2 Simplifications

The back-door and front-door criterion provide conditions for intervention queries being equal to conditional queries. We can proof them in the tensor network formalism based on network separations, which contribution to contractions are scalar multiplications, which therefore are dropped in normations.

2 Learning

Given a parametrization of hypothesis distribution by the states of selection variables, we can include the intervention variables. The cores to be trivialized by intervention can depend on the state of the selection variables. For example, a selection variable might select whether the cores direction is from X_0 to X_1 or in the other direction. In both cases only one of the to intervention variables D_0 and D_1 influences the core.