tnreason FOR PARITY-CHECK CODES

RESEARCH NOTES IN THE ENEXA PROJECT

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Parity-Check Codes encode a set of bits with the help of their parity

$$y = \bigoplus_{v \in \mathcal{V}} x_v$$

where $\oplus$ denoted summation in mod2.

Denoting the parity bit by Y the relation is expressed as

$$\rho^{\oplus\,|\nu} \left[Y, X_{\mathcal{V}}\right] = \sum_{\times_{v \in \mathcal{V}} [m_v]} e_{\bigoplus_{v \in \mathcal{V}} x_v} \left[Y\right] \otimes e_{x_{\mathcal{V}}} \left[X_{\mathcal{V}}\right] .$$

Efficient tensor network decompositions can be derived from the commutativity of $\oplus$, which enables decomposing $\rho^{\oplus\,|\nu}$ into a network of smaller parity calculating tensors. The smallest number of tensor cores can thereby be achieved in an HT architecture.