
MESSAGE PASSING IN LOGICAL INFERENCE

RESEARCH NOTES IN THE ENEXA PROJECT

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1 Unit Clause Propagation (UCP)

Unit clauses are assignments to single variables. They are propagated between clusters of formulas

1.1 Forward Chaining for Horn-SAT

Forward chaining is a linear time and complete satisfiability checker on Horn Logic (a subset of Propositional Logic where each clause has at most one positive literal).

- Messages passed are the one-hot encodings of assignment to single variables (unit clauses).
- Clusters are the Horn clauses, which receive messages for their negative literals. A Cluster sends a nontrivial message to a variable, if the variable is the only unassigned literal in the clause and the clause has not been satisfied yet.

1.2 UCP for 2-SAT

2-SAT is in P and can be solved by the message passing algorithm Unit Clause Propagation (UCP).

- Messages passed are the one-hot encodings of assignment to single variables (unit clauses).
- Any clause that receives a message has only a single literal left and either gets directly trivial (if the message coincides with the literal) or assigns the remaining variable and passes further.

1.3 DPLL for generic SAT

DPLL combines backtracking search with unit clause propagation (UCP). A form of message passing is applied to reduce the clauses given the current partial assignment: When guessed an assignment to a variable, the variable is removed from all clauses, either making the clause trivial (coinciding assignment) or smaller. If only one literal remains in a clause, the variable would be assigned accordingly (unit propagation). This can be directly done in the intermediate message passing scheme, or understood as the next backtracking step ("Find-Unit-Clause" in Figure 7.17 in ?).

2 WalkSAT

WalkSAT is a stochastic local search algorithm for SAT. It starts with a random assignment and iteratively flips variables to reduce the number of unsatisfied clauses. This can be understood as a (modified) Gibbs sampling algorithm, where the number of unsatisfied clauses is the energy function to be minimized. The modifications are:

- Selection of variable to be resampled: Typically chosen by looking at unsatisfied clauses and picking a variable that minimizes the number of newly unsatisfied clauses (whereas in Gibbs sampling one follows a fixed variable order).
- Marginal probability: Typically fixed by a mixing parameter, whereas in Gibbs sampling would be sensitive to the energy differences.