tnreason FOR PORTFOLIO OPTIMIZATION

RESEARCH NOTES IN THE ENEXA PROJECT

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1 The portfolio optimization problem

Given n products and the covariance matrices $\Sigma[L_0, L_1] \in \mathbb{R}^{d\times d}$, the expectations $\mu[L_0]$ and the risk appetite parameter $q > 0$, we state the portfolio optimization problem

$$\operatorname{argmin}_{x_{[d]}} q \left(\sum_{l_0,l_1\in[n],\, l_0\neq l_1} \Sigma[L_0 = l_0, L_1 = l_1] \cdot e_1\left[X_{l_0}\right] \cdot e_1\left[X_{l_1}\right] \right) - \sum_{l\in[n]} \mu[L = l] \cdot e_1\left[X_l\right] . \tag{1}$$

Remark 1 (Connection with tnreason)**.**

- *The portfolio optimization problem is a mode search in an exponential family of atomic variables and terms of pairs.*
- *The objective of the optimization is the (negative of its) energy tensor E.*
- *It has a monomial decomposition with order constraint $r = 2$.*

2 Connection with tnreason

Let us now show the connection with tnreason in more precision.

2.1 Exponential family of features

The portfolio optimization problem is a mode search in an exponential family of atomic variables and terms of pairs.

2.2 Energy Tensor and corresponding distribution

The energy tensor to be maximized is

$$E = -q \left(\sum_{l_0,l_1\in[n],\, l_0\neq l_1} \Sigma[L_0 = l_0, L_1 = l_1] \cdot e_1\left[X_{l_0}\right] \cdot e_1\left[X_{l_1}\right] \right) + \sum_{l\in[n]} \mu[L = l] \cdot e_1\left[X_l\right] .$$

We can reconstruct a member of the corresponding Markov Logic Network to the by the temperature $\beta > 0$ scaled energy as

$$\mathbb{P}^{\beta}\left[X_{[d]}\right] = \langle \exp\left[\beta \cdot E\right] \rangle \left[X_{[d]} | \varnothing\right] .$$

Sampling from these members in the low temperature limit $\beta \to 0$ is equivalent to the portfolio optimization problem.

2.3 Representation by a monomial selection architecture

The energy tensor E has a monomial decomposition with order constraint $r = 2$.