Pricing Strategies for Online Dating Platforms

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Online dating is the most common way for new couples to meet, with three-in-ten Americans having used dating apps, and revenues from dating apps swelling to more than five billion annually. The majority of these dating apps earn revenue via subscription based pricing, where subscriptions for access to the app are sold at a fixed price. Subscription based pricing is a ubiquitous way to monetize mobile apps, however in the context of online dating is controversial as it potentially misaligns the incentives of the platform and its users. Another, less popular but more traditional monetization strategy is the contract based model, in which the dating app is contracted by the user to facilitate a search for a partner at some agreed upon one time price. The purpose of this work is to understand the profit and welfare trade-offs associated with either pricing strategy for online dating platforms.

We present a natural and novel model for the operation of an online dating platform. In our model, we show that subscription pricing always achieves at least 36.7% of the profit earned by contract pricing for all market parameters, explaining its prevalence in practice. We then take a fine-grained approach and establish profit dominance relations between the two strategies when the marginal cost of operation is small or large, respectively. We show that in online settings contract pricing is guaranteed to yield higher profit. Further, under a natural slow matching condition, we show that in online settings profit maximizing contract pricing leads to a higher percentage of the user-base getting matched. Finally, we show that contract pricing allows the platform to incorporate user preference information in a way that aligns the interest of the platform and user, solving the potential incentive issues that plague subscription pricing.

Key words: online dating, matchmaking, subscription, contract, pricing
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1. Introduction

An increasing share of people, especially young people, are meeting and falling in love online. Over the past two decades, the internet has displaced conventional mediums such as family, school, or the workplace, to become the most common way for new couples to meet (Shashkevich 2019). As of 2019, three-in-ten U.S. adults say they have used a dating site or app before, and that percentage rises to 48% for 18-to-29 year-olds (Pew Research Center 2020b). Further, dating apps have been enormously successful in connecting people left out of traditional dating culture. A full two-thirds of lesbian, gay, or bisexual Americans report using dating apps (Pew Research Center 2020a) and, with the COVID-19 pandemic severely restricting the venues in which people can meet, many leading dating apps are reporting record numbers of users and subscribers (Meisenzahl 2020).

The companies that run these apps have in turn grown with the increased demand. The dating services industry has swelled over the last five years, with an annualized growth rate of 12.9% (IBISWorld 2021), and revenues projected to rise 9.3% in 2021 to $5.3 billion as mobile services expand. In the United States, dating services are largely consolidated under one corporate entity, the Match Group, which as of 2020, is estimated to have cornered 60% of the dating app market with its suite of apps, including Tinder, Hinge, OkCupid, and Match (Meisenzahl 2020).

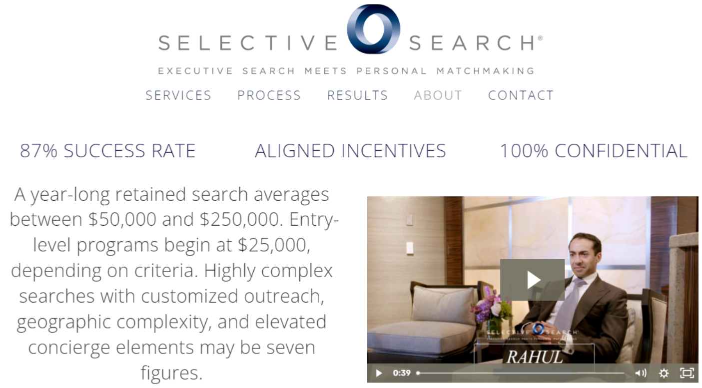
The majority of dating apps in the United States, including all apps owned by the Match Group, primarily follow a subscription based revenue model where users pay a subscription fee per period to use the platform[1] (e.g. month, see Fig. 1). While subscription based revenue models are an extremely common way to monetize apps, their use for dating platforms specifically is questionable. In the subscription model, the matchmaker's interest inherently conflicts with users'. As (Wu et al. 2019) note, a profit-maximizing subscription based dating app will strive to retain all of its subscribers, however upon finding a compatible partner, users will terminate their subscription and leave the platform. Thus, the platform has an incentive not to provide the best potential matches to their users. Framed another way, if the subscription based platform could divine a user's perfect match, they would have a fiduciary obligation to never reveal that match to the user. Of course, the

[1] Many of these apps have a paid and free version of the platform, where the unpaid version of site has reduced search features and requires users to see advertisements. In the free version of the site, the subscription price the user pays is equivalently the value of the advertisement displayed.

Figure 1 Examples of subscription and contract pricing in online dating platforms.

Membership	Term	Price Per Month
Basic	~	Free
Premium	1 month	$29.95
Premium	3 months	$19.98
Premium	6 months	$12.49
Premium	12 months	$12.49

zoosk



Note. Depicted are pricing descriptions for two online dating platforms. On the left is an example of subscription price at `https://www.zoosk.com/`. On the right is an example of contract pricing at `https://www.selectivesearch.com/pricing`.

subscription based model is not the only way to monetize the services of a dating app. Another, less popular but more traditional monetization strategy is the contract based model, in which the dating app is contracted by the user to facilitate a search for a partner at some agreed upon price. Traditional (offline) matchmakers in India and China typically use some forms of contract prices (Larmer 2013). In the context of online dating, some high-end matchmaking sites, such as `selectivesearch.com`, have already adopted the contract based model. In the pricing section of the selective service site[2] they state "*While most dating apps and services are incentivized to keep members paying ongoing fees, Selective Search works with clients through a defined contract. Each contract is for a finite number of introductions over a defined period of time.*". Ostensibly, the contract based model can align the interests of matchmakers and users. In Fig. 1 examples of both subscription priced and contract priced dating apps are shown for reference.

In light of the above, in this work we study the operations of a dating platform run by a profit maximizing monopolist engaged in one of two pricing strategies: (i) *subscription pricing* (SP), where the matchmaker commits to a fixed price p, and users pay the price in a continuous fashion until they leave the platform, and (ii) *contract pricing* (CP), where the matchmaker commits to a fixed, one time price p. (SP) is very flexible, and requires no commitment between the platform and user making it easy to implement, understand, and deploy, even if it leads to confusing incentives. (CP) is less flexible, especially in

[2] `https://www.selectivesearch.com/pricing`

the online setting where it requires a large upfront payment, and specific mechanics to enforce that the contracts are carried out in good faith. To illustrate the point, suppose the contract requires the user to pay the price upfront, then without some sort of regulation or commitment mechanism the platform extracts full profit from the user at the outset and thus has no incentive to work on their behalf. If the contract requires the user to pay only after meeting their match, then the matchmaker has the difficult problem of enforcing payment at the end of the user's time on the platform.

Given the advantages and disadvantages of both (SP) and (CP) for online dating platforms, the purpose of this work will be to understand the profit and welfare trade-offs associated with each. A summary of our key contributions and findings is as follows:

1. We give a novel, natural model to describe the operations of an online dating platform. In our model, we characterize the optimal profit a matchmaker can obtain under two important pricing paradigms, subscription pricing and contract pricing. We then study the relative profits which can be achieved, and prove tight bounds on the profit ratio of (SP) to (CP). Specifically, we show (SP) always achieves at least $1/e$ of the profit earned by (CP) (c.f. Theorem 1), offering a principled justification for the ubiquity of (SP) in practice. We also identify two counter-intuitive instances when the profit of subscription pricing or contract pricing dominates the other, namely when costs are vanishingly small, or sufficiently large (c.f. Theorem 2).
2. We next study the implications of the choice of (SP) or (CP) on the users of the platform. We look at which pricing strategy, when optimized to maximize profit, leads to a higher proportion of matches among the user-base. When model parameters satisfy a natural slow matching condition, we show a sharp relationship between the optimal subscription price and the optimal contract price (c.f. Lemma 3). Using this price characterization, when marginal costs are low (as may be assumed in an online setting) we prove that not only is (SP) less profitable than (CP), but also a smaller percentage of the user-base is matched under (SP) compared to (CP) (c.f. Theorem 3). Thus in online settings the use of contract pricing exhibits a win-win for the platform and the users.
3. Finally, we consider the scenario where the platform can incorporate heterogeneous potential match information to vary the matching rate. We show under (SP), the profit maximizing matchmaker offers the users the worst possible potential matches

to keep them on the platform longer, whereas under (CP), the situation is reversed, formalizing the intuition of Wu et al. (2019). Moreover, not only does (CP) create an incentive for the platform to match the user as quickly as possible, the ability to learn user preferences also induces the platform to offer a lower contract price (c.f. Theorem 4).

1.1. Literature Review

Our work is related to several streams of literature in economics, computer science, and operations. Here, we overview some of these streams and connect them to our work.

Platform design in operations management Our work contributes to a deep literature dealing with aspects of platform design using models from operations management. The most relevant paper to our work is (Wu et al. 2018), who study competing matchmakers in a two-period, two-user model with Hotelling valuations. They model dating platforms as strategically investing in matching technologies, investigating the interaction between competition, and providing the best service for their users. (Wu et al. 2018) does not resolve the suitor-matchmaker incentive issues but does argue that via competition and perfect information about match quality, matchmaking platforms can be induced to act in the agents' best interests. (Ellison and Ellison 2009, Ellison and Wolitzky 2012, Dukes and Liu 2016, Halaburda et al. 2018, Basu et al. 2019) also study online platform incentives to provide less-than-perfect services. As the US dating market is largely non-competitive (Gilbert 2019), we instead focus on changing the pricing structure itself to address incentive issues, and in a significantly more general model.

Pricing strategy in operations management The closest to our paper, in terms of the framework and style of pricing strategy analysis, is (Ladas et al. 2021). The authors also consider two business models, pay-per-use selling, and product selling, which roughly correspond to our subscription pricing and contract pricing, respectively. They focus primarily on an equilibria analysis of the business model choice under duopoly. Their work explores the scenarios in which the pay-per-use model is more profitable than product selling. Other analyses of pay-per-use selling and product selling can be seen in (Varian 2000, Sundararajan 2004, Agrawal et al. 2012, Balasubramanian et al. 2015). Analysis of similar business models can be seen in (Niculescu and Wu 2014). In our work, we analyze the pricing strategies of profit maximizing monopolists.

Approximation analysis for online platforms To understand the optimal design of matching platforms, (Kanoria and Saban 2021) introduces a stylized dynamic fluid model for the two-sided matching with strategic agents. They find, in unbalanced markets, the platform should force the short side to initiate contact with potential partners, therefore mitigating wasted searching effort. However, they focus on a setting without prices. (Johari et al. 2019, Immorlica et al. 2021) investigate the information disclosure problem for online platforms in two-sided matching markets. (Aouad and Saritaç 2020, Aouad and Saban 2021) shift their interests to optimization for online matching platforms in dynamic settings.

Matchmaking in other markets Outside of dating/marriage, online matchmaking is also a fundamental problem for labor markets (Bimpikis et al. 2020, Belavina et al. 2020) and in the video game industry. (Chen et al. 2021) study the problem of maximizing player engagement in video games through improved matchmaking. They focus on a stylized model with different skill levels of players, and where winning or losing influences the players' willingness to stay on the platform. (Chen et al. 2017, Huang et al. 2019, Deng et al. 2021) also investigate how to improve players' engagement through matching.

1.2. Paper Outline

In Section 2, we introduce our notation and provide preliminary results about (SP) and (CP). In Section 3, we compare the achievable profit of the two selling strategies. In Section 4 we study the implications of committing to (SP) or (CP) for the platform user-base. Finally, in Section 5 we discuss the implications of our work for matchmakers, users, and regulators, and highlight interesting avenues for future research.

2. Model and Preliminaries

We consider a profit maximizing matchmaker running an online platform for users seeking permanent partners, which we refer to as a *match*[3]. A random user's valuation for matching is described by a non-negative random variable V drawn from a distribution F with density f. We use the notation $\overline{F}(x) := 1 - F(x)$ to denote the survival function. We model the matching process as a continuous stream of interactions with potential match candidates,

[3] In online dating contexts, a match can refer to a candidate partner for the user, or someone who's shown preliminary interest but has not met the user. In this work, we will use match to mean the successful formation of a long-term partnership culminating in a departure from the platform.

indexed by t, all with homogeneous match rate q, such that, over any period of time Δ on the platform, the probability of a user matching is an exponential random variable[4] with rate q, *i.e.*, $\Pr(\text{Match} \in [t, t+\Delta)) = 1 - e^{-q\Delta}$. This formulation is the natural continuous time model for the operation of swipe apps like Tinder, Hinge, or Bumble, where users can continuously swipe through platform-prepared candidate matches. We assume users are time-sensitive and their valuation for matching decays at a constant rate $\delta \in (0,1)$ as they spend time on the platform, so user's valuation for matching after t time on the platform is $V_t = \delta^t V$.

On the platform side, we assume the matchmaker commits to some fixed pricing strategy, either a continuously charged subscription price or a one-time contracted price. In either case, we assume users are rational, and will pay the price (potentially in a continuous fashion) if their expected utility from payment is non-negative. The platform prepares potential matches for the user at marginal operating cost c, paid continuously throughout the user's time on the platform. We assume the length of time the user can spend on the platform is upper bounded by T, which can be thought of as the size of the pool of potential matches (T may be nearly infinite in a city, or quite restrictive outside of cities), or as the maximum possible time the user can spend on the platform. When a user reaches time T on the platform they leave, and we say the platform has been *exhausted*.

We consider two pricing strategies which we term subscription pricing (SP), where the user continuously pays a subscription price to participate on the platform, and contract pricing (CP), where the user and platform enter into a contract in which the user pays a fixed fee, and in return uses the platform until matched, or the platform is exhausted. We now formally describe the matchmakers' pricing strategies. We characterize their profit in terms of model parameters in Section 2.1.

Subscription Pricing (SP): In *subscription* pricing, the platform commits to a fixed price p and each user pays the price in a continuous fashion until they either match, their expected utility from further time on the platform drops below 0, or the platform is exhausted. Let $\mathcal{R}_{SP}(p,c,F)$ be the expected profit the matchmaker earns using subscription price p, then

$$\mathcal{R}_{SP}(p,c,F) := (p-c)\,\mathbb{E}\left[\text{Time on platform}|(\text{SP})\right].$$

[4] For a discrete matching process, if the probability of matching in each period is q, then a user's time on the platform is geometrically distributed. The exponential distribution is the continuous analogue of the geometric distribution of the discrete matching process.

Let $\mathcal{R}_{SP}(c,F) := \max_p \mathcal{R}_{SP}(p,c,F)$ denote the maximum achievable profit under subscription pricing.

Contract Pricing (CP): In *contract* pricing, if the user agrees to a fixed, one-time payment p, the matchmaker commits to displaying potential matches until the user matches and leaves, or exhausts the platform. Let $\mathcal{R}_{CP}(p,c,F)$ be the expected profit the matchmaker earns using the contract price p, *i.e.*,

$$\mathcal{R}_{CP}(p,c,F) := p - c \times \mathbb{E}\left[\text{Time on platform}|(\text{CP})\right].$$

Let $\mathcal{R}_{CP}(c,F) := \max_p \mathcal{R}_{CP}(p,c,F)$ denote the maximum achievable profit under contract pricing.

Note, (SP) and (CP) represent two ends of the spectrum of pricing strategies parameterized by the time commitment required of the user at payment. In (SP), the user pays a continuous subscription rate and can immediately stop payment at any time, and thus no commitment is required on the part of the user or matchmaker. In (CP), a user contracts with the platform to pay a one-time price for unlimited access to the platform until they either match or exhaust the candidate pool, requiring full commitment from the user if the payment is made upfront, or full commitment from the platform if the payment is made at the end. For the purposes of our model, the timing of the payment via the contract is irrelevant.

2.1. Preliminaries

In this subsection, we give two expressions for the profits of (SP) and (CP), in terms of the model primitives. These expressions follow from integral representations for their profit based on value differentials and time differentials, respectively. We then define conditions for when the profit maximizing prices are unique for each strategy. Finally, we define a useful market condition used repeatedly throughout the work.

Profit of Subscription Pricing.

To compute the profit for subscription pricing, first consider a user with fixed valuation v facing a fixed subscription price p. To derive when a user will pay the continuously charged subscription price p, fix a discrete period of length Δ. The user will pay $p\Delta$ at time t to use the platform for Δ more time if $p\Delta \le \int_0^\Delta v\delta^{t+x} q e^{-qx} dx$, where $\int_0^\Delta v\delta^{t+x} q e^{-qx} dx$ is user's expected valuation for getting matched in the period. Dividing through and letting Δ tend to 0, the limiting condition is then,

$$p \leq \lim_{\Delta \to 0} \frac{\int_0^{\Delta} v\delta^{t+x} q e^{-qx} dx}{\Delta} = v\delta^t q. \tag{1}$$

Eq. (1) is the stopping criteria for the user. The user will stay on the platform until they either match and leave, which is distributed as an exponential random variable with rate q, or until their valuation falls to the point Eq. (1) is not satisfied which occurs at $\tau := \max\{\frac{\log\left(\frac{p}{vq}\right)}{\log(\delta)}, 0\}$, or until they exhaust the platform at time T. Let $X_S(v,p)$ be the random variable representing the users time on the platform when the subscription price is p. Note $X_S(v,p) = \min\{\text{Exp}(q), \tau, T\}$, and the expectation over the exponential matching randomness is then,

$$\begin{aligned}
\mathbb{E}[X_S(v,p)] &= \int_0^{\min\{\tau,T\}} qte^{-qt} dt + \min\{\tau,T\} e^{-q\min\{\tau,T\}} \\
&= \frac{1 - e^{-q\min\{\tau,T\}}(1 + q\min\{\tau,T\})}{q} + \min\{\tau,T\} e^{-q\min\{\tau,T\}} \\
&= \frac{1 - e^{-q\min\{\tau,T\}}}{q}.
\end{aligned} \tag{2}$$

Now taking expectation over the valuations, the average users expected time on the platform is then $E_{V\sim F}[X_S(V,p)]$ and the expected profit for a given subscription price p is,

$$\begin{aligned}
\mathcal{R}_{SP}(p,c,F) &= (p-c)\mathbb{E}[\text{Time on platform}|(\text{SP})] \\
&= \left(\frac{p-c}{q}\right) \int_{\frac{p}{q}}^{\infty} \min\left\{1 - \left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}, 1 - e^{-qT}\right\} f(v) dv.
\end{aligned} \tag{3}$$

Note Eq. (3) expresses the profit as an integral over the valuations. Next we derive a similar expression for (CP).

Profit of Contract Pricing.

To compute the profit for contract pricing, again consider a user with fixed valuation v and note that the user will pay to participate only if their time discounted expected valuation for a match exceeds the contract price, p. If the user pays, they will stay on the platform until they either match and leave, which again is distributed as an exponential random variable with rate q, or until they exhaust the platform at time T. Let $X_C(q)$ be the random variable representing the time the user spends on the platform assuming they paid the contract price, then $X_C(q) = \min\{\mathrm{Exp}(q), T\}$. A user with valuation v for matching will pay if $v\mathbb{E}[\delta^{X_C(q)}\mathbf{1}_{X_C(q)<T}] \geq p$ where $\mathbf{1}_{X_C(q)<T}$ is the indicator function which is 1 when the user matches, and 0 when the user exhausts the platform. The expectation can then be evaluated as,

$$\mathbb{E}[\delta^{X_C(q)}\mathbf{1}_{X_C(q)<T}] = q\int_0^T \delta^t e^{-qt}dt = \frac{q\left(1-\delta^T e^{-qT}\right)}{q-\log(\delta)}.$$

If a user pays the contract price, the expected operating costs of the platform will be c times the expected time to users stay on the platform, *i.e.*,

$$c\mathbb{E}[X_C(q)] = c\left(\int_0^T qte^{-qt}dt + Te^{-qT}\right) = \frac{c(1-e^{-qT})}{q} \tag{4}$$

Now taking expectation over the valuations, the expected profit earned by offering contract price p is,

$$\begin{aligned}\mathcal{R}_{CP}(p,c,F) &= \int_{\frac{p(q-\log(\delta))}{q(1-\delta^T e^{-qT})}}^{\infty} \left(p - \frac{c(1-e^{-qT})}{q}\right) f(v)dv \\ &= \left(p - \frac{c(1-e^{-qT})}{q}\right)\overline{F}\left(\frac{p\,(q-\log(\delta))}{q\,(1-\delta^T e^{-qT})}\right)\end{aligned} \tag{5}$$

Note Eq. (5) also expresses the profit as an integral over the valuations.

In this work, we will find it convenient to alternatively represent the profit from (SP) and (CP) as integrals over the time on the platform. In the following lemma, we introduce equivalent integral formulations of Eq. (3) and Eq. (5). We defer the proofs to the appendix.

Lemma 1 **(Integral Formulations of $\mathcal{R}_{SP}$ and $\mathcal{R}_{CP}$).** *For all positive valued distributions F, parameters $c, q, T > 0$, and $\delta \in (0,1)$,*

$$\mathcal{R}_{SP}(p,c,F) = \int_0^T (p-c)\overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt, \tag{6}$$

$$\mathcal{R}_{CP}(p,c,F) = \int_0^T \left(\frac{p\delta^t\,(q-\log(\delta))}{1-\delta^T e^{-qT}} - c\right)\overline{F}\left(\frac{p\,(q-\log(\delta))}{q\,(1-\delta^T e^{-qT})}\right)e^{-qt}dt. \tag{7}$$

We can interpret the integral formulations in Lemma 1 as describing the state of the platform at each time t. For (SP), $\overline{F}(p/q\delta^{-t}) \times e^{-qt}$ is the fraction of the market at time t that is still not priced out, times the fraction of those users who still have not matched. For (CP), $\overline{F}\left(\frac{p(q-\log(\delta))}{q(1-\delta^T e^{-qT})}\right) \times e^{-qt}$ is similarly the fraction of the market not priced out, times the fraction of those users who still have not matched at each time t, now paying a time dependant price.

While Eqs. (3) and (5) to (7) allow us to express the profit for fixed subscription/contract prices, in this work our focus is on the profit achieved using the optimal prices. Unfortunately, these profit functions can generally be quite badly behaved with multiple locally/globally optimal prices. To allow for tractable analysis, we will focus on a natural class of valuation distributions that simplifies the price optimization of (SP) and (CP). Specifically, we look at monotone hazard rate distributions (MHR).

DEFINITION 1 (MONOTONE HAZARD RATE (MHR) DISTRIBUTIONS). A random variable $V \sim F$ with density f is MHR if $\frac{\overline{F}(x)}{f(x)}$ is non-increasing.

MHR distributions are commonly used to model valuations, where they strike the appropriate balance between structure (MHR distributions have sub-exponential tails) and generality. MHR includes many common distributions, including Normal, Uniform, Exponential, and more. In Lemma 2 we show that when valuations are MHR, the optimal subscription/contract prices are unique and further, as c increases the prices increase.

LEMMA 2 **(Uniqueness of Optimal Prices)**. *For all positive valued, MHR distributions F, and parameters $c, q, T > 0$, and $\delta \in (0,1)$, both the optimal subscription price and the optimal contract price are unique, and increasing in c.*

2.1.1. Slow Matching Condition. Finally, we introduce one condition that will be helpful when thinking about dating markets. Specifically, in this paper, it will be useful to focus on markets that do not quickly match users relative to the users' patience, and the size of the pool of potential matches. We describe such markets with the following condition.

CONDITION 1 ((C1) SLOW MATCHING CONDITION). When the market parameters q, δ, and T are such that,

$$\left(1 - \frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}} \geq \left(\frac{q}{q - \log(\delta)}\right)\left(\frac{1 - \delta^T e^{-qT}}{1 - e^{-qT}}\right),$$

we say the market satisfies a *slow matching* condition.

To build intuition for (C1), consider the case when the pool of potential matches is large, *i.e.*, $T = \infty$. In this case, the condition reduces to $q \leq -\log(\delta)$. Recall, q is the rate at which users leave the platform due to matching, and also recall that user's valuations decay exponentially in δ. Condition (C1) compares these rates and implies that over any period t, assuming the market is not exhausted, the rate of user's valuation decay (and thus departures when valuations dip below the price) exceeds the rate of departure due to successful matching. Thus, we understand (C1) as markets where more users leave due to lack of patience or market exhaustion than because they match.

According to Pew Research Center (2020b), while three-in-ten Americans have tried dating apps, less than one-in-ten have found a serious relationship using from them, suggesting that matching is less likely to cause users to leave the platform than other causes of user churn like loss of patience, or exhausting local options. Thus, we believe (C1) to be a natural condition in our contexts of interest and likely to hold in practice.

3. Profit Guarantees for (SP) vs. (CP)

This section studies the optimal profits a matchmaker can earn under (SP) and (CP). Note, neither (SP) nor (CP) always yields more profit for all parameters and valuation distributions in our setting. Moreover, there are scenarios for which (SP) earns positive profit whereas (CP) earns nothing when the costs are prohibitively high (see Fig. 3 for example when valuations are exponentially distributed). Further, note that the reverse is not true, there is never an instance where (SP) earns nothing but (CP) earns positive profit (we prove this fact in Theorem 2(b)). Thus, in order then to understand when each pricing strategy is desirable for the matchmaker, we will attempt lower bound the ratio of the optimal profit achievable by (SP) to (CP), $\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)}$.

Specifically, we prove that no matter the market conditions, subscription based pricing always earns at least 36.7% of the profit of contract pricing, and for almost all parameters, the guarantee is even stronger.

THEOREM 1 (**$\mathcal{R}_{SP}$ Approximates $\mathcal{R}_{CP}$**). *For all positive valued distributions F, parameters $c,q,T>0$, and $\delta \in (0,1)$, then:*

$$\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)} \geq \left(1 - \frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}.$$

Taking the minimum over $\frac{q}{-\log(\delta)}$ yields a constant factor approximation,

$$\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)} \geq \frac{1}{e}.$$

Further, this bound is tight.

Discussion of Theorem 1 Theorem 1 demonstrates that in all parameter regimes, subscription pricing guarantees a substantial fraction of the profit garnered by contract pricing. This guarantee is of particular interest as contract pricing requires commitments between the users and the platform, and so is not necessarily implementable in an online setting. As subscription pricing requires no extraordinary market powers or commitments, it may be the only available pricing mechanism for the matchmaker, and justifies the prevalence of subscription based pricing in US online dating markets.

Further, we note that our bound is parametric and often guarantees more than $1/e$ of the optimal profit without considering anything about the valuation distribution. In Fig. 2, we show for fixed valuations how the guaranteed fraction changes with the operating cost c. For most cases, the fraction is much higher than our lower bound of $1/e$. Additionally, from the proof of the theorem we note that the worst case occurs when user valuations have no variation, a condition which is quite unlikely (c.f. Example EC.1 for tight examples). Thus in practice, we expect (SP) to earn an even greater fraction of the profit than the worst case guarantee in Theorem 1.

3.1. Proof Sketch of Theorem 1

In this subsection we describe the main ideas for the proof of Theorem 1 in the case when the number of potential candidates is large *i.e.* $T = \infty$ (for the full proof see Section B.2). The proof follows in three steps. First, we reduce the problem to the case where user's valuations are fixed and deterministic. Then, for fixed valuations $V \sim F_v$ where F_v is the distribution of a point mass on v, we consider two feasible subscription prices, neither of which individually imply the guarantee but together the best of which always guarantee 36.7% of the optimal contract pricing profit. To see where these two feasible prices come from, consider the ratio $\mathcal{R}_{SP}(c, F_v)/\mathcal{R}_{CP}(c, F_v)$ as a function of c, and let $p^*(c) = \arg\max_p \mathcal{R}_{SP}(p, c, F_v)$ be the optimal subscription price of $\mathcal{R}_{SP}(c, F_v)$. The derivative of $\mathcal{R}_{SP}(c, F_v)$ in c is,

$$\begin{aligned}\frac{\partial \mathcal{R}_{SP}(c, F_v)}{\partial c} &= \frac{1}{q}\left(\left(\frac{\partial p^*(c)}{\partial c} - 1\right)\left(1 - \left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}}\right) + \left(\frac{p^*(c) - c}{vq\log(\delta)}\right)\left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}-1}\frac{\partial p^*(c)}{\partial c}\right)\\ &= -\frac{1}{q}\left(1 - \left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}}\right),\end{aligned}$$

where the second equality follows from the optimality condition of $p^*(c)$,

$$\frac{1}{q}\left(\left(1-\left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}}\right)+\left(\frac{p^*(c)-c}{vq\log(\delta)}\right)\left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}-1}\right)=0.$$

Using this, the derivative of $\mathcal{R}_{SP}(c,F_v)/\mathcal{R}_{CP}(c,F_v)$ is then,

$$\frac{\partial \mathcal{R}_{SP}(c,F_v)/\mathcal{R}_{CP}(c,F_v)}{\partial c}=\frac{\frac{1}{q}\left(1-\left(\frac{p^*(c)}{vq}\right)^{-\frac{q}{\log(\delta)}}\right)\left(\frac{p^*(c)}{vq}-\frac{q}{q-\log(\delta)}\right)}{\mathcal{R}^2_{CP}(c,F_v)} \tag{8}$$

It can be checked that Eq. (8) is always positive when (C1) holds, and thus the worst case ratio occurs when $c=0$. For this case, we analyze the profit of the price that maximizes (SP) when $c=0$, $p^*(0)$, (we compute $p^*(0)$ in Example EC.3) to achieve the guarantee. When (C1) does not hold, the worst case for the ratio no longer occurs at $c=0$. Since by Lemma 2 $p^*(c)$ is monotonically increasing in c, Eq. (8) is negative and then positive, as a function of c. The minimum of $\mathcal{R}_{SP}(c,F_v)/\mathcal{R}_{CP}(c,F_v)$ thus occurs at an intermediate c^* such that, $\frac{p^*(c^*)}{vq}-\frac{q}{q-\log(\delta)}=0$, or equivalently when,

$$p^*(c^*)=v\left(\frac{q^2}{q-\log(\delta)}\right).$$

Analyzing this second feasible price achieves the guarantee when (C1) does not hold, completing the proof. A graphical representation of the two cases can be seen in Fig. 2.

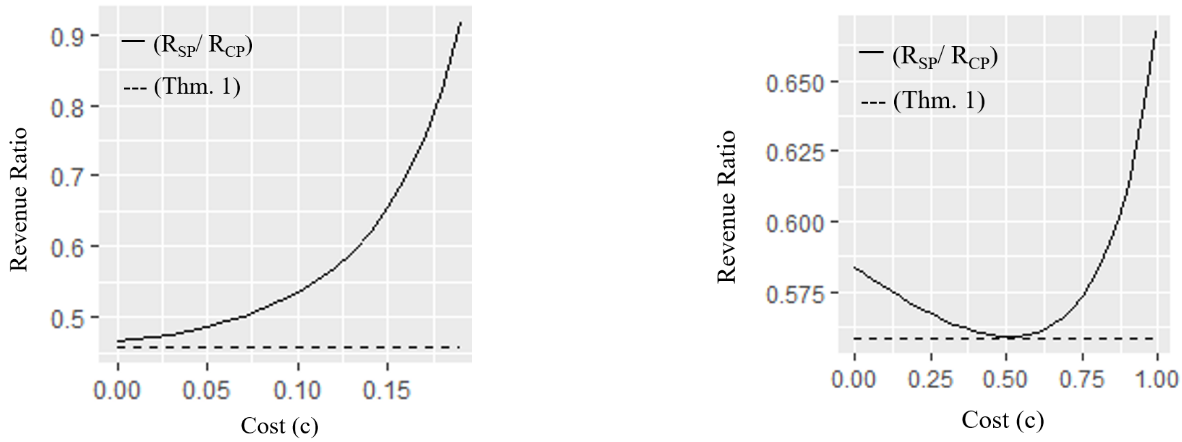
3.2. Fine-Grained Profit Analysis

In Theorem 1 we prove the profit of (SP) always approximates the profit of (CP). One interesting consequence of the Theorem 1 is that, when the match rate q is fixed and users are very patient *i.e.* $\delta\approx 1$, then the parameter $\frac{q}{-\log(\delta)}$ becomes very large, and the guarantee tends to 1 implying that when users are patient, subscription pricing achieves essentially as much profit as contract pricing regardless of cost or valuation distribution. In fact, when $\delta=1$, $\mathcal{R}_{SP}(c,F)=\mathcal{R}_{CP}(c,F)$ (see Example EC.2 for full derivation) and thus Theorem 1 lets us understand the performance of the strategies in markets with less than perfectly patient users.

In Theorem 2 we look at two other instances where extremal versions of a market parameter, namely the marginal operating cost c, similarly imply profit relations.

THEOREM 2 **(Profit Relationships as Cost Varies)**. *For all positive valued, MHR distributions F, parameters $q,T>0$, and $\delta\in(0,1)$, then:*

Figure 2 **Bounds on $\frac{\mathcal{R}_{SP}}{\mathcal{R}_{CP}}$.**



Note. Here we plot the guarantee in Theorem 1 when valuations are fixed and equal to 1, $T=\infty$ and $\delta=0.4$ as c varies. On the left, we let $q=-\log(d)-0.1$ so that (C1) holds. On the right, we let $q=-\log(\delta)+1$ so that (C1) does not hold.

a) When $c=0$, $\mathcal{R}_{CP}(0,F)\geq\mathcal{R}_{SP}(0,F)$.

b) When c *sufficiently large,* $\mathcal{R}_{CP}(c,F)\leq\mathcal{R}_{SP}(c,F)$.

In Theorem 2 we find that the profit from contract pricing always dominates the profit of optimal subscription pricing as the marginal cost of operating the platform tends to zero. This result is somewhat surprising. In online markets one can expect the marginal cost of operating with one more user to be small, and thus although (SP) is vastly more common for online dating, Theorem 2(a) suggests that (CP) may be more profitable for the platform (if they can implement it). On the other hand, when the marginal cost of operation is high, as might be expected in the case of a traditional matchmaker working directly with users, we find the reverse is true. The profit of the optimal subscription price dominates. Thus both (SP) and (CP) are, in some sense, not best suited for the markets in which they are the dominant paradigm. In the next section continue on this track, and study the impact of (SP) vs (CP) on users when marginal costs are low.

4. Impact and Incentives of Profit maximizing Online Matchmakers

In this section, we look beyond profit and study the structure, social welfare, and specific incentives induced by each pricing strategy. In Section 4.1 we prove a sharp relation between the optimal prices under (SP) and (CP) when (C1) holds, and use that relation to study the proportion of the market that ends up matched when the profit maximizing matchmaker chooses (SP) or (CP). In Section 4.2 we relax our assumption that the population of potential matches are all equally likely to match with the user. In a natural extension, we show that under (SP) the matchmaker has an incentive to show the user their least likely matches first, whereas under (CP) the matchmaker has the opposite incentive. Moreover, we find that under (CP) not only does the matchmaker display the most likely matches first, but they also do so at a lower price to the user.

4.1. Proportion of the Market Matched

In this subsection, we will examine the consequences profit maximizing sales practices have for users. First, we study which strategy leads to more of the population being matched. To help us characterize the percent of the market that matches under (SP) or (CP), we will first prove a structural condition on the prices.

LEMMA 3 **(Price Dominance for (SP) and (CP))**. *For all positive valued, MHR distributions F, parameters $c,q,T>0$, and $\delta\in(0,1)$ satisfying (C1), then:*

a) The optimal subscription price is greater than or equal to the optimal contract price divided by the expected amount of time a user who pays the contract price spends on the platform, $q^{-1}\left(1-e^{-qT}\right)$, i.e.,

$$\arg\max_p \mathcal{R}_{SP}(p,c,F) \geq \left(\frac{q}{1-e^{-qT}}\right)\arg\max_p \mathcal{R}_{CP}(p,c,F).$$

b) The ratio $\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)}$ is increasing, and there exists unique c^* such that $\mathcal{R}_{SP}(c^*,F)=\mathcal{R}_{CP}(c^*,F)$.

To build intuition for the condition in Lemma 3(a), consider a user who pays some contract price p_C. From Eq. (4), the expected time they will stay on the platform is $\mathbb{E}[X_C(v,p)\mathbf{1}_{X_C(v,p)\leq T}]=\frac{1-e^{-qT}}{q}$. Thus, $\left(\frac{q}{1-e^{-qT}}\right)\arg\max_p \mathcal{R}_{CP}(p,c,F)$ is the expected price per unit of time paid on the platform, and the claim is that under (SP) users pay a higher price per unit time than users under (CP), in expectation. Lemma 3(a) follows from an argument similar to the one outlined in the proof sketch of Theorem 1, and Lemma 3(b) is an easy consequence of the price dominance in (a), and allows us to get a handle on how the relative profits change as costs increase.

Armed with this lemma, we will now investigate whether users are better off under (SP) or (CP). Let $\mathcal{M}_{SP}(p,c,F)$ and $\mathcal{M}_{CP}(p,c,F)$ be the proportion of the market that ultimately gets matched under subscription pricing or contract pricing, with price p, respectively. Note, this is the proportion of users who first pay the price, times the probability of then getting matched eventually. Thus, like the profit, the match proportion can be expressed as a function of the model parameters. Formally,

$$\mathcal{M}_{SP}(p,c,F) = \int_{\frac{p}{q}}^{\infty} \underbrace{\min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}}_{\text{Prob. of matching for valuation } v} f(v)dv, \tag{9}$$

$$\mathcal{M}_{CP}(p,c,F) = \underbrace{\left(1-e^{-qT}\right)}_{\text{Prob. of matching}} \underbrace{\overline{F}\left(\frac{p\,(q-\log(\delta))}{q\,(1-\delta^T e^{-qT})}\right)}_{\text{Prob. of paying price } p}, \tag{10}$$

Note $\mathcal{M}_{SP}(p,c,F)$ and $\mathcal{M}_{CP}(p,c,F)$ are trivially monotone decreasing in p. Let $p_S^* = \arg\max_p \mathcal{R}_{SP}(p,c,F)$, $p_C^* = \arg\max_p \mathcal{R}_{CP}(p,c,F)$, and let $\mathcal{M}_{SP}(c,F) = \mathcal{M}_{SP}(p_S^*,c,F)$, $\mathcal{M}_{CP}(c,F) = \mathcal{M}_{CP}(p_C^*,c,F)$ denote the proportion of the user-based matched under the profit maximizing prices. The optimal profit of the two strategies can be written as,

$$\begin{aligned}
\mathcal{R}_{SP}(c,F) &= \max_p \left(\frac{p-c}{q}\right) \int_{\frac{p}{q}}^{\infty} \min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\} f(v)dv \\
&= \left(\frac{p_S^*-c}{q}\right) \mathcal{M}_{SP}(c,F) \\
\mathcal{R}_{CP}(c,F) &= \max_p \left(p - \frac{c(1-e^{-qT})}{q}\right) \overline{F}\left(\frac{p\,(q-\log(\delta))}{q\,(1-\delta^T e^{-qT})}\right) \\
&= \left(\frac{p_C^*}{1-e^{-qT}} - \frac{c}{q}\right) \mathcal{M}_{CP}(c,F).
\end{aligned}$$

As with profit, there is no universal match proportion relation between the two strategies. When costs are sufficiently high, (CP) is not economically viable and thus matches none of the market, whereas (SP) stays in and still matches at least some users. However, as mentioned above, for online dating one can expect the marginal cost of operating a platform to be relatively small. We will assume this for our next result.

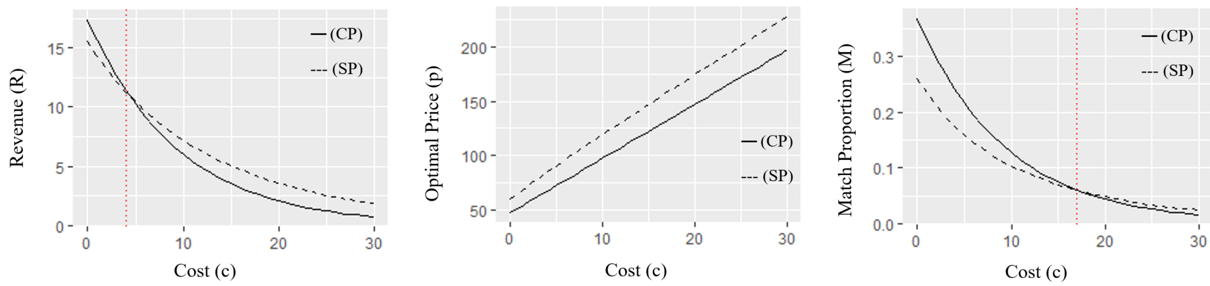
THEOREM 3 **($\mathcal{M}_{CP}$ Dominates $\mathcal{M}_{SP}$ when Costs are Low)**. *For all positive valued, MHR distributions F, and parameters q, T, and δ satisfying (C1), if c is less than c^*, then:*

$$\mathcal{M}_{CP}(c,F) \geq \mathcal{M}_{SP}(c,F).$$

Proof of Theorem 3 By the definition of c^* in Lemma 3(b), for any $c \leq c^*$, $\mathcal{R}_{SP}(c,F) \leq \mathcal{R}_{CP}(c,F)$. Further, by Lemma 3(a) if p_S^* and p_C^* are the profit maximizing subscription and contract price, respectively, then $\frac{p_S^*-c}{q} \geq \frac{p_C^*}{1-e^{-qT}} - \frac{c}{q}$. Therefore, by Eqs. (9) and (10)

$$\mathcal{M}_{CP}(c,F) = \frac{\mathcal{R}_{CP}(c,F)}{\frac{p_C^*}{(1-e^{-qT})} - \frac{c}{q}} \geq \frac{q\mathcal{R}_{CP}(c,F)}{p_S^*-c} \geq \frac{q\mathcal{R}_{SP}(c,F)}{p_S^*-c} = \mathcal{M}_{SP}(c,F).$$

Figure 3 Relations between optimal price, profit, and match proportion when valuations are exponential.



Note. Here we plot the profit, optimal prices, and match proportions under (SP) and (CP) when valuations are drawn from an exponential(100) distribution, and where $T=\infty$, $\delta=0.8$, $q=0.2$, and c varies. In the left panel, we plot the profits of (SP) and (CP), and the note relative profit ordering switches from $\mathcal{R}_{CP}>\mathcal{R}_{SP}$ when $c\leq 4$, to $\mathcal{R}_{CP}>\mathcal{R}_{SP}$ for $c>4$. In the middle panel, we plot the optimal contract price and the optimal subscription times a factor of $1/q$, as in Lemma 3. In the right panel, we plot the proportion of the market that gets matched under (SP) and (CP) and note that $\mathcal{M}_{CP}$ dominates $\mathcal{M}_{SP}$ for $c\leq 17$.

□

Theorem 3 gives generic conditions for when (CP) matches a higher proportion of the market than (SP). Specifically, it requires that the marginal costs be relatively small and (C1) hold, both of which describe the current operational conditions of online dating platforms. Moreover, by Lemma 3(b) in these same conditions, the profit the platform earns from contract pricing exceeds the profit of subscription price. Thus in online dating markets, (CP) is a rare win-win. More users are matched, and more profit is made than under (SP). In Fig. 3 we plot the relative profits, induced optimal prices, and match proportions as the cost varies for market parameters satisfying (C1) and valuations drawn from an exponential distribution. We note that, in these numerics, c^* occurs quite far from 0, the prices are close in the sense of Lemma 3(a), and (CP) matches significantly more of the market, even when c is larger than c^* suggesting the result in Theorem 3 is relatively conservative.

4.2. Incentive Considerations for Online Matchmaking

Previously, we assumed q, the rate at which a user matches on the platform, was fixed and constant for all potential matches. In this subsection, we will extend our framework to heterogeneous populations of potential matches, which we model as having varying match rates. Many dating platforms, including some of the most popular apps like Hinge, Tinder, Okcupid etc., implement features to segregate high probability and low probability potential matches (e.g. Hinge roses, Tinder top picks, etc.). It is then clear, users have preferences over potential match that change the match rate, and platforms are able to learn these match rates and use them to determine the order in which potential matches are displayed.

Specifically, we assume there are k possible matching rates $\{q_1,...,q_k\}$, where $q_1 \leq q_2 \leq ... \leq q_k$, and the matchmaker can order the potential matches based on matching rate. The population size of potential matches with rate q_i for the user is t_i, $i=1,...,k$, where $t_1+...+t_k=T$. Without loss of generality, we assume the user's belief about the matching rate on the platform is $q=\frac{\sum_{i=1}^k q_i t_i}{T}$, which is correct in expectation. We use $\mathcal{R}_{SP}(p,c,F,\{t_j,...,t_i\},\{q_j,...,q_i\})$ and $\mathcal{R}_{CP}(p,c,F,\{t_j,...,t_i\},\{q_j,...,q_i\})$ to denote the profit of (SP) and (CP), respectively, when potential matches are shown following match rate order $\{q_j,\ldots,q_i\}$. For an example of the revenues under specific orders, see Example EC.4.

In Theorem 4 we characterize how a profit maximizing platform orders potential matches under (SP) and (CP) respectively, and how this ordering affects the contract price.

THEOREM 4 **(Strategic Matchmakers)**. *For all positive valued distributions F, parameters $c,q,T>0$, $\delta\in(0,1)$, and every k element permutation $\sigma\in\Sigma_k$,*

a) $\mathcal{R}_{CP}(c,F,\{t_k,...,t_1\},\{q_k,...,q_1\}) \geq \mathcal{R}_{CP}(c,F,\sigma(\{t_k,...,t_1\})),\sigma(\{q_k,...,q_1\}))$,

b) $\mathcal{R}_{SP}(c,F,\{t_1,...,t_k\}),\{q_1,...,q_k\}) \leq \mathcal{R}_{SP}(c,F,\sigma(\{t_1,...,t_k\})),\sigma(\{q_1,...,q_k\}))$,

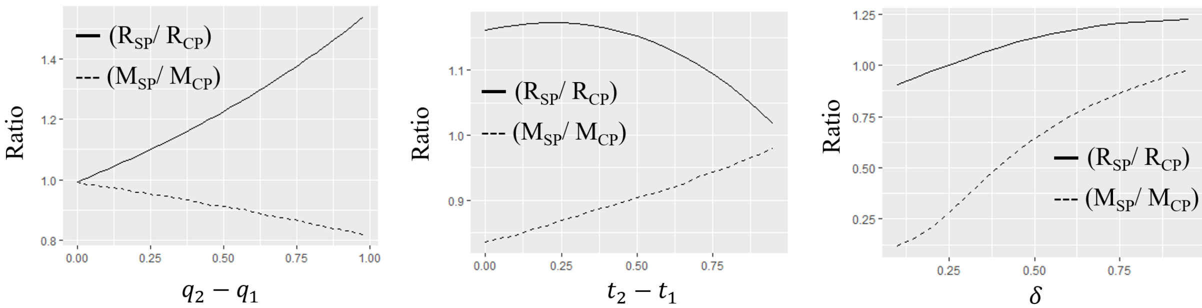
Further, when F is MHR,

c) The optimal contract price is decreasing when the matchmaker can manipulate the order of potential match candidates, i.e.,

$$\arg\max_p \mathcal{R}_{CP}(p,c,F,\{t_k,...,t_1\},\{q_k,...,q_1\}) \leq \arg\max_p \mathcal{R}_{CP}(p,c,F,\{T\},\{q\})$$

On dating apps, as the user interacts with the platform, the platform learns which potential matches the user would likely prefer. As mentioned in the introduction, ostensibly (SP) incentivizes the platform to hold likely matches back from the user, whereas (CP) incentivizes the platform to try and match the user as soon as possible. Theorem 4 formalizes this intuition and further shows that not only does (CP) incentivize the platform to use information about user preferences to help the user match, but also that information induces a profit maximizing platform to lower the contract price. This significantly simplifies the strategic considerations between the user and platform as their incentives are perfectly aligned.

Figure 4



Note. We plot the profit ratio $\frac{\mathcal{R}_{SP}}{\mathcal{R}_{CP}}$ and the matching proportion ratio $\frac{\mathcal{M}_{SP}}{\mathcal{M}_{CP}}$ when valuations are exponentially distributed with rate 100, and $T = 2$. In the left panel, we let $c = 10$, $\delta = 0.8$, and consider two matching rates, q_1 and q_2, $q_1 < q_2$, in equal proportion i.e. $t_1 = t_2 = 1$. We plot the ratios as the difference in rate $q_2 - q_1$ increases, holding the average matching rate $\overline{q}$ fixed at 1. In the middle panel, again we let $c = 10$, $\delta = 0.8$, and consider two matching rates, $q_1 = .1$ and $q_2 = .9$, as the proportions, t_1, and t_2, $t_1 + t_2 = 2$, shift. In the right panel, we let $c = 0$, $t_1 = t_2 = 1$, $q_1 = .1$ and $q_2 = .9$, as plot the ratios as δ varies.

In Fig. 4, we plot the profit and match ratios of (SP) to (CP) when valuations are exponentially distributed and $T = 2$, as other market parameters shift. In the left panel of Fig. 4, we let $c = 10$, $\delta = 0.8$, and consider two matching rates, q_1 and q_2, $q_1 < q_2$, in equal proportion i.e. $t_1 = t_2 = 1$. We plot the ratios as the difference in rate $q_2 - q_1$ increases, holding the average matching rate $\overline{q}$ fixed at 1. We note that as the difference between low and high probability match candidates increases, the profit from subscription pricing begins to outstrip contract pricing, at the expense of the proportion of the market that ends up matched. This suggests that unregulated manipulation on the basis of match probability is quite profitable to the subscription priced dating platform, but this profit comes directly at the expense of the user-base. In the middle panel, again let $c = 10$, $\delta = 0.8$, and consider two matching rates, $q_1 = .1$ and $q_2 = .9$, as the proportions, t_1, and t_2, $t_1 + t_2 = 2$, shift. Here we note a similar effect as in the left panel, but emphasize that the difference in revenue and match proportion are most pronounced when the pool of potential matches are primarily low matching rate (as is probably the case in practice). Finally, in the right panel, we let $c = 0$, $t_1 = t_2 = 1$, $q_1 = .1$ and $q_2 = .9$, as plot the ratios as δ varies. As δ tends to 1, which can be thought as the patience of the user-base, we note that the profit of (SP) continues to exceed (CP), but the overall match proportion begins to close between the strategies. The reason is that the platform engaged in (SP) can prolong users' stay by providing low match rate candidates first, causing patient users to pay the subscription price for a long period before finally reaching the higher probability matches. Thus, while match proportion may be similar in these instances, overall welfare to the user-base is much higher under (CP) than under (SP).

5. Conclusions

Our work yields a number of insights for market designers looking to understand and improve operations for online dating platforms. Our approximation results give a compelling answer to the so called “strategy puzzle” of online dating Wu et al. (2019). Online matchmakers prefer (SP) because not only is it easier to implement, but also because (SP) garners provably near optimal revenue for the platform. However, while subscription pricing is practical and profitable as a first approach, the theory in our paper provides strong motivation for further improving performance by attempting contract pricing in this space.

When marginal costs are low (as is the case online) and matching is a slow and noisy process, we prove that (CP) is both more profitable for the matchmaker, and more effective at matching a significant portion of the user-base. Of course, as mentioned in the introduction, (CP) is more difficult to implement in an online environment than (SP). However we believe this a difficulty worth surmounting. One potential avenue for implementing (CP) is by working in collaboration with government agencies that can track marriage records, and thus enforce contracts where users pay after matching. In fact, given the strong social value of efficient matchmaking, nationalized dating apps are being tested our overseas in countries like Japan and Singapore (AKITA 2019, Afp 2010). For such platforms, (CP) could reasonably be implemented and may yield superior results.

For users, we shed light on the consequences of the pricing strategies dating apps commit to. We note our model has some limitations that may be mitigated in practice. For instance, our work describes users looking for their life-long match, however on sites like Tinder or Grindr, many users may only be looking for short relationships. In this case, the incentives for the site are different, and even under (SP) the platform may still try to match users in the hopes of soliciting their repeat business. This heterogeneous population of user interests may provide some salve to the analysis presented in this paper, and would be interesting to study in future work. We also note that our paper studies two extremes in terms of the commitment required of the user at payment. Many apps offer intermediate length subscriptions, where users can pay for a moderate amount of time on the site (e.g. six months or a year). In future work, it would be interesting to study how the incentives and performance of dating apps change as the commitment required from the user varies between (SP) and (CP).

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Appendix A: Omitted Examples

EXAMPLE EC.1 (TIGHTNESS OF THEOREM 1). In this example, we give an instance of our model such that Theorem 1 is tight. Specifically, suppose valuations are fixed and drawn from a point-mass distribution on v, let $c=0$, $T=\infty$, and let q and δ be arbitrary. In this case the optimal subscription price is $p^* = vq\left(\frac{\log(\delta)-q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$, and the profit of optimal subscription pricing is,

$$\mathcal{R}_{SP}(0,F_v) = \left(\frac{vq}{q-\log(\delta)}\right)\left(\frac{\log(\delta)-q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}.$$

Similarly, the profit of optimal contract pricing is,

$$\mathcal{R}_{CP}(0,F_v) = \left(\frac{vq}{q-\log(\delta)}\right).$$

The ratio between $\mathcal{R}_{SP}(0,F_v)$ and $\mathcal{R}_{CP}(0,F_v)$ is then,

$$\frac{\mathcal{R}_{SP}(0,F_v)}{\mathcal{R}_{CP}(0,F_v)} = \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}},$$

therefore, the approximation ratio is tight. □

EXAMPLE EC.2 (TIGHTNESS OF LEMMA 3). In this example, we give an instance of our model such that the price dominance in Lemma 3 is tight. Specifically, let $\delta=1$ and the F, c, q, and T be arbitrary. In this case the profit of (SP) and (CP) can be written as,

$$\mathcal{R}_{SP}(c,F) = \max_p \int_0^T (p-c)\overline{F}(pq^{-1})e^{-qt}dt = \max_p (p-c)\overline{F}(pq^{-1})\int_0^T e^{-qt}dt$$
$$\mathcal{R}_{CP}(c,F) = \max_p \int_0^T \left(\frac{pq}{1-e^{-qT}}-c\right)\overline{F}\left(\frac{p}{1-e^{-qT}}\right)e^{-qt}dt = \max_p \left(\frac{pq}{1-e^{-qT}}-c\right)\overline{F}\left(\frac{p}{1-e^{-qT}}\right)\int_0^T e^{-qt}dt$$

Let $p_S^* = \arg\max_p \mathcal{R}_{SP}(p,c,F)$, $p_C^* = \arg\max_p \mathcal{R}_{CP}(p,c,F)$, then $(1-e^{-qT})\,p_S^* = qp_C^*$, and $\mathcal{R}_{SP}(c,F) = \mathcal{R}_{CP}(c,F)$. □

EXAMPLE EC.3 (TIGHTNESS OF THEOREM 3). In this example we describe the simple case where users have fixed valuation $v=1$, there are no costs $c=0$, and the pool of potential matches is large $T=\infty$. For these parameters, the optimal subscription price solves

$$\max_p \quad \frac{p}{q}\left(1-\left(\frac{p}{q}\right)^{-q/\log(d)}\right),$$

which yields the optimal subscription price

$$p_S^* = q\left(\frac{\log(d)-q}{\log(d)}\right)^{\log(d)/q}.$$

Under the optimal subscription price, the proportion of the market that gets matched is,

$$\mathcal{M}_{SP}\left(p_S^*,c,F\right) = \left(\frac{\log(d)-q}{\log(d)}\right)^{\log(d)/q}.$$

Similarly, in this case the optimal contract price is

$$p_C^* = \frac{q}{q-\log(d)},$$

the corresponding matching proportion $\mathcal{M}_{CP}(c,F)$ is always 1. Thus the contract price leads to a greater portion of the market getting matched. □

Example EC.4 (Manipulation Gap). In this example, we consider (SP) and (CP) when the platform has access to two types of potential matches it can display. Suppose valuations are fixed and drawn from a point-mass distribution on v, let $c=0$, $T=\infty$, $\delta=1$, and suppose $q_1=0$, $q_2=1$, and $t_1=t_2=1$. Then the matching rate perceived by users is $q=\frac{q_1+q_2}{2}=0.5$. The revenue of contract pricing that chooses to show type 2 potential matches first is,

$$\mathcal{R}_{CP}(0,F_v,\{q_2,q_1\})=v\left(1-e^{-qT}\right)=1-\frac{1}{e},$$

The revenue of contract pricing that chooses to show type 2 potential matches first is,

$$\mathcal{R}_{SP}(0,F_v,\{q_1,q_2\})=vq\left(\frac{1-e^{-t_1q_1}}{q_1}+e^{-t_1q_1}\left(\frac{1-e^{-t_2q_2}}{q_2}\right)\right)=1-\frac{1}{2e}.$$

By Example EC.2, when the match homogeneous the profit of (SP) and (CP) was the same. Now, when the platform is allowed to choose the order, the difference in profit is,

$$\mathcal{R}_{SP}(0,F_v,\{q_1,q_2\})-\mathcal{R}_{CP}(0,F_v,\{q_2,q_1\})=\frac{1}{2e}.$$

□

Appendix B: Omitted Proofs

B.1. Omitted Proofs from Section 2

Proof of Lemma 1 First, we derive the expression for (SP). Consider a platform offering subscription p to a user with fixed valuation v. Recall $X_S(v,p)$ is the random variable representing the time the user spends on the platform. It's expectation is,

$$\mathbb{E}[X_S(v,p)]=\int_0^{\min\{\tau,T\}}e^{-qt}dt,$$

where $\tau=\frac{\log\left(\frac{p}{vq}\right)}{\log(\delta)}$. The expected revenue over random users with subscription price p is then,

$$\begin{aligned}\mathcal{R}_{SP}(p,c,F)&=\int_{\frac{p}{q}}^{\infty}(p-c)\left(\int_0^{\min\{\tau,T\}}e^{-qt}dt\right)f(v)dv\\&=\int_0^T\left(\int_{\frac{p}{q\delta^t}}^{\infty}(p-c)f(v)dv\right)e^{-qt}dt\\&=\int_0^T(p-c)\overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt,\end{aligned}$$

where the second equality follows from Fubini's theorem, for $t\le\frac{\log\left(\frac{p}{vq}\right)}{\log(\delta)}$ which rearranged is $v\ge\frac{p}{q\delta^t}$. Thus we obtain our desired expression for (SP).

Now, consider a contract pricing with price p and let $\underline{v}$ be the minimum valuation for which a user will pay the contract price. Recall $X(v,p)$ is the random variable representing the time the user spends on the platform. Then,

$$p=\underline{v}\mathbb{E}[\delta^{X(\underline{v},p)}\mathbf{1}_{X_C(\underline{v},p)<T}]=\int_0^T\underline{v}q\delta^te^{-qt}dt=\frac{\underline{v}\left(1-\delta^Te^{-qT}\right)}{1-\frac{\log(\delta)}{q}}.\tag{EC.1}$$

The expected revenue earned by contract price p can then be written as,

$$\begin{aligned}\mathcal{R}_{CP}(p,c,F) &= \left(p - c\left(\int_0^T tqe^{-qt}dt + Te^{-qT}\right)\right)\overline{F}(\underline{v})\\ &= \left(\int_0^T \underline{v}\delta^t qe^{-qt}dt - c\left(-te^{-qt}|_0^T + \int_0^T e^{-qt}dt + Te^{-qT}\right)\right)\overline{F}(\underline{v})\\ &= \int_0^T \left(\underline{v}\delta^t q - c\right)\overline{F}(\underline{v})e^{-qt}dt\\ &= \int_0^T \left(p\delta^t\left(q-\log(\delta)\right)/(1-\delta^T e^{-qT}) - c\right)\overline{F}\left(p\left(1-\frac{\log(\delta)}{q}\right)/\left(1-\delta^T e^{-qT}\right)\right)e^{-qt}dt,\end{aligned}$$

the second equality comes from integration by parts and the final inequality follows from Eq. (EC.1). □

Proof of Lemma 2 First, we consider the optimal subscription price. Using Lemma 1, we can upper bound the revenue obtained a price p by,

$$\mathcal{R}_{SP}(p,c,F) = \int_0^T (p-c)\,\overline{F}\left(pq^{-1}\delta^{-t}\right)e^{-qt}dt \le \int_0^T (p-c)\,\overline{F}\,(p)\,e^{-qt}dt,$$

since $\overline{F}$ is decreasing. When p tends to infinity, the upper bound on the subscription revenue goes to 0, which implies the optimal subscription price is finite. Now, we show optimal subscription price is unique. To do so, first, consider equations of the form

$$p - c = pg(p) \tag{EC.2}$$

where $g(p)$ is decreasing, $c \ge 0$. Eq. (EC.2) has at most one positive solution in p, denote it p^*, since $g(p) = \frac{p-c}{p} \le 1$ for all $c, p \ge 0$, and $g(p) < g(p^*) \le 1$ for all $p > p^*$, when $g(p)$ is decreasing. Further, the first-order derivative of $pg(p)$ is

$$\frac{\partial pg(p)}{\partial p} = g(p) - pg'(p) \le g(p) < 1$$

for all $p > p^*$. If there is another solution $\bar{p} > p^*$ of Eq. (EC.2), by the mean value theorem, there exists $p \in (p^*, \bar{p})$, such that

$$g(p) - pg'(p) = 1,$$

which contradicts with the fact $g(p) - pg'(p) < 1$ for all $p > p^*$. Therefore, we conclude Eq. (EC.2) has at most one solution for $p > 0$, $c \ge 0$, when function $g(p)$ is decreasing in p. Rearrange Eq. (EC.2), we get

$$p\,(1 - g(p)) = c,$$

where $p\,(1-g(p))$ is increasing in p since $g(p)$ is decreasing in p. Therefore, the solution of Eq. (EC.2) is increasing in c.

Here, we show the first-order condition for the optimal subscription price is exactly of the form of Eq. (EC.2) implying uniqueness. Consider

$$\frac{\partial \mathcal{R}_{SP}(p,c,F)}{\partial p} = \int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt - (p-c)\int_0^T q^{-1}\delta^{-t}f(pq^{-1}\delta^{-t})e^{-qt}dt = 0. \tag{EC.3}$$

There are two ways the above equation can be zero. The first is if p is such that $V \le p$ almost surely. In this case, both integrals are zero, and while p is a critical point, such a p cannot be the revenue optimal subscription price as it earns no revenue. Assume $\overline{F}(pq^{-1}) > 0$, then both integrals are positive and we can rearrange the expression Eq. (EC.3) to be

$$\begin{aligned} p - c &= \frac{\int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt}{\int_0^T q^{-1}\delta^{-t} f(pq^{-1}\delta^{-t})e^{-qt}dt} \\ &= \frac{\int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt}{\int_0^T \frac{e^{-qt}}{p\log(\delta)} d\overline{F}(pq^{-1}\delta^{-t})} \\ &= \frac{p\log(\delta)\int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt}{0 - \overline{F}(pq^{-1}) + \int_0^T q\overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt} \\ &= \frac{p\log(\delta)}{q - \overline{F}(pq^{-1})\left(\int_0^T \overline{F}\left(pq^{-1}\delta^{-t}\right)e^{-qt}dt\right)^{-1}}, \end{aligned}$$

where the second follows from the identity $d\overline{F}(pq^{-1}\delta^{-t}) = f(pq^{-1}\delta^{-t})pq^{-1}\log(\delta)\delta^{-t}dt$, the third equation follows from integration by parts, and the fourth from simplifying. Now to apply Eq. (EC.2), we require the $\frac{\log(\delta)}{q - \overline{F}(pq^{-1})\left(\int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt\right)^{-1}}$ to be decreasing in p. To show this, let $h(p) = \overline{F}(pq^{-1})\left(\int_0^T \overline{F}\left(pq^{-1}\delta^{-t}\right)e^{-qt}dt\right)^{-1}$, consider the derivative of the $h^{-1}(p)$,

$$\frac{\partial \int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}\left(\overline{F}(pq^{-1})\right)^{-1}dt}{\partial p} = \int_0^T \frac{-\delta^{-t} f(pq^{-1}\delta^{-t})\overline{F}(pq^{-1}) + \overline{F}(pq^{-1}\delta^{-t})f(pq^{-1})}{q\left(\overline{F}(pq^{-1})\right)^2} e^{-qt}dt.$$

To show this derivative is negative, consider

$$\begin{aligned} -\delta^{-t} f(pq^{-1}\delta^{-t})\overline{F}(pq^{-1}) + \overline{F}(pq^{-1}\delta^{-t})f(pq^{-1}) &\le -f(pq^{-1}\delta^{-t})\overline{F}(pq^{-1}) + \overline{F}(pq^{-1}\delta^{-t})f(pq^{-1}) \\ &= f\left(pq^{-1}\right) f\left(pq^{-1}\delta^{-t}\right)\left(-\frac{\overline{F}(pq^{-1})}{f(pq^{-1})} + \frac{\overline{F}(pq^{-1}\delta^{-t})}{f(pq^{-1}\delta^{-t})}\right) \\ &\le 0 \end{aligned}$$

where the first inequality follows from $\delta^{-t} \ge 1$ and the second inequality from the fact that F is MHR. Then, $h^{-1}(p)$ is decreasing in p. Consequently $\frac{\log(\delta)}{q - \overline{F}(pq^{-1})\left(\int_0^T \overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt\right)^{-1}}$ is also decreasing in p. Therefore, the first-order condition of the optimal subscription price is of the form in Eq. (EC.2). Combining with the existence of finite optimal subscription price, we conclude that the optimal subscription price is unique and increasing in c, and the revenue of subscription pricing is unimodal.

The uniqueness of the optimal contract price comes from MHR directly. By Eq. (5), the first-order condition for the optimal contract price is

$$\overline{F}\left(\frac{p\left(q - \log(\delta)\right)}{q\left(1 - \delta^T e^{-qT}\right)}\right) - \left(p - \frac{c(1 - e^{-qT})}{q}\right)\left(\frac{q - \log(\delta)}{q\left(1 - \delta^T e^{-qT}\right)}\right) f\left(\frac{p\left(q - \log(\delta)\right)}{q\left(1 - \delta^T e^{-qT}\right)}\right) = 0. \tag{EC.4}$$

As above, assuming p is such that $\overline{F}\left(\frac{p(q - \log(\delta))}{q\left(1 - \delta^T e^{-qT}\right)}\right) > 0$, we can rearrange Eq. (EC.4) to be

$$p - \frac{c(1 - e^{-qT})}{q} = \left(\frac{q\left(1 - \delta^T e^{-qT}\right)}{q - \log(\delta)}\right) \frac{\overline{F}\left(\frac{p(q - \log(\delta))}{q\left(1 - \delta^T e^{-qT}\right)}\right)}{f\left(\frac{p(q - \log(\delta))}{q\left(1 - \delta^T e^{-qT}\right)}\right)}.$$

Note $\overline{F}\left(\frac{p(q - \log(\delta))}{q\left(1 - \delta^T e^{-qT}\right)}\right) f\left(\frac{p(q - \log(\delta))}{q\left(1 - \delta^T e^{-qT}\right)}\right)^{-1}$ is non-increasing in p since F is MHR, while $p - \frac{c(1 - e^{-qT})}{q}$ increasing in p. Therefore, we can conclude the optimal contract price is unique and increasing in c. $\square$

B.2. Omitted Proofs from Section 3

Proof of Theorem 1 Our proof will follow in three steps. First, we reduce the problem to the case where valuations are fixed and deterministic. Next, we bound the ratio for fixed valuations when (C1) holds by analyzing a feasible price, which is the optimal subscription price when $c=0$. Finally, we bound the ratio when (C1) does not hold by analyzing a second feasible price, which is the average price of the contract pricing.

Step 1: Reduction to fixed valuations.

Define $\gamma(v) := \min\left\{1-\left(\frac{p}{qv}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}$, and note $\gamma(v)$ is a non-decreasing function of v. Now, by Eq. (3) the revenue of (SP) for some fixed price p is,

$$\begin{aligned}\mathcal{R}_{SP}(p,c,F) &= \left(\frac{p-c}{q}\right)\int_{p/q}^{\infty}\min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}f(v)dv \\ &= \left(\frac{p-c}{q}\right)\int_{p/q}^{\infty}\gamma(v)f(v)dv \\ &= \left(\frac{p-c}{q}\right)\mathbb{E}[\gamma(V)\mathbf{1}_{V\geq p/q}].\end{aligned}$$

By the generalized Markov's inequality, for any $a\in[p/q,\infty)$, we have,

$$\mathbb{E}[\gamma(V)\mathbf{1}_{V\geq p/q}] \geq \overline{F}(a)\gamma(a).$$

Applying the inequality we obtain,

$$\mathcal{R}_{SP}(p,c,F) \geq \left(\frac{p-c}{q}\right)\min\left\{1-\left(\frac{p}{xq}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}\overline{F}(x), \quad \text{for any } x\geq\frac{p}{q}. \tag{EC.5}$$

Now, suppose the optimal price for $\mathcal{R}_{CP}(p,c,F)$ is p^*, and let $v^* = \frac{p^*(q-\log(\delta))}{q\left(1-\delta^T e^{-qT}\right)}$. Taking x as v^* in Eq. (EC.5) yields,

$$\mathcal{R}_{SP}(p,c,F) \geq \left(\frac{p-c}{q}\right)\min\left\{1-\left(\frac{p}{v^*q}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}\overline{F}(v^*). \tag{EC.6}$$

Therefore,

$$\begin{aligned}\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)} &\geq \frac{\mathcal{R}_{SP}(p,c,F)}{\mathcal{R}_{CP}(p^*,c,F)} \\ &\geq \frac{\max_p\left(\frac{p-c}{q}\right)\min\left\{1-\left(\frac{p}{v^*q}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}\overline{F}(v^*)}{\left(\frac{v^*}{1-\frac{\log(\delta)}{q}}\left(1-\delta^T e^{-qT}\right)-\frac{c}{q}\left(1-e^{-qT}\right)\right)\overline{F}(v^*)} \\ &= \frac{\max_p\left(\frac{p-c}{q}\right)\min\left\{1-\left(\frac{p}{v^*q}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}}{\left(\frac{v^*}{1-\frac{\log(\delta)}{q}}\left(1-\delta^T e^{-qT}\right)-\frac{c}{q}\left(1-e^{-qT}\right)\right)}\end{aligned} \tag{EC.7}$$

where the first inequality follows from using the optimal price for (CP) and the second inequality follows by applying Eq. (EC.6) and plugging in the revenue for (CP) from Eq. (5). Define F_v to be the point mass distribution for a random variable that is equal to some constant v with probability one. Note, then that the ratio in Eq. (EC.7) is exactly the same as the ratio for F_{v^*}, i.e.:

$$\frac{\max_p \left(\frac{p-c}{q}\right) \min\left\{1-\left(\frac{p}{v^* q}\right)^{\frac{-q}{\log(\delta)}}, 1-e^{-qT}\right\}}{\left(\frac{v^*}{1-\frac{\log(\delta)}{q}}\left(1-\delta^T e^{-qT}\right)-\frac{c}{q}\left(1-e^{-qT}\right)\right)} = \frac{\mathcal{R}_{SP}(c,F_{v^*})}{\mathcal{R}_{CP}(c,F_{v^*})} \geq \inf_v \frac{\mathcal{R}_{SP}(c,F_v)}{\mathcal{R}_{CP}(c,F_v)}. \tag{EC.8}$$

For the remainder of our proof, we will lower bound Eq. (EC.8) by finding the worst case ratio over all point mass valuations v.

Step 2: Bounds when (C1) holds.

Fix some point mass valuation v. First assume, that $\delta^T > \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$, and consider the feasible subscription price be $\tilde{p} = vq\delta^T$. The corresponding subscription revenue is

$$\begin{aligned}\mathcal{R}_{SP}(\tilde{p},c,F_v) &= \left(v\delta^T - \frac{c}{q}\right)\left(1-e^{-qT}\right)\\ &= \mathcal{R}_{CP}(c,F_v)\frac{\delta^T\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)} + \frac{c\left(1-e^{-qT}\right)}{q}\left[\frac{\delta^T\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)} - 1\right].\end{aligned}$$

Rearranging we have,

$$\begin{aligned}\frac{\mathcal{R}_{SP}(\tilde{p},c,F_v)}{\mathcal{R}_{CP}(p^*,c,F_v)} &= \frac{\delta^T\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)} + \frac{c\left(1-e^{-qT}\right)}{q}\left[\frac{\delta^T\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)} - 1\right]\\ &\geq \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(\frac{\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)}\right)\\ &\quad + \frac{c\left(1-e^{-qT}\right)}{q}\left[\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(\frac{\left(1-e^{-qT}\right)}{\frac{q}{q-\log(\delta)}\left(1-\delta^T e^{-qT}\right)}\right) - 1\right]\\ &\geq 1 + \frac{c\left(1-e^{-qT}\right)}{q}\left[1-1\right] \geq 1.\end{aligned}$$

where the first inequality follows from our assumption, and the second inequality follows from rearranging (C1).

Next consider the alternative assumption, $\delta^T \leq \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$. For this case, consider the feasible subscription price $\tilde{p} = vq\left(\frac{\log(\delta)-q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$. By Eq. (3), the revenue generated by $\tilde{p}$ is,

$$\begin{aligned}\mathcal{R}_{SP}\left(\tilde{p},c,F_v\right) &= \frac{vq}{q-\log(\delta)}\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}} - \frac{c}{q-\log(\delta)}\\ &= \frac{\mathcal{R}_{CP}(c,F_v)}{1-\delta^T e^{-qT}}\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}} + \frac{c}{q}\left[\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(\frac{1-e^{-qT}}{1-\delta^T e^{-qT}}\right) - \frac{q}{q-\log(\delta)}\right].\end{aligned}$$

By (C1), we always have

$$\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(\frac{1-e^{-qT}}{1-\delta^T e^{-qT}}\right) - \frac{q}{q-\log(\delta)} \geq 0.$$

Therefore,

$$\mathcal{R}_{SP}(c,F_v) \geq \mathcal{R}_{CP}(c,F_v)\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(1-\delta^T e^{-qT}\right)^{-1},$$

as desired. Finally, letting $x = \frac{q}{-\log(\delta)}$, and minimizing the above expression for $0 < x < 1$, we have $\min_{x\in(0,1)}(1+x)^{\frac{-1}{x}} \geq \frac{1}{e}$ with the minimum occurring as x tends to 0. Thus in this case $\frac{\mathcal{R}_{SP}}{\mathcal{R}_{CP}} \geq \frac{1}{e}$.

Step 3: Bounds when (C1) does not hold.

Again fix some point mass valuation v, and now consider the feasible subscription price, $\tilde{p} = vq^2(1-\delta^T e^{-qT})\left((q-\log(\delta))(1-e^{-qT})\right)^{-1}$. By Eq. (3), the revenue generated by $\tilde{p}$ is,

$$\begin{aligned}\mathcal{R}_{SP}(\tilde{p},c,F_v) &= \left(\frac{vq(1-\delta^T e^{-qT})}{(q-\log(\delta))(1-e^{-qT})}-\frac{c}{q}\right)\left[1-\left(\left(\frac{1}{1-\frac{\log(\delta)}{q}}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)\right)^{-\frac{q}{\log(\delta)}}\right]\\ &= \frac{\mathcal{R}_{CP}(c,F_v)}{1-e^{-qT}}\left[1-\left(\left(\frac{1}{1-\frac{\log(\delta)}{q}}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)\right)^{-\frac{q}{\log(\delta)}}\right]\end{aligned}$$

Rearranging we have,

$$\frac{\mathcal{R}_{SP}(\tilde{p},c,F_v)}{\mathcal{R}_{CP}(c,F_v)} \geq \underbrace{\frac{1}{1-e^{-qT}}\left[1-\left(\left(\frac{1}{1-\frac{\log(\delta)}{q}}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)\right)^{-\frac{q}{\log(\delta)}}\right]}_{\phi(T)}.$$

The derivative of $\phi(T)$ with respect to T is,

$$\frac{\partial\phi(T)}{\partial T} = \left(\frac{qe^{-qT}}{(1-e^{-qT})^2}\right)\left(\left(1-\frac{q}{\log(\delta)}\right)\left(\frac{1-\delta^T}{1-\delta^T e^{-qT}}\right)\left(\frac{1-\delta^T e^{-qT}}{\left(1-\frac{\log(\delta)}{q}\right)(1-e^{-qT})}\right)^{-\frac{q}{\log(\delta)}}-1\right).$$

When $\frac{q}{-\log(\delta)} \geq 1$, $\frac{\partial\phi(T)}{\partial T} \geq 0$, and thus,

$$\frac{\mathcal{R}_{SP}(\tilde{p},c,F_v)}{\mathcal{R}_{CP}(c,F_v)} \geq \lim_{T\to 0}\phi(T) = \frac{1}{2}.$$

When $\frac{q}{-\log(\delta)} < 1$, $\frac{\partial\phi(T)}{\partial T} < 0$. Define T^* such that,

$$\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}} = \left(\frac{q}{q-\log(\delta)}\right)\left(\frac{1-\delta^{T^*}e^{-qT^*}}{1-e^{-qT^*}}\right),$$

in this case, (C1) does not hold only for $T < T^*$. Thus to complete the proof we can consider

$$\frac{\mathcal{R}_{SP}(\tilde{p},c,F_v)}{\mathcal{R}_{CP}(c,F_v)} \geq \lim_{T\to T^*}\phi(T) = \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\left(1-\delta^{T^*}e^{-qT^*}\right)^{-1}.$$

Combining across all cases and taking the minimum yields the claimed bound. For tightness, consider the case when $c = 0$, $T = \infty$, and valuations are a point mass v. In this case the optimal price for (SP) can be computed as $p^* = vq\left(\frac{\log(\delta)-q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$ yielding optimal revenue

$$\mathcal{R}_{SP}(0,F_v) = \frac{vq}{q-\log(\delta)}\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}.$$

Then the ratio between (SP) and (CP) is

$$\frac{\mathcal{R}_{SP}(0,F_v)}{\mathcal{R}_{CP}(0,F_v)} = \left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}$$

matching the guarantee. Taking $\frac{q}{-\log(\delta)} \to 0$ gives the $1/e$ constant factor. □

Proof of Theorem 2 We will prove the two parts separately.

Part a)

First note that when c = 0, the optimal contract revenue is

$$\mathcal{R}_{CP}(0,F)=\max_p p\overline{F}\left(\frac{q-\log(\delta)}{q\left(1-\delta^T e^{-qT}\right)}p\right)=\frac{q\left(1-\delta^T e^{-qT}\right)}{q-\log(\delta)}p^*\overline{F}(p^*),$$

where p^* is the solution of $\max_p p\overline{F}(p)$. Now, using Lemma 1, we can upper bound $\mathcal{R}_{SP}(0,F)$ as,

$$\begin{aligned}\mathcal{R}_{SP}(0,F)&=\max_p\int_0^T p\overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt\\&\le\int_0^T\max_p p\overline{F}(pq^{-1}\delta^{-t})e^{-qt}dt\\&=\int_0^T p^*\overline{F}(p^*)q\delta^t e^{-qt}dt\\&=\frac{q\left(1-\delta^T e^{-qT}\right)}{q-\log(\delta)}p^*\overline{F}(p^*)=\mathcal{R}_{CP}(0,F),\end{aligned}$$

as desired. □

Part b) Without loss of generality, assume $f(x)>0$ for all $x>0$, and let $\underline{f}(x)=\min\{f(y):y\le x\}$. By definition, $\underline{f}(x)$ is non-increasing in x. For any MHR distribution F, $\frac{\overline{F}(x)}{f(x)}$ is non-increasing, and $\frac{\overline{F}(x)}{\underline{f}(x)}$ is also non-increasing. Now recall by Eq. (5), the revenue of optimal contract pricing is

$$\mathcal{R}_{CP}(c,F)=\max_p\left(p-\frac{c}{q}\left(1-e^{-qT}\right)\right)\overline{F}\left(\left(\frac{q-\log(\delta)}{q\left(1-\delta^T e^{-qT}\right)}\right)p\right).$$

Let optimal contract price for cost c be $p^*(c)$, by Lemma 2 $p^*(c)$ increases with and is lower bounded by c. Let $\tilde{p}(c)=\frac{qp^*(c)}{\left(1-e^{-qT}\right)}$, $v^*=\left(\frac{q-\log(\delta)}{q\left(1-\delta^T e^{-qT}\right)}\right)p^*(c)$, and $\tilde{v}=\frac{1}{2}\left(\frac{p^*(c)}{1-e^{-qT}}+v^*\right)$. Using Eq. (3) we can lower bound the revenue of the optimal subscription pricing by,

$$\begin{aligned}\mathcal{R}_{SP}(c,F)&=\max_p\left(\frac{p-c}{q}\right)\int_{\frac{p}{q}}^\infty\min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}},1-e^{-qT}\right\}f(v)dv\\&\ge\left(\frac{\tilde{p}(c)-c}{q}\right)\int_{\frac{\tilde{p}(c)}{q}}^\infty\min\left\{1-\left(\frac{\tilde{p}(c)}{vq}\right)^{\frac{-q}{\log(\delta)}},1-e^{-qT}\right\}f(v)dv\\&=\left(1-e^{-qT}\right)\left(\frac{\tilde{p}(c)-c}{q}\right)\int_{\frac{\tilde{p}(c)}{q}}^\infty\min\left\{\left(1-\left(\frac{\tilde{p}(c)}{vq}\right)^{\frac{-q}{\log(\delta)}}\right)\left(1-e^{-qT}\right)^{-1},1\right\}f(v)dv\\&\ge\left(1-e^{-qT}\right)\left(\frac{\tilde{p}(c)-c}{q}\right)\int_{\frac{\tilde{p}(c)}{q}}^\infty\left(1-\left(\frac{\tilde{p}(c)}{vq}\right)^{\frac{-q}{\log(\delta)}}\right)f(v)dv\\&=\left(p^*(c)-\frac{c}{q}\left(1-e^{-qT}\right)\right)\int_{\frac{p^*(c)}{1-e^{-qT}}}^\infty\left(1-\left(\frac{p^*(c)}{v\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)f(v)dv\\&\ge\left(p^*(c)-\frac{c}{q}\left(1-e^{-qT}\right)\right)\int_{\tilde{v}}^{v^*}\left(1-\left(\frac{p^*(c)}{v\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)\underline{f}(v)dv\\&\ge\left(p^*(c)-\frac{c}{q}\left(1-e^{-qT}\right)\right)\left(\left(1-\left(\frac{p^*(c)}{\tilde{v}\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)\underline{f}\left(v^*\right)\left(v^*-\tilde{v}\right)\right),\end{aligned}$$

where the first inequality follows from the fact that $\tilde{p}(c)=\frac{p^*(c)}{\left(1-e^{-qT}\right)}$ is only a feasible subscription price, the second equality follows from taking $\left(1-e^{-qT}\right)$ out of the minimum, the second inequality follows from $\left(1-\left(\frac{\tilde{p}(c)}{vq}\right)^{\frac{-q}{\log(\delta)}}\right)\le\left(1-\left(\frac{\tilde{p}(c)}{vq}\right)^{\frac{-q}{\log(\delta)}}\right)\left(1-e^{-qT}\right)^{-1}$, the third inequality follows from $\underline{f}(v)\le f(v)$

and $\left(1-\left(\frac{p^*(c)}{v\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)f(v)\geq 0$, the fourth inequality follows since $\underline{f}(x)$ is non-increasing and $1-\left(\frac{p^*(c)}{v\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}$ is increasing in v. Therefore,

$$\begin{aligned}\frac{\mathcal{R}_{SP}(c,F)}{\mathcal{R}_{CP}(c,F)}&\geq\left(\left(1-\left(\frac{p^*(c)}{\tilde{v}\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)(v^*-\tilde{v})\right)\frac{\underline{f}(v^*)}{\overline{\overline{F}}(v^*)}\\&\geq\left(\left(1-\left(\frac{p^*(c)}{\tilde{v}\left(1-e^{-qT}\right)}\right)^{\frac{-q}{\log(\delta)}}\right)(v^*-\tilde{v})\right)\frac{\underline{f}(0)}{\overline{\overline{F}}(0)},\end{aligned}$$

the second inequality follows from F is MHR. Note $\frac{p^*(c)}{\tilde{v}}$ is a constant with respect to q,δ,T, and $v^*-\tilde{v}$ tends infinity as $p^*(c)$ tends to infinity. Therefore, the ratio will be larger than 1 when $p^*(c)$ is sufficiently large, implying $\mathcal{R}_{CP}(c,F)\leq\mathcal{R}_{SP}(c,F)$ for all c sufficiently large, as desired.

□

B.3. Omitted Proofs of Section 4

Proof of Lemma 3 First, let $\underline{v}$ be the lowest valuation a user will pay the contract price, we can rewrite the revenue of the contract pricing as

$$\mathcal{R}_{CP}(c,F)=\left(\frac{\underline{v}q\left(1-\delta^Te^{-qT}\right)}{q-\log(\delta)}-\frac{c}{q}\left(1-e^{-qT}\right)\right)\overline{F}(\underline{v}),$$

The first-order condition for the optimal contract price is then,

$$\frac{\partial\mathcal{R}_{CP}(c,F)}{\partial\underline{v}}=\frac{q\left(1-\delta^Te^{-qT}\right)}{q-\log(\delta)}\overline{F}(\underline{v})-\left(\frac{\underline{v}q}{q-\log(\delta)}-\frac{c}{q}\left(1-e^{-qT}\right)\right)f(\underline{v})=0. \tag{EC.9}$$

By Lemma 2 the optimal contract price is unique in p and thus its unique in $\underline{v}$, so let $\underline{v}^*$ be the solution of Eq. (EC.9), and $\tilde{p}=\left(\frac{v^*q^2}{q-\log(\delta)}\right)\left(\frac{1-\delta^Te^{-qT}}{1-e^{-qT}}\right)$ be a feasible subscription price. In the proof for the uniqueness of optimal subscription price (Lemma 2), we also show that the subscription revenue is unimodal over the subscription price p. If the first order derivative of subscription revenue is larger than 0 at $\tilde{p}$, by gradient method, the optimal subscription price should be higher than $\tilde{p}$, then we have the price dominance.

Now, by Eq. (3) the revenue of optimal subscription pricing is,

$$\mathcal{R}_{SP}(c,F)=\max_p\left(\frac{p-c}{q}\right)\int_{\frac{p}{q}}^{\infty}\min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}},1-e^{-qT}\right\}f(v)dv.$$

For the case $1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}\leq 1-e^{-qT}$, then $\min\left\{1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}},1-e^{-qT}\right\}=1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}}$, and the first order derivative of Eq. (3) at $\tilde{p}$ is

$$\begin{aligned}\frac{\partial\mathcal{R}_{SP(c,F)}}{\partial p}|_{\tilde{p}}&=\frac{1}{q}\int_{\tilde{p}q^{-1}}^{\infty}\left[\left(1-\left(\frac{\tilde{p}}{vq}\right)^{\frac{-q}{\log(\delta)}}\right)+\frac{q}{\log(\delta)}\left(\frac{\tilde{p}-c}{\tilde{p}}\right)\left(\frac{\tilde{p}}{vq}\right)^{\frac{-q}{\log(\delta)}}\right]f(v)dv\\&\geq\frac{1}{q}\int_{\tilde{p}q^{-1}}^{\infty}\left[1-\left(1-\frac{q}{\log(\delta)}\right)\left(\frac{\tilde{p}}{vq}\right)^{\frac{-q}{\log(\delta)}}\right]f(v)dv\\&\geq\frac{1}{q}\left[1-\left(1-\frac{q}{\log(\delta)}\right)\left(\frac{\tilde{p}}{v^*q}\right)^{\frac{-q}{\log(\delta)}}\right]\overline{F}(v^*)\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{q}\left[1-\left(1-\frac{q}{\log(\delta)}\right)\left(\left(\frac{q}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)\right)^{\frac{-q}{\log(\delta)}}\right]\overline{F}(v^*)\\
&\geq \frac{1}{q}\left[1-\left(1-\frac{q}{\log(\delta)}\right)\left(\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}}\right)^{\frac{-q}{\log(\delta)}}\right]\overline{F}(v^*)\\
&\geq 0,
\end{aligned}
$$

the first inequality is induced by $\frac{\tilde{p}-c}{\tilde{p}} \leq 1$ when $c \geq 0$, the second inequality follows from the fact that $\left[1-\left(1-\frac{q}{\log(\delta)}\right)\left(\frac{\tilde{p}}{vq}\right)^{\frac{-q}{\log(\delta)}}\right]$ is increasing in v and the generalized Markov's inequality, the second equality follows from plugging in the subscription price is $\tilde{p}$, the third inequality is derived from (C1), namely $\left(1-\frac{q}{\log(\delta)}\right)^{\frac{\log(\delta)}{q}} \geq \left(\frac{q}{q-\log(\delta)}\right)\left(\frac{1-\delta^t e^{-qt}}{1-e^{-qt}}\right)$, by simplifying we obtain the final inequality.

For the opposite case $1-\left(\frac{p}{vq}\right)^{\frac{-q}{\log(\delta)}} \geq 1-e^{-qT}$, and let $\tilde{v}=\frac{\tilde{p}}{q}=\left(\frac{v^* q}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)$, the first order derivative of Eq. (3) at $\tilde{p}$ is

$$
\begin{aligned}
\frac{\partial \mathcal{R}_{SP(c,F)}}{\partial p}|_{\tilde{p}} &= \left(\frac{1-e^{-qT}}{q}\right)\left(\overline{F}\left(\frac{\tilde{p}}{q}\right)-\left(\frac{\tilde{p}-c}{q}\right)f\left(\frac{\tilde{p}}{q}\right)\right)\\
&= \left(\frac{1-e^{-qT}}{q}\right)\left(\overline{F}\left(\tilde{v}\right)-\left(\left(\frac{v^* q}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)-\frac{c}{q}\right)f\left(\tilde{v}\right)\right)\\
&= f(\tilde{v})\left(\frac{1-e^{-qT}}{q}\right)\left(\frac{\overline{F}\left(\tilde{v}\right)}{f\left(\tilde{v}\right)}-\left(\frac{q}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)\left(\frac{\overline{F}(v^*)}{f(v^*)}\right)\right)\\
&\geq 0.
\end{aligned}
$$

where the second equality follows from plugging $\tilde{p}$, the third follows from Eq. (EC.9), the inequality follows from the fact that $\frac{\overline{F}(x)}{f(x)}$ is non-increasing, $\tilde{v} \leq v^*$, and $\left(\frac{q}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right) \leq 1$.

Thus in both cases, the first order derivative of the subscription revenue at $\tilde{p}=\left(\frac{v^* q^2}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)$ is larger than 0. Thus the optimal subscription price should be higher than $\tilde{p}=\left(\frac{v^* q^2}{q-\log(\delta)}\right)\left(\frac{1-\delta^T e^{-qT}}{1-e^{-qT}}\right)$, which is in turn higher than the optimal contract price $\left(\frac{v^* q}{q-\log(\delta)}\right)(1-\delta^T e^{-qT})$. Hence, we show the price dominance

$$
\arg\max_p \mathcal{R}_{SP}(p,c,F) \geq \left(\frac{q}{1-e^{-qT}}\right)\arg\max_p \mathcal{R}_{CP}(p,c,F).
$$

when (C1) holds.

□

B.4. Omitted Proofs from Section 4.2

Proof of Theorem 4. We'll prove each part separately.

Part a) First, we show the result for two matching rates $\{q_1, q_2\}$ with population sizes $\{t_1, t_2\}$. The revenue of (CP) with contract price p is,

$$
\mathcal{R}_{CP}(c,F) = (p - c\mathbb{E}[\text{Time on platform}|(\text{CP})])\,\overline{F}\left(\left(\frac{q-\log(\delta)}{q-q\delta^T e^{-qT}}\right)p\right).
$$

For any price p, the proportion of users which will pay the price depends only on the user supposed match rate q and is thus independent of the order in which candidate matches are shown. Thus, to compare the revenue of the two possible orderings, $\{q_2, q_1\}$ and $\{q_1, q_2\}$, we only need to compare the expected matching cost to the platform from each user, or equivalently, the expected time on the platform. Recall $X_C(q)$ was the expected time on the platform for a user who paid the contract price. When the platform chooses matching order $\{q_1, q_2\}$, the expected time on the platform for a user who paid the contract price is,

$$\begin{aligned} X_C(\{q_1,q_2\}) &= \mathbb{E}[\text{Time on platform}|(\text{CP})] \\ &= \int_0^{t_1} tq_1 e^{-tq_1}dt + e^{-t_1q_1}\int_0^{t_2}(t+t_1)\,q_2e^{-tq_2}dt + Te^{-(t_1q_1+t_2q_2)} \\ &= \int_0^{t_1} tq_1 e^{-tq_1}dt + e^{-t_1q_1}\left(\int_0^{t_2} tq_2e^{-tq_2}dt + t_1\left(1-e^{-q_2t_2}\right)\right) + Te^{-(t_1q_1+t_2q_2)}. \end{aligned}$$

Similarly, when the matching order is $\{q_2, q_1\}$, the expected time on the platform for a user who paid the contract price is

$$\begin{aligned} X_C(\{q_2,q_1\}) &= \mathbb{E}[\text{Time on platform}|(\text{CP})] \\ &= \int_0^{t_2} tq_2 e^{-tq_2}dt + e^{-t_2q_2}\int_0^{t_1}(t+t_2)\,q_1e^{-tq_1}dt + Te^{-(t_1q_1+t_2q_2)} \\ &= \int_0^{t_2} tq_2 e^{-tq_2}dt + e^{-t_2q_2}\left(\int_0^{t_1} tq_1e^{-tq_1}dt + t_2\left(1-e^{-t_2q_2}\right)\right) + Te^{-(t_1q_1+t_1q_1)}. \end{aligned}$$

Then,

$$\begin{aligned} &X_C(\{q_1,q_2\}) - X_C(\{q_2,q_1\}) \\ &= \left(1-e^{-t_2q_2}\right)\left(\int_0^{t_1} tq_1e^{-tq_1}dt + t_1e^{-t_1q_1}\right) - \left(1-e^{-t_1q_1}\right)\left(\int_0^{t_2} tq_2e^{-tq_2}dt + t_2e^{-t_2q_2}\right), \\ &= \frac{\left(1-e^{-t_1q_1}\right)\left(1-e^{-t_2q_2}\right)}{q_1} - \frac{\left(1-e^{-t_1q_1}\right)\left(1-e^{-t_2q_2}\right)}{q_2} \ge 0, \end{aligned}$$

where the final inequality follows from $q_2 \ge q_1$. Therefore, we conclude for any distribution F, $\mathcal{R}_{CP}(c, F, \{q_2, q_1\}) \ge \mathcal{R}_{CP}(c, F, \{q_1, q_2\})$.

Now, for k matching rates $\{q_1, \ldots, q_k\}$, with associated populations $\{t_1, \ldots, t_k\}$ let $T_i = \sum_{j=1}^{i} t_j$, for $i = 1, \ldots k$, and $T_0 = 0$. The expected time on the platform for a user who paid the contract price is

$$X_C(\{q_1,\ldots,q_k\}) = \sum_{i=1}^{k}\left(\int_0^{t_i}(t+T_{i-1})\,e^{-q_i(t+T_{i-1})}dt\right) + Te^{-qT}.$$

If we swap q_j and q_{j+1} in $\{q_1, \ldots, q_k\}$ where $1 \le j \le k-1$, the expected time on the platform for a user who paid the contract price becomes

$$\begin{aligned} X_C(\{q_1,\ldots q_{j+1},q_j,\ldots,q_k\}) = Te^{-qT} &+ \sum_{i=1}^{j-1}\left(\int_0^{t_i}(t+T_{i-1})\,e^{-q_i(t+T_{i-1})}dt\right) + \int_0^{t_{j+1}}(t+T_{j-1})e^{-q_{j+}(t+T_{j-1})}dt \\ &+ \int_0^{t_j}(t+T_{j-1}+t_{j+1})e^{-q_{j+1}\left(t+T_{j-1}+t_{j+1}\right)}dt + \sum_{i=j+2}^{k}\left(\int_0^{t_i}(t+T_{i-1})\,e^{-q_i(t+T_{i-1})}dt\right) \end{aligned}$$

Note all the other parts for the integrations won't change if we only swap two matching rates next to each other. Therefore, we can generalize the proof for order $\{q_1, \ldots, q_k\}$ by switching any two reverse orders that are next to each other. $\square$

Part b) As in part a), we first show the result for two matching rates $\{q_1,q_2\}$ with population sizes $\{t_1,t_2\}$. The revenue of (SP) with subscription price p is,

$$\mathcal{R}_{SP}(c,F)=\int_{\frac{p}{q}}^{\infty}(p-c)\times\mathbb{E}\left[\text{Time on platform}|(\text{SP})\right]f(v)dv.$$

The revenue comparison $\mathcal{R}_{SP}(c,F,\{t_1,...,t_k\}),\{q_1,...,q_K\})\leq\mathcal{R}_{SP}(c,F,\sigma(\{t_1,...,t_k\})),\sigma(\{q_1,...,q_k\}))$ will follow if we can show that for each user, the expected time on the platform will be longer for $\{q_1,...,q_K\}$. Therefore, we only need to show that for price p and fixed user valuation v, the expected time the user will stay on platform satisfies

$$X_S\left(v,p,\{q_1,q_2\}\right)\geq X_S\left(v,p,\{q_2,q_1\}\right).$$

If T is binding, the expected time on platform will be the same as contract model, therefore the conclusion is the same. Otherwise, let $\tau=\frac{\log(p/vq)}{\log(\delta)}$, when $\tau\leq\min\{t_1,t_2\}$,

$$X_S\left(v,p,\{q_1,q_2\}\right)=\int_0^{\tau}e^{-tq_1}dt,$$
$$X_S\left(v,p,\{q_2,q_1\}\right)=\int_0^{\tau}e^{-tq_2}dt.$$

Note that $e^{-tq_1}\geq e^{-tq_2}$, therefore,

$$X_S(v,p,\{q_1,q_2\})\geq X_S(v,p,\{q_2,q_1\}).$$

When $t_1\leq\tau\leq t_2$

$$X_S(v,p,\{q_1,q_2\})=\int_0^{t_1}e^{-tq_1}dt+e^{-qt_1}\int_0^{\tau-t_1}e^{-tq_2}dt,$$
$$X_S(v,p,\{q_2,q_1\})=\int_0^{t_1}e^{-tq_2}dt+e^{-qt_2}\int_0^{\tau-t_1}e^{-tq_2}dt.$$

Similarly, we can show that

$$X_S(v,p,\{q_1,q_2\})\geq X_S(v,p,\{q_2,q_1\})$$

for $t_2\leq\tau\leq t_2$ or $\max\{t_1,t_2\}\leq\tau\leq t_1+t_2$. Therefore, we can conclude that for any distribution F, $\mathcal{R}_{SP}(c,F,\{q_2,q_1\})\leq\mathcal{R}_{SP}(c,F,\{q_1,q_2\})$.

For matching order $\{q_1,\dots,q_k\}$, let $T_i=\sum_{j=1}^{i}t_j$, for $i=1,\dots k$, and $T_0=0$. If T is binding, the expected time on the platform for a user whose valuation is v under the subscription price p is

$$X_S(v,p,\{q_1,\dots,q_k\})=\sum_{i=1}^{k}\left(\int_0^{t_i}(t+T_{i-1})\,e^{-q_i(t+T_{i-1})}dt\right)+Te^{-qT},$$

and we can apply the proof in part a). Otherwise, let $q_{k'}$ be the k'-th matching rate such that $T_{k'}\leq\tau\leq T_{k'+1}$, the expected time on the platform for a user whose valuation is v under the subscription price p is

$$\begin{aligned}X_S(v,p,\{q_1,\dots,q_k\})=&\sum_{i=1}^{k'}\left(\int_0^{t_i}(t+T_{i-1})\,e^{q_i(t+T_{i-1})}dt\right)\\&+\int_0^{\tau-T_{k'}}(t+T_{k'})\,e^{q_i\left(t+T_{k'}\right)}dt+\tau e^{-\left(\sum_{i=1}^{k'}q_it_i+q_{k'+1}\left(\tau-T_{k'}\right)\right)}.\end{aligned}$$

If we swap matching rates q_j and q_{j+1} where $j<k'$, it will be the same as T is binding, if we swap matching rates q_j and q_{j+1} where $j\geq k'+1$, it will cause no difference for user's the expected time on the platform, if we swap $q_{k'}$ and $q_{k'+1}$, the expected before k' will stay as $\sum_{i=1}^{k'-1}\left(\int_0^{t_i}(t+T_{i-1})\,e^{q_i(t+T_{i-1})}dt\right)$, we only need to consider the difference in $q_{k'}$ and $q_{k'+1}$, which is analyzed above. Therefore, we can generalize the proof for order $\{q_1,...,q_k\}$ by switching any two reverse orders that next to each other. $\square$

Part c) As in part (a), let $X_C(\{q\})$ be the expected time a user who pays the contract price will spend on the platform when the match rate is q, and let $X_C(\{q_k,\dots,q_1\})$ be the expected time a user who pays the contract price will spend on the platform when the matching rate order is $\{q_k,\dots,q_1\}$. First we will show $X_C(\{q_k,\dots,q_1\}) \le X_C(\{q\})$, i.e., compared with uniform matching rate q, users will leave the platform sooner when match rates are in descending order. To this end, the revenue of contract pricing is

$$\mathcal{R}_{CP}(p,c,F,\{t_k,\dots,t_1\},\{q_k,\dots,q_1\}) = (p - cX_C(\{q_k,\dots,q_1\}))\,\overline{F}\left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}p\right).$$

Let p^* be the optimal contract price when the match rate is q, i.e. p^* such that,

$$\begin{aligned}\frac{\partial \mathcal{R}_{CP}(p,c,F,\{T\},\{q\})}{\partial p}|_{p^*} &= \overline{F}\left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}p^*\right) - \left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}\right)(p^* - cX_C(\{q\}))\,f\left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}p^*\right)\\ &= 0.\end{aligned}$$

Let $v^* = \frac{q-\log(\delta)}{q\left(1-\delta^T e^{-qT}\right)}p^*$, then, the first order derivative of $\mathcal{R}_{CP}(c,F,\{t_k,\dots,t_1\},\{q_k,\dots,q_1\})$ at p^* is

$$\begin{aligned}&\frac{\partial \mathcal{R}_{CP}(c,F,\{t_k,\dots,t_1\},\{q_k,\dots,q_1\})}{\partial p}|_{p^*}\\ &= \overline{F}(v^*) - \left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}\right)(p^* - cX_C(\{q_k,\dots,q_1\}))\,f(v^*)\\ &= \frac{\partial \mathcal{R}_{CP}(p,c,F,\{T\},\{q\})}{\partial p}|_{p^*} + c\left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}\right)(X_C(\{q\}) - X_C(\{q_k,\dots,q_1\})\,f(v^*)\\ &= c\left(\frac{q-\log(\delta)}{q\,(1-\delta^T e^{-qT})}\right)(X_C(\{q\}) - X_C(\{q_k,\dots,q_1\}))\,f(v^*) \ge 0,\end{aligned}$$

where the second equality follows from minus $X_C(\{q\})$, then add $X_C(\{q\})$ back, the third equality follows from $\frac{\partial \mathcal{R}_{CP}(p,c,F,\{T\},\{q\})}{\partial p}|_{p^*} = 0$, the inequality follows from $X_C(\{q_k,\dots,q_1\}) \le X_C(\{q\})$. Thus to maximize the revenue, the optimal contract price for $\mathcal{R}_{CP}(c,F,\{t_k,\dots,t_1\},\{q_k,\dots,q_1\})$ should be higher than p^*. $\square$