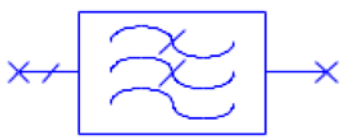
Elliptic-Function Lowpass Filter (Closed Form): LPFE

Symbol



Summary

LPFE models represent lumped-element elliptic-function (Cauer) lowpass filters. The insertion loss ripples between zero and a specified maximum in the passband. The stopband attenuation increases rapidly between the passband edge and the stopband edge, and ripples between a specified minimum stopband attenuation and infinite attenuation. This type of filter offers optimum selectivity at the expense of increased complexity and poor group delay flatness.

Parameters

Name	Description	Unit Type	Default
ID	Element ID	Text	LPFE1
N	Order of the filter.		3
FP	Passband corner frequency (when Qu is infinite).	Frequency	1 GHz
AP	Maximum Passband Insertion Loss (when Qu is infinite).	DB	0.1 dB
AS	Minimum Stopband Attenuation(when Qu is infinite).	DB	20 dB
*FS	Stopband corner frequency (when Qu is infinite).	Frequency	0 GHz
*DM	Where to put design margin:0=AP, 1=AS, 2=FP, 3=FS.		1
*RS	Source resistance.	Resistance	50 ohm

Name	Description	Unit Type	Default
*RL	Load resistance	Resistance	50 ohm
*QU	Average unloaded Q of reactive elements in the filter.		1e12

** indicates a secondary parameter*

Parameter Details

An elliptic function lowpass prototype filter is completely specified by any four of the five parameters: N, FP, FS, AP, and AS. That is, the value of any parameter is dependent on the value of the other four parameters.

- If zero is specified for any one of these five parameters, then the model computes its value from the value of the other four parameters.
- If a value is specified for all five parameters, then the specified value of the last parameter (AS) is ignored and is computed by the model from the values of the other four parameters.
- It is an error to specify zero for more than one of these five parameters.

N. In mathematical terms, N is defined as the highest exponent of the complex frequency variable s in the transfer function, S21(s0), of the filter's normalized lowpass prototype, or, equivalently, the highest exponent of s in the transfer function of the lowpass filter. And, in terms of a measurable electrical characteristic, the number of positive-frequency passband reflection (|S11|) zeros corresponds to N/2 for N even and (N+1)/2 for N odd.

DM. If zero is specified for N, the model will determine N. But, since N must be an integer, there will typically be some design margin available, and this margin must be assigned to some parameter other than N. The value of DM determines where the model will assign this design margin.

- DM=0. After determining N, the model will recompute AP, such that 0 < new AP ≤ AP
- DM=1. After determining N, the model will recompute AS, such that AS ≤ new AS < ∞
- DM=2. After determining N, the model will recompute FP, such that FS < new FP ≤ FP
- DM=3. After determining N, the model will recompute FS, such that FS ≤ new FS < FP

Parameter Restrictions and Recommendations

1. N can be zero if FP, FS, AP, and AS are not, otherwise 0 < N < 27.
2. FP can be zero if N, FS, AP, and AS are not, otherwise 0 < FP, and, if FS is not zero, then FS < FP.

   FS can be zero if N, FP, AP, and AS are not, otherwise 0 < FS, and, if FP is not zero, then FS < FP.
3. AP can be zero if N, FP, FS, and AS are not, otherwise 0 < AP, and, if AS is not zero, then AP < AS.

   Recommend AP greater than or equal to 0.001 dB.
4. AS can be zero if N, FP, FS, and AP are not, otherwise AP < AS.
5. If N is set to 0, then 0 ≤ DM < 4
6. 0 < RS.

0 < RL

7. QU > 0 specifies a finite unloaded Q (recommend QU ≤ 1e12).

   QU = 0 specifies an infinite unloaded Q.

8. 0 ≤ DM < 4

Implementation Details

The model is implemented as a short-circuit admittance matrix,

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

, whose equivalent normalized lowpass prototype transfer function, S21(s), is:

$$|\ S_{21}(s)|^2 = \frac{1}{1+\varepsilon^2 R_n^2\left(\frac{s}{j}\right)} = \frac{f(s)^2}{|\ g(s)|^2} = \frac{f(s)^2}{1+\ |\ h(s)|^2}$$

where Rn is the Elliptic Rational Function, and

$$|\ S_{21}(s)|^2 = \frac{1}{1+\varepsilon^2 R_n^2\left(\frac{s}{j}\right)} = \frac{f(s)^2}{|\ g(s)|^2} = \frac{f(s)^2}{1+\ |\ h(s)|^2}$$

where Rn is the Elliptic Rational Function, and

$$\varepsilon^2 = 10^{\mathrm{AP}/10} - 1$$

$$R_n(\omega) = C \times \omega^{\Pi} \times \prod_{i=1}^{\frac{N-\Pi}{2}} \left(\frac{\omega^2 - \omega_i^2}{\omega^2 - \frac{\omega_s^2}{\omega_i^2}} \right)$$

$$C = \prod_{i=1}^{\frac{N-\Pi}{2}} \left(\frac{1 - \frac{\omega_s^2}{\omega_i^2}}{1 - \omega_i^2} \right)$$

$$\omega_i = \mathrm{cd}\left(\frac{2i-1}{N} K\left(\frac{1}{\omega_s^2} \right), \frac{1}{\omega_s^2} \right)$$

$$s = 1 \Big/ \left(\frac{\sqrt{\mathrm{FP2} \times \mathrm{FP1}}}{\mathrm{QU} \times\ |\ \mathrm{FP2} - \mathrm{FP1}\ |} + j\left(\frac{-1}{\omega} \right) \right)$$

$$s = \frac{1}{\mathrm{QU}} + j\omega$$

$$j = \sqrt{-1}$$

where cd is a Jacobian elliptic function, K is Legendre's complete elliptic integral of the first kind, and a lowpass-to-lowpass frequency transformation has been applied:

$$\omega = \frac{\_FREQ}{FP}$$

and

$$\omega_s = \frac{FS}{FP}$$

_FREQ is the variable containing the project frequency. The parameters of the elliptic-function filter (N, FP, FS, AP, AS) are related by "the degree equation":

$$N = \left(\frac{K(m)}{K(1-m)}\right)\left(\frac{K(1-m_o)}{K(m_o)}\right)$$

where

$$m_o = \frac{10^{AP/10}-1}{10^{AS/10}-1}$$

and

$$m = \left(\frac{FP}{FS}\right)^2$$

The admittances are given by:

$$y_{11} = \left(\frac{1}{RS}\right)\frac{g(s)+g(-s)-h(s)-h(-s)}{g(s)-g(-s)+h(s)-h(-s)}$$

$$y_{22} = \left(\frac{1}{RL}\right)\frac{g(s)+g(-s)+h(s)+h(-s)}{g(s)-g(-s)+h(s)-h(-s)}$$

$$m = \left(\frac{FS}{FP}\right)^2$$

Layout

This element does not have an assigned layout cell. You can assign artwork cells to any element. See “Assigning Artwork Cells to Layout of Schematic Elements” for details.

Recommendations for Use

Note that this model behaves as if it has ideal impedance transformers at its ports, so there is no attenuation due to mismatched source and load impedances. The model expects that the source impedance equals RS and that the load impedance equals RL, but RS need not have any special relationship to RL for ideal transmission (as would normally be the case).

References

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[8] Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables, edited by Milton Abramowitz and Irene A. Stegun, (U. S. National Bureau of Standards, 1964), Chapters 16 and 17 by L. M. Milne-Thomson.

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