Appendix D

Useful tables and formulae

D.1 The deciBel

The deciBel (dB) represents a logarithmic ratio between two quantities. Of itself it is unitless. If the ratio is referred to a specific quantity (P_2, V_2 or I_2 below) this is indicated by a suffix, e.g. dBμV is referred to 1μV, dBm is referred to 1mW.

Common suffixes

suffix	refers to	suffix	refers to
dBV	1 volt	dBA	1 amp
dBmV	1 millivolt	dBμA	1 microamp
dBμV	1 microvolt	dBμA/m	1 microamp per metre
dBV/m	1 volt per metre	dBW	1 watt
dBμV/m	1 microvolt per metre	dBm	1 milliwatt
		dBμW	1 microwatt

Originally the dB was conceived as a power ratio, hence it is given by:

$$\text{dB} = 10 \log_{10} (P_1/P_2)$$

Power is proportional to voltage squared, hence the ratio of voltages or currents across a constant impedance is given by:

$$\text{dB} = 20 \log_{10} (V_1/V_2) \quad \text{or} \quad 20 \log_{10} (I_1/I_2)$$

Conversion between voltage in dBμV and power in dBm for a given impedance Z ohms is:

$$V(\text{dB}\mu\text{V}) = 90 + 10 \log_{10} (Z) + P(\text{dBm})$$

dBμV versus dBm for Z = 50Ω

dBμV	μV	dBm	pW	dBμV	mV	dBm	nW
–20	0.1	–127	0.0002	30	0.03162	–77	0.02
–10	0.316	–117	0.002	40	0.10	–67	0.2
				50	0.3162	–57	2.0
0	**1.0**	**–107**	**0.02**	60	1.0	–47	20.0
							μW
5	1.778	–102	0.063	70	3.162	–37	0.2
7	2.239	–100	0.1	80	10.0	–27	2.0
10	3.162	–97	0.2	90	31.62	–17	20.0
15	5.623	–92	0.632	100	100.0	–7	200.0
20	10.0	–87	2.0	**120**	**1.0V**	**+13**	**20mW**

Actual voltage, current or power can be derived from the antilog of the dB value:

V = $\log^{-1}$ (dBV/20) volts
I = $\log^{-1}$ (dBA/20) amps
P = $\log^{-1}$ (dBW/10) watts

Table of ratios

dB	Voltage or current ratio	Power ratio	dB	Voltage or current ratio	Power ratio
–30	0.0316	0.001	12	3.981	15.849
–20	0.1	0.01	14	5.012	25.120
–10	0.3162	0.1	16	6.310	39.811
–6	0.501	0.251	18	7.943	63.096
–3	0.708	0.501	20	10.000	100.00
0	**1.000**	**1.000**	25	17.783	316.2
1	1.122	1.259	30	31.62	1000
2	1.259	1.585	35	56.23	3162
3	1.413	1.995	40	100.0	10,000
4	1.585	2.512	45	177.8	31,623
5	1.778	3.162	50	316.2	10^5
6	1.995	3.981	60	1000	10^6
7	2.239	5.012	70	3162	10^7
8	2.512	6.310	80	10,000	10^8
9	2.818	7.943	90	31,623	10^9
10	3.162	10.000	100	10^5	10^{10}
			120	10^6	10^{12}

D.2 Antennas

D.2.1 Antenna factor

AF = E – V
where AF = antenna factor, dB/m
E = field strength at the antenna, dBμV/m
V = voltage at antenna terminals, dBμV

D.2.2 Gain versus antenna factor

G = 20 log F – 29.79 – AF
where G = gain over isotropic antenna, dBi
F = frequency, MHz
AF = antenna factor, dB/m

D.2.3 Dipoles

Gain of a λ/2 dipole over an isotropic radiator:

G = 1.64 or 2.15dB

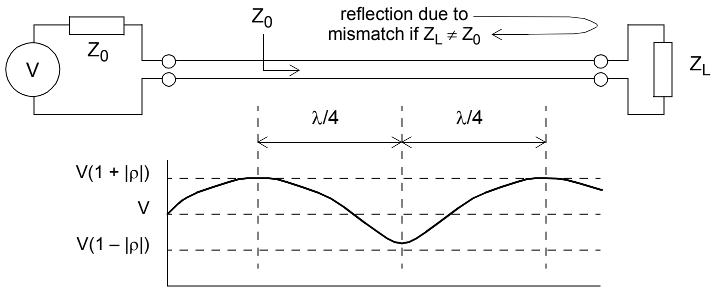
Input resistance of short dipoles of length L [20]:

$0 < L < \lambda/4$: $R_{in} = 20 \cdot \pi^2 \cdot (L/\lambda)^2$ ohms

$\lambda/4 < L < \lambda/2$: $R_{in} = 24.7 \cdot (\pi \cdot L/\lambda)^{2.4}$ ohms

D.2.4 VSWR

The term Voltage Standing Wave Ratio (VSWR) describes the degree of mismatch between a transmission line and its source or load. It also describes the amplitude of the standing wave that exists along the line as a result of the mismatch.



VSWR K $= (1 + |\rho|)/(1 - |\rho|) = (Z_0/Z_L)$ or (Z_L/Z_0)

Reflection coefficient $|\rho| = (K - 1)/(K + 1)$

D.3 Fields

D.3.1 The wave impedance

In free space:

$$Z_0 = (\mu_0/\varepsilon_0)^{0.5} = E/H = 377\Omega \text{ or } 120\pi$$

μ_0 = permeability of free space = $4\pi \cdot 10^{-7}$ Henries per metre
ε_0 = permittivity of free space = $8.85 \cdot 10^{-12}$ Farads per metre

D.3.2 Near field / far field

$d < \lambda/2\pi$: near field; $d > \lambda/2\pi$: far field

(see Figure 10.9 on page 231)

In the near field, the impedance is either higher or lower than Z_0 depending on its source. For a high-impedance field of F Hz at distance d metres due to an electric dipole:

$$|Z| = 1/(2\pi F \cdot \varepsilon \cdot d)$$

For a low-impedance field due to a current loop:

$$|Z| = 2\pi F \cdot \mu \cdot d$$

D.3.3 Power density

Conversion from field strength to power density in the far field:

$$P = E^2/(120\cdot\pi)$$

where P = power density, mW/cm^2
E is field strength, volts/metre

or for an isotropic antenna:

$$P = P_T/4\pi \cdot R^2$$

where R is distance in metres from source of power P_T watts

D.3.4 Field strength

For an equivalent radiated power of P_T, the field strength in free space at R metres from the transmitter is:

$$E = (30 \cdot P_T)^{0.5} / R$$

or $$E\ (\text{mV/m}) = 173 \cdot (P_T \text{ in kW})^{0.5} / (R \text{ in km})$$

Propagation near the ground is attenuated at a greater rate than 1/R. For the frequency range between 30 and 300MHz and distances greater than 30 metres, the median field strength varies as $1/R^n$ where n ranges from about 1.3 for open country to 2.8 for heavily built-up urban areas [161].

D.3.5 Field strength from a small loop or monopole [11]

"Small" in this context means substantially shorter than $\lambda/4$. For a loop in free space of area A m^2 carrying current I amps at a frequency f Hz, electric field at distance R metres and an elevation angle θ is:

$$E = 131.6 \cdot 10^{-16} (f^2 \cdot A \cdot I)/R \cdot \sin\theta \quad \text{volts/metre}$$

Correcting for ground reflection (x2) with a measuring distance of 10m at maximum orientation:

$$E = 26.3 \cdot 10^{-16} (f^2 \cdot A \cdot I) \quad \text{volts/metre}$$

Short monopole of length L (<< $\lambda/2$) over ground plane at distance R driven by common mode current I:

$$E = 4\pi \cdot 10^{-7} \cdot (f \cdot I \cdot L)/R \cdot \sin\theta \quad \text{volts/metre}$$

Maximum orientation at 10m:

$$E = 1.26 \cdot 10^{-7} \cdot (f \cdot I \cdot L) \quad \text{volts/metre}$$

D.3.6 Field strength from a resonant cable [17]

When cable length approaches or exceeds a wavelength, the resonances drastically change the emission patterns resulting in multiple lobes depending on the L/λ ratio. Maximum field intensity occurs when the radiator length is $\lambda/2$. Now the field strength (uncorrected for ground reflections, since these are unpredictable) is:

$$E_\theta = \{(60 \cdot I)/R\} \cdot \{\cos(\beta \cdot L \cdot \cos\theta/2) - \cos(\beta \cdot L/2)\}/\sin\theta$$

where β is the phase constant $2\pi/\lambda$.

D.3.7 Electric versus magnetic field strength

In the far field the electric and magnetic field strengths are related by the impedance of free space, Z_o (377Ω):

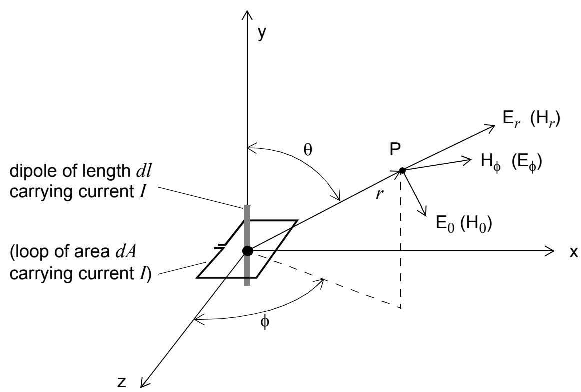
$$E(\text{dB}\mu\text{V/m}) = H(\text{dB}\mu\text{A/m}) + 51.5$$

H can be expressed in Amps per metre, Tesla or Gauss:

1 Gauss	=	100μT	=	79.5 A/m
1 A/m	=	$4\pi \cdot 10^{-7}$ T		

D.3.8 The field equations [8]

The following equations characterize the E and H fields at a point P due to an elementary electric dipole (current filament) and an elementary magnetic dipole (current loop). They use the spherical co-ordinate system shown below.



For the electric dipole:

$$E_r = Idl\cos\theta\left(\frac{\beta^3}{2\pi\omega\varepsilon_0}\right)\left(\frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right)e^{-j\beta r}$$

$$E_\theta = Idl\sin\theta\left(\frac{\beta^3}{4\pi\omega\varepsilon_0}\right)\left(\frac{j}{(\beta r)} + \frac{1}{(\beta r)^2} - \frac{j}{(\beta r)^3}\right)e^{-j\beta r}$$

$$H_\phi = Idl\sin\theta\left(\frac{\beta^2}{4\pi\omega}\right)\left(\frac{j}{(\beta r)} + \frac{1}{(\beta r)^2}\right)e^{-j\beta r}$$

For the magnetic dipole:

$$H_\theta = IdA\sin\theta\left(\frac{\beta^3}{4\pi}\right)\left(\frac{-1}{(\beta r)} + \frac{j}{(\beta r)^2} + \frac{1}{(\beta r)^3}\right)e^{-j\beta r}$$

$$H_r = IdA\cos\theta\left(\frac{\beta^3}{2\pi}\right)\left(\frac{j}{(\beta r)^2} + \frac{1}{(\beta r)^3}\right)e^{-j\beta r}$$

$$E_\phi = IdA \sin\theta\left(\frac{\beta^4}{4\pi\omega\varepsilon_0}\right)\left(\frac{1}{(\beta r)} - \frac{j}{(\beta r)^2}\right)e^{-j\beta r}$$

In all the above,

β = the phase constant $2\pi/\lambda$
ω = the angular frequency of I in rad/s
ε_0 = the permittivity of free space (see D.3.1)
r, θ describe the co-ordinates of point P
E_r, E_θ, E_ϕ are the electric field vectors in V/m
H_r, H_θ, H_ϕ are the magnetic field vectors in A/m

These equations show that:

a) for $\beta r << 1$ (the near field) the higher order terms dominate with E varying as $1/r^3$ and H as $1/r^2$ for the electric dipole, and vice versa for the magnetic. The $1/r^2$ terms are known as the induction field.

b) for $\beta r >> 1$ (the far field) the radial term (E_r or H_r) becomes insignificant and the transverse terms (θ and ϕ) propagate as a plane wave, varying as $1/r$.

D.4 Shielding

D.4.1 Skin depth

δ = $(\pi \cdot F \cdot \mu \cdot \sigma)^{-0.5}$ metres

For a conductor with permeability μ_r and conductivity σ_r, F in Hz:

δ = $0.0661 \cdot (F \cdot \mu_r \cdot \sigma_r)^{-0.5}$ metres
or $2.602 \cdot (F \cdot \mu_r \cdot \sigma_r)^{-0.5}$ inches

Typical skin depth for copper ($\mu_r = \sigma_r = 1$) is 66µm ($6.6.10^{-5}$ m) at 1MHz, 6.6µm at 100MHz

D.4.2 Reflection loss (R)

The magnitude of reflection loss depends on the ratio of barrier impedance to wave impedance, which in turn depends on its distance from the source and whether the field is electric or magnetic (in the near field) or whether it is a plane wave (in the far field). The following expressions are for F in Hz, r in metres and μ_r and σ_r as shown above.

R	=	$168 - 10 \cdot \log_{10}((\mu_r/\sigma_r) \cdot F)$ dB	Plane wave
R_E	=	$322 - 10 \cdot \log_{10}((\mu_r/\sigma_r) \cdot F^3 \cdot r^2)$ dB	Electric field
R_H	=	$14.6 - 10 \cdot \log_{10}((\mu_r/\sigma_r)/F \cdot r^2)$ dB	Magnetic field

D.4.3 Absorption loss (A)

A = $8.69 \cdot (t/\delta)$ dB
where t is barrier thickness, δ is skin depth

D.4.4 Re-reflection loss (B)

B = $20 \cdot \log_{10}(1 - e^{-2\sqrt{2}(t/\delta)})$ dB

B is negligible unless material thickness t is less than the skin depth δ;
e.g. if $t = \delta$, $B = -0.53$dB; if $t = 2\delta$, $B = -0.03$dB
B is always a negative value, since multiple reflections degrade shielding effectiveness

D.4.5 Shielding effectiveness (see section 14.1.1)

Intrinsic shielding effectiveness of a homogeneous conducting barrier of infinite extent:

$$SE_{dB} = R_{dB} + A_{dB} + B_{dB}$$

Properties of typical conductors [14]

Material	Relative conductivity σ_r (copper = 1)†	Relative permeability @ 1kHz * μ_r
Silver	1.08	1
Copper	1.00	1
Gold	0.70	1
Chromium	0.66	1
Aluminium	0.61	1
Zinc	0.30	1
Tin	0.15	1
Nickel	0.22	50–60
Mild steel	0.10	300–600
Mu-metal	0.03	20,000

*: relative permeability approaches 1 above 1MHz for most materials
†: absolute conductivity of copper is $5.8 \cdot 10^7$ mhos

D.5 Capacitance, inductance and PCB layout

D.5.1 Capacitance

Capacitance between two plates of area A cm^2 spaced d cm apart in free space:

$$C = 0.0885 \cdot A/d \text{ pF}$$

The self-capacitance of a sphere of radius r cm:

$$C = 4\pi \cdot 0.0885 \cdot r = 1.1 \cdot r \text{ pF}$$

The capacitance per unit length between concentric circular cylinders of inner radius r_1, outer radius r_2 in free space:

$$C = 2\pi \cdot 0.0885 / \ln(r_2/r_1) \text{ pF/cm}$$

The capacitance per unit length between two conductors of diameter d spaced D apart in free space:

$$C = \pi \cdot 0.0885 / \cosh^{-1}(D/d) \text{ pF/cm}$$

The factor 0.0885 in each of the above equations is due to the permittivity of free space ε_0 (see D.3.1); multiply by the dielectric constant or relative permittivity ε_r for other materials.

Relative permittivities ε_r of some dielectrics

Air	1.0
PTFE (Teflon)	2.1
Polyethylene	2.3
Polystyrene	2.5
PVC, Polycarbonate	3.2
Polyimide	3.4
Epoxy glass	4.2–4.7
Glass (borosilicate)	5.0
Porcelain	5.5
Phenolic resin fabric	5.5
Alumina (pure)	8.5
Methanol @ 900MHz	31
De-ionised water	80

D.5.2 Inductance [6]

The inductance of a straight length of wire of length l and diameter d:

$$L = 0.0051 \cdot l \cdot (\ln(4l/d) - 0.75)\ \mu\text{H for } l\text{, d in inches, or}$$
$$0.002 \cdot l \cdot (\ln(4l/d) - 0.75)\ \mu\text{H for } l\text{, d in cm}$$

A useful rule of thumb is 20nH/inch.

The inductance of a return circuit of parallel round conductors of length l cm, diameter d and distance apart D, for $D/l << 1$:

$$L = 0.004 \cdot l \cdot (\ln(2D/d) - D/l + 0.25)\ \mu\text{H}$$

The mutual inductance between two parallel straight wires of length l cm and distance apart D, for $D/l << 1$:

$$M = 0.002 \cdot l \cdot (\ln(2l/D) - 1 + D/l)\ \mu\text{H}$$

The mutual inductance between two conductors spaced D apart at height h over a ground plane carrying its return current [98]:

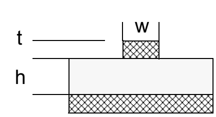
$$M = 0.001 \cdot \ln(1 + (2h/D)^2)\ \mu\text{H/cm}$$

The inductance of a single wire of diameter d at height h over a ground plane carrying its return current [98]

$$L = 0.002 \cdot \ln(4h/d)\ \mu\text{H/cm}$$

D.5.3 PCB track propagation delay and characteristic impedance [100]

<u>Surface microstrip</u>



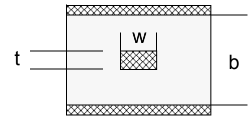
$$T_{pd} = 1.017 \cdot \sqrt{(0.475 \cdot \varepsilon_r + 0.67)}\ \text{ns/ft}$$

$$Z_0 = (87/\sqrt{(\varepsilon_r + 1.41)}) \cdot \ln[5.98h/(0.8w + t)]\ \Omega$$

e.g. for h = 1.6mm, w = 0.3mm, $\varepsilon_r = 4.2$ and t << w in surface microstrip, $Z_0 = 130\Omega$

NB [42] shows that this equation (which is widely referenced) is inaccurate compared to numerical model results for lower values of Z_0, i.e. large values of w/h, and gives a more accurate but more complex equation derived from Wadell [137]

<u>Embedded stripline</u>



T_{pd} = $1.017 \cdot \sqrt{\varepsilon_r}$ ns/ft

Z_0 = $(60/\sqrt{\varepsilon_r}) \cdot \ln[4b/(0.67\pi \cdot (0.8w + t))]\ \Omega$

For FR4 epoxy fibreglass PCB material at high frequencies ε_r is typically 4.2 which gives a propagation delay T_{pd} of 1.7ns/ft (56ps/cm) for surface microstrip and 2.1ns/ft (69ps/cm) for embedded stripline.

e.g. for b = 1.6mm, w = 0.3mm, ε_r = 4.2 and t << w in embedded stripline, Z_0 is 74Ω.

When a track is loaded with devices, their capacitances modify the track's propagation delay and Z_0 as follows:

T_{pd}' = $T_{pd} \cdot \sqrt{(1 + C_D/C_o)}$

Z_0' = $Z_0/\sqrt{(1 + C_D/C_o)}$

where C_D is the distributed device capacitance per unit length, i.e. the total load capacitance divided by the track length, and C_o is the intrinsic capacitance of the track calculated from:

C_o = $1000 \cdot (T_{pd}/Z_0)$ pF/length

D.5.4 *Distributed coupling [19]*

For a two-wire transmission line coupled to a plane wave electromagnetic field:

s = line length in m, b = vertical wire separation in m, a = wire diameter in m

Z_1 = source end impedance in ohms, Z_2 = load end impedance in ohms, Z_0 is line characteristic impedance, derived from geometry: $Z_0 = 276 \cdot \log(2b/a)$

β is phase constant = 2π/λ

D is "denominator function":

$$D = (Z_0 \cdot Z_1 + Z_0 \cdot Z_2) \cdot \cosh(j \cdot \beta \cdot s) + (Z_0^2 + Z_1 \cdot Z_2) \cdot \sinh(j \cdot \beta \cdot s)$$

There can be three equations for different wave conditions (variable X is the coupling factor, V/E, for the load end):

(a) E vertical, travelling towards line

$$X = Z_2 \cdot b/D \cdot (Z_0 \cdot (1 - \cos\beta s) + Z_1 \cdot j \cdot \sin\beta s)$$

(b) E vertical, travelling along line

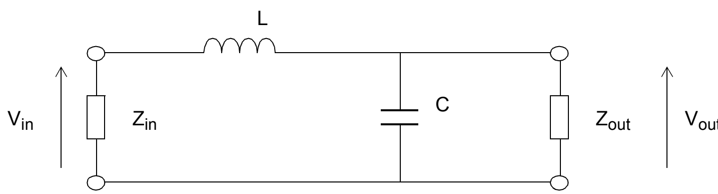
$$X = Z_2 \cdot \frac{b \cdot (Z_0 - Z_1)}{2D} \cdot ((1 - \cos 2\beta s) + j \cdot \sin 2\beta s)$$

(c) E horizontal, travelling towards line

$$X = Z_2 \cdot \left(\frac{-2j \cdot \sin\beta\frac{b}{2}}{D \cdot \beta} \right) \cdot (Z_0 \cdot \sin\beta s + Z_1 \cdot j \cdot (1 - \cos\beta s))$$

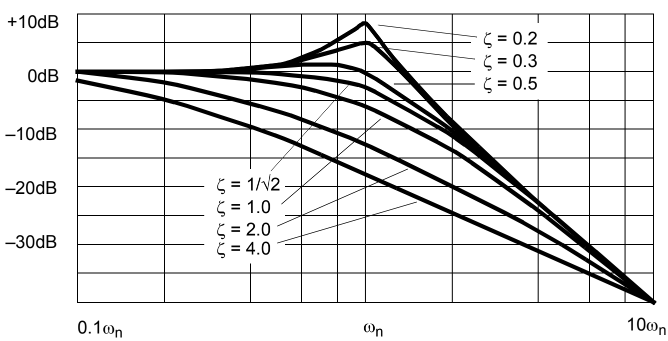
D.6 Filters

D.6.1 Second order low-pass filter [124]



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + 2j\zeta(\omega/\omega_n) - (\omega/\omega_n)^2} \qquad \omega_n = 1/(\sqrt{LC})$$

$$\zeta = \frac{L}{2Z_{out}\sqrt{LC}}$$



The damping factor ζ describes both the insertion loss at the corner frequency and the frequency response of the filter. ζ is affected by the load impedance and low values may cause insertion gain around the corner frequency.

The following design procedure may be applied to any low-pass LC filter and to the typical mains filter configuration (see Figure 13.27 on page 369) to design both the differential components and the common mode components, remembering that the latter are symmetrical about earth and can therefore be treated as two separate circuits.

1. Identify the required cut-off (corner) frequency ω_n: the second order filter rolls off at 40dB/decade, so the desired attenuation A_{dB} at some higher frequency F will put the corner frequency at:

   $\omega_n = 2\pi F / \log^{-1}(A/40)$

2. Identify the load resistance Z_{out} and desired damping factor ζ. A value for ζ between 0.7 and 1 will normally be adequate if Z_{out} is reasonably well specified. Values much larger than 1 will cause excessive low frequency attenuation while much less than 0.7 will cause ringing and insertion gain.

3. From these calculate the required component values:

L = $2 \cdot Z_{out} \cdot \zeta / \omega_n$

C = $1/(L \cdot \omega_n^2)$

4. Iterate as required to obtain useable standard component values.

D.6.2 Filter insertion loss vs. impedance

Standard filters are nearly always characterized between 50Ω resistive impedances. This is unlikely to match the actual circuit impedance. However, if you know the actual circuit impedances and they are also resistive, you can calculate the expected insertion loss from the published 50Ω value. First, derive the transfer impedance Z_T of the filter:

Z_T = $25/\{\text{antilog}(IL_{dB}/20) - 1\}$ where IL_{dB} is the 50Ω published insertion loss

Now the insertion loss between other resistive impedances Z_S (source) and Z_L (load) is:

IL_{dB} = $20 \log \{1 + (Z_S \cdot Z_L)/(Z_T \cdot (Z_S + Z_L))\}$

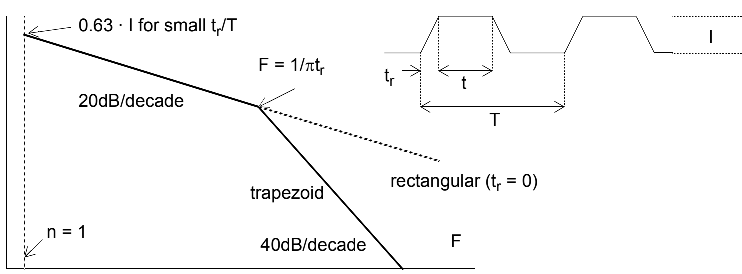
Reactive source or load impedances modify the performance and this equation is no longer applicable.

D.7 Fourier series

For a symmetrical trapezoidal wave of rise time t_r, period T and peak-to-peak amplitude I, the harmonic current at harmonic number n is:

$$I(n) = 2I((t+t_r)/T)\left(\frac{\sin n\pi((t+t_r)/T)}{n\pi((t+t_r)/T)}\right)\left(\frac{\sin n\pi(t_r/T)}{n\pi(t_r/T)}\right)$$

This gives the envelope shown below.



Straight line envelope of harmonic amplitudes

The general form [2] of the Fourier Series is:

$$f(t) = 0.5A_0 + \sum_{n=1}^{\infty} (A_n \cos\omega_n t + B_n \sin\omega_n t)$$

where the coefficients A_n and B_n are:

$$A_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos\omega_n t\,dt$$

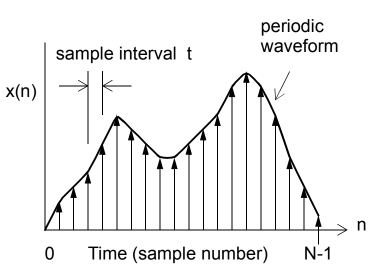
$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin\omega_n t\,dt$$

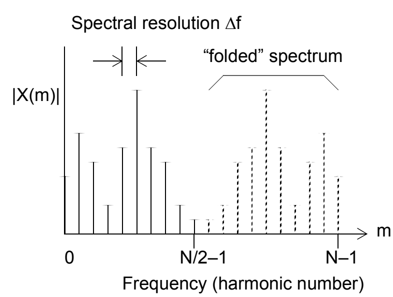
Any arbitrary waveform can be analysed by sampling it at discrete time intervals and taking the discrete Fourier transform (DFT) [4]. This is achieved by replacing the integrals above by a finite weighted summation:

$$A(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cos 2\pi m\left(\frac{n}{N}\right)$$

$$B(m) = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\sin 2\pi m\left(\frac{n}{N}\right)$$

Here the time axis is given by (n/N) where N is the total number of samples and x(n) is the sample value at the nth sample. m represents the frequency axis and the DFT calculates A(m) and B(m) for each discrete frequency from m = 0 (DC) through to m = (N/2) – 1.





The spectral resolution Δf in the frequency domain is the reciprocal of the total sample time, 1/Nt, which is equivalent to the period of the time domain waveform. m = 1 represents the fundamental frequency, m = 2 the second harmonic, and so forth. The spectrum image is folded about m = N/2 and therefore the maximum harmonic frequency that can be analysed is half the number of samples times the fundamental, or 1/2t.

For EMC work A(0) and B(0) are normally neglected (they represent the DC component) as also is phase information, represented by the phase angle between the real and imaginary components arg(A(m) + jB(m)). The mean square amplitude:

$$|X(m)|^2 = |A(m)|^2 + |B(m)|^2$$

represents the power in the mth harmonic. A simple program to calculate X(m) for an array of samples x(n) in the time domain is easily written.