MPF objective function for an RBM

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This document derives the MPF objective function for the case of an RBM, and connections to all states which differ by a single bit flip.

The energy function over the visible units for an RBM is found by marginalizing out the hidden units. This gives an energy function of:

$$E(\mathbf{x}) = -\sum_i \log(1 + \exp(-W_i \mathbf{x})) \tag{1}$$

where W_i is a vector of coupling parameters and $\mathbf{x}$ is the binary input vector. The MPF objective function for this is

$$K = \sum_{\mathbf{x} \in \mathcal{D}} \sum_n \exp\left(\frac{1}{2}\left[E(\mathbf{x}) - E(\mathbf{x} + \mathbf{d}(\mathbf{x}, n))\right]\right) \tag{2}$$

where the sum over n indicates a sum over all data dimensions, and the function $\mathbf{d}(\mathbf{x}, n)$ is

$$\mathbf{d}(\mathbf{x}, n)_i = \left\{ \begin{array}{cc} 0 & i \neq n \\ -(2x_i - 1) & i = n \end{array} \right. \tag{3}$$

Substituting into the objective function

$$K = \sum_{\mathbf{x} \in \mathcal{D}} \sum_n \exp\left(\frac{1}{2}\left[-\sum_i \log(1 + \exp(-W_i \mathbf{x})) + \sum_i \log(1 + \exp(-W_i \mathbf{x} + W_i \mathbf{d}(\mathbf{x}, n)))\right]\right) \tag{4}$$

The released matlab code implements the sum over n in a for loop, and calculates the change in $W_i \mathbf{x}$ caused by $W_i \mathbf{d}(\mathbf{x}, n)$ for all samples simultaneously. (This is a minor savings - only one column of W needs to be visited). (Note that the for loop could also be done over samples, with the change induced by each bit flip being calculated by matrix operations. If the code is run with a small batch size, this implementation would be faster.) Though I could not see a way to replace both for loops with matrix operations, a more clever programmer might!