Graph Theory

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You are advised to go through all the questions in "A walk through Combinatorics" Chapter 9-13. Here are a few practice problems from there.

- Prove that in any simple graph there are two vertices with same degree.

- Count the number of spanning trees in a complete graph.

- Let G be a graph on 10 vertices and 28 edges. Show that G has a cycle with 4 edges.

- Let G be a graph such that every vertex is of degree 4. Show that it is possible to color the edges with two colors such that every vertex has exactly 2 edges of one color and 2 of the other.

- Is a similar statement true if every vertex is of degree 3?

- Count the number of simple graphs on n vertices.

- Two graphs are called *isomorphic* if they can be changed from one to another by vertex renaming. Show that there are at least 5000 non isomorphic graphs on 8 vertices.

- Construct a graph on 10 vertices without triangles. What is the maximum number edges it can have?

- A *Hamiltonian cycle* in a graph G is a cycle in which every vertex appears exactly once. A *Hamiltonian path* is a path which contains every vertex exactly once. Consider a complete graph in which each vertex has been assigned a direction. Such a graph is called a *tournament*. Show that every tournament has a Hamiltonian path. Show that every tournament has a Hamiltonian cycle iff it is strongly connected.