



$$\frac{(\text{mass in})}{\Delta t} - \frac{(\text{mass out})}{\Delta t} + \frac{\text{generation/consumption}}{\Delta t} = \frac{\text{accumulation}}{\Delta t}$$

$$(\text{rate of mass in}) - (\text{rate of mass out}) + (\text{rate gen/cons.}) = \text{rate of acc.}$$

$\dot{m} = \rho u$

$$\cancel{\rho A u} \rightarrow \left(\cancel{\rho A u} + \frac{\partial(\rho u) A}{\partial x} dx \right) + \tilde{m} = \frac{\partial(\rho \phi A dx)}{\partial t}$$

mass conservation continuity eqn.

$$\left(- \frac{\partial(\rho u)}{\partial x} = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m} \right)$$

Additional notes in red: $\frac{M}{L^2} \frac{1}{T} = \checkmark$ and $\frac{M}{L^2} \frac{1}{T}$ with a red 'X' over it.

3D Cartesian coords.

$$-\left[\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} \right] = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}$$

$$-\nabla \cdot (\rho \vec{u}) = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m}$$

$$a = [a_1 \ a_2 \ a_3]$$

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right]$$

$$u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

$$\nabla \cdot u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z}$$

$$\left[-\frac{\partial(\rho u)}{\partial x} = \frac{\partial(\rho \phi)}{\partial t} - \tilde{m} \right] \frac{1}{\rho_{sc}}$$

Darcy's law

$$u = -\frac{k}{\mu} \frac{\partial p}{\partial x}$$

Formation volume factor

$$B_L = \frac{V_{L, RC}}{V_{L, sc}} = \frac{\rho_{L, sc}}{\rho_{L, RC}}$$

$$\vec{u} = -\frac{\bar{K}}{\mu} (\nabla p + \rho g \vec{z})$$

gas $\ll 1$

oil. > 1

water ≈ 1

$$\phi = \phi(p)$$

$$\frac{\partial}{\partial x} \left[\frac{k}{B_w \mu} \frac{\partial p}{\partial x} \right] = \frac{\partial}{\partial t} \left(\frac{\phi}{B_w} \right) - \frac{\beta z}{\rho_{sc}}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p} \frac{\partial p}{\partial t}$$

$$\frac{\partial}{\partial t} \left[\frac{\phi}{B_w} \right] = \frac{\partial}{\partial p} \left(\frac{\phi}{B_w} \right) \frac{\partial p}{\partial t} = \left[\frac{1}{B_w} \frac{\partial \phi}{\partial p} + \phi \frac{\partial}{\partial p} \left(\frac{1}{B_w} \right) \right] \frac{\partial p}{\partial t}$$

$$c_R = \frac{1}{\phi} \frac{\partial \phi}{\partial p}$$

$$c_f = B_w \frac{\partial}{\partial p} \left(\frac{1}{B_w} \right)$$

$$\frac{\partial}{\partial t} \left(\frac{\phi}{B_w} \right) = \frac{\phi}{B_w} \left[\underbrace{\frac{1}{\phi} \frac{\partial \phi}{\partial p}}_{C_R} + B_w \frac{\partial}{\partial p} \left(\frac{1}{B_w} \right) \right] \frac{\partial p}{\partial t} = \frac{\phi}{B_w} C_T \frac{\partial p}{\partial t}$$

$$C_R + C_I = C_T$$

$$\frac{\partial}{\partial x} \left(\frac{k}{\mu B_w} \frac{\partial p}{\partial x} \right) = \frac{\phi}{B_w} C_T \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{k}{\mu} \left[\frac{1}{B_w} \frac{\partial^2 p}{\partial x^2} + \frac{\partial}{\partial x} \left(\frac{1}{B_w} \right) \frac{\partial p}{\partial x} \right] = \frac{\phi}{B_w} C_T \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{k}{\mu} \left[\frac{1}{B_w} \frac{\partial^2 p}{\partial x^2} + \frac{\partial}{\partial p} \left(\frac{1}{B_w} \right) \left(\frac{\partial p}{\partial x} \right)^2 \right] = \frac{\phi}{B_w} C_T \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{C_R}{B_w} \approx 0$$

$$\boxed{\frac{k}{\mu B_w} \frac{\partial^2 p}{\partial x^2} = \frac{\phi C_T}{B_w} \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}}$$

$$\alpha = \frac{k}{\mu \phi C_T}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{\mu \phi C_T}{k} \frac{\partial p}{\partial t} - \frac{\tilde{m}}{\rho_{sc}}$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{\alpha} \frac{\partial p}{\partial t} \quad \leftarrow \begin{array}{l} \text{Diffusivity} \\ \text{eqn.} \\ \text{Heat con.} \end{array}$$