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Chapter 7, Part 2: Near-wall Models

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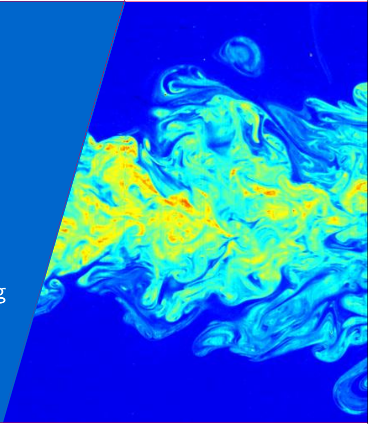


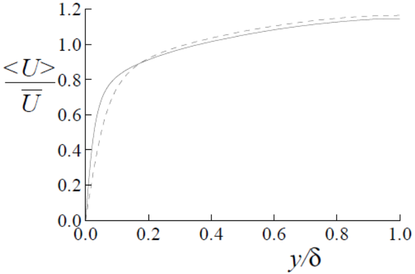
Outline

Wall effect on turbulent flows

The overall effect of wall on turbulent flows:

- Low Reynolds number: The turbulence Reynolds number ($Re_L = k^2/(\epsilon\nu)$) goes to zero when approaching the wall.
- High shear rate: Highest mean shear rate $\partial U/\partial y$ occurs at the wall
- Two-component turbulence: fluctuation in the wall-normal direction is damped more rapidly than the other two directions
- Wall-blocking.

These effects needs to be considered in the near wall region and the general forms of turbulence models need to be modified.



source: S. Pope, Turbulent Flows

Wall effect on turbulent flows

- Introduction of damping functions. For example, in standard $k - \epsilon$ models, the eddy viscosity is evaluated as

$$\nu_T = C_\mu \frac{k^2}{\epsilon},$$

which overestimates the turbulence eddy viscosity in the near-wall region. A damping function f_μ

$$\nu_T = f_\mu C_\mu \frac{k^2}{\epsilon},$$

where the damping function is defined as a function of the turbulence Reynolds number. There are several proposals, for example

$$f_\mu = e^{\frac{-2.5}{1+Re/50}} \quad \text{Jones and Launder (1972)}$$

$$f_\mu = 1 - e^{(-0.0002y^+ - 0.00065y^{+2})} \quad \text{Rodi and Mansour (1993)}$$

Near-wall region

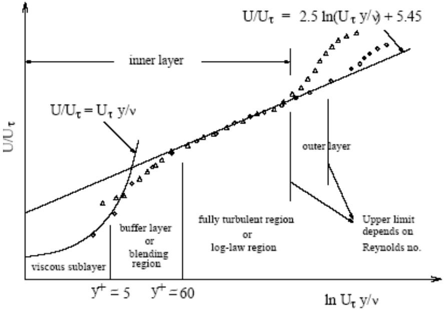
Characteristics of near-wall flow ($y^+ < 30$ for example)

- Viscous damping close to wall reduces the tangential velocity fluctuations
- Wall blocking reduces the normal velocity fluctuations
- Toward the outer part of the near-wall region, the turbulence is rapidly augmented by the production of T.K.E. due to large mean velocity gradient.
- Steep profiles of velocity, k, ϵ, concentration, temperature, etc. (high gradient); most vigorous transport of mass and momentum
- Basic turbulence models (e.g., $k - \epsilon$ and LES models) are only valid for the core region away from the wall; require addition or modification.
- However, some turbulence models (e.g. Spalart-Allmara and $k - \omega$ models are designed to be applied throughout the boundary layer, provided the near-wall mesh resolution is sufficient.
- Observation: if the mean flow is approximately parallel to the boundary, the log-law holds

Law of the wall

In the near-wall region, there are three-layers:

- viscous sublayer ($0 < y^+ < 5$): laminar; viscosity plays a dominant role
- buffer layer (interim layer, ($5 < y^+ < 30$)): viscosity and turbulence are equally important
- log-layer (outer layer, ($30 < y^+$)): fully turbulent. The range ($30 < y^+ < 200$) also called inertial sub-layer



source: Ansys CFX documentation

Law of the wall

In the log layer the velocity profile can be estimated with the log law:

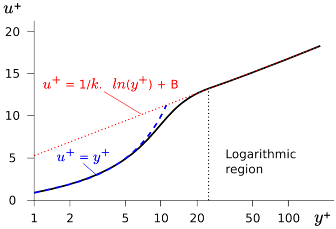
$$u^+ = \frac{1}{\kappa}\ln(y^+) + B \tag{1}$$

and close to the wall in the viscous sublayer

$$u^+ = y^+ \tag{2}$$

where:

- u^+ Dimensionless velocity in wall unit
- y^+ Dimensionless wall distance in wall unit
- κ von Karman's constant (≈ 0.41)
- B Constant (≈ 5.1)



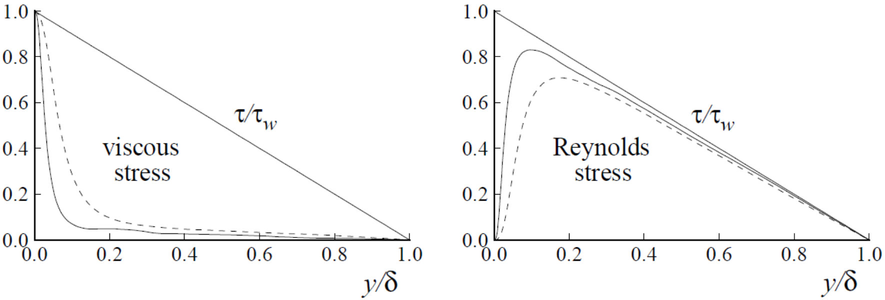
source: http://www.cfd-online.com/Wiki

Derivation of the wall law

The total shear stress is

$$\tau = \mu \frac{\partial U}{\partial y} - \rho \overline{u' v'}$$

The partition of the viscous part and the turbulence part in the channel



For small y, e.g., $y < 0.1\delta$, the total shear stress τ is almost constant and approximated equal to the wall shear stress. That is the reason this near-wall layer is also called the constant-stress layer.

$$\tau \approx \tau_w$$

Derivation of the wall law

We can define a very important velocity scale, i.e., the friction velocity u_τ by

$$\tau_w = \rho u_\tau^2 \rightarrow u_\tau = \sqrt{\frac{\tau_w}{\rho}}.$$

Different scales: Near-wall:

- very close to the wall, the important parameters are the kinematic viscosity ν and the wall shear stress τ_w
- As a result, the velocity scale should be the friction velocity u_τ and the length scale should be the viscous length scale

$$\delta_\nu = \frac{\nu}{u_\tau}$$

- We can make everything dimensionless using these scale, such as

$$U^+ = \frac{U}{u_\tau}, \qquad y^+ = \frac{y}{\delta_\nu} = \frac{u_\tau y}{\nu}$$

It is obvious to see that the wall distance in wall units y^+ is a local Reynolds number which measures the relative importance of viscous and turbulent transport at different distance from the wall.

Derivation of the wall law

Dimensional analysis:

- In the inner layer, the important parameters are U, y, τ_w,ρ, ν, but not δ
- Dimensional analysis (5 parameters and 3 independent dimensions) mandates 2 independent dimensionless groups: $U^+ = U/u_\tau$ and $y^+ = u_\tau y/\nu$
- Their relationship should be $U^+ = f_w(y^+)$,which should be a universal function.
- The inner layer should be roughly located in the region $y/\delta < 0.1$, or where the shear stress is approximately constant.

Derivation of the wall law

Dimensional analysis:

- In the outer layer, the important parameters are U, y, τ_w,ρ, δ, but not ν
- Dimensional analysis (5 parameters and 3 independent dimensions) mandates 2 independent dimensionless groups: $\frac{U_e-U}{u_\tau}$ and $\eta = y/\delta$
- Their relationship should be $\frac{U_e-U}{u_\tau} = f_o(\eta)$,which unfortunately is not a universal function.

Derivation of the wall law

In the overlap layer:

- In the inner layer: $U^+ = f_w(y^+)$.
- In the outer layer: $U_e^+ - U^+ = f_o(\eta)$

Let $\delta^+ = \delta u_\tau / \nu$. Consequently, that $y^+ = \eta\delta^+$. In the overlap region, we can add the inner and outer equations together to have

$$U_e^+(\delta^+) = f_o(\eta) + f_w(\eta\delta^+)$$

Differentiate the previous equation w.r.t δ^+:

$$U_e^{+'}(\delta^+) = 0 + \eta f_w^{'}(\eta\delta^+)$$

Derivation of the wall law

Differentiate again w.r.t η:

$$\begin{aligned} 0 &= f_w^{'}(\eta\delta^+) + \eta\delta^+ f_w^{''}(\eta\delta^+) \\ &= f_w^{'}(y^+) + y^+ f_w^{''}(y^+) \\ &= \frac{d}{dy^+}\left(y^+ \frac{df_w}{dy^+} \right) \end{aligned}$$

Thus $y^+ \frac{df_w}{dy^+}$ should be a constant, which is $1/\kappa$, i.e.

$$\frac{df_w}{dy^+} = \frac{1}{\kappa y^+}.$$

It can be integrated to give

$$f_w = U^+ = \frac{1}{\kappa} \ln y^+ + B = \frac{1}{\kappa} \ln E y^+$$

Derivation of the wall law

Some comments on log-law:

- the constants: $\kappa \approx 0.41, \quad \frac{1}{\kappa} = 2.44$, and $B = 5.0, \quad (B = 7.76)$
- in the log-law region

$$\frac{\partial U}{\partial y} = \frac{u_\tau}{\kappa y}, \quad \frac{y}{u_\tau}\frac{\partial U}{\partial y} = y^+ \frac{\partial U^+}{\partial y^+} = \text{constant}$$

Derivation of the wall law

In the viscous sublayer, the turbulent fluctuations are damped by the viscosity and the wall shear stress is purely due to viscous effect

$$\mu \frac{\partial U}{\partial y} = \tau_w$$

which can be integrated to give a linear velocity profile

$$U(y) = \frac{\tau_w}{\mu} y, \qquad \text{Note we have used the no-slip BC at wall:} U(y = 0) = 0$$

This linear profile can be written in wall units as simple form of

$$U^+ = y^+$$

DNS and experiments have shown the linear viscous sublayer lies roughly within $y^+ < 5$.

Derivation of the wall law

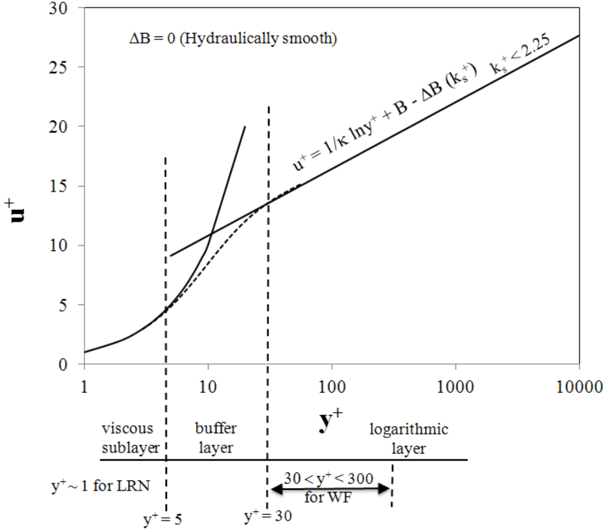
Short summary on different regions of wall-bounded channel flow:

- Inner layer (roughly $y/\delta < 0.1$) - velocity scales on u_τ and y^+, not on δ
- Overlap region: the mean-velocity profile must be logarithmic.
- Outer layer (roughly $y^+ > 50$) - viscosity can be neglected

The log-law can in fact be extend beyond the overlay region. So there is another way to demarcate the regions:

- Viscous sublayer: $y^+ < 5$ linear velocity profile
- Buffer layer: $5 < y^+ < 30$
- Log-law layer: $y^+ > 30$ logarithmic velocity profile

Derivation of the wall law



Velocity distribution for smooth wall channel flows

Derivation of the wall law

Effect of roughness:

- For many engineering applications, the walls are not smooth.
- The pioneering work of Nikuradse on artificially roughened pipe flows.
- The roughness height (size) k_s can be written in wall units:

$$k_s^+ = \frac{u_\tau k_s}{\nu} = \frac{k_s}{\delta_\nu} \tag{3}$$

So k_s^+ is the ratio between roughness height and the viscous sublayer depth. It looks like a Reynolds number.

Derivation of the wall law

- Whether the wall is smooth, rough, or somewhere in between, depends on the value of k_s^+
  - Hydraulically smooth: $k_s^+ < 5$, k_s is less than the viscous sublayer depth
  - Transition: $5 < k_s^+ < 70$, the effects of roughness and viscosity are comparable.
  - Hydraulically rough: $k_s^+ > 70$, pressure drag due to the roughness elements is the predominant mechanism for momentum transfer and viscous effect is minimal. For sufficiently large Re, the wall friction is independent of Reynolds number.From dimensional analysis

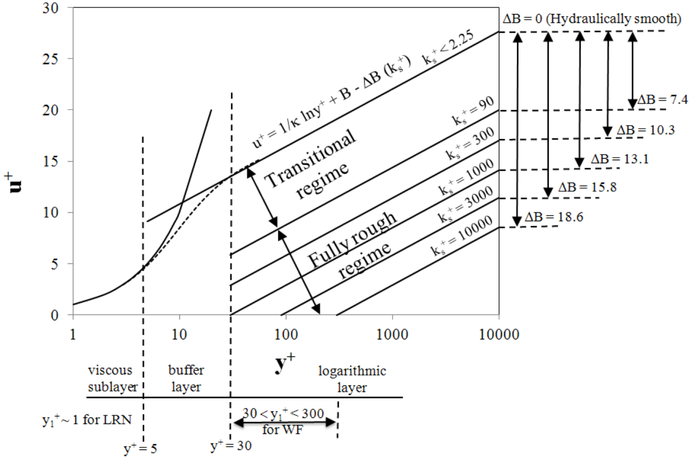
$$U^+ = \frac{1}{\kappa} \ln \frac{y}{k_s} + B_k, \quad \text{where } B_s \approx 8.5$$

The limits on k_s^+ for different regimes are approximate. Different source will have different values.

Derivation of the wall law

Intuitively, hydraulically smooth means that the roughness elements are hidden within the viscous sublayer. They have a much different effect on the turbulent law of the wall velocity profile than if they are sticking out into the main part of the flow.

Derivation of the wall law



Velocity distribution for smooth and rough wall channel flows

Derivation of the wall law

Considering the roughness effect, the log-law has the form of

$$u^+ = \frac{1}{\kappa} \ln\left(y^+\right) + B - \Delta B\left(k_s^+\right), \quad u^+ = \frac{u}{u_*}, \quad y^+ = \frac{u_* y}{\nu}, \tag{4}$$

where B is a constant (=5.2) corresponding to smooth wall, $k_s^+ = u_\tau k_s/\nu$ is the dimensionless roughness height, and k_s is Nikuradse's roughness height. The roughness function ΔB depends on k_s^+ and represents the downward shift of the logarithmic velocity profile. Cebeci and Bradshaw (1977) proposed a functional form for ΔB

$$\Delta B = \begin{cases} 0 & \text{if} \quad k_s^+ < 2.25, \\ \left[B - 8.5 + \frac{1}{\kappa}\ln k_s^+\right] \sin\left[0.4258\left(\ln k_s^+ - 0.811\right)\right] & \text{if} \quad 2.25 \le k_s^+ < 90, \\ B - 8.5 + \frac{1}{\kappa}\ln k_s^+ & \text{if} \quad k_s^+ \ge 90. \end{cases} \tag{5}$$

Derivation of the wall law

In an equilibrium boundary layer, an alternative formula can be used (Fluent, 2009; OpenFOAM, 2012)

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{Ey^+}{1 + C_s k_s^+} \right) \tag{6}$$

where E is the smooth wall constant ($=e^{\kappa B} \approx 8.432$) and $(1 + C_s k_s^+)$ represents the modification due to roughness. To match Equations 4 and 6, it is easy to verify that

$$1 + C_s k_s^+ = e^{\kappa \Delta B \left(k_s^+ \right)}. \tag{7}$$

C_s is a constant which takes into account the roughness types and ranges from 0.2 to 1.

Derivation of the wall law

There is no general guideline in choosing the value of C_s. In many CFD codes, the default value is set to 0.5. However, for fully rough conditions where $C_s k_s^+ \gg 1$, Equation 6 can be simplified as

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{E y^+}{C_s k_s^+} \right). \tag{8}$$

To match Equations 4 and 8 in the fully rough regime, the following relationship holds

$$\frac{1}{\kappa} \ln \left(\frac{E}{C_s} \right) = 8.5, \tag{9}$$

which gives $C_s \approx 0.258$. In dimensional form, the rough wall log-law can also be expressed as

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \left(\frac{30y}{k_s} \right) = \frac{1}{\kappa} \ln \left(\frac{y}{y_0} \right), \tag{10}$$

where $y_0 = k_s/30$ is the roughness length. Note that y_0 is only a fraction of the roughness height k_s.

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Derivation of the wall law

For each near-wall cell, given the cell center velocity u_1, the friction velocity u_τ is solved iteratively from Equation 6.

$$u^+ = \frac{1}{\kappa} \ln \left(\frac{Ey^+}{1 + C_s k_s^+} \right)$$

Note that both sides of the equation depends on u_τ.

The wall effect on the momentum balance is through a modified effective viscosity approach, which ensures correct wall friction even when the velocity gradient in the wall function approach is erroneous (Bredberg, 2000).

Derivation of the wall law

For a typical $k - \epsilon$ model with wall-function simulation:

- The near-wall cell center value of ϵ is based on the assumption of equilibrium between production and dissipation of turbulent kinetic energy and has the form

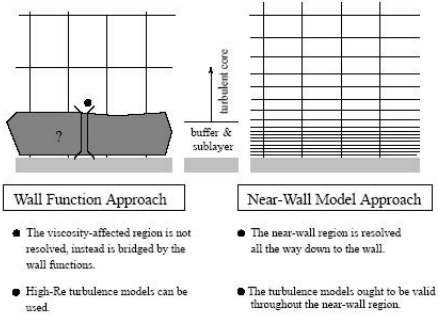
$$\epsilon_1 = \frac{C_\mu^{3/4} k_1^{3/2}}{\kappa y_1}. \tag{11}$$

- For k, the wall normal gradient is set to zero. It has been verified that this boundary condition can produce reasonable results. Another option is to set the near wall cell center value of k as in Celik and Rodi (1988).

Modeling near-wall region

Two approaches to modeling the near-wall region:

- wall functions: also termed as High-Reynolds Number (HRN) model; the inner region (viscous sublayer and buffer layer) is not resolved.
- near-wall modeling: also termed as Low-Reynolds Number (LRN) model: inner region resolved (with sufficiently fine mesh)



source: Ansys CFX documentation

Modeling near-wall region

The idea of "wall functions" (Launder and Spalding, 1972):

- ignore the details inside the viscous sublayer and buffer layer (if any)
- apply boundary condition (based on log-law) some distance away from the wall. The first near-wall cell center should be in the log-law layer ($30 < y^+ < 200$). Check your mesh!
- bridge between the wall and the first near-wall cell enter (which lies in the log-law layer)
- wall functions are very attractive: simplifications and economies; widely used in engineering problems
- drawbacks: not exactly applicable to conditions such as strong pressure gradient, separated or impinging flow. Their physical basis is not certain and performance is poor.

Major components of "wall functions" approach:

- Laws-of-the-wall for the mean velocity and other scalars (temperature, concentration, etc.)
- Formulae or B.C.s for other near-wall quantities (k, ϵ, ω)

Modeling near-wall region

How to implement the "wall functions": three steps:

- solve the momentum equation with a modified wall friction, either through an added source term or via a modified effective viscosity
- set k at the first near-wall cell center iteratively with the use of the law-of-the-wall
- set ϵ or ω with the calculated k value

Wall functions

How to implement the "wall functions":

- Standard wall functions
- Launder-Spalding methodology
- Chieng-Launder model

Standard wall functions

Standard wall functions (from the limiting behavior near a wall):

$$U = \frac{u_\tau}{\kappa} \ln\left(\frac{y u_\tau}{\nu}\right) + B, \quad k = \frac{u_\tau^2}{\sqrt{\beta^*}}, \omega = \frac{u_\tau}{\sqrt{\beta^*}\kappa y_p} \tag{12}$$

In a wall-function mesh, to predict the wall shear τ_w, you should not use simple finite difference

$$\tau_w = \mu \frac{\partial U}{\partial y}|_w > \mu \frac{U_p}{y_p} \tag{13}$$

where the subscript p denotes the first near wall cell center.
The modification should be either

- an added momentum source term simulating the correct wall friction
- or a modified viscosity, an effective viscosity ν_e, that ensures the correct friction even though the velocity gradient is erroneous.

Standard wall functions

Through the law-of-the-wall

$$\frac{U}{u_\tau} = \frac{1}{\kappa} \ln\left(Ey^+\right) \tag{14}$$

the wall friction is computed as

$$\tau_w = \rho u_\tau^2 = \frac{\rho u_\tau U \kappa}{\ln\left(Ey^+\right)} \tag{15}$$

- If an added momentum source term is used to simulate the correct wall friction, then the term should be $S_u = \tau_w \cdot A$.
- If a modified effective wall viscosity is used (as in OpenFOAM),

$$\mu_e \frac{U_p}{y_p} = \tau_w = \rho u_\tau^2 = \frac{\rho u_\tau U \kappa}{\ln\left(Ey^+\right)} \quad \Rightarrow \quad \mu_e = \frac{\rho u_\tau y_p \kappa}{\ln\left(Ey^+\right)} \tag{16}$$

Standard wall functions

So how do we get the shear velocity u_τ: solve from the law-of-wall through ***iteration***.

Assume the tangential velocity U_p at the first near-wall cell center is known at a given time step, then

$$u_\tau = \frac{U_p \kappa}{\ln\left(E y_p^+\right)} \quad \textit{where} \quad y_p^+ = \frac{y_p u_\tau}{\nu} \tag{17}$$

The iteration is repeated until convergence (usually very fast).

Limitation of standard wall functions: can not be used when $u_\tau = 0$, such a re-attachment point in re-circulating flows.

Launder-Spalding methodology

To overcome the singularity limitation of standard wall functions, Launder and Spalding (1974) proposed a modification:

- Solve the momentum equation with a modified wall viscosity
- Solver the T.K.E., with modified integrated production and dissipation terms
- Set ϵ using the predicted k

For the modified viscosity, instead of calculating u_τ through iteration from the log-law, the following identity is used:

$$u_\tau = C_\mu^{1/4}\sqrt{k} \tag{18}$$

thus

$$\mu_e = \frac{\rho C_\mu^{1/4}\sqrt{k_p} y_p \kappa}{\ln\left(E y_p^*\right)} \quad \textit{where} \quad y_p^* = \frac{y C_\mu^{1/4}\sqrt{k_p}}{\nu} \tag{19}$$

Modeling near-wall region

Wilcox (1988) $k-\omega$ model:

$$\nu_T = \frac{k}{\omega} \tag{20}$$

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(\nu + \sigma^* \nu_T)\frac{\partial k}{\partial x_j}\right] \tag{21}$$

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha \frac{\omega}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \sigma \nu_T)\frac{\partial \omega}{\partial x_j}\right] \tag{22}$$

$$\alpha = \frac{5}{9}, \quad \beta = \frac{3}{40}, \quad \beta^* = \frac{9}{100}, \quad \sigma = \frac{1}{2}, \quad \sigma^* = \frac{1}{2}, \quad \varepsilon = \beta^* \omega k \tag{23}$$

Reference: Wilcox, D.C. (1988). Re-assessment of the scale-determining equation for advanced turbulence models. AIAA Journal, vol. 26, no. 11, pp. 1299-1310

Modeling near-wall region

Wilcox (2004) modified $k-\omega$ model:

$$\nu_T = \frac{k}{\omega} \tag{24}$$

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j} \left[(\nu + \sigma^* \nu_T) \frac{\partial k}{\partial x_j} \right] \tag{25}$$

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \alpha \frac{\omega}{k} \tau_{ij} \frac{\partial U_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[(\nu + \sigma \nu_T) \frac{\partial \omega}{\partial x_j} \right] \tag{26}$$

$$\alpha = \frac{13}{25}, \quad \beta = \beta_0 f_\beta,, \quad \beta^* = \beta_0^* f_{\beta*}, \quad \sigma = \frac{1}{2}, \quad \sigma^* = \frac{1}{2}, \quad \beta_0 = \frac{9}{125} \tag{27}$$

$$f_\beta = \frac{1+70\chi_\omega}{1+80\chi_\omega}, \quad \chi_\omega = \left| \frac{\Omega_{ij}\Omega_{jk}S_{ki}}{(\beta_0^*\omega)^3} \right|, \quad \beta_0^* = \frac{9}{100}, \quad f_{\beta*} = \begin{cases} 1, & \chi_k \le 0 \\ \frac{1+680\chi_k^2}{1+80\chi_k^2}, & \chi_k > 0 \end{cases} \tag{28}$$

$$\chi_k \equiv \frac{1}{\omega^3} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}, \quad \varepsilon = \beta^* \omega k, \quad l = \frac{k^{\frac{1}{2}}}{\omega} \tag{29}$$

References: Wilcox, D.C. (2004), Turbulence Modeling for CFD, ISBN 1-928729-10-X, 2nd Ed., DCW Industries, Inc..

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Modeling near-wall region

$k-\omega$ SST model (Menter, 1993, 1994):

- a popular two-equation eddy-viscosity model.
- The shear stress transport (SST) formulation has good properties
  - The use of a $k-\omega$ formulation in the inner parts of the boundary layer makes the model directly usable all the way down to the wall through the viscous sub-layer, hence the SST $k-\omega$ model can be used as a Low-Re turbulence model without any extra damping functions.
  - The SST formulation also switches to a $k-\epsilon$ behaviour in the free-stream and thereby avoids the common $k-\omega$ problem that the model is too sensitive to the inlet free-stream turbulence properties.
  - Good behavior in adverse pressure gradients and separating flow.
- Drawback:too large turbulence levels in regions with large normal strain, like stagnation regions and regions with strong acceleration.

References:

Menter, F. R. (1993), "Zonal Two Equation k-ęØ Turbulence Models for Aerodynamic Flows", AIAA Paper 93-2906.

Menter, F. R. (1994), "Two-Equation Eddy-Viscosity Turbulence Models for Engineering Applications", AIAA Journal, vol. 32, no 8. pp. 1598-1605.

Modeling near-wall region

$k-\omega$ SST model (Menter, 1993, 1994):

$$\nu_T = \frac{a_1 k}{\max(a_1\omega, SF_2)} \tag{30}$$

$$\frac{\partial k}{\partial t} + U_j\frac{\partial k}{\partial x_j} = P_k - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[(\nu + \sigma_k \nu_T)\frac{\partial k}{\partial x_j}\right] \tag{31}$$

$$\frac{\partial \omega}{\partial t} + U_j\frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[(\nu + \sigma_\omega \nu_T)\frac{\partial \omega}{\partial x_j}\right] + 2(1-F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i} \tag{32}$$

$$F_2 = \tanh\left[\left[\max\left(\frac{2\sqrt{k}}{\beta^*\omega y}, \frac{500\nu}{y^2\omega}\right)\right]^2\right], \quad P_k = \min\left(\tau_{ij}\frac{\partial U_i}{\partial x_j}, 10\beta^* k\omega\right) \tag{33}$$

$$F_1 = \tanh\left\{\left\{\min\left[\max\left(\frac{\sqrt{k}}{\beta^*\omega y}, \frac{500\nu}{y^2\omega}\right), \frac{4\sigma_{\omega 2}k}{CD_{k\omega}y^2}\right]\right\}^4\right\}, \quad CD_{k\omega} = \max\left(2\rho\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 10^-\right. \tag{34}$$

$$\phi = \phi_1 F_1 + \phi_2(1-F_1), \quad \alpha_1 = \frac{5}{9}, \alpha_2 = 0.44, \quad \beta_1 = \frac{3}{40}, \beta_2 = 0.0828, \quad \beta^* = \frac{9}{100}, \quad \sigma_{k1} = 0.85, \tag{35}$$

Modeling near-wall region

LRN models:

- LRN models: a turbulent model can be integrated all the way to the wall through "damping functions" for certain terms in the model equations.
- "Damping functions" are introduced to represent the viscous effects near a wall
- The "damping functions" should reproduce the asymptotic behavior in the limit of a wall
- There are exceptions: HRN $k - \omega$ model (Wilcox, 1988) can be integrated to the wall without the need of damping function. In other versions of the $k - \omega$ models (Wilcox, 1993, 2006), noted as LRN $k - \omega$ models, damping functions are used.

LRN models

Low-Re $k - \epsilon$ models (Patel (1995) and Rodi (1993)):

- There are many different low-*Re* $k - \epsilon$ models in the literature
- The most common and classical models are listed below

Model	**Reference**	**Description**
Chien Model	Chien (1982)	A very common model in turbo-machinery applications. Has nice numerical properties.
Launder-Sharma Model	Launder (1974)	An old classical model which has attracted some attention for its ability to in model cases predict by-pass transition.
Nagano-Tagawa Model	Nagano (1990)	A model originally developed for heat-transfer applications.

LRN models

These Low-Re $k-\epsilon$ models can be written in a general form like:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}\left[\rho k u_j - \left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] = P - \rho\epsilon - \rho D$$

$$\frac{\partial}{\partial t}(\rho \epsilon) + \frac{\partial}{\partial x_j}\left[\rho \epsilon u_j - \left(\mu + \frac{\mu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial x_j}\right] = (C_{\epsilon_1} f_1 P - C_{\epsilon_2} f_2 \rho\epsilon)\frac{\epsilon}{k} + \rho E$$

$$\mu_t = C_\mu f_\mu \rho \frac{k^2}{\epsilon} \qquad P = \tau_{ij}^{turb}\frac{\partial u_i}{\partial x_j}$$

Where C_{ϵ_1}, C_{ϵ_2}, C_μ, σ_k and σ_ϵ are model constants.

- Comparing to HRN version, the difference are the damping functions f_μ, f_1 and f_2 and the extra source terms D and E
- They are only active close to solid walls and makes it possible to solve k and ϵ down to the viscous sublayer.

	Chien	Launder-Sharma	Nagano-Tagawa
c_μ	0.09	0.09	0.09
σ_k	1	1	1.4
σ_ϵ	1.3	1.3	1.3
D	$2\,\nu\,\frac{k}{y^2}$	$2\,\nu\,\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$	0
E	$-\frac{2\nu\epsilon}{y^2}\,e^{-0.5y^+}$	$2\,\nu\,\nu_t\,\left(\frac{\partial^2 u}{\partial y^2}\right)^2$	0
ϵ_{wall}	0	0	$\nu\,\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$
C_{ϵ_1}	1.35	1.44	1.45
C_{ϵ_2}	1.8	1.92	1.9
f_μ	$1-e^{(-0.0115y^+)}$	$e^{\frac{-3.4}{(1+R_t/50)^2}}$	$\left(1-e^{\frac{-y^+}{26}}\right)^2\left(1+\frac{4.1}{Re_t^{3/4}}\right)$
f_1	1	1	1
f_2	$1-0.22e^{-\left(\frac{Re_t}{6}\right)^2}$	$1-0.3e^{-Re_t^2}$	$\left(1-0.3e^{-\left(\frac{Re_t}{6.5}\right)^2}\right)\left(1-e^{\frac{-y^+}{6}}\right)^2$

where $Re_t \equiv \frac{k^2}{\nu\epsilon}$, $y^+ \equiv \frac{u^* y}{\nu}$ and $k_{wall} = 0$.

LRN models

OpenFOAM implements the following LRN $k - \epsilon$ model for incompressible flows:

- Launder-Sharma low-*Re* $k - \epsilon$ model : *LaunderSharmaKE*
- Lam-Bremhorst low-*Re* $k - \epsilon$ model : *LamBremhorstKE*
- Lien cubic low-*Re* $k - \epsilon$ model : *LienCubicKELowRe*
- Lien-Leschziner low-*Re* $k - \epsilon$ model : *LienLeschzinerLowRe*

Near-wall resolution

Near-wall resolution is critical for a successful CFD simulation of wall-bounded flows:

- We have shown the importance of using an appropriate y^+ value in combination with a given turbulence modelling approach.
- How to calculate the correct first cell height (Δy_1) based on your desired y^+ value.
- This is an important first step as the global mesh resolution parameters will also be influenced by this near-wall mesh as well as the Reynolds number.

Near-wall resolution

Two main choices we have in choosing a near-wall modelling strategy:

- Resolving the Viscous Sublayer
  - involves the full resolution of the boundary layer
  - required where wall-bounded effects are of high priority (adverse pressure gradients, drag force, heat and mass transfer, etc.)
  - wall adjacent grid height must be order $y^+ \sim \mathcal{O}(1)$
  - must use an appropriate low-Re number turbulence model (i.e. $k - \omega$ SST model)
- Adopting a Wall Function Approach
  - involves modelling the boundary layer using a log-law wall function.
  - suitable for cases where wall-bounded effects are secondary
  - wall adjacent grid height should ideally reside in the log-law region where $y^+ > 11$
  - HRN version of most turbulence models are applicable

Near-wall resolution

How to calculate the first cell height for a desired y^+ value?

- Calculate the Reynolds number $Re = \frac{UL}{\nu}$, where U and L are characteristic velocity and length scales, respectively
- According to the definition of y^+, first cell center height should be

$$y^+ = \frac{U_\tau \Delta y_1}{\nu} \rightarrow \Delta y_1 = \frac{y^+ \nu}{U_\tau}$$

- Given the desired y^+, if we know the friction velocity U_τ, then we can estimate Δy_1. The friction velocity is defined as

$$U_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

- The wall shear stress τ_w can be calculated from skin friction coefficient C_f, i.e.,

$$\tau_w = \frac{1}{2} C_f \rho U^2$$

- The friction coefficient C_f can be estimated using one of many empirical formulas. For example, the Schlichting skin-friction correlation

$$C_f = [2\, log_{10}(Re) - 0.65]^{-2.3} \quad \text{for} \quad Re < 10^9$$

Near-wall resolution

Difficulties in specifying near-wall resolution:

- Flow might not be uniform, i.e., hard do define a uniform mean flow velocity U and length scale L for the whole domain.
- For unsteady flows, the turbulent flow field changes with time. Thus the Δy_1 should be re-estimated and adjusted during the simulation. This is relatively hard to do. There are some CFD packages adopting this approach (ref. CFX).
- Hard to choose a "good" formula for the skin-friction coefficient C_f.

Good online calculators:

- For setup boundary condition: http://www.cfd-online.com/Tools/turbulence.php
- For estimate near wall mesh size: http://www.cfd-online.com/Tools/yplus.php

Questions?