COMPUTER PROJECT 1

Loc Luong
ID: 900353083

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1 Problem

Consider a reservoir with six wells is 100% filled with an incompressible fluid. The spatial distributions of the reservoir properties are shown in figures below (figure 1). Note that the permeability along N-S direction is the same as the permeability along E-W direction. The implementation plan for the project is:

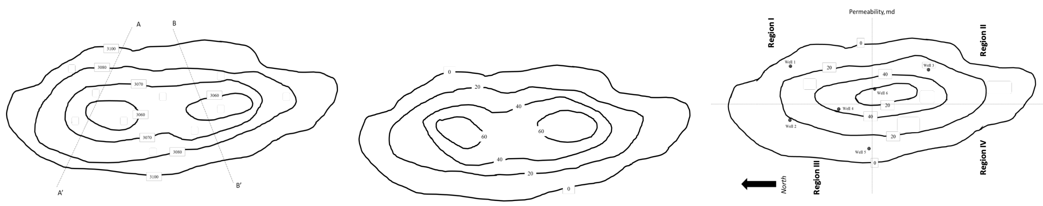
- Well 1, 2 and 4 are injectors; Well 3, 5 and 6 are producers.
- Injection rates of Well 1 and 2 are kept constant as 1,500 STB/D; Well 4 is injecting under constant sand face pressure.
- Well 3, 5 and 6 are producing under constant sand-face pressure (psf) of 14.7 psia.

Fluid and wellbore properties are summarized below:

Fluid compressibility (psi^{-1})	0
Viscosity (cp)	1
Density (lbm/ft^3)	62.4
Formation Volume Factor (RB/STB)	1
r_w (ft)	0.25
Skin of well	-1

2 Reservoir Properties

The reservoir is sealed and the permeability of boundaries is zero. Thickness distribution is from 0 to 60 ft. Reservoir is 3000 ft under the ground. There are six well in the reservoir, three of them are producer (well 3, 5, 6) and the other are injector (well 1, 2, 4).



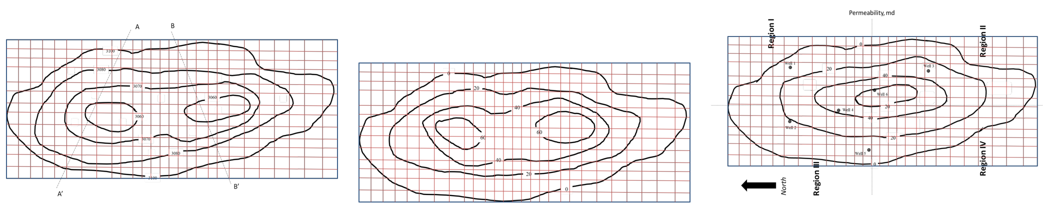
(a) Reservoir top (b) Reservoir thickness (c) Reservoir permeability

Figure 1: Reservoir properties distribution map

3 Methodology

3.1 Digitalize data

A web based tool (WebPlotDigitizer) have been used to extract data from images. The figure below is created to get the data at each gird point from WebPlotDigitizer.

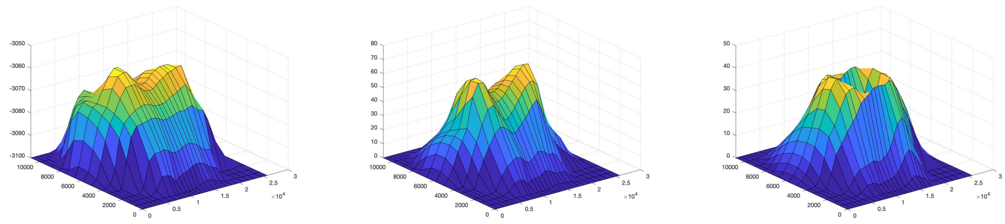


(a) Reservoir top (b) Reservoir thickness (c) Reservoir permeability

Figure 2: Reservoir properties distribution map before digitalizing

3.2 Mesh grid system building

After digitalizing and getting data from images, a mesh grid system is built to cover the reservoir. The reservoir is divided in 351 blocks and each grid point is assigned with specific value of permeability, thickness and top by using harmonic spline interpolation function. Because the permeability and thickness outside boundaries of the reservoir are zeros, the data got from interpolation method need to be filtered in order to represent the reservoir more accurate. Specifically, permeability and thickness values outside the boundaries of reservoir are assigned to zeros. Top values outside are assigned to 3100 because the boundary circle of reservoir is 3100 ft.



(a) Reservoir top (b) Reservoir thickness (c) Reservoir permeability

Figure 3: Reservoir properties distribution map after digitalizing

3.3 Define active block by numbering

The number of active blocks which have non-negative value of transmissibility is count by a for loop in python. There are 282 active blocks in the reservoir with properties values is greater then zero. Next, a boundary with all zeros values is established to calculate the transmissibility of each block.

3.4 Partial differential equation governing reservoir flow

Single-phase 2D with incompressible fluid reservoir flow equation is shown:

$$\frac{\partial}{\partial x}\left(\frac{A_x\beta_c k_x}{\mu B}\frac{\partial\Phi}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{A_y\beta_c k_y}{\mu B}\frac{\partial\Phi}{\partial y}\right)\Delta y+q_{STB/D}=0 \tag{1}$$

With depth gradient, the equation become:

$$
\begin{aligned}
&\beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i+\frac{1}{2},j}(p_{i+1,j}-p_{i,j}) + \beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i-\frac{1}{2},j}(p_{i-1,j}-p_{i,j}) + \\
&\beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j+\frac{1}{2}}(p_{i,j+1}-p_{i,j}) + \beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j-\frac{1}{2}}(p_{i,j-1}-p_{i,j}) + q_{STB/D} \\
&= \frac{1}{144}\frac{g}{g_c}\Bigg[\beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i+\frac{1}{2},j}(G_{i+1,j}-G_{i,j}) + \beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i-\frac{1}{2},j}(G_{i-1,j}-G_{i,j}) + \\
&\beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j+\frac{1}{2}}(G_{i,j+1}-G_{i,j}) + \beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j-\frac{1}{2}}(G_{i,j-1}-G_{i,j})\Bigg]
\end{aligned}
\tag{2}
$$

3.5 Transmissibility calculation in North, South, West and East directions

Because using interpolation for generating reservoir property values, each point in the grid map have unique value lead to the fact that a block have four values of permeability, thickness and top. So, the centered value which describe the properties of a block will be calculated by averaging four values of permeability, thickness and top numbers.
Transmissibility between two block is calculated by harmonic averaging:

$$
\frac{2\beta_c}{\mu B\left[\dfrac{1}{\left(\dfrac{A_x k_x}{\Delta x}\right)_{i,j}} + \dfrac{1}{\left(\dfrac{A_x k_x}{\Delta x}\right)_{i+1,j}}\right]} \tag{3}
$$

3.6 Well properties definition and calculation

Productivity index of each well is calculated by the equation:

$$
\Omega_{i,j,k} = \left[\frac{2\pi \overline{k} \Delta L}{\mu B(ln(\frac{r_e}{r_w}) + s)}\right] \tag{4}
$$

3.7 Coefficient matrix construction

The reservoir has 282 active blocks, so a system of equations of size 282×282 need to be built by assigning the transmissibility values to the system of equations. A typical for loop is used to create the cofficient matrix.

4 Results

4.1 Pressure distribution

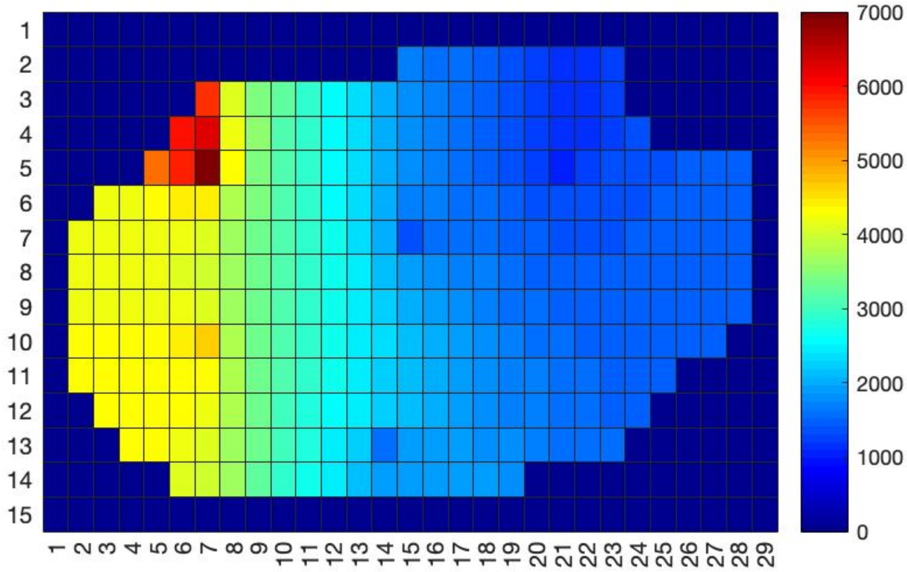


Figure 4: Pressure distribution after the convergence is achieved

4.2 Well-block properties

Table 1: Well block properties

Well #	**Well Type**	**Flow Rate (STB/D)**	**Productivity index, Ω STB/(day-psi)**	p_{sf} **(psi)**	**Well Block Pressure (psi)**
1	Injector	1500	**0.1801**	**15330.78**	**7002.08**
2	Injector	1500	**0.7105**	**6790.11**	**4678.92**
3	Producer	**-1048.25**	**1.0656**	14.7	**998.40**
4	Injector	**197.05**	**2.8670**	3000	**2931.27**
5	Producer	**-197.47**	**0.1216**	14.7	**1638.69**
6	Producer	**-1951.32**	**1.3937**	14.7	**1414.78**

4.3 Material balance and residual check

Flow rate of six well in reservoir:

$$\mathbf{Q} = \begin{pmatrix} 1500.0 \\ 1500.0 \\ -1048.2528447232612 \\ 197.0506396907944 \\ -197.47125354051602 \\ -1951.3265414269713 \end{pmatrix}$$

Material balance check is sum of the flow rates of six well.

$$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 = 4.5929482439532876e - 11$$

Residual check of the calculation followed the equation below:

$$R = Q_{i,j} - (S_{i,j}p_{i+1,j} + N_{i,j}p_{i-1,j} + E_{i,j+1}p_{i,j+1} + W_{i,j}p_{i,j-1} + C_{i,j}p_{i,j}) \quad (5)$$

Residual check at wells:

$$\mathbf{Q} = \begin{pmatrix} -4.54747351e - 13 \\ -2.66786593e - 12 \\ -1.45305989e - 12 \\ 7.27595761e - 12 \\ -3.08197912e - 13 \\ -1.29674049e - 12 \end{pmatrix}$$

The residual check of all blocks can be performed by running the file "**cp1_solve.py**".

5 My implementation

I have used python to solve the computer project 1.
The main file is "**cp1_solve.py**".
The file "**griddata.py**" is to define Biharmonic spline interpolation function which is same with matlab.
The file "**boundaries_zero.py**" is to create a zeros boundaries outside the reservoir for calculating the N, W, E, and S transmissibility.