NUMERICAL RESERVOIR SIMULATION
COMPUTER PROJECT 1

Loc Luong
ID: 900353083

February 9, 2020

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1 Introduction

Consider a reservoir with six wells is 100% filled with slightly compressible fluid. The spatial distributions of the reservoir properties are shown in figures below (figure 1). The permeability along N-S direction is twice as the permeability along E-W direction. The implementation plan for the project is:

- There are 6 wells in the reservoir put on production.
- All the wells put on production for 2 years at constant sand-face pressure at 1000 psia.
- Starting from third year, well 1 and 2 are at shut-in condition, the remain wells are still on production shedule.

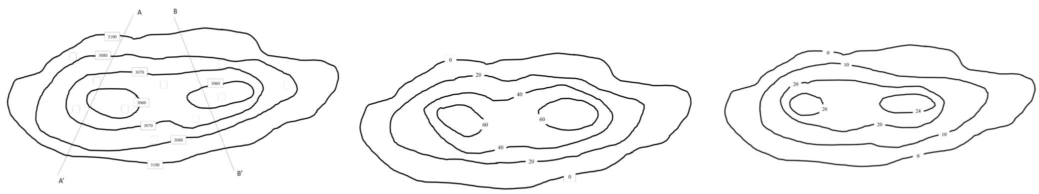
Reservoir, well and fluid properties necessary for the project is shown in the table below:

Table 1: Reservoir properties

	Parameters	**Unit**	**Parameters**	**Unit**
1	$\mu_o = \mu_{sc} + 0.00134 ln(p)$	cp	$p_{sc} = 14.7$	psia
2	$\mu_{sc} = 0.6$	cp	$r_w = 0.25$	ft
3	$B_o = (1 + c_o(p - p_{sc}))^{-1}$	RB/STB	$p_{initial} = 5000$	psia
4	$\rho_o = \rho_{sc}(1 + c_o(p - p_{sc}))^{-1}$	lb/ft3	$c_\phi = 10^{-6}$	$psia^{-1}$
5	$\rho_{sc} = 42$	lb/ft3	Skin = 0 for all well	
6	$c_o = 3 \times 10^{-5}$	$psia^{-1}$	$p_{sf} = 1000$	psia

2 Reservoir and fluid properties

The reservoir is sealed and the permeability of boundaries is zero. Thickness distribution is from 0 to 60 ft. Reservoir is 3000 ft under the ground. Porosity distribution is from 0 to 26 percentage.



(a) Reservoir top (b) Reservoir thickness (c) Reservoir porosity

Figure 1: Reservoir properties distribution map

Reservoir permeability is shown in the figure below, and the permeability distirbution is re-calibrated with multiplying factor as shown in the table 2.

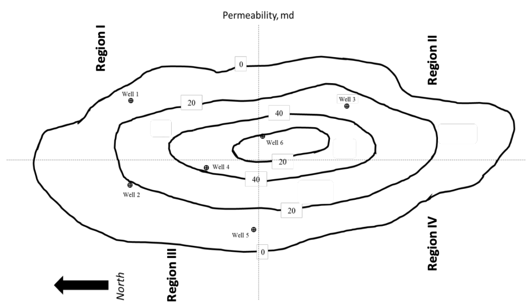


Figure 2: Reservoir permeability distribution

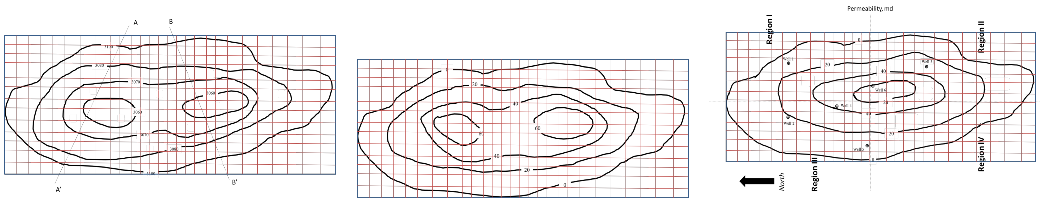
Table 2: Multiplying factor for permeability

Region 1	0.2
Region 2	0.3
Region 3	1
Region 4	3.0

3 Methodology

3.1 Digitalize data

A web based tool (WebPlotDigitizer) have been used to extract data from images. The figure below is created to get the data at each gird point from WebPlotDigitizer.

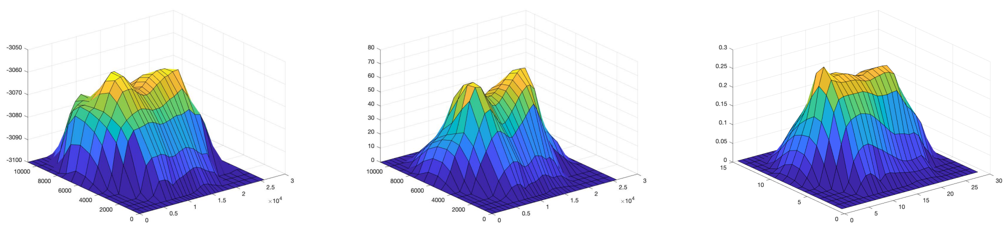


(a) Reservoir top (b) Reservoir thickness (c) Reservoir permeability

Figure 3: Reservoir properties distribution map before digitalizing

3.2 Mesh grid system building

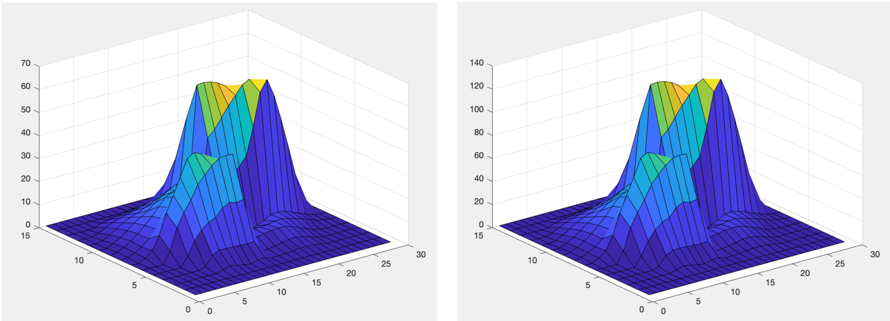
After digitalizing and getting data from images, a mesh grid system is built to cover the reservoir. The reservoir is divided in 351 blocks and each grid point is assigned with specific value of permeability, thickness, porosity and top by using harmonic spline interpolation function. Because the permeability and thickness outside boundaries of the reservoir are zeros, the data got from interpolation method need to be filtered in order to represent the reservoir more accurate. Specifically, permeability and thickness values outside the boundaries of reservoir are assigned to zeros. Top values outside are assigned to 3100 because the boundary circle of reservoir is 3100 ft.



(a) Reservoir top (b) Reservoir thickness (c) Reservoir porosity

Figure 4: Reservoir properties distribution map after digitalizing

The permeability distribution after calibration is shown as :



(a) Permeability along W-E direction (b) Permeability along N-S direction

Figure 5: Reservoir permeability distribution map in N-S and W-E directions.

3.3 Define active block by numbering

The number of active blocks which have non-negative value of transmissibility is count by a for loop in python. There are 247 active blocks in the reservoir with properties values is greater then zero. Next, a boundary with all zeros values is established to calculate the transmissibility of each block.

3.4 Partial differential equation governing reservoir flow

Single-phase 2D with slightly compressible fluid reservoir flow equation is shown:

$$\frac{\partial}{\partial x}\left(\frac{A_x\beta_c k_x}{\mu B}\frac{\partial \Phi}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(\frac{A_y\beta_c k_y}{\mu B}\frac{\partial \Phi}{\partial y}\right)\Delta y + q_{STB/D} = \frac{V_b}{\alpha_c}\frac{\partial}{\partial t}(\frac{\phi}{B})$$
$$\cong \frac{V_b\phi_t c_t}{\alpha_c B^0}\frac{\partial p}{t} \cong \left(\frac{V_b\phi_t c_t}{\alpha_c B^0}\right)_{i,j,k}\frac{p^{n+1}_{i,j,k}-p^{n}_{i,j,k}}{\Delta t}$$

With depth gradient, the equation become:

$$
\begin{aligned}
\beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i+\frac{1}{2},j} (p_{i+1,j} - p_{i,j}) + \beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i-\frac{1}{2},j} (p_{i-1,j} - p_{i,j}) + \\
\beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j+\frac{1}{2}} (p_{i,j+1} - p_{i,j}) + \beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j-\frac{1}{2}} (p_{i,j-1} - p_{i,j}) + q_{STB/D} \\
= \left(\frac{V_b \phi_t c_t}{\alpha_c B^0}\right)_{i,j,k} \frac{p^{n+1}_{i,j,k} - p^n_{i,j,k}}{\Delta t} + \\
\frac{1}{144}\frac{g}{g_c}\left[\beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i+\frac{1}{2},j} (G_{i+1,j} - G_{i,j}) + \beta_c \frac{A_x k_x}{\mu B \Delta x}\bigg|_{i-\frac{1}{2},j} (G_{i-1,j} - G_{i,j}) + \right. \\
\left. \beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j+\frac{1}{2}} (G_{i,j+1} - G_{i,j}) + \beta_c \frac{A_y k_y}{\mu B \Delta y}\bigg|_{i,j-\frac{1}{2}} (G_{i,j-1} - G_{i,j})\right]
\end{aligned}
$$

3.5 Transmissibility calculation in North, South, West and East directions

Because using interpolation for generating reservoir property values, each point in the grid map have unique value lead to the fact that a block have four values of permeability, thickness and top. Block center grid method is used to implement in this project.
Transmissibility between two block is calculated by harmonic averaging:

$$
\frac{2\beta_c}{\mu B \left[\dfrac{1}{\left(\dfrac{A_x k_x}{\Delta x}\right)_{i,j}} + \dfrac{1}{\left(\dfrac{A_x k_x}{\Delta x}\right)_{i+1,j}}\right]}
$$

The viscosity, formation volume factor and density is calculated using pressure values from 2 neighbor blocks.

$$
\mu^n_{i+\frac{1}{2},j,k} = \mu^n\left(p^n_{i+\frac{1}{2},j,k}\right)
$$

$$
B^n_{i+\frac{1}{2},j,k} = B^n\left(p^n_{i+\frac{1}{2},j,k}\right)
$$

$$
\rho^n_{i+\frac{1}{2},j,k} = \rho^n\left(p^n_{i+\frac{1}{2},j,k}\right)
$$

3.6 Well properties definition and calculation

Productivity index of each well is calculated by the equation:

$$\Omega_{i,j,k} = \left[\frac{2\pi\bar{k}\Delta L}{\mu B(ln(\frac{r_e}{r_w}) + s)} \right]$$

3.7 Coefficient matrix construction

The reservoir has 247 active blocks, so a system of equations of size 247×247 need to be built by assigning the transmissibility values to the system of equations. A typical for loop is used to create the cofficient matrix.

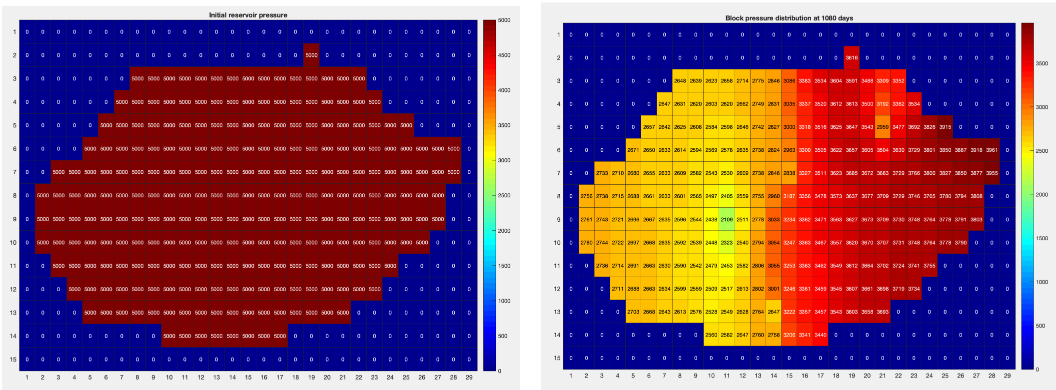
In this project, I have used three different programming language for implementation. In Python, the linear algebrea library is used for solving the system of equations, default solver in Matlab is also used and eigen library in C++ (LU decomposition).

4 Result

4.1 Reservoir block pressure after 3 years

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	3615.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	2648.03	2639.09	2623.15	2657.84	2713.59	2775.21	2846.39	3085.51	3382.64	3534.26	3604.25	3590.71	3488.28	3309.08	3352.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	2647.25	2631.32	2620.34	2602.93	2620.06	2661.83	2749.17	2830.62	3034.52	3337.46	3519.54	3612.38	3612.62	3500.06	3192.46	3362.06	3533.65	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	2657.50	2641.81	2625.49	2608.09	2583.79	2598.13	2645.93	2742.20	2826.94	3000.23	3318.00	3515.85	3624.78	3646.77	3543.49	2958.99	3477.14	3692.28	3825.83	3914.69	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	2671.45	2650.03	2632.71	2614.19	2594.16	2569.27	2578.29	2634.66	2737.98	2824.44	2963.09	3299.72	3505.03	3621.81	3656.54	3605.44	3504.17	3629.91	3728.63	3801.10	3849.91	3887.03	3918.11	3961.05	0.00
0.00	0.00	2733.27	2709.65	2680.36	2654.60	2632.71	2608.71	2582.38	2543.06	2529.88	2609.15	2738.42	2845.96	2837.54	3326.67	3510.76	3622.89	3684.69	3671.67	3683.35	3729.25	3766.18	3799.98	3826.64	3850.08	3877.08	3954.74	0.00
0.00	2756.32	2737.60	2714.81	2688.07	2660.72	2632.87	2601.47	2564.68	2496.93	2405.40	2558.97	2754.98	2959.66	3187.34	3355.75	3478.08	3572.54	3636.83	3676.62	3709.19	3729.16	3746.32	3764.92	3780.06	3794.06	3807.63	0.00	0.00
0.00	2761.10	2743.06	2721.14	2695.80	2667.00	2634.58	2595.53	2544.46	2438.26	2109.35	2511.39	2777.51	3033.45	3234.34	3361.74	3471.39	3563.01	3626.80	3672.72	3708.58	3730.09	3747.77	3763.79	3778.34	3791.29	3802.66	0.00	0.00
0.00	2779.94	2743.86	2721.76	2697.34	2668.29	2634.73	2592.46	2538.58	2447.53	2323.14	2540.33	2794.05	3053.53	3247.34	3362.60	3466.72	3556.78	3620.32	3669.89	3707.44	3730.59	3748.24	3763.64	3778.17	3790.11	0.00	0.00	0.00
0.00	0.00	2736.09	2714.22	2691.28	2663.30	2629.71	2589.79	2542.37	2478.82	2452.58	2582.11	2805.83	3055.15	3252.72	3362.63	3461.65	3549.03	3611.67	3664.21	3701.87	3724.40	3740.78	3755.21	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	2711.00	2688.46	2663.04	2633.52	2599.31	2558.62	2508.54	2516.80	2612.78	2801.57	3001.09	3246.30	3360.96	3458.71	3544.69	3606.95	3660.92	3698.48	3718.89	3734.06	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	2703.25	2667.72	2643.14	2613.08	2575.86	2528.26	2548.51	2627.76	2763.91	2646.85	3221.97	3356.69	3456.85	3542.87	3603.25	3657.59	3692.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2560.32	2582.12	2647.27	2759.85	2758.22	3206.04	3340.67	3439.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 6: Reservoir pressure distribution after 3 years

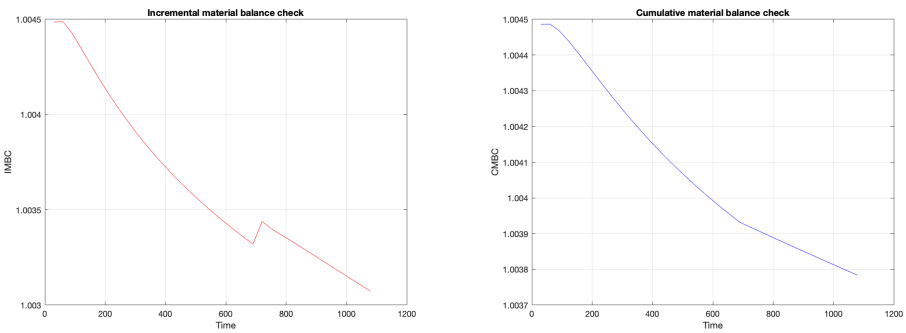


(a) Initial reservoir pressure (b) Reservoir pressure at 3 years

Figure 7: Reservoir pressure distribution

4.2 Material balance check

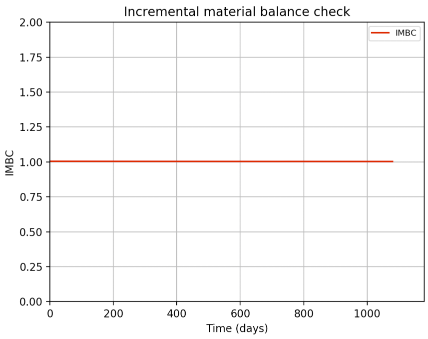
1. Material balance check from Matlab



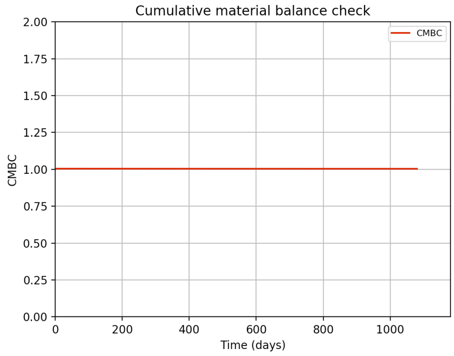
(a) Incremental material balance check (b) Cumulative material balanace check

Figure 8: Material balance checks in Matlab

2. Material balance check from Python



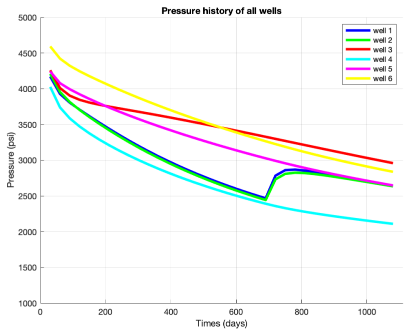
(a) Incremental material balance check



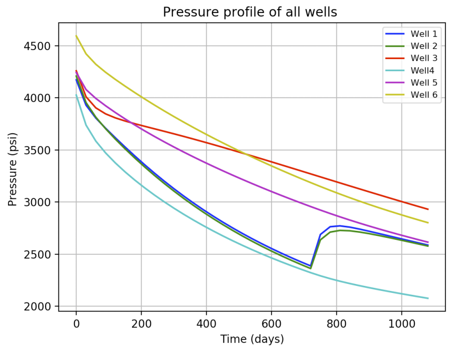
(b) Cumulative material balanace check

Figure 9: Material balance checks in Python

4.3 Production history



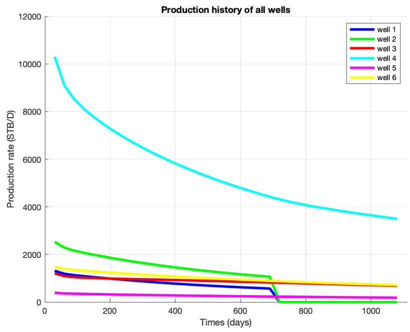
(a) Production history in Matlab



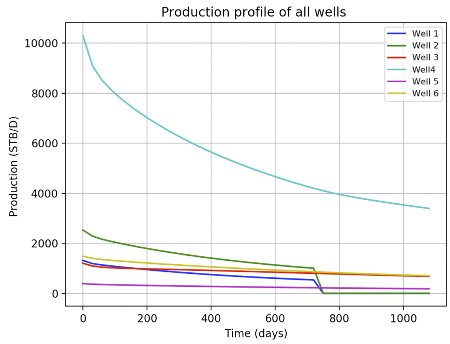
(b) Production history in Python

Figure 10: Production history for all wells

4.4 Well block pressure history



(a) Well pressure in Matlab



(b) Well pressure in Python

Figure 11: Well pressure for all wells

4.5 Additional result

In this project, I have compared runtime of three different languages which are Python, Matlab and C++. The table below is the comparison of runtime among them (in case of just running simulation and no plots)

Table 3: Runtime comparison between Python, Matlab and C++

	Languages	**Runtime**
1	Python	0.60501
2	Matlab (average)	0.5124
3	C++	0.6891

5 Summary and conclusion

This project is constructed for single phase with slightly compressible fluid in 2-D. The heterogeneity of reservoir is shown through different distribution in permeability in both X and Y directions, diverse distribution in porostiby, thickness and grid dimensions.
In this project, using different programming languages and Webplot digitizing tools, the reservoir model is generated. A simulation is run for a total of 3 years with schedule of shut-in well 1 and well at the end of second years.
Through the project, we can have a deeper understanding about reservoir

simulation and get familiar with implementation of commercial simulator softwares.

My implementation

I have used mainly python to solve the computer project 1.
The main file is "**cp1_solve.py**" with OOP programming principle.
The file "**cp1_matlab.m**" my implementation on Matlab
The file "**cp1_cpp.cpp**" my implementation on C++.

References

[1] Numerical reservoir simulation slides. Dr-Miao Zhang

[2] Turgay Ertekin, Jamal H. Abou-Kassem, Gregory R.King. *Basic applied reservoir simulation.* Chaper 8.3, Single phase slightly compressible flow, 186–210.