A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method

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BGCE Student Project

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Multiphase flow

Examples

- Liquid-liquid mixtures (e.g. oil in water)
- Gas-liquid mixtures (e.g. bubble dynamics)

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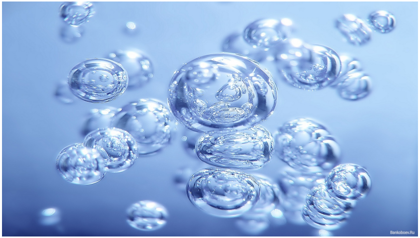

Macroscopic fluid mechanics

- N immiscible fluids.
- Each has own ρ_i, ν_i
- Hydrodynamics described by (incompressible) NSE

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$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{1}{\varrho_i} \nabla p + \nu_i \nabla^2 \vec{v}$$

Mathematic foundation

Interface conditions

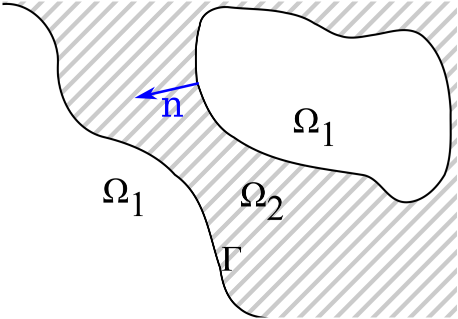


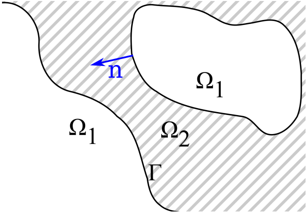
Figure: Two fluid domains Ω_i and interface Γ inbetween

- Velocity across interface is C_0-continous

$$[v] = \lim_{\epsilon \to 0} (\vec{v}\,(x + \epsilon\vec{n}) - \vec{v}(x - \epsilon\vec{n})) = 0$$

Mathematic foundation

Interface conditions cont'd



- Normal stress is balanced by surface tension → pressure jump

$$[T] \cdot \vec{n} = \lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon \vec{n}) - \mathbf{T}_1(x - \epsilon \vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

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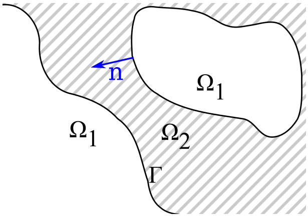

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where $\mathbf{T}_i$ is the stress tensor $\mathbf{T}_i = 2\nu_i \rho_i \mathbf{S}_i - p\mathbb{I}$ and κ is the curvature of the interface $\nabla \cdot \vec{n}$. $\mathbf{S}_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$.

To solve the two-phase problem we need to:

- solve the flow equations $\rightarrow$ LBM
- compute the motion of the interface $\rightarrow$ Level Set Method
- couple the two algorithms $\rightarrow$ according BC's

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Interface capturing

The interface between fluid phases is captured by a Level-Set Method. I.e. a *level set function* $\varphi := \varphi(x, t) \to \mathbb{R}$ is tracked through the fluid domain. The interface is given (implicitly) by the zero-isosurface of this function. It holds:

$$\frac{\partial \varphi}{\partial t} + \vec{v} \cdot \nabla \varphi = 0$$

where $\vec{v}$ is the velocity at the interface, given by NSE.
Benefit of Level-Set Method:

- Implicit surface description eases bubble coalescence/breakup in code
- High density and viscosity ratios ($> 10^3$) possible for simulation

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Interface implementation (Methods Coupling)

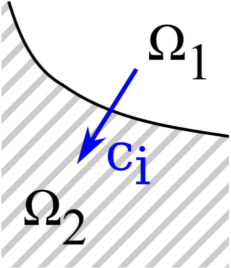
Hydrodynamics are solved by LBM (D2Q9, SRT for our first test)

- Interface becomes a new boundary condition for LBM

$$f_i(x, t+1) = f_{i^*}^{+}(x, t) + 6\Delta x f_i^* c_i \cdot \tilde{v} + R_i$$

- $\tilde{v}$ is the velocity on the interface along the direction c_i

- R_i ensures the jump conditions of the normal stress and corrects the error terms resulting from the bounce back treatment



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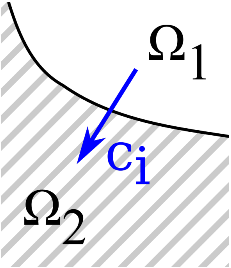

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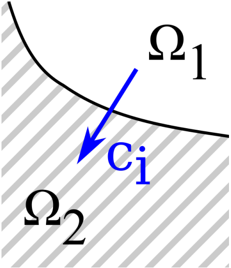

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- with:

$$\Lambda_i = c_i \otimes c_i - \frac{1}{3}|c_i|^2 \mathbb{I}$$

$$A = -q(1 - q)[S] - (q - 1/2)S^2 + O(\Delta x)$$

- $S^{(k)}$ velocity gradient at x_k
- $[S] = \lim_{\epsilon \to 0}(S(x + \epsilon\vec{n}) - S(x - \epsilon\vec{n}))$ jump of velocity gradient at the interface. Depends on normal, tangent and curvature.

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- Run level set method to calculate surface description
- Run LBM for a prescribed number of steps
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Validation

Validation setups

TODO: Hier könnte man Bilder von unserer ersten Simulation zeigen. Leider können wir die Richtigkeit bisher nicht mit Zahlen belegen.

Outlook

$\rightarrow$ *Conclusion habe ich rausgenommen. Ich wüsste zumindest nicht was wir da reinschreiben könnten. Wenn du was weißt, darfst du das aber sehr gerne wieder einfügen.*

Outlook:

- Add correction term to prevent mass loss
- Reduce computational effort: Store Level-Set function only in narrow band around interface, Adalsteinsson and Sethian TODO: Quellen als Footnotes + Uebersichtsfolie
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References

- Thömmes, Guido, et al. "A lattice Boltzmann method for immiscible multiphase flow simulations using the level set method." Journal of Computational Physics 228.4 (2009): 1139-1156
- Mitchell, Ian, A Toolbox of Level Set Methods