A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method

Lorenz Hufnagel, Daniel Zint

BGCE Student Project

x y, 2015

Multiphase flow - Examples

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- e.g. Oil in water
- ...

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Macroscopic fluid mechanics

- N immiscible fluids.
- Each has own ρ_i, ν_i
- Hydrodynamics described by (incompressible) NSE

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$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{1}{\rho_i} \nabla p + \nu_i \nabla^2 \vec{v}$$

Mathematic foundation

Interface conditions

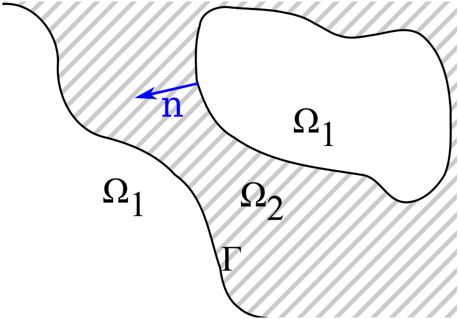


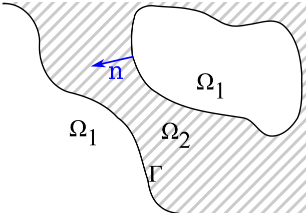
Figure: Two fluid domains Ω_i and interface Γ inbetween

- Velocity across interface is C_0-continous

$$\lim_{\epsilon \to 0} (\vec{v}\,(x + \epsilon \vec{n}) - \vec{v}(x - \epsilon \vec{n})) = 0$$

Mathematic foundation

Interface conditions cont'd



- Normal stress is balanced by surface tension

$$\lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon \vec{n}) - \mathbf{T}_1(x - \epsilon \vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

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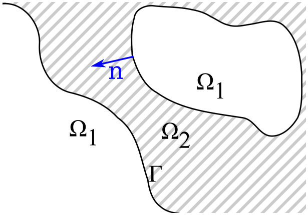
Interface conditions cont'd



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$$\lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon \vec{n}) - \mathbf{T}_1(x - \epsilon \vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

where $\mathbf{T}_i$ is the stress tensor $\mathbf{T}_i = 2\nu_i \rho_i \mathbf{S}_i - p\mathbf{Id}$ and κ is the curvature of the interface $\nabla \cdot \vec{n}$. $\mathbf{S} = \frac{1}{2}(\partial_{x_i} v_j + \partial_{x_j} v_i)$

Mathematic foundation

Interface capturing

The interface between fluid phases is captured by a Level-Set Method.
I.e. a *level set function* $\phi := \phi(x, t) \to \mathbb{R}$ is tracked through the fluid domain. The interface is given by the zero-isosurface of this function. It holds:

$$\frac{\partial \varphi}{\partial t} + \vec{v} \cdot \nabla \varphi = 0$$

Mathematic foundation

Interface capturing

Hydrodynamics are solved by LBM.

- Interface becomes a new boundary condition for LBM

$$f_i(x, t+1) = f_{i^*}^{+}(x, t) + 6hf_i^* c_i \cdot \tilde{u} + R_i$$

- $\tilde{u}$ is the velocity on the interface along the direction c_i
- R_i ensures the jump conditions of the normal stress and corrects the error terms resulting from the bounce back treatment

TODO: Bild vom Interface <Coupling und BC's erklären!!...>

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Algorithm for LBM with level set

- Create initial interface
- Run level set method to calculate surface description
- Run LBM for a prescribed number of steps
- Run level set method for the same number of steps
- Repeat till end of simulation

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Validation

Validation setups

Conclusion & Outlook

$\rightarrow \ldots$

Outlook:

- Add correction term to prevent mass loss
- Reduce computational effort: Store Level-Set function only in narrow band around interface, Adalsteinsson and Sethian TODO: Quellen als Footnotes + Uebersichtsfolie
- Include thermal flow (simulate e.g. lava lamp) / Include gravity
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References

Thömmes, Guido, et al. ”A lattice Boltzmann method for immiscible multiphase flow simulations using the level set method.” Journal of Computational Physics 228.4 (2009): 1139-1156