A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method

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BGCE Student Project

x y, 2015

Multiphase flow - Examples

- Examples
- e.g. Oil in water
- ...

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Macroscopic fluid mechanics

- N immiscible fluids.
- Each has own ρ_i, ν_i
- Hydrodynamics described by (incompressible) NSE

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$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} = -\frac{1}{\rho_i} \nabla p + \nu_i \nabla^2 \vec{v}$$

Mathematic foundation

Interface conditions

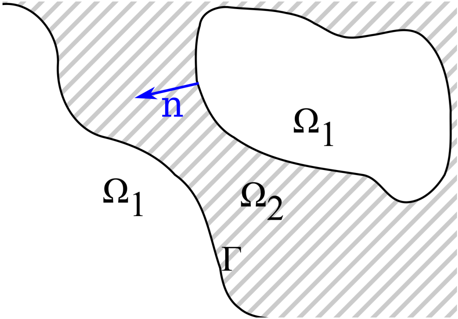


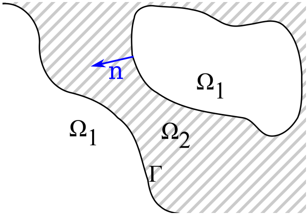
Figure: Two fluid domains Ω_i and interface Γ inbetween

- Velocity across interface is C_0-continous

$$\lim_{\epsilon \to 0} (\vec{v}\,(x + \epsilon \vec{n}) - \vec{v}(x - \epsilon \vec{n})) = 0$$

Mathematic foundation

Interface conditions cont'd



- Normal stress is balanced by surface tension

$$\lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon \vec{n}) - \mathbf{T}_1(x - \epsilon \vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

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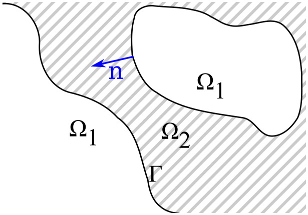

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$$\lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon \vec{n}) - \mathbf{T}_1(x - \epsilon \vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

where $\mathbf{T}_i$ is the stress tensor $\mathbf{T}_i = 2\nu_i\rho_i\mathbf{S}_i - p\mathbf{Id}$ and κ is the curvature of the interface $\nabla \cdot \vec{n}$. $\mathbf{S} = \frac{1}{2}(\partial_{x_i} v_j + \partial_{x_j} v_i)$

Mathematic foundation

Interface capturing

The interface between fluid phases is captured by a Level-Set Method.
I.e. a *level set function* $\phi := \phi(x,t) \to \mathbb{R}$ is tracked through the fluid domain. The interface is given by the zero-isosurface of this function. It holds:

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0.$$

Mathematic foundation

Interface capturing

Level-Set function only stored in narrow band around interface, Adalsteinsson and Sethian TODO: Quellen als Footnotes + Uebersichtsfolie

Interface properties (curvature, normal) are obtained from discrete level-set function by weighted least-squares method.

Mathematic foundation

Level Set

- The level set equation of Osher and Sethian

$$\varphi_t + v \cdot \nabla \varphi = 0$$

- with the level set function φ
- we will use the signed distance function as level set function and therefore we get

$$n = \nabla \varphi$$

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Mathematic foundation

Interface capturing

Hydrodynamics are solved by LBM.
<Coupling und BC's erklären!!...>

Mathematic foundation

Coupling of LBM and level set method

- New boundary condition at the interface

$$f_i(x, t+1) = f_{i^*}^{+}(x, t) + 6hf_i^* c_i \cdot \tilde{u} + R_i$$

Validation

Validation setups

Conclusion & Outlook

$\rightarrow \ldots$

Outlook:

- Include thermal flow (simulate e.g. lava lamp)
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References

Thömmes, Guido, et al. ”A lattice Boltzmann method for immiscible multiphase flow simulations using the level set method.” Journal of Computational Physics 228.4 (2009): 1139-1156