A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method

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BGCE Student Project

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Multiphase flow

Examples

- Liquid-liquid mixtures (e.g. oil in water)
- Gas-liquid mixtures (e.g. bubble dynamics)

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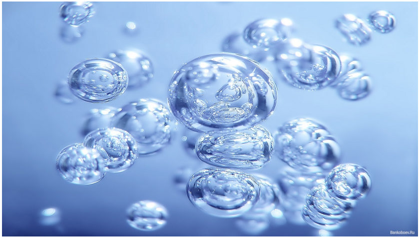

Macroscopic fluid mechanics

- N immiscible fluids.
- Each has own ρ_i, ν_i
- Hydrodynamics described by (incompressible) NSE

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$$\nabla \cdot \vec{v}_i = 0 \text{ in } \Omega_i$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_i \cdot \nabla)\, \vec{v}_i = -\frac{1}{\varrho_i} \nabla p_i + \nu_i \nabla^2 \vec{v}_i \text{ in } \Omega_i$$

Mathematic foundation

Interface conditions

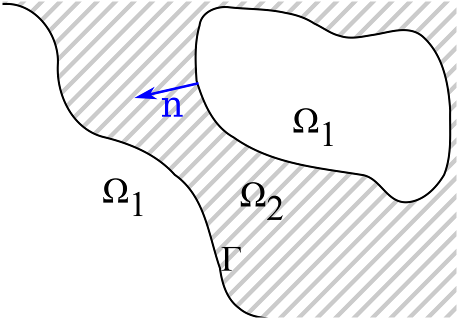


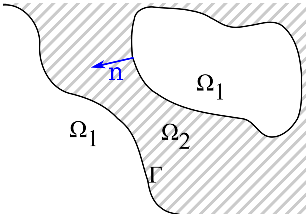
Figure : Two fluid domains Ω_i and interface Γ inbetween

- Velocity across interface is C_0-continous

$$[v] = \lim_{\epsilon \to 0} (\vec{v}\,(x + \epsilon\vec{n}) - \vec{v}(x - \epsilon\vec{n})) = 0$$

Mathematic foundation

Interface conditions cont'd



- Normal stress is balanced by surface tension → pressure jump

$$[T] \cdot \vec{n} = \lim_{\epsilon \to 0} (\mathbf{T}_2(x + \epsilon\vec{n}) - \mathbf{T}_1(x - \epsilon\vec{n})) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$

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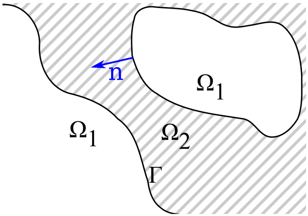

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where $\mathbf{T}_i$ is the stress tensor $\mathbf{T}_i = 2\nu_i\rho_i\mathbf{S}_i - p\mathbb{I}$ and κ is the curvature of the interface $\nabla \cdot \vec{n}$. $\mathbf{S}_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$.

Mathematic foundation

Solvers for the two-phase problem

- Colour gradient method of Gunstensen and Rothman
  - only for small density and viscosity differences
- Method of Shan and Chen
  - models miscible fluids
  - immiscible flows only approximatly
- Free surface methods
  - only for big density and viscosity differences
- Method of Thoemmes and Becker
  - using Level Set Method for interface motion
  - small and big density/viscosity differences possible
  - mass loss in Level Set Method

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Ingredients for solver with Level Set Method

To solve the two-phase problem we need to:

- solve the flow equations $\rightarrow$ LBM
- compute the motion of the interface $\rightarrow$ Level Set Method
- couple the two algorithms $\rightarrow$ according BC's

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LBM

- Stream

$$f_i(x + c_i, t + 1) = f_i(x, t) - \frac{1}{\tau}(f_i - f_i^{eq}) + G_i$$

- Collide (Bhatnagar-Gross-Krook approximation)

$$f_i^{eq}(\rho, v) = f_i^*(\rho + 3c_i \cdot v + \frac{9}{2}(c_i \cdot v)^2 - \frac{3}{2}v^2)$$

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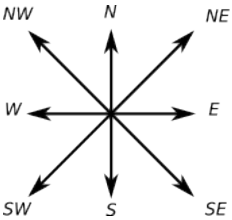
LBM

the corresponding D2Q9 weight factors



$$f_i^* = \begin{cases} \frac{4}{9} & i = C \\ \frac{1}{9} & i = N, E, S, W \\ \frac{1}{36} & i = NE, SE, SW, NW \end{cases}$$

Mathematic foundation

LBM

we can compute for

- Density

$$\rho(x,t) = \sum f_i(x,t)$$

- Velocity

$$u = \sum f_i(x,t) c_i$$

Mathematic foundation

Interface capturing

Interface between fluid phases is captured by level set method

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- Level set function: $\varphi := \varphi(x, t) \to \mathbb{R}$
  Interface is the zero-isosurface of this function
- Level set equation:

$$\frac{\partial \varphi}{\partial t} + \vec{v} \cdot \nabla \varphi = 0$$

where $\vec{v}$ is the velocity at the interface, given by NSE.

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- Implicit surface description eases bubble coalescence/breakup in code
- High density and viscosity ratios ($> 10^3$) possible for simulation

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Interface implementation (Methods Coupling)

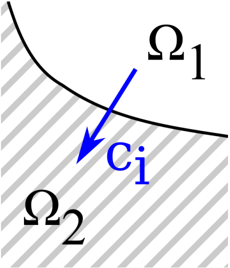
Hydrodynamics are solved by LBM (Stream-Collide, D2Q9, SRT for our first test)

- Interface becomes a new boundary condition for LBM

$$f_i(x, t+1) = f_{i^*}^{+}(x, t) + 6\Delta x f_i^* c_i \cdot \tilde{v} + R_i$$

- $\tilde{v}$ is the velocity on the interface along the direction c_i
- calculated by linear interpolation:

$$\tilde{v} = qv(x_2, t) + (1-q)v(x_1, t)$$



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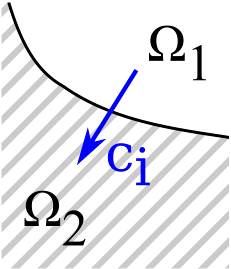

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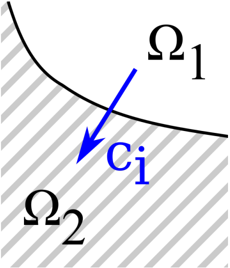

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Interface implementation (Methods Coupling)

R_i ensures the jump conditions of the normal stress and corrects the error terms resulting from the bounce back treatment.

- $R_i = 6\Delta x^2 f_i^* \Lambda_i : A$,
  with $A : B = \sum_{i,j} A_{ij} B_{ij}$.
- $\Lambda_i = c_i \otimes c_i - \frac{1}{3}|c_i|^2 \mathbb{I}$,
  with $c_i \otimes c_i = c_i c_i^T$.
- $A = -q(1-q)[S] - (q - 1/2)S^{(k)}$
  - $S^{(k)}$ velocity gradient at x_k
  - $[S]$ jump of velocity gradient at the interface. Depends on $\vec{n}$, $\vec{t}$ and κ.

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Algorithm for LBM with level set

- Create initial interface
- Run level set method to calculate surface description
- Run LBM for a prescribed number of steps
- Run level set method for the same time interval
- Repeat until end of simulation

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- Implementation in Matlab
- Own LBM-solver
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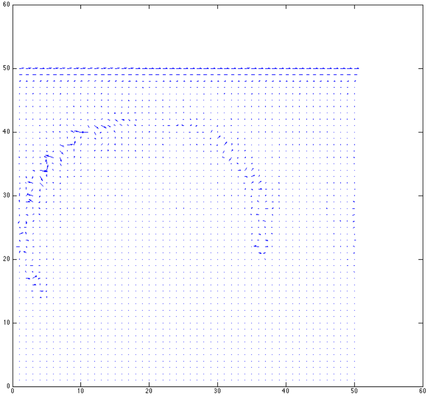

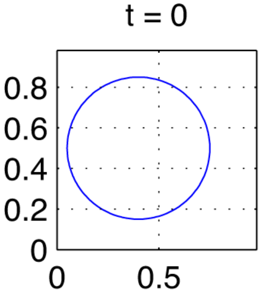

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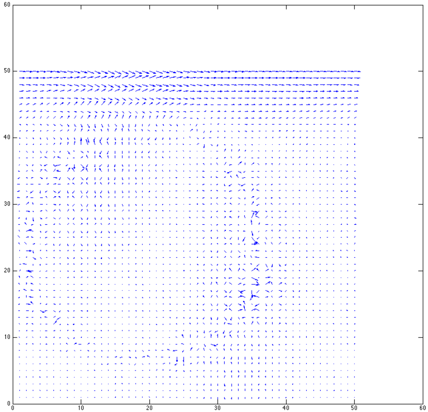

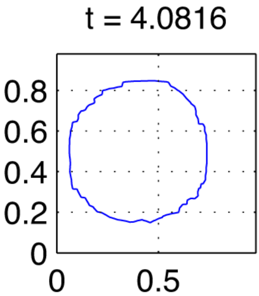

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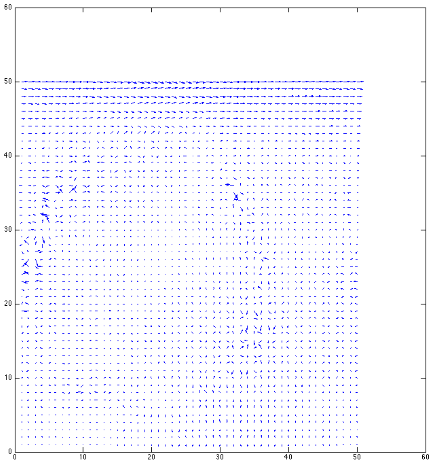

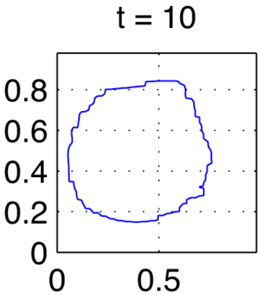

Outlook

. . . so far intermediate results

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- Create more examples
- Add correction term to prevent mass loss
- Reduce computational effort: Store Level-Set function only in narrow band around interface, Adalsteinsson and Sethian

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- D. Adalsteinsson, and J. A. Sethian. A fast level set method for propagating interfaces. J. Comput. Phys., 118(2):269-277, 1995
- Mitchell, Ian, A Toolbox of Level Set Methods