A Lattice Boltzmann Method for immiscible multiphase flow simulations using the Level Set Method

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BGCE Student Project

x y, 2015

Multiphase flow - Examples

- Examples
- e.g. Oil in water
- . . .

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Macroscopic fluid mechanics

- N immiscible fluids.
- Each has own ρ_i, ν_i
- Hydrodynamics described by (incompressible) NSE

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$$\nabla \cdot \vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\,\vec{v} = -\frac{1}{\rho_i}\nabla p + \nu_i \nabla^2 \vec{v}$$
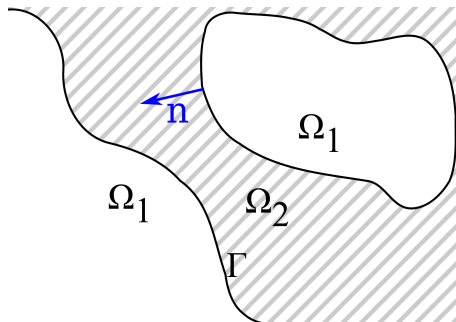
Mathematic foundation
Interface conditions

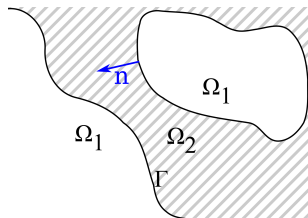Figure: Two fluid domains Ω_i and interface Γ inbetween

- Velocity across interface is C_0-continous

$$\lim_{\epsilon \to 0} \left(\vec{v} \left(x + \epsilon \vec{n} \right) - \vec{v}(x - \epsilon \vec{n}) \right) = 0$$

Mathematic foundation
Interface conditions cont'd

- Normal stress is balanced by surface tension

$$\lim_{\epsilon \to 0} \left(\mathbf{T}_2(x + \epsilon\vec{n}) - \mathbf{T}_1(x - \epsilon\vec{n}) \right) \cdot \vec{n} = 2\sigma\kappa\vec{n}$$
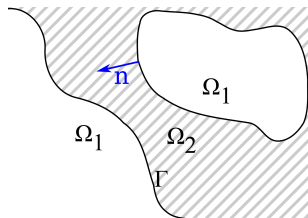
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where $\mathbf{T}_i$ is the stress tensor $\mathbf{T}_i = 2\nu_i \rho_i \mathbf{S}_i - p\mathbf{Id}$ and κ is the curvature of the interface $\nabla \cdot \vec{n}$. $\mathbf{S} = \frac{1}{2}(\partial_{x_i} v_j + \partial_{x_j} v_i)$

Mathematic foundation
Interface capturing

The interface between fluid phases is captured by a Level-Set Method.

I.e. a *level set function* $\phi := \phi(x,t) \rightarrow \mathbb{R}$ is tracked through the fluid domain. The interface is given by the zero-isosurface of this function. It holds

$$\frac{\partial \phi}{\partial t} + \vec{v} \cdot \nabla \phi = 0$$

Mathematic foundation
Interface capturing

Level-Set function only stored in narrow band around interface, Adalsteinsson and Sethian
Interface properties (curvature, normal) are obtained from discrete level-set function by weighted least-squares method.

Mathematic foundation
Interface capturing

Hydrodynamics are solved by LBM.
$<$Coupling und BC's erklären!!... $>$

Validation

Validation setups

Conclusion & Outlook

$\rightarrow$ ...

Outlook:

- Include thermal flow (simulate e.g. lava lamp)
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References

Thömmes, Guido, et al. "A lattice Boltzmann method for immiscible multiphase flow simulations using the level set method." Journal of Computational Physics 228.4 (2009): 1139-1156