Marching Algorithm

Computational algorithm workflow

1 read all the input data for the current pipe segment:

oil.rt, gas.rt, oil.sg, gas.sg, L, diam, angle, temp.grad and initial pressure

2 set the calculation increment

$$dL = L/n$$

Usually n = 30

3 guess initial outlet pressure.

Can assume 0.002 psi/ft for the gradient

$$\frac{dP}{dL} = 0.002$$

4 calculate the average pressure:

$$p_{avg} = \frac{(p_0 + p_1)}{2}$$

5 calculate the fluid properties at P and T for P_{avg}

oil.fvf, gas.fvf, oil.rt, gas.rt, oil.supvel, gas.supvel

$$B_o, B_g Q_o, Q_g V_{SL}, V_{SG}\rho_o, \rho_g \nu_o, \nu_g z Re, f$$

6 calculate the pressure gradient dp.dz (-dP/dL)

$$\left(\frac{dp}{dL}\right) = f(P_{avg})$$

7 calculate the outlet-calculated pressure

$$p_{i+1(C)} = p_i - \left(\frac{dP}{dL}\right)_i dL_i$$

8 compare the guessed and calculated outlet pressures:

(p.guess - p.calc) / p.calc should be less than the tolerance otherwise, increase iteration and make p.guess = p.calc

$$|(p_{i+1(G)} - p_{i+1(C)})/p_{i+1(C)}| < \epsilon$$

9 Repeat 1 to 6 until convergence achieved.

Ten iterations is the usual.

10 when convergence is achive, move to the next pipe increment

p2.inlet = p1.outlet

11 Repeat for all pipe increments

and calculate p and dp.dz for the current segment

12 If there are more pipe segments, repeat calculations

1-11

Implementation of marching algorithm for well gradient

For demo purposes, only using a dummy function, $log(P_{avg})^{-1}$. The next thing to do is generating a dataframe with the data. Actually, it could be two dataframes, one for the main results for each pipe segment; and the second dataframe -with more detail-, showing the iterations.

```
library(latex2exp)
library(ggplot2)

tolerance = 0.00001
thp       = 200          # initial pressure (tubing head pressure)
depth_wh  = 0            # depth at wellhead
depth_bh  = 9700         # depth at bottomhole
segments  = 30           # calculation segments

# rows have to be greater than segments to allocate the zero or initial value
# consider that in length.out parameter in the sequence below
depth   <- seq.int(from = depth_wh, to = depth_bh, length.out = segments+1)
n       <- length(depth)   # depth points same as # rows or (segments+1)

# dummy function that represents a lot of subsurface calculations
fPa <- function(x) 9e-02 + 1e-04 * x + 5e-08  * x^2 - 2e-11 * x^3

depth_top <- depth_wh
dp_dz     <- 0.002                    # 1st approximation of the gradient
p_in      <- thp                      # the initial pressure
output <- vector("list")
for (i in seq_len(n)) {               # n: is the number of depths or # rows
    depth_prev <- ifelse(i == 1, depth_top, depth[i-1])
    dL = depth[i] - depth_prev              # calculate dL
    p_out = p_in + dp_dz * dL               # calculate outlet pressure
    cat(sprintf("i=%2d depth=%8.0f dL=%8.1f segment=%d \n",  # header outer loop
                i, depth[i], dL, i-1))
    cat(sprintf("%8s %6s %6s %8s %8s %8s %10s \n",           # header inner loop
            "iter", "p_in", "p_out", "p_avg", "p_calc", "dp/dz", "eps"))
    epsilon <- 1   # initial values before inner loop
    iter <- 1
    # here we start iterating for the pressure gradient
    while (epsilon > tolerance) {       # loop until AE greater than tolerance
      p_avg <- (p_in + p_out) / 2       # calculate average pressure
      dp_dz <- fPa(p_avg)   # calculate gradient as function of average pressure
      p_calc <- p_in - (-dp_dz) * dL
      epsilon <- abs( (p_out - p_calc) / p_calc )  # absolute error
      cat(sprintf("%8d %6.1f %6.1f %8.2f %8.2f %8.5f %10.8f \n",
                  iter, p_in, p_out, p_avg, p_calc, dp_dz, epsilon))

      if (epsilon >= tolerance) p_out = p_calc # if error too big, iterate again
```

```
      iter <- iter + 1                          # with a new pressure
    } # end of while
    p_in = p_out      # assign p_out to the inlet pressure of new segment, p_in
    output[[i]] <- list(depth = depth[i], p_calc = p_calc,   # values to list
                        dp_dz = dp_dz, p_avg = p_avg)
} # end-for
#> i= 1 depth=       0 dL=     0.0 segment=0
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  200.0  200.0   200.00   200.00  0.11184 0.00000000
#> i= 2 depth=     323 dL=   323.3 segment=1
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  200.0  236.2   218.08   236.85  0.11398 0.00291946
#>        2  200.0  236.9   218.43   236.87  0.11402 0.00005615
#>        3  200.0  236.9   218.43   236.87  0.11402 0.00000108
#> i= 3 depth=     647 dL=   323.3 segment=2
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  236.9  273.7   255.30   275.17  0.11846 0.00521195
#>        2  236.9  275.2   256.02   275.20  0.11854 0.00010249
#>        3  236.9  275.2   256.03   275.20  0.11855 0.00000202
#> i= 4 depth=     970 dL=   323.3 segment=3
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  275.2  313.5   294.36   315.05  0.12326 0.00483722
#>        2  275.2  315.0   295.12   315.08  0.12335 0.00009717
#>        3  275.2  315.1   295.14   315.08  0.12335 0.00000195
#> i= 5 depth=    1293 dL=   323.3 segment=4
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  315.1  355.0   335.02   356.58  0.12836 0.00454033
#>        2  315.1  356.6   335.83   356.62  0.12846 0.00009306
#>        3  315.1  356.6   335.85   356.62  0.12847 0.00000191
#> i= 6 depth=    1617 dL=   323.3 segment=5
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  356.6  398.2   377.39   399.87  0.13378 0.00429984
#>        2  356.6  399.9   378.24   399.91  0.13390 0.00008982
#>        3  356.6  399.9   378.26   399.91  0.13390 0.00000188
#> i= 7 depth=    1940 dL=   323.3 segment=6
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  399.9  443.2   421.56   445.03  0.13954 0.00410123
#>        2  399.9  445.0   422.47   445.07  0.13966 0.00008719
#>        3  399.9  445.1   422.49   445.07  0.13967 0.00000185
#> i= 8 depth=    2263 dL=   323.3 segment=7
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  445.1  490.2   467.65   492.16  0.14565 0.00393422
#>        2  445.1  492.2   468.61   492.20  0.14578 0.00008501
#>        3  445.1  492.2   468.64   492.20  0.14579 0.00000184
#> i= 9 depth=    2587 dL=   323.3 segment=8
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  492.2  539.3   515.77   541.39  0.15213 0.00379124
#>        2  492.2  541.4   516.80   541.44  0.15227 0.00008313
#>        3  492.2  541.4   516.82   541.44  0.15228 0.00000182
#> i=10 depth=    2910 dL=   323.3 segment=9
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  541.4  590.7   566.06   592.85  0.15900 0.00366651
#>        2  541.4  592.8   567.14   592.90  0.15915 0.00008144
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#>        3  541.4  592.9   567.17   592.90  0.15915 0.00000181
#> i=11 depth=    3233 dL=   323.3 segment=10
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  592.9  644.4   618.63   646.65  0.16626 0.00355546
#>        2  592.9  646.7   619.78   646.71  0.16642 0.00007984
#>        3  592.9  646.7   619.80   646.71  0.16643 0.00000179
#> i=12 depth=    3557 dL=   323.3 segment=11
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  646.7  700.5   673.61   702.95  0.17394 0.00345433
#>        2  646.7  702.9   674.83   703.00  0.17411 0.00007826
#>        3  646.7  703.0   674.85   703.00  0.17411 0.00000177
#> i=13 depth=    3880 dL=   323.3 segment=12
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  703.0  759.3   731.15   761.86  0.18203 0.00335992
#>        2  703.0  761.9   732.43   761.91  0.18221 0.00007661
#>        3  703.0  761.9   732.46   761.92  0.18221 0.00000175
#> i=14 depth=    4203 dL=   323.3 segment=13
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  761.9  820.8   791.37   823.52  0.19054 0.00326942
#>        2  761.9  823.5   792.72   823.58  0.19073 0.00007482
#>        3  761.9  823.6   792.75   823.58  0.19073 0.00000171
#> i=15 depth=    4527 dL=   323.3 segment=14
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  823.6  885.3   854.42   888.08  0.19947 0.00318027
#>        2  823.6  888.1   855.83   888.14  0.19967 0.00007282
#>        3  823.6  888.1   855.86   888.14  0.19967 0.00000167
#> i=16 depth=    4850 dL=   323.3 segment=15
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  888.1  952.7   920.42   955.66  0.20881 0.00309004
#>        2  888.1  955.7   921.90   955.72  0.20901 0.00007053
#>        3  888.1  955.7   921.93   955.73  0.20902 0.00000161
#> i=17 depth=    5173 dL=   323.3 segment=16
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1  955.7 1023.3   989.52  1026.38  0.21853 0.00299644
#>        2  955.7 1026.4   991.05  1026.45  0.21875 0.00006791
#>        3  955.7 1026.5   991.09  1026.45  0.21875 0.00000154
#> i=18 depth=    5497 dL=   323.3 segment=17
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1026.5 1097.2  1061.82  1100.37  0.22861 0.00289723
#>        2 1026.5 1100.4  1063.41  1100.44  0.22883 0.00006487
#>        3 1026.5 1100.4  1063.45  1100.44  0.22884 0.00000145
#> i=19 depth=    5820 dL=   323.3 segment=18
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1100.4 1174.4  1137.44  1177.72  0.23900 0.00279021
#>        2 1100.4 1177.7  1139.08  1177.79  0.23922 0.00006138
#>        3 1100.4 1177.8  1139.12  1177.79  0.23923 0.00000135
#> i=20 depth=    6143 dL=   323.3 segment=19
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1177.8 1255.1  1216.47  1258.51  0.24963 0.00267322
#>        2 1177.8 1258.5  1218.15  1258.58  0.24986 0.00005740
#>        3 1177.8 1258.6  1218.18  1258.58  0.24986 0.00000123
#> i=21 depth=    6467 dL=   323.3 segment=20
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
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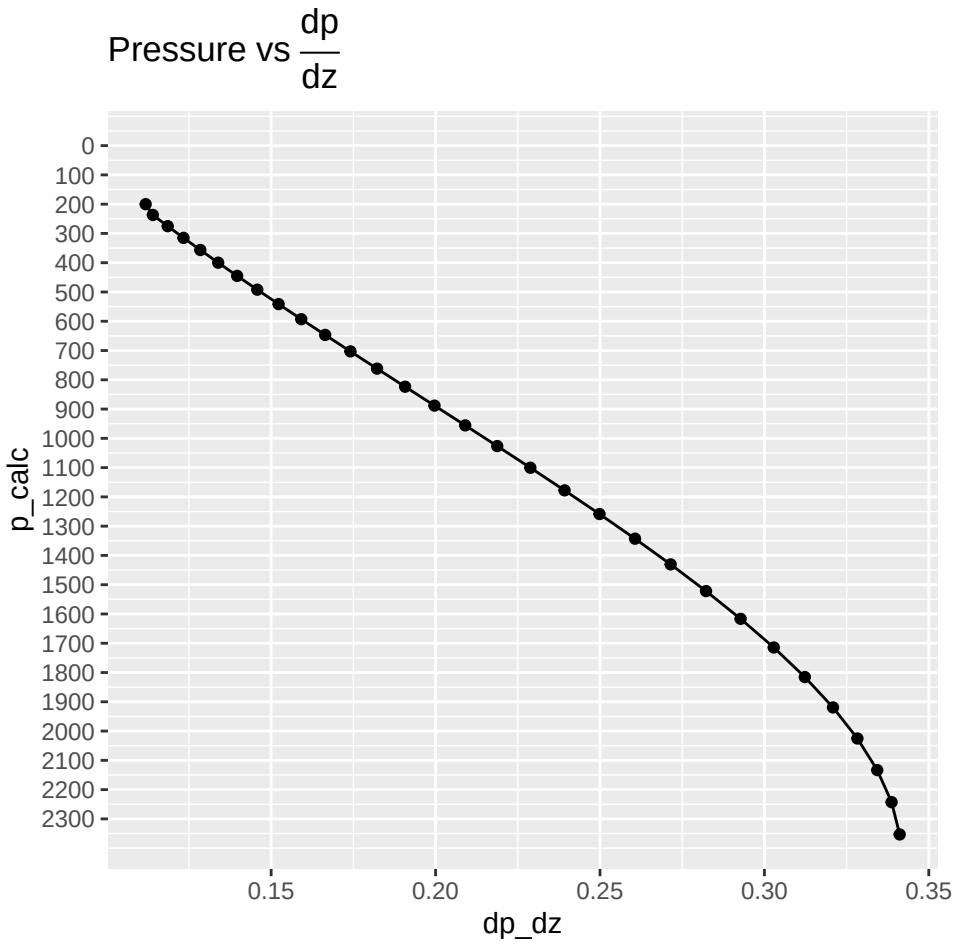
```
#>        1 1258.6 1339.4  1298.97  1342.78  0.26043 0.00254417
#>        2 1258.6 1342.8  1300.68  1342.85  0.26065 0.00005290
#>        3 1258.6 1342.9  1300.72  1342.86  0.26065 0.00000110
#> i=22 depth=    6790 dL=   323.3 segment=21
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1342.9 1427.1  1384.99  1430.57  0.27128 0.00240109
#>        2 1342.9 1430.6  1386.71  1430.63  0.27149 0.00004788
#>        3 1342.9 1430.6  1386.74  1430.64  0.27149 0.00000095
#> i=23 depth=    7113 dL=   323.3 segment=22
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1430.6 1518.4  1474.53  1521.83  0.28204 0.00224219
#>        2 1430.6 1521.8  1476.23  1521.89  0.28224 0.00004238
#>        3 1430.6 1521.9  1476.26  1521.89  0.28225 0.00000080
#> i=24 depth=    7437 dL=   323.3 segment=23
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1521.9 1613.2  1567.52  1616.49  0.29258 0.00206594
#>        2 1521.9 1616.5  1569.19  1616.55  0.29276 0.00003649
#>        3 1521.9 1616.6  1569.22  1616.55  0.29276 0.00000064
#> i=25 depth=    7760 dL=   323.3 segment=24
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1616.6 1711.2  1663.88  1714.42  0.30268 0.00187121
#>        2 1616.6 1714.4  1665.49  1714.47  0.30284 0.00003031
#>        3 1616.6 1714.5  1665.51  1714.47  0.30285 0.00000049
#> i=26 depth=    8083 dL=   323.3 segment=25
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1714.5 1812.4  1763.43  1815.40  0.31215 0.00165741
#>        2 1714.5 1815.4  1764.94  1815.45  0.31229 0.00002403
#>        3 1714.5 1815.4  1764.96  1815.45  0.31229 0.00000035
#> i=27 depth=    8407 dL=   323.3 segment=26
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1815.4 1916.4  1865.93  1919.15  0.32075 0.00142461
#>        2 1815.4 1919.2  1867.30  1919.19  0.32085 0.00001787
#>        3 1815.4 1919.2  1867.32  1919.19  0.32085 0.00000022
#> i=28 depth=    8730 dL=   323.3 segment=27
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 1919.2 2022.9  1971.06  2025.31  0.32821 0.00117372
#>        2 1919.2 2025.3  1972.25  2025.33  0.32828 0.00001213
#>        3 1919.2 2025.3  1972.26  2025.33  0.32828 0.00000013
#> i=29 depth=    9053 dL=   323.3 segment=28
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 2025.3 2131.5  2078.40  2133.41  0.33426 0.00090664
#>        2 2025.3 2133.4  2079.37  2133.43  0.33431 0.00000712
#> i=30 depth=    9377 dL=   323.3 segment=29
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 2133.4 2241.5  2187.46  2242.91  0.33866 0.00062631
#>        2 2133.4 2242.9  2188.16  2242.92  0.33868 0.00000320
#> i=31 depth=    9700 dL=   323.3 segment=30
#>     iter   p_in  p_out    p_avg   p_calc    dp/dz        eps
#>        1 2242.9 2352.4  2297.66  2353.21  0.34113 0.00033691
#>        2 2242.9 2353.2  2298.06  2353.21  0.34114 0.00000071

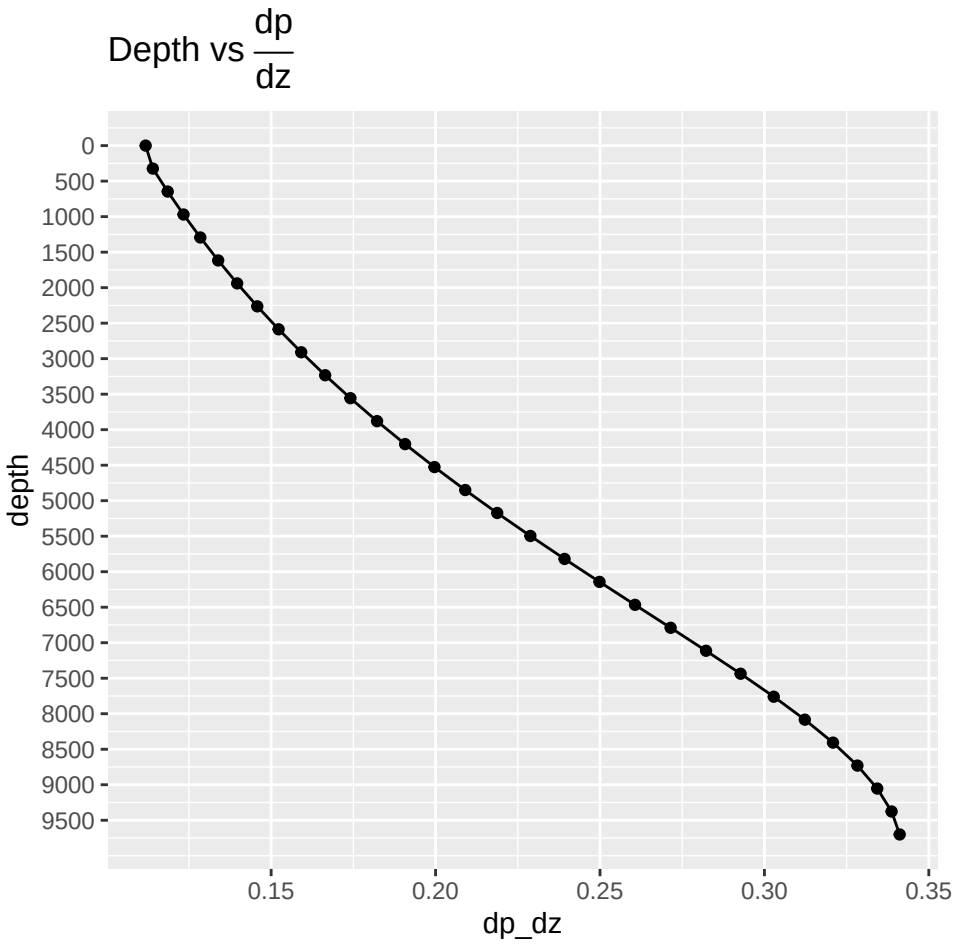
out_df <- data.table::rbindlist(output)    # convert list to table
out_df
```

```
#>          depth    p_calc     dp_dz     p_avg
#>  1:    0.0000  200.0000 0.1118400  200.0000
#>  2:  323.3333  236.8666 0.1140205  218.4332
#>  3:  646.6667  275.1959 0.1185450  256.0309
#>  4:  970.0000  315.0801 0.1233549  295.1374
#>  5: 1293.3333  356.6171 0.1284669  335.8480
#>  6: 1616.6667  399.9101 0.1338980  378.2629
#>  7: 1940.0000  445.0679 0.1396654  422.4882
#>  8: 2263.3333  492.2045 0.1457861  468.6353
#>  9: 2586.6667  541.4397 0.1522764  516.8211
#> 10: 2910.0000  592.8978 0.1591518  567.1677
#> 11: 3233.3333  646.7077 0.1664259  619.8016
#> 12: 3556.6667  703.0021 0.1741098  674.8537
#> 13: 3880.0000  761.9158 0.1822113  732.4576
#> 14: 4203.3333  823.5849 0.1907334  792.7490
#> 15: 4526.6667  888.1444 0.1996730  855.8632
#> 16: 4850.0000  955.7259 0.2090193  921.9336
#> 17: 5173.3333 1026.4540 0.2187516  991.0884
#> 18: 5496.6667 1100.4431 0.2288372 1063.4470
#> 19: 5820.0000 1177.7922 0.2392289 1139.1161
#> 20: 6143.3333 1258.5793 0.2498620 1218.1842
#> 21: 6466.6667 1342.8553 0.2606520 1300.7158
#> 22: 6790.0000 1430.6361 0.2714916 1386.7442
#> 23: 7113.3333 1521.8949 0.2822481 1476.2642
#> 24: 7436.6667 1616.5535 0.2927623 1569.2231
#> 25: 7760.0000 1714.4732 0.3028475 1665.5124
#> 26: 8083.3333 1815.4461 0.3122901 1764.9589
#> 27: 8406.6667 1919.1881 0.3208534 1867.3166
#> 28: 8730.0000 2025.3322 0.3282821 1972.2598
#> 29: 9053.3333 2133.4260 0.3343113 2079.3714
#> 30: 9376.6667 2242.9167 0.3386781 2188.1601
#> 31: 9700.0000 2353.2099 0.3411352 2298.0589
#>          depth    p_calc     dp_dz     p_avg
```

```
ggplot(out_df, aes(x=dp_dz, y=p_calc)) +
    scale_y_reverse(limits = c(max(out_df$p_calc), 0),
                    breaks = seq(0, max(out_df$p_calc), 100)) +
  geom_line() + geom_point() +
  labs(title = TeX("Pressure vs $\\frac{dp}{dz}$"))
```




```
ggplot(out_df, aes(x=dp_dz, y=depth)) +
    scale_y_reverse(limits = c(max(out_df$depth), 0),
                    breaks = seq(0, max(out_df$depth), 500)) +
    geom_line() +
    geom_point() + labs(title = TeX("Depth vs $\\frac{dp}{dz}$"))
```


Depth vs dp/dz
depth
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
9500
0.15
0.20
0.25
0.30
0.35
dp_dz