CHAPTER 19

Statistical Process Control and Process Capability

GENERAL DISCUSSION OF CONTROL CHARTS

Variation in any process is due to *common causes* or *special causes*. The natural variation that exists in materials, machinery, and people give rise to common causes of variation. In industrial settings, special causes, also known as *assignable causes*, are due to excessive tool wear, a new operator, a change of materials, a new supplier, etc. One of the purposes of *control charts* is to locate and, if possible, eliminate special causes of variation. The general structure of a control chart consists of *control limits* and a *centerline* as shown in Figure 19-1. There are two control limits, called an *upper control limit* or UCL and a *lower control limit* or LCL.

When a point on the control chart falls outside the control limits, the process is said to be out of statistical control. There are other anomalous patterns besides a point outside the control limits that also

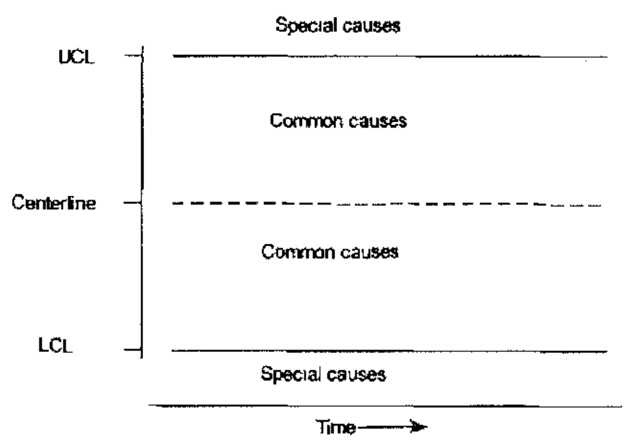

Fig. 19-1

indicate a process that is out of control. These will be discussed later. It is desirable for a process to be in control so that its behavior is predictable.

VARIABLES AND ATTRIBUTES CONTROL CHARTS

Control charts may be divided into either *variable control charts* or *attribute control charts*. The terms "variables" and "attributes" are associated with the type of data being collected on the process. When measuring characteristics such as time, weight, volume, length, pressure drop, concentration, etc., we consider such data to be continuous and refer to it as *variable data*. When counting the number of defective items in a sample or the number of defects associated with a particular type of item, the resulting data are called *attribute data*. Variable data are considered to be of a higher level than attribute data. Table 19.1 gives the names of many of the various variable and attribute control charts and the statistics plotted on the chart.

Table 19.1

Chart Type	Statistics Plotted
X-bar and R chart	Averages and ranges of subgroups of variable data
X-bar and Sigma chart	Averages and standard deviations of subgroups of variable data
Median chart	Median of subgroups of variable data
Individual chart	Individual measurements
Cusum chart	Cumulative sum of each $\bar{X}$ minus the nominal
Zone chart	Zone weights
EWMA chart	Exponentially weighted moving average
P-chart	Ratio of defective items to total number inspected
NP-chart	Actual number of defective items
C-chart	Number of defects per item for a constant sample size
U-chart	Number of defects per item for varying sample size

The charts above the line in Table 19.1 are variable control charts and the charts below the line are attribute control charts. We shall discuss some of the more basic charts. Minitab will be used to construct the charts. Today, charting is almost always accomplished by the use of statistical software such as Minitab. MINITAB is a registered trademark of Minitab Inc., 3081 Enterprise Drive, State College, PA 16801. Phone: 814-238-3280; fax: 814-238-4383. Telex: 881612. The author would like to thank Minitab Inc. for permission to use output from Minitab throughout the outline.

X-BAR AND *R* CHARTS

The general idea of an X-bar chart can be understood by considering a process having mean μ and standard deviation σ. Suppose the process is monitored by taking periodic samples, called *subgroups*, of size n and computing the sample mean, $\bar{X}$, for each sample. The central limit theorem assures us that the mean of the sample mean is μ and the standard deviation of the sample mean is $\sigma/\sqrt{n}$. The centerline for the sample means is taken to be μ and the upper and lower control limits are taken to be $3\,(\sigma/\sqrt{n})$ above and below the centerline. The lower control limit is given by equation (*1*):

$$\text{LCL} = \mu - 3(\sigma/\sqrt{n}) \tag{1}$$

The upper control limit is given by equation (*2*):

$$\text{UCL} = \mu + 3(\sigma/\sqrt{n}) \qquad (2)$$

For a normally distributed process, a subgroup mean will fall between the limits, given in (*1*) and (*2*), 99.7% of the time. In practice, the process mean and the process standard deviation are unknown and need to be estimated. The process mean is estimated by using the mean of the periodic sample means. This is given by equation (*3*), where m is the number of periodic samples of size n selected:

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{m} \qquad (3)$$

The mean, $\bar{\bar{X}}$, can also be found by summing all the data and then dividing by mn. The process standard deviation is estimated by pooling the subgroup variances, averaging the subgroup standard deviations or ranges, or by sometimes using a historical value of σ.

EXAMPLE 1. Data are obtained on the width of a product. Five observations per period are sampled for 20 periods. The data are shown below in Table 19.2. The number of periodic samples is $m = 20$, the sample size or subgroup size is $n = 5$, the sum of all the data is 199.84, and the centerline is $\bar{\bar{X}} = 1.998$. The Minitab pull-down menus **Stat → Control charts → Xbar** were used to produce the control chart shown in Fig. 19-2. The data in Table 19.2 are stacked into a single column before applying the above pull-down menu sequence.

Table 19.2

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
2.000	2.007	1.987	1.989	1.997	1.983	1.966	2.004	2.009	1.991
1.988	1.988	1.983	1.989	2.018	1.972	1.982	1.998	1.994	1.989
1.975	2.002	2.006	1.997	1.999	2.002	1.995	2.011	2.020	2.000
1.994	1.978	2.019	1.976	1.990	1.991	2.020	1.991	2.000	2.016
1.991	2.012	2.021	2.007	2.003	1.997	2.008	1.972	2.006	2.037

Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17	Period 18	Period 19	Period 20
2.004	1.988	1.996	1.999	2.018	1.986	2.002	1.988	2.011	1.998
1.980	1.991	2.005	1.984	2.009	2.010	1.969	2.031	1.976	2.003
1.998	2.003	1.996	1.988	2.023	2.012	2.018	1.978	1.998	2.016
1.994	1.997	2.008	2.011	2.010	2.013	1.984	1.987	2.023	1.996
2.006	1.985	2.007	2.005	1.993	1.988	1.990	1.990	1.998	2.009

The standard deviation for the process may be estimated in four different ways. The standard deviation may be estimated by using the average of the 20 subgroup ranges, by using the average of the 20 subgroup standard deviations, by pooling the 20 subgroup variances, or by using a historical value for σ, if one is known. Minitab allows for all four options. The 20 means for the samples shown in Table 19.2 are plotted in Fig. 19-2. The chart indicates a process that is in control. The individual means randomly vary about the centerline and none fall outside the control limits.

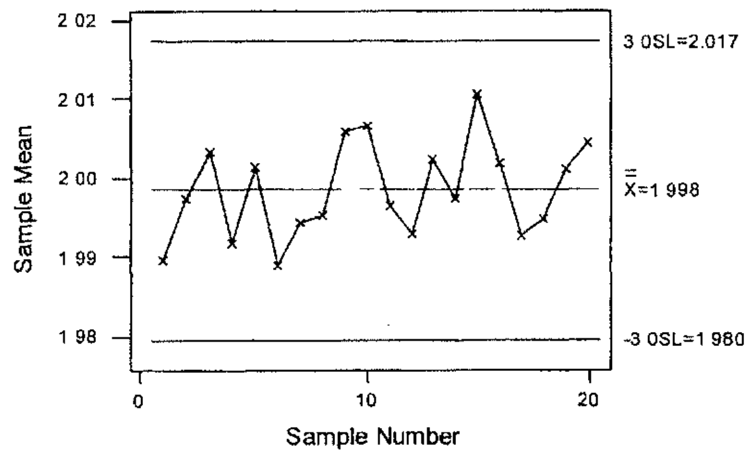

Fig. 19-2 X-bar chart for width.

The *R chart* is used to track process variation. The range, R, is computed for each of the m subgroups. The centerline for the R chart is given by equation (*4*):

$$\bar{R} = \frac{\sum R}{m} \tag{4}$$

As with the X-bar chart, several different methods are used to estimate the standard deviation of the process.

EXAMPLE 2. For the data in Table 19.2, the range of the first subgroup is $R_1 = 2.000 - 1.975 = 0.025$ and the range for the second subgroup is $R_2 = 2.012 - 1.978 = 0.034$. The 20 ranges are: 0.025, 0.034, 0.038, 0.031, 0.028, 0.030, 0.054, 0.039, 0.026, 0.048, 0.026, 0.018, 0.012, 0.027, 0.030, 0.027, 0.049, 0.053, 0.047, and 0.020. The mean of these 20 ranges is 0.0331. A Minitab plot of these ranges is shown in Fig. 19-3. The R chart does not indicate any unusual patterns with respect to variability.

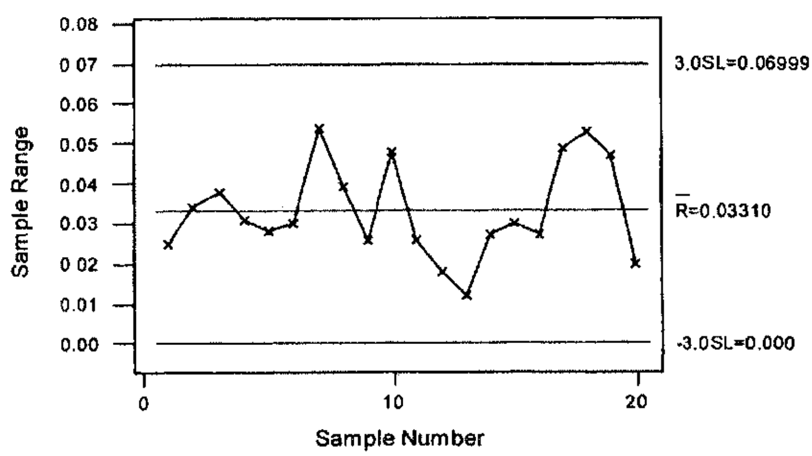

Fig. 19-3 R chart for width.

TESTS FOR SPECIAL CAUSES

In addition to a point falling outside the control limits of a control chart, there are other indications that are suggestive of non-randomness of a process caused by special effects. Table 19.3 gives eight tests for special causes.

Table 19.3 Tests for Special Causes

1. One point more than 3 sigmas from centerline
2. Nine points in a row on same side of centerline
3. Six points in a row, all increasing or all decreasing
4. Fourteen points in a row, alternating up and down
5. Two out of three points more than 2 sigmas from centerline (same side)
6. Four out of five points more than 1 sigma from centerline (same side)
7. Fifteen points in a row within 1 sigma of centerline (either side)
8. Eight points in a row more than 1 sigma from centerline (either side)

PROCESS CAPABILITY

To perform a capability analysis on a process, the process needs to be in statistical control. It is usually assumed that the process characteristic being measured is normally distributed. This may be checked out using tests for normality such as the Kolmogorov–Smirnov test, the Ryan–Joiner test, or the Anderson–Darling test. Process capability compares process performance with process requirements. Process requirements determine *specification limits*. LSL and USL represent the *lower specification limit* and the *upper specification limit*.

The data used to determine whether a process is in statistical control may be used to do the capability analysis. The 3-sigma distance on either side of the mean is called the *process spread*. The mean and standard deviation for the process characteristic may be estimated from the data gathered for the statistical process control study.

EXAMPLE 3. As we saw in Example 2, the data in Table 19.2 come from a process that is in statistical control. We found the estimate of the process mean to be 1.9984. The standard deviation of the 100 observations is found to equal 0.013931. Suppose the specification limits are LSL = 1.970 and USL = 2.030. The Kolmogorov Smirnov test for normality is applied by using Minitab and it is found that we do not reject the normality of the process characteristic. The *nonconformance rates* are computed as follows. The proportion above the USL = $P(X > 2.030) = P((X - 1.9984)/0.013931 > (2.030 - 1.9984)/0.013931) = P(Z > 2.27) = 0.0116$. That is, there are 0.0116(1,000,000) = 11,600 *parts per million (ppm)* above the USL that are nonconforming. Note that $P(Z > 2.27)$ may be found using Minitab rather than by looking it up in the standard normal tables. This is shown as follows.

```
MTB > cdf 2.27;
SUBC> normal 0 1.

Normal with mean = 0 and standard deviation = 1.00000

     x      P(X <= x)
2.2700       0.9884
```

We have $P(Z < 2.27) = 0.9884$ and therefore $P(Z > 2.27) = 1 - 0.9884 = 0.0116$. The proportion below the LSL = $P(X < 1.970) = P(Z < -2.04) = 0.0207$. There are 20,700 ppm below the LSL that are nonconforming. Again, Minitab is used to find the area to the left of -2.04 under the standard normal curve.

```
MTB > cdf -2.04;
SUBC> normal 0 1.

Normal with mean = 0 and standard deviation = 1.00000

      x       P(X <= x)
-2.0400       0.0207
```

The total number of nonconforming units is 11,600 + 20,700 = 32,300 ppm. This is of course an unacceptably high number of nonconforming units.

Suppose $\hat{\mu}$ represents the estimated mean for the process characteristic and $\hat{\sigma}$ represents the estimated standard deviation for the process characteristic, then the nonconformance rates are estimated as follows: the proportion above the USL equals

$$P(X > \text{USL}) = P\left(Z > \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right)$$

and the proportion below the LSL equals

$$P(X < \text{LSL}) = P\left(Z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right)$$

The *process capability index* measures the process's potential for meeting specifications, and is defined as follows:

$$C_{\text{P}} = \frac{\text{allowable spread}}{\text{measured spread}} = \frac{\text{USL} - \text{LSL}}{6\hat{\sigma}} \tag{5}$$

EXAMPLE 4. For the process data in Table 19.2, USL − LSL = 2.030 − 1.970 = 0.060, $6\hat{\sigma} = 6(0.013931) = 0.083586$, and $C_{\text{P}} = 0.060/0.083586 = 0.72$.

The C_{PK} *index* measures the process performance, and is defined as follows:

$$C_{\text{PK}} = \text{minimum}\left\{\frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}},\ \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}}\right\} \tag{6}$$

EXAMPLE 5. For the process data in example 1,

$$C_{\text{PK}} = \text{minimum}\left\{\frac{2.030 - 1.9984}{3(0.013931)},\ \frac{1.9984 - 1.970}{3(0.013931)}\right\} = \text{Minimum}\,\{0.76, 0.68\} = 0.68$$

For processes with only a lower specification limit, the *lower capability index* C_{PL} is defined as follows:

$$C_{\text{PL}} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} \tag{7}$$

For processes with only an upper specification limit, the *upper capability index* C_{PU} is defined as follows.

$$C_{\text{PU}} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} \tag{8}$$

Then C_{PK} may be defined in terms of C_{PL} and C_{PU} as follows:

$$C_{\text{PK}} = \min\{C_{\text{PL}}, C_{\text{PU}}\} \tag{9}$$

The relationship between nonconformance rates and C_{PL} and C_{PU} are obtained as follows:

$$P(X < \text{LSL}) = P\left(Z < \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}\right) = P(Z < -3C_{\text{PL}}),\ \text{since}\ -3C_{\text{PL}} = \frac{\text{LSL} - \hat{\mu}}{\hat{\sigma}}$$

$$P(X > \text{USL}) = P\left(Z > \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}\right) = P(Z > 3C_{\text{PU}}),\ \text{since}\ 3C_{\text{PU}} = \frac{\text{USL} - \hat{\mu}}{\hat{\sigma}}$$

EXAMPLE 6. Suppose that $C_{PL} = 1.1$, then the proportion nonconforming is $P(Z < -3(1.1)) = P(Z < -3.3)$. This may be found using Minitab as follows.

```
MTB > cdf -3.3 put into c1;
SUBC> normal 0 1.
MTB > print c1;
SUBC> format (f10.8).

0.00048348
```

There would be $1{,}000{,}000 \times 0.00048348 = 483$ ppm nonconforming. Using this technique, a table relating nonconformance rate to capability index can be constructed. This is given in Table 19.4.

Table 19.4

C_{PL} or C_{PU}	Proportion nonconforming	ppm
0.1	0.38208867	382089
0.2	0.27425308	274253
0.3	0.18406010	184060
0.4	0.11506974	115070
0.5	0.06680723	66807
0.6	0.03593027	35930
0.7	0.01786436	17864
0.8	0.00819753	8198
0.9	0.00346702	3467
1.0	0.00134997	1350
1.1	0.00048348	483
1.2	0.00015915	159
1.3	0.00004812	48
1.4	0.00001335	13
1.5	0.00000340	3
1.6	0.00000079	1
1.7	0.00000017	0
1.8	0.00000003	0
1.9	0.00000001	0
2.0	0.00000000	0

EXAMPLE 7. A capability analysis using Minitab and the data in Table 19.2 may be obtained by using the following pull-down menus in Minitab: **Stat → Quality tools → Capability Analysis (Normal)**. The Minitab output is shown in Fig. 19-4. The output gives nonconformance rates, capability indexes, and several other measures. The quantities found in Examples 3, 4, and 5 are very close to the corresponding measures shown in the figure. The differences are due to round-off error as well as different methods of estimating certain parameters. The graph is very instructive. It shows the distribution of sample measurements as a histogram. The population distribution of process measurements is shown as the normal curve. The tail areas under the normal curve to the right of the USL and to the left of the LSL represent the percentage of nonconforming products. By multiplying the sum of these percentages by one million, we get the ppm nonconformance rate for the process.

P- AND *NP*-CHARTS

When mass-produced products are categorized or classified, the resulting data are called *attribute data*. After establishing standards that a product must satisfy, specifications are determined. An item not

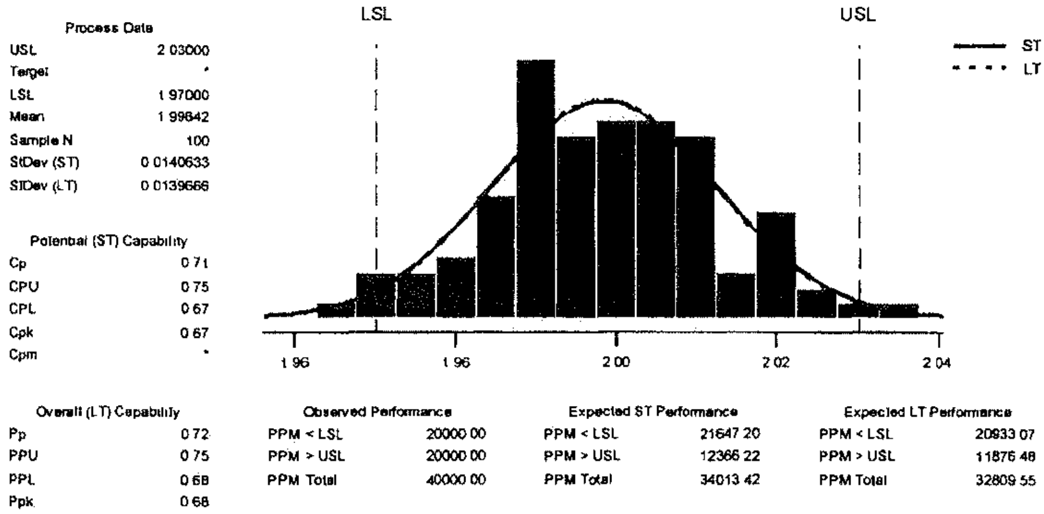

Fig. 19-4 Process capability analysis for width.

meeting specifications is called a *nonconforming item*. A nonconforming item that is not usable is called a *defective item*. A defective item is considered to be more serious than a nonconforming item. An item might be nonconforming because of a scratch or a discoloration, but not be a defective item. The failure of a performance test would likely cause the product to be classified as defective as well as nonconforming. Flaws found on a single item are called *nonconformities*. Nonrepairable flaws are called *defects*. A defect is considered to be more serious than a nonconformity.

Four different control charts are used when dealing with attribute data. The four charts are the *P*-, *NP*-, *C*-, and *U*-charts. The *P*- and *NP*-charts are based on the binomial distribution and the *C*- and *U*-charts are based on the Poisson distribution. The *P*-chart is used to monitor the proportion of non-conforming items being produced by a process. The *P*-chart and the notation used to describe it are illustrated in Example 8.

EXAMPLE 8. Suppose 20 respirator masks are examined every thirty minutes and the numbers of defective units are recorded per 8-hour shift. The total number examined on a given shift is equal to $n = 20(16) = 320$. Table 19.5 gives the results for 30 such shifts. The centerline for the *P*-chart is equal to the proportion of defectives for the 30 shifts, and is given by the total number of defectives divided by the total number examined for the 30 shifts, or

$$\bar{p} = 72/9600 = 0.0075$$

The standard deviation associated with the binomial distribution, which underlies this chart, is

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.0075 \times 0.9925}{320}} = 0.004823$$

The 3-sigma control limits for this process are

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-p)}{n}} \tag{10}$$

The lower control limit is LCL $= 0.0075 - 3\,(0.004823) = -0.006969$. When the LCL is negative, it is taken to be zero since the proportion defective in a sample can never be negative. The upper control limit is UCL $= 0.0075 + 3\,(0.004823) = 0.021969$.

The Minitab solution to obtaining the *P*-chart for this process is obtained by using the following pull-down menus: **Stat** → **Control charts** → **P**. The *P*-chart is shown in Fig. 19-5. Even though it appears that samples 15 and 20 indicate the presence of a special cause, when the proportion defective for samples 15 and 20 (both equal to 0.021875) are compared with the UCL = 0.021969, it is seen that the points are not beyond the UCL.

Table 19.5

Shift #	Number Defective X_i	Proportion Defective $P_i = X/n$	Shift #	Number Defective X_i	Proportion Defective $P_i = X/n$
1	1	0.003125	16	2	0.006250
2	2	0.006250	17	0	0.000000
3	2	0.006250	18	4	0.012500
4	0	0.000000	19	1	0.003125
5	4	0.012500	20	7	0.021875
6	4	0.012500	21	4	0.012500
7	4	0.012500	22	1	0.003125
8	6	0.018750	23	0	0.000000
9	4	0.012500	24	4	0.012500
10	0	0.000000	25	4	0.012500
11	0	0.000000	26	3	0.009375
12	0	0.000000	27	2	0.006250
13	1	0.003125	28	0	0.000000
14	0	0.000000	29	0	0.000000
15	7	0.021875	30	5	0.015625

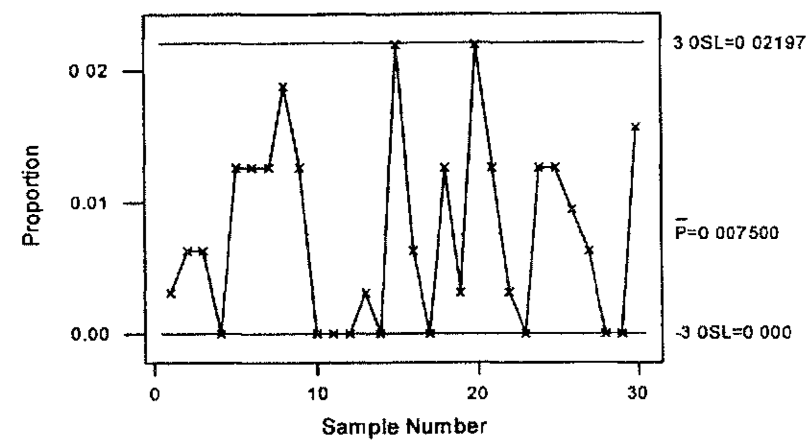

Fig. 19-5 *P*-chart for number defective.

The *NP*-chart monitors the number of defectives rather than the proportion of defectives. The *NP*-chart is considered by many to be preferable to the *P*-chart because the number defective is easier for quality technicians and operators to understand than is the proportion defective. The centerline for the *NP*-chart is given by $n\bar{p}$ and the 3-sigma control limits are

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})} \tag{11}$$

EXAMPLE 9. For the data in Table 19.5, the centerline is given by $n\bar{p} = 320(0.0075) = 2.4$ and the control limits are LCL $= 2.4 - 4.63 = -2.23$, which we take as 0, and UCL $= 2.4 + 4.63 = 7.03$. If 8 or more defectives are found on a given shift, the process is out of control. The Minitab solution is found by using the pull-down sequence: **Stat → Control charts → NP.**

The number defective per sample needs to be entered in some column of the worksheet prior to executing the pull down sequence. The Minitab *NP*-chart is shown in Figure 19-6.

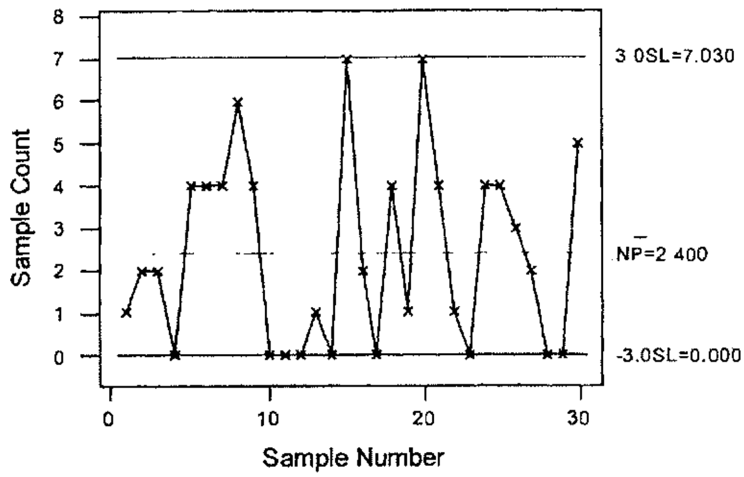

Fig. 19-6 *NP*-chart for number defective.

OTHER CONTROL CHARTS

This chapter serves as only an introduction to the use of control charts to assist in statistical process control. Table 19.1 gives a listing of many of the various control charts in use in industrial settings today. To expedite calculations on the shop floor, the *median chart* is sometimes used. The medians of the samples are plotted rather than the means of the samples. If the sample size is odd, then the median is simply the middle value in the ordered sample values.

For low-volume production runs, *individuals charts* are often used. In this case, the subgroup or sample consists of a single observation. Individuals charts are sometimes referred to as X charts.

A *zone chart* is divided into four zones. Zone 1 is defined as values within 1 standard deviation of the mean, zone 2 is defined as values between 1 and 2 standard deviations of the mean, zone 3 is defined as values between 2 and 3 standard deviations of the mean, and zone 4 as values 3 or more standard deviations from the mean. Weights are assigned to the four zones. Weights for points on the same side of the centerline are added. When a cumulative sum is equal to or greater than the weight assigned to zone 4, this is taken as a signal that the process is out of control. The cumulative sum is set equal to 0 after signaling a process out of control, or when the next plotted point crosses the centerline.

The exponentially weighted moving average (*EWMA chart*) is an alternative to the individuals or X-bar chart that provides a quicker response to a shift in the process average. The EWMA chart incorporates information from all previous subgroups, not only the current subgroup.

Cumulative sums of deviations from a process target value are utilized by a *Cusum chart*. Both the EWMA chart and the Cusum chart allow for quick detection of process shifts.

When we are concerned with the number of nonconformities or defects in a product rather than simply determining whether the product is defective or non-defective, we use a *C-chart* or a *U-chart*. When using these charts, it is important to define an *inspection unit*. The inspection unit is defined as the fixed unit of output to be sampled and examined for nonconformities. When there is only one inspection unit per sample, the C-chart is used, and when the number of inspection units per sample vary, the U-chart is used.

Solved Problems

X-BAR AND R CHARTS

19.1 An industrial process fills containers with breakfast oats. The mean fill for the process is 510 grams and the standard deviation of fills is known to equal 5 grams. Four containers are selected every hour and the mean weight of the subgroup of four weights is used to monitor the process for special causes and to help keep the process in statistical control. Find the lower and upper control limits for the X-bar control chart.

SOLUTION

In this problem, we are assuming that μ and σ are known and equal 510 and 5 respectively. When μ and σ are unknown, they must be estimated. The lower control limit is $\text{LCL} = \mu - 3(\sigma/\sqrt{n}) = 510 - 3(2.5) = 502.5$ and the upper control limit is $\text{UCL} = \mu + 3(\sigma/\sqrt{n}) = 510 + 3(2.5) = 517.5$.

19.2 Table 19.6 contains the widths of a product taken at 20 time periods. The control limits for an X-bar chart are LCL = 1.981 and UCL = 2.018. Are any of the subgroup means outside the control limits?

Table 19.6

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
2.000	2.007	1.987	1.989	1.997	1.983	1.966	2.004	2.009	1.991
1.988	1.988	1.983	1.989	2.018	1.972	1.982	1.998	1.994	1.989
1.975	2.002	2.006	1.997	1.999	2.002	1.995	2.011	2.020	2.000
1.994	1.978	2.019	1.976	1.990	1.991	2.020	1.991	2.000	2.016
1.991	2.012	2.021	2.007	2.003	1.997	2.008	1.972	2.006	2.037

Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17	Period 18	Period 19	Period 20
2.004	1.988	1.996	1.999	2.018	2.025	2.002	1.988	2.011	1.998
1.980	1.991	2.005	1.984	2.009	2.022	1.969	2.031	1.976	2.003
1.998	2.003	1.996	1.988	2.023	2.035	2.018	1.978	1.998	2.016
1.994	1.997	2.008	2.011	2.010	2.013	1.984	1.987	2.023	1.996
2.006	1.985	2.007	2.005	1.993	2.020	1.990	1.990	1.998	2.009

SOLUTION

The means for the 20 subgroups are 1.9896, 1.9974, 2.0032, 1.9916, 2.0014, 1.9890, 1.9942, 1.9952, 2.0058, 2.0066, 1.9964, 1.9928, 2.0024, 1.9974, 2.0106, **2.0230**, 1.9926, 1.9948, 2.0012, and 2.0044 respectively. The sixteenth mean, 2.0230, is outside the upper control limit. All others are within the control limits.

19.3 Refer to Problem 19.2. It was determined that a spill occurred on the shop floor just before the sixteenth subgroup was selected. This subgroup was eliminated and the control limits were re-computed and found to be LCL = 1.979 and UCL = 2.017. Are there any of the means other than the mean for subgroup 16 outside the new limits?

SOLUTION

None of the means given in Problem 19.2 other than the sixteenth one fall outside the new limits. Assuming that the new chart does not fail any of the other tests for special causes given in Table 19.3, the control limits given in this problem could be used to monitor the process.

19.4 Verify the control limits given in Problem 19.2. Estimate the standard deviation of the process by pooling the 20 sample variances.

SOLUTION

The mean of the 100 sample observations is 1.999. One way to find the pooled variance for the 20 samples is to treat the 20 samples, each consisting of 5 observations, as a one-way classification. The within or error mean square is equal to the pooled variance of the 20 samples. The Minitab analysis as a one-way design gave the following analysis of variance table.

```
Analysis of Variance
Source  DF        SS        MS      F       P
Factor  19  0.006342  0.000334   1.75   0.044
Error   80  0.015245  0.000191
Total   99  0.021587
```

The estimate of the standard deviation is $\sqrt{0.000191} = 0.01382$. The lower control limit is LCL = $1.999 - 3(0.01382/\sqrt{5}) = 1.981$ and the upper control limit is UCL = $1.999 + 3(0.01382/\sqrt{5}) = 2.018$.

TESTS FOR SPECIAL CAUSES

19.5 Table 19.7 contains data from 20 subgroups, each of size 5. The X-bar chart is given in Fig. 19-7. What effect did a change to a new supplier at time period 10 have on the process? Which test for special causes in Table 19.3 did the process fail?

Table 19.7

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
2.000	2.007	1.987	1.989	1.997	1.983	1.966	2.004	2.009	1.991
1.988	1.988	1.983	1.989	2.018	1.972	1.982	1.998	1.994	1.989
1.975	2.002	2.006	1.997	1.999	2.002	1.995	2.011	2.020	2.000
1.994	1.978	2.019	1.976	1.990	1.991	2.020	1.991	2.000	2.016
1.991	2.012	2.021	2.007	2.003	1.997	2.008	1.972	2.006	2.037

Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17	Period 18	Period 19	Period 20
2.014	1.998	2.006	2.009	2.028	1.996	2.012	1.998	2.021	2.008
1.990	2.001	2.015	1.994	2.019	2.020	1.979	2.041	1.986	2.013
2.008	2.013	2.006	1.998	2.033	2.022	2.028	1.988	2.008	2.026
2.004	2.007	2.018	2.021	2.020	2.023	1.994	1.997	2.033	2.006
2.016	1.995	2.017	2.015	2.003	1.998	2.000	2.000	2.008	2.019

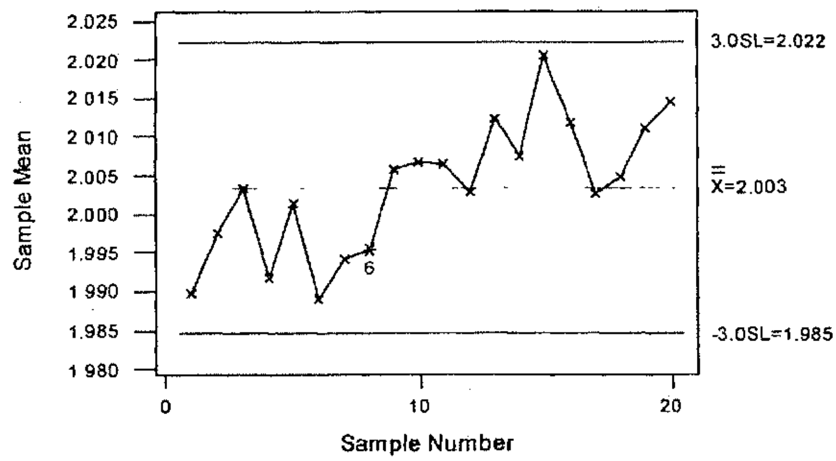

Fig. 19-7 X-bar chart for width.

SOLUTION

The control chart in Fig. 19-7 shows that the change to the new supplier caused an increase in the width. This shift after time period 10 is apparent. The 6 shown on the graph in Fig. 19-7 indicates test 6 given in Table 19.3 was failed. Four out of five points were more than one sigma from the centerline (same side). The five points correspond to subgroups 4 through 8.

PROCESS CAPABILITY

19.6 Refer to Problem 19.2. After determining that a special cause was associated with subgroup 16, we eliminate this subgroup. The mean width is estimated by finding the mean of the data from the other 19 subgroups and the standard deviation is estimated by finding the standard deviation of the same data. If the specification limits are LSL = 1.960 and USL = 2.040, find the lower capability index, the upper capability index, and the C_{PK} index.

SOLUTION

Using the 95 measurements after excluding subgroup 16, we find that $\hat{\mu} = 1.9982$ and $\hat{\sigma} = 0.01400$. The lower capability index is

$$C_{PL} = \frac{\hat{\mu} - \text{LSL}}{3\hat{\sigma}} = \frac{1.9982 - 1.960}{0.0420} = 0.910$$

the upper capability index is

$$C_{PU} = \frac{\text{USL} - \hat{\mu}}{3\hat{\sigma}} = \frac{2.040 - 1.9982}{0.042} = 0.995$$

and $C_{PK} = \min\{C_{PL}, C_{PU}\} = 0.91$.

19.7 Refer to Problem 19.1. (*a*) Find the percentage nonconforming if LSL = 495 and USL = 525. (*b*) Find the percentage nonconforming if LSL = 490 and USL = 530.

SOLUTION

(*a*) Assuming the fills are normally distributed, the area under the normal curve below the LSL is found as follows.

```
MTB > cdf 495 c1;
SUBC> normal mean = 510 sigma = 5.
```

```
MTB > print c1;
SUBC> format (f10.6).

    0.001350
```

By symmetry, the area under the normal curve above the USL is also 0.001350. The total area outside the specification limits is 0.002700. The ppm nonconforming is 0.002700(1,000,000) = 2700.

(*b*) The ppm nonconforming for LSL = 490 and USL = 530 is found similarly.

```
MTB > cdf 490 c1;
SUBC> normal mean = 510 sigma = 5.
MTB > print c1;
SUBC> format (f10.6).

    0.000032
```

The ppm nonconforming is 0.000064(1,000,000) = 64.

P- AND *NP*-CHARTS

19.8 Printed circuit boards are inspected for defective soldering. Five hundred circuit boards per day are tested for a 30-day period. The number defective per day are shown in Table 19.8. Construct a *P*-chart and locate any special causes.

Table 19.8

Day	1	2	3	4	5	6	7	8	9	10
# Defective	2	0	2	5	2	4	5	1	2	3
Day	11	12	13	14	15	16	17	18	19	20
# Defective	3	2	0	4	3	8	10	4	4	5
Day	21	22	23	24	25	26	27	28	29	30
# Defective	2	4	3	2	3	3	2	1	1	2

SOLUTION

The confidence limits are

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The centerline is $\bar{p} = 92/15{,}000 = 0.00613$ and the standard deviation is

$$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{(0.00613)(0.99387)}{500}} = 0.00349$$

The lower control limit is 0.00613 − 0.01047 = −0.00434, and is taken to equal 0, since proportions cannot be negative. The upper control limit is 0.00613 + 0.01047 = 0.0166. The proportion of defectives on day 17 is equal to $P_{17} = 10/500 = 0.02$ and is the only daily proportion to exceed the upper limit.

19.9 Give the control limits for an *NP*-chart for the data in Problem 19.8.

SOLUTION

The control limits for the number defective are $n\bar{p} \pm 3\sqrt{n\bar{p}(1-p)}$. The centerline is $n\bar{p} = 3.067$. The lower limit is 0 and the upper limit is 8.304.

19.10 Suppose respirator masks are packaged in boxes of either 25 or 50 per box. At each 30 minute interval during a shift a box is randomly chosen and the number of defectives in the box determined. The box may either contain 25 or 50 masks. The number checked per shift will vary between 400 and 800. The data are shown in Table 19.9. Use Minitab to find the control chart for the proportion defective.

Table 19.9

Shift #	Sample Size n_i	Number Defective X_i	Proportion Defective $P_i = X_i/n_i$
1	400	3	0.0075
2	575	7	0.0122
3	400	1	0.0025
4	800	7	0.0088
5	475	2	0.0042
6	575	0	0.0000
7	400	8	0.0200
8	625	1	0.0016
9	775	10	0.0129
10	425	8	0.0188
11	400	7	0.0175
12	400	3	0.0075
13	625	6	0.0096
14	800	5	0.0063
15	800	4	0.0050
16	800	7	0.0088
17	475	9	0.0189
18	800	9	0.0113
19	750	9	0.0120
20	475	2	0.0042

SOLUTION

When the sample sizes vary in monitoring a process for defectives, the centerline remains the same, that is, it is the proportion of defectives over all samples. The standard deviation, however, changes from sample to sample and gives control limits consisting of stair-stepped control limits. The control limits are

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-p)}{n_i}}$$

The centerline is $p = 108/11{,}775 = 0.009172$. For the first subgroup, we have $n_i = 400$.

$$\sqrt{\frac{p(1-\bar{p})}{n_i}} = \sqrt{\frac{(0.009172)(0.990828)}{400}} = 0.004767$$

and $3(0.004767) = 0.014301$. The lower limit for subgroup 1 is 0 and the upper limit is $0.009172 + 0.014301 = 0.023473$. The limits for the remaining shifts are determined similarly. These changing limits give rise to the stair-stepped upper control limits shown in Fig. 19-8.

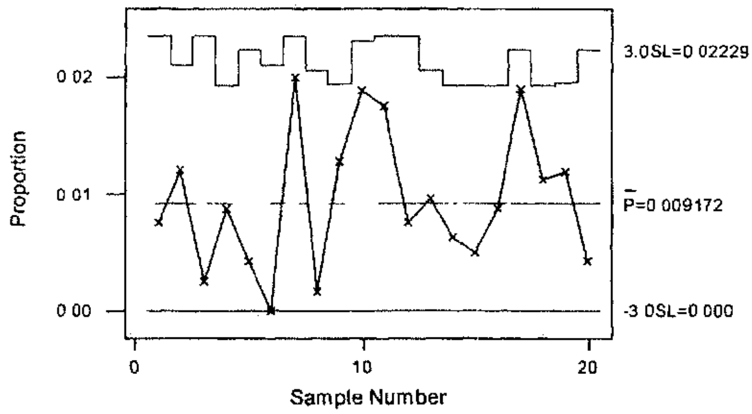

Fig. 19-8 P-chart for defectives.

OTHER CONTROL CHARTS

19.11 When measurements are expensive, data are available at a slow rate, or when output at any point is fairly homogeneous, an *individuals chart with moving range* may be indicated. The data consists of single measurements taken at different points in time. The centerline is the mean of all the individual measurements, and variation is estimated by using *moving ranges*. Traditionally, moving ranges have been calculated by subtracting adjacent data values and taking the absolute value of the result. Table 19.10 gives the coded breaking strength measurements of an expensive cable used in aircraft. One cable per day is selected from the production process and tested. Give the Minitab-generated individuals chart and interpret the output.

Table 19.10

Day	1	2	3	4	5	6	7	8	9	10
Strength	491.5	502.0	505.5	499.6	504.1	501.3	503.5	504.3	498.5	508.8
Day	11	12	13	14	15	16	17	18	19	20
Strength	515.4	508.0	506.0	510.9	507.6	519.1	506.9	510.9	503.9	507.4

SOLUTION

The following pull-down menus are used: **Stat → Control charts → Individuals.**

Figure 19-9 shows the individuals chart for the data in Table 19.10. The individual values in Table 19.10 are plotted on the control chart. The 2 that is shown on the control chart for weeks 9 and 18 corresponds to the second test for special causes given in Table 19.3. This indication of a special cause corresponds to nine points in a row on the same side of the centerline. An increase in the process temperature at time period 10 resulted in an increase in breaking strength. This change in breaking strength resulted in points below the centerline prior to period 10 and mostly above the centerline after period 10.

19.12 The *exponentially weighted moving average, EWMA, chart*, is used to detect small shifts from a target value, t. The points on the EWMA chart are given by the following equation:

$$\hat{x}_t = w\bar{x}_t + (1 - w)\hat{x}_{t-1}$$

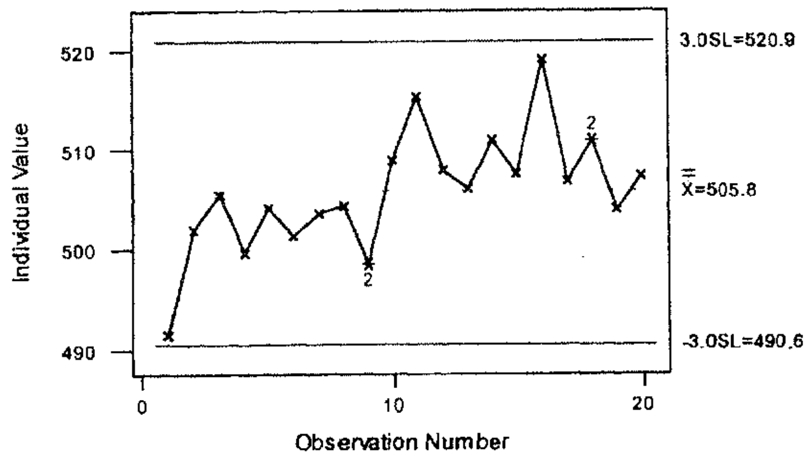

Fig. 19-9 Individuals chart for strength.

To illustrate the use of this equation, suppose the data in Table 19.7 were selected from a process that has target value equal to 2.000. The starting value $\hat{x}_0$ is chosen to equal the target value, 2.000. The weight w is usually chosen to be between 0.10 and 0.30. Minitab uses the value 0.20 as a default. The first point on the EWMA chart would be $\hat{x}_1 = w\bar{x}_1 + (1 - w)\hat{x}_0 = 0.20(1.9896) + 0.80(2.000) = 1.9979$. The second point on the chart would be $\hat{x}_2 = w\bar{x}_2 + (1 - w)\hat{x}_1 = 0.20(1.9974) + 0.80(1.9979) = 1.9978$, and so forth. The Minitab analysis is obtained by using the following pull-down menu: **Stat → Control charts → EWMA**. The target value is supplied to Minitab. The output is shown in Fig. 19-10. By referring to Fig. 19-10, determine for which subgroups the process shifted from the target value.

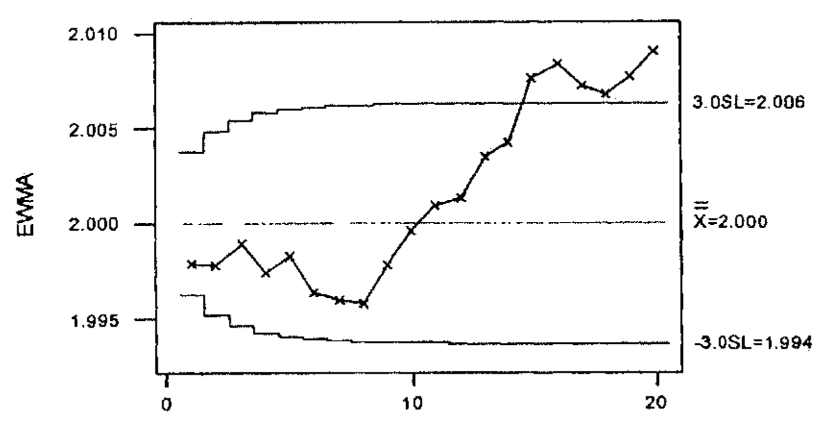

Fig. 19-10 EWMA chart for width.

SOLUTION

The graph of the $\hat{x}_i$ values crosses the upper control limit at the time point 15. This is the point at which we would conclude that the process had shifted away from the target value. Note that the EWMA chart has stair-stepped control limits.

19.13 A *zone chart* is divided into four zones. Zone 1 is defined as values within 1 standard deviation of the mean, zone 2 is defined as values between 1 and 2 standard deviations of the mean, zone 3 is defined as values between 2 and 3 standard deviations of the mean, and zone 4 as values 3 or more standard deviations from the mean. The default weights assigned to the zones by Minitab are 0, 2, 4, and 8 for zones 1 through 4. Weights for points on the same side of the centerline are added. When a cumulative sum is equal to or greater than the weight assigned to zone 4, this is taken as a signal that the process is out of control. The cumulative sum is set equal to 0 after signaling a process out of control, or when the next plotted point crosses the centerline. Figure 19-11 shows the Minitab analysis using a zone chart for the data in Table 19.6. The pull-down menus needed to produce this chart are: **Stat** → **Control charts** → **Zone**. What out of control points does the zone chart find?

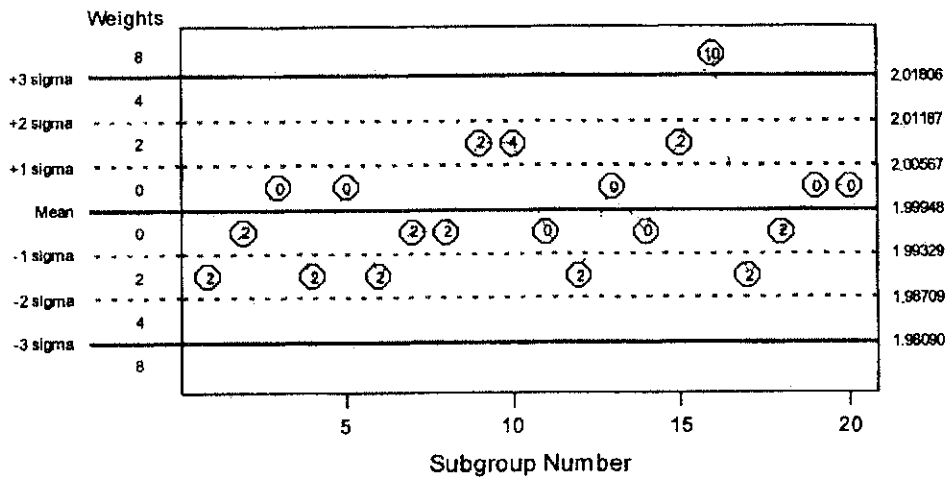

Fig. 19-11 Zone chart for width.

SOLUTION

Subgroup 16 corresponds to an out of control point. The zone score corresponding to subgroup 16 is 10, and since this exceeds the score assigned to zone 4, this locates an out of control time period in the process.

19.14 When we are concerned with the number of nonconformities or defects in a product rather than simply determining whether the product is defective or nondefective, we use a *C-chart* or a *U-chart*. When using these charts, it is important to define an *inspection unit*. The inspection unit is defined as the fixed unit of output to be sampled and examined for nonconformities. When there is only one inspection unit per sample, the C-chart is used; when the number of inspection units per sample varies, the U-chart is used.

One area of application for C- and U-charts is in the manufacture of roll products such as paper, films, plastics, textiles, and so forth. Nonconformities or defects, such as the occurrence of black spots in photographic film, as well as the occurrence of fiber bundles, dirt spots, pinholes, static electricity marks, and agglomerates in various other roll products, always occur at some level in the manufacture of roll products. The purpose of the C- or U-chart is to make sure that the process output remains within an acceptable level of occurrence of such nonconformities. These nonconformities often occur randomly and independently of one another over the total area of the roll product. In such cases, the Poisson distribution is used to form the control chart. The centerline for the C-chart is located at $\bar{c}$, the mean number of nonconformities over all subgroups. The standard deviation of the Poisson distribution is $\sqrt{\bar{c}}$ and therefore the 3 sigma control limits are $\bar{c} \pm 3\sqrt{\bar{c}}$. That is, the lower limit is $\text{LCL} = \bar{c} - 3\sqrt{\bar{c}}$ and the upper limit is $\text{UCL} = \bar{c} + 3\sqrt{\bar{c}}$.

When a coating is applied to a material, small nonconformities called agglomerates sometimes occur. The number of agglomerates in a length of 5 feet are recorded for a jumbo roll of product. The results for 24 such rolls are given in Table 19.11. Are there any points outside the 3-sigma control limits?

Table 19.11

Jumbo roll #	1	2	3	4	5	6	7	8	9	10	11	12
Agglomerates	3	3	6	0	7	5	3	6	3	5	2	2
Jumbo roll #	13	14	15	16	17	18	19	20	21	22	23	24
Agglomerates	2	7	6	4	7	8	5	13	7	3	3	7

SOLUTION

The mean number of agglomerates per Jumbo roll is equal to the total number of agglomerates divided by 24 or $\bar{c} = 117/24 = 4.875$. The standard deviation is $\sqrt{\bar{c}} = 2.208$. The lower control limit is $\text{LCL} = 4.875 - 3(2.208) = -1.749$. Since it is negative, we take the lower limit to be 0. The upper limit is $\text{UCL} = 4.875 + 3(2.208)$ or 11.499. An out of control condition is indicated for Jumbo roll #20 since the number of agglomerates, 13, exceeds the upper control limit, 11.499.

19.15 This problem is a follow up to Problem 19.14. You should review Problem 19.14 before attempting this problem. Table 19.12 gives the data for 20 Jumbo rolls. The table gives the roll number, the length of roll inspected for agglomerates, the number of inspection units (recall from Problem

Table 19.12

Jumbo roll #	Length Inspected	# of Inspection Units, n_i	# of Agglomerates	$u_i =$ Col. 4/Col. 3
1	5.0	1.0	6	6.00
2	5.0	1.0	4	4.00
3	5.0	1.0	6	6.00
4	5.0	1.0	2	2.00
5	5.0	1.0	3	3.00
6	10.0	2.0	8	4.00
7	7.5	1.5	6	4.00
8	15.0	3.0	6	2.00
9	10.0	2.0	10	5.00
10	7.5	1.5	6	4.00
11	5.0	1.0	4	4.00
12	5.0	1.0	7	7.00
13	5.0	1.0	5	5.00
14	15.0	3.0	8	2.67
15	5.0	1.0	3	3.00
16	5.0	1.0	5	5.00
17	15.0	3.0	10	3.33
18	5.0	1.0	1	1.00
19	15.0	3.0	8	2.67
20	15.0	3.0	15	5.00

19.14 that 5 feet constitutes an inspection unit), the number of agglomerates found in the length inspected, and the number of agglomerates per inspection unit. The centerline for the U-chart is $\bar{u}$, the sum of column 4 divided by the sum of column 3. The standard deviation, however, changes from sample to sample and gives control limits consisting of stair-stepped control limits. The lower control limit for sample i is $\text{LCL} = \bar{u} - 3\sqrt{\bar{u}/n_i}$ and the upper control limit for sample i is $\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n_i}$.

Use Minitab to construct the control chart for this problem and determine whether the process is in control.

SOLUTION

The centerline for the U-chart is $\bar{u}$, the sum of column 4 divided by the sum of column 3. The standard deviation, however, changes from sample to sample and gives control limits consisting of stair-stepped control limits. The lower control limit for sample i is $\text{LCL} = \bar{u} \quad 3\sqrt{\bar{u}/n_i}$ and the upper control limit for sample i is $\text{UCL} = \bar{u} + 3\sqrt{\bar{u}/n_i}$. The centerline for the above data is $\bar{u} = 123/33 = 3.73$. The Minitab solution is obtained by the pull-down sequence **Stat** → **Control charts** → **U**. The information required by Minitab to create the U-chart is that given in columns 3 and 4 of Table 19.12. The U-chart for the data in Table 19.12 is shown in Fig. 19-12. The control chart does not indicate any out of control points.

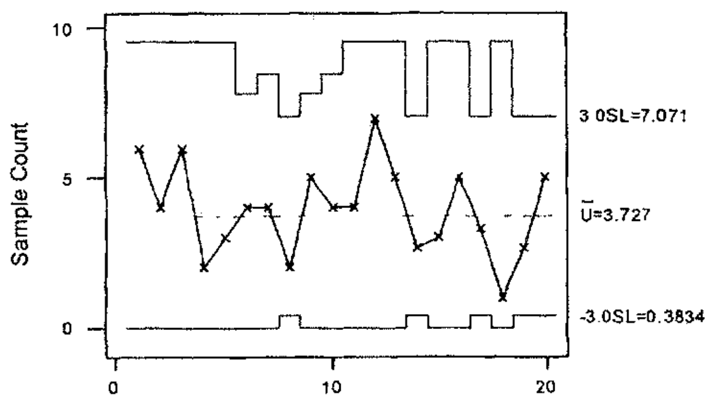

Fig. 19-12 U-chart for agglomerates.

Supplementary Problems

X-BAR AND R CHARTS

19.16 The data from ten subgroups, each of size 4, is shown in Table 19.13. Compute $\bar{X}$ and R for each subgroup as well as $\bar{\bar{X}}$ and $\bar{R}$. Plot the $\bar{X}$ values on a graph along with the centerline corresponding to $\bar{\bar{X}}$. On another graph, plot the R-values along with the centerline corresponding to $\bar{R}$.

19.17 A frozen food company packages 1 pound packages (454 grams) of green beans. Every two hours, 4 of the packages are selected and the weight is determined to the nearest tenth of a gram. Table 19.14 gives the data for a one-week period.

Use the method discussed in Problem 19.4 to estimate the standard deviation by pooling the variances of the 20 samples. Use this estimate to find the control limits for an X-bar chart. Are any of the 20 subgroup means outside the control limits?

Table 19.13

Subgroup	Subgroup observations			
1	13	11	13	16
2	11	12	20	15
3	16	18	20	15
4	13	15	18	12
5	12	19	11	12
6	14	10	19	16
7	12	13	20	10
8	17	17	12	14
9	15	12	16	17
10	20	13	18	17

Table 19.14

Mon. 10:00	Mon. 12:00	Mon. 2:00	Mon. 4:00	Tue. 10:00	Tue. 12:00	Tue. 2:00	Tue. 4:00	Wed. 10:00	Wed. 12:00
453.0	451.6	452.0	455.4	454.8	452.6	453.6	453.2	453.0	451.6
454.5	455.0	451.5	453.0	450.9	452.8	456.1	455.8	451.4	456.0
452.6	452.8	450.8	454.3	455.0	455.5	453.9	452.0	452.5	455.0
451.8	453.5	454.8	450.6	453.6	454.8	454.8	453.5	452.1	453.0

Wed. 2:00	Wed. 4:00	Thur. 10:00	Thur. 12:00	Thur. 2:00	Thur. 4:00	Fri. 10:00	Fri. 12:00	Fri. 2:00	Fri. 4:00
454.7	451.1	452.2	454.0	455.7	455.3	454.2	451.1	455.7	450.7
451.4	452.6	448.9	452.8	451.8	452.4	452.9	453.8	455.3	452.5
450.9	448.5	455.3	455.5	451.2	452.3	451.5	452.4	455.4	454.1
455.8	454.4	453.9	453.8	452.8	452.3	455.8	454.3	453.7	454.2

19.18 The control limits for the R chart for the data in Table 19.14 are LCL = 0 and UCL = 8.205. Are any of the subgroup ranges outside the 3 sigma limits?

19.19 The process that fills the 1 pound packages of green beans discussed in Problem 19.17 is modified in hopes of reducing the variability in the weights of the packages. After the modification was implemented and in use for a short time, a new set of weekly data was collected and the ranges of the new subgroups were plotted using the control limits given in Problem 19.18. The new data are given in Table 19.15. Does it appear that the variability has been reduced? If the variability has been reduced, find new control limits for the X-bar chart using the data in Table 19.15.

TESTS FOR SPECIAL CAUSES

19.20 Operators making adjustments to machinery too frequently is a problem in industrial processes. Table 19.16 contains a set of data in which this is the case. Find the control limits for an X-bar chart and then form the X-bar chart and check the 8 tests for special causes given in Table 19.3.

Table 19.15

Mon. 10:00	Mon. 12:00	Mon. 2:00	Mon. 4:00	Tue. 10:00	Tue. 12:00	Tue. 2:00	Tue. 4:00	Wed. 10:00	Wed. 12:00
454.9	454.2	454.4	454.7	454.3	454.2	454.6	453.6	454.4	454.6
452.7	453.6	453.6	453.9	454.2	452.8	454.5	453.2	455.0	454.1
457.0	454.4	453.6	454.6	454.2	453.3	454.3	453.6	454.6	453.3
454.2	453.9	454.3	453.9	453.4	453.3	454.9	453.1	454.1	454.3

Wed. 2:00	Wed. 4:00	Thur. 10:00	Thur. 12:00	Thur. 2:00	Thur. 4:00	Fri. 10:00	Fri. 12:00	Fri. 2:00	Fri. 4:00
453.0	453.9	453.8	455.1	454.2	454.4	455.1	455.7	452.2	455.4
454.0	454.2	453.6	453.3	453.0	452.6	454.6	452.8	453.7	452.8
452.9	454.3	454.1	454.7	453.8	454.9	454.1	453.8	454.4	454.7
454.2	454.7	454.7	453.9	453.9	454.2	454.6	454.9	454.5	455.1

Table 19.16

Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9	Period 10
2.006	2.001	1.993	1.983	2.003	1.977	1.972	1.998	2.015	1.985
1.994	1.982	1.989	1.983	2.024	1.966	1.988	1.992	2.000	1.983
1.981	1.996	2.012	1.991	2.005	1.996	2.001	2.005	2.026	1.994
2.000	1.972	2.025	1.970	1.996	1.985	2.026	1.985	2.006	2.010
1.997	2.006	2.027	2.001	2.009	1.991	2.014	1.966	2.012	2.031

Period 11	Period 12	Period 13	Period 14	Period 15	Period 16	Period 17	Period 18	Period 19	Period 20
2.010	1.982	2.002	1.993	2.024	1.980	2.008	1.982	2.017	1.992
1.986	1.985	2.011	1.978	2.015	2.004	1.975	2.025	1.982	1.997
2.004	1.997	2.002	1.982	2.029	2.006	2.024	1.972	2.004	2.010
2.000	1.991	2.014	2.005	2.016	2.007	1.990	1.981	2.029	1.990
2.012	1.979	2.013	1.999	1.999	1.982	1.996	1.984	2.004	2.003

PROCESS CAPABILITY

19.21 Suppose the specification limits for the frozen food packages in Problem 19.17 are LSL = 450 grams and USL = 458 grams. Use the estimates of μ and σ obtained in Problem 19.17 to find C_{PK}. Also estimate the ppm not meeting the specifications.

19.22 In Problem 19.21, compute C_{PK} and estimate the ppm nonconforming after the modifications in Problem 19.19 have been made.

P- AND *NP*-CHARTS

19.23 A company produces fuses for automobile electrical systems. Five hundred of the fuses are tested per day for 30 days. Table 19.17 gives the number of defective fuses found per day for the 30 days. Determine the centerline and the upper and lower control limits for a *P*-chart. Does the process appear to be in statistical control? If the process is in statistical control, give a point estimate for the ppm defective rate.

Table 19.17

Day	1	2	3	4	5	6	7	8	9	10
# Defective	3	3	3	3	1	1	1	1	6	1
Day	11	12	13	14	15	16	17	18	19	20
# Defective	1	1	5	4	6	3	6	2	7	3
Day	21	22	23	24	25	26	27	28	29	30
# Defective	2	3	6	1	2	3	1	4	4	5

19.24 Suppose in Problem 19.23, the fuse manufacturer decided to use an *NP*-chart rather than a *P*-chart. Find the centerline and the upper and lower control limits for the chart.

19.25 Scottie Long, the manager of the meat department of a large grocery chain store, is interested in the percentage of packages of hamburger meat that have a slight discoloration. Varying numbers of packages are inspected on a daily basis and the number with a slight discoloration is recorded. The data are shown in Table 19.18. Give the stair-stepped upper control limits for the 20 subgroups.

Table 19.18

Day	Subgroup size	Number discolored	Percent discolored
1	100	1	1.00
2	150	1	0.67
3	100	0	0.00
4	200	1	0.50
5	200	1	0.50
6	150	0	0.00
7	100	0	0.00
8	100	0	0.00
9	150	0	0.00
10	200	2	1.00
11	100	1	1.00
12	200	1	0.50
13	150	3	2.00
14	200	2	1.00
15	150	1	0.67
16	200	1	0.50
17	150	4	2.67
18	150	0	0.00
19	150	0	0.00
20	150	2	1.33

OTHER CONTROL CHARTS

19.26 Review Problem 19.11 prior to working this problem. Hourly readings of the temperature of an oven, used for bread making, are obtained for 24 hours. The baking temperature is critical to the process and the oven is operated constantly during each shift. The data are shown in Table 19.19. An individuals chart is used to help monitor the temperature of the process. Find the centerline and the moving ranges corresponding to using adjacent pairs of measurements. How are the control limits found?

Table 19.19

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Temperature	350.0	350.0	349.8	350.4	349.6	350.0	349.7	349.8	349.4	349.8	350.7	350.9
Hour	13	14	15	16	17	18	19	20	21	22	23	24
Temperature	349.8	350.3	348.8	351.6	350.0	349.7	349.8	348.6	350.5	350.3	349.1	350.0

19.27 Review Problem 19.12 prior to working this problem. Use Minitab to construct an EWMA chart for the data in Table 19.14. Using a target value of 454 grams, what does the chart indicate concerning the process?

19.28 Review the discussion of a zone chart in Problem 19.13 before working this problem. Construct a zone chart for the data in Table 19.16. Does the zone chart indicate any out of control conditions? What short coming of the zone chart does this problem show?

19.29 Work Problem 19.15 prior to working this problem. Construct the stair-stepped control limits for the U-chart in Problem 19.15.

19.30 A *Pareto chart* is often used in quality control. A Pareto chart is a bar graph that lists the defects that are observed in descending order. The most frequently occurring defects are listed first, followed by those that occur less frequently. By the use of such charts, areas of concern can be identified and efforts made to correct those defects that account for the largest percentage of defects. The following defects are noted for respirator masks inspected during a given time period: discoloration, loose strap, dents, tears, and pinholes. The results are shown in Table 19.20.

Table 19.20

discoloration	discoloration	discoloration
strap	strap	strap
discoloration	dent	strap
discoloration	strap	discoloration
strap	discoloration	discoloration
discoloration	discoloration	dent
discoloration	dent	tear
tear	pinhole	discoloration
dent	discoloration	pinhole
discoloration	tear	tear

Figure 19-13 is a Pareto chart generated by Minitab. The data given in Table 19.20 are entered into column 1 of the worksheet. The pull-down menus needed to construct this chart are as follows: **Stat → Quality tools → Pareto charts**. By referring to the Pareto chart, what type of defect should receive the most attention? What two types of defect should receive the most attention?

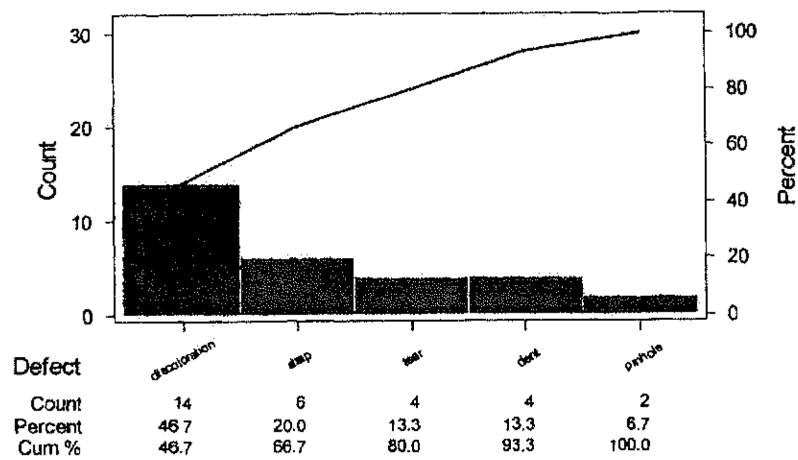

Fig. 19-13 Respirator defects.