

Contents

1	Statistical Process Control	4
1.1	Introduction to Statistical Process Control	4
1.2	What is Statistical Process Control	5
1.3	Statistical Quality Control	5
1.4	7 Basic Tools of Quality	5
1.5	Ten R Packages that every Data Scientist Should know	6
1.5.1	10 R packages I wish I knew about earlier	6
1.5.2	Using the <code>summary</code> command	15
1.5.3	Using R	17
1.6	Causes of Variation	18
1.7	Review Questions (Theory Questions)	19
1.8	Some Remarks on Multivariate Techniques	19
2	Control Charts	20
2.1	Control Charts	20
2.1.1	Control Charts	21
2.2	Nelson Rules for Interpreting Control Charts	21
2.2.1	Descriptions of Tests	21
2.2.2	Interpreting Control Charts	25
2.2.3	Control Limits	25
2.3	Quality Control Charts	26
2.4	More on Control Charts	27
2.4.1	Control Chart Selection	27
2.4.2	Attribute Control Charts	28
2.4.3	Types of Attributes Control Charts	28
2.4.4	p-charts	29
2.4.5	np-charts	30
2.4.6	The c-chart	31
2.4.7	The u-chart	32

3	Multivariate Normality	33
3.1	Multivariate Normal	33
3.1.1	Testing for Normality	34
3.2	Multivariate Control Charts	40
3.2.1	Multivariate Control Charts	40
3.2.2	The MSQC package	40
3.2.3	The <code>mult.chart</code> Function	41
3.2.4	T2 control chart	41
3.2.5	<code>mqcc</code> Example	42
3.3	The <code>qcc</code> R package - The 7QC tools revisited	45
3.3.1	<code>qcc</code> : Quality Control Charts	45
3.3.2	Types of Control Chart supported by <code>qcc</code>	45
3.3.3	Pareto Chart Analysis.	46
3.3.4	Pareto Analysis (Implementation with <code>qcc</code> package)	46
3.3.5	Cause and Effect Diagrams	48
3.3.6	Constructing Process Maps	50
3.4	The <code>qcc</code> R package - Other Types of Graph	53
3.4.1	Operating Characteristic (OC) Curves	53
3.4.2	Moving Average (MA) Chart	55
3.4.3	CUSUM charts	59
3.4.4	Individual/Moving-Range chart	61
4	Process Capability	62
4.1	Process Capability	62
4.1.1	Capability Study	62
4.1.2	What is Process Capability?	63
4.1.3	Capability Indices	63
4.1.4	Interpreting Capability Indices	65
4.2	Process Capability	66
4.2.1	Capability Study	67
4.2.2	What is Process Capability?	67
4.2.3	Capability Indices	68
4.2.4	Interpreting Capability Indices	69
4.2.5	Process Capability Analysis	69
4.2.6	Intepreting Process Capability Indices	70
4.3	Process Capability Analysis	71
4.4	Worked Examples	71
4.4.1	MA6001 Autumn 2007/2008 Question 4	71
4.4.2	MA6001 Autumn 2008/2009 Question 4	72

5	SixSigma	73
5.1	Six Sigma - Background to Six Sigma	73
5.1.1	Why six-sigma?	74
5.2	Six Sigma R Package	75
5.2.1	DMAIC Process	75
5.2.2	Data Set - <code>ss.data.ca</code>	75
5.2.3	<code>ss.ca.z</code> Capability Indices	77
5.2.4	Loss Fucntion Aanalysis	78
5.3	Other R Packages	79
5.3.1	<code>spc</code> : Statistical Process Control	80

Chapter 1

Statistical Process Control

Statistical process control (SPC) is a method of quality control which uses statistical methods. SPC is applied in order to monitor and control a process. Monitoring and controlling the process ensures that it operates at its full potential. At its full potential, the process can make as much conforming product as possible with a minimum (if not an elimination) of waste (rework or trash). SPC can be applied to any process where the "conforming product" (product meeting specifications) output can be measured. Some key tools are used in SPC. These include control charts; a focus on continuous improvement; and the design of experiments.

1.1 Introduction to Statistical Process Control

- The term Statistical Process Control (SPC) is typically used in context of manufacturing processes (although it may also pertain to services and other activities), and it denotes statistical methods used to monitor and improve the quality of the respective operations.
- By gathering information about the various stages of the process and performing statistical analysis on that information, the SPC engineer is able to take necessary action (often preventive) to ensure that the overall process stays in-control and to allow the product to meet all desired specifications.
- SPC involves monitoring processes, identifying problem areas, recommending methods to reduce variation and verifying that they work, optimizing the process, assessing the reliability of parts, and other analytic operations.
- SPC uses such basic statistical quality control methods as quality control charts (Shewhart, Pareto, and others), capability analysis, gage repeatability/reproducibility analysis, and reliability analysis.

1.2 What is Statistical Process Control

- The concepts of Statistical Process Control (SPC) were initially developed by Dr. Walter Shewhart of Bell Laboratories in the 1920's, and were expanded upon by Dr. W. Edwards Deming, who introduced SPC to Japanese industry after WWII.
- After early successful adoption by Japanese firms, Statistical Process Control has now been incorporated by organizations around the world as a primary tool to improve product quality by reducing process variation.
- Dr. Shewhart identified two sources of process variation:

Chance variation that is inherent in process, and stable over time,

Assignable variation , or Uncontrolled variation, which is unstable over time - the result of specific events outside the system.

- Dr. Deming relabeled chance variation as **Common Cause** variation, and assignable variation as **Special Cause** variation.
- Based on experience with many types of process data, and supported by the laws of statistics and probability, Dr. Shewhart devised control charts used to plot data over time and identify both Common Cause variation and Special Cause variation.

1.3 Statistical Quality Control

There is little difference between Statistical Quality Control (SQC) and Statistical Process Control (SPC). At one time, there might have been some philosophical separation, but today, they exist as general synonyms. Some prefer SQC because the idea of quality is larger and more encompassing than that of process. Others counter this by pointing out that the term process is problematic by nature, whereas a focus on quality is symptomatic in character. Still others looked at SQC as the management version of SPC. The bottom line is simple both approaches get the job done.

As a notable extension of this discussion, we should acknowledge that some quality professionals are still trying to argue that Six Sigma Quality (SSQ) is just another parallax of Total Quality Management (TQM). To better understand the true parallaxes of Six Sigma, let us consider its three primary domains. In this manner, we can better understand how the ideas of SQC and SPC support the aims of SSQ.

1.4 7 Basic Tools of Quality

These are 7 QC tools also known as Ishikawa's **7QC** tools

Cause-and-effect diagram : Identifies many possible causes for an effect or problem and sorts ideas into useful categories. (also called *Ishikawa* or *fishbone chart*)

Check sheet : A structured, prepared form for collecting and analyzing data; a generic tool that can be adapted for a wide variety of purposes.

Control charts : Graphs used to study how a process changes over time.

Histogram : The most commonly used graph for showing frequency distributions, or how often each different value in a set of data occurs.

Pareto chart : Shows on a bar graph which factors are more significant.

Scatter diagram : Graphs pairs of numerical data, one variable on each axis, to look for a relationship.

Stratification : A technique that separates data gathered from a variety of sources so that patterns can be seen (some lists replace stratification with flowchart or run chart).

For the sake of brevity, we will only look at a couple of these.

1.5 Ten R Packages that every Data Scientist Should know

1.5.1 10 R packages I wish I knew about earlier

- *Following material written by Drew Conway, and was published on the Yhat blog Feb 2013*
- <http://blog.yhathq.com/posts/10-R-packages-I-wish-I-knew-about-earlier.html>
- **qcc** is a library for statistical quality control. Back in the 1950s, the now defunct Western Electric Company was looking for a better way to detect problems with telephone and electrical lines.

Remark: *We will discuss these rules shortly*

- They came up with a set of rules to help them identify problematic lines. The rules look at the historical mean of a series of datapoints and based on the standard deviation, the rules help judge whether a new set of points is experiencing a mean shift.
- The classic example is monitoring a machine that produces lug nuts. Let's say the machine is supposed to produce 2.5 inch long lug nuts. We measure a series of lug nuts: 2.48, 2.47, 2.51, 2.52, 2.54, 2.42, 2.52, 2.58, 2.51.
- Is the machine broken? Well it's hard to tell, but the *Western Electric Rules* can help.

- While you might not be monitoring telephone lines, gcc can help you monitor transaction volumes, visitors or logins on your website, database operations, and lots of other processes.

Other Remarks

- Quality Control and quality assurance are important functions in most businesses from manufacturing to software development.
- For most, this means that one or more people are meticulously inspecting what's coming out of the factory, looking for imperfections and validating that requirements for products and services produced are satisfied.
- Often times QC and QA are performed manually by a select few specialists, and determining suitable quality can be extremely complex and error-prone.


```
# install.package(qcc)
library(qcc)

# series of value w/ mean of 10 with a little random noise added in
x <- rep(10, 100) + rnorm(100)

# a test series w/ a mean of 11
new.x <- rep(11, 15) + rnorm(15)

# qcc will flag the new points
qcc(x, newdata=new.x, type="xbar.one")
```

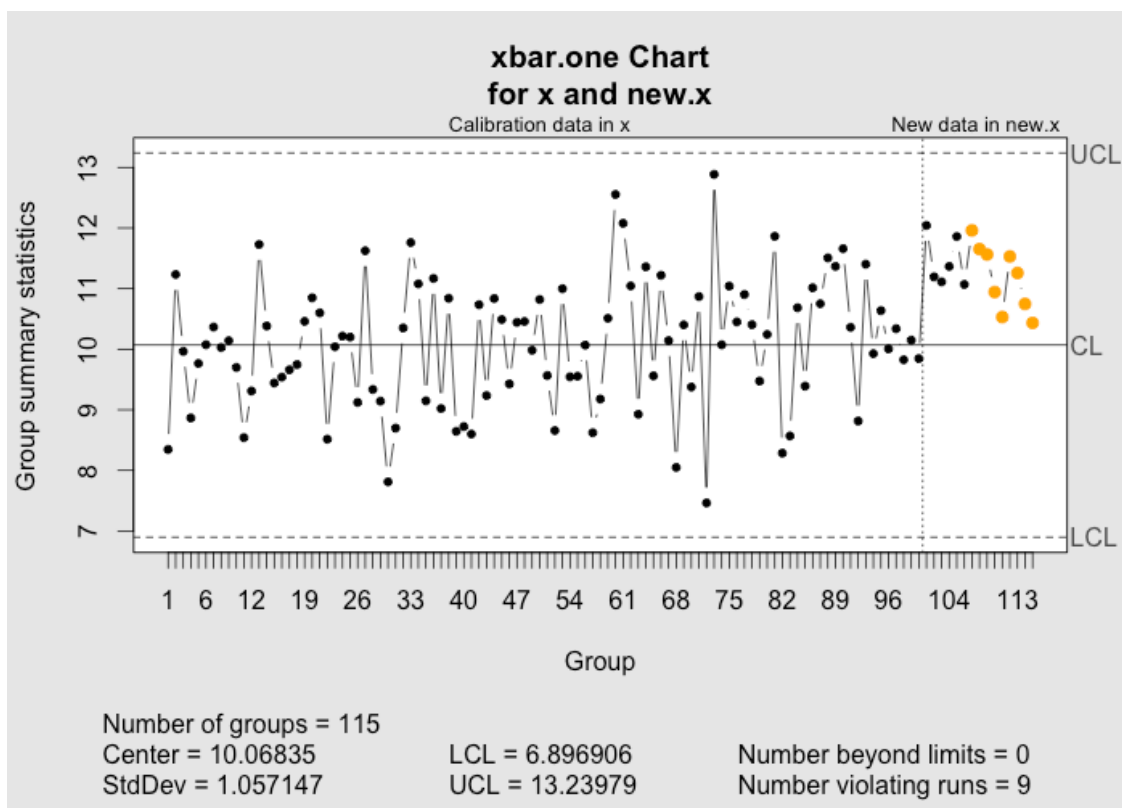
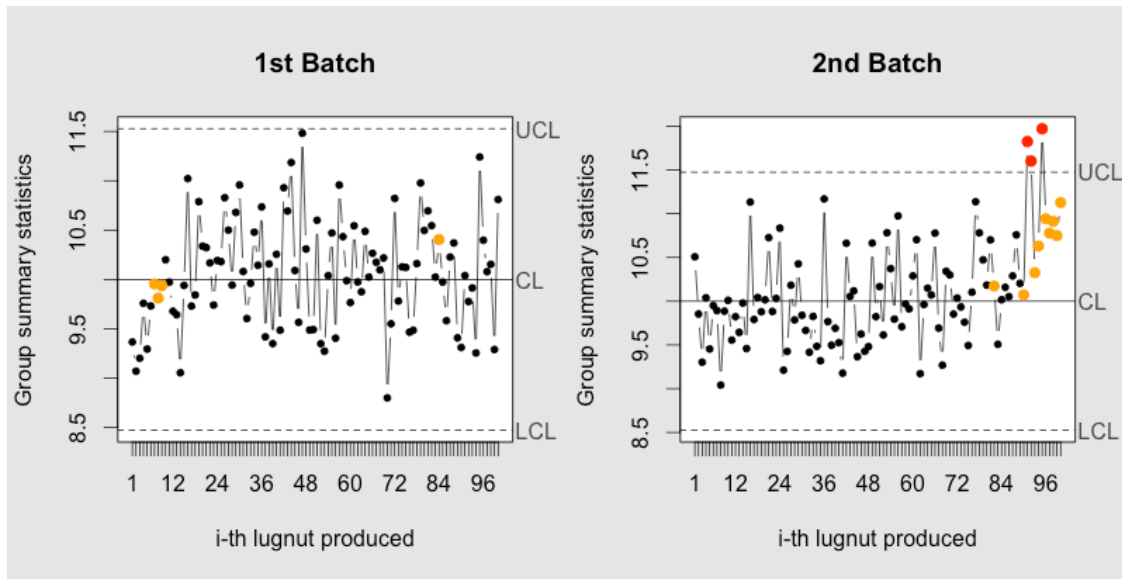


Figure 1.1:



```
library(qcc)
#make 2 plots in 1 figure
par(mfrow=c(1,2))

#points have base value of 10 w/ normally distributed error
lugnuts <- rep(10, 100) + rnorm(100, mean=0, sd=0.5)
qcc(lugnuts, type="xbar.one", center=10, add.stats=FALSE,
    title="1st Batch",
    xlab="i-th lugnut produced")
```

Second Batch

- First 90 points have mean value of 10 with normally distributed error,
- Last 10 points have mean value of 11 with normally distributed error

```
lugnuts <- c(rep(10, 90), rep(11, 10)) + rnorm(100, mean=0, sd=0.5)
qcc(lugnuts, type="xbar.one", center=10, add.stats=FALSE,
title="2nd Batch",
xlab="i-th lugnut produced")
```

```

> set.seed(1234)
> lugnuts <- rep(10, 100) + rnorm(100, mean=0, sd=0.5)
> summary(lugnuts)
Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
8.827   9.552   9.808   9.922  10.240  11.270
> length(lugnuts)
[1] 100
> newLugnuts <- rep(11, 10) + rnorm(10, mean=0, sd=0.5)
> summary(newLugnuts)
Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
10.55  10.75   11.00   10.92  11.08   11.21
> length(newLugnuts)
[1] 10

```

```

qcc1 <- qcc(lugnuts, type="xbar.one", center=10, add.stats=TRUE,
title="1st Batch of 100",
xlab="i-th lugnut produced")

```

```

qcc2 <- qcc(lugnuts, newdata=newLugnuts,
type="xbar.one", center=10,
add.stats=TRUE, title="All Lugnuts",
xlab="i-th lugnut produced")

```

```

mode(qcc1)
class(qcc1)
names(qcc1)

```

```

> mode(qcc1)
[1] "list"
> class(qcc1)
[1] "qcc"
> names(qcc1)
[1] "call"          "type"          "data.name"     "data"          "statistics"
[6] "sizes"         "center"        "std.dev"       "nsigmas"       "limits"
[11] "violations"

```

```
> qcc1$violations
$beyond.limits
integer(0)

$violating.runs
[1] 13 38 39 40 48 49 50 51 52 53 54 55
```

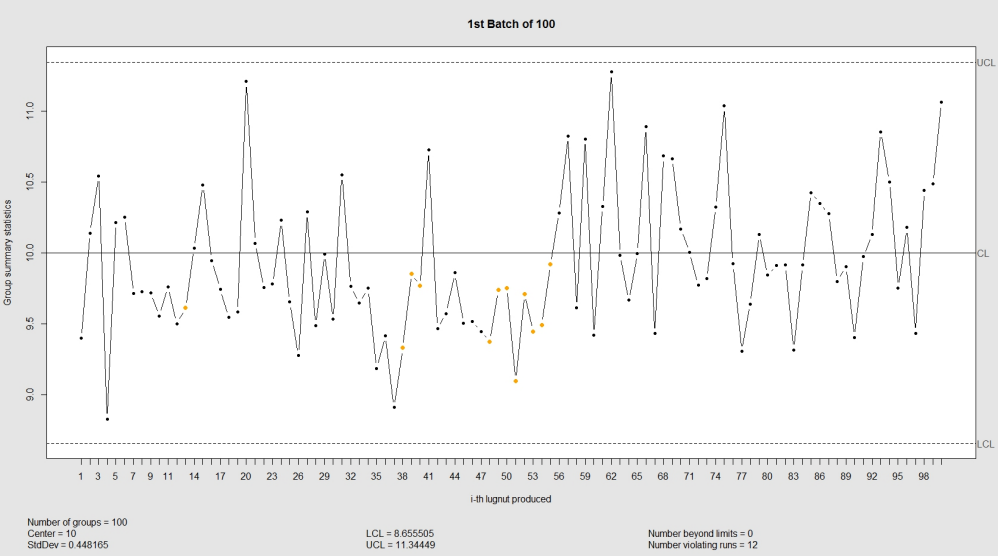


Figure 1.2:

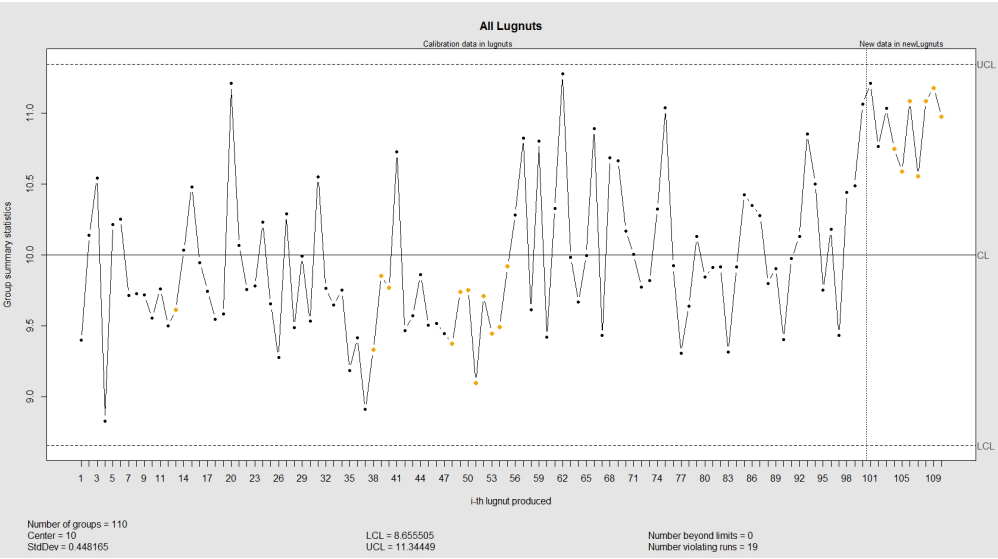


Figure 1.3:

1.5.2 Using the summary command

```
Call:
qcc(data = lugnuts, type = "xbar.one", center = 10, add.stats = TRUE,
title = "1st Batch of 100", xlab = "i-th lugnut produced")
```

xbar.one chart for lugnuts

Summary of group statistics:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
8.827	9.552	9.808	9.922	10.240	11.270

Group sample size: 1

Number of groups: 100

Center of group statistics: 10

Standard deviation: 0.448165

Control limits:

LCL	UCL
-----	-----

8.655505	11.34449
----------	----------

```
Call:
```

```
qcc(data = lugnuts, type = "xbar.one", center = 10, newdata = newLugnuts,
add.stats = TRUE, title = "All Lugnuts", xlab = "i-th lugnut produced")
```

xbar.one chart for lugnuts

.....

Summary of group statistics in newLugnuts:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
10.55	10.75	11.00	10.92	11.08	11.21

Group sample size: 1

Number of groups: 10

Control limits:

LCL UCL

8.655505 11.34449

Example used by Drew Conway

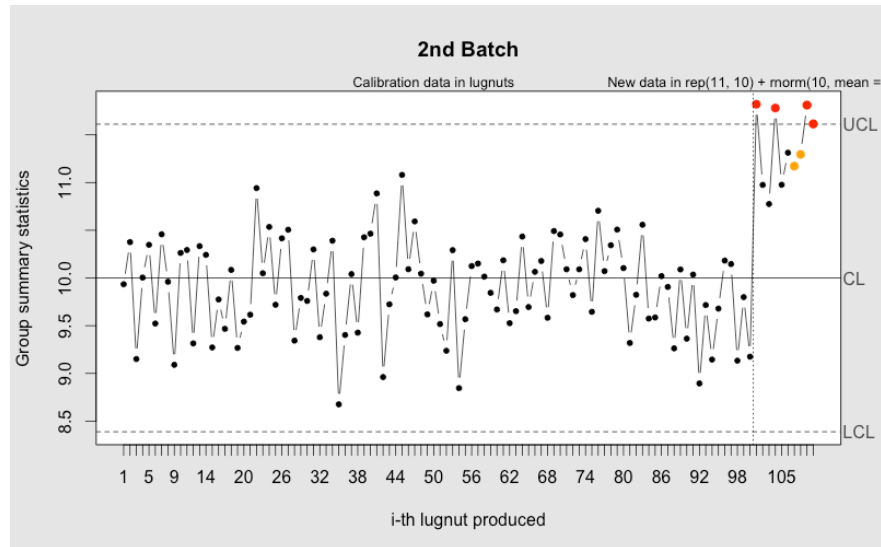


Figure 1.4:

```
lugnuts <- rep(10, 100) + rnorm(100, mean=0, sd=0.5)
qcc(lugnuts, newdata=rep(11, 10) + rnorm(10, mean=0, sd=0.5),
type="xbar.one", center=10,
add.stats=FALSE, title="2nd Batch",
xlab="i-th lugnut produced")
```

Remarks

- newdata
- add.stats

1.5.3 Using R

Advantages of using R statistical program along with the **qcc** package:

- There are several packages for interfacing with databases, RODBC being a common and useful one on MS windows.

- Allows of automation: you can program a regular event loop to check for new data and run a new set of charts and notifications if there is new data.
- The **mail** and **sendmailR** packages were designed to automatically send e-mails with regular reports and warning messages.
- It produces the standard SPC charts, these can go to the screen or a file to be sent out.
- Bespokes tests can be program for out of control signals
- Full programming language with common (and uncommon) statistics so you can preprocess you data in many ways to reduce dimension.
- You can have multiple instances running on multiple or a single computer, each processing for a single department, or you can combine it all into one script to run for all the departments.

1.6 Causes of Variation

- The purpose is to control the quality of product or service outputs from a process by maintaining control of the process.
- When a process is described as being in control, it means that the amount of variation in the output is relatively constant and within established limits that are deemed acceptable.
- There are two kinds of causes of variation in a process: Common causes, or chance causes, of variation are due to factors that are inherent in the design of the system, and reflect the usual amount of variation to be expected. Assignable causes, or special causes, of variation are due to unusual factors that are not part of the process design and not ordinarily part of the process.
- A stable process is one in which only common causes of variation affect the output quality. Such a process can also be described as being in a state of statistical control. An unstable process is one in which both assignable causes and common causes affect the output quality. (Note that, by definition, the common causes are always present.) Such a process can also be described as being out of control, particularly when the assignable cause is controllable.
- The way we set out to improve the quality of output for a process depends on the source of process variation.

- For a process that is stable, improvement can take place only by improving the design of the process. A pervasive error in process management is tampering, which is to take actions (such as making machine adjustments) that presume that a process is not in control, when in fact it is stable.
- Such actions only increase variability, and are analogous to the over-correcting that new drivers do in learning to steer a car. For a process that is unstable, improvement can be achieved by identifying and correcting assignable causes.

1.7 Review Questions (Theory Questions)

1. Differentiate common (or chance) causes of variation in the quality of process output from assignable (or special) causes.
2. Differentiate a stable process from an unstable process.
3. Other than applying the 3-sigma rule for detecting the presence of an assignable cause, what else do we look for when studying a control chart?
4. Describe how the output of a stable process can be improved. What actions do not improve a stable process, but rather, make the output more variable?
5. What is the purpose of maintaining control charts?
6. What is tampering in the context of process control?

1.8 Some Remarks on Multivariate Techniques

- Nowadays, the intensive use of an automatic data acquisition systems and the use of on-line computers for process monitoring have led to an increased occurrence of industrial processes with two or more correlated quality characteristics, in which the statistical process control and the capability analysis should be performed using multivariate methodologies. (*Edgar Santos-Fernandez*)

Chapter 2

Control Charts

2.1 Control Charts

Control Charts A run chart is a time series plot that plots levels of output quality on the vertical axis, with respect to a sequence of time periods on the horizontal axis. In the context of statistical process control the measurements that are graphed are typically sample data that have been obtained by the method of rational subgroups. A control chart is a run chart that includes the lower and upper control limits that identify the range of variation that can be ascribed to common causes. Any outputs that are outside of the control limits suggest the existence of assignable-cause variation. The control limits are determined either by process parameters having been specified, or by observing sample outcomes during a period of time in which the process is deemed to be in a stable condition. The statistical methods of process control are based on the concepts of hypothesis testing (see Table) The null hypothesis is that the process is stable and only common causes of variation exist. The alternative hypothesis is that the process is unstable and includes assignable-cause variation. Thus, the lower and upper control limits on a control chart are the critical values with respect to rejecting or not rejecting the null hypothesis that the process is stable and in control.

The standard practice is to place the control limits at three standard error units above and below the hypothesized value. This 3-sigma rule is conventionally applied for most control chart procedures. If a process is stable, not only should all sample statistics be within the control limits, but there also should be no discernable pattern in the sequence of the sample statistics. We look at the methods for determining control limits and we describe the interpretation of control charts for the process mean, process range process standard deviation, process range, process proportion.

Of these four, control charts for the mean, standard deviation, and range are designated as control charts for variables, because measurements are involved. Control charts for the proportion are designated as control charts for attributes, because counts (which are discrete

rather than continuous variables) are involved.

2.1.1 Control Charts

- The control chart is a graph used to study how a process changes over time. Data are plotted in time order. A control chart always has a central line for the average, an upper line for the upper control limit and a lower line for the lower control limit.
- Units are usually the means of small samples.
- These lines are determined from historical data. By comparing current data to these lines, you can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).
- There are several types of control chart. In the short term, we will look at the **x-bar** chart (related to mean of values from sample). Other types of chart include R charts and S charts (related to range and standard deviation of the values from each sample)

2.2 Nelson Rules for Interpreting Control Charts

- The eight tests used in statistical process control were developed by Lloyd S. Nelson, a process control expert. They are based on his determination that the identified patterns are very unlikely to occur in stable processes.
- Thus the existence of any of these patterns in an \bar{X} chart indicates that the process may be unstable, and that one or more assignable causes may exist.
- The table on the next page contains examples of test failure for each of the eight tests, with a description for each graph as to what is required for the illustrated test failure.
- In practice, tests 1,2 and 7 are considered the three most useful.

2.2.1 Descriptions of Tests

Test 1 - 3 sigma rule Identifies points outside of the control limits

Test 1 identifies points that are more standard deviations from the center line. Test 1 is universally recognized as necessary for detecting out-of-control situations. It has a false alarm rate of only 0.27%.

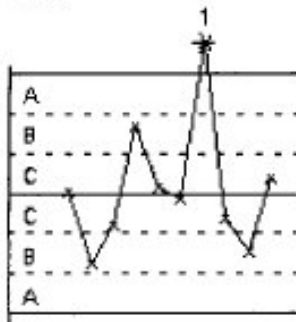
Test 2 Identifies shifts in the means

Test 2 signals when 9 points in a row fall on the same side of the center line. The use of Test 2 significantly increases the sensitivity of the chart to detect small shifts in the mean.

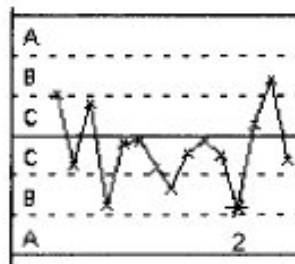
When test 1 and test 2 are used together, significantly fewer subgroups are needed to detect a small shift in the mean than are needed when test 1 is used alone. Therefore, adding test 2 helps to detect common out-of-control situations and increases sensitivity enough to warrant a slight increase in the false alarm rate.

Test 1

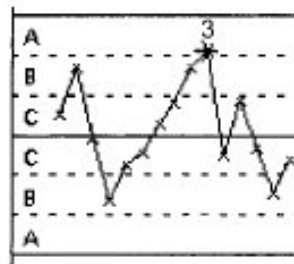
One point more than 3 sigmas from center line

**Test 2**

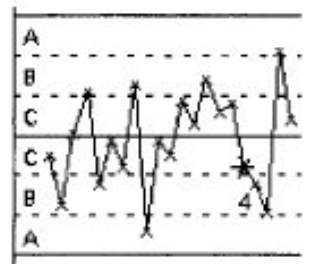
Nine points in a row on same side of center line

**Test 3**

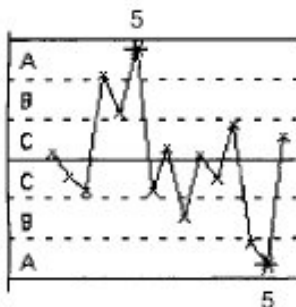
Six points in a row, all increasing or all decreasing

**Test 4**

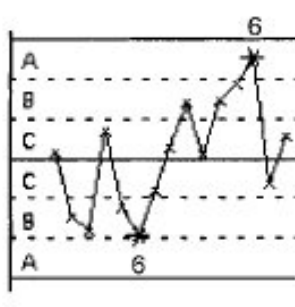
Fourteen points in a row, alternating up and down

**Test 5**

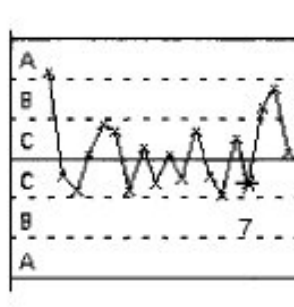
Two out of three points in a row more than 2 sigmas from center line (same side)

**Test 6**

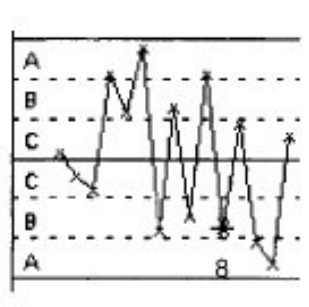
Four out of five points in a row more than 1 sigma from center line (same side)

**Test 7**

Fifteen points in a row within 1 sigma of center line (either side)

**Test 8**

Eight points in a row more than 1 sigma from center line (either side)



Test 3 k points in a row, all increasing or all decreasing

Test 3 is designed to detect drifts in the process mean.

However, when test 3 is used in addition to test 1 and test 2, it does not significantly increase the sensitivity of the chart to detect drifts in the process mean.

Test 4 k points in a row, alternating up and down

Although this pattern can occur in practice, it is recommended to search for any unusual trends or patterns rather than test for one specific pattern.

Test 5 k out of $k+1$ points > 2 standard deviations from center line

This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

Test 6 k out of $k+1$ points > 1 standard deviation from the center line

This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

Test 7 Identifies control limits that are too wide

Test 7 signals when 12 or 15 points in a row fall within 1 standard deviation of the center line.

Test 7 is used only for the \bar{X} chart when the control limits are estimated from the data. When this test fails, the cause is usually a systemic source of variation (stratification) within a subgroup, which is often the result of not forming rational subgroups.

Test 8 k points in a row > 1 standard deviation from center line (either side)

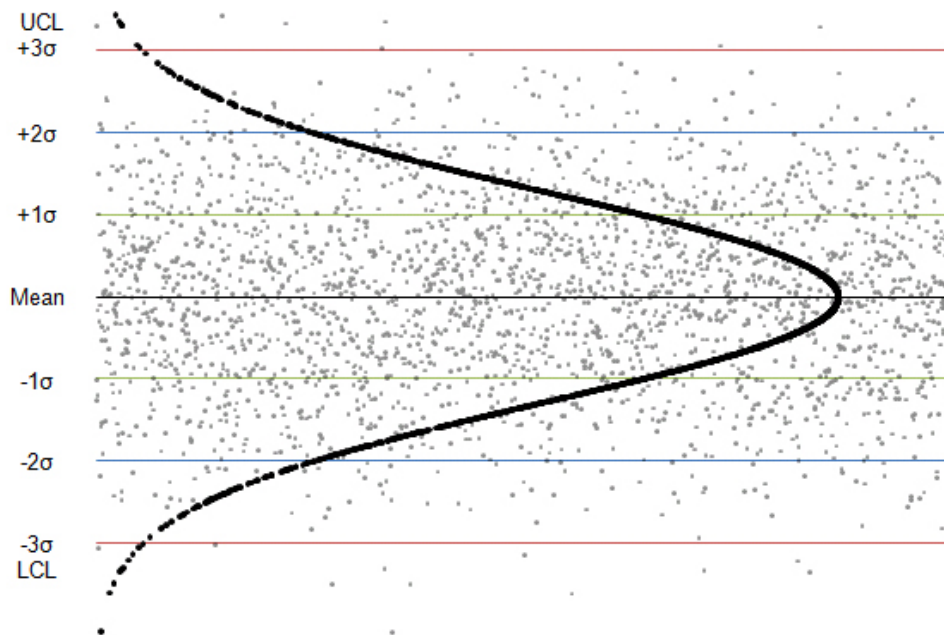
This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

2.2.2 Interpreting Control Charts

- The most obvious test in terms of its rationale is Test 1, which simply requires that at least one value of \bar{X} be beyond Zone A. For a sample mean to be beyond three standard errors from the centre-line is of course very unlikely in a stable process.
- The other seven patterns also are very unlikely. The one test whose rationale is not obvious is Test 7, which requires that 15 consecutive values of \bar{X} be in Zone C, which is within one standard error of the centre-line.
- Although this kind of pattern would seem to be very desirable, it is also very unlikely. It may, for example, indicate that the process standard deviation was overstated in the process specifications or that the sample measurements are in error. Whatever the cause, the results are too good to be true and therefore require investigation.

2.2.3 Control Limits

- Statistical tables have been developed for various types of distributions that quantify the area under the curve for a given number of standard deviations from the mean (based on the *normal distribution*).
- Shewhart found that control limits placed at ***three standard deviations from the mean*** in either direction provide an economical tradeoff between the risk of reacting to a false signal and the risk of not reacting to a true signal - regardless the shape of the underlying process distribution.
- If the process has a normal distribution, 99.7% of the population is captured by the curve at three standard deviations from the mean. Stated another way, there is only a 100-99.7%, or 0.3% chance of finding a value beyond 3 standard deviations. Therefore, a measurement value beyond 3 standard deviations indicates that the process has either shifted or become unstable (more variability).



2.3 Quality Control Charts

- In the Lab classes, we will look at using the R package qcc, which stands for quality control charts.
- This package can be used to create the following. Shewhart quality control charts for continuous, attribute and count data.
 1. CUSUM and EWMA charts.
 2. Operating characteristic curves.
 3. Process capability analysis.
 4. Pareto chart and cause-and-effect chart.
 5. Multivariate control charts

(We wont have time to get through all of this material)

- There are two inbuilt data sets, available from the qcc package, that we will use
 1. pistonrings
 2. orangejuice

The pistonrings data set (diameter) The first data set describes piston rings for an automotive engine are produced by a forging process.

- The inside diameter of the rings manufactured by the process is measured on 25 samples, each of size 5, drawn from a process being considered 'in control'. There are further 15 sets of observation (i.e. 40 batches in total) to demonstrate alternate outcomes. The dataset is restructured as a data set called diameter. The orangejuice data set The second data set describes frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel.

2.4 More on Control Charts

2.4.1 Control Chart Selection

- Correct control chart selection is a critical part of creating a control chart. If the wrong control chart is selected, the control limits will not be correct for the data.
- The type of control chart required is determined by the type of data to be plotted and the format in which it is collected.
- Data collected is either in variables or attributes format, and the amount of data contained in each sample (subgroup) collected is specified.
- **Variables data** is defined as a measurement such as height, weight, time, or length. Monetary values are also variables data.
 - * Generally, a measuring device such as a weighing scale, vernier, or clock produces this data.
 - * Another characteristic of variables data is that it can contain decimal places e.g. 3.4, 8.2.
- **Attributes data** is defined as a count such as the number of employees, the number of errors, the number of defective products, or the number of phone calls. A standard is set, and then an assessment is made to establish if the standard has been met.
 - * The number of times the standard is either met or not is the count. Attributes data never contains decimal places when it is collected, it is always whole numbers, e.g. 2, 15.

2.4.2 Attribute Control Charts

- The Shewhart control chart plots quality characteristics that can be measured and expressed numerically. We measure weight, height, position, thickness, etc. If we cannot represent a particular quality characteristic numerically, or if it is impractical to do so, we then often resort to using a quality characteristic to sort or classify an item that is inspected into one of two "buckets".
- An example of a common quality characteristic classification would be designating units as "conforming units" or "nonconforming units".
- Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called **attributes**.
- *Note that there is a difference between "nonconforming to an engineering specification" and "defective" – a nonconforming unit may function just fine and be, in fact, not defective at all, while a part can be "in spec" and not function as desired (i.e., be defective).*
- Examples of quality characteristics that are attributes are the number of failures in a production run, the proportion of malfunctioning wafers in a lot, the number of people eating in the cafeteria on a given day, etc.

2.4.3 Types of Attributes Control Charts

- Control charts dealing with the proportion or fraction of defective product are called **p-charts** (for proportion).
- Control charts dealing with the number of defective product are called **np-charts**.
- Control charts dealing with the number of defects or nonconformities are called **c-charts** (for count).
- There is another chart which handles defects per unit, called the **u-chart** (for unit). This applies when we wish to work with the average number of nonconformities per unit of product.

2.4.4 p-charts

- The p-chart is a type of control chart used to monitor the **proportion of nonconforming units** in a sample, where the sample proportion nonconforming is defined as the ratio of the number of nonconforming units to the sample size, n .
- The p-chart only accommodates dichotomous PASS/FAIL-type inspection as determined by a series of tests, effectively applying the specifications to the data before they are plotted on the chart.
- Other types of control charts display the magnitude of the quality characteristic under study, making troubleshooting possible directly from those charts.
- A p-chart is an attributes control chart used with data collected in subgroups of varying sizes. Because the subgroup size can vary, it shows a proportion on nonconforming items rather than the actual count.
- *p-charts show how the process changes over time. The process attribute (or characteristic) is always described in a yes/no, pass/fail, go/no go form.*
- Example: use a p-chart to plot the proportion of incomplete insurance claim forms received weekly. The subgroup would vary, depending on the total number of claims each week.

2.4.5 np-charts

The np-chart is a type of control chart used to monitor the number of nonconforming units in a sample. An np-chart is an *attributes* control chart used with data collected in subgroups that are the **same size**.

It is an adaptation of the p-chart and used in situations where personnel find it easier to interpret process performance in terms of concrete numbers of units rather than the somewhat more abstract proportion.

The np-chart differs from the p-chart in only the three following aspects:

- The control limits are

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1 - \bar{p})}$$

, where n is the sample size and \bar{p} is the estimate of the long-term process mean established during control-chart setup.

- The number nonconforming (np), rather than the fraction nonconforming (p), is plotted against the control limits.
- The sample size, n , is constant.

2.4.6 The c-chart

- In this chart, we plot the number of defectives (per batch, per day, per machine, per 100 feet of pipe, etc.).
- This chart assumes that defects of the quality attribute are rare, and the control limits in this chart are computed based on the Poisson distribution (distribution of rare events).
- The c-chart is a type of control chart used to monitor "count"-type data, typically total number of nonconformities per unit. It is also occasionally used to monitor the total number of events occurring in a given unit of time.
- The c-chart differs from the p-chart in that it accounts for the possibility of more than one nonconformity per inspection unit, and that (unlike the p-chart and u-chart) it requires a fixed sample size.
- The p-chart models "pass"/"fail"-type inspection only, while the c-chart (and u-chart) give the ability to distinguish between (for example) 2 items which fail inspection because of one fault each and the same two items failing inspection with 5 faults each; in the former case, the p-chart will show two non-conformant items, while the c-chart will show 10 faults.
- The Poisson distribution is the basis for the chart and requires the following assumptions:
 - * The number of opportunities or potential locations for nonconformities is very large
 - * The probability of nonconformity at any location is small and constant
 - * The inspection procedure is same for each sample and is carried out consistently from sample to sample

2.4.7 The u-chart

- In this chart we plot the rate of defectives, that is, the number of defectives divided by the number of units inspected (the n ; e.g., feet of pipe, number of batches).
- Unlike the C chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.
- The u-chart is a type of control chart used to monitor "count"-type data where the sample size is greater than one, typically the average number of nonconformities per unit.
- The u-chart differs from the c-chart in that it accounts for the possibility that the number or size of inspection units for which nonconformities are to be counted may vary. Larger samples may be an economic necessity or may be necessary to increase the area of opportunity in order to track very low nonconformity levels.

Chapter 3

Multivariate Normality

3.1 Multivariate Normal

- The multivariate normal distribution or multivariate Gaussian distribution, is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions.
- One possible definition is that a random vector is said to be k -variate normally distributed if every linear combination of its k components has a univariate normal distribution.
- The multivariate normal distribution is often used to describe, at least approximately, any set of (possibly) correlated real-valued random variables each of which clusters around a mean value.

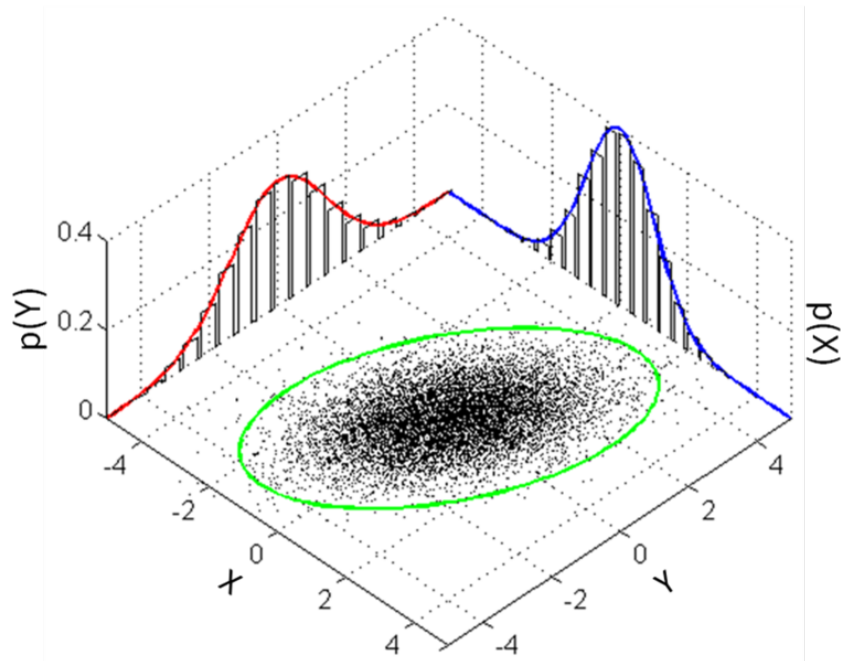


Figure 3.1:

3.1.1 Testing for Normality

Graphical Methods

- Histograms
- Normal Probability Plots

Hypothesis Tests for Univariate Data

- Shapiro-Wilk Test (inbuilt with R)
- D'Agostino Test (MSQC package)

Hypothesis Tests for Multivariate Data

- Mardia Test (MSQC package)
- Henze and Zirkler (MSQC package)
- Royston Test (MSQC package)

The bimetal data set (MSQC package)

- Bimetal thermostat has innumerable practical uses. These types of thermostats hold a bimetallic strip composed by two strips of different metals that convert the changing of temperature in mechanical displacement due to the difference in thermal expansion.
- Certain type of strip composed of brass and steel is analyzed in a quality laboratory by testing the deflection, curvature, resistivity, and hardness in low and high expansion sides.

```
> tail(bimetal1)
      deflection curvature resistivity Hardness low side Hardness high side
[23,]      20.76      39.98      14.98             22.29             26.03
[24,]      21.00      40.11      15.17             22.04             25.99
[25,]      20.57      39.73      14.35             22.02             25.80
[26,]      20.78      39.83      15.27             21.60             25.89
[27,]      20.96      40.03      15.26             21.98             25.94
[28,]      21.14      39.93      14.98             21.84             25.98
```

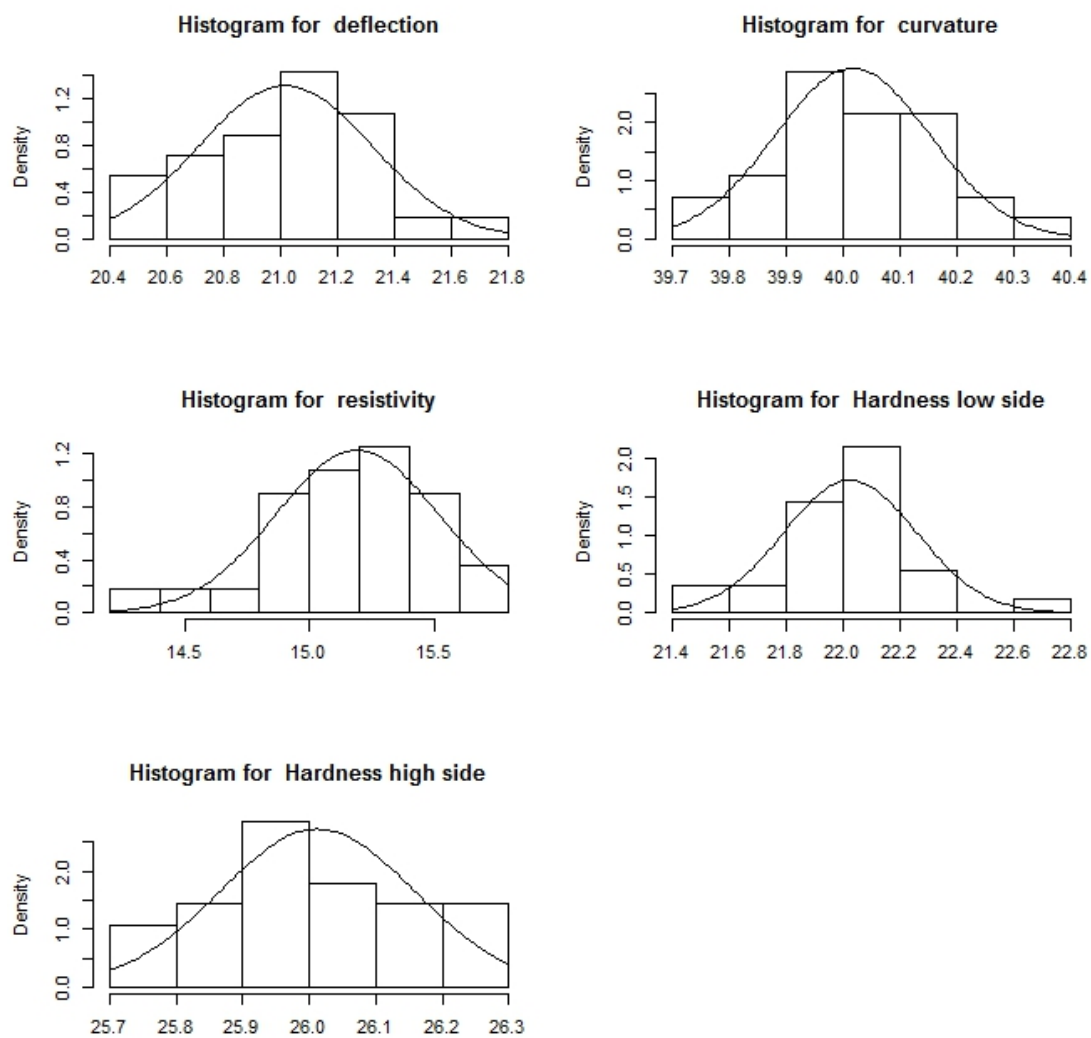


Figure 3.2:

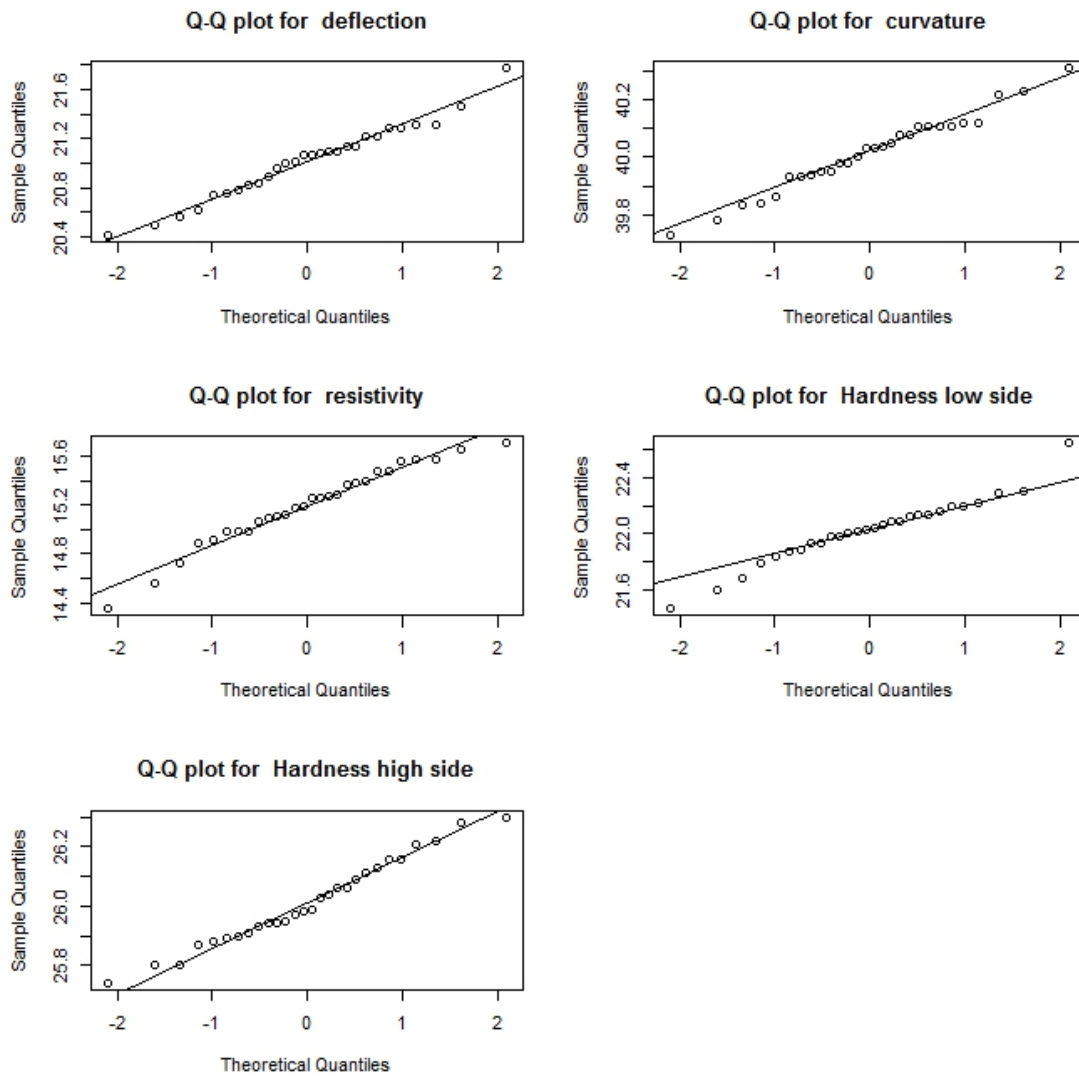


Figure 3.3:

D'Agostino Test (MSQC Pacakge)

- Using the bimetal1 data set in MSQC package

```
> for (i in 1 : 5){  
+   DAGOSTINO(bimetal1[,i])  
+ }  
D'Agostino Test  
Skewness  
Skewness coefficient: 0.0831225  
Statistics: 0.2117358  
p-value: 0.8323131  
Kurtosis  
The kurtosis coefficient: 3.0422  
Statistics: 0.591983  
p-value: 0.553862  
Omnibus Test  
Chi-squared: 0.3952759  
Degree of freedom: 2  
p-value: 0.8206669  
....  
....  
D'Agostino Test  
Skewness  
Skewness coefficient: -0.04173762  
Statistics: -0.1063873  
p-value: 0.9152751  
Kurtosis  
The kurtosis coefficient: 4.162062  
Statistics: 1.675258  
p-value: 0.09388364  
Omnibus Test  
Chi-squared: 2.817807  
Degree of freedom: 2  
p-value: 0.2444111
```

Some Multivariate (MSQC Package)

```
> MardiaTest(bimetal1)
$skewness
[1] 6.982112

$p.value
[1] 0.585327

$kurtosis
[1] 33.77373

$p.value
[1] 0.3490892

>
>
>
> HZ.test(bimetal1)
[1] 0.6068650 0.7709586
>
>
> Royston.test(bimetal1)
test.statistic      p.value
1.1814742          0.9364221
```

Box Cox Transformation

- The Box-Cox transforms nonnormally distributed data to a set of data that has approximately normal distribution.

3.2 Multivariate Control Charts

- With the enhancements in data acquisition systems it is usual to deal with processes with more than one correlated quality characteristic to be monitored.
- A common practice is to control the stability of the process using univariate control charts.
- This practice increases the probability of false alarm of special cause of variation.
- Therefore, the analysis should be performed through a multivariate approach; that is, the variables must be analyzed together, not independently.

3.2.1 Multivariate Control Charts

- Multivariate control charts monitor multiple process characteristics. Independent variables can be charted individually, but if the variables are correlated, a multivariate chart is needed to determine whether the process is in control.
- Multivariate control charts can detect shifts in the mean or the relationship between several related variables.
- The multivariate control chart plots Hotellings T^2 statistic. The calculation for the control limit differs based on whether targets have been specified.

3.2.2 The MSQC package

In his book, Edgar Santos-Fernandez present the multivariate normal distribution, the data structure of the multivariate problems dealt in this book, the `mult.chart` function that allows the computation in R, and the most used multivariate control charts:

- The control ellipsoid or w^2 control chart
- The T^2 or Hotelling chart
- The Multivariate Exponentially Weighted Moving Average (MEWMA) chart
- The Multivariate Cumulative Sum (MCUSUM) chart
- The chart based on Principal Components Analysis (PCA)

3.2.3 The `mult.chart` Function

The performing of the multivariate control chart in R can be carried out with the function `mult.chart` which is a general function that allows to compute the most accepted and diversified continuous multivariate chart such as

- χ^2
- Hotelling T^2
- MEWMA
- MCUSUM according to Crosier (1988)
- MCUSUM by Pignatiello and Runger (1990)

Finally the function `mult.chart` returns:

- The T^2 statistics
- The Upper Control Limit (UCL)
- The sample covariance matrix (S)
- The mean vector (Xmv)
- And if any point falls outside of the UCL and its decomposition

```
mult.chart(dowell1, type = "chi", alpha = 0.05)
```

3.2.4 T^2 control chart

The origin of the T^2 control chart dates back to the pioneer works of Harold Hotelling who applied this method to the bombsight problem in Second World War. The Hotelling (1947) procedure has become without doubt the most applied in multivariate process control and it is the multivariate analogous of the Shewhart control chart. For that reason, it is also known as multivariate Shewhart control chart.

```
data("carbon1")
mult.chart(type = "t2", carbon1)
mult.chart(type = "t2", carbon1)$t2
```

3.2.5 mqcc Example

```
# library(mqcc)
# Ryan (2000, Table 9.2) data with p = 2 variables,
# m = 20 samples, n = 4 sample size:

X1 = matrix(c(72, 56, 55, 44, 97, 83, 47, 88, 57, 26, 46,
49, 71, 71, 67, 55, 49, 72, 61, 35, 84, 87, 73, 80, 26, 89, 66,
50, 47, 39, 27, 62, 63, 58, 69, 63, 51, 80, 74, 38, 79, 33, 22,
54, 48, 91, 53, 84, 41, 52, 63, 78, 82, 69, 70, 72, 55, 61, 62,
41, 49, 42, 60, 74, 58, 62, 58, 69, 46, 48, 34, 87, 55, 70, 94,
49, 76, 59, 57, 46), ncol = 4)

X2 = matrix(c(23, 14, 13, 9, 36, 30, 12, 31, 14, 7, 10,
11, 22, 21, 18, 15, 13, 22, 19, 10, 30, 31, 22, 28, 10, 35, 18,
11, 10, 11, 8, 20, 16, 19, 19, 16, 14, 28, 20, 11, 28, 8, 6,
15, 14, 36, 14, 30, 8, 35, 19, 27, 31, 17, 18, 20, 16, 18, 16,
13, 10, 9, 16, 25, 15, 18, 16, 19, 10, 30, 9, 31, 15, 20, 35,
12, 26, 17, 14, 16), ncol = 4)

X = list(X1 = X1, X2 = X2)
q = mqcc(X, type = "T2")
summary(q)
```

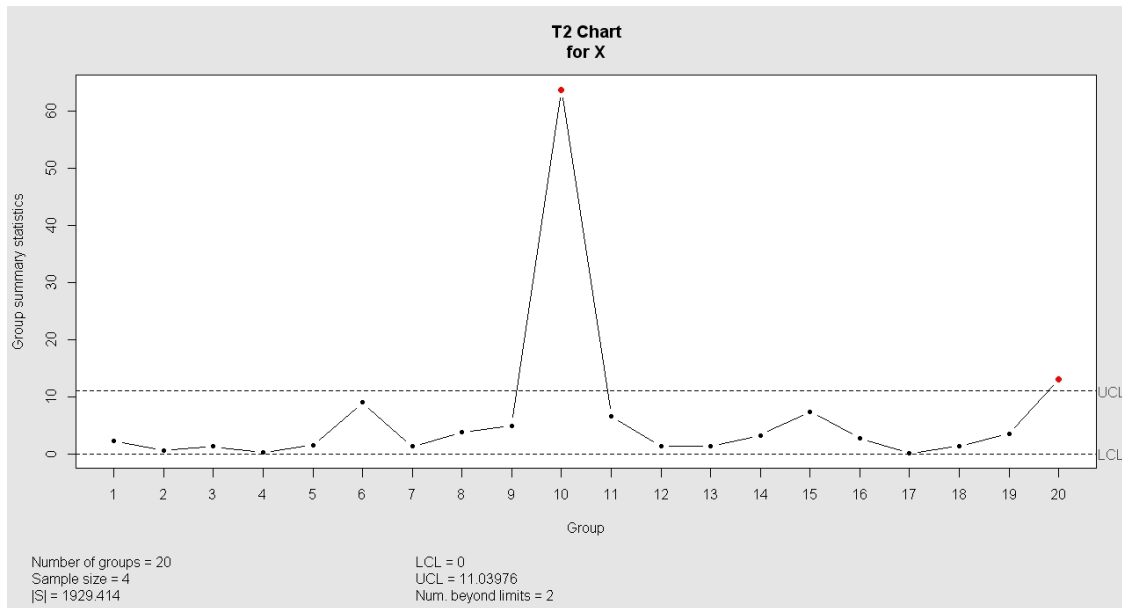


Figure 3.4:

Call:

```
mqcc(data = X, type = "T2")
```

T2 chart for X

Summary of group statistics:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.1243	1.3250	2.5030	6.4700	5.3490	63.7600

Number of variables: 2

Number of groups: 20

Group sample size: 4

Center:

X1	X2
60.3750	18.4875

Covariance matrix:

	X1	X2
X1	222.0333	103.11667

X2 103.1167 56.57917
|S|: 1929.414

Control limits:

LCL	UCL
0	11.03976

3.3 The qcc R package - The 7QC tools revisited

- The **qcc** package was built by Luca Scrucca for nothing but statistical quality control.
- It's extremely easy to use. You provide it with data and it tells you which points are considered to be outliers based on the Shewart Rules.
- It even color codes them based on how irregular each point is.
- Even though statistical quality control an old topic, statistical quality control is still highly relevant. There are probably have lots of jobs, processes, logs, or database metric tha could be monitored using control charts.

3.3.1 qcc : Quality Control Charts

Some Remarks

- Shewhart quality control charts for continuous, attribute and count data.
- Cusum and EWMA charts.
- Operating characteristic curves.
- Process capability analysis.
- Pareto chart and cause-and-effect chart.
- Multivariate control charts.

3.3.2 Types of Control Chart supported by qcc

"xbar" mean - means of a continuous process variable

"R" range ranges of a continuous process variable

"S" standard deviation standard deviations of a continuous variable

"xbar.one" mean one-at-time data of a continuous process variable

"p" proportion proportion of nonconforming units

"np" count number of nonconforming units

"c" count nonconformities per unit

"u" count average nonconformities per unit

"g" count number of non-events between events

3.3.3 Pareto Chart Analysis.

- A Pareto chart is a barplot where the categories are ordered in non increasing order, and a line is also added to show the cumulative sum.
- Quality problems are rarely spread evenly across the different aspects of the production process or different plants. Rather, a few "bad apples" often account for the majority of problems.
- This principle has come to be known as the Pareto principle, which basically states that quality losses are mal-distributed in such a way that a small percentage of possible causes are responsible for the majority of the quality problems.
- For example, a relatively small number of "dirty" cars are probably responsible for the majority of air pollution; the majority of losses in most companies result from the failure of only one or two products. To illustrate the "bad apples", one plots the Pareto chart,

3.3.4 Pareto Analysis (Implementation with qcc package)

```
defect <- c(80, 27, 66, 94, 33)

names(defect) <- c("price code", "schedule date",
  "supplier code", "contact num.", "part num.")

# 1
pareto.chart(defect, ylab = "Error frequency")

#2
pareto.chart(defect, ylab = "Error frequency", xlab = "Error causes", las=1)

#3
pareto.chart(defect, ylab = "Error frequency", col=rainbow(length(defect)))

#4
pareto.chart(defect, cumperc = seq(0, 100, by = 5),
  ylab2 = "A finer tickmarks grid")
```

Output to accompany graphs

Pareto chart analysis for defect

	Frequency	Cum.Freq.	Percentage	Cum.Percent.
contact num.	94	94	31.33333	31.33333
price code	80	174	26.66667	58.00000
supplier code	66	240	22.00000	80.00000
part num.	33	273	11.00000	91.00000
schedule date	27	300	9.00000	100.00000

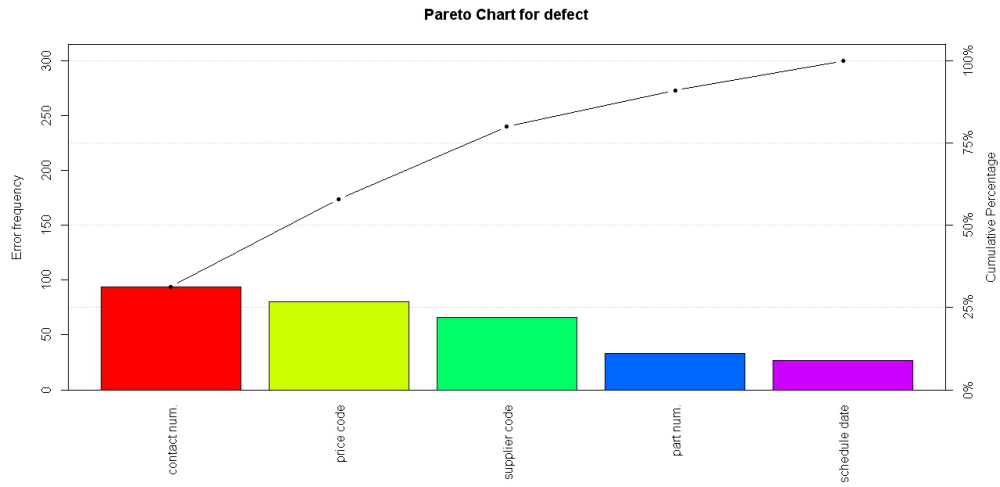


Figure 3.5: Third Implementation

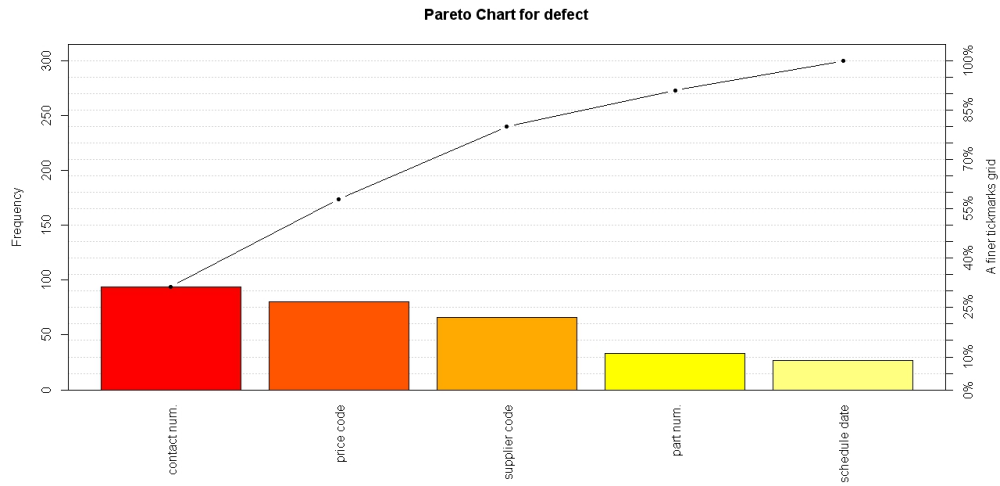


Figure 3.6: Fourth Implementation

3.3.5 Cause and Effect Diagrams

The cause and effect diagram is also known as “Ishikawa diagram”, and has been widely used in Quality Management. It is one of the Seven Basic Tools of Quality.


```

cause.and.effect(cause=list(
  Measurements=c("Micrometers", "Microscopes", "Inspectors"),
  Materials=c("Alloys", "Lubricants", "Suppliers"),
  Personnel=c("Shofts", "Supervisors", "Training", "Operators"),
  Environment=c("Condensation", "Moisture"),
  Methods=c("Brake", "Engager", "Angle"),
  Machines=c("Speed", "Lathes", "Bits", "Sockets")),
effect="Surface Flaws")

```

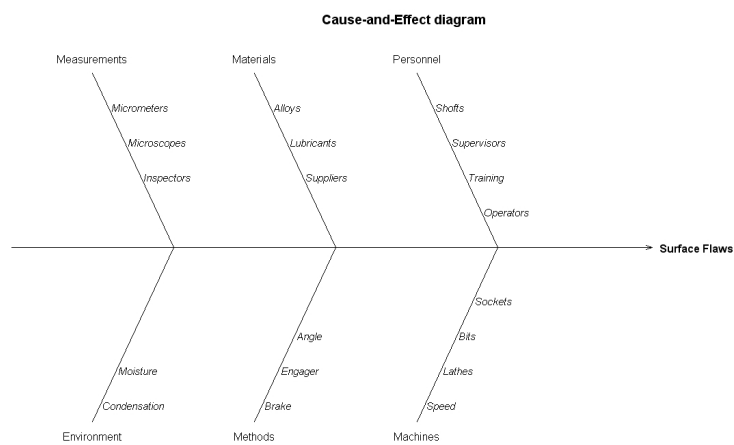


Figure 3.7:

Implementation with Six Sigma Package

```
effect <- "Flight Time"
causes.gr <- c("Operator", "Environment", "Tools", "Design",
"Raw.Material", "Measure.Tool")
causes <- vector(mode = "list", length = length(causes.gr))
causes[1] <- list(c("operator #1", "operator #2", "operator #3"))
causes[2] <- list(c("height", "cleaning"))
causes[3] <- list(c("scissors", "tape"))
causes[4] <- list(c("rotor.length", "rotor.width2", "paperclip"))
causes[5] <- list(c("thickness", "marks"))
causes[6] <- list(c("calibrate", "model"))
ss.ceDiag(effect, causes.gr, causes, sub = "Paper Helicopter Project")
```

3.3.6 Constructing Process Maps

```
inputs.overall<-c("operators", "tools", "raw material", "facilities")
outputs.overall<-c("helicopter")
steps<-c("INSPECTION", "ASSEMBLY", "TEST", "LABELING")
```

```
#Inputs of process "i" are inputs of process "i+1"
input.output<-vector(mode="list",length=length(steps))
input.output[1]<-list(c("sheets", "..."))
input.output[2]<-list(c("sheets"))
input.output[3]<-list(c("helicopter"))
input.output[4]<-list(c("helicopter"))
```

Parameters of each process

```
x.parameters<-vector(mode="list",length=length(steps))
```

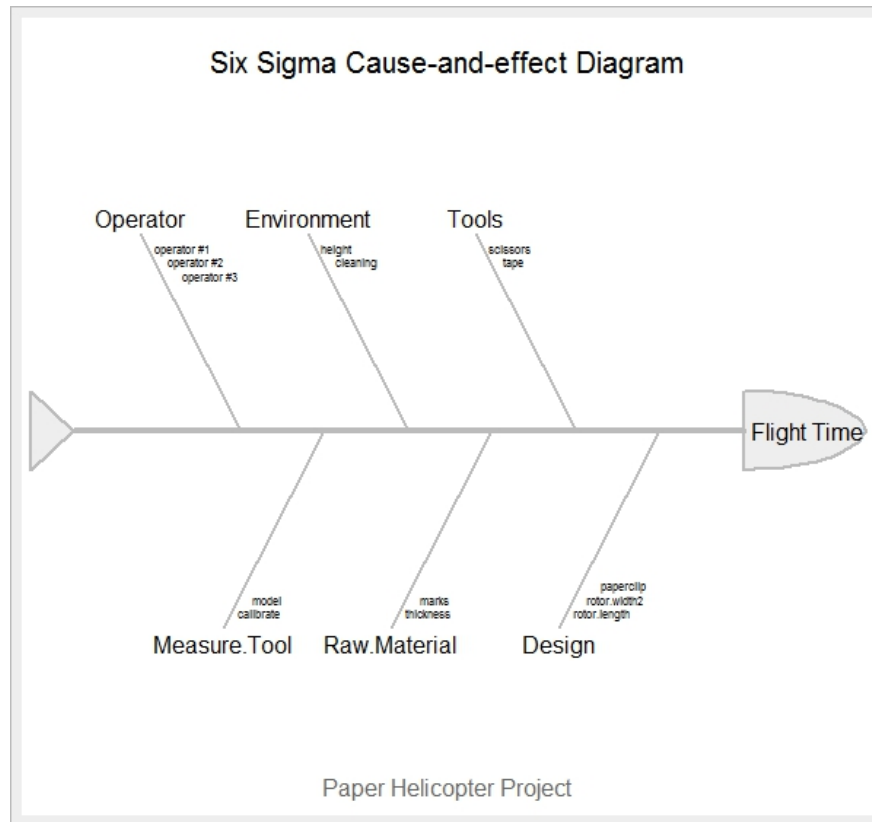


Figure 3.8:

```
x.parameters[1]<-list(c(list(c("width", "NC")),list(c("operator", "C")),
                        list(c("Measure pattern", "P"))), list(c("discard", "P"))))
x.parameters[2]<-list(c(list(c("operator", "C")),list(c("cut", "P")),
                        list(c("fix", "P")), list(c("rotor.width", "C")),
                        list(c("rotor.length",
                                list(c("paperclip", "C"))
                                list(c("discard", "P")),
                                list(c("environment", "N")))))
x.parameters[4]<-list(c(list(c("operator", "C")),
                        list(c("label", "P"))))
```

```
x.parameters
```

```
#Features of each process
y.features<-vector(mode="list",length=length(steps))
y.features[1]<-list(c(list(c("ok", "Cr"))))
y.features[2]<-list(c(list(c("weight", "Cr"))))
y.features[3]<-list(c(list(c("time", "Cr"))))
y.features[4]<-list(c(list(c("label", "Cr"))))
y.features
ss.pMap(steps, inputs.overall, outputs.overall,
        input.output, x.parameters, y.features,
        sub="Paper Helicopter Project")
```

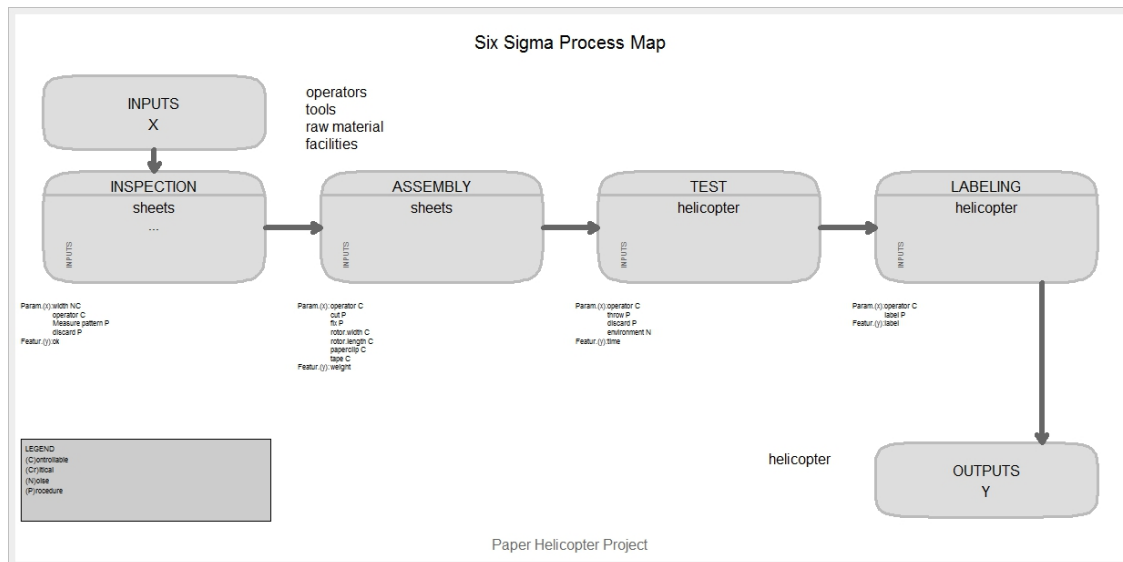


Figure 3.9:

3.4 The qcc R package - Other Types of Graph

3.4.1 Operating Characteristic (OC) Curves

- The OC Curve is used in sampling inspection. It plots the probability of accepting a batch of items against the quality level of the batch.
- The figure below shows an 'OC' (Operating Characteristic) Curve for a sample of 50 items taken from a batch of 2000 and using a critical acceptance number 'c' of 2 (the batch will be accepted if there are two or less defectives in the sample).
- From the curve you can see that there is about a 23% probability of accepting a batch that contains 8% of defective items.
- When designing a sampling plan it is usual to decide on two points, the AQL and LQL and the associated Producer's Risk and Consumer's Risk. The necessary sample size and acceptance number for the curve to pass through these points is then calculated and hence the shape of the curve.
- OC Curves are mainly associated with sampling inspection but they are also used to find the Average Run Length in control charts.
- A common supplementary plot to standard quality control charts is the so-called operating characteristic or OC curve (see example below). One question that comes to mind when using standard variable or attribute charts is how sensitive is the current quality control procedure? Put in more specific terms, how likely is it that you will not find a sample (e.g., mean in an X-bar chart) outside the control limits (i.e., accept the production process as "in control"), when, in fact, it has shifted by a certain amount?
- This probability is usually referred to as the (beta) error probability, that is, the probability of erroneously accepting a process (mean, mean proportion, mean rate defectives, etc.) as being "in control."
- Note that operating characteristic curves pertain to the false-acceptance probability using the sample-outside-of- control-limits criterion only, and not the runs tests described earlier.
- Operating characteristic curves are extremely useful for exploring the power of our quality control procedure. The actual decision concerning sample sizes should depend not only on the cost of implementing the plan (e.g., cost per item sampled), but also on the costs resulting from not detecting quality problems. The OC curve allows the engineer to estimate the probabilities of not detecting shifts of certain sizes in the production quality.

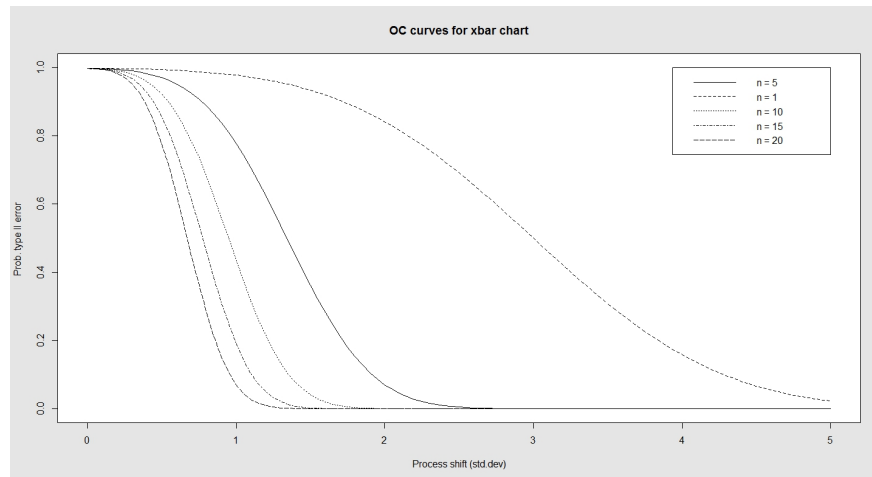


Figure 3.10:

pistonrings Data

```
data(pistonrings); attach(pistonrings);

diameter <- qcc.groups(diameter, sample)
beta <- oc.curves.xbar(qcc(diameter, type="xbar", nsigmas=3, plot=FALSE))
print(round(beta, digits=4))

# or to identify points on the plot use
## Not run: oc.curves.xbar(qcc(diameter,
    type="xbar", nsigmas=3, plot=FALSE), identify=TRUE)

detach(pistonrings)
```

3.4.2 Moving Average (MA) Chart

- To return to the piston ring example, suppose we are mostly interested in detecting small trends across successive sample means.
- For example, we may be particularly concerned about machine wear, leading to a slow but constant deterioration of quality (i.e., deviation from specification).
- Another way is to use some weighting scheme that summarizes the means of several successive samples; moving such a weighted mean across the samples will produce a moving average chart (as shown in the following graph).

GExponentially-weighted MA (EWMA) Chart

```
data(pistonrings)
attach(pistonrings)
diameter <- qcc.groups(diameter, sample)
q <- ewma(diameter[1:25,], lambda=0.2, nsigmas=3)
summary(q)

q <- ewma(diameter[1:25,], lambda=0.2, nsigmas=2.7,
newdata=diameter[26:40,], plot = FALSE)
summary(q)

plot(q)
```

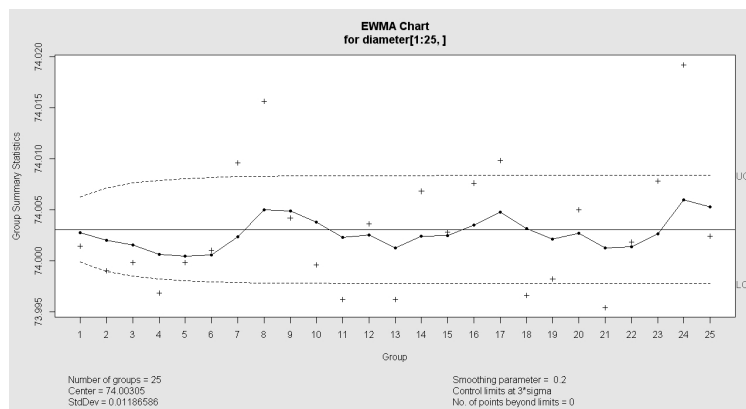


Figure 3.11:

```
> summary(q)
```

Call:

```
ewma(data = diameter[1:25, ], lambda = 0.2, nsigmas = 2.7,  
newdata = diameter[26:40, ], plot = FALSE)
```

ewma chart for diameter[1:25,]

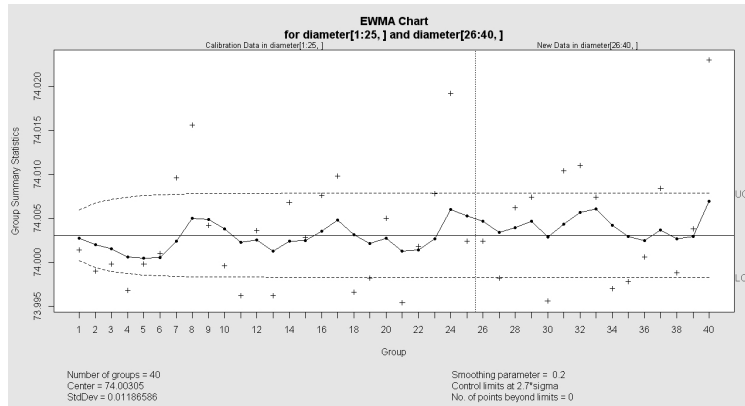


Figure 3.12:

Summary of group statistics:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
74.00	74.00	74.00	74.00	74.01	74.02

Group sample size: 5

Number of groups: 25

Center of group statistics: 74.00305

Standard deviation: 0.01186586

Summary of group statistics in diameter[26:40,]:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
74.00	74.00	74.00	74.00	74.01	74.02

Group sample size: 5

Number of groups: 15

Smoothing parameter: 0.2

Control limits:

	LCL	UCL
1	74.00018	74.00591
2	73.99938	74.00672
...		
40	73.99827	74.00782

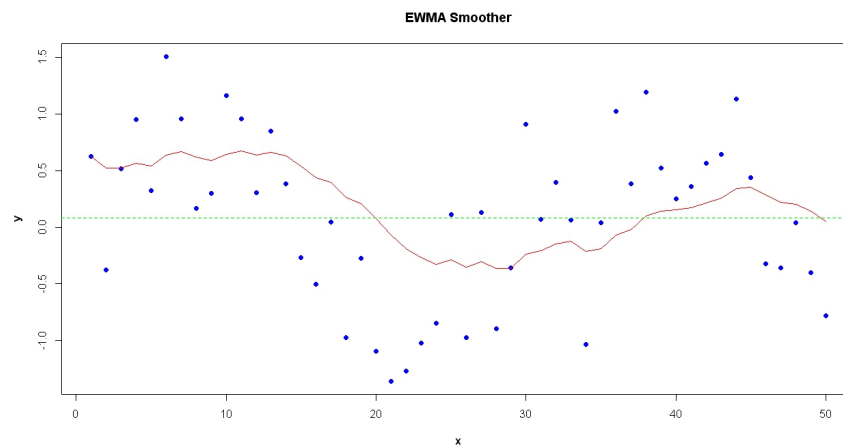


Figure 3.13:

EWMA : Individual observations

```
x <- c(33.75, 33.05, 34, 33.81, 33.46, 34.02, 33.68, 33.27, 33.49, 33.20,
33.62, 33.00, 33.54, 33.12, 33.84) # viscosity data (Montgomery, pag. 242)
q <- ewma(x, lambda=0.2, nsigmas=2.7)
summary(q)
```

```
x <- 1:50
y <- rnorm(50, sin(x/5), 0.5)
plot(x,y,pch=16,col="blue",font.lab=2)
lines(ewmaSmooth(x,y,lambda=0.1), col="red")
abline(h=mean(y),col="green",lty=2)
title("EWMA Smoother")
```

3.4.3 CUSUM charts

- CUSUM charts, while not as intuitive and simple to operate as Shewhart charts, have been shown to be more efficient in detecting small shifts in the mean of a process.
- In particular, analyzing ARL's for CUSUM control charts shows that they are better than Shewhart control charts when it is desired to detect shifts in the mean that are 2 sigma or less.

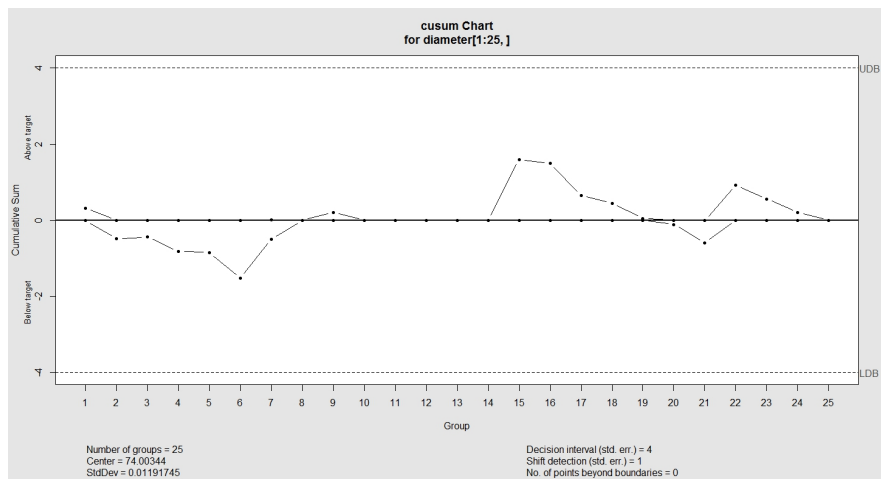


Figure 3.14:

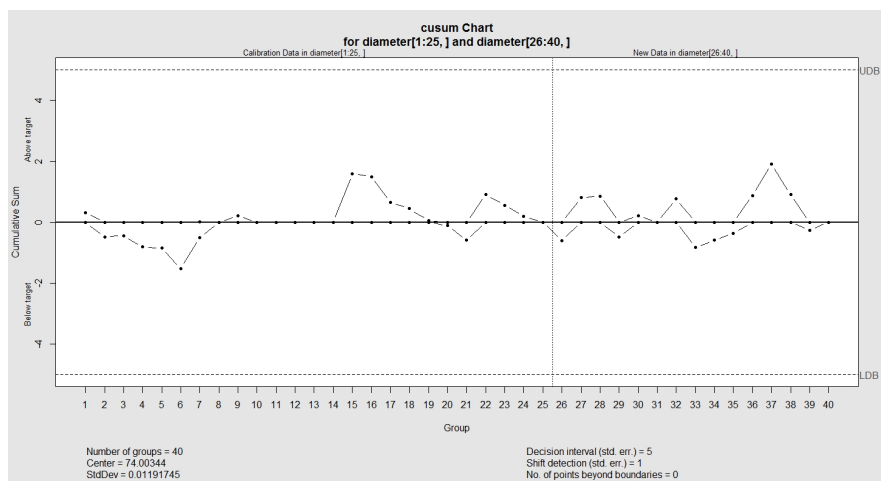


Figure 3.15:

3.4.4 Individual/Moving-Range chart

- The individual/moving-range chart is a type of control chart used to monitor variables data from a business or industrial process for which it is impractical to use rational subgroups.
- The chart is necessary in the following situations:
 - * Where automation allows inspection of each unit, so rational subgrouping has less benefit.
 - * Where production is slow so that waiting for enough samples to make a rational subgroup unacceptably delays monitoring
 - * For processes that produce homogeneous batches (e.g., chemical) where repeat measurements vary primarily because of measurement error
- The "chart" actually consists of a pair of charts: one, the individuals chart, displays the individual measured values; the other, the moving range chart, displays the difference from one point to the next.
- As with other control charts, these two charts enable the user to monitor a process for shifts in the process that alter the mean or variance of the measured statistic.

Chapter 4

Process Capability

4.1 Process Capability

Process capability is the measure of process performance. Capability refers to the ability of a process to make parts that are well within engineering specifications. A capability study is done to answer the questions, “*Does the process need to be improved?*” and “*How much does the process need to be improved?*”

To define the study of process capability from another perspective, a capability study is a technique for analyzing the random variability found in a production process. In every manufacturing process there is variability. This variability may be large or small, but it is always present. It can be divided into two types:

- Variability due to common (random) causes
- Variability due to assignable (special) causes

The first type of variability can be expected to occur naturally within a process. It is attributed to common causes that behave like a constant system of chances. These chances form a unique and describable distribution. This variability can never be completely eliminated from a process. Variability due to assignable causes, on the other hand, refers to the variation that can be linked to specific or special causes. If these causes, or factors, are modified or controlled properly, the process variability associated with them can be eliminated. Assignable causes cannot be described by a single distribution.

4.1.1 Capability Study

A capability study measures the performance potential of a process when no assignable causes are present (when it is in statistical control). Since the inherent variability of the process can be described by a unique distribution, usually a normal distribution, capability can be

evaluated by utilizing this distributions properties. Simply put, capability is expressed as the proportion of in-specification process output to total process input.

Capability calculations allow predictions to be made regarding quality, enabling manufacturers to take a preventive approach to defects. This statistical approach contrasts to the traditional approach to manufacturing, which is a two-step process: production personnel make the product, and quality control personnel inspect and eliminate those products that do not meet specifications. This is wasteful and expensive, since it allows time and materials to be invested in products that are not always usable. It is also unreliable, since even 100% inspection would fail to catch all defective products. Control Limits are Not an Indication of Capability

Those new to SPC often believe they dont need capability indices. They think they can compare the control limits to the specification limits instead. This is not true, because control limits look at the distribution of averages and capability indices look at the distribution of individuals. The distribution of individuals will always spread out further than the distribution of averages (see figure below).

4.1.2 What is Process Capability?

Distribution of averages compared to distribution of individuals, for the same sample data. Control limits (based on averages) would probably be inside specification limits, even though many parts are out of specification. This shows why you should not compare control limits to specification limits.

Therefore, the control limits are often within the specification limits, but the ± 3 Sigma distribution of parts is not. Subgroup averages follow more closely a normal distribution. This is why we can create control charts for processes that are not normally distributed. But averages cannot be used for capability calculations, because capability concerns itself with individual parts, or samples from a process. After all, parts, not averages, get shipped.

4.1.3 Capability Indices

Capability The uniformity of product which a process is capable of producing. Can be expressed numerically using CP, CR, CpK, and Zmax/3 when the data is normally distributed.

CP For process capability studies: CP is a capability index defined by the formula. CP shows the process capability potential but does not consider how centered the process is. CP may range in value from 0 to infinity, with a large value indicating greater potential capability. A value of 1.33 or greater is usually desired.

CR For process capability studies: the inverse of CP, CR can range from 0 to infinity in value, with a smaller value indicating a more capable process.

CpK For process capability studies: an index combining CP and K to indicate whether the process will produce units within the tolerance limits. CpK has a value equal to CP if the process is centered on the nominal; if CpK is negative, the process mean is outside the specification limits; if CpK is between 0 and 1, then some of the 6 sigma spread falls outside the tolerance limits. If CpK is larger than 1, the 6 sigma spread is completely within the tolerance limits. A value of 1.33 or greater is usually desired.

4.1.4 Interpreting Capability Indices

- The greater the CpK value, the better. A CpK greater than 1.0 means that the $6\sigma(\pm 3\sigma)$ spread of the data falls completely within the specification limits. A CpK of 1.0 means that one end of the 6σ spread falls on a specification limit. A CpK between 0 and 1 means that part of the 6σ spread falls outside the specification limits. A negative CpK indicates that the mean of the data is not between the specification limits.
- Since a CpK of 1.0 indicates that 99.73% of the parts produced are within specification limits, in this process it is likely that only about 3 out of 1,000 need to be scrapped or rejected. Why bother to improve the process beyond this point, since it will produce no reduction in scrap or reject costs? Improvement beyond just meeting specification may greatly improve product performance, cut warranty costs, or avoid assembly problems.
- Many companies are demanding CpK indexes of 1.33 or 2.0 of their suppliers products. A CpK of 1.33 means that the difference between the mean and specification limit is 4σ (since 1.33 is $4/3$). With a CpK of 1.33, 99.994% of the product is within specification. Similarly a CpK of 2.0 is 6σ between the mean and specification limit (since 2.0 is $6/3$).
- This improvement from 1.33 to 2.0 or better is sometimes justified to produce more product near the optimal target. Depending on the process or part, this may improve product performance, product life, customer satisfaction, or reduce warranty costs or assembly problems.
- Continually higher CpK indexes for every part or process is not the goal, since that is almost never economically justifiable. A cost/benefit analysis that includes customer satisfaction and other true costs of quality is recommended to determine which processes should be improved and how much improvement is economically attractive.

4.2 Process Capability

Process capability is the measure of process performance. Capability refers to the ability of a process to make parts that are well within engineering specifications. A capability study is done to answer the questions, “*Does the process need to be improved?*” and “*How much does the process need to be improved?*”

To define the study of process capability from another perspective, a capability study is a technique for analyzing the random variability found in a production process. In every manufacturing process there is variability. This variability may be large or small, but it is always present. It can be divided into two types:

- Variability due to common (random) causes
- Variability due to assignable (special) causes

The first type of variability can be expected to occur naturally within a process. It is attributed to common causes that behave like a constant system of chances. These chances form a unique and describable distribution. This variability can never be completely eliminated from a process. Variability due to assignable causes, on the other hand, refers to the variation that can be linked to specific or special causes. If these causes, or factors, are modified or controlled properly, the process variability associated with them can be eliminated. Assignable causes cannot be described by a single distribution.

4.2.1 Capability Study

- A capability study measures the performance potential of a process when no assignable causes are present (when it is in statistical control). Since the inherent variability of the process can be described by a unique distribution, usually a normal distribution, capability can be evaluated by utilizing this distributions properties.
- Simply put, capability is expressed as the proportion of in-specification process output to total process input.
- Capability calculations allow predictions to be made regarding quality, enabling manufacturers to take a preventive approach to defects. This statistical approach contrasts to the traditional approach to manufacturing, which is a two-step process: production personnel make the product, and quality control personnel inspect and eliminate those products that do not meet specifications.
- This is wasteful and expensive, since it allows time and materials to be invested in products that are not always usable. It is also unreliable, since even 100% inspection would fail to catch all defective products.
- Control Limits are Not an Indication of Capability
- Those new to SPC often believe they dont need capability indices. They think they can compare the control limits to the specification limits instead.
- This is not true, because control limits look at the distribution of averages and capability indices look at the distribution of individuals. The distribution of individuals will always spread out further than the distribution of averages.

4.2.2 What is Process Capability?

Distribution of averages compared to distribution of individuals, for the same sample data. Control limits (based on averages) would probably be inside specification limits, even though many parts are out of specification. This shows why you should not compare control limits to specification limits.

Therefore, the control limits are often within the specification limits, but the ± 3 Sigma distribution of parts is not. Subgroup averages follow more closely a normal distribution. This is why we can create control charts for processes that are not normally distributed. But averages cannot be used for capability calculations, because capability concerns itself with individual parts, or samples from a process. After all, parts, not averages, get shipped.

4.2.3 Capability Indices

Capability The uniformity of product which a process is capable of producing. Can be expressed numerically using CP, CR, CpK, and $Z_{\max}/3$ when the data is normally distributed.

CP For process capability studies: CP is a capability index defined by the formula. CP shows the process capability potential but does not consider how centered the process is. CP may range in value from 0 to infinity, with a large value indicating greater potential capability. A value of 1.33 or greater is usually desired.

CR For process capability studies: the inverse of CP, CR can range from 0 to infinity in value, with a smaller value indicating a more capable process.

CpK For process capability studies: an index combining CP and K to indicate whether the process will produce units within the tolerance limits. CpK has a value equal to CP if the process is centered on the nominal; if CpK is negative, the process mean is outside the specification limits; if CpK is between 0 and 1, then some of the 6 sigma spread falls outside the tolerance limits. If CpK is larger than 1, the 6 sigma spread is completely within the tolerance limits. A value of 1.33 or greater is usually desired.

4.2.4 Interpreting Capability Indices

- The greater the CpK value, the better. A CpK greater than 1.0 means that the $6\sigma(\pm 3\sigma)$ spread of the data falls completely within the specification limits. A CpK of 1.0 means that one end of the 6σ spread falls on a specification limit. A CpK between 0 and 1 means that part of the 6σ spread falls outside the specification limits. A negative CpK indicates that the mean of the data is not between the specification limits.
- Since a CpK of 1.0 indicates that 99.73% of the parts produced are within specification limits, in this process it is likely that only about 3 out of 1,000 need to be scrapped or rejected. Why bother to improve the process beyond this point, since it will produce no reduction in scrap or reject costs? Improvement beyond just meeting specification may greatly improve product performance, cut warranty costs, or avoid assembly problems.
- Many companies are demanding CpK indexes of 1.33 or 2.0 of their suppliers products. A CpK of 1.33 means that the difference between the mean and specification limit is 4σ (since 1.33 is $4/3$). With a CpK of 1.33, 99.994% of the product is within specification. Similarly a CpK of 2.0 is 6σ between the mean and specification limit (since 2.0 is $6/3$).
- This improvement from 1.33 to 2.0 or better is sometimes justified to produce more product near the optimal target. Depending on the process or part, this may improve product performance, product life, customer satisfaction, or reduce warranty costs or assembly problems.
- Continually higher CpK indexes for every part or process is not the goal, since that is almost never economically justifiable. A cost/benefit analysis that includes customer satisfaction and other true costs of quality is recommended to determine which processes should be improved and how much improvement is economically attractive.

4.2.5 Process Capability Analysis

- Process capability compares the output of an in-control process to the specification limits by using capability indices.
- The comparison is made by forming the ratio of the spread between the process specifications (the specification "width") to the spread of the process values, as measured by 6 process standard deviation units (the process "width").

4.2.6 Interpreting Process Capability Indices

- **CP**

Historically, this is one of the first capability indexes used. The "natural tolerance" of the process is computed as $6s$. The index simply makes a direct comparison of the process natural tolerance to the engineering requirements. Assuming the process distribution is normal and the process average is exactly centered between the engineering requirements, a CP index of 1 would give a "capable process." However, to allow a bit of room for process drift, the generally accepted minimum value for CP is 1.33. In general, the larger CP is, the better. The CP index has two major shortcomings. First, it cannot be used unless there are both upper and lower specifications. Second, it does not account for process centering. If the process average is not exactly centered relative to the engineering requirements, the CP index will give misleading results. In recent years, the CP index has largely been replaced by CPK (see below).

- **CPM**

A CPM of at least 1 is required, and 1.33 is preferred. CPM is closely related to CP. The difference represents the potential gain to be obtained by moving the process mean closer to the target. Unlike CPK, the target need not be the center of the specification range.

4.3 Process Capability Analysis

Process Capability Indices for a characteristic of interest from a continuous process can be obtained using the command `process.capability`.

Lower and Upper Specification Limits must be specified.

$$c_p = \frac{USL - LSL}{6 \times s} = \frac{12}{6 \times 1.956} = 1.02$$
$$c_{pk}(\text{upper}) = \frac{USL - \bar{x}}{3 \times s} = \frac{506 - 500.38}{3 \times 1.956} = 0.96$$
$$c_{pk}(\text{lower}) = \frac{\bar{x} - USL}{3 \times s} = \frac{500.38 - 494}{3 \times 1.956} = 1.04$$

4.4 Worked Examples

4.4.1 MA6001 Autumn 2007/2008 Question 4

Over several weeks of normal, or in-control, operation, 20 samples of 150 packages each of synthetic-gut tennis strings were tested for breaking strength. A total of 141 packages of the 3000 tested failed to conform to the manufacturer's specifications.

- (i.) What is an estimate of the process proportion defective when the system is in control?
- (ii.) Compute the upper and lower control limits for an np chart.
- (iii.) Using the results of part (ii) above, what conclusion should be made about the process if tests on a new sample of 150 packages find 12 defective?

A quality control process monitors the weight per carton of laundry detergent. Control limits for the mean are set at $UCL = 20.12$ ounces and $LCL = 19.90$ ounces. Samples of size 5 are used for the sampling and inspection process.

- (i.) What are the process mean and process standard deviation for the manufacturing operation?
- (ii.) If the process mean shifts to 20.25 ounces, what is the probability of a type II error?

For a single sampling plan 50(0, 1) what is the probability of accepting the batch when

- (i.) $p = 0.01$
- (ii.) $p = 0.02$

4.4.2 MA6001 Autumn 2008/2009 Question 4

Lear Seating of Kitchener, Ontario, manufactures seats for Chrysler, Ford, and General Motors cars. Several years ago, Lear instituted statistical process control, which has resulted in improved quality and lower costs. One of the components of a front-seat cushion is a wire spring, produced from 4-mm steel wire. A machine is employed to bend the wire so that the spring's length is 500mm. If the springs are longer than 500mm, they will loosen and eventually fall out. If they are too short, they won't easily fit into position. (In fact, in the past, when there was a relatively large number of short springs, workers incurred arm and hand injuries when attempting to install the springs.)

To determine if the process is under control, random samples of four springs are taken every two hours. The results of 25 samples taken over the last week gave the following overall results:

- (i.) Calculate the centreline and control limits for the $\bar{\bar{x}}$ and s charts.
- (ii.) Assuming the process is in control and given that the tolerances are set at $500 \pm 6\text{mm}$, calculate capability indices and comment.
- (iii.) Calculate the probability of a type II error if the process mean shifts to 501mm.
item[(iv.)] What is the average run length (ARL) for the situation outlined at (iii) above?

Chapter 5

SixSigma

5.1 Six Sigma - Background to Six Sigma

- Motorola, one of the worlds leading manufacturers and suppliers of semiconductors and electronic equipment systems for civil and military applications, introduced the concept of six-sigma process quality to enhance the reliability and quality of their products, and cut product cycle times and expenditure on test/repair. Motorola used the following statement to explain:
- Sigma is a statistical unit of measurement that describes the distribution about the mean of any process or procedure. A process or procedure that can achieve plus or minus six-sigma capability can be expected to have a defect rate of no more than a few parts per million, even allowing for some shift in the mean. In statistical terms, this approaches zero defects.
- The approach was championed by Motorolas chief executive officer at the time, Bob Galvin, to help improve competitiveness. The six-sigma approach became widely publicized when Motorola won the US Baldrige National Quality Award in 1988.
- Six-sigma is a disciplined approach for improving performance by focusing on producing better products and services faster and cheaper. The emphasis is on improving the capability of processes through rigorous data gathering, analysis and action, and:
 - enhancing value for the customer;
 - eliminating costs which add no value (waste).

Unlike simple cost-cutting programmes six-sigma delivers cost savings whilst retaining or even improving value to the customers.

5.1.1 Why six-sigma?

In a process in which the characteristic of interest is a variable, defects are usually defined as the values which fall outside the specification limits (LSLUSL). Assuming and using a normal distribution of the variable, the percentage and/or parts per million defects can be found.

For example, in a centred process with a specification set at $x \pm 3\sigma$ there will be 0.27 per cent or 2700 ppm defects. This may be referred to as an unshifted $\pm 3\sigma$ process and the quality called 3 sigma quality. In an unshifted 6 sigma process, the specification range is 6σ and it produces only 0.002 ppm defects.

It is difficult in the real world, however, to control a process so that the mean is always set at the nominal target value in the centre of the specification. Some shift in the process mean is expected.

5.2 Six Sigma R Package



5.2.1 DMAIC Process

- DMAIC (an abbreviation for Define, Measure, Analyze, Improve and Control) refers to a data-driven improvement cycle used for improving, optimizing and stabilizing business processes and designs.
- The DMAIC improvement cycle is the core tool used to drive Six Sigma projects.
- However, DMAIC is not exclusive to Six Sigma and can be used as the framework for other improvement applications.

5.2.2 Data Set - `ss.data.ca`

This data set is the volume measured in 20 bottles for a filling process in a winery

```
ss.data.ca
```

```
> ss.data.ca
  Volume
1  755.81
2  750.54
```

.....
.....
19 750.26
20 751.29

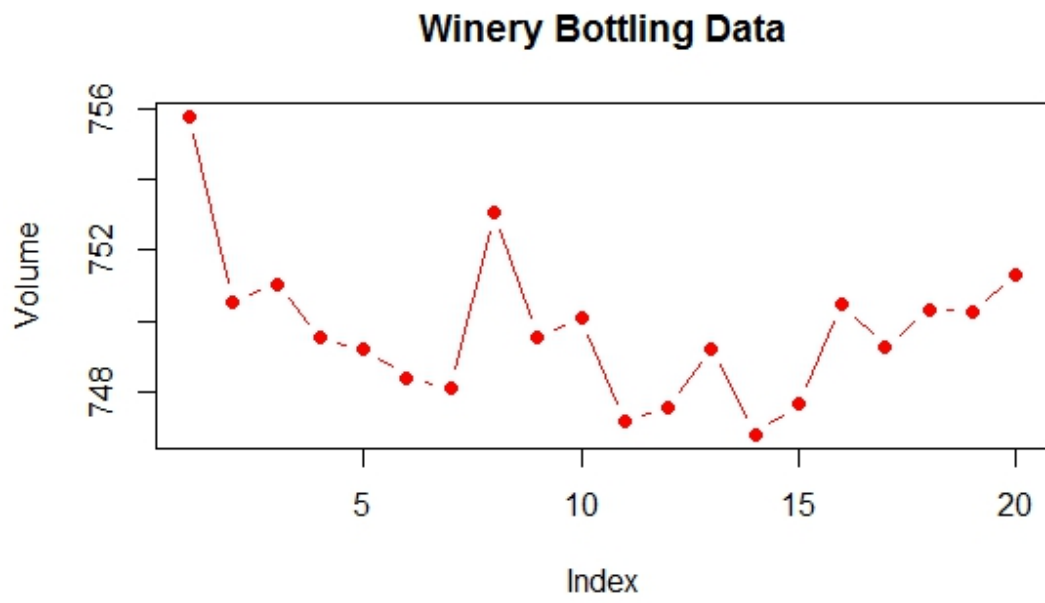


Figure 5.1:

5.2.3 ss.ca.z Capability Indices

Compute the Capability Indices of a process, Z (Sigma Score), C_p and C_{pk} .

```
> ss.ca.cp(ss.data.ca$Volume,740, 760)
[1] 1.584136
> ss.ca.cpk(ss.data.ca$Volume,740, 760)
[1] 1.546513
> ss.ca.z(ss.data.ca$Volume,740,760)
[1] 3.139539
```

5.2.4 Loss Function Analysis

The Taguchi Loss Function is graphical depiction of loss developed by the Japanese business statistician Genichi Taguchi to describe a phenomenon affecting the value of products produced by a company. Praised by Dr. W. Edwards Deming (the business guru of the 1980s American quality movement),[1] it made clear the concept that quality does not suddenly plummet when, for instance, a machinist exceeds a rigid blueprint tolerance. Instead "loss" in value progressively increases as variation increases from the intended condition. This was considered a breakthrough in describing quality, and helped fuel the continuous improvement movement that since has become known as lean manufacturing.

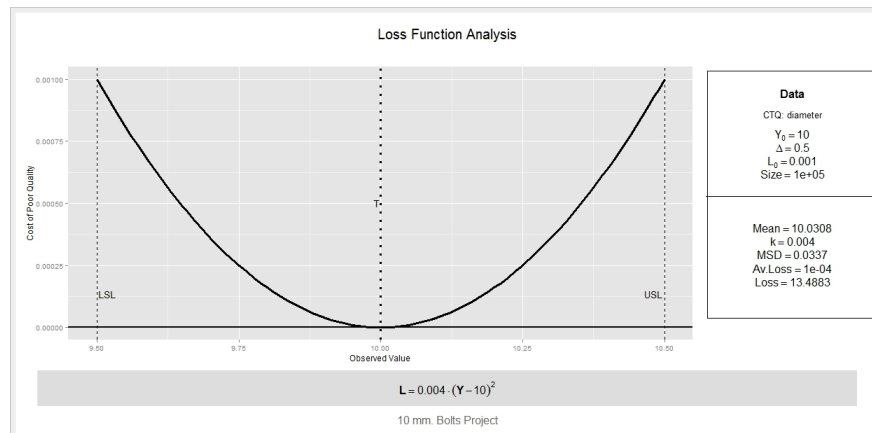


Figure 5.2:

5.3 Other R Packages

1. mpci
2. mspc
3. qualityTools
4. spc
5. SixSigma with R (Emilio Cano)

5.3.1 spc: Statistical Process Control

- Evaluation of control charts by means of the zero-state, steady-state ARL (Average Run Length) and RL quantiles.
- Setting up control charts for given in-control ARL.
- The control charts under consideration are one- and two-sided EWMA, CUSUM, and Shiryaev-Roberts schemes for monitoring the mean of normally distributed independent data.
- ARL calculation of the same set of schemes under drift are added.
- Other charts and parameters are in preparation.
- Further SPC areas will be covered as well (sampling plans, capability indices ...)

R package spc provides

```
xcusum.ad steady-state ARLs of CUSUM charts
xcusum.arl (zero-state) ARLs of CUSUM charts
xcusum.crit decision intervals of CUSUM charts
xewma.ad steady-state ARLs of EWMA charts
xewma.arl (zero-state) ARLs of EWMA charts
xewma.crit critical values of EWMA charts
```