0.1 Definition of Statistical Process Control

Application of statistical methods and procedures (such as control charts) to analyze the inherentvariability of a process or its outputs to achieve and maintain a state of statistical control, and to improve the process capability.

1 Deming's Lectures (source unknown)

This article attempts to clarify the role played by W. Edwards Deming at the beginning of the modern Japanese quality control movement by summarizing and analyzing the actual content of the series of quality control lectures he gave in Japan during the summer of 1950. The primary source documents are the published lecture transcripts that Deming considered authentic.Analysis of the transcripts shows that Deming spent most of the eight-day lecture series discussing statistical process control.

However, he opened the lectures with extended remarks that contain a core of the philosophy for which he later became famous. Yet, significant elements of what is now known as the Deming method or Deming philosophy did not appear in the lecture series. Deming included in the lectures an extended discussion of sampling inspection that revealed his ambivalence to the subject. The transcripts show that Deming introduced to the Japanese a product design cycle of Shewhart that is distinct from the management process that the Japanese later came to call the plan-do-check-act cycle.

1.1 Control Charts

During the Measure phase, one of the first things the Back Belt wants to do is to determine whether the process is in control with respect to the major 'Y'. The primary tool for this is a control chart. In many cases, the process may already keep control charts; many do. But there are large number of way in which control charts are produced, and a great many pitfalls, so the Black Belt would be well advised to examine the procedures used for the control chart and ensure they are appropriate for his purposes.

The simplest control chart consists of a simple plot of the observed variable versus time, with the control limits marked on the chart, and sometimes, the specification limit.

The control limits are typically set at +/- three standard deviations. It is important to remember that the control limits should not be recalculated each time the control

chart is redrawn. Rather, they should be set once, and then changed because of a change in the process.

2 Attribute Control Charts

- The Shewhart control chart plots quality characteristics that can be measured and expressed numerically. We measure weight, height, position, thickness, etc. If we cannot represent a particular quality characteristic numerically, or if it is impractical to do so, we then often resort to using a quality characteristic to sort or classify an item that is inspected into one of two "buckets".
- An example of a common quality characteristic classification would be designating units as "conforming units" or "nonconforming units".
- Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called ***attributes***.
- *Note that there is a difference between "nonconforming to an engineering specification" and "defective" – a nonconforming unit may function just fine and be, in fact, not defective at all, while a part can be "in spec" and not fucntion as desired (i.e., be defective).*
- Examples of quality characteristics that are attributes are the number of failures in a production run, the proportion of malfunctioning wafers in a lot, the number of people eating in the cafeteria on a given day, etc.

2.1 Types of Attributes Control Charts

- Control charts dealing with the proportion or fraction of defective product are called ***p-charts*** (for proportion).
- Control charts dealing with the number of defective product are called ***np-charts***.
- Control charts dealing with the number of defects or nonconformities are called ***c-charts*** (for count).
- There is another chart which handles defects per unit, called the ***u-chart*** (for unit). This applies when we wish to work with the average number of nonconformities per unit of product.

2.2 p-charts

- The p-chart is a type of control chart used to monitor the **proportion of nonconforming units** in a sample, where the sample proportion nonconforming is defined as the ratio of the number of nonconforming units to the sample size, n.
- The p-chart only accommodates dichotomous PASS/FAIL-type inspection as determined by a series of tests, effectively applying the specifications to the data before they are plotted on the chart.
- Other types of control charts display the magnitude of the quality characteristic under study, making troubleshooting possible directly from those charts.
- A p-chart is an attributes control chart used with data collected in subgroups of varying sizes. Because the subgroup size can vary, it shows a proportion on nonconforming items rather than the actual count.
- *p-charts show how the process changes over time. The process attribute (or characteristic) is always described in a yes/no, pass/fail, go/no go form.*
- Example: use a p-chart to plot the proportion of incomplete insurance claim forms received weekly. The subgroup would vary, depending on the total number of claims each week.

2.3 np-charts

The np-chart is a type of control chart used to monitor the number of nonconforming units in a sample. An np-chart is an *attributes* control chart used with data collected in subgroups that are the **same size**.

It is an adaptation of the p-chart and used in situations where personnel find it easier to interpret process performance in terms of concrete numbers of units rather than the somewhat more abstract proportion.

The np-chart differs from the p-chart in only the three following aspects:

- The control limits are
$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$
, where n is the sample size and $\bar{p}$ is the estimate of the long-term process mean established during control-chart setup.
- The number nonconforming (np), rather than the fraction nonconforming (p), is plotted against the control limits.
- The sample size, n, is constant.

2.4 The c-chart

- In this chart, we plot the number of defectives (per batch, per day, per machine, per 100 feet of pipe, etc.).
- This chart assumes that defects of the quality attribute are rare, and the control limits in this chart are computed based on the Poisson distribution (distribution of rare events).
- The c-chart is a type of control chart used to monitor "count"-type data, typically total number of nonconformities per unit.It is also occasionally used to monitor the total number of events occurring in a given unit of time.
- The c-chart differs from the p-chart in that it accounts for the possibility of more than one nonconformity per inspection unit, and that (unlike the p-chart and u-chart) it requires a fixed sample size.
- The p-chart models "pass"/"fail"-type inspection only, while the c-chart (and u-chart) give the ability to distinguish between (for example) 2 items which fail inspection because of one fault each and the same two items failing inspection with 5 faults each; in the former case, the p-chart will show two non-conformant items, while the c-chart will show 10 faults.
- The Poisson distribution is the basis for the chart and requires the following assumptions:
    - The number of opportunities or potential locations for nonconformities is very large
    - The probability of nonconformity at any location is small and constant
    - The inspection procedure is same for each sample and is carried out consistently from sample to sample

2.5 The u-chart

- In this chart we plot the rate of defectives, that is, the number of defectives divided by the number of units inspected (the n; e.g., feet of pipe, number of batches).
- Unlike the C chart, this chart does not require a constant number of units, and it can be used, for example, when the batches (samples) are of different sizes.
- The u-chart is a type of control chart used to monitor "count"-type data where the sample size is greater than one, typically the average number of nonconformities per unit.
- The u-chart differs from the c-chart in that it accounts for the possibility that the number or size of inspection units for which nonconformities are to be counted may vary. Larger samples may be an economic necessity or may be necessary to increase the area of opportunity in order to track very low nonconformity levels.

3 Multivariate Control Charts

- With the enhancements in data acquisition systems it is usual to deal with processes with more than one correlated quality characteristic to be monitored.
- A common practice is to control the stability of the process using univariate control charts.
- This practice increases the probability of false alarm of special cause of variation.
- Therefore, the analysis should be performed through a multivariate approach; that is, the variables must be analyzed together, not independently.

3.1 Multivariate Control Charts

- Multivariate control charts monitor multiple process characteristics. Independent variables can be charted individually, but if the variables are correlated, a multivariate chart is needed to determine whether the process is in control.
- Multivariate control charts can detect shifts in the mean or the relationship between several related variables.
- The multivariate control chart plots Hotelling's T2 statistic. The calculation for the control limit differs based on whether targets have been specified.

3.2 The MSQC package

In his book, Edgar Santos-Fernandez present the multivariate normal distribution, the data structure of the multivariate problems dealt in this book, the mult.chart function that allows the computation in R, and the most used multivariate control charts:

- The control ellipsoid or w2 control chart
- The T2 or Hotelling chart
- The Multivariate Exponentially Weighted Moving Average (MEWMA) chart
- The Multivariate Cumulative Sum (MCUSUM) chart
- The chart based on Principal Components Analysis (PCA)

3.3 The `mult.chart` Function

The performing of the multivariate control chart in R can be carried out with the function mult.chart which is a general function that allows to compute the most accepted and diversified continuous multivariate chart such as

- χ^2
- Hotelling T^2
- MEWMA
- MCUSUM according to Crosier (1988)
- MCUSUM by Pignatiello and Runger (1990)

Finally the function `mult.chart` returns:

- The T2 statistics
- The Upper Control Limit (UCL)
- The sample covariance matrix (S)
- The mean vector (Xmv)
- And if any point falls outside of the UCL and its decomposition

```
mult.chart(dowel1, type = "chi", alpha = 0.05)
```

3.4 T2 control chart

The origin of the T2 control chart dates back to the pioneer works of Harold Hotelling who applied this method to the bombsight problem in Second World War. The Hotelling (1947) procedure has become without doubt the most applied in multivariate process control and it is the multivariate analogous of the Shewhart control chart. For that reason, it is also known as multivariate Shewhart control chart.

```
data("carbon1")
mult.chart(type = "t2", carbon1)
mult.chart(type = "t2", carbon1)$t2
```

3.5 mqcc Example

```
# library(mqcc)
# Ryan (2000, Table 9.2) data with p = 2 variables,
#  m = 20 samples, n = 4 sample size:

X1 = matrix(c(72, 56, 55, 44, 97, 83, 47, 88, 57, 26, 46,
49, 71, 71, 67, 55, 49, 72, 61, 35, 84, 87, 73, 80, 26, 89, 66,
50, 47, 39, 27, 62, 63, 58, 69, 63, 51, 80, 74, 38, 79, 33, 22,
54, 48, 91, 53, 84, 41, 52, 63, 78, 82, 69, 70, 72, 55, 61, 62,
41, 49, 42, 60, 74, 58, 62, 58, 69, 46, 48, 34, 87, 55, 70, 94,
49, 76, 59, 57, 46), ncol = 4)

X2 = matrix(c(23, 14, 13, 9, 36, 30, 12, 31, 14, 7, 10,
11, 22, 21, 18, 15, 13, 22, 19, 10, 30, 31, 22, 28, 10, 35, 18,
11, 10, 11, 8, 20, 16, 19, 19, 16, 14, 28, 20, 11, 28, 8, 6,
15, 14, 36, 14, 30, 8, 35, 19, 27, 31, 17, 18, 20, 16, 18, 16,
13, 10, 9, 16, 25, 15, 18, 16, 19, 10, 30, 9, 31, 15, 20, 35,
12, 26, 17, 14, 16), ncol = 4)

X = list(X1 = X1, X2 = X2)
q = mqcc(X, type = "T2")
summary(q)
```

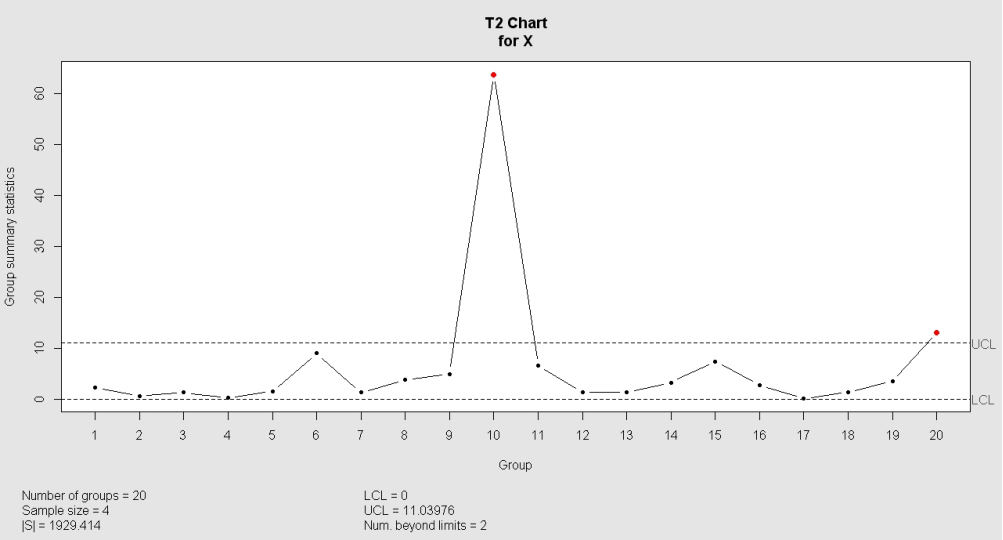

Figure 1

```
Call:
mqcc(data = X, type = "T2")

T2 chart for X

Summary of group statistics:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.1243  1.3250  2.5030  6.4700  5.3490 63.7600

Number of variables:  2
Number of groups:  20
Group sample size:  4

Center:
     X1      X2
60.3750 18.4875

Covariance matrix:
         X1        X2
X1 222.0333 103.11667
X2 103.1167  56.57917
|S|:  1929.414

Control limits:
```

```
LCL      UCL
  0 11.03976
```

4 Process Capability

Process capability is the measure of process performance. Capability refers to the ability of a process to make parts that are well within engineering specifications. A capability study is done to answer the questions, *"Does the process need to be improved?"* and *"How much does the process need to be improved?"*

To define the study of process capability from another perspective, a capability study is a technique for analyzing the random variability found in a production process. In every manufacturing process there is variability. This variability may be large or small, but it is always present. It can be divided into two types:

- Variability due to common (random) causes
- Variability due to assignable (special) causes

4.1 Types of Variability

The first type of variability can be expected to occur naturally within a process. It is attributed to common causes that behave like a constant system of chances. These chances form a unique and describable distribution. This variability can never be completely eliminated from a process. Variability due to assignable causes, on the other hand, refers to the variation that can be linked to specific or special causes. If these causes, or factors, are modified or controlled properly, the process variability associated with them can be eliminated. Assignable causes cannot be described by a single distribution.

4.2 Capability Study

- A capability study measures the performance potential of a process when no assignable causes are present (when it is in statistical control). Since the inherent variability of the process can be described by a unique distribution, usually a normal distribution, capability can be evaluated by utilizing this distribution's properties.
- Simply put, capability is expressed as the proportion of in-specification process output to total process input.
- Capability calculations allow predictions to be made regarding quality, enabling manufacturers to take a preventive approach to defects. This statistical approach contrasts to the traditional approach to manufacturing, which is a two-step process: production personnel make the product, and quality control personnel inspect and eliminate those products that do not meet specifications.
- This is wasteful and expensive, since it allows time and materials to be invested in products that are not always usable. It is also unreliable, since even 100% inspection would fail to catch all defective products.
- Control Limits are Not an Indication of Capability
- Those new to SPC often believe they don't need capability indices. They think they can compare the control limits to the specification limits instead.
- This is not true, because control limits look at the distribution of averages and capability indices look at the distribution of individuals. The distribution of individuals will always spread out further than the distribution of averages.

4.3 What is Process Capability?

Distribution of averages compared to distribution of individuals, for the same sample data. Control limits (based on averages) would probably be inside specification limits, even though many parts are out of specification. This shows why you should not compare control limits to specification limits.

Therefore, the control limits are often within the specification limits, but the ± 3 Sigma distribution of parts is not. Subgroup averages follow more closely a normal distribution. This is why we can create control charts for processes that are not normally distributed. But averages cannot be used for capability calculations, because capability concerns itself with individual parts, or samples from a process. After all, parts, not averages, get shipped.

4.4 Capability Indices

Capability — The uniformity of product which a process is capable of producing. Can be expressed numerically using CP, CR, CpK, and Zmax/3 when the data is normally distributed.

CP — For process capability studies: CP is a capability index defined by the formula. CP shows the process capability potential but does not consider how centered the process is. CP may range in value from 0 to infinity, with a large value indicating greater potential capability. A value of 1.33 or greater is usually desired.

CR — For process capability studies: the inverse of CP, CR can range from 0 to infinity in value, with a smaller value indicating a more capable process.

CpK — For process capability studies: an index combining CP and K to indicate whether the process will produce units within the tolerance limits. CpK has a value equal to CP if the process is centered on the nominal; if CpK is negative, the process mean is outside the specification limits; if CpK is between 0 and 1, then some of the 6 sigma spread falls outside the tolerance limits. If CpK is larger than 1, the 6 sigma spread is completely within the tolerance limits. A value of 1.33 or greater is usually desired.

4.5 Interpreting Capability Indices

- The greater the CpK value, the better. A CpK greater than 1.0 means that the $6\sigma(\pm 3\sigma)$ spread of the data falls completely within the specification limits. A CpK of 1.0 means that one end of the 6σ spread falls on a specification limit. A CpK between 0 and 1 means that part of the 6σ spread falls outside the specification limits. A negative CpK indicates that the mean of the data is not between the specification limits.

- Since a CpK of 1.0 indicates that 99.73% of the parts produced are within specification limits, in this process it is likely that only about 3 out of 1,000 need to be scrapped or rejected. Why bother to improve the process beyond this point, since it will produce no reduction in scrap or reject costs? Improvement beyond just meeting specification may greatly improve product performance, cut warranty costs, or avoid assembly problems.

- Many companies are demanding CpK indexes of 1.33 or 2.0 of their suppliers' products. A CpK of 1.33 means that the difference between the mean and specification limit is 4σ (since 1.33 is 4/3). With a CpK of 1.33, 99.994% of the product is within specification. Similarly a CpK of 2.0 is 6σ between the mean and specification limit (since 2.0 is 6/3).

- This improvement from 1.33 to 2.0 or better is sometimes justified to produce more product near the optimal target. Depending on the process or part, this may improve product performance, product life, customer satisfaction, or reduce warranty costs or assembly problems.

- Continually higher CpK indexes for every part or process is not the goal, since that is almost never economically justifiable. A cost/benefit analysis that includes customer satisfaction and other true costs of quality is recommended to determine which processes should be improved and how much improvement is economically attractive.

4.6 Process Capability Analysis

- Process capability compares the output of an in-control process to the specification limits by using capability indices.

- The comparison is made by forming the ratio of the spread between the process specifications (the specification "width") to the spread of the process values, as measured by 6 process standard deviation units (the process "width").

- **CP**
  Historically, this is one of the first capability indexes used. The "natural tolerance" of the process is computed as 6s . The index simply makes a direct comparison of the process natural tolerance to the engineering requirements. Assuming the process distribution is normal and the process average is exactly centered between the engineering requirements, a CP index of 1 would give a "capable process." However, to allow a bit of room for process drift, the generally accepted minimum value for CP is 1.33. In general, the larger CP is, the better. The CP index has two major shortcomings. First, it cannot be used unless there are both upper and lower specifications. Second, it does not account for process centering. If the process average is not exactly centered relative to the engineering requirements, the CP index will give misleading results. In recent years, the CP index has largely been replaced by CPK (see below).

- **CPM**
  A CPM of at least 1 is required, and 1.33 is preferred. CPM is closely related to CP. The difference represents the potential gain to be obtained by moving the process mean closer to the target. Unlike CPK, the target need not be the center of the specification range.