1. A certain process is observed and recorded daily.

    - How improbable is a point outside the 3σ limits when the process is in control?
    - How often will such a point occur when observed daily (find expected frequency in terms of days)?
    - How improbable is a point outside the 2σ limits when the process is in control?
    - How often will such a point occur?

    Solution: *We assume that when the process is under control, then it follows the normal models for which chances to go further than 3σ from the mean are 0.2699796% (this precise number was found by R-software but you could find approximately this value from the tables using value $z = 3$), it is less than 3 out of 1000 days when we should observe such a point on average. Deviating 2σ is more probable, namely, 4.550026% (again you can use tables to find this value). This mean that there should be on average less than 46 days out of 1000 that you would see values outside two sigma "belt".*

2. Suppose that for the above process $\bar{X}$ and R control charts have been created based on the subsample means of weekly observation (thus the subsample size is $n = 7$).

    - Using the AIAG chart that is presented in Figure 4.6 of the textbook, identify the values of A_2, D_3 and D_4 for this process (see also our lecture slides where these values have been presented).

    - After analysing twelve month data it has been found out that the process has been in control for the entire year and the average value of the range was 6.3 while the mean value was 78. Find the control limits for the $\bar{X}$ and R charts and present them on a graph for a weekly control chart.

    - From the obtained data calculate the standard deviation for this process.

    - Using the obtained values compute the percentage of the items that will not fall within the $\bar{X}$ control belt if the process is in control.

Solution for Part 1

- *It is easy to find out from the information at the bottom-right corner of the chart that for the sample size* $n = 7$ *the values are* $A_2 = .42$, $D_3 = .08$, $D_4 = 1.92$.

Solution for Part 2

- *The central line for* $\bar{X}$ *chart should be at the overall mean, i.e. at* 78 *and the upper and lower control limits are calculated as follows:* $UCL = 78+0.42*6.3 = 80.646$, $LCL = 78 - 0.42 * 6.3 = 75.354$, *respectively. Similarly, for the* R *chart, we get the center at* 6.3 *and the upper and lower control limits are* $UCL = 0.08 * 6.3 = 0.504$, $LCL = 1.92 * 6.3 = 12.096$, *respectively. The graphs are presented on the following figures*

Full Workings

- The control limits for the $\bar{X}$ control chart are given by the following equations:

$LCL_{\bar{X}} = \bar{X} - A_2\bar{R}$ $\qquad$ $UCL_{\bar{X}} = \bar{X} + A_2\bar{R}$

$\bar{X}$ = mean
$\bar{R}$ = average range

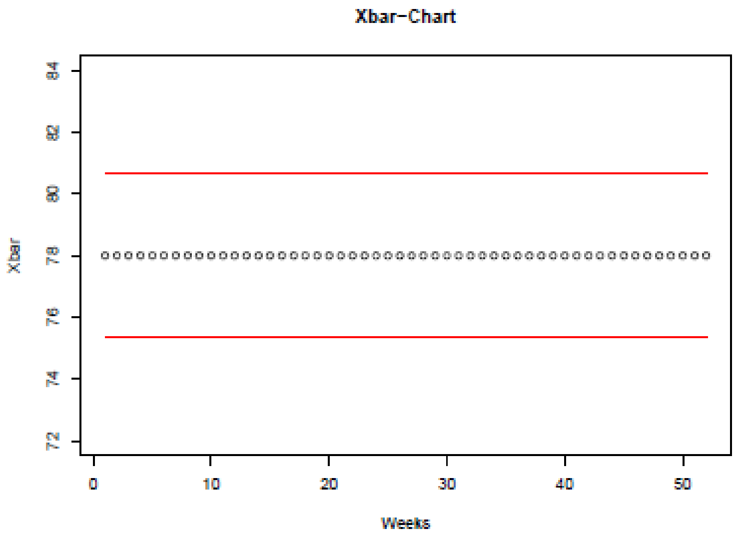
Since we are told the mean is 78 and the average range is 6.3, and since we know the value of A_2 is 0.42 for a subsample size of 7, the controls limits for the $\bar{X}$ control chart will be:

$LCL_{\bar{X}} = 78 - (0.42)(6.3)$ $\qquad$ $UCL_{\bar{X}} = 78 + (0.42)(6.3)$

$LCL_{\bar{X}} = 78 - (0.42)(6.3)$ $\qquad$ $UCL_{\bar{X}} = 78 + (0.42)(6.2)$

$LCL_{\bar{X}} = 78 - 2.646$ $\qquad$ $UCL_{\bar{X}} = 78 + 2.646$

$LCL_{\bar{X}} = 75.354$ $\qquad$ $UCL_{\bar{X}} = 80.646$



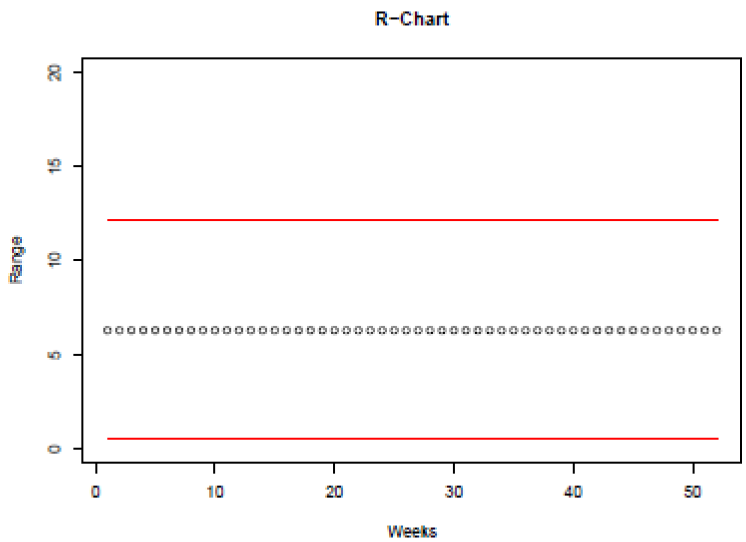
The control limits for the R control chart are given by the following equations:

$LCL_R = D_3\bar{R}$ $\qquad$ $UCL_R = D_4\bar{R}$

Since we are told the range is 6.3, and since we know the values of D_3 and D_4 are 0.08 and 1.92 respectively, for a sample size of 7, the controls limits for the R control chart will be:

$LCL_R = (0.08)(6.3)$ $\qquad$ $UCL_R = (1.92)(6.3)$

$LCL_R = 0.504$ $\qquad$ $UCL_R = 12.096$



- *We recall from the lecture that by the three-sigma rule we have the following relation*
$$A_2 * \bar{R} = 3 * \sigma/\sqrt{n}$$
which in our case translate to
$$0.42 * 6.3 = 3 * \sigma/\sqrt{7}$$
yielding $\sigma = 0.42 * 6.3 * \sqrt{(7)}/3 = 2.333553.$

- To determine the standard deviation of the process we use the following equation where *n* is the subsample size, in this case 7:

$$\hat{\sigma} = \frac{A_2\sqrt{n}}{3}\bar{R}$$

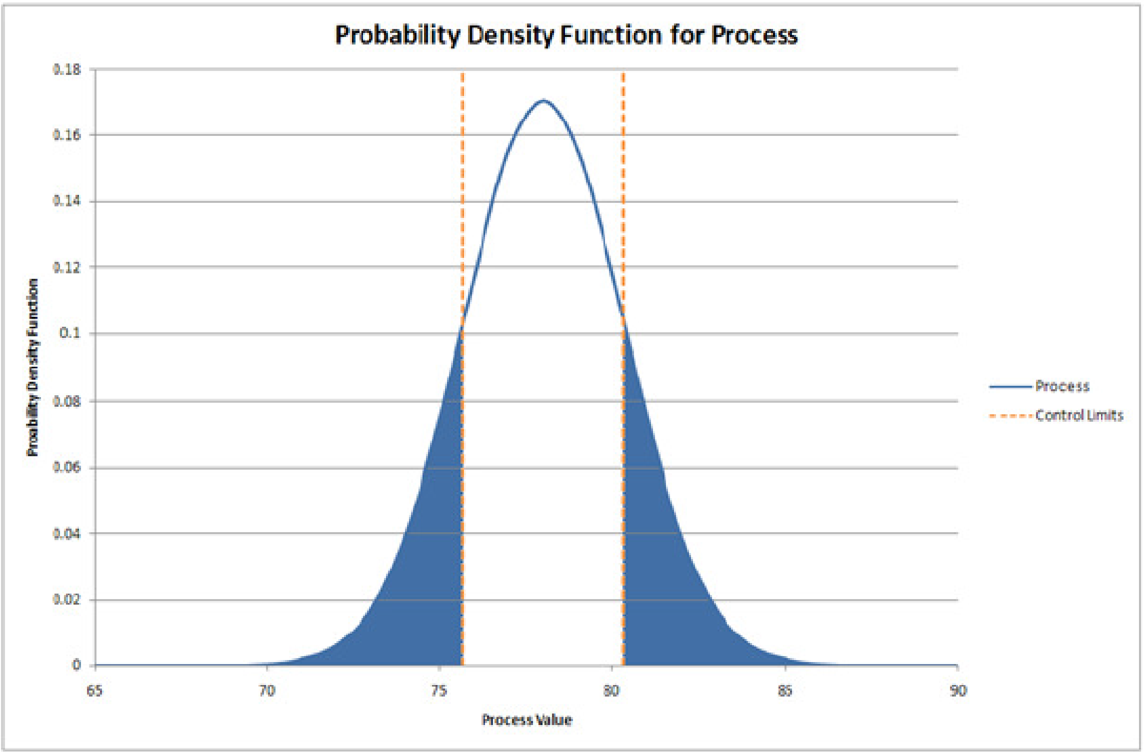
$$\hat{\sigma} = \frac{0.42\sqrt{7}}{3}6.3$$

$$\hat{\sigma} = 2.334$$

<u>Solutions to Part 4</u>

- *Finally, if a single item is represented by the value* X *following the normal model with the mean 78 and standard deviation* 2.33, *than chance for* X *to fall outside the control interval* $[75.354, 80.646]$ *are given as* $2(1-0.8719) = 0.2562$, *where by standardization* 0.8719 *is value from the table for* $z = (80.646 - 78)/2.33 = 1.14$. *We conclude that there will be more than 25% individual observations falling outside the control "belt".*

- We know now the mean of the process which is 78 and the standard deviation of the process which 2.334. The control belt is from 75.354 to 80.646. The non-conformance rate will be equivalent to the area under the curve outside the control limits for the below graph.



The standardised control limits for the process will therefore be given by[1]:

$$SLCL = \frac{75.354 - 78}{2.334} \qquad SUCL = \frac{80.646 - 78}{2.334}$$

$$SLCL = -1.13 \qquad SUCL = 1.13$$