

## 1 Multivariate Control Charts

- With the enhancements in data acquisition systems it is usual to deal with processes with more than one correlated quality characteristic to be monitored.
- A common practice is to control the stability of the process using univariate control charts.
- This practice increases the probability of false alarm of special cause of variation.
- Therefore, the analysis should be performed through a multivariate approach; that is, the variables must be analyzed together, not independently.

### 1.1 Multivariate Control Charts

- Multivariate control charts monitor multiple process characteristics. Independent variables can be charted individually, but if the variables are correlated, a multivariate chart is needed to determine whether the process is in control.
- Multivariate control charts can detect shifts in the mean or the relationship between several related variables.
- The multivariate control chart plots Hotelling's  $T^2$  statistic. The calculation for the control limit differs based on whether targets have been specified.

### 1.2 The MSQC package

In his book, Edgar Santos-Fernandez present the multivariate normal distribution, the data structure of the multivariate problems dealt in this book, the `mult.chart` function that allows the computation in R, and the most used multivariate control charts:

- The control ellipsoid or  $w^2$  control chart
- The  $T^2$  or Hotelling chart
- The Multivariate Exponentially Weighted Moving Average (MEWMA) chart
- The Multivariate Cumulative Sum (MCUSUM) chart
- The chart based on Principal Components Analysis (PCA)

### 1.3 The `mult.chart` Function

The performing of the multivariate control chart in R can be carried out with the function `mult.chart` which is a general function that allows to compute the most accepted and diversified continuous multivariate chart such as

- $\chi^2$
- Hotelling  $T^2$
- MEWMA
- MCUSUM according to Crosier (1988)
- MCUSUM by Pignatiello and Runger (1990)

Finally the function `mult.chart` returns:

- The  $T^2$  statistics
- The Upper Control Limit (UCL)
- The sample covariance matrix (S)
- The mean vector (Xmv)
- And if any point falls outside of the UCL and its decomposition

```
mult.chart(dowel1, type = "chi", alpha = 0.05)
```

### 1.4 $T^2$ control chart

The origin of the  $T^2$  control chart dates back to the pioneer works of Harold Hotelling who applied this method to the bombsight problem in Second World War. The Hotelling (1947) procedure has become without doubt the most applied in multivariate process control and it is the multivariate analogous of the Shewhart control chart. For that reason, it is also known as multivariate Shewhart control chart.

```
data("carbon1")  
mult.chart(type = "t2", carbon1)  
mult.chart(type = "t2", carbon1)$t2
```

## 1.5 mqcc Example

```
# library(mqcc)
# Ryan (2000, Table 9.2) data with p = 2 variables,
# m = 20 samples, n = 4 sample size:

X1 = matrix(c(72, 56, 55, 44, 97, 83, 47, 88, 57, 26, 46,
49, 71, 71, 67, 55, 49, 72, 61, 35, 84, 87, 73, 80, 26, 89, 66,
50, 47, 39, 27, 62, 63, 58, 69, 63, 51, 80, 74, 38, 79, 33, 22,
54, 48, 91, 53, 84, 41, 52, 63, 78, 82, 69, 70, 72, 55, 61, 62,
41, 49, 42, 60, 74, 58, 62, 58, 69, 46, 48, 34, 87, 55, 70, 94,
49, 76, 59, 57, 46), ncol = 4)

X2 = matrix(c(23, 14, 13, 9, 36, 30, 12, 31, 14, 7, 10,
11, 22, 21, 18, 15, 13, 22, 19, 10, 30, 31, 22, 28, 10, 35, 18,
11, 10, 11, 8, 20, 16, 19, 19, 16, 14, 28, 20, 11, 28, 8, 6,
15, 14, 36, 14, 30, 8, 35, 19, 27, 31, 17, 18, 20, 16, 18, 16,
13, 10, 9, 16, 25, 15, 18, 16, 19, 10, 30, 9, 31, 15, 20, 35,
12, 26, 17, 14, 16), ncol = 4)

X = list(X1 = X1, X2 = X2)
q = mqcc(X, type = "T2")
summary(q)
```

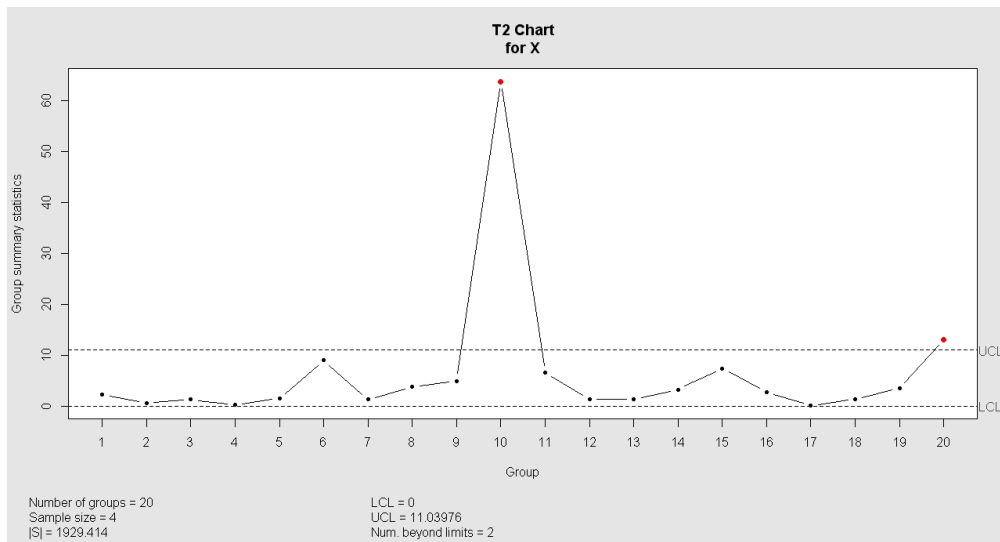


Figure 1

Call:

```
mqcc(data = X, type = "T2")
```

T2 chart for X

Summary of group statistics:

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	0.1243	1.3250	2.5030	6.4700	5.3490	63.7600

Number of variables: 2

Number of groups: 20

Group sample size: 4

Center:

	X1	X2
	60.3750	18.4875

Covariance matrix:

	X1	X2
X1	222.0333	103.1167
X2	103.1167	56.57917

|S|: 1929.414

Control limits:

LCL            UCL  
0 11.03976