Contents

Chapter 1

Statistical Process Control

1.1 Introduction to Statistical Process Control

- Statistical process control (SPC) is a method of quality control which uses statistical methods.
- The term Statistical Process Control (SPC) is typically used in context of manufacturing processes (although it may also pertain to services and other activities), and it denotes statistical methods used to monitor and improve the quality of the respective operations.
- SPC can be applied to any process where the "conforming product" (product meeting specifications) output can be measured.
- SPC is applied in order to monitor and control a process. Monitoring and controlling the process ensures that it operates at its full potential. At its full potential, the process can make as much conforming product as possible with a minimum (if not an elimination) of waste (rework or trash).
- By gathering information about the various stages of the process and performing statistical analysis on that information, the SPC engineer is able to take necessary action (often preventive) to ensure that the overall process stays in-control and to allow the product to meet all desired specifications.
- SPC involves monitoring processes, identifying problem areas, recommending methods to reduce variation and verifying that they work, optimizing the process, assessing the reliability of parts, and other analytic operations.
- SPC uses such basic statistical quality control methods as **quality control charts** (Shewart, Pareto, and others), capability analysis, gage repeatability/reproducibility analysis, and reliability analysis.

1.2 7 Basic Tools of Quality

These are 7 QC tools also known as Ishikawas **7QC** tools

Cause-and-effect diagram : Identifies many possible causes for an effect or problem and sorts ideas into useful categories. (also called *Ishikawa* or *fishbone chart*)

Check sheet : A structured, prepared form for collecting and analyzing data; a generic tool that can be adapted for a wide variety of purposes.

Control charts : Graphs used to study how a process changes over time.

Histogram : The most commonly used graph for showing frequency distributions, or how often each different value in a set of data occurs.

Pareto chart : Shows on a bar graph which factors are more significant.

Scatter diagram : Graphs pairs of numerical data, one variable on each axis, to look for a relationship.

Stratification : A technique that separates data gathered from a variety of sources so that patterns can be seen (some lists replace stratification with flowchart or run chart).

For the sake of brevity, we will only look at a couple of these.

1.3 Background to Statistical Process Control

- The concepts of Statistical Process Control (SPC) were initially developed by Dr. Walter Shewhart of Bell Laboratories in the 1920's, and were expanded upon by Dr. W. Edwards Deming, who introduced SPC to Japanese industry after WWII.
- After early successful adoption by Japanese firms, Statistical Process Control has now been incorporated by organizations around the world as a primary tool to improve product quality by reducing process variation.
- Dr. Shewhart identified two sources of process variation:

  Chance variation that is inherent in process, and stable over time,

  Assignable variation , or Uncontrolled variation, which is unstable over time - the result of specific events outside the system.
- Dr. Deming relabeled chance variation as **Common Cause** variation, and assignable variation as **Special Cause** variation.
- Based on experience with many types of process data, and supported by the laws of statistics and probability, Dr. Shewhart devised control charts used to plot data over time and identify both Common Cause variation and Special Cause variation.

1.4 Contemporary Context of SPC

1.4.1 10 R packages I wish I knew about earlier

- *Following material written by Drew Conway, and was published on the Yhat blog Feb 2013*
- *http://blog.yhathq.com/posts/10-R-packages-I-wish-I-knew-about-earlier.html*
- **qcc** is a library for statistical quality control. Back in the 1950s, the now defunct Western Electric Company was looking for a better way to detect problems with telephone and eletrical lines.

  ***Remark**: We will discuss these rules shortly*
- They came up with a set of rules to help them identify problematic lines. The rules look at the historical mean of a series of datapoints and based on the standard deviation, the rules help judge whether a new set of points is experiencing a mean shift.
- While you might not be monitoring telephone lines, qcc can help you monitor transaction volumes, visitors or logins on your website, database operations, and lots of other processes.

Other Remarks

- Quality Control and quality assurance are important functions in most businesses from manufacturing to software development.
- For most, this means that one or more people are meticulously inspecting what's coming out of the factory, looking for imperfections and validating that requirements for products and services produced are satisfied.
- Often times QC and QA are performed manually by a select few specialists, and determining suitable quality can be extremely complex and error-prone.

1.5 Some Remarks on Multivariate Techniques

- Nowadays, the intensive use of an automatic data acquisition systems and the use of online computers for process monitoring have led to an increased occurrence of industrial processes with two or more correlated quality characteristics, in which the statistical process control and the capability analysis should be performed using multivariate methodologies. *(Edgar Santos-Fernandez)*

1.6 Causes of Variation

- The purpose is to control the quality of product or service outputs from a process by maintaining control of the process.
- When a process is described as being "in control", it means that the amount of variation in the output is relatively constant and within established limits that are deemed acceptable.
- There are two kinds of causes of variation in a process:

  Common causes *(or chance causes)* of variation are due to factors that are inherent in the design of the system, and reflect the usual amount of variation to be expected.

  Assignable causes *(or special causes)* of variation are due to unusual factors that are not part of the process design and not ordinarily part of the process.

- A **stable process** is one in which only common causes of variation affect the output quality. Such a process can also be described as being in a state of statistical control.
- An **unstable process** is one in which both assignable causes and common causes affect the output quality. (Note that, by definition, the common causes are always present.)

Such a process can also be described as being out of control, particularly when the assignable cause is controllable.

- The way we set out to improve the quality of output for a process depends on the source of process variation.
- For a process that is stable, improvement can take place only by improving the design of the process.
- **(Important)** A pervasive error in process management is **tampering**, which is to take actions (such as making machine adjustments) that presume that a process is not in control, when in fact it is stable.
- Such actions only increase variability, and are analogous to the over-correcting that new drivers do in learning to steer a car. For a process that is unstable, improvement can be achieved by identifying and correcting assignable causes.

Chapter 2

Control Charts

2.1 Control Charts

- A **run chart** is a time series plot that plots levels of output quality on the vertical axis, with respect to a sequence of time periods on the horizontal axis. In the context of statistical process control the measurements that are graphed are typically sample data that have been obtained by the method of rational subgroups.
- A **control chart** is a run chart that includes the lower and upper control limits that identify the range of variation that can be ascribed to common causes. Any outputs that are outside of the control limits suggest the existence of assignablecause variation.
- The control limits are determined either by process parameters having been specified, or by observing sample outcomes during a period of time in which the process is deemed to be in a stable condition. The statistical methods of process control are based on the concepts of hypothesis testing
- The null hypothesis is that the process is stable and only common causes of variation exist.

		Condition of Process	
		H_0 True: Stable	H_0 False: Unstable
Decision	Continue Process	Correct decision	Type II error: Allowing an unstable process to continue
	Adjust Process	Type I error: Adjusting a stable process	Correct decision

- The alternative hypothesis is that the process is unstable and includes assignable-cause variation.
- Thus, the lower and upper control limits on a control chart are the critical values with respect to rejecting or not rejecting the null hypothesis that the process is stable and in control.
- The standard practice is to place the control limits at three standard error units above and below the hypothesized value. This **3-sigma rule** is conventionally applied for most control chart procedures.
- If a process is stable, not only should all sample statistics be within the control limits, but there also should be no discernable pattern in the sequence of the sample statistics.
- We look at the methods for determining control limits and we describe the interpretation of control charts for the
  1. process mean,
  2. process range
  3. process standard deviation,
  4. process range, process proportion.
- Of these four, control charts for the mean, standard deviation, and range are designated as control charts for variables, because measurements are involved.
- Control charts for the proportion are designated as control charts for attributes, because counts (which are discrete rather than continuous variables) are involved.

2.1.1 Control Charts

- The control chart is a graph used to study how a process changes over time. Data are plotted in time order. A control chart always has a central line for the average, an upper line for the upper control limit and a lower line for the lower control limit.
- Units are usually the summary statistics (i.e. means or ranges) of small samples.
- These lines are determined from historical data. By comparing current data to these lines, you can draw conclusions about whether the process variation is consistent (in control) or is unpredictable (out of control, affected by special causes of variation).
- (**Important**) There are several types of control chart. In the short term, we will look at the **x-bar** chart (related to mean of values from sample).
- (**Important**) Other types of chart include **R** charts and **S** charts (related to range and standard deviation of the values from each sample)

2.1.2 Data Used for Control Chart

4 The production of a certain type of red brick requires that the dimensions of the end product fall within specification limits set out by the production engineer. The target width for the bricks is 140 mm $\pm$ 2mm. Four bricks were selected at random every 5 minutes and their widths recorded. Some of these data are displayed below, along with the sample to which each brick belongs.

Sample					means	ranges
1	140.5	139.9	140.0	140.5	140.22	0.6
2	139.7	140.0	139.8	140.2	139.93	0.5
3	139.5	139.8	139.8	140.6	139.93	1.1
:	:	:	:	:	:	:
:	:	:	:	:	:	:
:	:	:	:	:	:	:
98	138.5	140.5	139.0	139.3	139.33	2.0
99	140.5	140.6	140.1	141.4	140.65	1.3
100	140.1	140.3	139.7	139.4	139.88	0.9
					$\bar{\bar{x}} = 140.00$	$\bar{R} = 1.21$

(a) Calculate the control limits for the mean and range charts. **4**

(b) The data are plotted on the final page of the exam paper, which can be detached and included in your answer sheet. Using the limits calculated in part (i), check for process control. In practice why are two "types" of chart required ? **4**

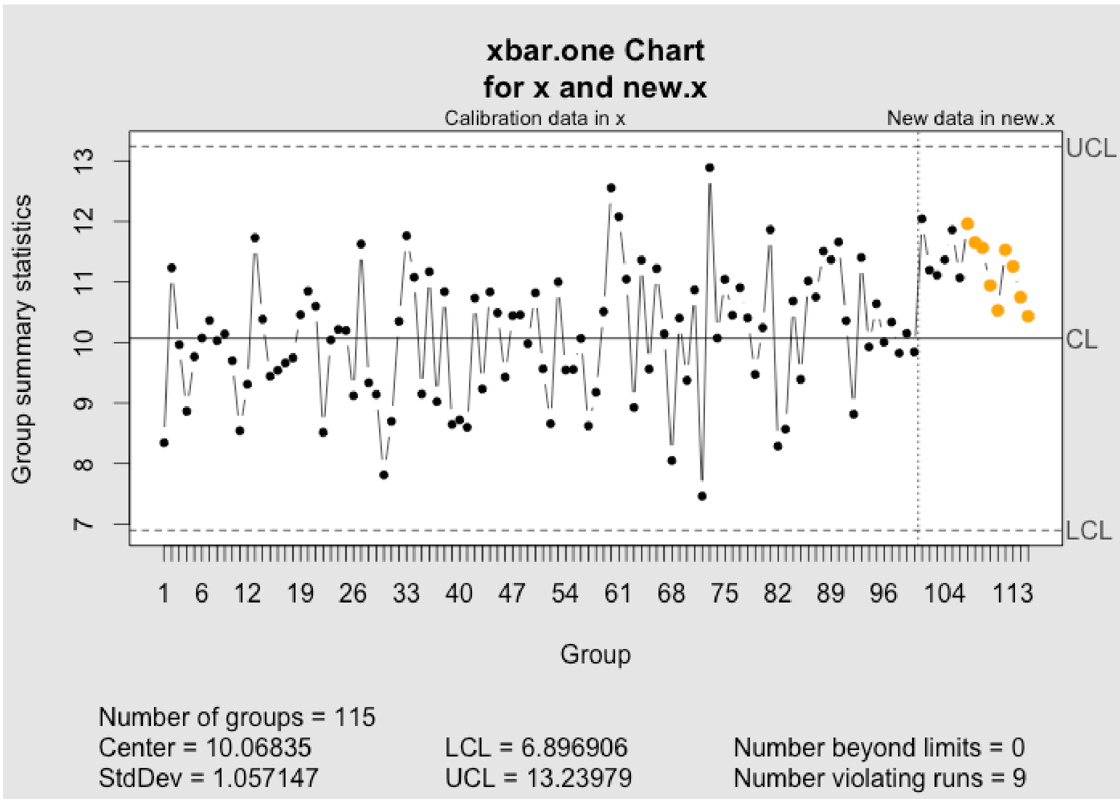
(c) Is the process capable? Give reasons. **4**

(d) Calculate the ARL i.e. average run length for a change of + 0.5 mm in the average. In words give a practical interpretation of the ARL. **4**

(e) What is a CUSUM chart ? What type of departures from the production target value is this type of chart useful for detecting? Explain how it works. **4**

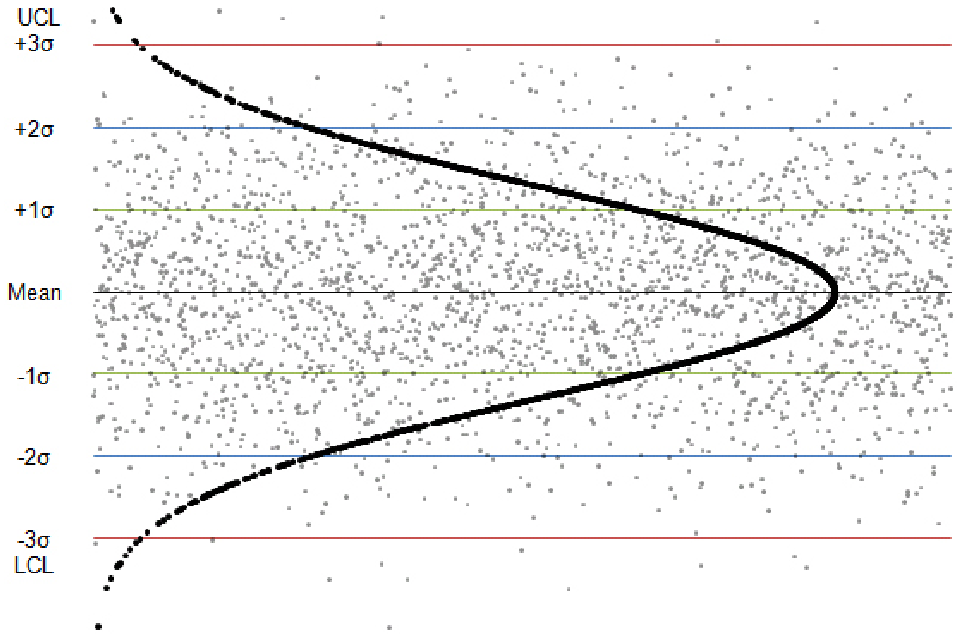
2.1.3 Example of a Control Chart

This relates to a process mean.



2.1.4 Control Limits

- Statistical tables have been developed for various types of distributions that quantify the area under the curve for a given number of standard deviations from the mean (based on the ***normal distribution***).
- Shewhart found that control limits placed at ***three standard deviations from the mean*** in either direction provide an economical tradeoff between the risk of reacting to a false signal and the risk of not reacting to a true signal - regardless the shape of the underlying process distribution.
- If the process has a normal distribution, 99.7% of the population is captured by the curve at three standard deviations from the mean. Stated another way, there is only a 100-99.7%, or 0.3% chance of finding a value beyond 3 standard deviations. Therefore, a measurement value beyond 3 standard deviations indicates that the process has either shifted or become unstable (more variability).


UCL
+3σ
+2σ
+1σ
Mean
-1σ
-2σ
-3σ
LCL

2.2 Computing Control Limits

Exam Paper Formulas for Control Limits

- Process Mean

$$\bar{\bar{x}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}}$$

- Process Standard Deviation

$$\bar{s} \pm 3\frac{c_5\bar{s}}{c_4}$$

- Process Range

$$[\bar{R}D_3, \bar{R}D_4]$$

Process Mean and Standard Deviation Known

The process mean and standard deviation would be known either because they are process specifications or are based on historical observations of the process when It was deemed to be stable. The centreline for the ***X.bar*** chart is set at the process mean.

Using the 3-sigma rule, the control limits are defined by the same method as that used for determining critical values for hypothesis testing.

$$\text{Control limits} = \mu \pm 3\frac{\sigma}{\sqrt{n}}$$

Process Mean and Standard Deviation Unknown.

When the process mean and standard deviation are unknown, the required assumption is that the samples came from a stable process. Thus, recent sample results are used as the basis for determining the stability of the process as it continues. First the overall mean of the ***k*** sample means and the mean of the ***k*** sample standard deviations are determined:

X.double-bar	***s.bar***
$\bar{\bar{X}} = \frac{\Sigma \bar{X}}{k}$	$\bar{s} = \frac{\Sigma s}{k}$

Although ***X.double-bar*** is an unbiased estimator of μ, ***s.bar*** is a biased estimator of σ. This is true even though s^2 is an unbiased estimator of σ^2.

Therefore, the formula for the control limits incorporates a correction for the biasedness in ***s.bar.***

$$\text{Control limits} = \bar{\bar{X}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}}$$

2.3 Control Limits: Worked Example

	item 1	item 2	item 3	item 4	mean	sd.dev	range
1	15.01	15.16	14.98	14.8	14.99	0.148	0.36
2	15.09	15.08	15.14	15.03	15.08	0.045	0.11
3	15.04	14.93	15.1	15.13	15.05	0.088	0.2
4	14.9	14.94	15.03	14.92	14.95	0.057	0.13
5	15.04	15.08	15.05	14.98	15.04	0.042	0.1
6	14.96	14.96	14.81	14.91	14.91	0.071	0.15
7	15.01	14.9	15.1	15.03	15.01	0.083	0.2
8	14.71	14.77	14.92	14.95	14.84	0.116	0.24
9	14.81	14.64	14.8	14.95	14.8	0.127	0.31
10	15.03	14.99	14.89	15.03	14.98	0.066	0.14
11	15.16	14.95	14.91	14.83	14.96	0.141	0.33
12	14.92	15.01	15.05	15.02	15	0.056	0.13
13	15.06	14.95	15.03	15.02	15.02	0.047	0.11
14	14.99	15.04	15.14	15.11	15.07	0.068	0.15
15	14.94	14.9	15.08	15.17	15.02	0.125	0.27

Overall Mean and Mean of standard deviations

$$\text{Centerline} = \bar{\bar{X}} = \frac{\Sigma \bar{X}}{k} = \frac{224.72}{15} = 14.98$$

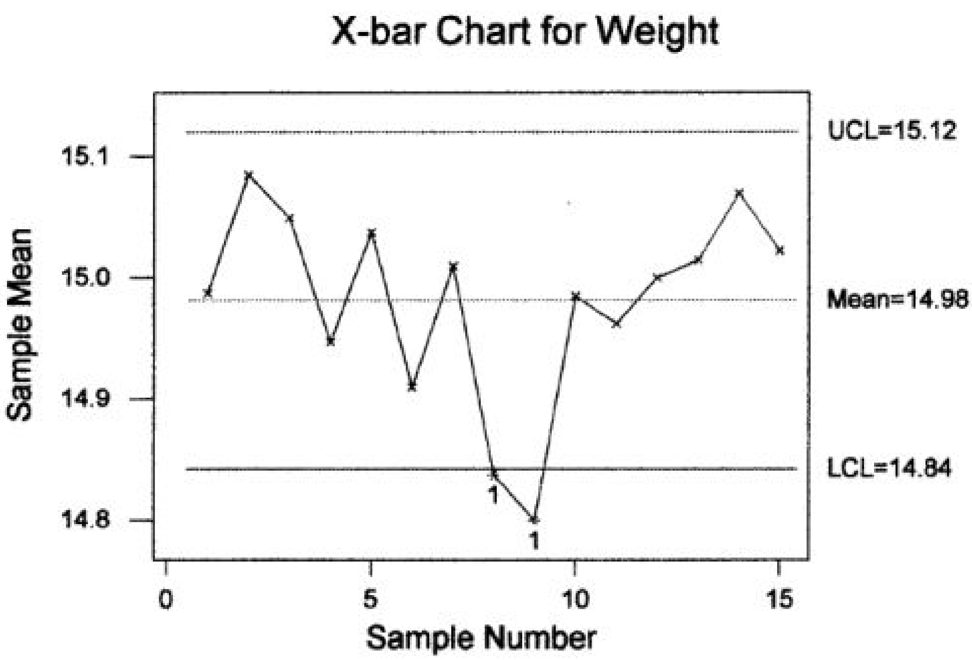
$$\bar{s} = \frac{\Sigma s}{k} = \frac{1.280}{15} = 0.08551$$

$$\text{Control limits} = \bar{\bar{X}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}}$$

The value for c_4 is computed using a special table, and is dependent on the sample size. For a batch sample size of 4, the value for c_4 is 0.9213.

$$\begin{aligned}\text{Control limits} &= \bar{\bar{X}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}} \\ &= 19.98 \pm 3\frac{0.08551}{0.9213\sqrt{4}} \\ &= 19.98 \pm 0.14 \\ &= 14.84 \text{ and } 15.12\,\text{oz}\end{aligned}$$

$$\text{Control limits} = \bar{\bar{X}} \pm 3\frac{\bar{s}}{c_4\sqrt{n}}$$
$$= 19.98 \pm 3\frac{0.08551}{0.9213\sqrt{4}}$$
$$= 19.98 \pm 0.14$$
$$= 14.84 \text{ and } 15.12 \text{ oz}$$



X-bar Chart: Weight

```
TEST 1. One point more than 3.00 sigmas from center line.
Test Failed at points: 8 9
```

2.4 Nelson Rules for Interpreting Control Charts

- The eight tests used in statistical process control were developed by Lloyd S. Nelson, a process control expert. They are based on his determination that the identified patterns are very unlikely to occur in stable processes.
- Thus the existence of any of these patterns in an $\bar{X}$ chart indicates that the process may be unstable, and that one or more assignable causes may exist.
- The table on the next page contains examples of test failure for each of the eight tests, with a description for each graph as to what is required for the illustrated test failure.
- In practice, tests 1,2 and 7 are considered the three most useful.

2.4.1 Descriptions of Tests

Test 1 - 3 sigma rule Identifies points outside of the control limits

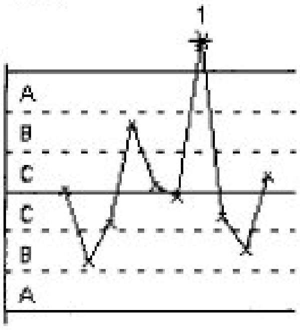
Test 1 identifies points that are more standard deviations from the center line. Test 1 is universally recognized as necessary for detecting out-of-control situations. It has a false alarm rate of only 0.27%.

Test 2 Identifies shifts in the means

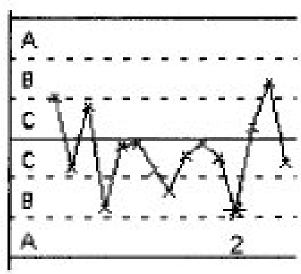
Test 2 signals when 9 points in a row fall on the same side of the center line. The use of Test 2 significantly increases the sensitivity of the chart to detect small shifts in the mean.

When test 1 and test 2 are used together, significantly fewer subgroups are needed to detect a small shift in the mean than are needed when test 1 is used alone. Therefore, adding test 2 helps to detect common out-of-control situations and increases sensitivity enough to warrant a slight increase in the false alarm rate.

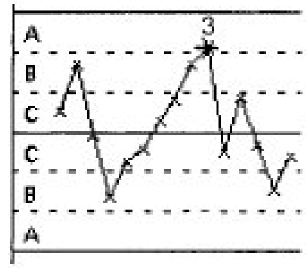
Test 1
One point more than 3 sigmas from center line



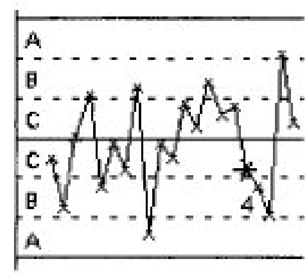
Test 2
Nine points in a row on same side of center line



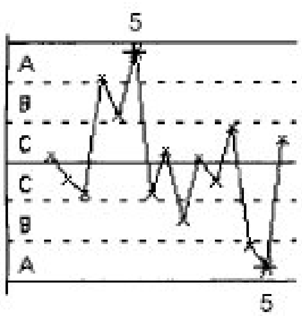
Test 3
Six points in a row, all increasing or all decreasing



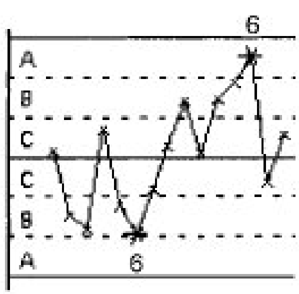
Test 4
Fourteen points in a row, alternating up and down



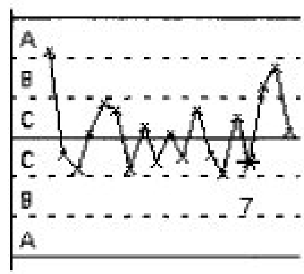
Test 5
Two out of three points in a row more than 2 sigmas from center line (same side)



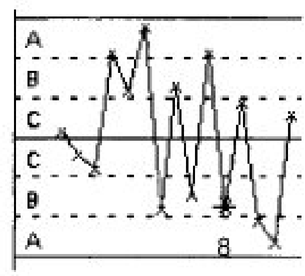
Test 6
Four out of five points in a row more than 1 sigma from center line (same side)



Test 7
Fifteen points in a row within 1 sigma of center line (either side)



Test 8
Eight points in a row more than 1 sigma from center line (either side)



Test 3 k points in a row, all increasing or all decreasing

Test 3 is designed to detect drifts in the process mean.

However, when test 3 is used in addition to test 1 and test 2, it does not significantly increase the sensitivity of the chart to detect drifts in the process mean.

Test 4 k points in a row, alternating up and down

Although this pattern can occur in practice, it is recommended to search for any unusual trends or patterns rather than test for one specific pattern.

Test 5 k out of k=1 points $>$ 2 standard deviations from center line

This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

Test 6 k out of k+1 points $>$ 1 standard deviation from the center line

This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

Test 7 Identifies control limits that are too wide

Test 7 signals when 12 or 15 points in a row fall within 1 standard deviation of the center line.

Test 7 is used only for the $\bar{X}$ chart when the control limits are estimated from the data. When this test fails, the cause is usually a systemic source of variation (stratification) within a subgroup, which is often the result of not forming rational subgroups.

Test 8 k points in a row $>$ 1 standard deviation from center line (either side)

This test is not quite as informative because it did not uniquely identify special cause situations that are common in practice.

2.4.2 Interpreting Control Charts

- The most obvious test in terms of its rationale is Test 1, which simply requires that at least one value of **X.bar** be beyond Zone A. For a sample mean to be beyond three standard errors from the centre-line is of course very unlikely in a stable process.
- The other seven patterns also are very unlikely. The one test whose rationale is not obvious is Test 7, which requires that 15 consecutive values of **X.bar** be in Zone C, which is within one standard error of the centre-line.
- Although this kind of pattern would seem to be very desirable, it is also very unlikely. It may, for example, indicate that the process standard deviation was overstated in the process specifications or that the sample measurements are in error. Whatever the cause, the results are too good to be true and therefore require investigation.

2.5 Multivariate Normal

- The multivariate normal distribution or multivariate Gaussian distribution, is a generalization of the one-dimensional (univariate) normal distribution to higher dimensions.
- One possible definition is that a random vector is said to be k-variate normally distributed if every linear combination of its k components has a univariate normal distribution.
- The multivariate normal distribution is often used to describe, at least approximately, any set of (possibly) correlated real-valued random variables each of which clusters around a mean value.

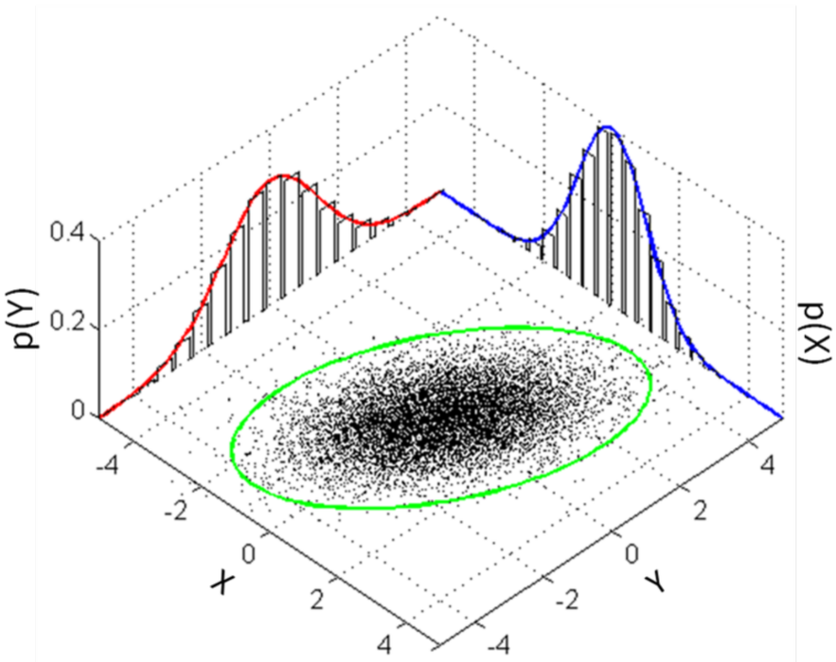


Figure 2.1:

2.5.1 Testing for Normality

Graphical Methods

- Histograms
- Normal Probability Plots

Hypothesis Tests for Univariate Data

- Shapiro-Wilk Test (inbuilt with `R`)
- D'Agostino Test (MSQC package)

Hypothesis Tests for Multivariate Data

- Mardia Test (MSQC package)
- Henze and Zirkler (MSQC package)
- Royston Test (MSQC package)

The bimetal data set (MSQC package)

- Bimetal thermostat has innumerable practical uses. These types of thermostats hold a bimetallic strip composed by two strips of different metals that convert the changing of temperature in mechanical displacement due to the difference in thermal expansion.
- Certain type of strip composed of brass and steel is analyzed in a quality laboratory by testing the deflection, curvature, resistivity, and hardness in low and high expansion sides.

```
> tail(bimetal1)
deflection curvature resistivity Hardness low side Hardness high side
[23,]      20.76     39.98       14.98             22.29              26.03
[24,]      21.00     40.11       15.17             22.04              25.99
[25,]      20.57     39.73       14.35             22.02              25.80
[26,]      20.78     39.83       15.27             21.60              25.89
[27,]      20.96     40.03       15.26             21.98              25.94
[28,]      21.14     39.93       14.98             21.84              25.98
```

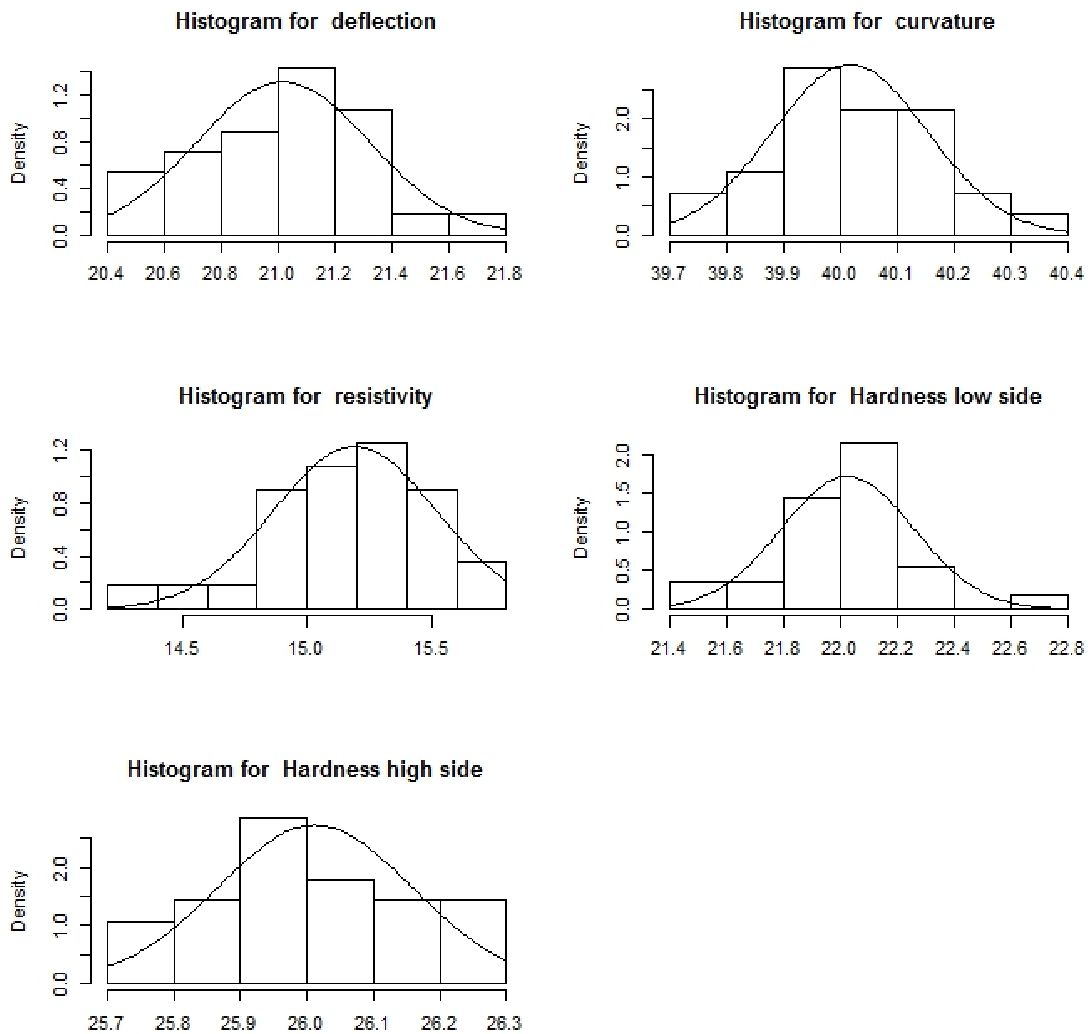

Histogram for deflection
Density
0.0 0.4 0.8 1.2
20.4 20.6 20.8 21.0 21.2 21.4 21.6 21.8
Histogram for curvature
Density
0.0 1.0 2.0
39.7 39.8 39.9 40.0 40.1 40.2 40.3 40.4
Histogram for resistivity
Density
0.0 0.4 0.8 1.2
14.5 15.0 15.5
Histogram for Hardness low side
Density
0.0 0.5 1.0 1.5 2.0
21.4 21.6 21.8 22.0 22.2 22.4 22.6 22.8
Histogram for Hardness high side
Density
0.0 1.0 2.0
25.7 25.8 25.9 26.0 26.1 26.2 26.3

Figure 2.2:

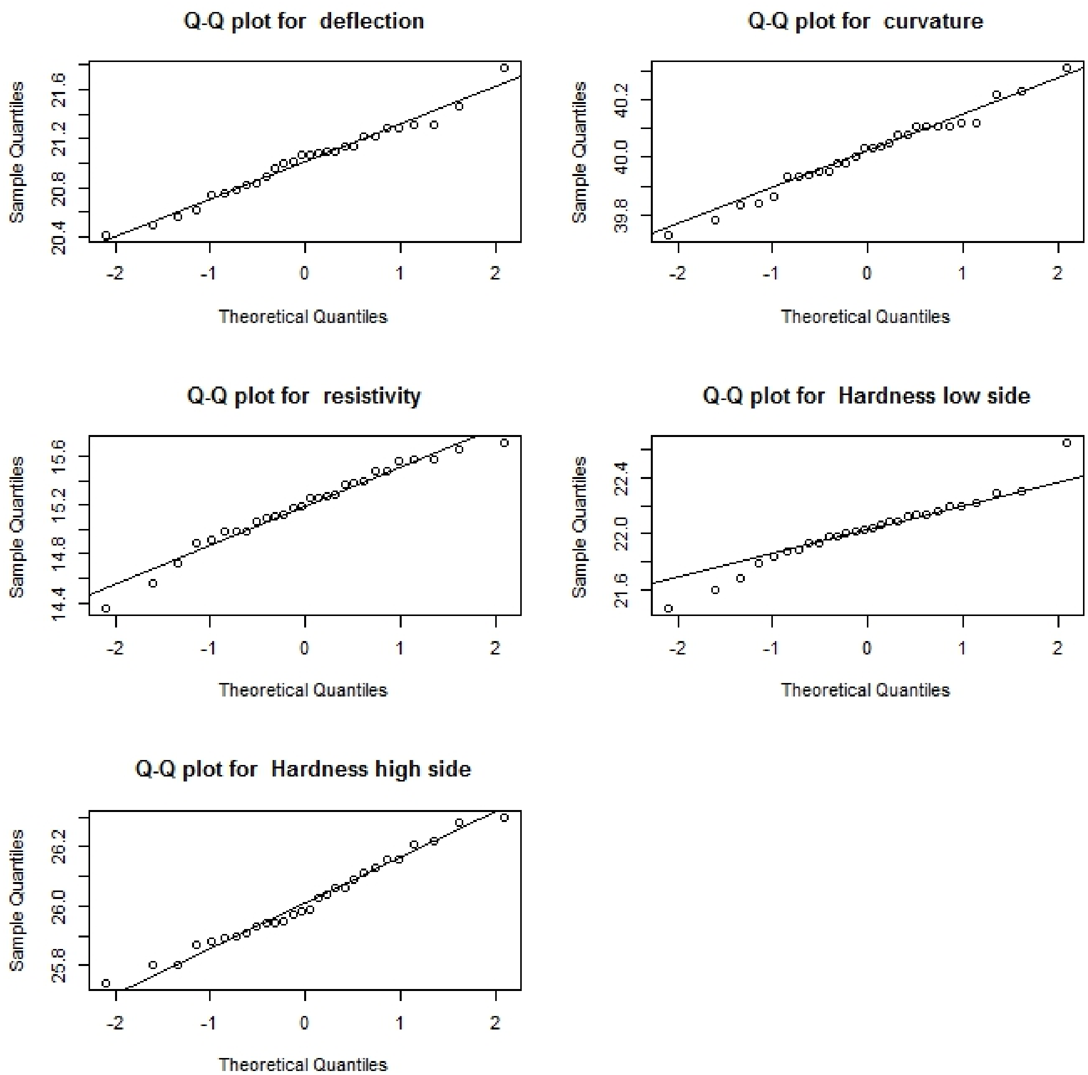

Q-Q plot for deflection
Sample Quantiles
20.4
20.8
21.2
21.6
Theoretical Quantiles
-2
-1
0
1
2
Q-Q plot for curvature
Sample Quantiles
39.8
40.0
40.2
Theoretical Quantiles
Q-Q plot for resistivity
Sample Quantiles
14.4
14.8
15.2
15.6
Theoretical Quantiles
Q-Q plot for Hardness low side
Sample Quantiles
21.6
22.0
22.4
Theoretical Quantiles
Q-Q plot for Hardness high side
Sample Quantiles
25.8
26.0
26.2
Theoretical Quantiles

Figure 2.3:

D'Agostino Test (MSQC Pacakge)

- Using the bimetal1 data set in MSQC package

```
> for (i in 1 : 5){
+  DAGOSTINO(bimetal1[,i])
+  }
D'Agostino Test
Skewness
Skewness coefficient: 0.0831225
Statistics: 0.2117358
p-value: 0.8323131
Kurtosis
The kurtosis coefficient: 3.0422
Statistics: 0.591983
p-value: 0.553862
Omnibus Test
Chi-squared: 0.3952759
Degree of freedom: 2
p-value: 0.8206669
....
....
D'Agostino Test
Skewness
Skewness coefficient: -0.04173762
Statistics: -0.1063873
p-value: 0.9152751
Kurtosis
The kurtosis coefficient: 4.162062
Statistics: 1.675258
p-value: 0.09388364
Omnibus Test
Chi-squared: 2.817807
Degree of freedom: 2
p-value: 0.2444111
```

Some Multivariate (MSQC Pacakge)

```
> MardiaTest(bimetal1)
$skewness
[1] 6.982112

$p.value
[1] 0.585327

$kurtosis
[1] 33.77373

$p.value
[1] 0.3490892

>
>
>
> HZ.test(bimetal1)
[1] 0.6068650 0.7709586
>
>
> Royston.test(bimetal1)
test.statistic        p.value
1.1814742      0.9364221
```

Box Cox Transformation

- The Box-Cox transforms nonnormally distributed data to a set of data that has approximately normal distribution.

Chapter 3

Review Questions (Theory Questions)

1. Differentiate common (or chance) causes of variation in the quality of process output from assignable (or special) causes.
2. Differentiate a stable process from an unstable process.
3. Other than applying the 3-sigma rule for detecting the presence of an assignable cause, what else do we look for when studying a control chart?
4. Describe how the output of a stable process can be improved. What actions do not improve a stable process, but rather, make the output more variable?
5. What is the purpose of maintaining control charts?
6. What is tampering in the context of process control?

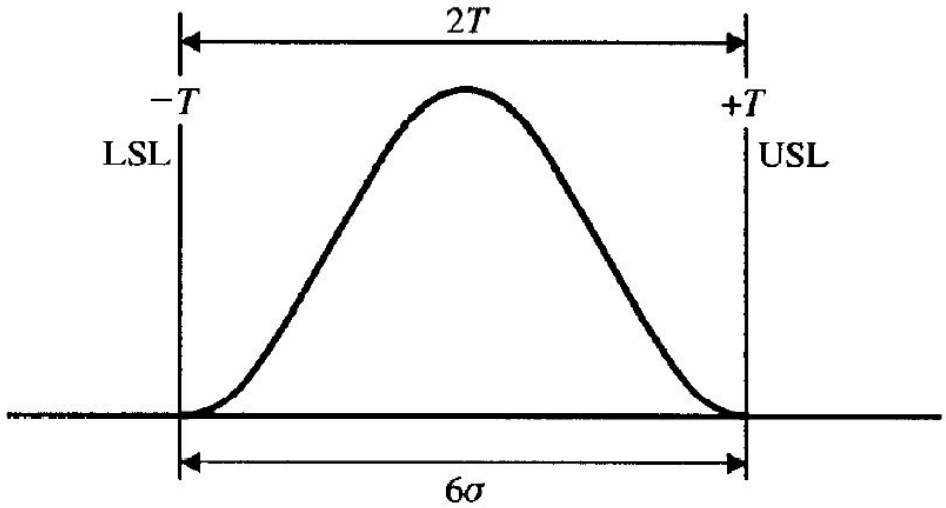
Chapter 4

Process Capability Indices

4.1 Process Capability

- In managing variables the usual aim is not to achieve exactly the same diameter for every piston, the same weight for every tablet, sales figures exactly as forecast, etc but to reduce the variation of products and process parameters around a **target value**.
- No adjustment of a process is called for as long as there has been no identified change in its accuracy or precision. This means that, in controlling a process, it is necessary to establish first that it is in statistical control, and then to compare its centering and spread with the specified target value and specification tolerance.
- We have seen previously that, if a process is not in statistical control, special causes of variation may be identified with the aid of ***control charts***.
- Only when all the special causes have been accounted for, or eliminated, can process capability be sensibly assessed. The variation due to common causes may then be examined and the "*natural specification*" compared with any imposed specification or tolerance zone.
- The relationship between process variability and tolerances may be formalized by consideration of the standard deviation, σ, of the process.
- In order to manufacture within the specification, the distance between the **upper specification limit** (USL) or upper tolerance ($+T$) and **lower specification limit** (LSL) or lower tolerance ($-T$), i.e. ($USL - LSL$) or 2T must be equal to or greater than the width of the base of the process bell, i.e. 6σ.

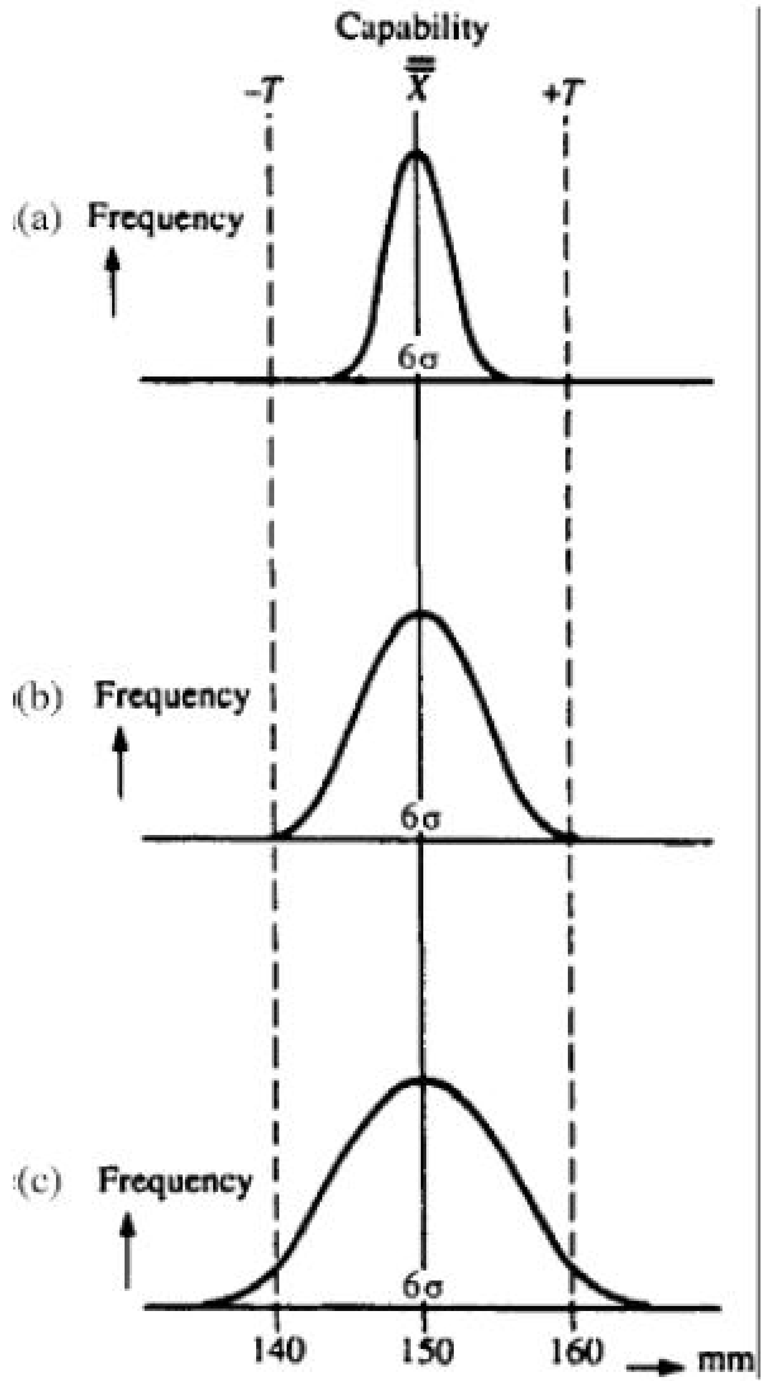
The relationship between $USL - LSL$ (i.e. 2T) and 6σ gives rise to three levels of precision of the process (Figure below):



(a) **High Relative Precision**, where the tolerance band is very much greater than 6σ ($2T >> 6\sigma$)

(b) **Medium Relative Precision**, where the tolerance band is just greater than 6σ ($2T > 6\sigma$)

(c) **Low Relative Precision**, where the tolerance band is less than 6σ ($2T < 6\sigma$)



Capability
$\bar{\bar{X}}$
$-T$
$+T$
(a) Frequency
6σ
(b) Frequency
6σ
(c) Frequency
6σ
140
150
160
mm

4.1.1 Process Capability Indices

- A process capability index is a measure relating the actual performance of a process to its specified performance, where processes are considered to be a combination of the plant or equipment, the method itself, the people, the materials and the environment.
- These indices assumes process output is approximately normally distributed.
- The absolute minimum requirement is that three process standard deviations each side of the process mean are contained within the specification limits.
- This means that approximately 99.7 per cent of output will be within the tolerances. A more stringent requirement is often stipulated to ensure that produce of the correct quality is consistently obtained over the long term.
- When a process is under statistical control (i.e. only random or common causes of variation are present), a process capability index may be calculated.
- Process capability indices are simply a means of indicating the variability of a process relative to the product specification tolerance.

C_p index

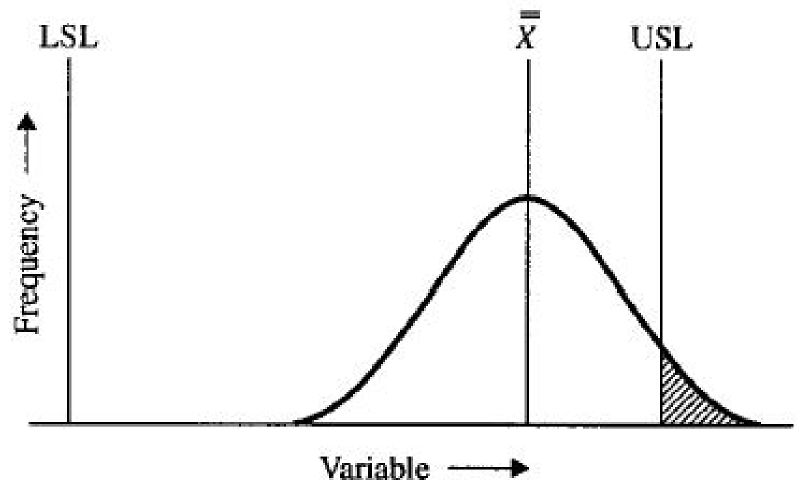
- In order to manufacture within a specification, the difference between the USL and the LSL must be less than the total process variation.
- A comparison of 6σ with (USLLSL) or 2T gives an obvious process capability index, known as the C_p of the process:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}}$$

- This estimates what the process is capable of producing if the process mean were to be centered between the specification limits. Clearly, any value of C_p below 1 means that the process variation is greater than the specified tolerance band so the process is incapable.
- For increasing values of C_p the process becomes increasingly capable. The C_p index makes no comment about the centring of the process, it is a simple comparison of total variation with tolerances.

C_{pk} **index**

- It is possible to envisage a relatively wide tolerance band with a relatively small process variation, but in which a significant proportion of the process output lies outside the tolerance band (Figure below).



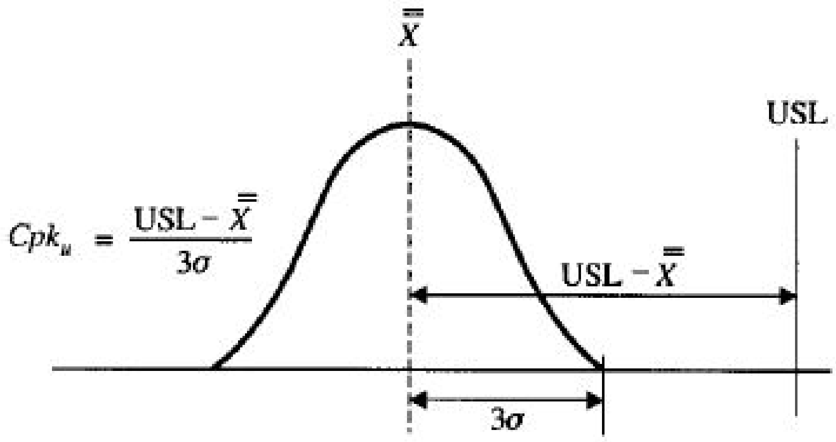
- This does not invalidate the use of Cp as an index to measure the potential capability of a process when centred, but suggests the need for another index which takes account of both the process variation and the centring. Such an index is the C_{pk}, which is widely accepted as a means of communicating process capability.

$$Cpk_{\mathrm{u}} = \frac{\mathrm{USL} - \overline{X}}{3\sigma} \qquad Cpk_{\mathrm{l}} = \frac{\overline{X} - \mathrm{LSL}}{3\sigma}.$$

- For upper and lower specification limits, there are two C_{pk} values, C_{pku} and C_{pkl}. These relate the difference between the process mean and the upper and the lower specification limits respectively, to 3σ (half the total process variation).

- The overall process C_{pk} is the lower value of C_{pku} and C_{pkl}. A C_{pk} of 1 or less means that the process variation and its centring is such that at least one of the tolerance limits will be exceeded and the process is incapable. As in the case of Cp, increasing values of C_{pk} correspond to increasing capability.

- It may be possible to increase the C_{pk} value by centring the process so that its mean value and the mid-specification or target, coincide. A comparison of the Cp and the C_{pk} will show zero difference if the process is centred on the target value.



- The C_{pk} can be used when there is only one specification limit, upper or lower a one-sided specification. This occurs quite frequently and the Cp index cannot be used in this situation.

Example 1

Example 1.

In tablet manufacture, the process parameters from 20 samples of size $n = 4$ are:

Mean Range ($\overline{R}$) = 91 mg, Process mean ($\overline{X}$) = 2500 mg

Specified requirements USL = 2650 mg, LSL = 2350 mg

Standard Deviation (s) = 44.2

$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{or} \quad \frac{2T}{6\sigma} = \frac{2650 - 2350}{6 \times 44.2} = \frac{300}{265.2} = 1.13$$

$$Cpk = \text{lesser of } \frac{\text{USL} - \overline{X}}{3\sigma} \quad \text{or} \quad \frac{\overline{X} - \text{LSL}}{3\sigma}$$

$$= \frac{2650 - 2500}{3 \times 44.2} \quad \text{or} \quad \frac{2500 - 2350}{3 \times 44.2} = 1.13.$$

Example 2

Example 2.

If the process parameters from 20 samples of size $n = 4$ are:

Mean range $(\overline{R})$ = 91 mg, Process mean $(\overline{\overline{X}})$ = 2650 mg

Specified requirements USL = 2750 mg, LSL = 2250 mg

Standard Deviation (s) = 44.2

$$Cp = \frac{\text{USL} - \text{LSL}}{6\sigma} \quad \text{or} \quad \frac{2T}{6\sigma} = \frac{2750 - 2250}{6 \times 44.2} = \frac{500}{265.2} = 1.89$$

$$Cpk = \text{lesser of } \frac{2750 - 2650}{3 \times 44.2} \quad \text{or} \quad \frac{2650 - 2250}{3 \times 44.2}$$

$$= \text{lesser of } 0.75 \text{ or } 3.02 = 0.75.$$

Conclusion – the Cp at 1.89 indicates a potential for higher capability than in example (i), but the low Cpk shows that this potential is not being realized because the process is not centred.

It is important to emphasize that in the calculation of all process capability indices, no matter how precise they may appear, the results are only ever approximations we never actually know anything, progress lies in obtaining successively closer approximations to the truth. In the case of the process capability this is true because:

- there is always some variation due to sampling;
- no process is ever fully in statistical control;
- no output exactly follows the normal distribution or indeed any other standard distribution.

Interpreting process capability indices without knowledge of the source of the data on which they are based can give rise to serious misinterpretation.

Interpreting capability indices - IMPORTANT

In the calculation of process capability indices so far, we have derived the standard deviation, σ, and recognized that this estimates the short-term variations within the process. This short term is the period over which the process remains relatively stable, but we know that processes do not remain stable for all time and so we need to allow within the specified tolerance limits for:

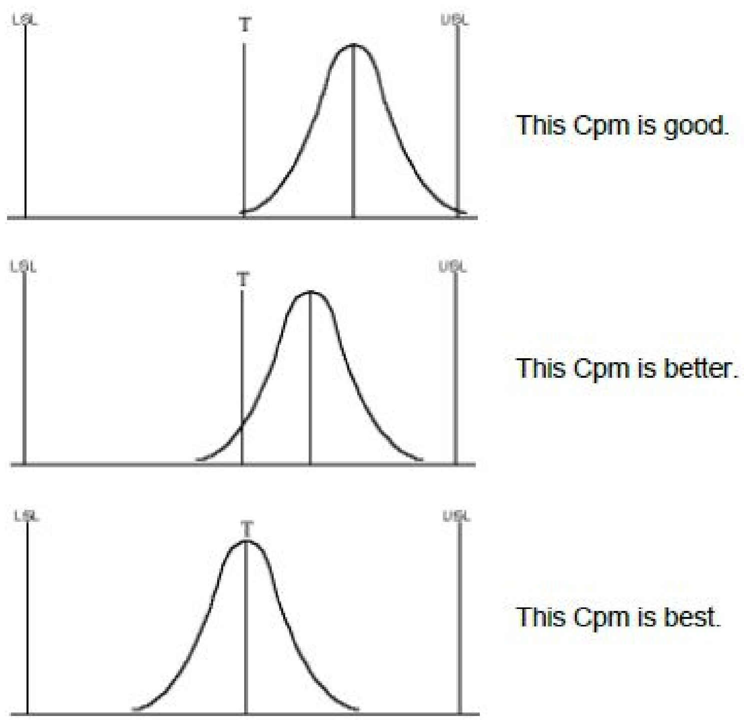
- some movement of the mean;
- the detection of changes of the mean;
- possible changes in the scatter (range);
- the detection of changes in the scatter;
- the possible complications of non-normal distributions.

Taking these into account, the following values of the C_{pk} index represent the given level of confidence in the process capability:

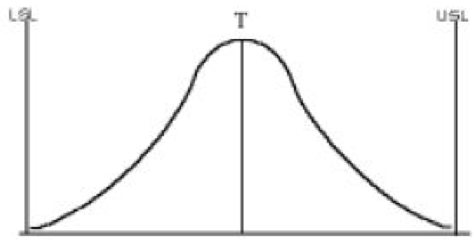
- $C_{pk} < 1$ A situation in which the production system is **not capable** and there will inevitably be non-conforming output from the process.
- $C_{pk} = 1$ A situation in which the production system is not really capable, since any change within the process will result in some undetected non-conforming output.
- $C_{pk} = 1.33$ A still far from acceptable situation since non-conformance is not likely to be detected by the process control charts.
- $C_{pk} = 1.5$ Not yet satisfactory since non-conforming output will occur and the chances of detecting it are still not good enough.
- $C_{pk} = 1.67$ Promising, non-conforming output will occur but there is a very good chance that it will be detected.
- $C_{pk} = 2$ High level of confidence in the production system, provided that control charts are in regular use.

The C_{pm} Index

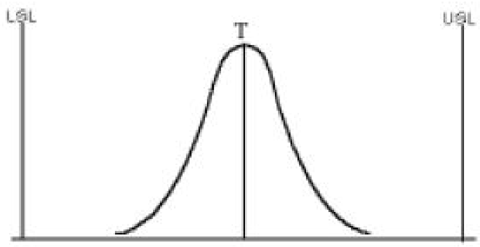
- Another Index C_{pm} incorporates the target when calculating the standard deviation. The standard error, denoted $\hat{\sigma}_{Cpm}$ compares each observation to a reference value.
- However, instead of comparing the data to the mean, the data is compared to the target. These differences are squared. Thus any observation that is different from the target observation will increase the $\hat{\sigma}_{Cpm}$ standard deviation.
- As this difference increases, so does the Cpm. And as this index becomes larger, the C_{pm} gets smaller.
- If the difference between the data and the target is small, so too is the sigma. And as this sigma gets smaller, the Cpm index becomes larger. The higher the C_{pm} index, the better the process.
- In the following charts the process is the same, but as the process becomes more centred, the C_{pm} gets better.



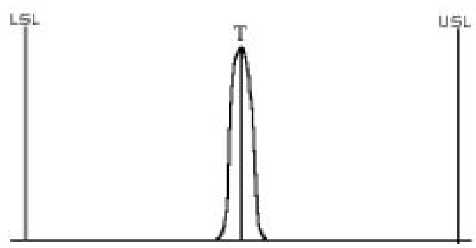
In these 3 charts, the process stays centred about the target, but as the variation is reduced, the Cpm gets better.



This Cpm is reasonably good.



This Cpm is better.



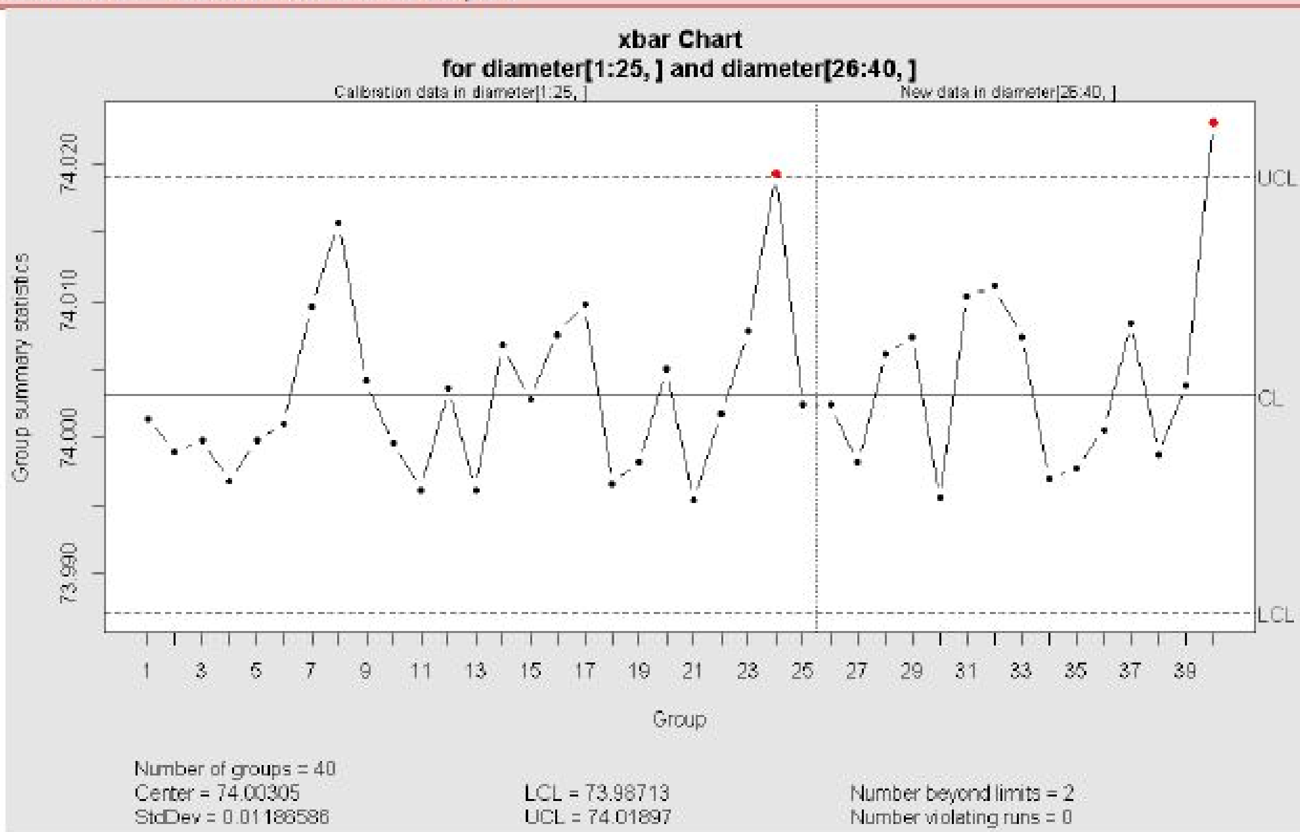
This Cpm is best.

	Population Known	Population Unknown
Cp	$C_p = \dfrac{USL - LSL}{6\sigma}$	$\hat{C}_p = \dfrac{USL - LSL}{6s}$
Cpk	$C_{pk} = \min\,[\dfrac{USL-\mu}{3\sigma}, \dfrac{\mu - LSL}{3\sigma}]$	$\hat{C}_{pk} = \min\,[\dfrac{USL-\bar{x}}{3s}, \dfrac{\bar{x} - LSL}{3s}]$
Cpm	$C_{pm} = \dfrac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$	$\hat{C}_{pm} = \dfrac{USL - LSL}{6\sqrt{s^2 + (\bar{x} - T)^2}}$

4.1.2 Worked Example with R

Data set used diameter (piston rings data set)
R code used previously, reminding ourselves about the data set.

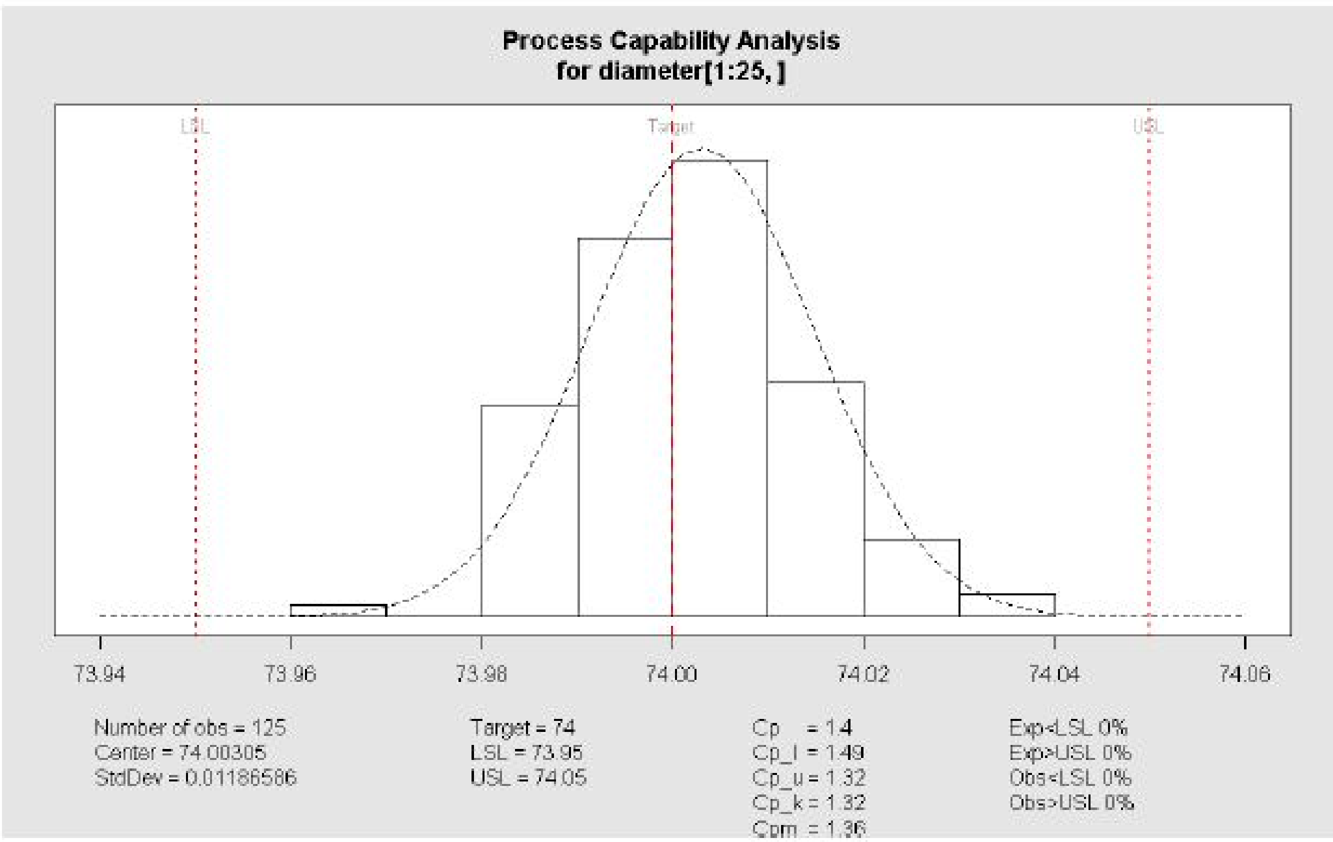
```
data(pistonrings)
attach(pistonrings)
dim(pistonrings)
diameter <- qcc.groups(diameter, sample)
obj <- qcc(diameter[1:25,], type="xbar",
newdata=diameter[26:40,])
```



4.1.3 Implementation of Process Capability Analysis

Indices and Confidence intervals for those indices.

```
> process.capability(obj, spec.limits=c(73.95,74.05))

Process Capability Analysis

Call:
process.capability(object = obj, spec.limits = c(73.95,
74.05))

Number of obs = 125            Target = 74
       Center = 74.00305          LSL = 73.95
       StdDev = 0.01186586        USL = 74.05

Capability indices:

      Value   2.5%  97.5%
Cp    1.405  1.230  1.579
Cp_l  1.490  1.327  1.653
Cp u  1.319  1.173  1.465
Cp k  1.319  1.145  1.493
Cpm   1.360  1.187  1.534

Exp<LSL 0%   Obs<LSL 0%
Exp>USL 0%   Obs>USL 0%
```



Process Capability Analysis
for diameter[1:25,]
LSL
Target
USL
73.94
73.96
73.98
74.00
74.02
74.04
74.06
Number of obs = 125
Center = 74.00305
StdDev = 0.01186586
Target = 74
LSL = 73.95
USL = 74.05
Cp = 1.4
Cp_l = 1.49
Cp_u = 1.32
Cp_k = 1.32
Cpm = 1.36
Exp<LSL 0%
Exp>USL 0%
Obs<LSL 0%
Obs>USL 0%