	Sequence	z - transform
1	$\delta[n]$	1
2	$u[n]$	$\frac{z}{z-1}$
3	b^n	$\frac{z}{z-b}$
4	$b^{n-1}\,u[n-1]$	$\frac{1}{z-b}$
5	$e^{a n}$	$\frac{z}{z-e^a}$
6	n	$\frac{z}{(z-1)^2}$
7	n^2	$\frac{z\,(z+1)}{(z-1)^3}$
8	$b^n\,n$	$\frac{b\,z}{(z-b)^2}$
9	$e^{a n}\,n$	$\frac{z\,e^a}{(z-e^a)^2}$
10	$\sin(a\,n)$	$\frac{\sin(a)\,z}{z^2-2\cos(a)\,z+1}$
11	$b^n\sin(a\,n)$	$\frac{\sin(a)\,b\,z}{z^2-2\cos(a)\,b\,z+b^2}$
12	$\cos(a\,n)$	$\frac{z\,(z-\cos(a))}{z^2-2\cos(a)\,z+1}$
13	$b^n\cos(a\,n)$	$\frac{z\,(z-b\cos(a))}{z^2-2\cos(a)\,b\,z+b^2}$

Table 9.1. z-transforms of some common sequences.

		Sequence	z - transform
	definition	$x_n = x[n]$	$X(z) = \sum_{n=0}^{\infty} x_n\,z^{-n}$
1	addition	$x_n + y_n$	$X(z) + Y(z)$
2	constant multiple	$c\,x_n$	$c\,X(z)$
3	linearity	$c\,x_n + d\,y_n$	$c\,X(z) + d\,Y(z)$
4	delayed unit step	$u[n-m]$	$\frac{z^{1-m}}{z-1}$
5	time delay 1 tap	$x_{n-1}\,u[n-1]$	$\frac{1}{z}\,X(z)$
6	time delayed shift	$x_{n-m}\,u[n-m]$	$z^{-m}\,X(z)$
7	forward 1 tap	x_{n+1}	$z\,(X(z) - x_0)$
8	forward 2 taps	x_{n+2}	$z^2\,(X(z) - x_0 - x_1\,z^{-1})$
9	time forward	x_{n+m}	$z^m\left(X(z) - \sum_{i=0}^{m-1} x_i\,z^{-i}\right)$
10	complex translation	$e^{a n}\,x_n$	$X(z\,e^{-a})$
11	frequency scale	$b^n\,x_n$	$X\left(\frac{z}{b}\right)$
12	differentiation	$n\,x_n$	$-z\,X'(z)$
13	integration	$\frac{1}{n}\,x_n$	$-\int \frac{X(z)}{z}\,dz$
14	integration shift	$\frac{1}{n+m}\,x_n$	$-z^{-m}\int \frac{X(z)}{z^{m+1}}\,dz$
15	discrete time convolution	$x_n * y_n = \sum_{i=0}^{n} x_i\,y_{n-i}$	$X(z)\,Y(z)$
16	convolution with $y_n = 1$	$\sum_{i=0}^{n} x_i$	$\frac{z}{z-1}\,X(z)$
17	initial time	x_0	$\lim_{z\to\infty} X(z)$
18	final value	$\lim_{n\to\infty} x_n$	$\lim_{z\to 1}\,(z-1)\,X(z)$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

$$L\{x\} = \bar{x} = X(s)$$
$$L\{\dot{x}\} = s\bar{x} - x_0$$
$$L\{\ddot{x}\} = s^2\bar{x} - sx_0 - x_1$$
$$L\{\dddot{x}\} = s^3\bar{x} - s^2x_0 - sx_1 - x_2$$

Matrices RULES

Addition/Subtraction of Matrices

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1n} \\ B_{21} & B_{22} & \dots & B_{2n} \\ \dots & \dots & \dots & \dots \\ B_{m1} & B_{m2} & \dots & B_{mn} \end{bmatrix} = \begin{bmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \dots & A_{1n}+B_{1n} \\ A_{21}+B_{21} & A_{22}+B_{22} & \dots & A_{2n}+B_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1}+B_{m1} & A_{m2}+B_{m2} & \dots & A_{mn}+B_{mn} \end{bmatrix}$$

Matrix Multiplication

Matrix can be Multiplied two ways,

- *Scalar Multiplication* **–** It involves multiplying a scalar quantity to the matrix. Every element inside the matrix is to be multiplied by the scalar quantity to form a new matrix.

For example-

$$5 \times \begin{bmatrix} 5 & 7 \\ 12 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 25 & 35 \\ 60 & 15 \\ 30 & 10 \end{bmatrix}$$

- *Multiplication of a matrix with another matrix:* Two matrices can be multiplied if the number of column of the first matrix is equal to the number of rows of the second matrix.

Consider two matrix **M1** & **M2,** having order of $m_1 \times n_1$ and $m_2 \times n_2$.

The matrices can be multiplied if and only if $n_1 = m_2$.

$$[1 \quad 2 \quad 3]_{1\times3} . \begin{bmatrix} 2 & 1 & 3 \\ 3 & 3 & 2 \\ 4 & 1 & 2 \end{bmatrix}_{3\times3} = [1.2+2.3+3.4 \quad 1.1+2.3+3.1 \quad 1.3+2.2+3.2]$$
$$= [20 \quad 10 \quad 13]_{1\times3}$$

The matrices, given above satisfies the condition for matrix multiplication, hence it is possible to multiply those matrices.

The resultant matrix obtained by multiplication of two matrices, is the order of m_1, n_2, where m_1 is the number of rows in the 1st matrix and n_2 is the number of column of the 2nd matrix.

Some rules:

The inverse rules of matrices are as follows:

- AI = IA = A
- $AA^{-1} = A^{-1}A = I$
- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
- $(A')^{-1} = (A^{-1})'$
- A+B=B+A
- c(A+B)=cA+cB
- A+(B+C)=(A+B)+C
- C(A+B)= CA+CB
- (A+B)C=AC+BC
- A(BC) = (AB)C
- AB≠BA

- A+B = B+A →Commutative Law of Addition
- A+B+C = A +(B+C) = (A+B)+C →Associative law of addition
- ABC = A(BC) = (AB)C →Associative law of multiplication
- A(B+C) = AB + AC →Distributive law of matrix algebra
- R(A+B) = RA + RB

Also, see here rules for transposition of matrices:

- (A’)’ = A
- (A+B)’ = A’+B’
- (AB)’ = B’A’
- (ABC) = C’B’A’

Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

A^{-1} = adj (A) / det(A) = adj (A) / |A|

$$|A| = 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

|A| = 2(4-1) -1(6-2)+1(3-4)=1

Adj(A): A_c

$$A_c = \begin{bmatrix} (4-1) & -(6-2) & (3-4) \\ -(2-1) & (4-2) & -(2-2) \\ (1-2) & -(2-3) & (4-3) \end{bmatrix} = \begin{bmatrix} 3 & -4 & -1 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

transpose of the cofactor matrix

$$A_c^T = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^k$	$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+\cdots+\frac{A_k}{(ax+b)^k}$, $k=1,2,3,\ldots$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_kx+B_k}{(ax^2+bx+c)^k}$, $k=1,2,3,\ldots$

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	• where x^2+bx+c cannot be factorised further	

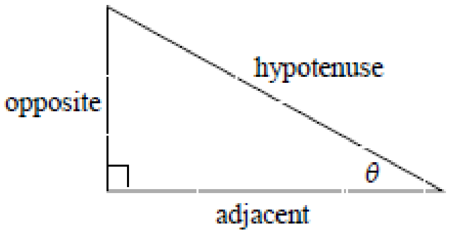
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



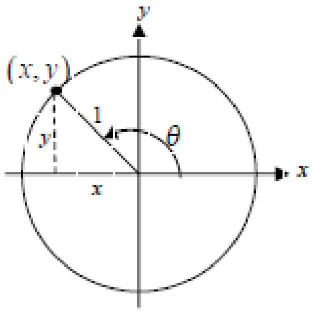
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1+\cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2}-\theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2}-\theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

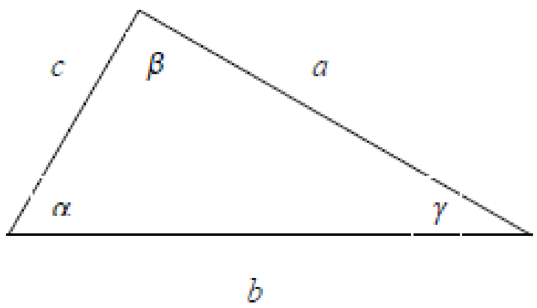
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

$$\log_b(M) = X \leftrightarrow b^X = M$$

$$\log_b(b) = 1$$

$$\log_b(1) = 0$$

$$\log_b(0) \rightarrow \text{not defined}$$

$$\log_b(b^k) = k \tag{1}$$

$$b \log_b(M) = M \tag{2}$$

$$\text{if} \log_b(M) = \log_b(N), \text{ then } M = N \tag{3}$$

$$\log_b(M^k) = k \cdot \log_b(M) \tag{4}$$

$$b^{\log_b(M)} = M \tag{5}$$

$$\log_b\left(\frac{1}{M}\right) = \log_b(M^{-1}) = -1 \cdot \log_b(M) \tag{6}$$

$$\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N) \tag{7}$$

$$\log_b\left(M \cdot N\right) = \log_b(M) + \log_b(N) \tag{8}$$

$$\log_b(M) = \frac{\log_c(M)}{\log_c(b)} \tag{9}$$

Exponential Laws

$$x^a \cdot x^b = x^{a+b}$$

$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{ab}$$

$$x^{-a} = \frac{1}{x^a}$$

$$x^0 = 1$$

n=power (result obtained by raising b to the power of a)

$$\log_b n = a \text{ and } b^a = n$$

b=base

a=exponent

Differential rules

In the following formulas, let *a, b, c, n* be constants, and $u = f(x); v = g(x); w = h(x)$		
General Formulas		
1	$\frac{d}{dx}c = 0$	**Constant Rule**
2	$\frac{d}{dx}cx = c$	**Factor Rule**
3	$\frac{d}{dx}(cu) = c\frac{du}{dx}$	**Factor Rule**
4	$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$	**Sum Rule**
5	$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$	**Product Rule**
6	$\frac{d}{dx}(uvw) = \frac{du}{dx}vw + u\frac{dv}{dx}w + uv\frac{dw}{dx}$	**Product Rule**
7	$\frac{d}{dx}\left(u/v\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	**Quotient Rule**
8	$\frac{du}{dx} = \frac{du}{dv}\frac{dv}{dx}$	**Chain Rule**
9	$\frac{d}{dx}x^n = nx^{n-1}$	**Power Rule**
10	$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$	**Power Rule**
11	$\frac{du}{dx} = \frac{1}{dx/du}$	

Exponential and Logarithmic Functions			
12	$\frac{d}{dx}e^x = e^x$	13	$\frac{d}{dx}a^x = a^x \ln a$
14	$\frac{d}{dx}\ln\lvert x\rvert = \frac{1}{x}$	15	$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$
16	$\frac{d}{dx}u^v = vu^{v-1}\frac{du}{dx} + u^v a^x \ln u\frac{dv}{dx}$		
Trigonometric Functions			
17	$\frac{d}{dx}\sin x = \cos x$	18	$\frac{d}{dx}\cos x = -\sin x$

19	$\frac{d}{dx}\tan x = \sec^2 x$	20	$\frac{d}{dx}\cot x = -\csc^2 x$
21	$\frac{d}{dx}\csc x = -\csc x \cot x$	22	$\frac{d}{dx}\sec x = \sec x \tan x$
Inverse Trigonometric Functions			
23	$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	24	$\frac{d}{dx}\cos^{-1} x = \frac{1}{\sqrt{1-x^2}}$
25	$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	26	$\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$
27	$\frac{d}{dx}\csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$	28	$\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$
Hyperbolic Functions			
29	$\frac{d}{dx}\sinh x = \cosh x$	30	$\frac{d}{dx}\cosh x = \sinh x$
31	$\frac{d}{dx}\tanh x = \operatorname{sech} x^2$	32	$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$
33	$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$	34	$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$
Inverse Hyperbolic Functions			
35	$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$	36	$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$
37	$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$	38	$\frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}$
39	$\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{\lvert x \rvert \sqrt{x^2+1}}$	40	$\frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$

In the following formulas, let *a*, *b*, *c*, *n* be constants, and
$u = f(x); v = g(x); w = h(x)$

General Formulas	
1	$\int a dx = ax + c$
2	$\int af(x)dx = a\int f(x)dx$
3	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \quad (n \neq -1)$
4	$\int (u \pm v \pm w)dx = \int u dx \pm \int v dx \pm \int w dx$
5	$\int u dv = uv - \int v du$ Integration by parts
6	$\int F(u)dx = \int \frac{F(u)}{u'} du$
7	$\int \frac{1}{x} dx = \ln\|x\| + c$
8	$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
9	$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\| + c$

Trigonometric Functions			
10	$\int \sin x dx = -\cos x + c$	11	$\int \cos x dx = \sin x + c$
12	$\int \tan x dx = \ln\|\sec x\| + c$	13	$\int \cot x dx = \ln\|\sin x\| + c$
14	$\int \sec^2 x dx = \tan x + c$	15	$\int \csc^2 x dx = -\cot x + c$
16	$\int \tan^2 x dx = \tan x - x + c$	17	$\int \cot^2 x dx = \cot x - x + c$
18	$\int \sec x \tan x dx = \sec x + c$	19	$\int \csc x \cot x dx = -\csc x + c$
20	$\int \sec x dx = \ln\|\sec x + \tan x\| + c$	21	$\int \csc x dx = \ln\|\csc x - \cot x\| + c$
22	$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$	23	$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$
24	$\int \sin^n x dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$		
25	$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x dx$		

Integration rules

Hyperbolic Functions

26	$\int \sinh x dx = \cosh x + c$	27	$\int \cosh x dx = \sinh x + c$
28	$\int \tanh x dx = \ln \cosh x + c$	29	$\int \coth x dx = \ln \mid \sinh x \mid + c$
30	$\int \operatorname{sech} x dx = \sin^{-1}(\tanh x) + c$	31	$\int \operatorname{csch} x dx = \ln \tanh \frac{x}{2} + c$
32	$\int \operatorname{sech}^2 x dx = \tanh x + c$	33	$\int \operatorname{csch}^2 x dx = -\coth x + c$
34	$\int \tanh^2 x dx = x - \tanh x + c$	35	$\int \coth^2 x dx = x - \coth x + c$
36	$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + c$	37	$\int \operatorname{csch} x \coth x dx = -\operatorname{csch} x + c$
38	$\int \sinh^2 x dx = \frac{\sinh 2x}{4} - \frac{x}{2} + c$	39	$\int \cosh^2 x dx = \frac{\sinh 2x}{4} + \frac{x}{2} + c$

Exponential and Logarithmic Functions

40	$\int e^x dx = e^x + c$	41	$\int a^x dx = \frac{a^x}{\ln a} + c$
42	$\int \ln x dx = x(\ln x - 1) + c$	43	$\int \log_a x dx = \frac{x}{\ln a}(\ln x - 1) + c$
44	$\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + c$	45	$\int \frac{e^{ax}}{x} dx = \ln \mid x \mid + \sum_{i=1}^{\infty} \frac{(ax)^i}{i \cdot i!} + c$
46	$\int x^2 e^{ax} dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) + c$		
47	$\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$		
48	$\int \frac{e^{ax}}{x^n} dx = \frac{1}{n-1}\left(-\frac{e^{ax}}{x^{n-1}} + a \int \frac{e^{ax}}{x^{n-1}} dx\right) + c \qquad (n \neq 1)$		
49	$\int x^n \ln x dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + c$		