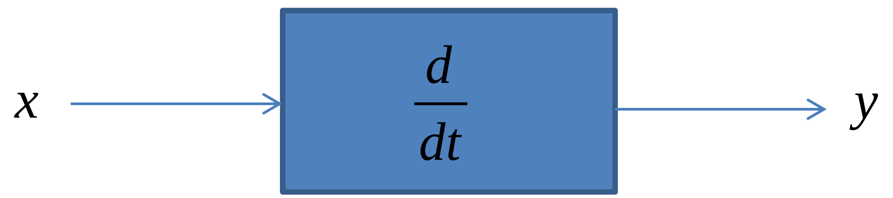
Automatic Control Systems

Lecture-5

Block Diagram Representation of Control Systems

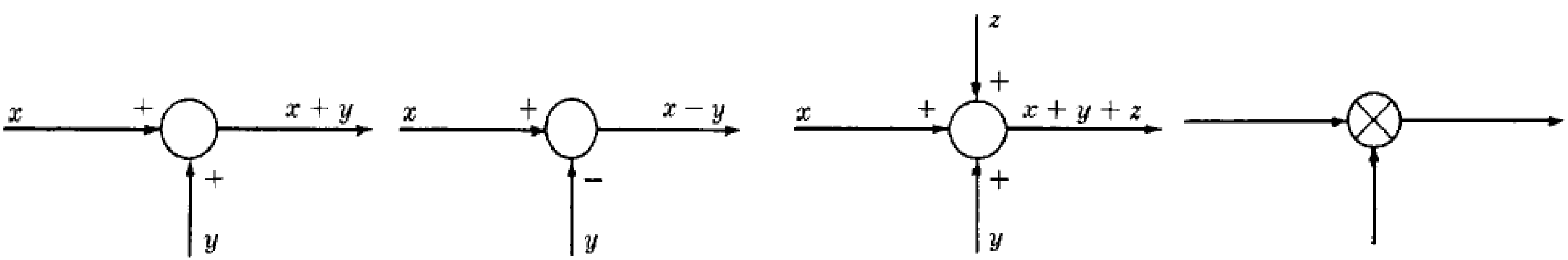
Introduction

- A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.
- The interior of the rectangle representing the block usually contains a description of or the name of the element, gain, or the symbol for the mathematical operation to be performed on the input to yield the output.
- The arrows represent the direction of information or signal flow.



Introduction

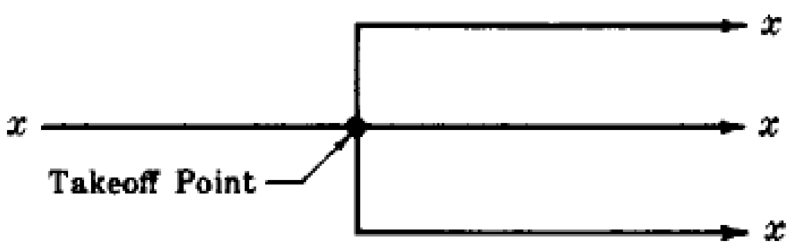
- The operations of addition and subtraction have a special representation.
- The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.
- The output is the algebraic sum of the inputs.
- Any number of inputs may enter a summing point.
- Some books put a cross in the circle.

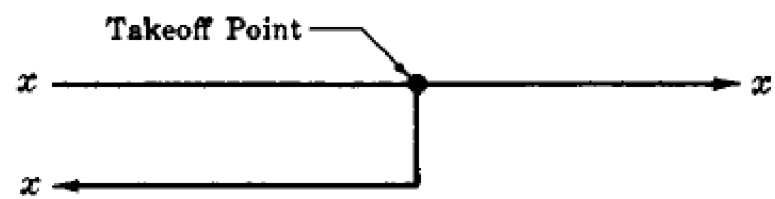


Introduction

- In order to have the same signal or variable be an input to more than one block or summing point, a takeoff (or pickoff) point is used.

- This permits the signal to proceed unaltered along several different paths to several destinations.

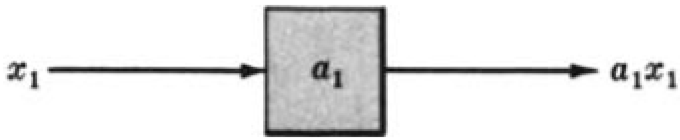


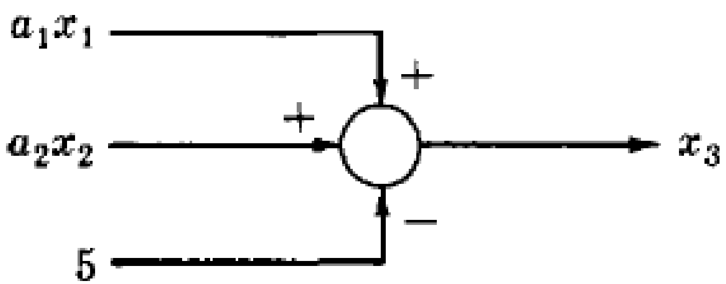


Example-1

- Consider the following equations in which x_1, x_2, x_3, are variables, and a_1, a_2 *are* general coefficients or mathematical operators.

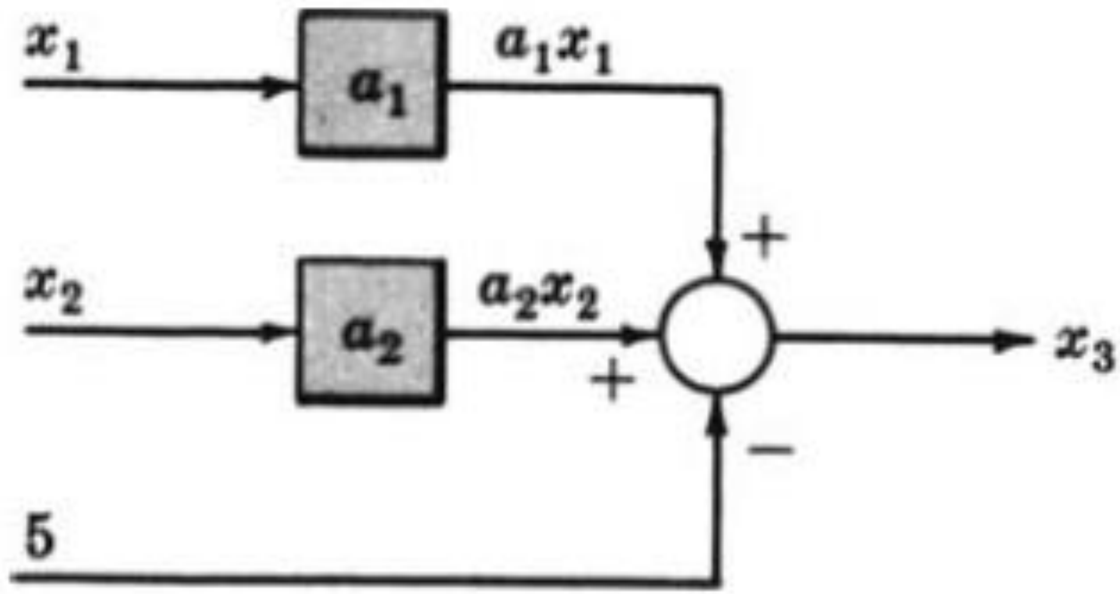
$$x_3 = a_1 x_1 + a_2 x_2 - 5$$





Example-1

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$



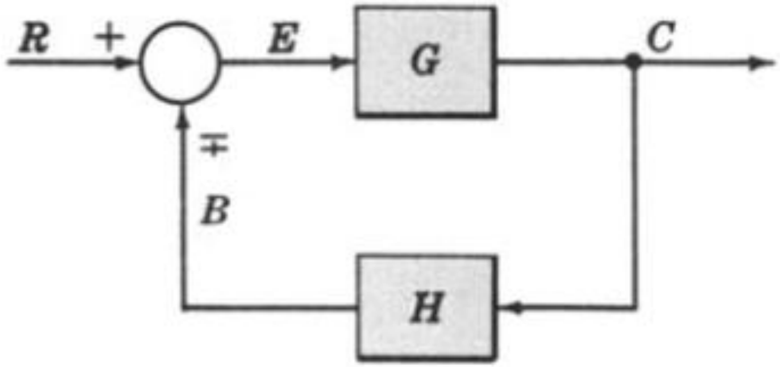
Example-2

- Draw the Block Diagrams of the following equations.

(1) $$x_2 = a_1 \frac{dx_1}{dt} + \frac{1}{b} \int x_1 dt$$

(2) $$x_3 = a_1 \frac{d^2 x_2}{dt^2} + 3 \frac{dx_1}{dt} - bx_1$$

Canonical Form of A Feedback Control System



$G \equiv$ direct transfer function $\equiv$ forward transfer function

$H \equiv$ feedback transfer function

$GH \equiv$ loop transfer function $\equiv$ open-loop transfer function

$C/R \equiv$ closed-loop transfer function $\equiv$ control ratio $\quad \dfrac{C}{R} = \dfrac{G}{1 \pm GH}$

$E/R \equiv$ actuating signal ratio $\equiv$ error ratio $\quad \dfrac{E}{R} = \dfrac{1}{1 \pm GH}$

$B/R \equiv$ primary feedback ratio $\quad \dfrac{B}{R} = \dfrac{GH}{1 \pm GH}$

Characteristic Equation

- The control ratio is the closed loop transfer function of the system.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

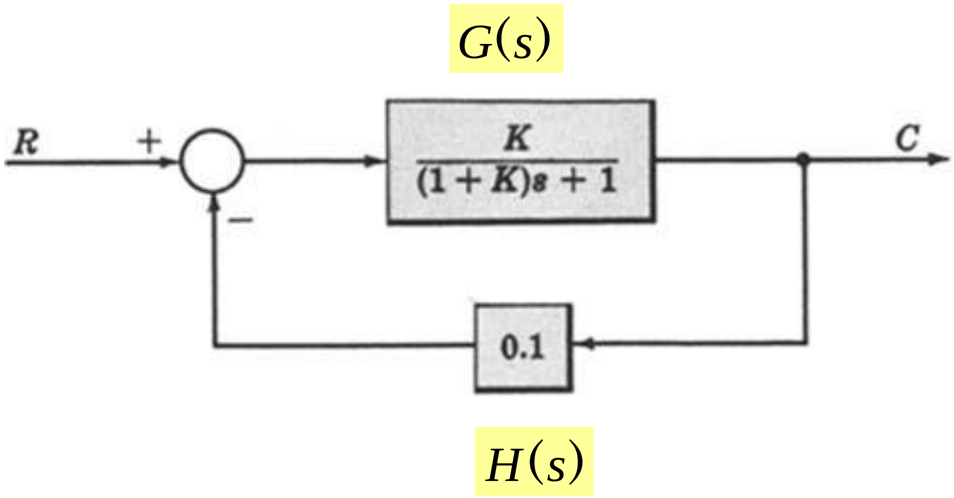
- The denominator of closed loop transfer function determines the characteristic equation of the system.

- Which is usually determined as:

$$1 \pm G(s)H(s) = 0$$

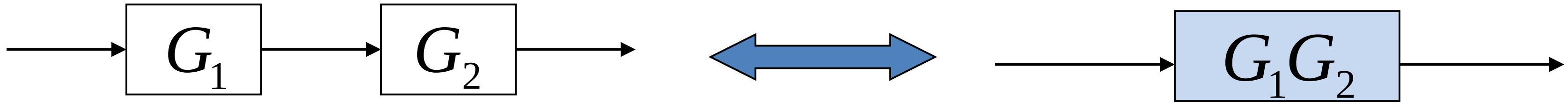
Example-3

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$
3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$
5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$
6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$
7. characteristic equation $1+G(s)H(s)=0$
8. Open loop poles and zeros if **9.** closed loop poles and zeros if K=10.

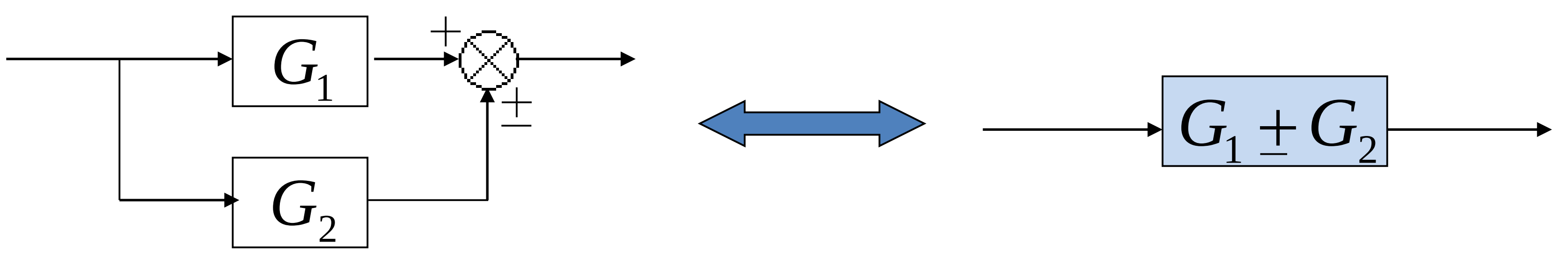


Reduction techniques

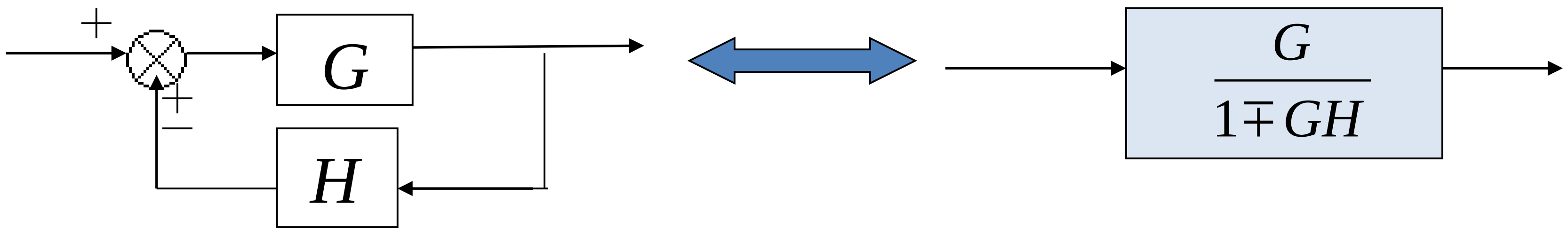
1. Combining blocks in cascade

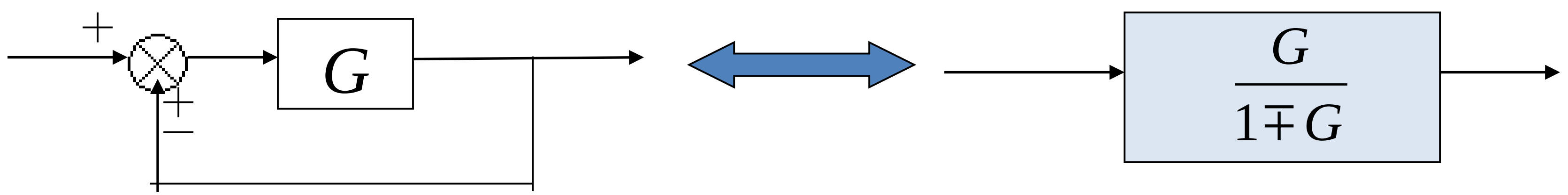


2. Combining blocks in parallel



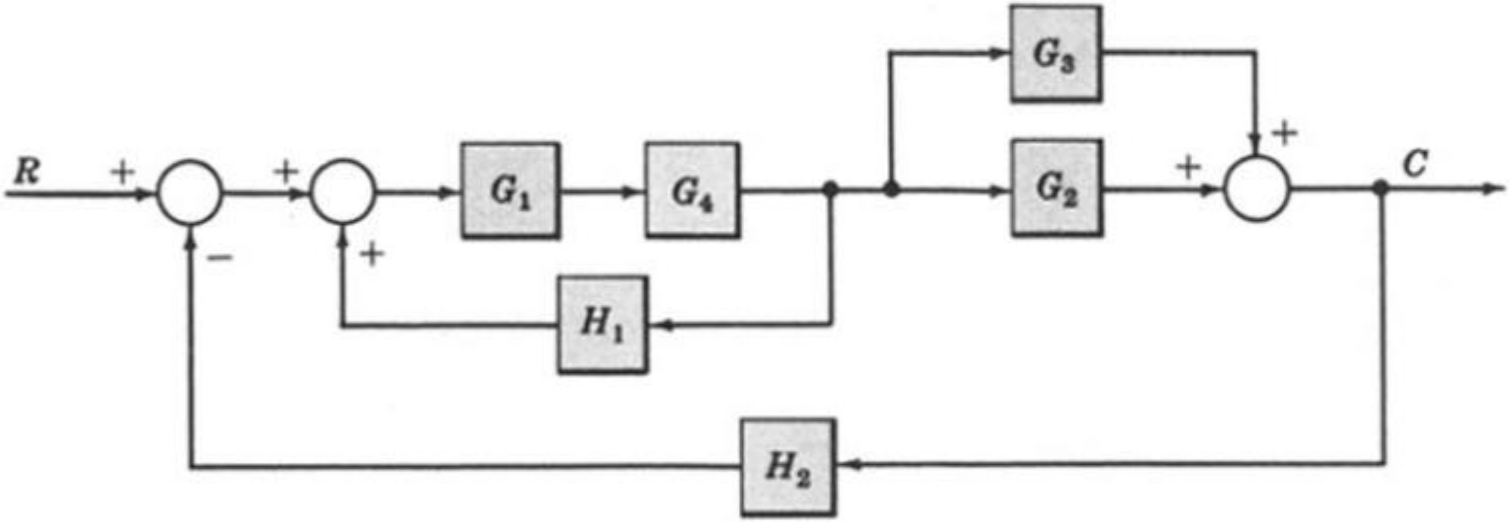
3. Eliminating a feedback loop





$$H = 1$$

Example-4: Reduce the Block Diagram to Canonical Form.

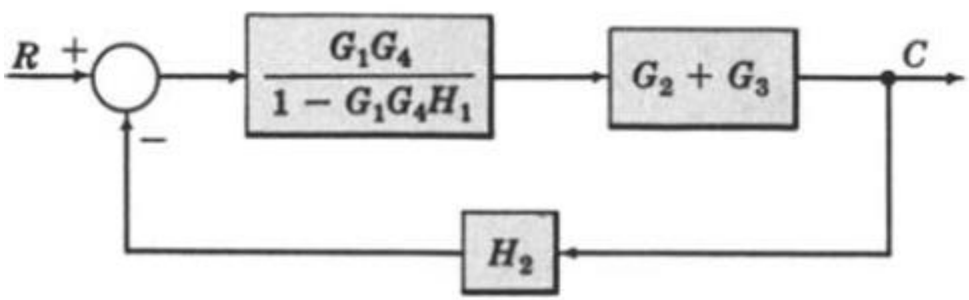


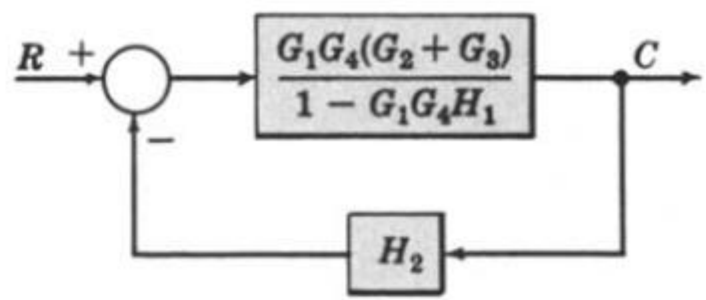
Step 1: Combine all cascade blocks

Step 2: Combine all parallel blocks

Example-4: Continue.

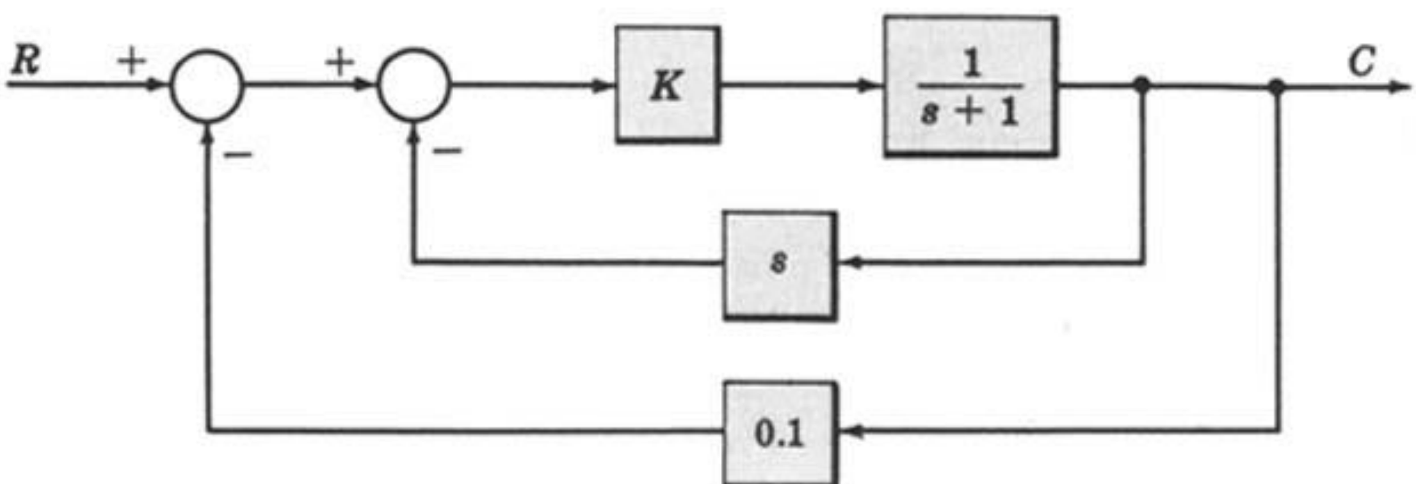
Step 3: Eliminate all minor feedback loops





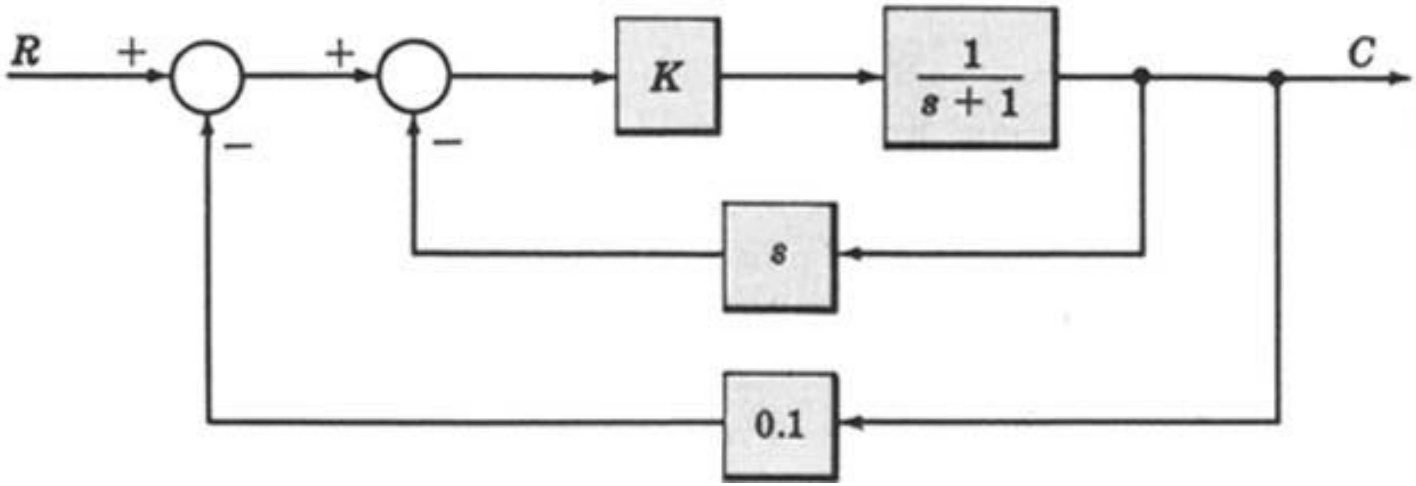
Example-5

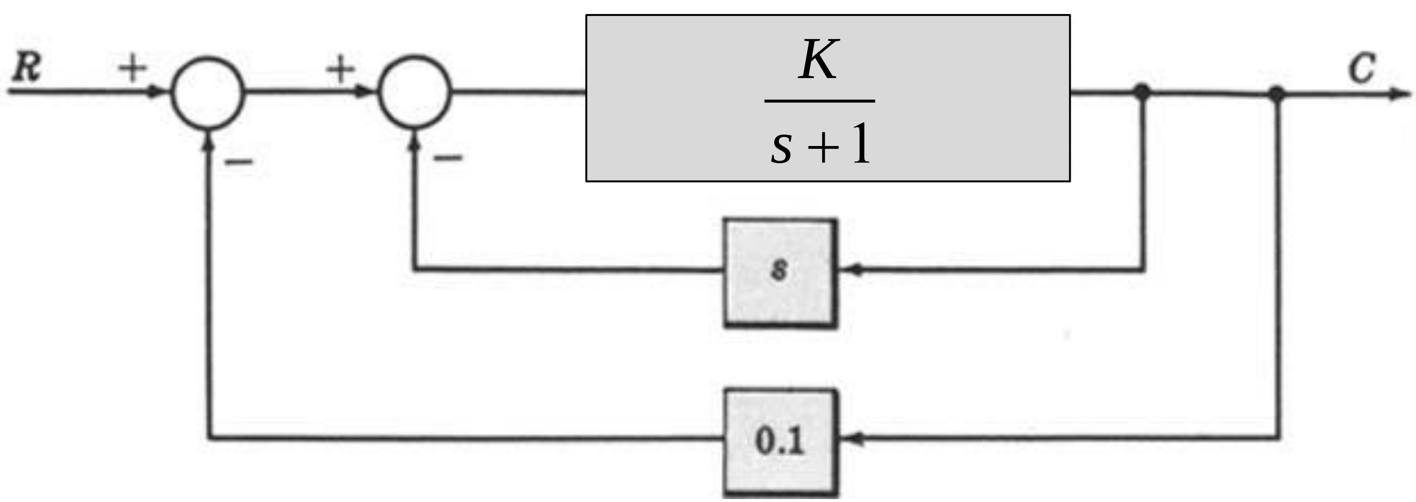
- For the system represented by the following block diagram determine:
  1. Open loop transfer function
  2. Feed Forward Transfer function
  3. control ratio
  4. feedback ratio
  5. error ratio
  6. closed loop transfer function
  7. characteristic equation
  8. closed loop poles and zeros if K=10.



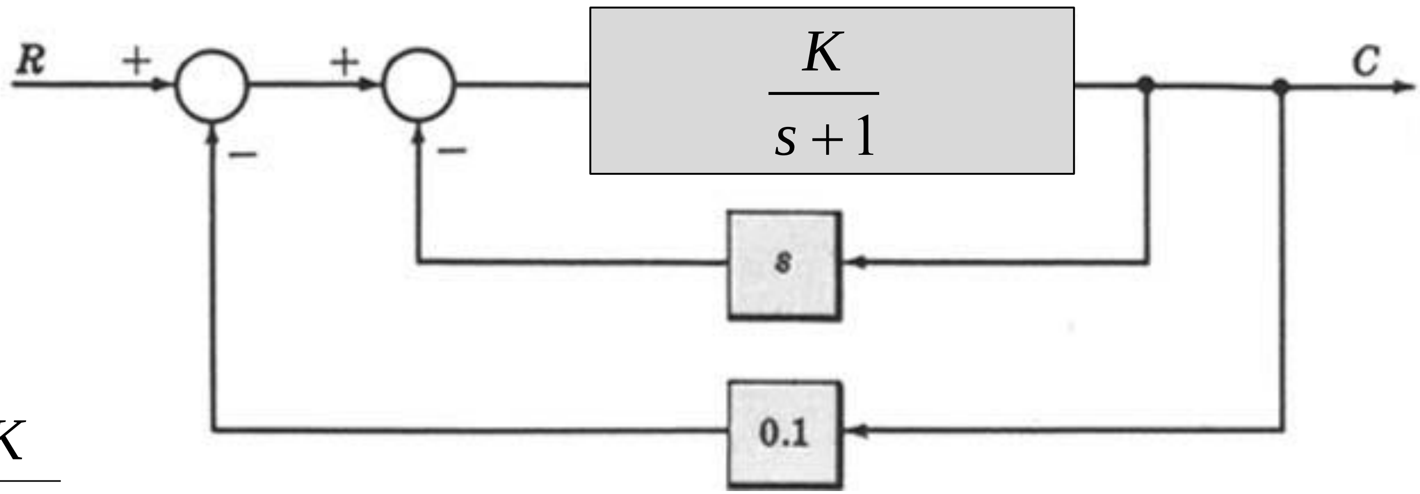
Example-5

– First we will reduce the given block diagram to canonical form

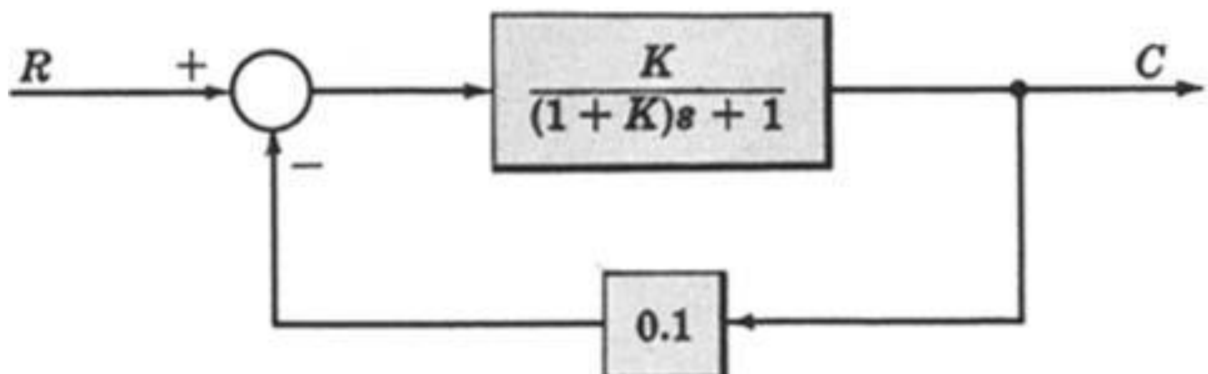




Example-5

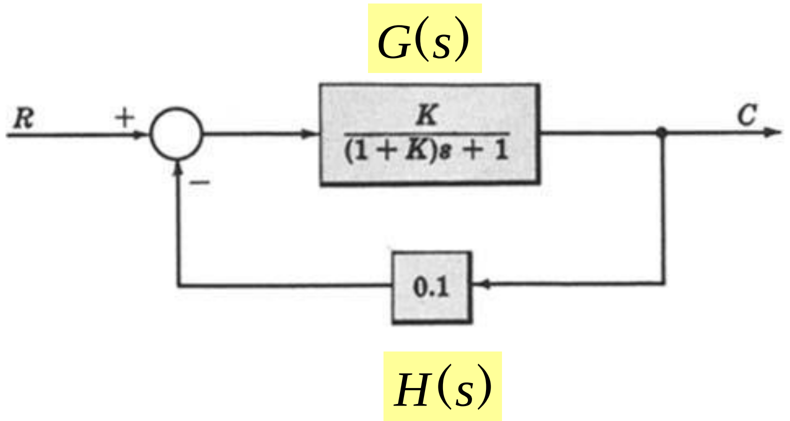


$$\frac{G}{1+GH} = \frac{\dfrac{K}{s+1}}{1+\dfrac{K}{s+1}s}$$



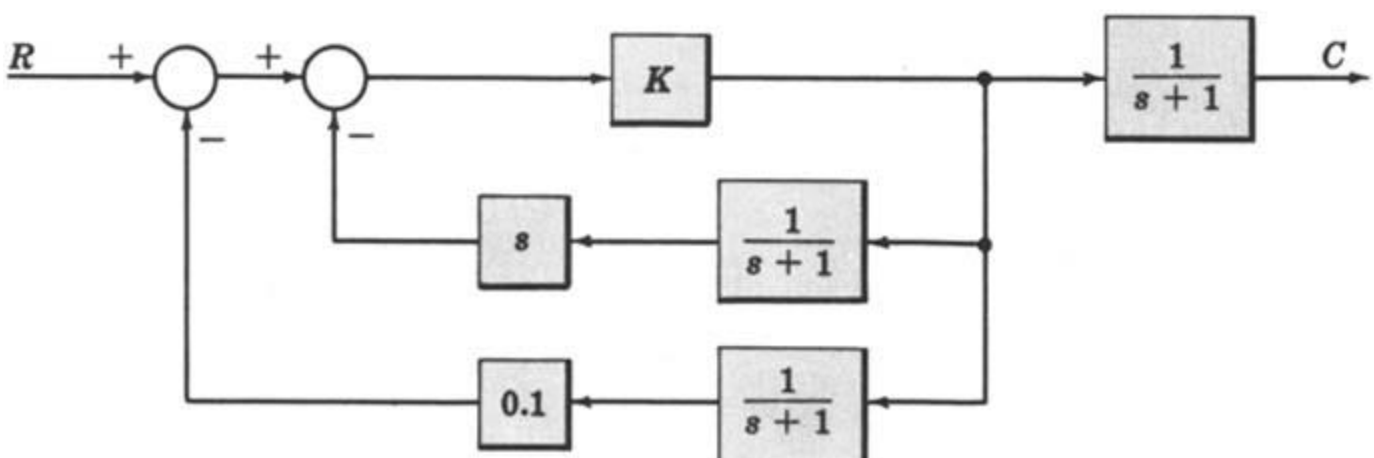
Example-5 (see example-3)

1. Open loop transfer function $\frac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer function $\frac{C(s)}{E(s)} = G(s)$
3. control ratio $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$
4. feedback ratio $\frac{B(s)}{R(s)} = \frac{G(s)H(s)}{1+G(s)H(s)}$
5. error ratio $\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$
6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$
7. characteristic equation $1+G(s)H(s) = 0$
8. closed loop poles and zeros if K=10.



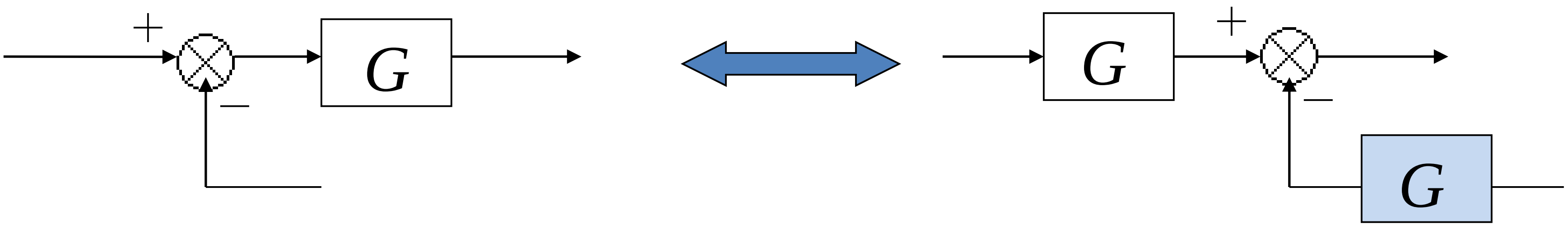
Example-6

- For the system represented by the following block diagram determine:
  1. Open loop transfer function
  2. Feed Forward Transfer function
  3. control ratio
  4. feedback ratio
  5. error ratio
  6. closed loop transfer function
  7. characteristic equation
  8. closed loop poles and zeros if K=100.

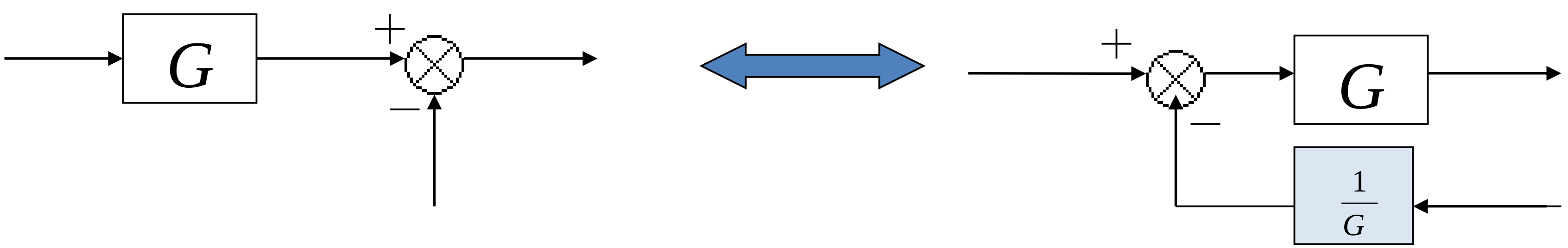


Reduction techniques

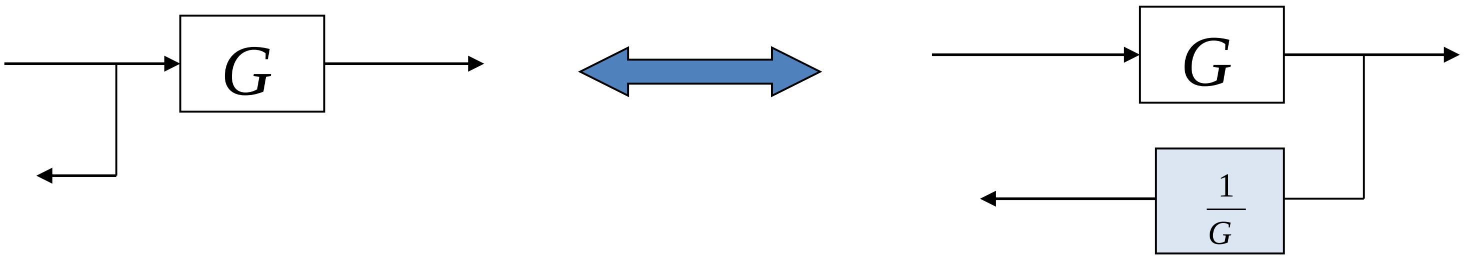
4. Moving a summing point behind a block



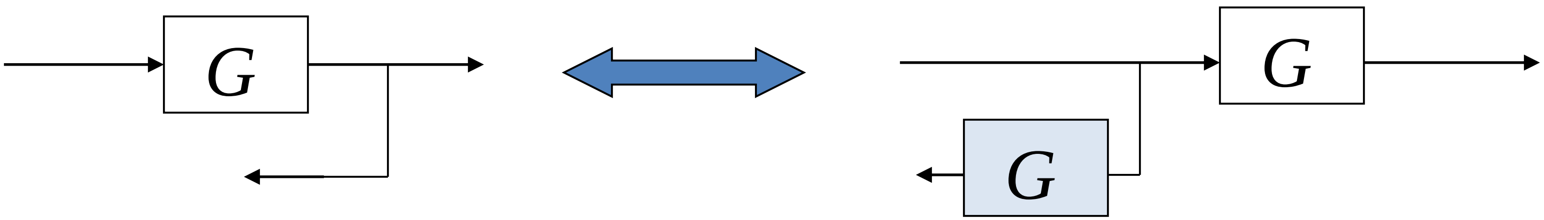
5. Moving a summing point ahead a block



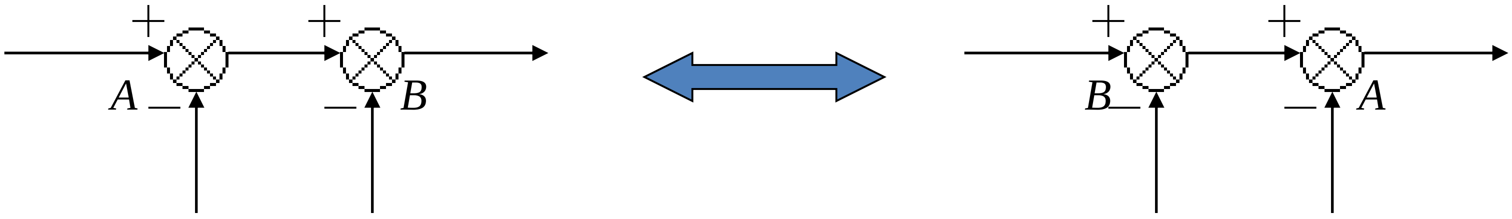
6. Moving a pickoff point behind a block



7. Moving a pickoff point ahead of a block

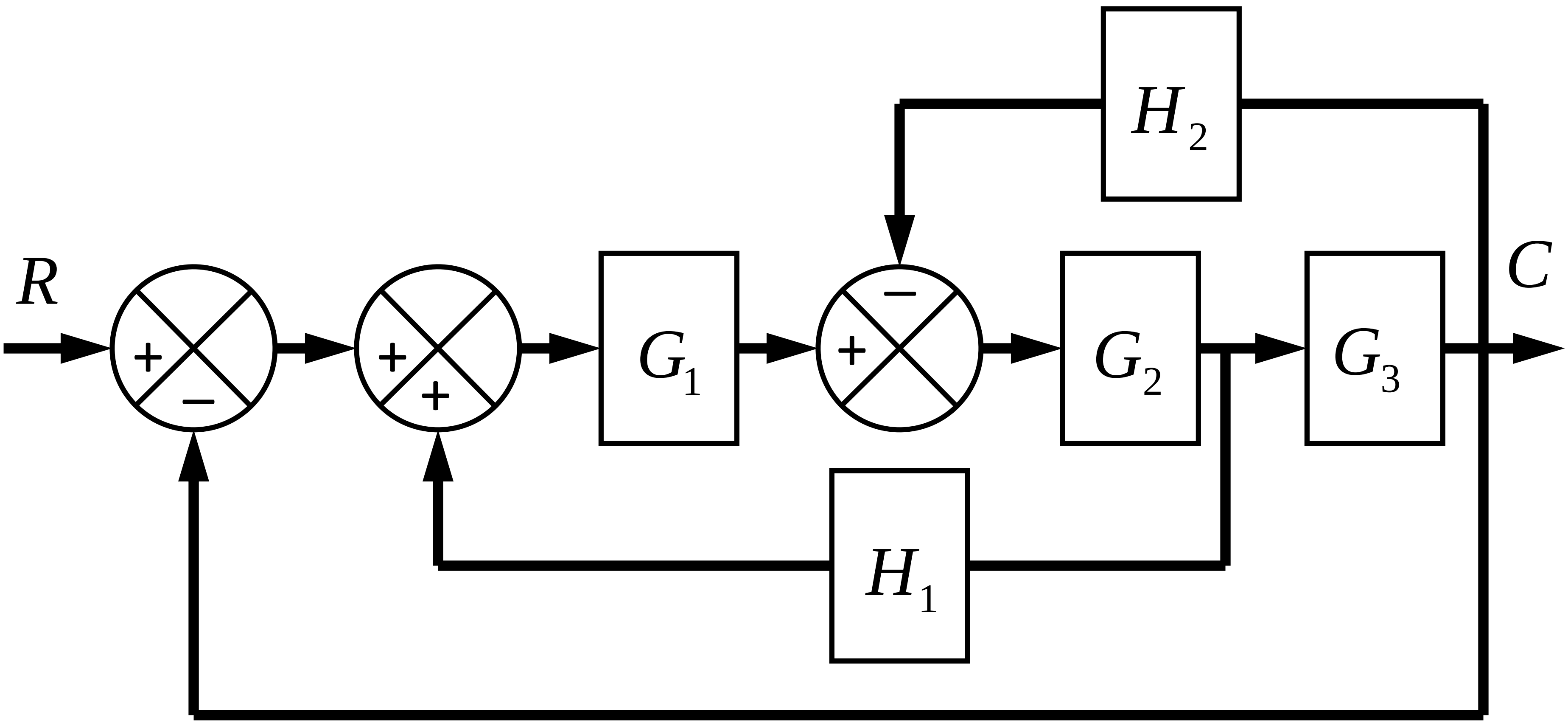


8. Swap with two neighboring summing points

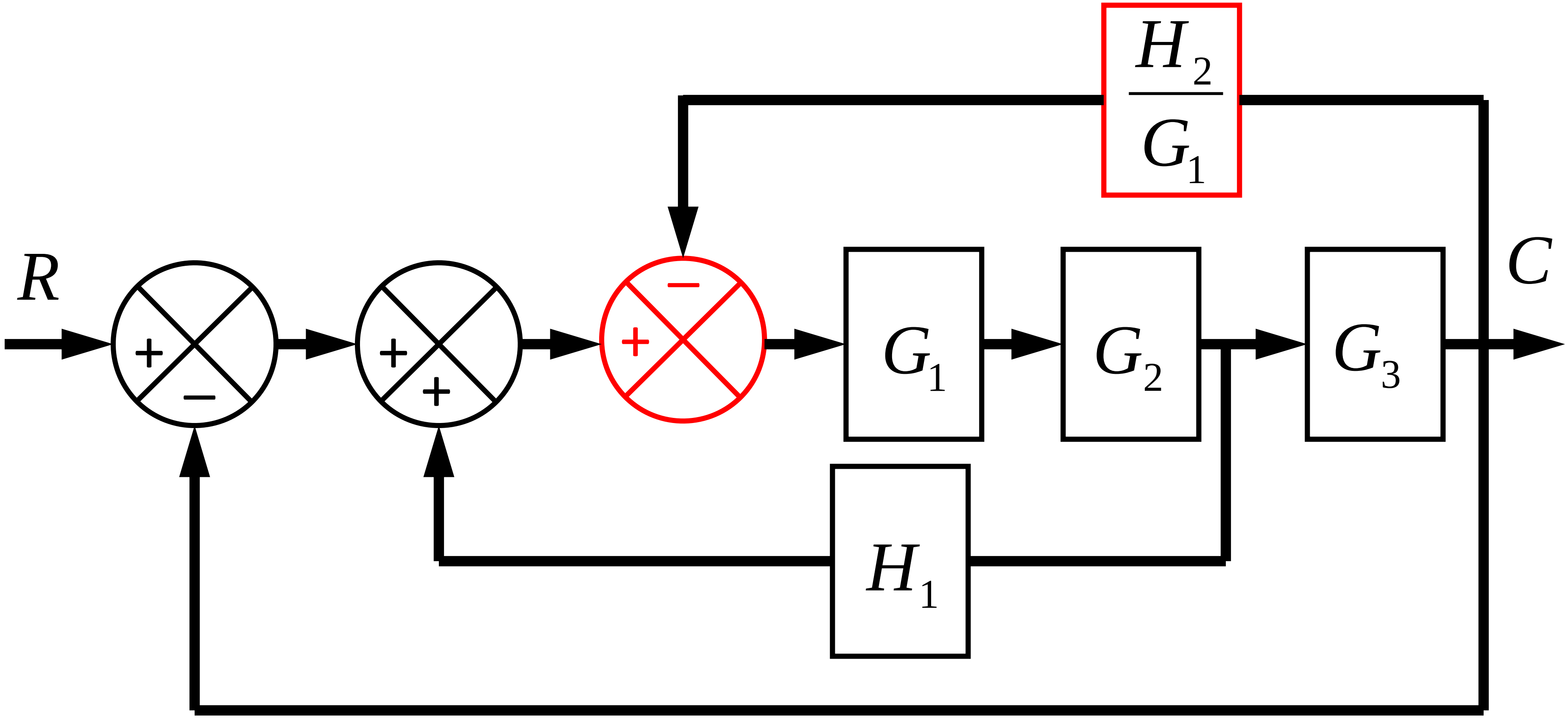


Example-7

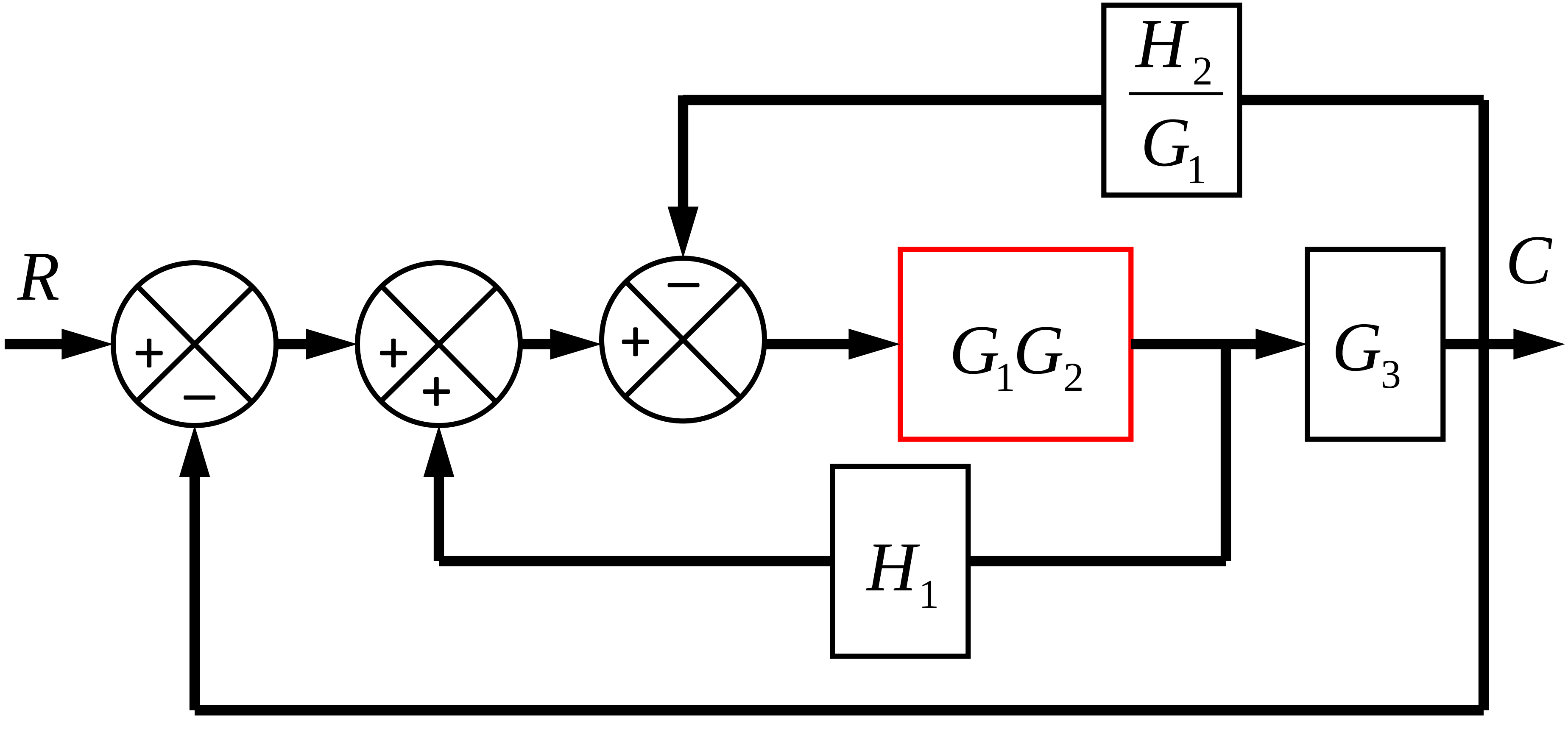
- Reduce the following block diagram to canonical form.



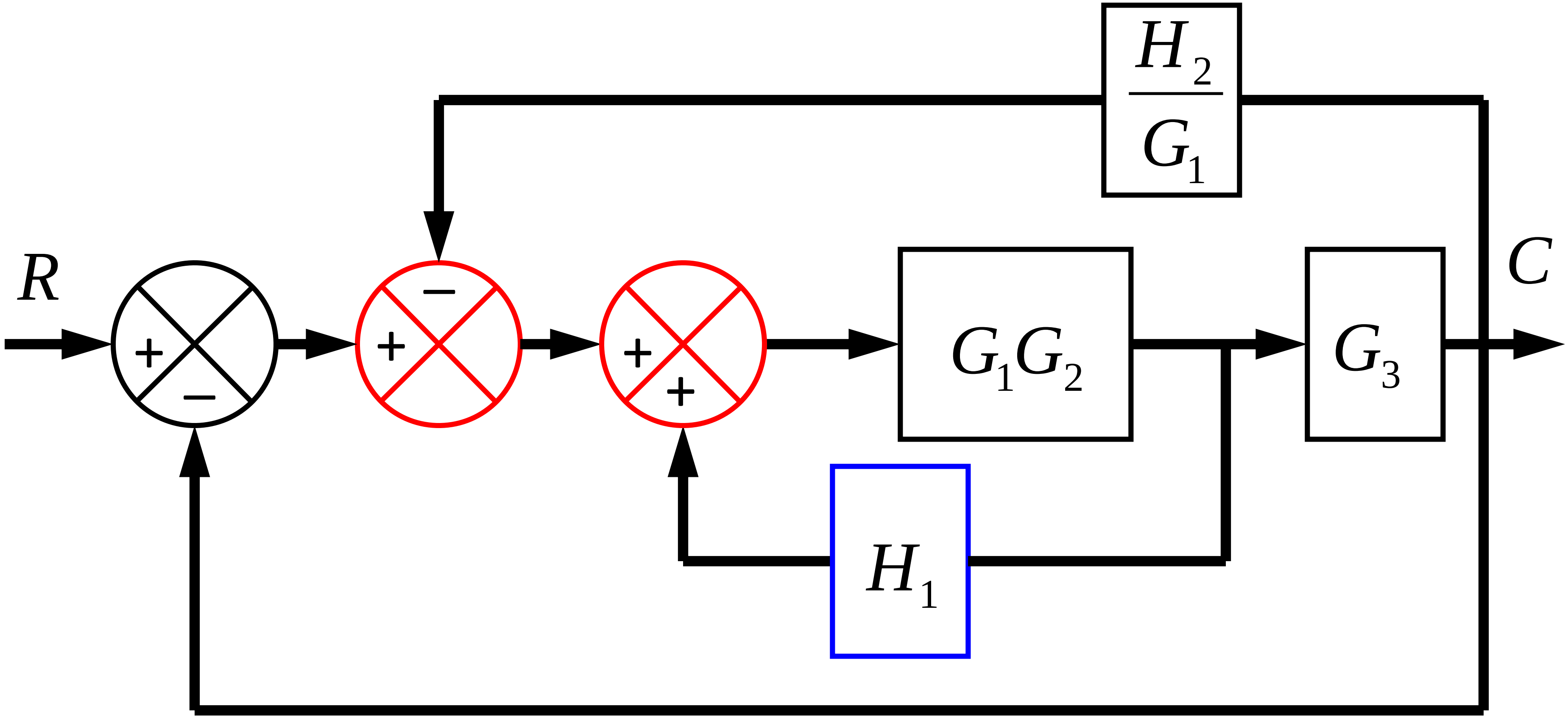
Example-7



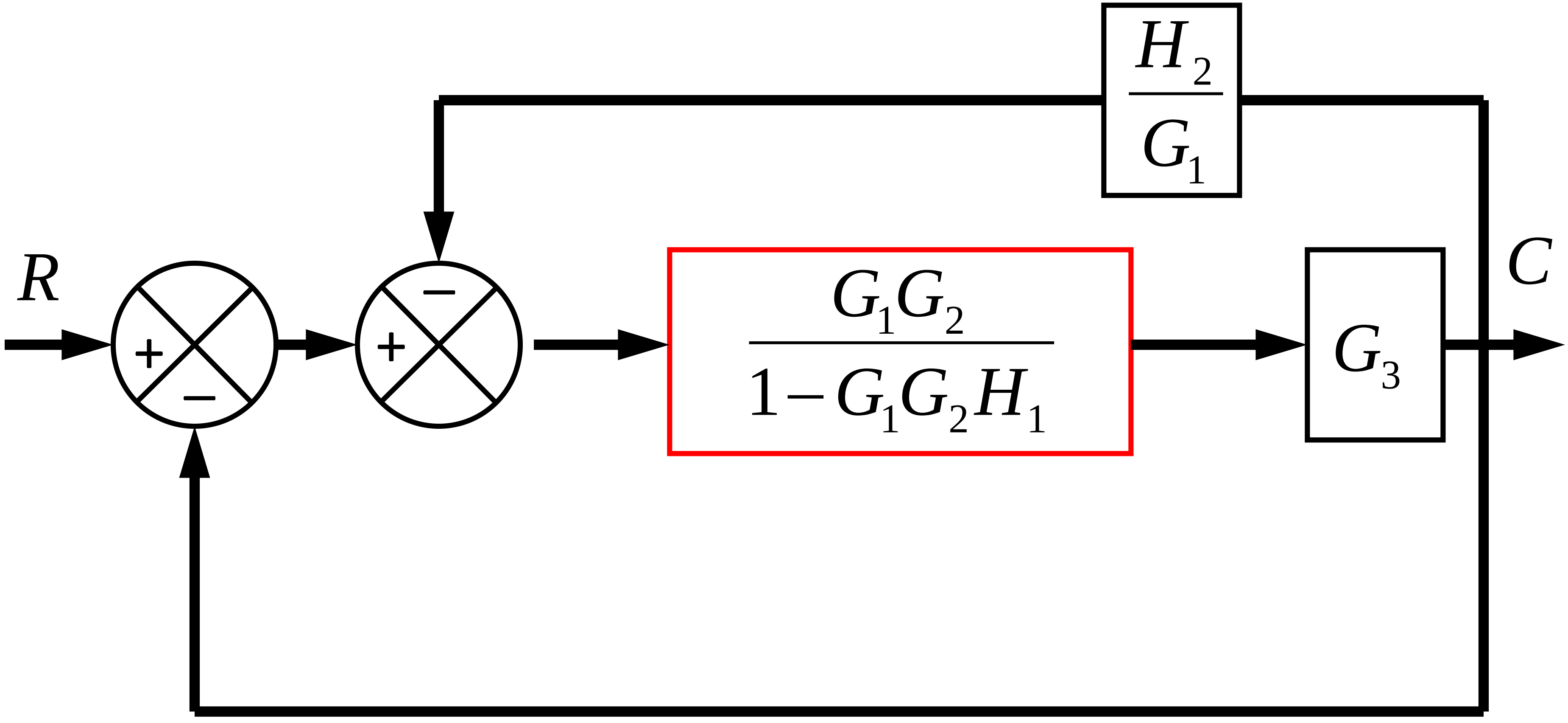
Example-7



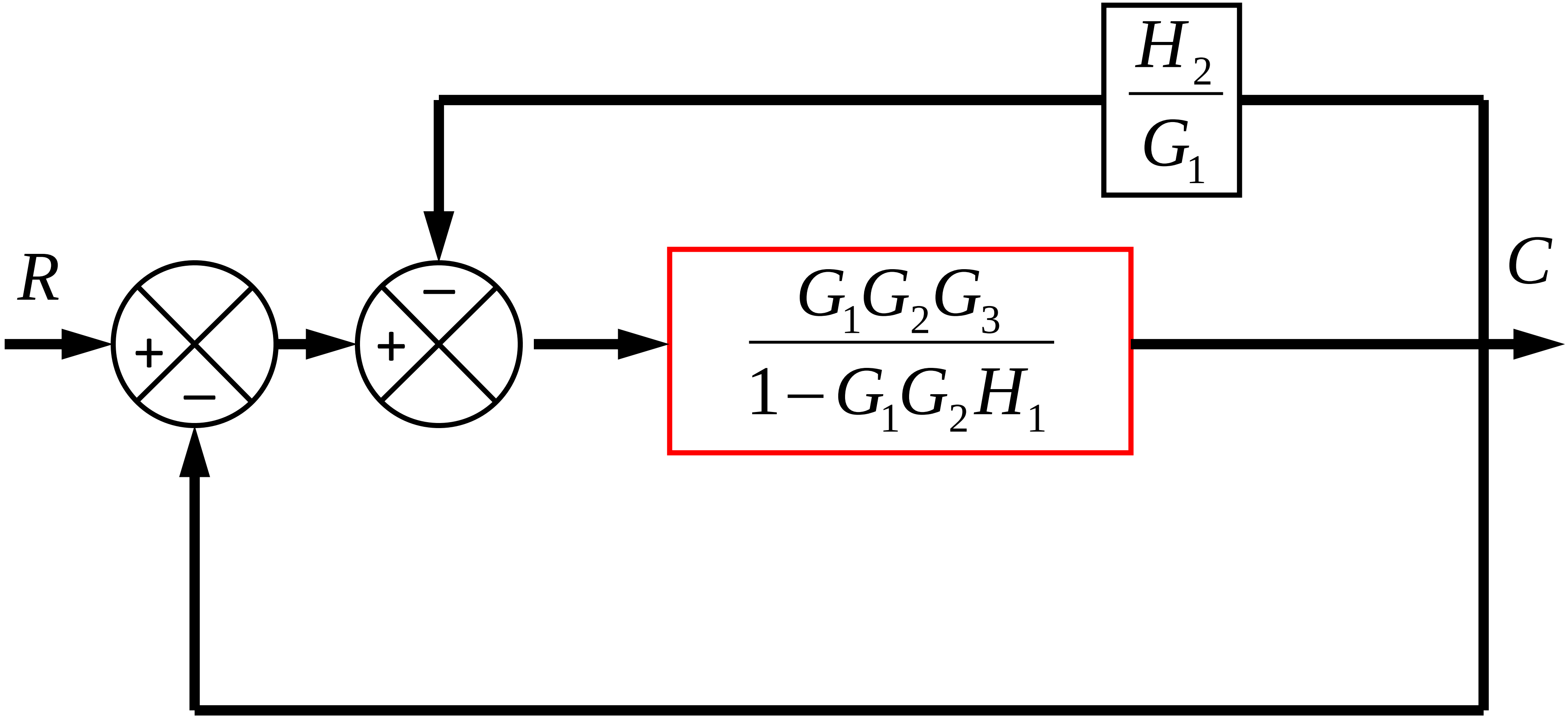
Example-7



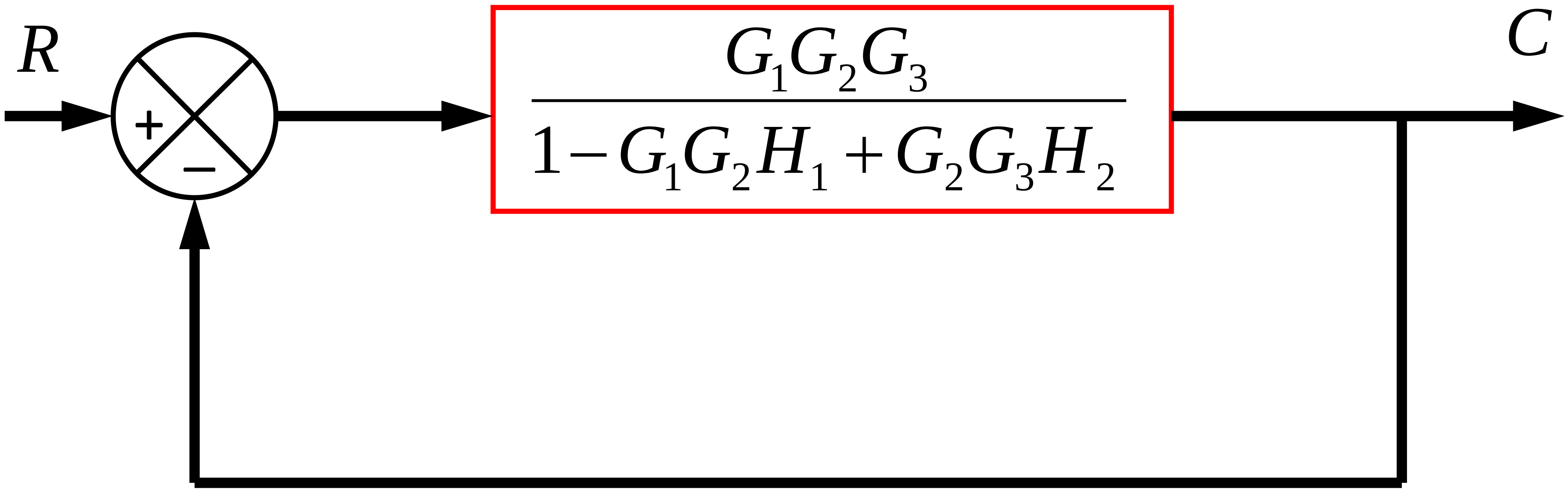
Example-7



Example-7

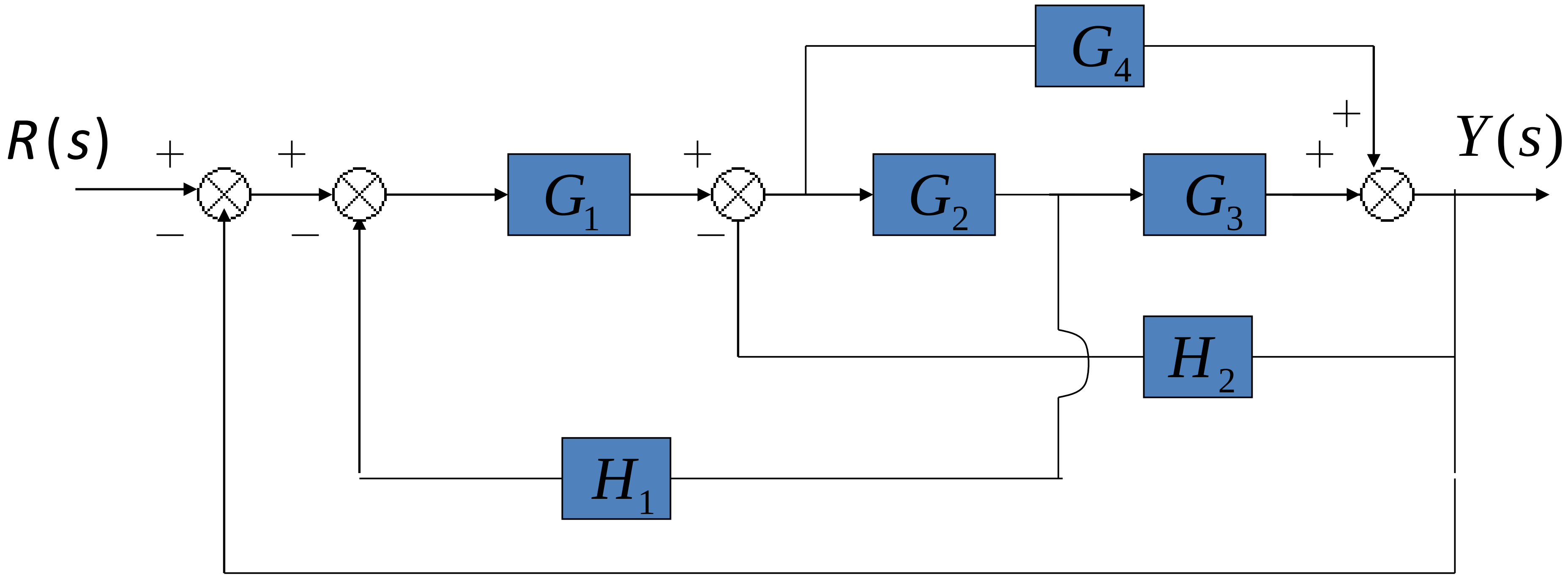


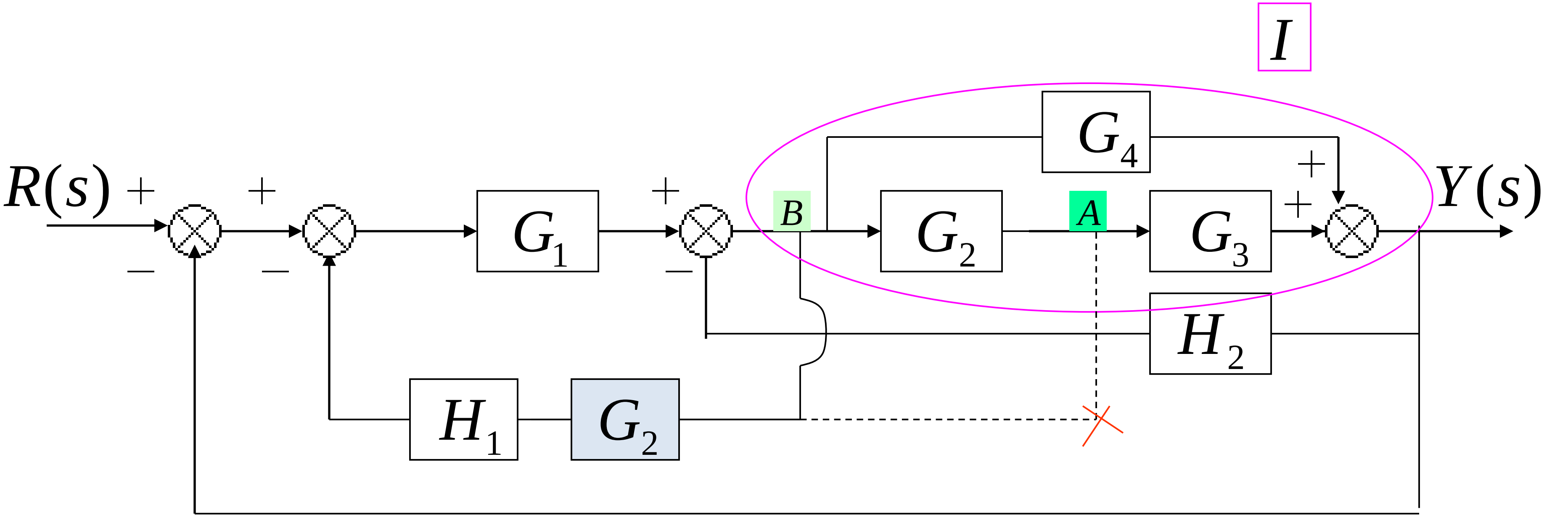
Example-7



Example 8

Find the transfer function of the following block diagram

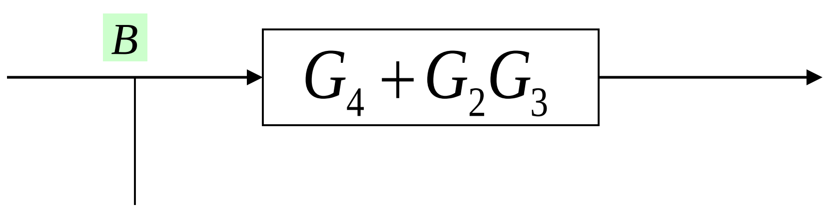


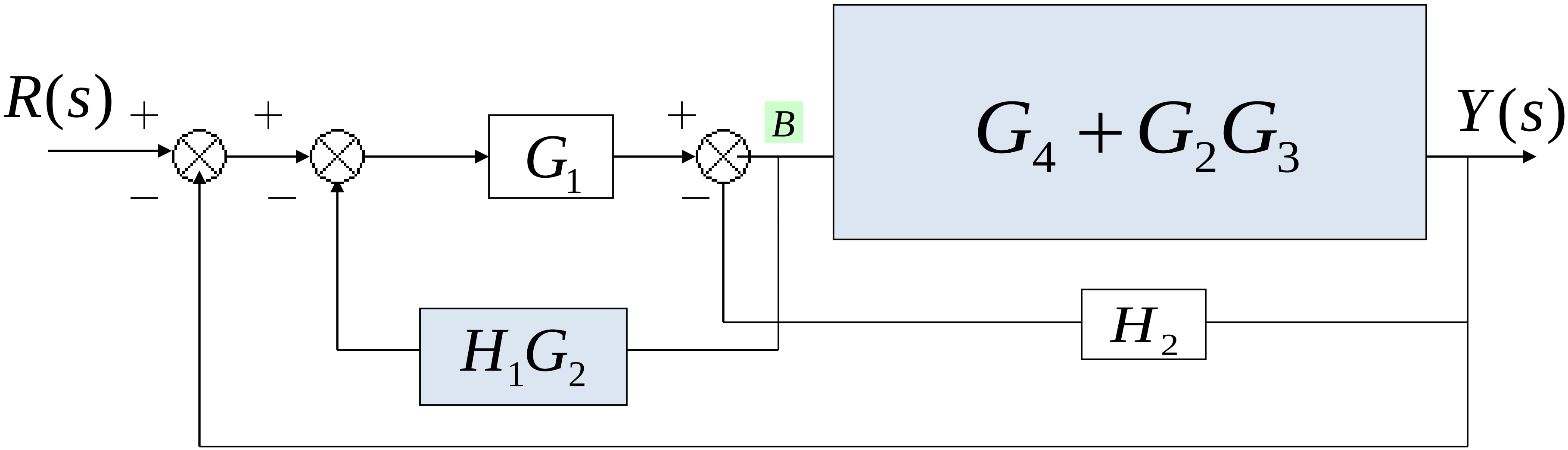


Solution:

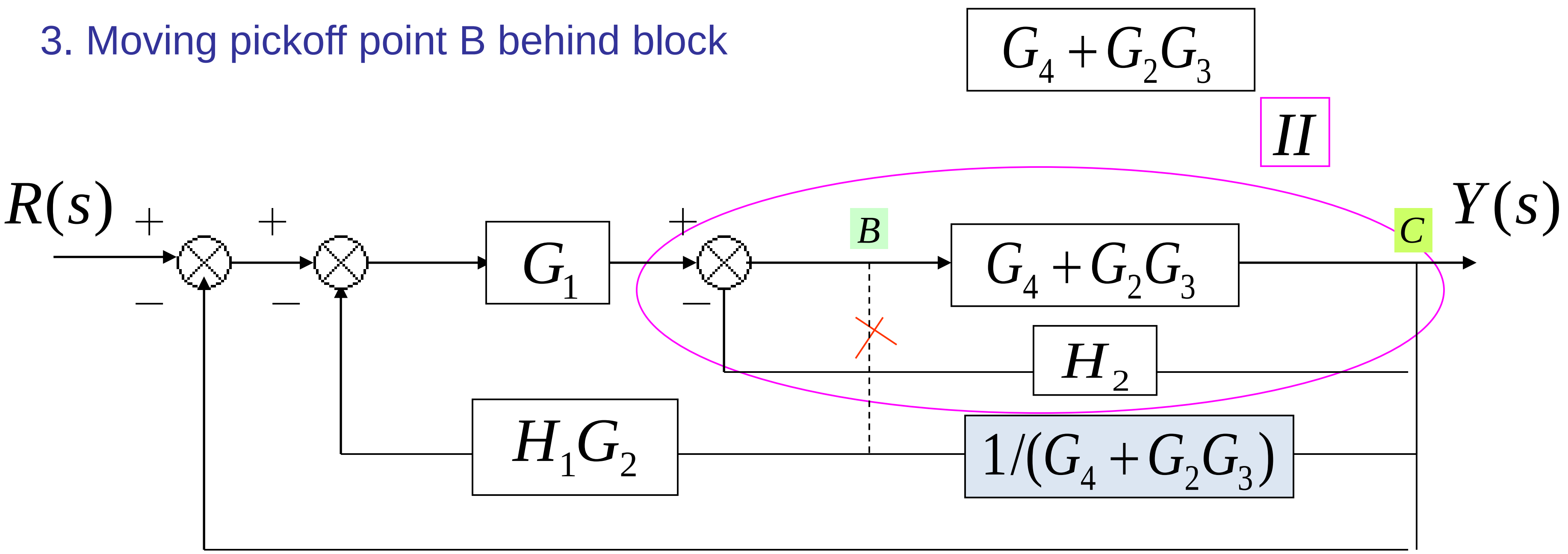
1. Moving pickoff point A ahead of block $\boxed{G_2}$

2. Eliminate loop I & simplify

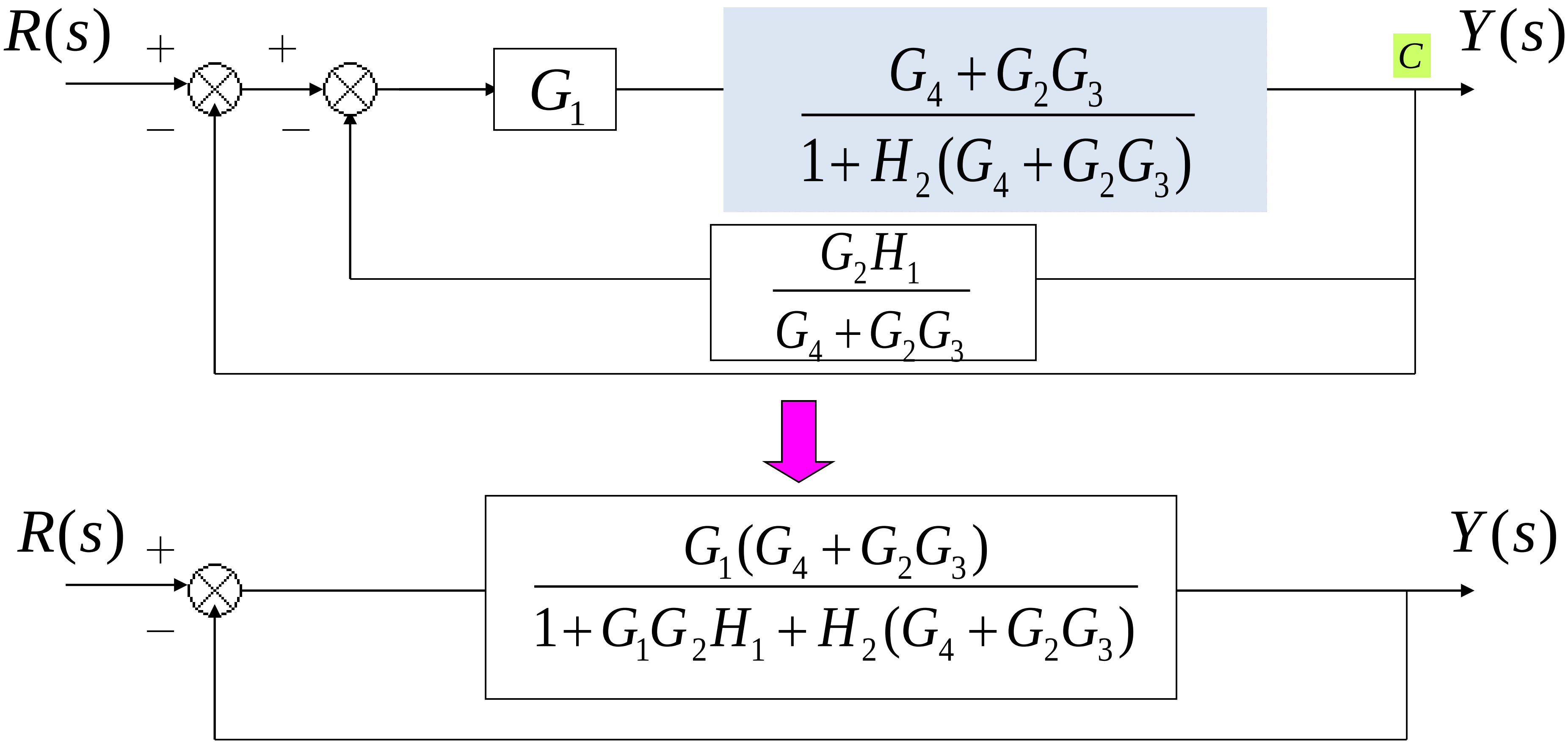




3. Moving pickoff point B behind block



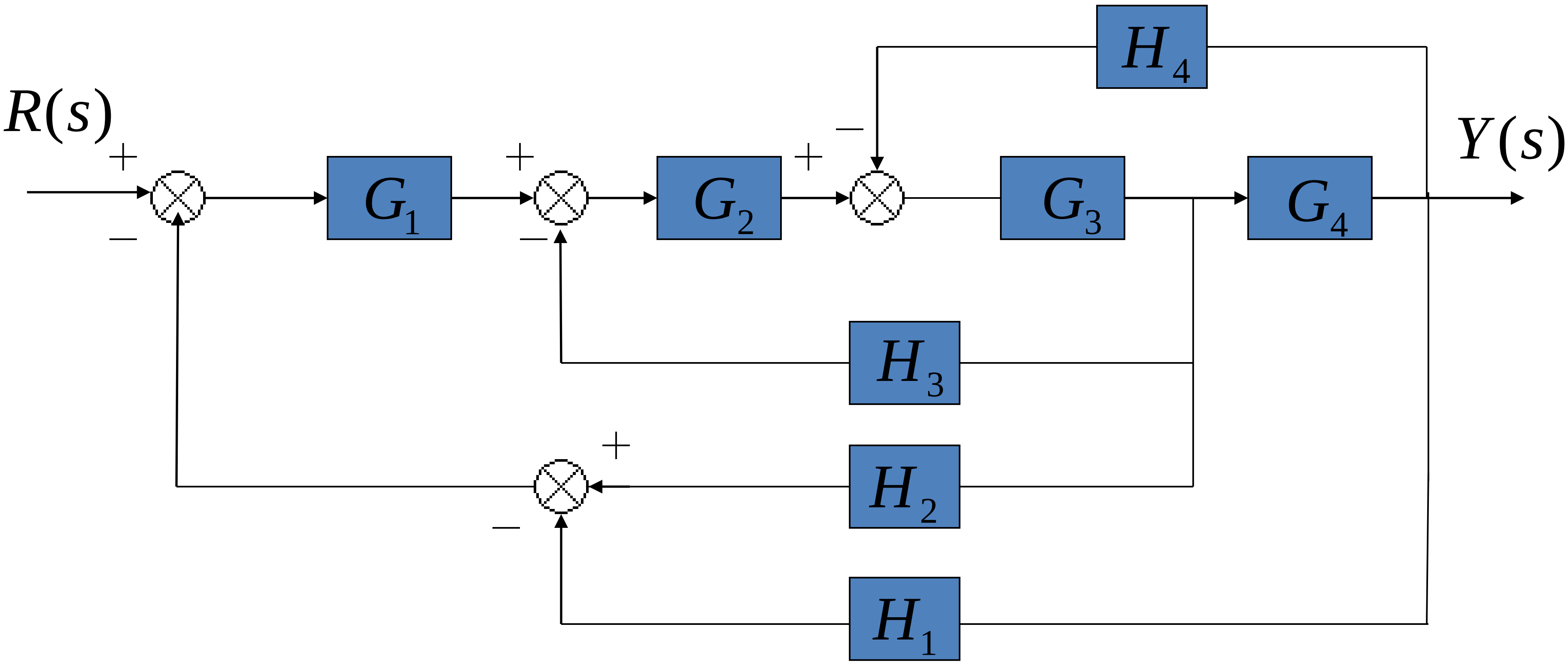
4. Eliminate loop III



$$\frac{Y(s)}{R(s)} = \frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1+H_2(G_4+G_2G_3)+G_1(G_4+G_2G_3)}$$

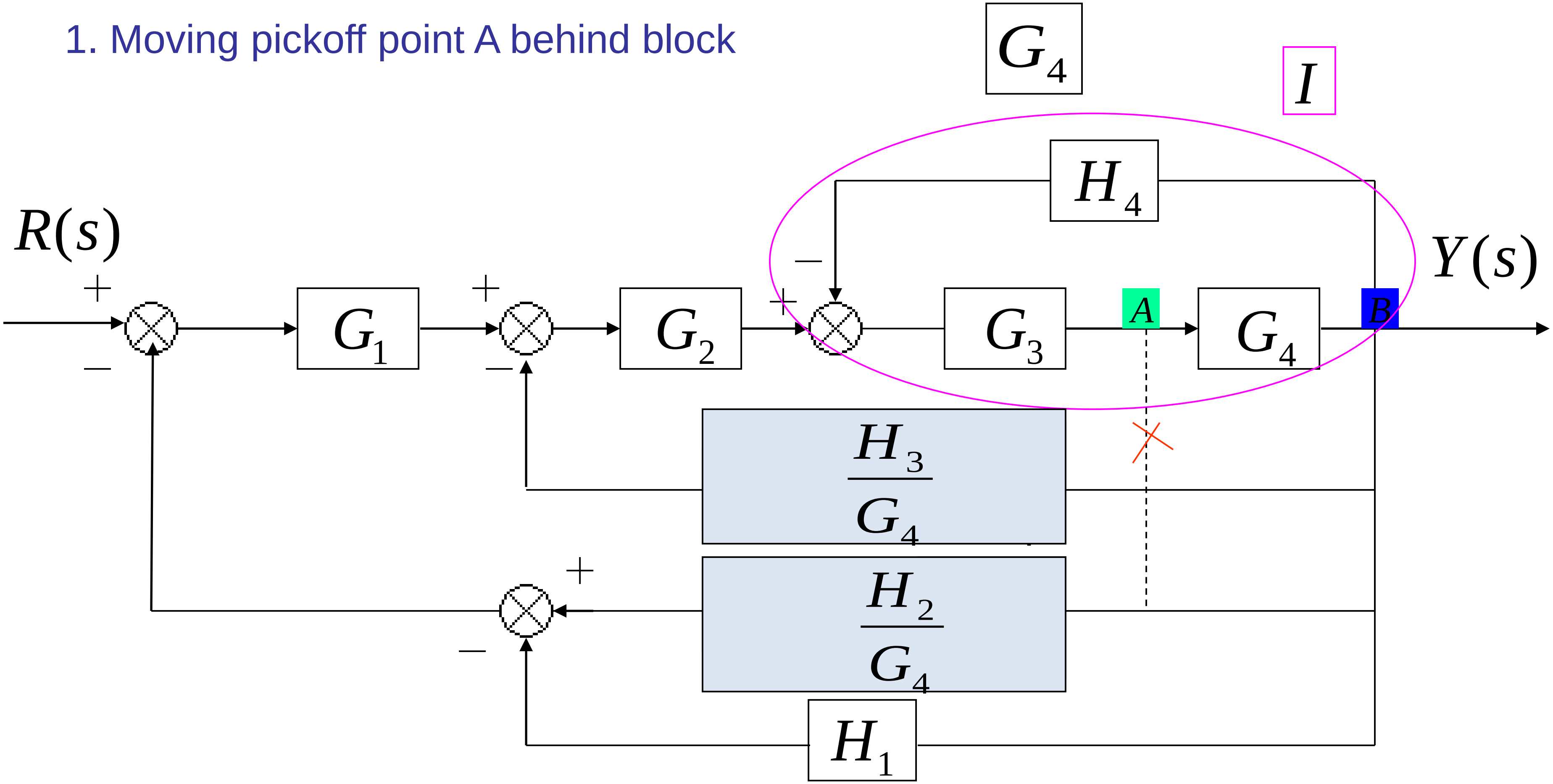
Example 9

Find the transfer function of the following block diagrams

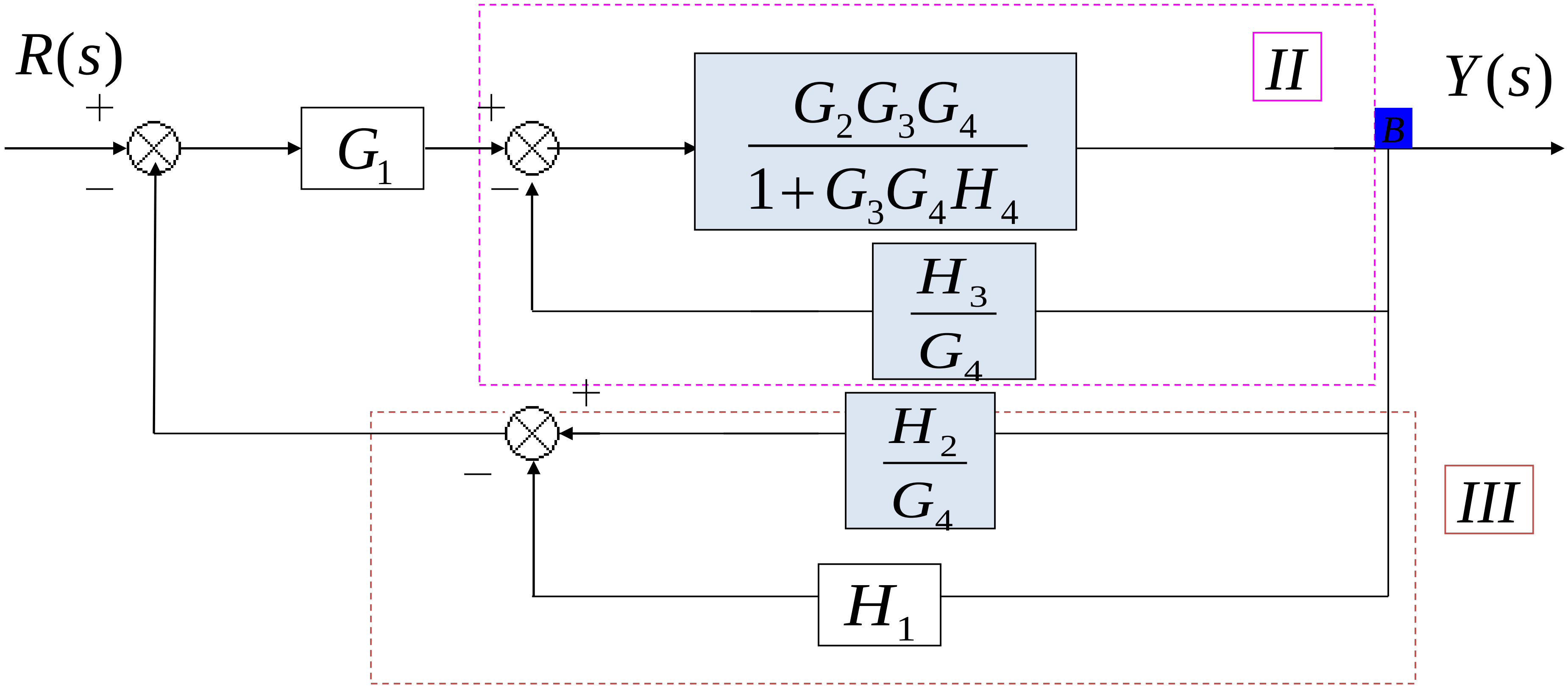


Solution:

1. Moving pickoff point A behind block



2. Eliminate loop I and Simplify



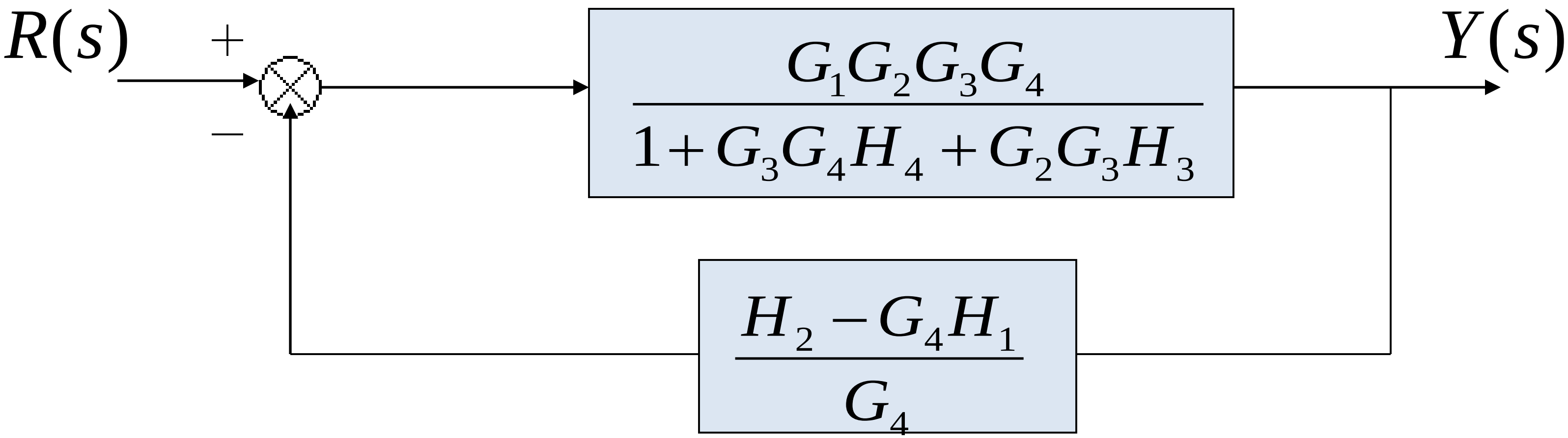
II feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III Not feedback

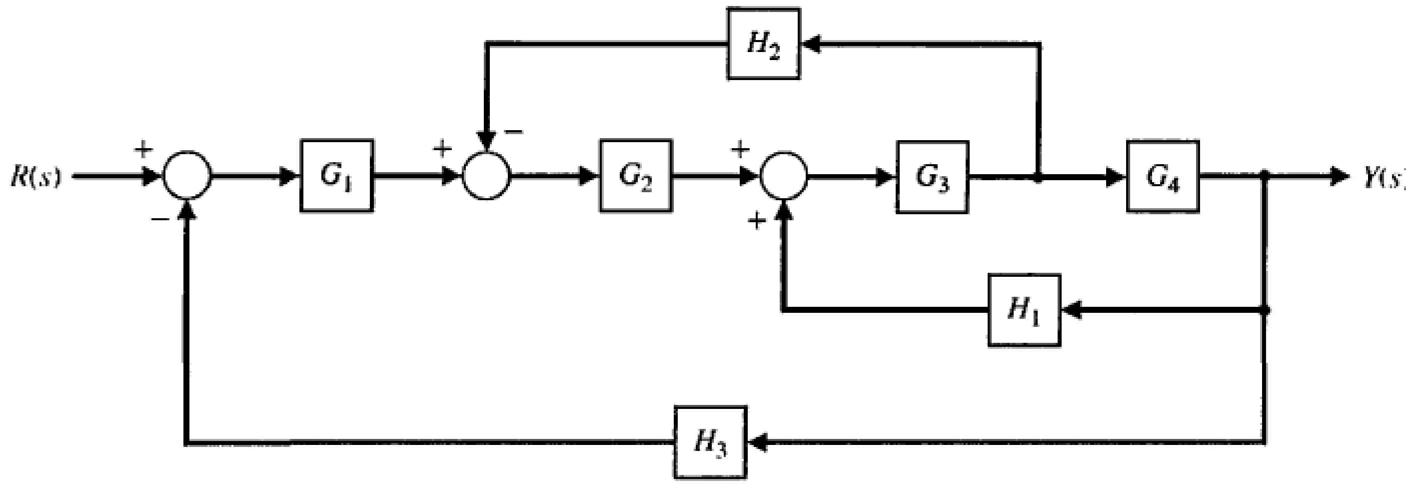
$$\frac{H_2-G_4H_1}{G_4}$$

3. Eliminate loop II & IIII

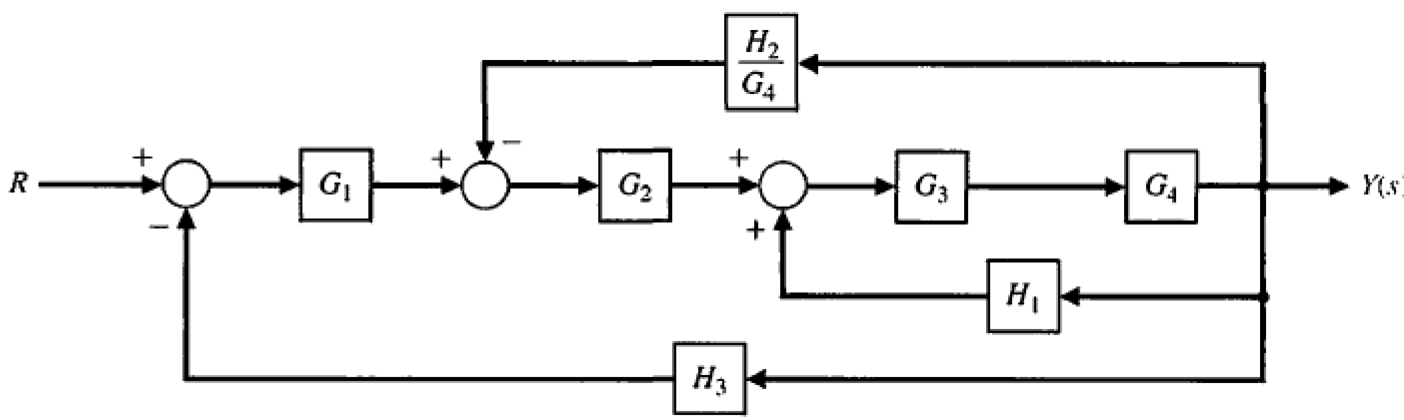


$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

Example-10: Reduce the Block Diagram.

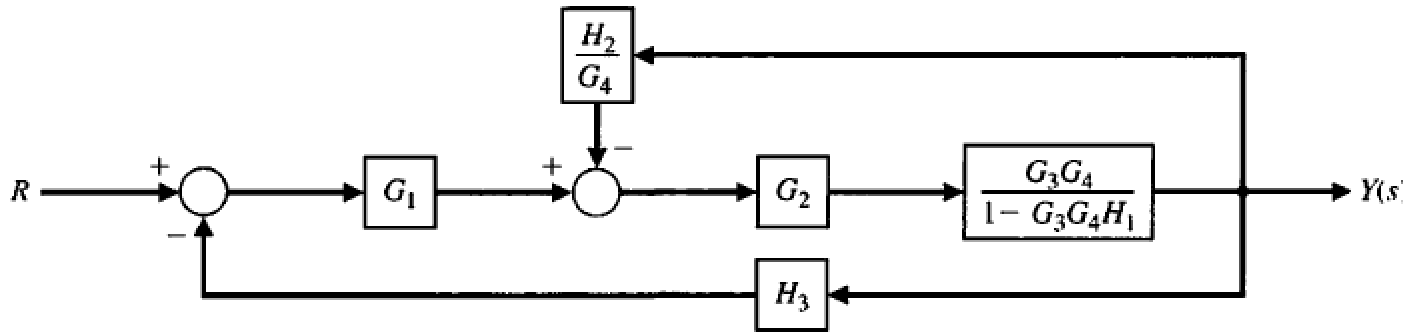


First, to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4

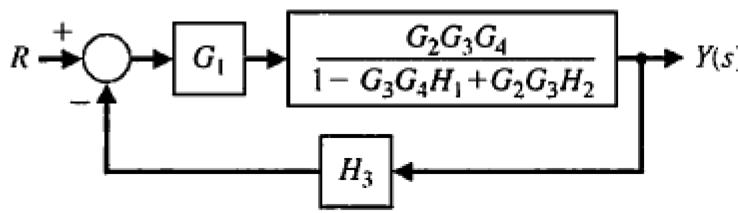


Example-10: Continue.

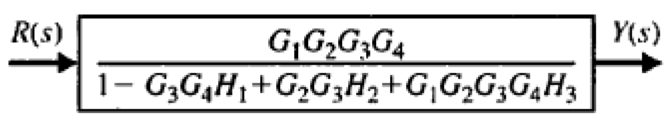
Eliminating the loop $G_3G_4H_1$ we obtain



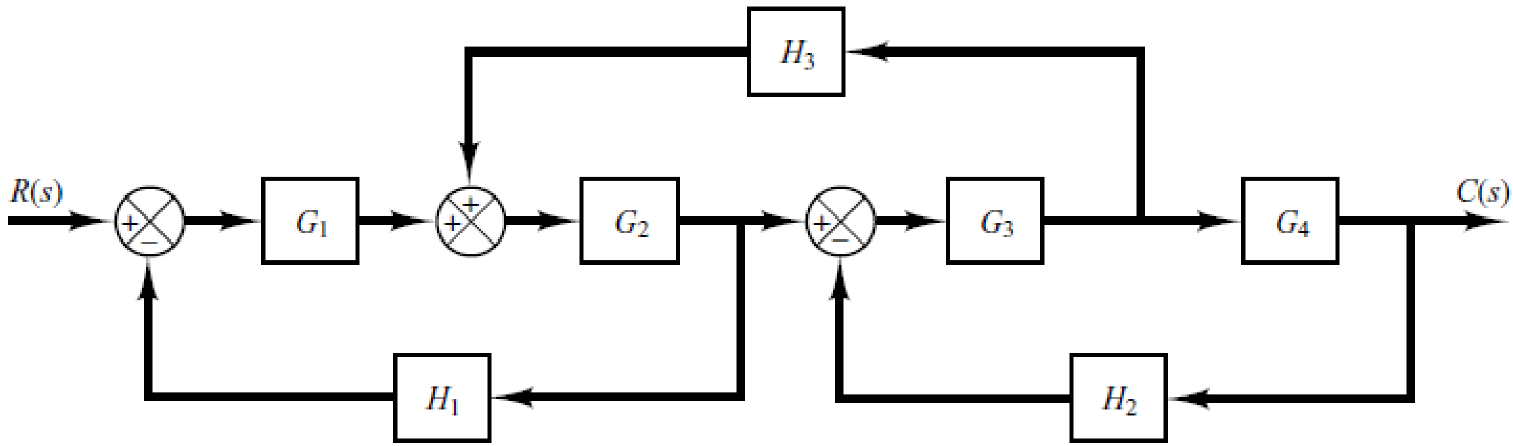
Then, eliminating the inner loop containing H_2/G_4, we obtain



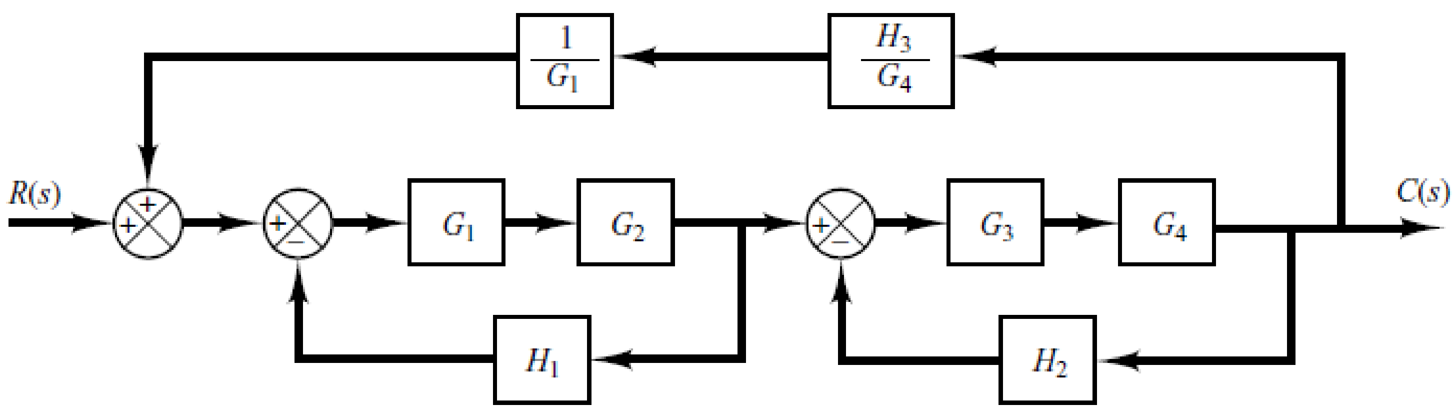
Finally, by reducing the loop containing H_3, we obtain



Example-11: Simplify the block diagram then obtain the close-loop transfer function C(S)/R(S). (from Ogata: Page-47)

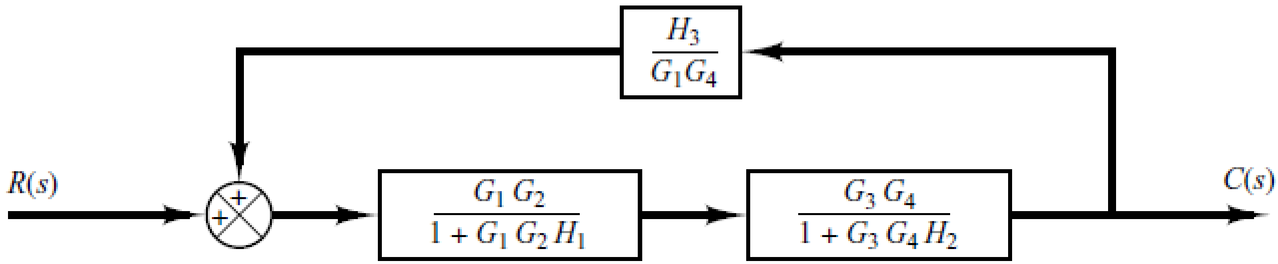


First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.

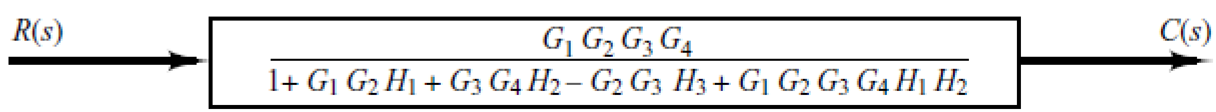


Example-11: Continue.

By simplifying each loop, the block diagram can be modified as



Further simplification results in



the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$$

Superposition of Multiple Inputs

Step 1: Set all inputs except one equal to zero.

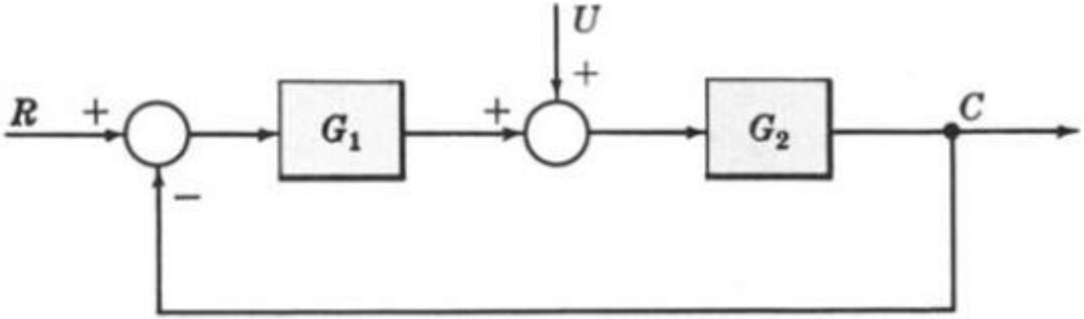
Step 2: Transform the block diagram to canonical form, using the transformations of Section 7.5.

Step 3: Calculate the response due to the chosen input acting alone.

Step 4: Repeat Steps 1 to 3 for each of the remaining inputs.

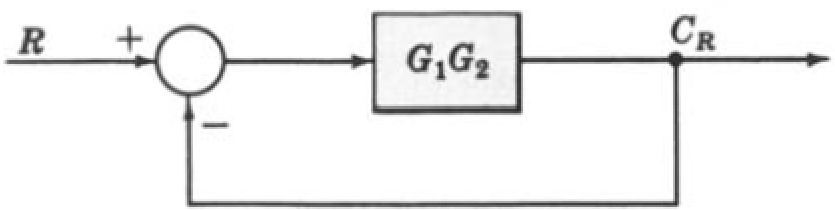
Step 5: Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

Example-12: Multiple Input System. Determine the output C due to inputs R and U using the Superposition Method.



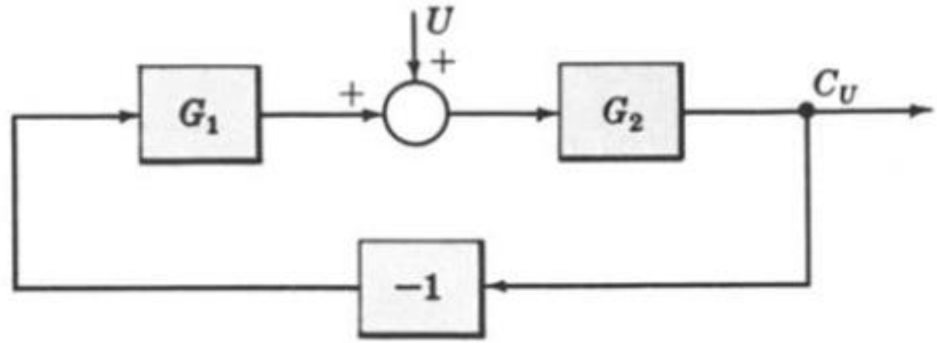
Step 1: Put $U \equiv 0$.

Step 2: The system reduces to



Step 3: the output C_R due to input R is $C_R = [G_1G_2/(1 + G_1G_2)]R$.

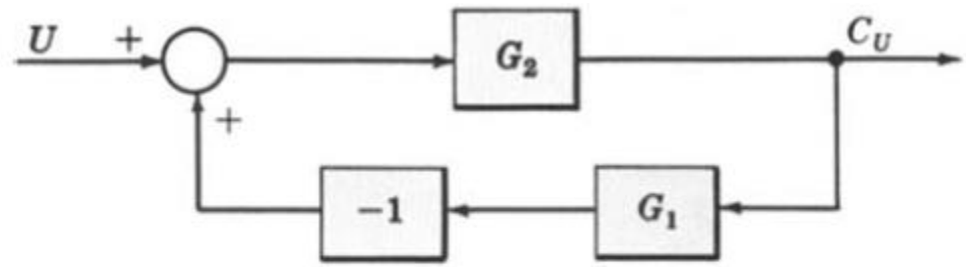
Example-12: Continue.



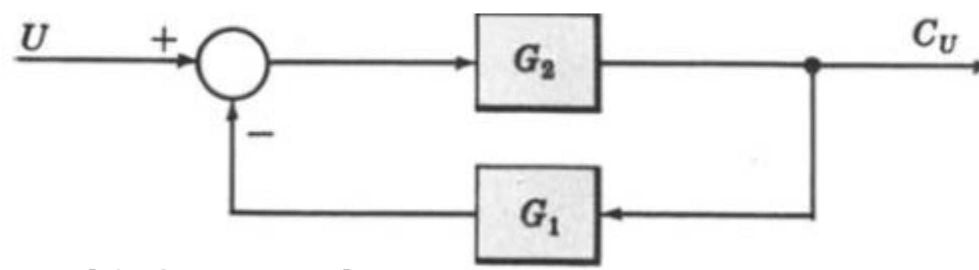
Step 4a: Put $R = 0$.

Step 4b: Put -1 into a block, representing the negative feedback effect:

Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: the output C_U due to input U is $C_U = [G_2/(1 + G_1G_2)]U$.

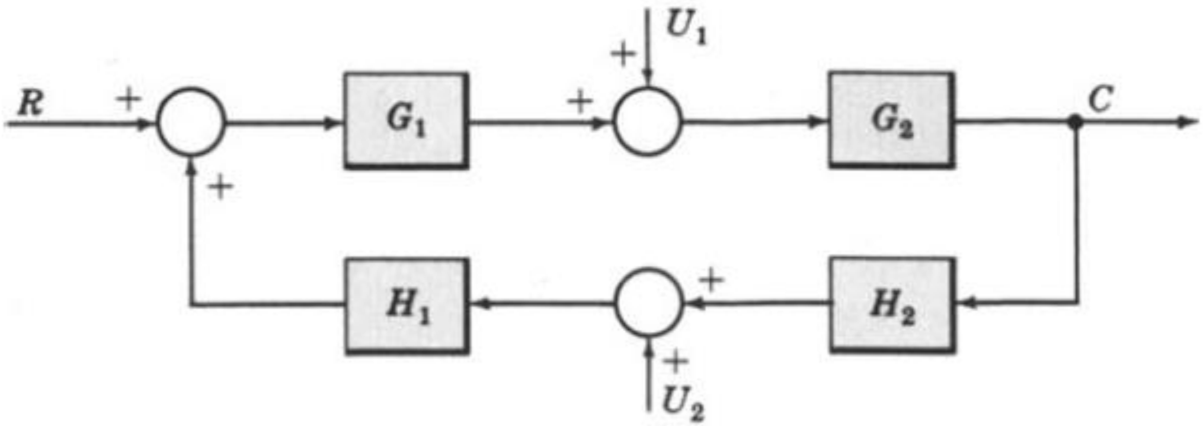
Example-12: Continue.

Step 5: The total output is $C = C_R + C_U$

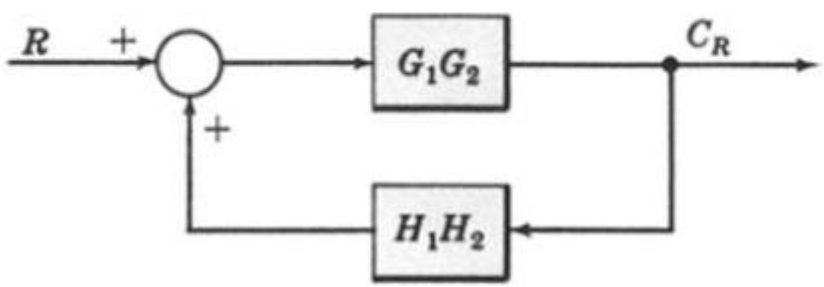
$$= \left[\frac{G_1G_2}{1 + G_1G_2}\right]R + \left[\frac{G_2}{1 + G_1G_2}\right]U$$

$$= \left[\frac{G_2}{1 + G_1G_2}\right][G_1R + U]$$

Example-13: Multiple-Input System. Determine the output C due to inputs R, U1 and U2 using the Superposition Method.



Let $U_1 = U_2 = 0$.

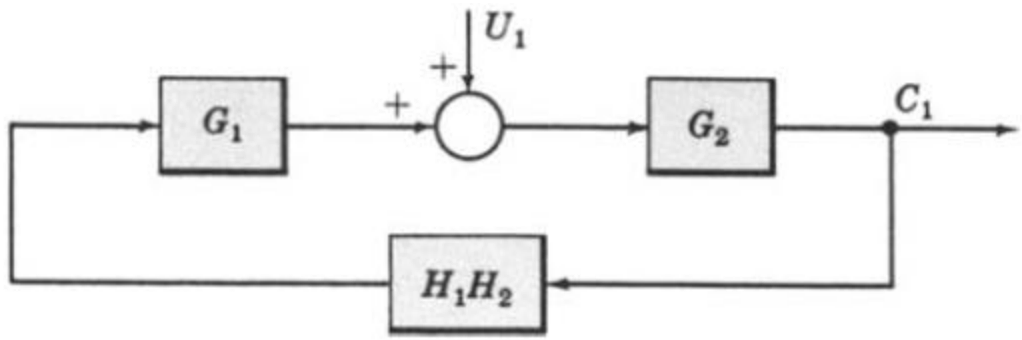


$$C_R = [G_1G_2/(1 - G_1G_2H_1H_2)]R$$

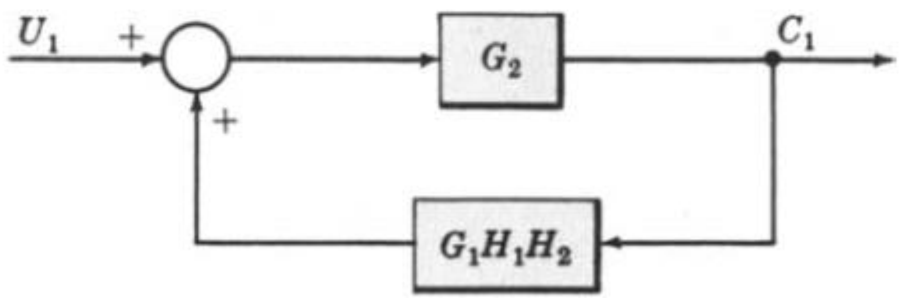
where C_R is the output due to R acting alone.

Example-13: Continue.

Now let $R = U_2 = 0$.



Rearranging the blocks, we get

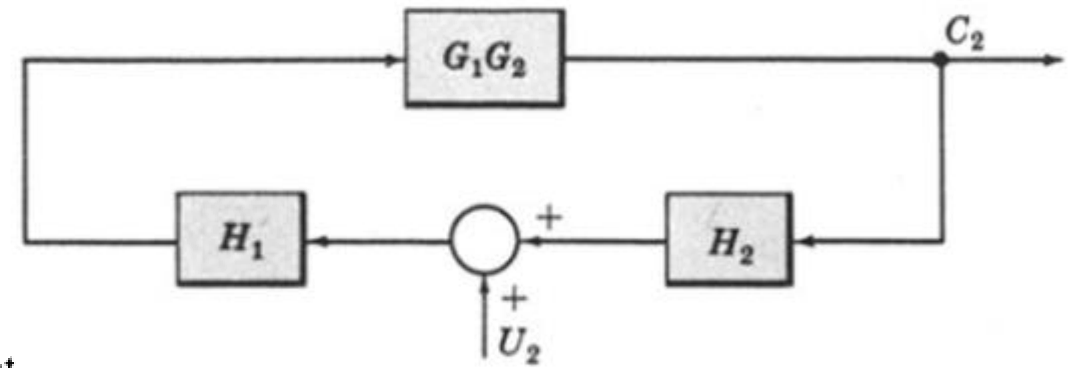


$$C_1 = [G_2/(1 - G_1G_2H_1H_2)]U_1$$

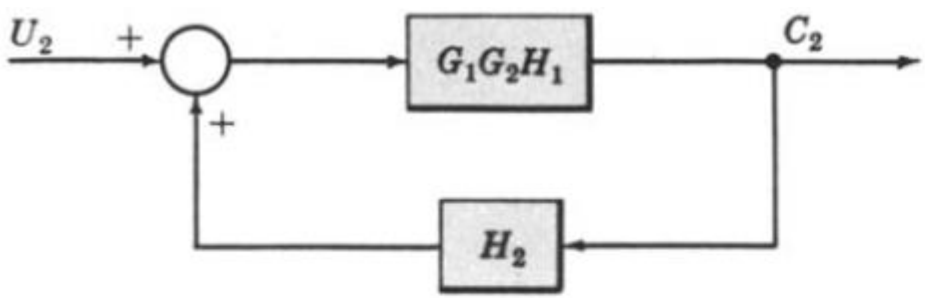
where C_1 is the response due to U_1 acting alone.

Example-13: Continue.

Finally, let $R = U_1 = 0$.



Rearranging the blocks, we get



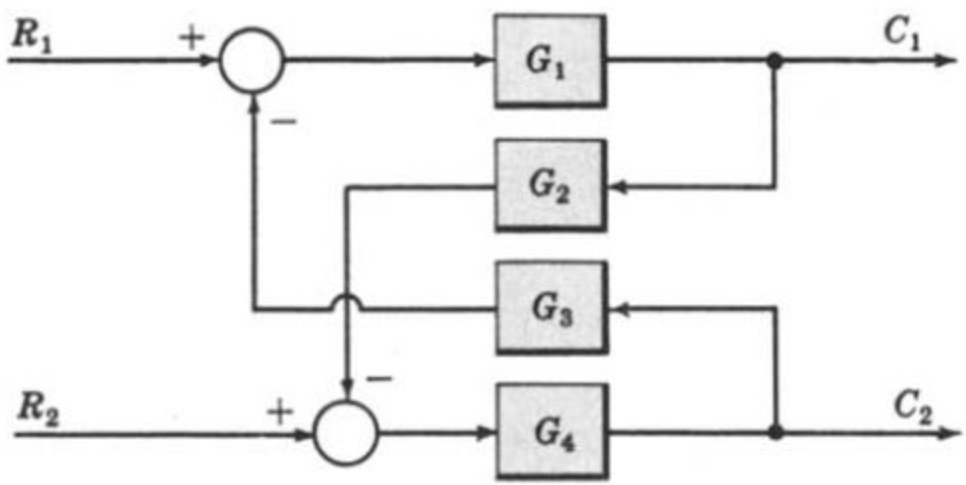
$$C_2 = [G_1G_2H_1/(1 - G_1G_2H_1H_2)]U_2$$

where C_2 is the response due to U_2 acting alone.

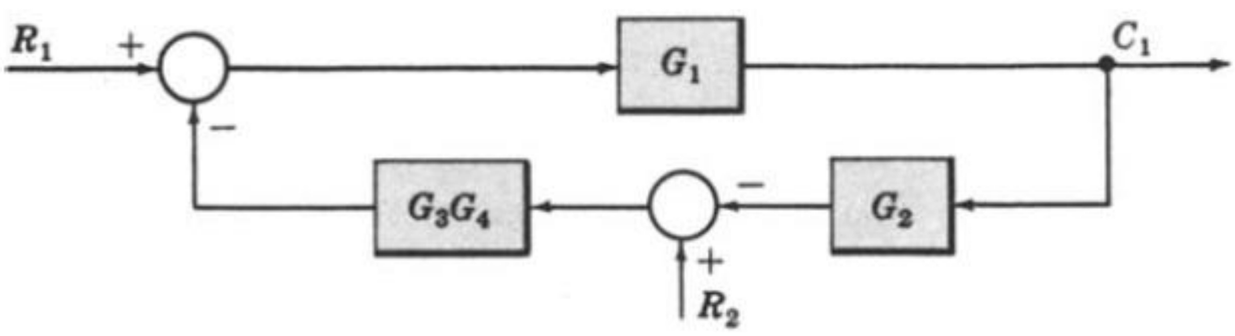
By superposition, the total output is

$$C = C_R + C_1 + C_2 = \frac{G_1G_2R + G_2U_1 + G_1G_2H_1U_2}{1 - G_1G_2H_1H_2}$$

Example-14: Multi-Input Multi-Output System. Determine C1 and C2 due to R1 and R2.

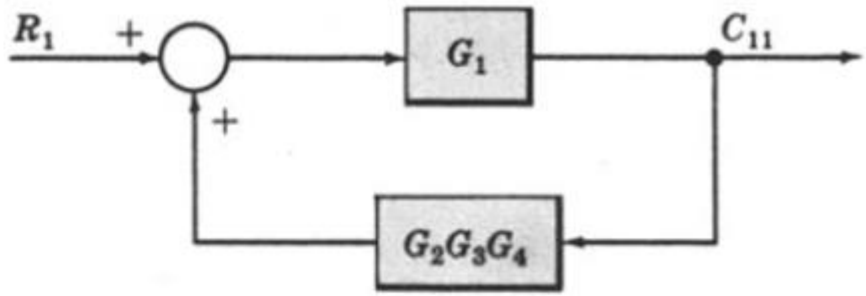


First ignoring the output C_2.



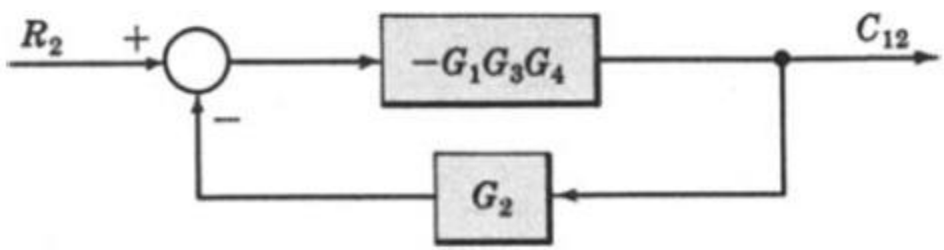
Example-14: Continue.

Letting $R_2 = 0$ and combining the summing points,



Hence C_{11}, the output at C_1 due to R_1 alone, is $C_{11} = G_1R_1/(1 - G_1G_2G_3G_4)$.

For $R_1 = 0$,

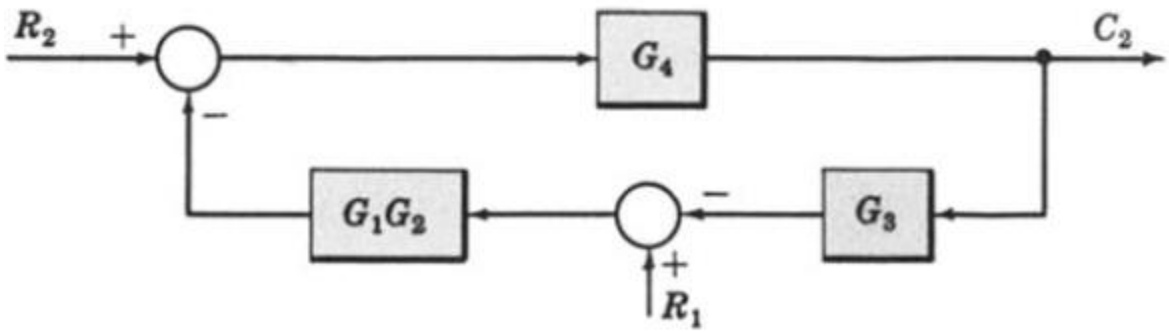


Hence $C_{12} = -G_1G_3G_4R_2/(1 - G_1G_2G_3G_4)$ is the output at C_1 due to R_2 alone.

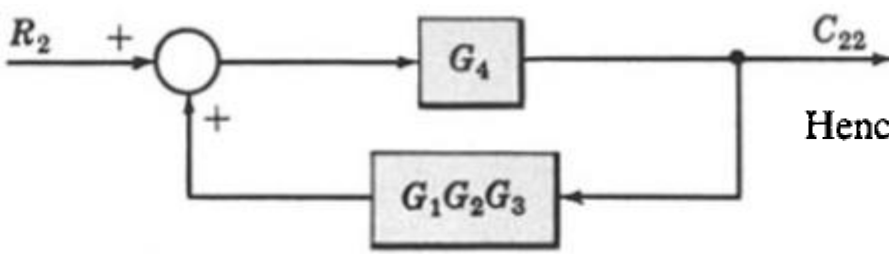
Thus $C_1 = C_{11} + C_{12} = (G_1R_1 - G_1G_3G_4R_2)/(1 - G_1G_2G_3G_4)$

Example-14: Continue.

Now we reduce the original block diagram, ignoring output C_1.

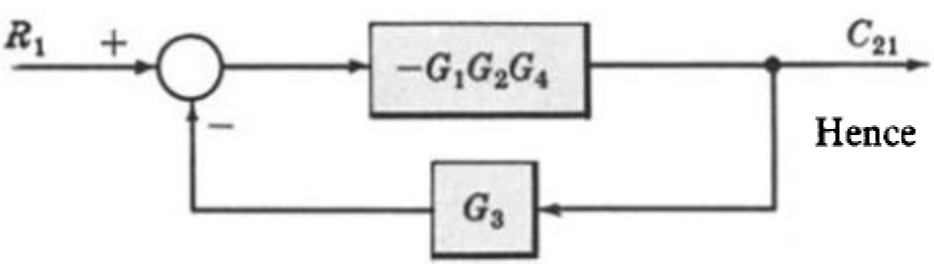


When R1 = 0,



Hence $C_{22} = G_4 R_2/(1 - G_1G_2G_3G_4)$

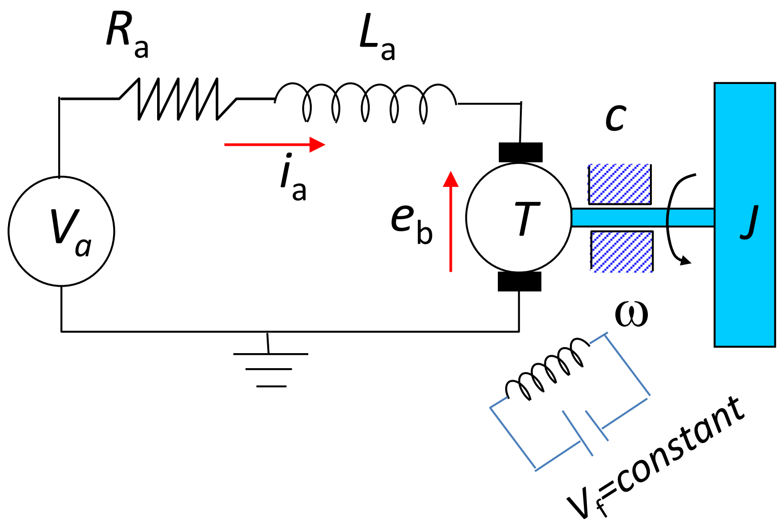
When R2 = 0,



Hence $C_{21} = -G_1G_2G_4R_1/(1 - G_1G_2G_3G_4)$

Finally, $C_2 = C_{22} + C_{21} = \ (G_4R_2 - G_1G_2G_4R_1)/(1 - G_1G_2G_3G_4)$

Block Diagram of Armature Controlled D.C Motor

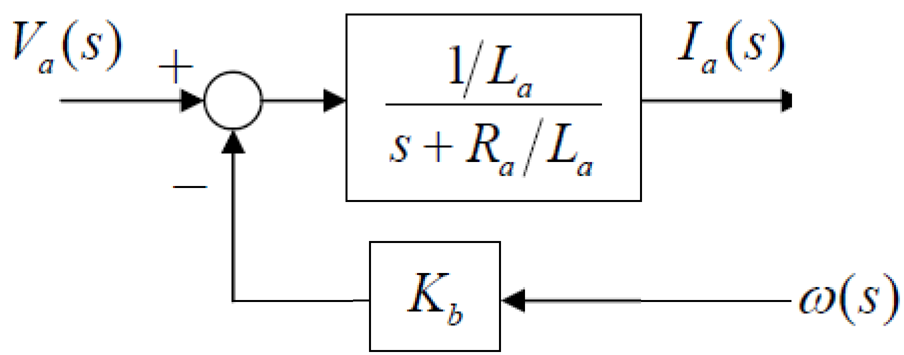


$$\left(L_a s + R_a\right) I_a(s) + K_b \omega(s) = V_a(s)$$

$$\left(Js + c\right)\omega(s) = K_m I_a(s)$$

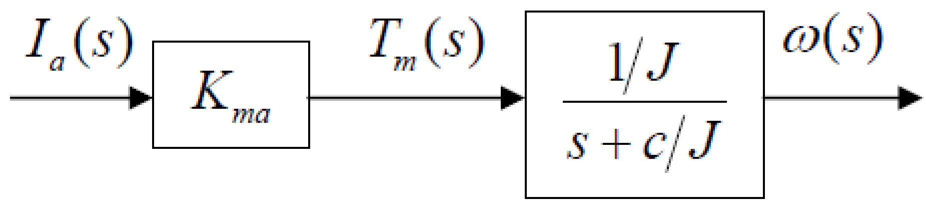
Block Diagram of Armature Controlled D.C Motor

$$\left(L_a s + R_a\right)I_a(s) + K_b\omega(s) = V_a(s)$$

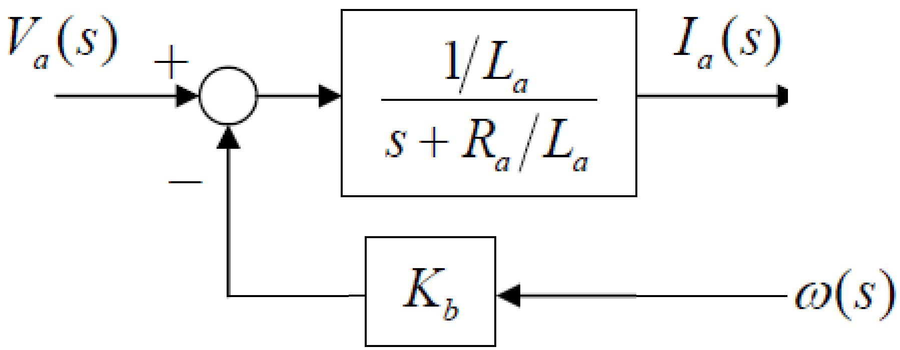


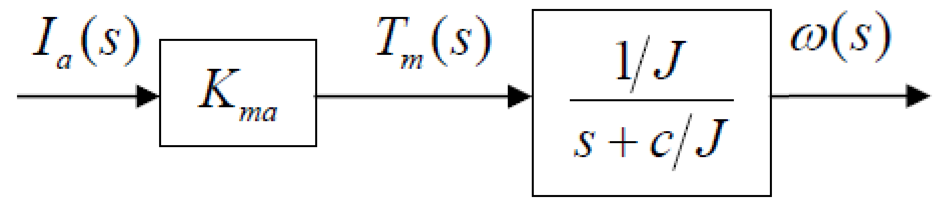
Block Diagram of Armature Controlled D.C Motor

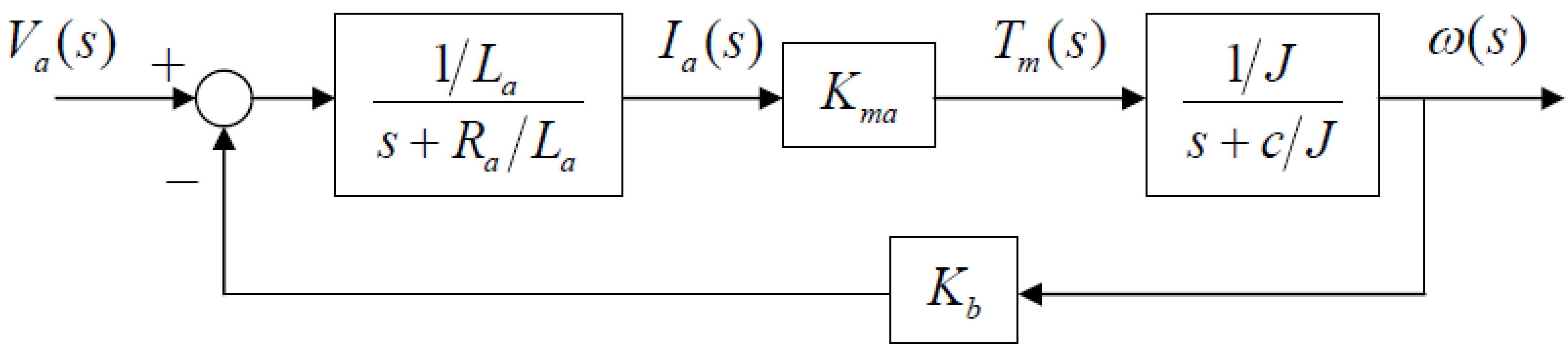
$$(Js + c)\omega(s) = K_{ma} I_a(s)$$



Block Diagram of Armature Controlled D.C Motor







For the Fig.1.1, The water level control system:

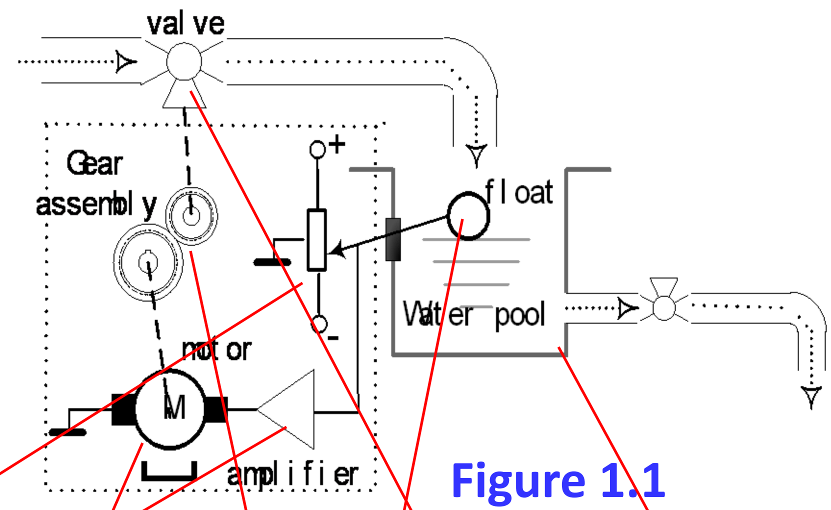


Figure 1.1

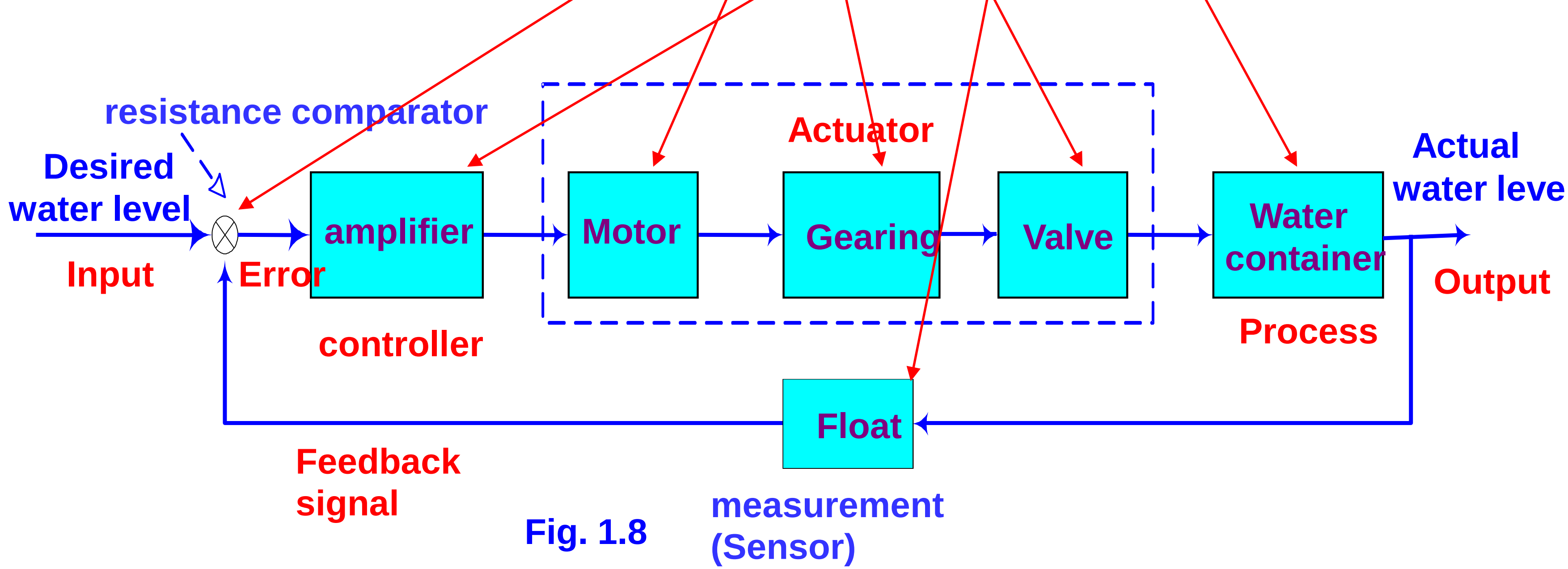


Fig. 1.8

The DC-Motor control system

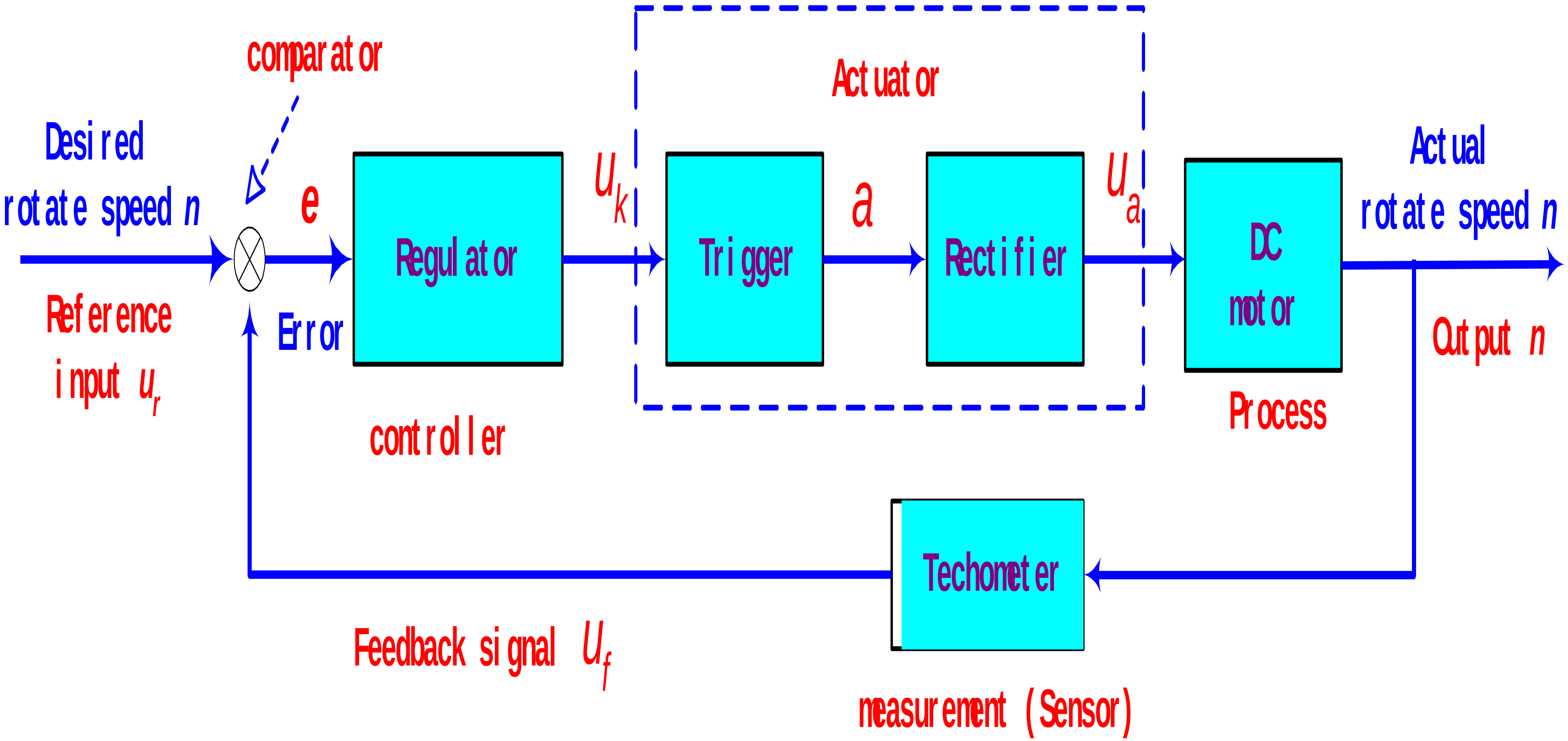


Fig. 1.9

Fundamental structure of control systems

1) Open loop control systems

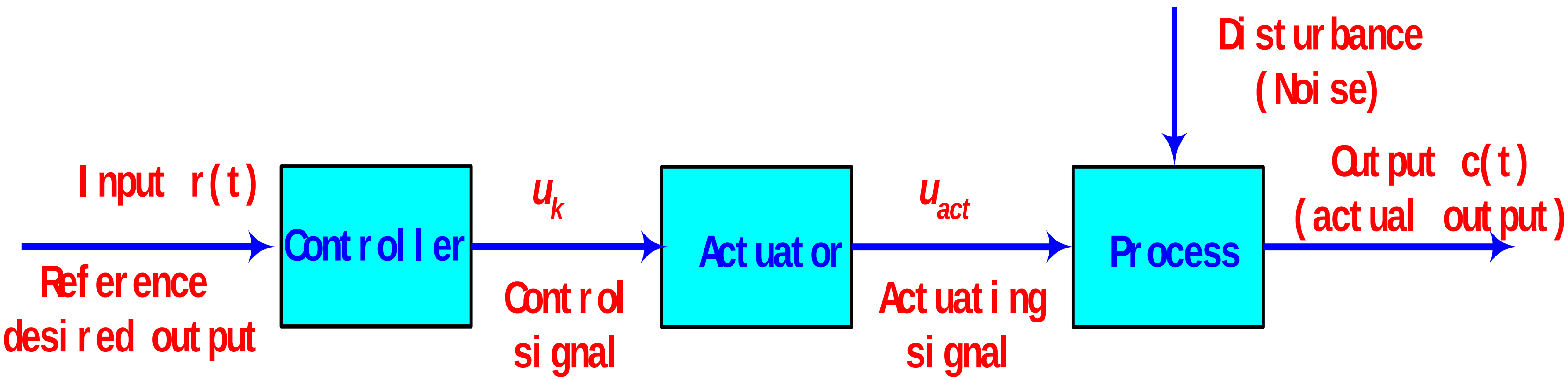


Fig. 1.10

Features: Only there is a forward action from the input to the output.

2) Closed loop (feedback) control systems

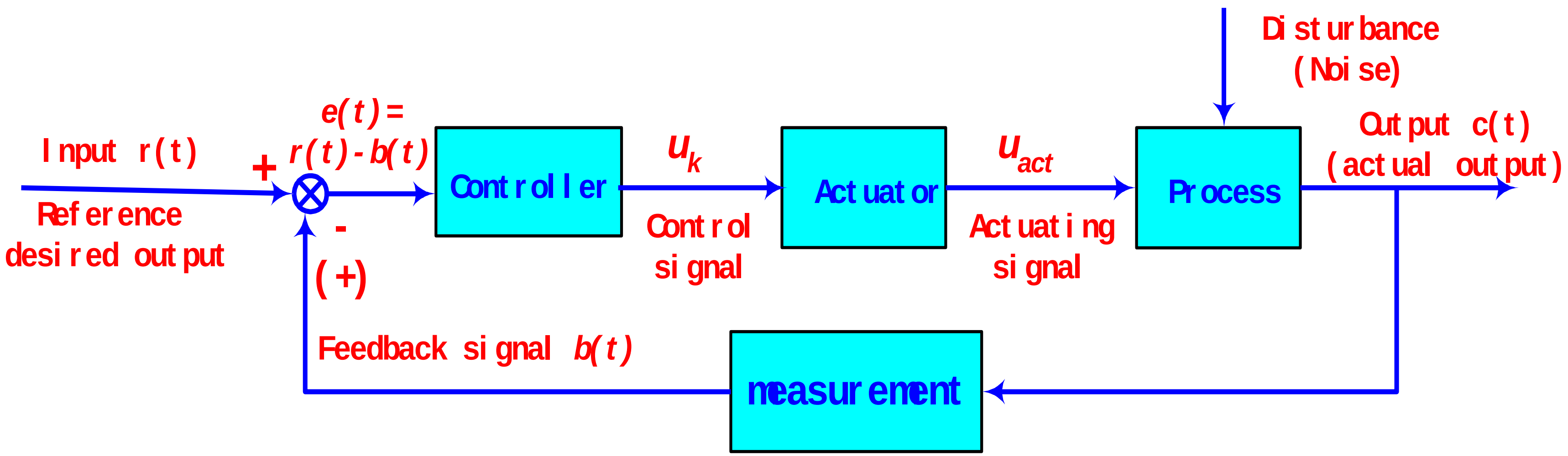


Fig. 1.11

Features:

not only there is a forward action , also a backward action between the output and the input (measuring the output and comparing it with the input).

1) measuring the output (controlled variable) . 2) Feedback.

Chapter 2 Mathematical models of systems

2.1 Introduction

2.1.1 Why?

1) Easy to discuss the full possible types of the control systems—in terms of the system's "mathematical characteristics".

2) The basis — analyzing or designing the control systems.

For example, we design a temperature Control system :

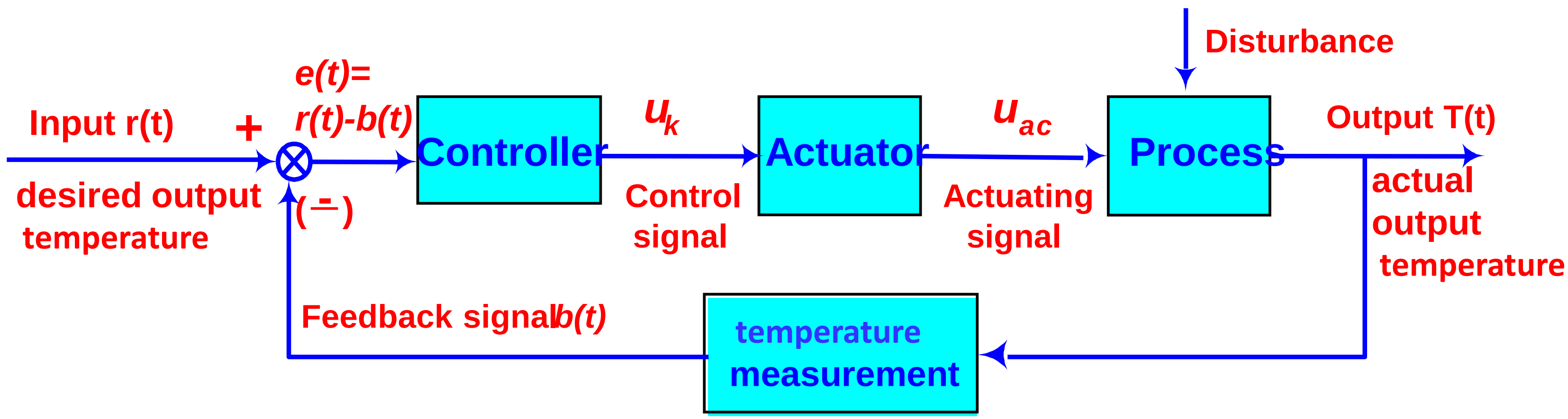


Fig. 2.1

The key — designing the controller → how produce u_k.