1 System Stability

1.1 Singularities of a Function

System is stable if for bounded input the output is bounded BIBO. System stability can be determined from the Transfer Function.

$$T.F = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1}s^{m-1} + \ldots + b_1 s + b_o}{a_n s^n \ + a_{n-1}s^{\ n-1} + \ldots + a_1 s + a_o}$$

Can be written in terms of factors:

$$T.F = \frac{N(s)}{D(s)} = K\frac{(s - Z_1)(s - Z_2)\ldots(s - Z_{m-1})(s - Z_m)}{(s - P_1)(s - P_2 \ \ldots(s - P_{\ n-1})(s - P_n \ )}$$

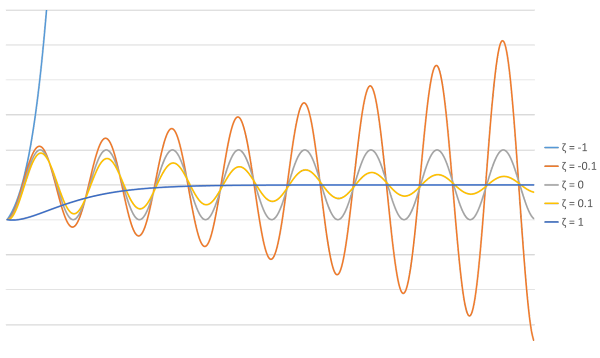
-> Roots of $D(s)$: $P_1, P_2, \ldots, P_n$ are called poles.
-> Roots of $N(s)$: $Z_1, Z_2, \ldots, Z_m$ are called zeros.

(where $m < n$)

All coeffieceints of $D(s), N(s)$ are real -> poles and zeros must be real or complex conjugate.

$$T.F = K(\frac{A_1}{S - P_1} + \frac{A_2}{S - P_2} + \ldots + \frac{A_{n-1}}{S - P_{n-1}} + \frac{A_n}{S - P_n})$$

For a second order systems: Root $(-\omega_n[\zeta \pm \sqrt{\zeta^2 - 1}])$



$$F(s) = \frac{A}{s}$$

Root is zero $(s = 0)$:

-> unstable: $\int_0^\infty f(t)\,dt$ is not bounded.

Stability Condition:

all poles lie in the left half of s-plane

-> all coefficients of $D(s)$ must be real and positive.

-> all powers of s from s^1 to s^n should be present, however all odd powers of s or all even power of s may be missing.

(necessary conditions, but not sufficient)

1.2 Routh's Stability Criterion

main