Ongoing Lectures Collection

CS241

Linear Control Systems

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Lectures Collection

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Colophon
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It is practically a big lie that LaTeX makes you focus on the content without bothering about the layout.

– Community Wiki Xport

Preface

I'm using LaTeX, hoping that this work will continue existing. I don't have much else to say, so I will just insert some blind text. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Ahmed Waleed

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Introduction | 1

1.1 Basic Information

Analysis of linear continuous system analysis of a system means simply checking the goodness of its measure of performance. Analysis could be done in two different ways:

- In the lab: by putting test input to the system and checking if the output satisfies the measure of performance.
- Using analytical techniques: which is our concern in this course.

The first step is to make a mathematical model to the system.

$$\Sigma F_x = m\ddot{x}$$
$$F - \mu\dot{x} - kx = m\ddot{x}$$
$$\therefore \boxed{F = m\ddot{x} + \mu\dot{x} + kx}$$

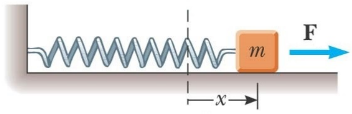


Figure 1.1: A block attached to a spring. ©

Then defining the measure of performance and studying how we can check these measure of performance.

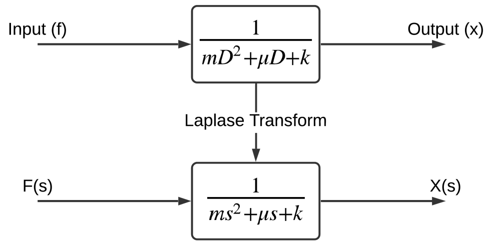


Figure 1.2: Since D is an operator (can't have a value), the transfer function is obtained by the Laplace transform of the first relation.

Transfer function ratio between Laplace transform of the output and Laplace transform of the input, assuming zero initial conditions.

1.2 Control Systems

A control system is an interconnection of components forming a system configuration that will provide a desired system response.

Open-loop control system (without feedback):

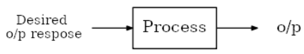


Figure 1.3: Its output does not track the input, and it is more affected by noise.

Closed-loop feedback control system (with feedback):

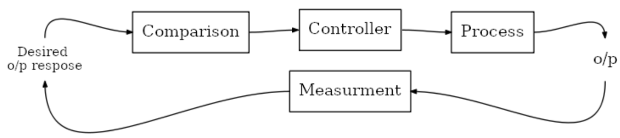


Figure 1.4: Closed loop control can improve accuracy, also the actuating signal is a function of the output.

1.3 Mathematical Model

Any linear continuous system can be represented either by a linear algebraic equation or an ordinary differential equation such as:

$$(mD^2 + \mu D + k)\,x(t) = y(t)$$

Solving the differential equation using Laplace transform assuming zero initial conditions made it possible to get the transfer function.

1.4 Block Diagram Reduction ➲

A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system. Control systems require the arithmetic manipulation in order to obtain the overall transfer function and this is the start point for the analysis of the system.

Table 1.1: Terminology

R	:	reference input / desired output response.
E	:	actuating / error signal.
G	:	control element and controlled system.
C	:	controlled variable / actual output.
H	:	feedback / backward transfer element.
B	:	primary feedback.
s	:	summation point.
t	:	takeoff point.

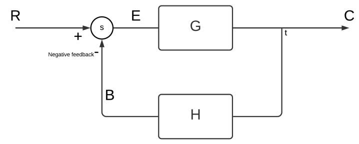
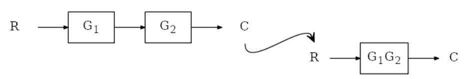
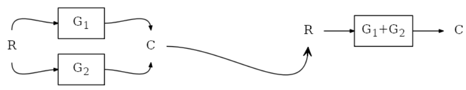


Figure 1.5: Canonical feedback loop.

Cascade connection :



Parallel connection :



Summation point :

a small circle, with plus or minus sign associated with the inputs, and the output is the algebraic sum of the inputs.

Take-off point :

a takeoff (or pickoff) point is used in order to have the same signal input to more than one block.

Overall transfer function (feedback loop elimination):

Applying reduction techniques mentioned above, we can obtain the overall transfer function of Figure 1.5

$$\begin{aligned}
@\,s: \quad & E = R - B && \text{(summation point)}\\
\because \quad & B = CH && \text{(block)}\\
\therefore \quad & E = R - CH\\
\because \quad & C = GE && \text{(block)}\\
\therefore \quad & C = G(R - CH)\\
& C + CGH = GR\\
& C(1+GH) = GR
\end{aligned}$$

$$\boxed{\frac{C}{R} = \frac{G}{(1+GH)}}$$

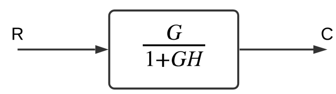


Figure 1.6: Feedback loop equivalent

Mathematical Models | 2

2.1 Introduction to Mathematical Models

We will be studying single output linear continuous systems. If a system has more than one input the superposition principle will be applied.

Super Position Principle ©

The superposition property, states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.

$$\boxed{F(x_1+x_2) = F(x_1)+F(x_2)}$$

(It can be used to prove linearity)

Simple Systems Equations ✉

The purpose of this section is to present methods of writing the differential equations for a variety of electrical and mechanical systems. This is the first step that must be mastered by the would-be control systems engineer.

Series Resistor-Inductor-Capacitor Circuit

$$v_L = (LD)\, i$$

$$v_R = R\, i$$

$$v_C = (\frac{1}{CD})\, i$$

$$\boxed{v = (LD + R + \frac{1}{CD})\, i}$$

$(R:\ resistor,\ L:\ inductor,\ C:\ capacitor)$

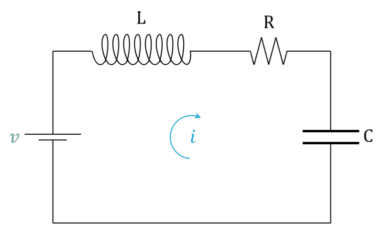


Figure 2.1: Simple electrical system.

Simple Mechanical Translation System

$$f_M = (MD^2)\, x$$

$$f_B = (BD)\, x$$

$$f_K = K\, x$$

$$\boxed{f = (MD^2 + BD + K)\, x}$$

$(M:\ mass,\ B:\ damping\ or\ viscous\ friction,\ K: elastance\ or\ stiffness)$

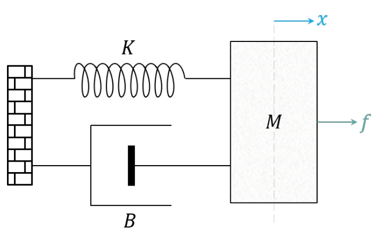


Figure 2.2: Mechanical translation system.

Simple Mechanical Rotational System

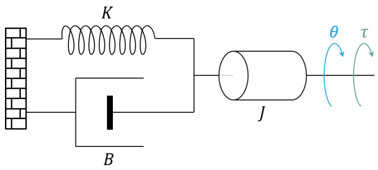


Figure 2.3: Mechanical rotational system.

$$\tau_J = (JD^2)\,\theta$$

$$\tau_B = (BD)\,\theta$$

$$\tau_K = K\,\theta$$

$$\boxed{\tau = (JD^2 + BD + K)\,\theta}$$

$(J:\ moment\ of\ inertia)$

Single-stage Rotating Amplifier

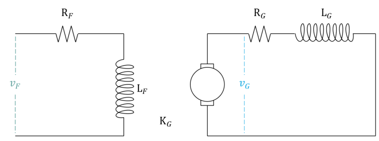


Figure 2.4: Field controlled generator.

$$v_F = (DL_F + R_F)\,i_F$$

$$v_G = K_G\,i_F$$

$$\boxed{v_F = (\frac{DL_F + R_F}{K_G})\,v_G}$$

$(F:\ field,\ G:\ generator)$

D-C Servomotor

Keep in mind the mechanical rotational system equation.

Armature Controlled Motor

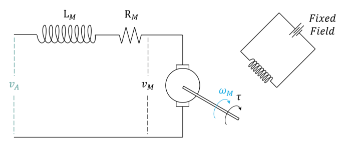


Figure 2.5: Armature controlled motor.

$$v_M = (K_b\,D)\,\theta_M$$

$$\tau = K_T\,i_M$$

$$i_M = (\frac{JD^2 + BD}{K_T})\,\theta_M$$

$$v_A = v_M + (L_M D + R_M)\,i_M$$

$$\boxed{v_A = [\frac{(L_M J)\,D^3 + (L_M B + R_M J)\,D^2 + (R_M B + K_b K_T)\,D}{K_T}]\,\theta_M}$$

$(M:\ motor,\ A:\ armature)$

Field Controlled Motor

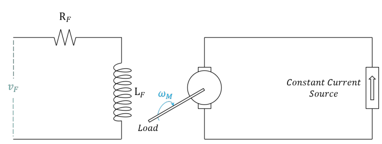


Figure 2.6: Field controlled motor.

$$\tau = K_F\,i_F$$

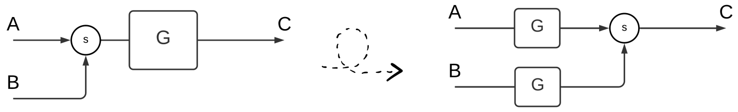
$$i_F = (\frac{JD^2 + BD}{K_F})\,\theta_M$$

$$v_F = (DL_F + R_F)\,i_F$$

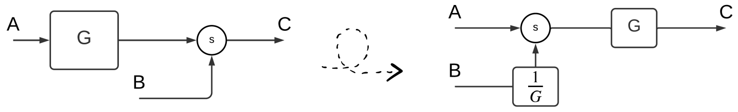
$$\boxed{v_F = [\frac{(JD^2 + BD)(DL_F + R_F)}{K_F}]\,\theta_M}$$

2.2 Reduction Techniques (Moving Points)

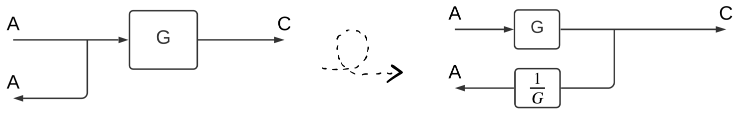
Summing point behind a block:



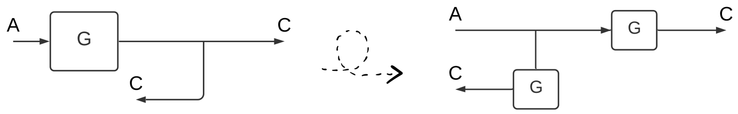
Summing point ahead a block:



Take-off point behind a block:



Take-off point ahead a block:



Homework Convert Motor-generator control schematic diagram to block diagram and simpify it.

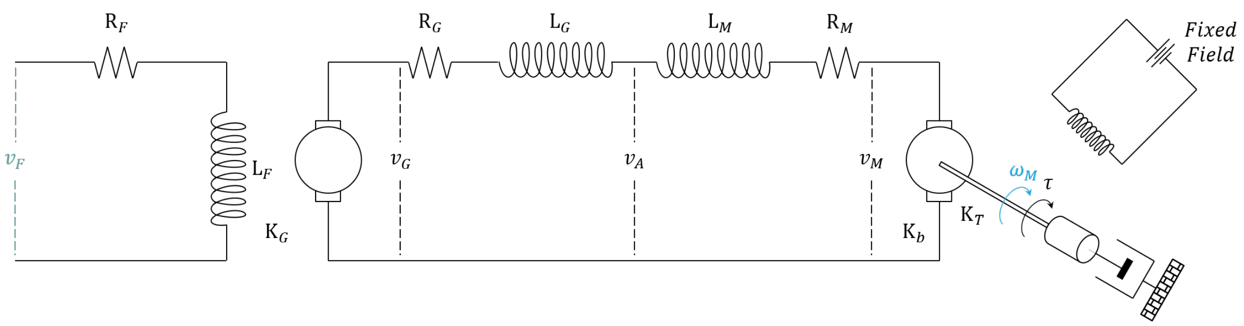
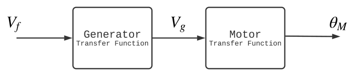


Figure 2.7: Motor-generator control.

Hint The last step should be:



Signal Flow Graphs | 3

3.1 Definitions

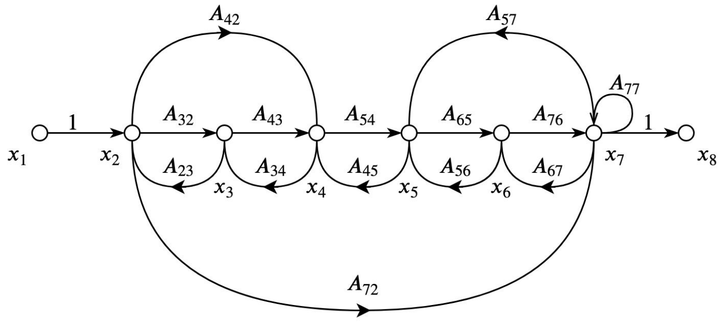


Figure 3.1: Signal flow general graph.

Nodes

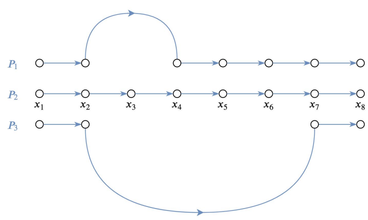
Source Nodes Represent independent variables and have only outgoing branches. (x_1)

Sink Nodes Represent dependent variables and have only incoming branches. (x_8)

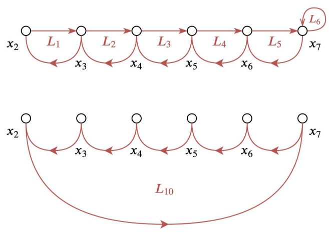
Mixed Nodes Have both incoming and outgoing branches. ($x_2 \longrightarrow x_7$)

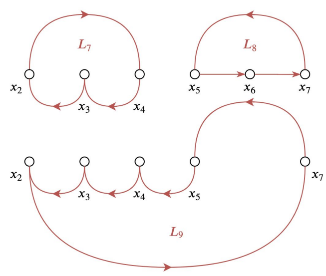
Paths

Forward Path From the input node to the output node.

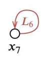


Feedback loop Originates and terminates on the same node.





Self loop A feedback loop consisting of a single branch.



Non-touching loops

Two at a time				Three at a time	Four at a time
L_1L_3	L_2L_4	L_3L_6	L_7L_8	$L_1L_3L_5$	
L_1L_4	L_2L_5	L_4L_6		$L_1L_3L_6$	
L_1L_5	L_2L_6	L_4L_7		$L_1L_4L_6$	$\dots$
L_1L_6	L_2L_8	L_5L_7		$L_2L_4L_6$	
L_1L_8	L_3L_5	L_6L_7		$L_4L_6L_7$	

Gain

Path Gain Product of branch gains encountered in a path. ($P_3 :\ A_{72}$)
Loop Gain Product of the branch gains of the loop. ($L_3 :\ A_{54}A_{45}$)

3.2 Block Diagram to Signal Flow Graph

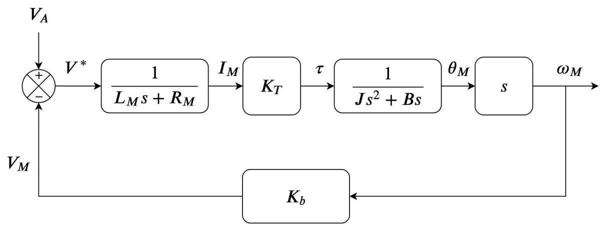


Figure 3.2: Motor block diagram.

Step 1 Represent all the signals, variables, summing points and take-off points of block diagram as nodes in signal flow graph.
Step 2 Represent the blocks of block diagram as branches in signal flow graph.
Step 3 Represent the transfer functions inside the blocks of block diagram as gains of the branches in signal flow graph.[1]

1: Connect the nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one.

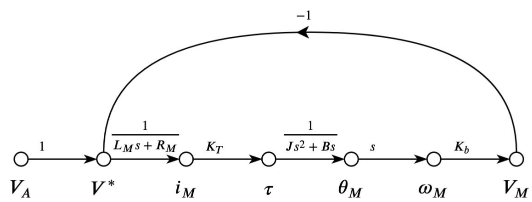


Figure 3.3: Motor flow-graph diagram.

3.3 Flow-graph Algebra

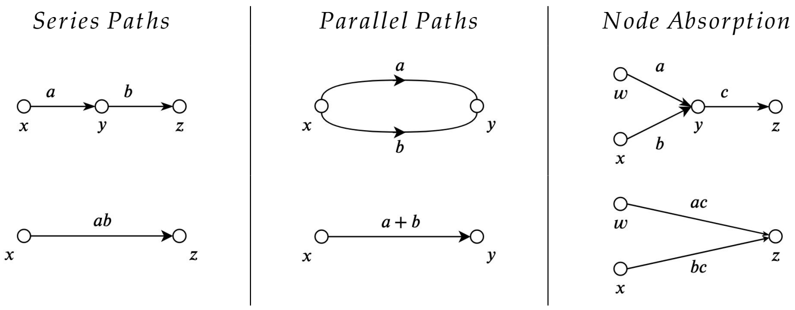


Table 3.1: Simplification rules.

Feedback paths

The equations for the feedback system of Figure 3.4 are:

$$C = GE$$
$$E = R - HC$$

Step 1 The node E can be eliminated to produce a graph with a self-loop.

Step 2 The final simplification is to eliminate the self-loop to produce the over-all transmittance.

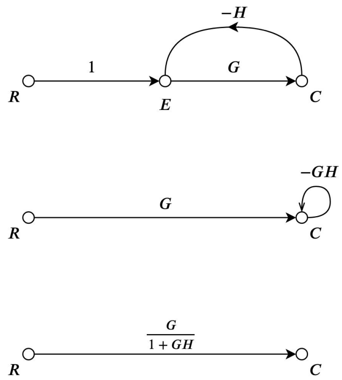


Figure 3.4: Reduction of a feedback path.

3.4 The Mason Rule

The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function. On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula.

$$\boxed{\frac{C(s)}{R(s)} = \frac{1}{\Delta} \cdot \sum_{i=1}^{n} P_i \, \Delta_i}$$

n Number of forward paths.
P_i The i^{th} forward path gain.
Δ Determinant of the system.
Δ_i Determinant of the i^{th} forward path.

Graph determinant Δ

$\boldsymbol{\Delta}$ = 1 − (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) − . . . and so forth with sums of higher number of non-touching loop gains.

$\boldsymbol{\Delta_i}$ = value of Δ for the part of the flow graph that does not touch the i^{th} forward path. [2]

2: $\Delta_i = 1$ if there are no non-touching loops to the i^{th} path.

Systematic approach

Step 1 Calculate forward path gain P_i for each i forward path.
Step 2 Calculate all loops transfer functions.
Step 3 Consider non-touching loops 2 at a time, 3 at a time, . . . etc.
Step 4 Calculate Δ from steps 2 and 3.
Step 5 Calculate Δ_i as portion of Δ not touching i forward path.

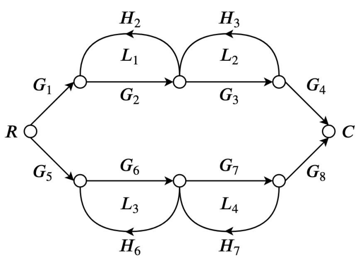
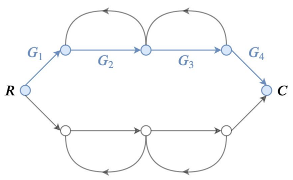
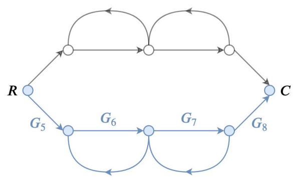


Figure 3.5: Mason rule example.

Step 1



$P_1 = G_1 G_2 G_3 G_4$



$P_2 = G_5 G_6 G_7 G_8$

Step 2

$$\begin{array}{|l|}\hline L_1 = G_2H_2 \\ L_2 = G_3H_3 \\ L_3 = G_6H_6 \\ L_4 = G_7H_7 \\ \hline\end{array}$$

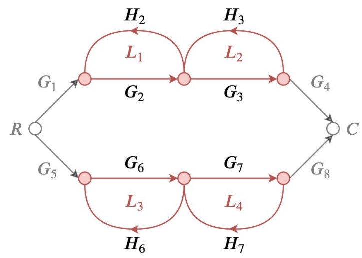


Figure 3.6: Loops considered.

Step 3

Two at a time	Three at a time
L_1L_3	
L_1L_4	...
L_2L_3	
L_2L_4	

Step 4

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$$

$$\begin{aligned}\Delta = {} & 1 - (G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7) \\ & + (G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7)\end{aligned}$$

Step 5

Eliminate forward path-1:

$$\Delta_1 = 1 - (L_3 + L_4)$$

$$\Delta_1 = 1 - (G_6H_6 + G_7H_7)$$

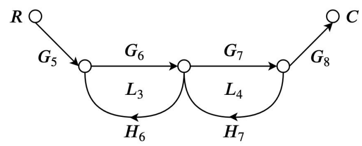


Figure 3.7: P_1 eliminated.

Eliminate forward path-2:

$$\Delta_2 = 1 - (L_1 + L_2)$$

$$\Delta_2 = 1 - (G_2H_2 + H_3G_3)$$

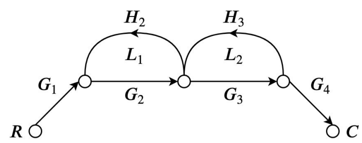


Figure 3.8: P_2 eliminated.

Applying Mason's rule

$$\frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4\left[1 - (G_6H_6 + G_7H_7)\right] + G_5G_6G_7G_8\left[1 - (G_2H_2 + H_3G_3)\right]}{1 - (G_2H_2 + H_3G_3 + G_6H_6 + G_7H_7) + (G_2H_2G_6H_6 + G_2H_2G_7H_7 + H_3G_3G_6H_6 + H_3G_3G_7H_7)}$$

4 System Stability

4.1 Singularities of a Function

System is stable if for bounded input the output is bounded BIBO.
System stability can be determined from the Transfer Function.

$$T.F = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_o}{a_n s^n \ + a_{n-1} s^{\ n-1} + \ldots + a_1 s + a_o}$$

Can be written in terms of factors:

$$T.F = \frac{N(s)}{D(s)} = K \frac{(s - Z_1)(s - Z_2) \ldots (s - Z_{m-1})(s - Z_m)}{(s - P_1)(s - P_2 \ \ldots (s - P_{\ n-1})(s - P_n \ )}$$

- Roots of $D(s)$: $P_1, P_2, \ldots, P_n$ are called poles.
- Roots of $N(s)$: $Z_1, Z_2, \ldots, Z_m$ are called zeros.

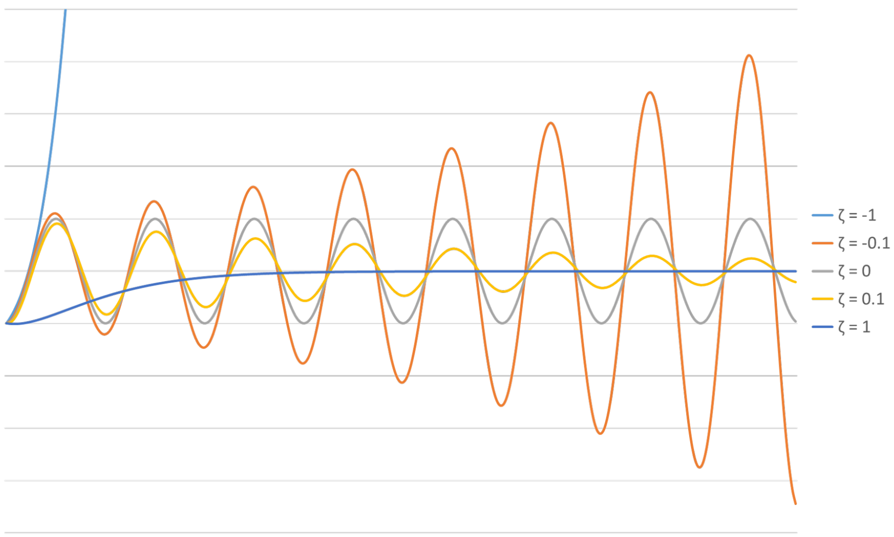
(where $m < n$)

since all coeffieceints of $D(s)$, $N(s)$ are real, poles and zeros must be real or complex conjugate.

Second order systems

$$T.F = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Root : $(s = -\omega_n[\zeta \pm \sqrt{\zeta^2 - 1}])$



where ζ is the damping ratio

Root : $(s = 0)$

$$\int_0^\infty f(t)\, dt \quad \text{is not bounded.}$$

(unstable)

Stability Condition

All poles should lie in the left half of s-plane, then it is necessary, but not sufficient that:

- All coefficients of $D(s)$ must be real and positive.
- All powers of s from s^1 to s^n should be present, however all odd powers of s or all even power of s may be missing.

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This is the last lecture, I will personally depend on

Dr. Imtiaz Hussain Lectures

Appendix

A

Laplace Transforms

Remember that we consider all functions are defined only on $t \geq 0$.

Table A.1: Theorems

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
$\grave{f}(t)$	$sF(s) - f(0)$
$\int_0^t f(x)dx$	$\frac{1}{s} F(s)$
$t f(t)$	$-\grave{F}(s)$
$\frac{1}{t} f(t)$	$\int_s^\infty F(x)dx\ ,\ if\ \lim_{t \to 0} \frac{1}{t} f(t)\ exists$
$e^{at} f(t)$	$F(s-a)$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as} F(s)$
$\int_0^t f(x)g(t-x)dx$	$F(s)\, G(s)$

Table A.2: General transforms ©

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
a	$\frac{a}{s}$
$\delta(t-a)$	e^{-as}
$\mathcal{U}(t-a)$	$\frac{1}{s}\, e^{-as}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
t^p	$\frac{\Gamma(p+1)}{s^{p+1}}\ ,\ p>-1$

Table A.3: Specific transforms ➲

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
$\frac{d^n}{dt^n}$	$s^{\boldsymbol{n}_{ote}} \rightsquigarrow a$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\int_{-\infty}^t f(x)dx$	$\frac{1}{s} F(s) + \frac{1}{s} \int_{-\infty}^0 f(x)dx$

[a] Since all the initial conditions are assumed to be zero.

$(a : constant, x : dummy\ variable, p : real\ number, n : integer)$

Gamma function which is defined as:

$$\Gamma\left(p\right) = \int\limits_0^\infty e^{-x} x^{p-1} dx$$ ➲

If n is a positive integer then,

$$\Gamma\left(n+1\right) = n!$$

B
Identification of Energy Functions

B.1 Electric Circuits

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inductance, L	$\frac{1}{2}Li^2$	—	—
Capacitance, C	—	$\frac{1}{2C}\left(\int i\ dt\right)^2$	—
Resistance, R	—	—	$\frac{1}{2}Ri^2$

Table B.1: Forcing function is $v\ voltage$

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inductance, L	—	$\frac{1}{2L}\left(\int v\ dt\right)^2$	—
Capacitance, C	$\frac{1}{2}Cv^2$	—	—
Conductance, G	—	—	$\frac{1}{2}Gv^2$

Table B.2: Forcing function is $i\ current$

B.2 Mechanics

Element	Kinetic energy T	Potential energy V	Dissipation function D
Mass, M	$\frac{1}{2}M\dot{x}^2$	—	—
Elastance, K	—	$\frac{1}{2}K(x_o - x)^2$	—
Damping, B	—	—	$\frac{1}{2}B(\dot{x}_o - \dot{x})^2$

Table B.3: Forcing function is $F\ force$

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inertia, J	$\frac{1}{2}J\dot{\theta}^2$	—	—
Elastance, K	—	$\frac{1}{2}K(\theta_o - \theta)^2$	—
Damping, B	—	—	$\frac{1}{2}B(\dot{\theta}_o - \dot{\theta})^2$

Table B.4: Forcing function is $\tau\ torque$

Reference Feedback control system analysis and synthesis.*

(John j. Dazzo, Constantine h. Houpis)

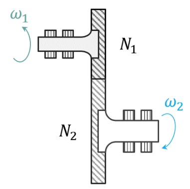
* Tables 2-6, 2-7, 2-8, and 2-9 "2nd Edition"

C

Not Simple Systems Equations

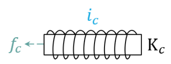
Gear train, rotational transformer

$$\frac{\omega_2}{\omega_1} = \frac{N_1}{N_2}$$



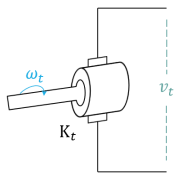
Solenoid, magnetic force

$$\frac{f_c}{i_c} = K_c$$



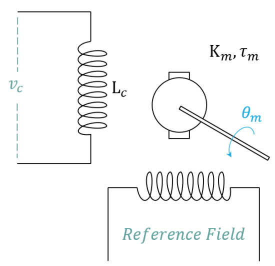
Tachometer, velocity sensor

$$\frac{v_t}{\omega_t} = K_t$$



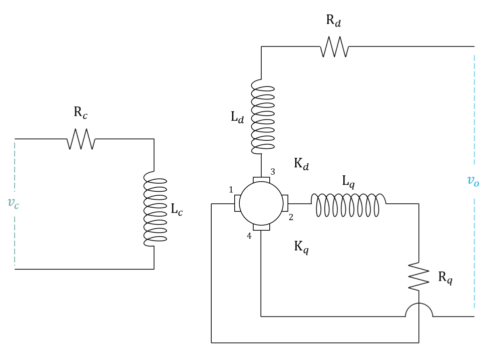
AC motor, two-phase control field

$$\frac{\Theta_m}{V_c} = \frac{K_m}{s(\tau_m s + 1)}$$



Amplidyne, rotary amplifier

$$\frac{V_o}{V_c} = \frac{K_q}{sL_c + R_c} \cdot \frac{K_d}{sL_q + R_q}$$



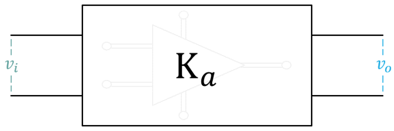
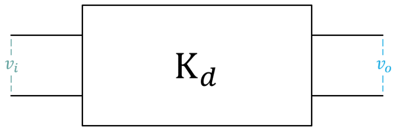
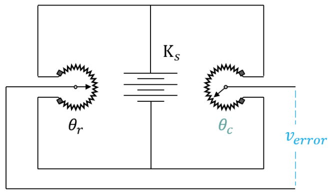
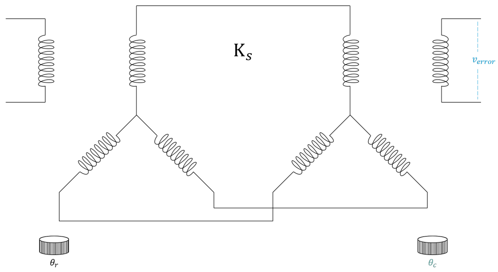
DC amplifier, 0 Hz amplifier	
$\frac{v_o}{v_i} = K_a$	
Demodulator, AC modulated signal to DC	
$\frac{v_o}{v_i} = K_d$	
Potentiometer, used in "Error detector bridge"	
$\frac{v_{error}}{\theta_r - \theta_c} = K_s$	
Synchro, as "Error detector"	
$\frac{v_{error}}{\theta_r - \theta_c} = K_s$	

Table C.1: Transfer functions

Reference Modern control systems.*

(Richard c. Dorf, Robert h.Bishop)

* Tables 2-4 "International Edition"

Greek Letters with Pronunciation

Character	Name	Character	Name
α	alpha *AL-fuh*	ν	nu *NEW*
β	beta *BAY-tuh*	ξ, Ξ	xi *KSIGH*
γ, Γ	gamma *GAM-muh*	o	omicron *OM-uh-CRON*
δ, Δ	delta *DEL-tuh*	π, Π	pi *PIE*
ϵ	epsilon *EP-suh-lon*	ρ	rho *ROW*
ζ	zeta *ZAY-tuh*	σ, Σ	sigma *SIG-muh*
η	eta *AY-tuh*	τ	tau *TOW (as in cow)*
θ, Θ	theta *THAY-tuh*	υ, Υ	upsilon *OOP-suh-LON*
ι	iota *eye-OH-tuh*	ϕ, Φ	phi *FEE, or FI (as in hi)*
κ	kappa *KAP-uh*	χ	chi *KI (as in hi)*
λ, Λ	lambda *LAM-duh*	ψ, Ψ	psi *SIGH, or PSIGH*
μ	mu *MEW*	ω, Ω	omega *oh-MAY-guh*

Capitals shown are the ones that differ from Roman capitals.