Ongoing Lectures Collection

CS241

Linear Control Systems

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Lectures Collection

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Colophon
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Preface

I'm using LaTeX, hoping that this work will continue existing. I don't have much else to say, so I will just insert some blind text. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Ahmed Waleed

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First Lecture 1

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1.1 Introduction

Analysis of linear continuous system analysis of a system means simply checking the goodness of its measure of performance. Analysis could be done in two different ways:

- In the lab: by putting test input to the system and checking if the output satisfies the measure of performance.
- Using analytical techniques: which is our concern in this course.

The first step is to make a mathematical model to the system.

$$\Sigma F_x = m\ddot{x}$$
$$F - \mu\dot{x} - kx = m\ddot{x}$$
$$\therefore \boxed{F = m\ddot{x} + \mu\dot{x} + kx}$$

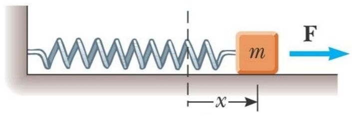


Figure 1.1: A block attached to a spring. ©

Then defining the measure of performance and studying how we can check these measure of performance.

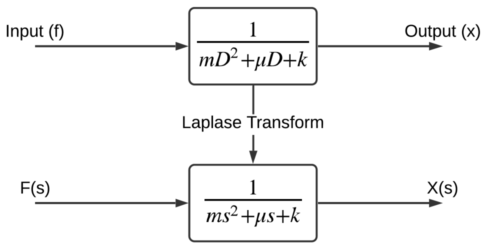


Figure 1.2: Since D is an operator (can't have a value), the transfer function is obtained by the Laplace transform of the first relation.

Transfer function ratio between Laplace transform of the output and Laplace transform of the input, assuming zero initial conditions.

1.2 Control Systems

A control system is an interconnection of components forming a system configuration that will provide a desired system response.

Open-loop control system (without feedback):

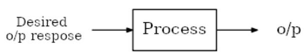


Figure 1.3: Its output does not track the input, and it is more affected by noise.

Closed-loop feedback control system (with feedback):

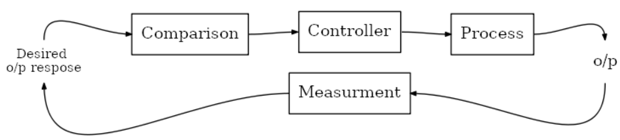


Figure 1.4: Closed loop control can improve accuracy, also the actuating signal is a function of the output.

1.3 Mathematical Model

Any linear continuous system can be represented either by a linear algebraic equation or an ordinary differential equation such as:

$$(mD^2 + \mu D + k)\, x(t) = y(t)$$

Solving the differential equation using Laplace transform assuming zero initial conditions made it possible to get the transfer function.

1.4 Block Diagram Reduction ➲

A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system. Control systems require the arithmetic manipulation in order to obtain the overall transfer function and this is the start point for the analysis of the system.

Table 1.1: Terminology

R	:	reference input / desired output response.
E	:	actuating / error signal.
G	:	control element and controlled system.
C	:	controlled variable / actual output.
H	:	feedback / backward transfer element.
B	:	primary feedback.
s	:	summation point.
t	:	takeoff point.

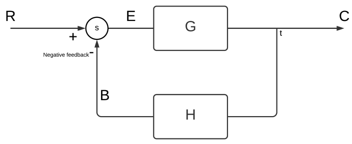
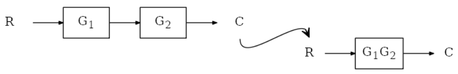
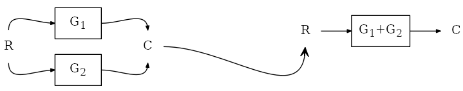


Figure 1.5: Canonical feedback loop.

Cascade connection :



Parallel connection :



Summation point : a small circle, with plus or minus sign associated with the inputs, and the output is the algebraic sum of the inputs.

Take-off point : a takeoff (or pickoff) point is used in order to have the same signal input to more than one block.

Overall transfer function (feedback loop elimination):

Applying reduction techniques mentioned above, we can obtain the overall transfer function of Figure **??**

$$
\begin{aligned}
@\,s:\quad & E = R - B && \text{(summation point)}\\
\because\quad & B = CH && \text{(block)}\\
\therefore\quad & E = R - CH &&\\
\because\quad & C = GE && \text{(block)}\\
\therefore\quad & C = G(R - CH) &&\\
& C + CGH = GR &&\\
& C(1 + GH) = GR &&
\end{aligned}
$$

$$\boxed{\frac{C}{R} = \frac{G}{(1 + GH)}}$$

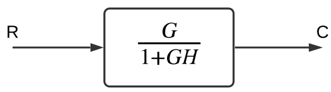


Figure 1.6: Feedback loop equivalent

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2.1 Introduction to Mathematical Models

We will be studying single output linear continuous systems. If a system has more than one input the superposition principle will be applied.

Super Position Principle ©

The superposition property, states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.

$$\boxed{F(x_1+x_2) = F(x_1)+F(x_2)}$$

(It can be used to prove linearity)

Simple Systems Equations ✉

The purpose of this section is to present methods of writing the differential equations for a variety of electrical and mechanical systems. This is the first step that must be mastered by the would-be control systems engineer.

Series Resistor-Inductor-Capacitor Circuit

$$v_L = (LD)\, i$$

$$v_R = R\, i$$

$$v_C = (\frac{1}{CD})\, i$$

$$\boxed{v = (LD + R + \frac{1}{CD})\, i}$$

$(R:\ resistor,\ L:\ inductor,\ C:\ capacitor)$

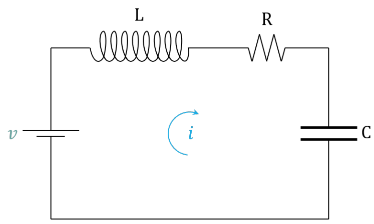


Figure 2.1: Simple electrical system.

Simple Mechanical Translation System

$$f_M = (MD^2)\, x$$

$$f_B = (BD)\, x$$

$$f_K = K\, x$$

$$\boxed{f = (MD^2 + BD + K)\, x}$$

$(M:\ mass,\ B:\ damping\ or\ viscous\ friction,\ K: elastance\ or\ stiffness)$

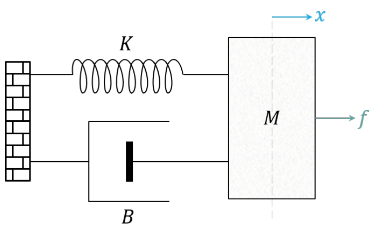


Figure 2.2: Mechanical translation system.

Simple Mechanical Rotational System

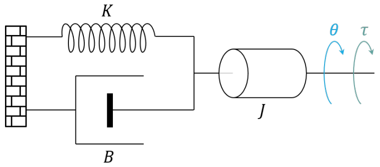


Figure 2.3: Mechanical rotational system.

$$\tau_J = (JD^2)\,\theta$$

$$\tau_B = (BD)\,\theta$$

$$\tau_K = K\,\theta$$

$$\boxed{\tau = (JD^2 + BD + K)\,\theta}$$

$(J:\ moment\ of\ inertia)$

Single-stage Rotating Amplifier

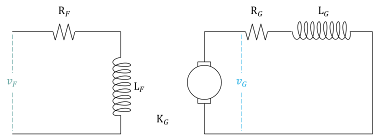


Figure 2.4: Field controlled generator.

$$v_F = (DL_F + R_F)\,i_F$$

$$v_G = K_G\,i_F$$

$$\boxed{v_F = \left(\frac{DL_F + R_F}{K_G}\right) v_G}$$

$(F:\ field,\ G:\ generator)$

D-C Servomotor

Keep in mind the mechanical rotational system equation.

Armature Controlled Motor

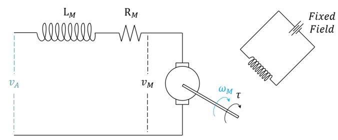


Figure 2.5: Armature controlled motor.

$$v_M = (K_b\,D)\,\theta_M$$

$$\tau = K_T\,i_M$$

$$i_M = \left(\frac{JD^2 + BD}{K_T}\right)\theta_M$$

$$v_A = v_M + (L_M D + R_M)\,i_M$$

$$\boxed{v_A = \left[\frac{(L_M J)\,D^3 + (L_M B + R_M J)\,D^2 + (R_M B + K_b K_T)\,D}{K_T}\right]\theta_M}$$

$(M:\ motor,\ A:\ armature)$

Field Controlled Motor

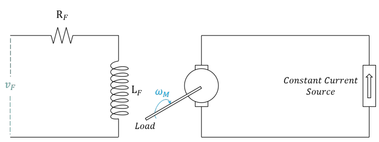


Figure 2.6: Field controlled motor.

$$\tau = K_F\,i_F$$

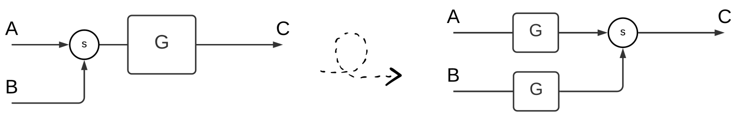
$$i_F = \left(\frac{JD^2 + BD}{K_F}\right)\theta_M$$

$$v_F = (DL_F + R_F)\,i_F$$

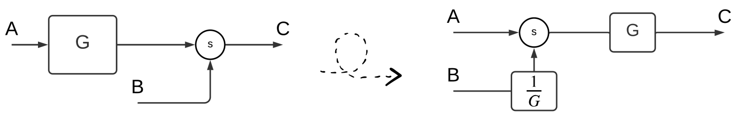
$$\boxed{v_F = \left[\frac{(JD^2 + BD)(DL_F + R_F)}{K_F}\right]\theta_M}$$

2.2 Reduction Techniques (Moving Points)

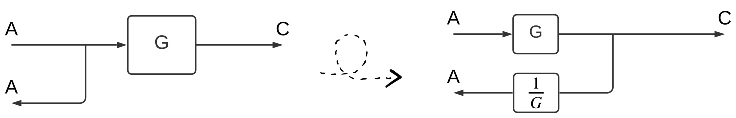
Summing point behind a block:



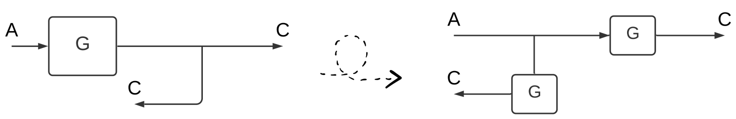
Summing point ahead a block:



Take-off point behind a block:



Take-off point ahead a block:



Homework Convert Motor-generator control schematic diagram to block diagram and simpify it.

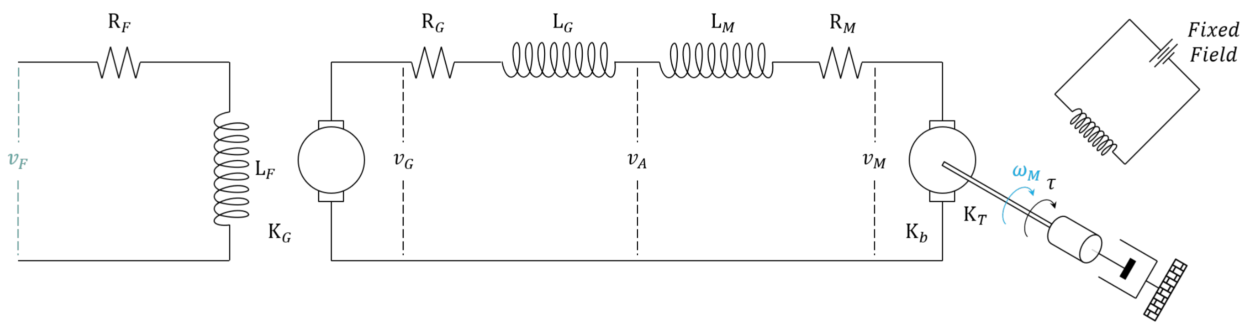
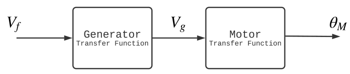


Figure 2.7: Motor-generator control.

Hint The last step should be:



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3.1 Signal Flow Graphs

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Appendix

A

Laplace Transforms

Remember that we consider all functions are defined only on $t \geq 0$.

Table A.1: Theorems

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
$f(at)$	$\frac{1}{a} F(\frac{s}{a})$
$\grave{f}(t)$	$sF(s) - f(0)$
$\int_0^t f(x) dx$	$\frac{1}{s} F(s)$
$t f(t)$	$-\grave{F}(s)$
$\frac{1}{t} f(t)$	$\int_s^\infty F(x) dx \ , \ if \ \lim_{t \to 0} \frac{1}{t} f(t) \ exists$
$e^{at} f(t)$	$F(s-a)$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as} F(s)$
$\int_0^t f(x) g(t-x) dx$	$F(s)\, G(s)$

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
a	$\frac{a}{s}$
$\delta(t-a)$	e^{-as}
$\mathcal{U}(t-a)$	$\frac{1}{s}\, e^{-as}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
t^p	$\frac{\Gamma(p+1)}{s^{p+1}} \ , \ p>-1$

Table A.2: General transforms ©

$f(t)$	$\mathscr{L}\left[f(t)\right] = \int_0^\infty f(t)\, e^{-st} dt = F(s)$
$\frac{d^n}{dt^n}$	$s^{n_{ote}} \rightsquigarrow a$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\int_{-\infty}^t f(x) dx$	$\frac{1}{s} F(s) + \frac{1}{s} \int_{-\infty}^0 f(x) dx$

Table A.3: Specific transforms ➲

[a] Since all the initial conditions are assumed to be zero.

$(a : constant, x : dummy\ variable, p : real\ number, n : integer)$

Gamma function which is defined as:

$$\Gamma\left(p\right) = \int_0^\infty e^{-x} x^{p-1} dx$$ ➲

If n is a positive integer then,

$$\Gamma\left(n+1\right) = n!$$

B

Identification of Energy Functions

B.1 Electric Circuits

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inductance, L	$\frac{1}{2}Li^2$	—	—
Capacitance, C	—	$\frac{1}{2C}\left(\int i\ dt\right)^2$	—
Resistance, R	—	—	$\frac{1}{2}Ri^2$

Table B.1: Forcing function is $v\ voltage$

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inductance, L	—	$\frac{1}{2L}\left(\int v\ dt\right)^2$	—
Capacitance, C	$\frac{1}{2}Cv^2$	—	—
Conductance, G	—	—	$\frac{1}{2}Gv^2$

Table B.2: Forcing function is $i\ current$

B.2 Mechanics

Element	Kinetic energy T	Potential energy V	Dissipation function D
Mass, M	$\frac{1}{2}M\dot{x}^2$	—	—
Elastance, K	—	$\frac{1}{2}K(x_o - x)^2$	—
Damping, B	—	—	$\frac{1}{2}B(\dot{x}_o - \dot{x})^2$

Table B.3: Forcing function is $F\ force$

Element	Kinetic energy T	Potential energy V	Dissipation function D
Inertia, J	$\frac{1}{2}J\dot{\theta}^2$	—	—
Elastance, K	—	$\frac{1}{2}K(\theta_o - \theta)^2$	—
Damping, B	—	—	$\frac{1}{2}B(\dot{\theta}_o - \dot{\theta})^2$

Table B.4: Forcing function is $\tau\ torque$

Reference Feedback control system analysis and synthesis.*

(John j. Dazzo, Constantine h. Houpis)

* Tables 2-6, 2-7, 2-8, and 2-9 "2nd Edition"