

# ALGORITHMS FUNDAMENTALS

## Introduction, Design & Types

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# Outline

1 Introduction to Algorithms



2 Algorithm Design



3 Algorithm Types



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3 Algorithm Types



# What is an Algorithm?

## Definition

An **algorithm** is a finite sequence of well-defined **instructions** that can be mechanically executed to solve a **computational** problem or perform a task.

## Key Components:

- **Input:** Zero or more quantities supplied externally
- **Output:** One or more quantities produced
- **Instructions:** Precise and unambiguous steps
- **Execution:** Performed mechanically



## Remember

An algorithm is a *method* for solving problems, not the implementation itself!



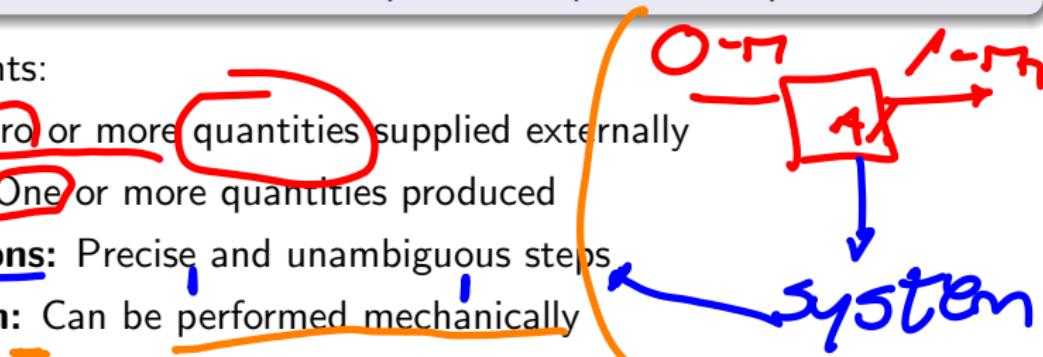
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design



# Algorithm Characteristics

## Essential Properties

- ① **Finiteness:** Must terminate after finite number of steps
  - ② **Definiteness:** Each step must be precisely defined *goal*
  - ③ **Input:** Zero or more input values
  - ④ **Output:** One or more output values
  - ⑤ **Effectiveness:** Operations must be basic and feasible
- avoids infinite loops*

⑤

②

## Example

### Making Coffee Algorithm:

- ① Boil water (definite action)
- ② Add coffee grounds (precise amount)
- ③ Wait 4 minutes (finite time)
- ④ Pour into cup (effective operation)

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Value → Constant  
Value

# Algorithm vs. Program vs. Process

**Algorithm *design***

- Abstract concept
- Language-independent
- Mathematical description
- Design phase

**Program *code***

- Concrete implementation
- Language-specific
- Code representation
- Implementation phase

**Process *FUJ***

- Runtime execution
- System-dependent
- Active computation
- Execution phase

*Works like*

## Example

terminate Example: Finding the maximum number

- **Algorithm:** "Compare each element with current maximum"
- **Program:** Python code implementing the comparison
- **Process:** Program running in memory, using CPU cycles

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*if replace*

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# Types of Algorithmic Thinking

## Fundamental Approaches

- ① **Sequential Thinking:** Step-by-step execution
- ② **Conditional Thinking:** Decision-based branching
- ③ **Iterative Thinking:** Repetitive operations
- ④ **Recursive Thinking:** Self-referential solutions
- ⑤ **Parallel Thinking:** Concurrent execution

## Example (Daily Life)

- Getting dressed (sequential)
- Choosing route to university (conditional)
- Studying until understanding (iterative)
- Recalling a recipe (recursive)
- Cooking multiple dishes (parallel)

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# Study Case: Euclidean GCD Algorithm

## Greatest Common Divisor Problem

**Input:** Two positive integers  $a$  and  $b$

**Output:** The largest integer that divides both  $a$  and  $b$

### Algorithm 1 Euclidean GCD Algorithm

**Input:** Integers  $a, b$  where  $a \geq b > 0$

**while**  $b \neq 0$  **do**

$r \leftarrow a \bmod b$

$a \leftarrow b$

$b \leftarrow r$

**end while**

**return**  $a$



# Study Case: Euclidean GCD Algorithm

## Greatest Common Divisor Problem

**Input:** Two positive integers  $a$  and  $b$

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### Algorithm 2 Euclidean GCD Algorithm

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# Exercise Time! [1]

## Exercise 1: Square Root Calculation

Design an algorithm to calculate the square root of a positive number using the Babylonian method:

- ① Start with an initial guess  $x_0$  (e.g.,  $S/2$  for number  $S$ )
- ② Improve the guess:  $x_{n+1} = \frac{1}{2}(x_n + \frac{S}{x_n})$
- ③ Repeat until convergence

## Exercise 2: Find Maximum in an Array

Write an algorithm that finds the maximum element in an array of integers.

**Consider:** What if the array is empty? What about duplicate maximums?



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# Exercise Time! [2]

## Exercise 3: Algorithms in Daily Life

Identify and describe three algorithms you use in your daily life. For each:

- Define clear inputs and outputs
- List the precise steps
- Verify all algorithm characteristics



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# Mathematical Notation for Algorithms [I]

## Mathematical Notation

Mathematical notation provides a formal way to express algorithms using symbols, formulas, and structures from mathematics.

## Why Mathematical Notation?

- **Precision:** Unambiguous specification
- **Universality:** Language-independent
- **Conciseness:** Compact representation
- **Analysis:** Enables formal verification



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# Mathematical Notation for Algorithms [II]

## Example

### Set Operations:

- $S = \{x \in \mathbb{N} : x \text{ is prime and } x < 20\}$
- $\forall x \in S, P(x)$  (for all elements in  $S$ , property  $P$  holds)
- $\exists x \in S : Q(x)$  (there exists an element in  $S$  with property  $Q$ )

## Example

### Function Definition:

$$f : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$$
$$f(a, b) = \begin{cases} a & \text{if } b = 0 \\ f(b, a \bmod b) & \text{otherwise} \end{cases}$$



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# Converting Mathematical Formulas to Pseudocode

## Mathematical Expression

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

## Another Example

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

```

1: sum ← 0
2: for i = 1 to n do
3:   sum ← sum + i2
4: end for
5: formula ←  $\frac{n(n+1)(2n+1)}{6}$ 
6: if sum = formula then
7:   return true
8: else
9:   return false
10: end if

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```

1: result ← 1
2: term ← 1
3: n ← 1
4: while |term| > ε do
5:   term ←  $\frac{term \cdot x}{n}$ 
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# Study Case: Factorial Using Different Notations

## Mathematical

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n - 1)! & \text{if } n > 0 \end{cases}$$

$$n! = \prod_{i=1}^n i$$

## Iterative Pseudocode

```

1: function FACTORIAL(n)
2:   result  $\leftarrow$  1
3:   for i = 1 to n do
4:     result  $\leftarrow$  result  $\times$  i
5:   end for
6:   return result
7: end function
```



# Pseudocode Standards and Conventions

## Basic Elements

- **Keywords:** if, then, else, while, for, return
- **Assignment:**  $x \leftarrow value$  or  $x = value$
- **Comparison:**  $=, \neq, <, >, \leq, \geq$
- **Logic:** and, or, not
- **Comments:** // Single line, /\* Multi-line \*/



# Algorithm Documentation Best Practices

## Essential Documentation Elements

- ① **Purpose:** What **does** the algorithm do?
- ② **Preconditions:** What must be true **before execution?**
- ③ **Postconditions:** What is guaranteed **after execution?**
- ④ **Parameters:** Input and **output** specifications
- ⑤ **Complexity:** Time and **space** requirements
- ⑥ **Examples:** Sample inputs and outputs



# Study Case: Fibonacci Sequence

## Problem Definition

The Fibonacci sequence:  $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$  for  $n \geq 2$

**Purpose:** Compute the nth Fibonacci number

**Preconditions:**  $n \geq 0$  (non-negative integer)

**Postconditions:** Returns  $F_n$  where  $F_0 = 0, F_1 = 1$

**Parameters:**

- $n$ : position in Fibonacci sequence

**Complexity:**  $O(n)$  time,  $O(1)$  space

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# Control Structures

## Sequential

- 1: Step 1
- 2: Step 2
- 3: Step 3
- 4: ...

**Execution:** One after another

## Conditional

- 1: if condition then
- 2: Action A
- 3: else
- 4: Action B
- 5: end if

**Execution:** Based on condition

## Iterative

- 1: while condition do
- 2: Action
- 3: end while
- 4: for  $i = 1$  to  $n$  do
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**Execution:** Repeated



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# Control Flow Concepts

- **Entry Point:** Where execution begins
- **Exit Point:** Where execution ends
- **Decision Points:** Where flow branches
- **Loop Control:** How iterations are managed



# Study Case: Decision-Making Algorithms

## Grade Classification Algorithm

**Problem:** Convert numerical grade to letter grade

### Decision Tree

- $\text{Score} \geq 90? \rightarrow A$
- $\text{Score} \geq 80? \rightarrow B$
- $\text{Score} \geq 70? \rightarrow C$
- $\text{Score} \geq 60? \rightarrow D$
- Otherwise  $\rightarrow F$

### Key Concepts

- Mutually exclusive conditions
- Order matters in if-elsif chains
- Default case handling



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# Greedy Algorithms: Philosophy and Approach

## Definition

A **greedy algorithm** makes locally optimal choices at each step, hoping to find a global optimum.

## Greedy Strategy

- ① **Greedy Choice:** At each step, choose the best available option
- ② **Optimal Substructure:** Optimal solution contains optimal solutions to subproblems
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# Greedy Algorithms: When to Use Them

## When Greedy Works:

- Activity selection
- Minimum spanning trees
- Huffman coding

## When Greedy Fails:

- 0/1 Knapsack problem
- Traveling salesman
- Graph coloring



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# Study Case: Coin Change Problem

## Problem Statement

**Given:** Coin denominations  $[d_1, d_2, \dots, d_k]$  and amount  $n$

**Goal:** Find minimum number of coins to make amount  $n$



# Study Case: Coin Change Problem [Solution]

**Denominations:** [25, 10, 5, 1] cents, **Amount:** 67 cents

```
1: Algorithm GREEDYCOINCHANGE(denominations, amount)
2: result  $\leftarrow \emptyset$ 
3: for each denomination d in descending order do
4:   while amount  $\geq d$  do
5:     result.append(d)
6:     amount  $\leftarrow$  amount  $- d$ 
7:   end while
8: end for
9: return result
```

**Solution:**  $67 = 25 + 25 + 10 + 5 + 1 + 1$  (6 coins)



# Divide and Conquer: Methodology

## Definition

**Divide and Conquer** breaks a problem into **smaller subproblems**, solves them **recursively**, then **combines solutions**.

## Three-Step Process

- ① **Divide:** Break problem into smaller subproblems
- ② **Conquer:** Solve subproblems recursively
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# Divide and Conquer: Algorithm Template

```
1: Algorithm DIVIDEANDCONQUER(problem)
2: if problem is small enough then
3:   return SOLVEDIRECTLY(problem)
4: else
5:   subproblems  $\leftarrow$  DIVIDE(problem)
6:   for each subproblem do
7:     solution  $\leftarrow$  DIVIDEANDCONQUER(subproblem)
8:   end for
9:   return COMBINE(solutions)
10: end if
```



# Problem Decomposition Strategies

- **Top-Down:** Start with the **main problem** and break it down into **subproblems**
- **Bottom-Up:** Solve smaller subproblems first and use their solutions to build up to the **main problem**
- **Recursive:** Define the problem in terms of smaller instances of itself



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# Study Case: Binary Search

## Problem

**Input:** Sorted array  $A[1..n]$  and search value  $x$

**Output:** Index of  $x$  in  $A$ , or  $-1$  if not found



# Study Case: Merge Sort Approach

## Sorting Problem

**Input:** Array  $A[1..n]$  of comparable elements

**Output:** Array sorted in ascending order



# Demo Time! Clean the House Exercise

## Problem

You need to clean your house efficiently. You have different rooms and different cleaning tasks.

**Greedy Strategy:** Always clean the dirtiest room first

- ① Assess dirt level of all rooms
- ② Choose room with highest dirt level
- ③ Clean that room completely
- ④ Repeat until all rooms clean

**Advantage:** Maximum immediate impact



**Divide & Conquer Strategy:**  
Split house systematically

- ① Divide house into sections (floors/wings)
- ② Recursively clean each section
- ③ Within each room, divide into areas
- ④ Clean areas systematically

**Advantage:** Systematic coverage



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# Exercise: Knapsack Problem

## Problem Statement

**Given:** Knapsack capacity  $W$ , items with weights  $w_i$  and values  $v_i$

**Goal:** Maximize total value without exceeding weight capacity



# Exercise: Knapsack Problem [Solution]

## 0/1 Knapsack (Greedy Approach)

Cannot take fractions - either take entire item or leave it

- **Strategy:** Sort by value/weight ratio, take whole items
- **Optimal:** No! (Greedy doesn't guarantee optimal solution)

- 1: Sort items by  $v_i/w_i$  descending
- 2: **for** each item  $i$  **do**
- 3:   **if**  $w_i \leq$  remaining capacity **then**
- 4:     Take entire item
- 5:     Update remaining capacity
- 6:   **end if**
- 7: **end for**



# Brute Force: Exhaustive Search

## Definition

**Brute force** algorithms try **all possible solutions** until finding the correct one.

## Characteristics:

- **Exhaustive:** Examines every possibility
- **Guaranteed:** Always finds optimal solution (if exists)
- **Expensive:** Often exponential time complexity
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# Brute Force: When to Use It

- ① Problem size is small
- ② No efficient algorithm is known
- ③ Correctness is more important than efficiency
- ④ As baseline for comparing other algorithms
- ⑤ When optimization overhead exceeds brute force cost



# Study Case: Password Cracking

## Problem

**Given:** Encrypted password hash and character set

**Goal:** Find the original password



# Study Case: Password Cracking [Solution]

```
1: Algorithm BRUTEFORCEPASSWORD(hash, maxlen)
2: for length = 1 to maxlength do
3:   for each possible string s of given length do
4:     if HASH(s) = hash then
5:       return s // Password found!
6:     end if
7:   end for
8: end for
9: return null // Password not found
```

Character set: [a-z] (26 characters)

Password length: 8 characters

Total attempts:  $26^8 \approx 208$  billion

Time estimate: Weeks on single computer!



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# Backtracking: Systematic Trial and Error

## Definition

**Backtracking** explores solution space systematically, **abandoning partial solutions** that cannot lead to valid solutions.

## Backtracking Strategy

- ① **Choose:** Make a choice from available options
- ② **Explore:** Recursively explore consequences
- ③ **Unchoose:** If path fails, backtrack and try another



# Backtracking: Systematic Trial and Error

## Definition

**Backtracking** explores solution space systematically, **abandoning partial solutions** that cannot lead to valid solutions.

## Backtracking Strategy

- ① **Choose:** Make a choice from available options
- ② **Explore:** Recursively explore consequences
- ③ **Unchoose:** If path fails, backtrack and try another



# Backtracking: Algorithm

```
1: Algorithm BACKTRACK(partial_solution)
2: if ISCOMPLETE(partial_solution) then
3:   return partial_solution
4: end if
5: for each possible choice c do
6:   if ISVALID(partial_solution, c) then
7:     partial_solution.add(c)
8:     result  $\leftarrow$  BACKTRACK(partial_solution)
9:     if result  $\neq$  null then
10:      return result
11:    end if
12:    partial_solution.remove(c) // Backtrack!
13:  end if
14: end for
15: return null
```



# Study Case: N-Queens Problem

## Problem

Place  $N$  queens on an  $N \times N$  chessboard such that no two queens attack each other.



# Study Case: N-Queens Problem [Solution]

```
1: Algorithm SOLVENQUEENS(board, row)
2: if row = N then
3:   return board // Solution found!
4: end if
5: for each column col in 0 to N – 1 do
6:   if ISSAFE(board, row, col) then
7:     board[row][col] ← Q // Place queen
8:     result ← SOLVENQUEENS(board, row + 1)
9:     if result ≠ null then
10:       return result
11:     end if
12:     board[row][col] ← . // Remove queen (backtrack)
13:   end if
14: end for
15: return null // No solution found
```



# Implementation Patterns for Backtracking

- **Recursive Structure:** Backtracking algorithms are typically implemented using recursion.
- **State Representation:** Maintain a representation of the current state (e.g., partial solution).
- **Choice Points:** Identify points where decisions are made (choices to explore).
- **Validity Checks:** Implement checks to determine if the current state is valid.
- **Base Case:** Define a base case for when a complete solution is found.
- **Backtracking Mechanism:** Ensure that the algorithm can revert to previous states when necessary.



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# Summary: Algorithm Types

## Key Takeaways

- **Greedy:** Make locally optimal choices (works when greedy choice property holds)
- **Divide & Conquer:** Break into subproblems, solve recursively, combine solutions
- **Brute Force:** Try all possibilities (guarantees correctness but expensive)
- **Backtracking:** Systematic search with pruning (intelligent brute force)



# Summary: Algorithm Types

## Algorithm Selection Guide

- **Small problem size?** → Consider brute force
- **Optimization problem with greedy choice property?** → Try greedy
- **Problem naturally divides into subproblems?** → Use divide & conquer
- **Constraint satisfaction or search problem?** → Apply backtracking



# Outline

1 Introduction to Algorithms

2 Algorithm Design

3 Algorithm Types



# Thanks!

## Questions?



Repo: <https://github.com/EngAndres/ud-public/tree/main/courses/computer-sciences-i>

