Algorithms Fundamentals

Introduction, Design & Types

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Outline

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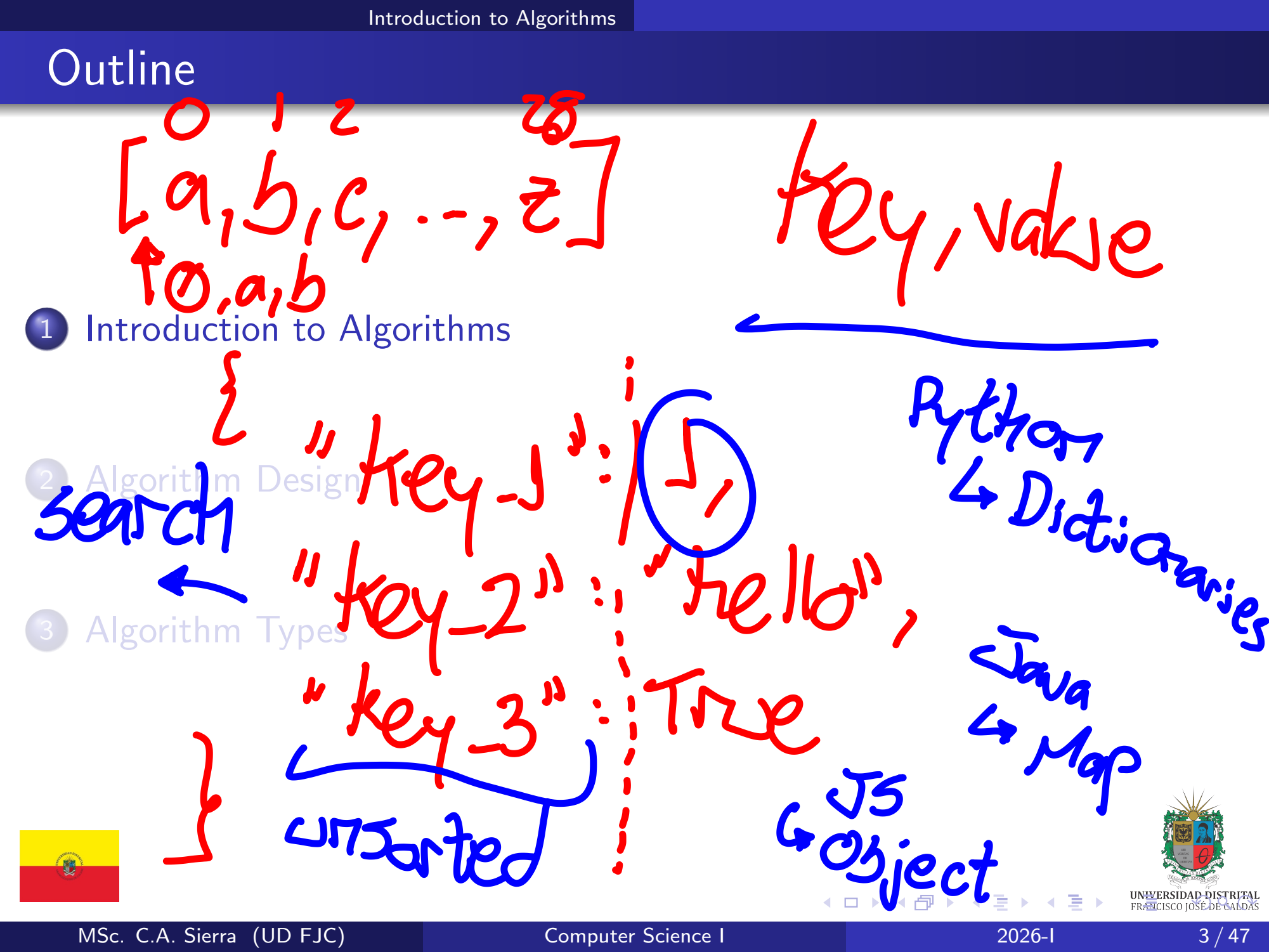


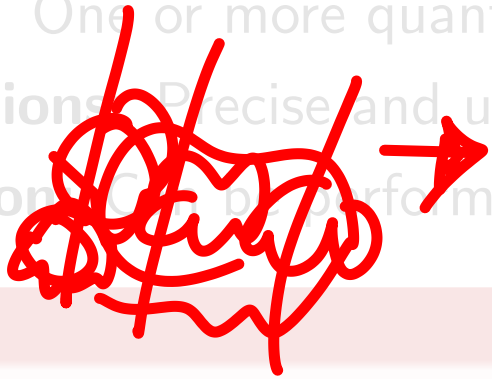
What is an Algorithm?

Definition

An algorithm is a finite sequence of well-defined instructions that can be mechanically executed to solve a computational problem or perform a task.

Key Components:

- **Input:** Zero or more quantities supplied externally
- **Output:** One or more quantities produced
- **Instructions:** Precise and unambiguous steps
- **Execution:** Can be performed mechanically



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An algorithm is a *method* for solving problems, not the implementation itself!

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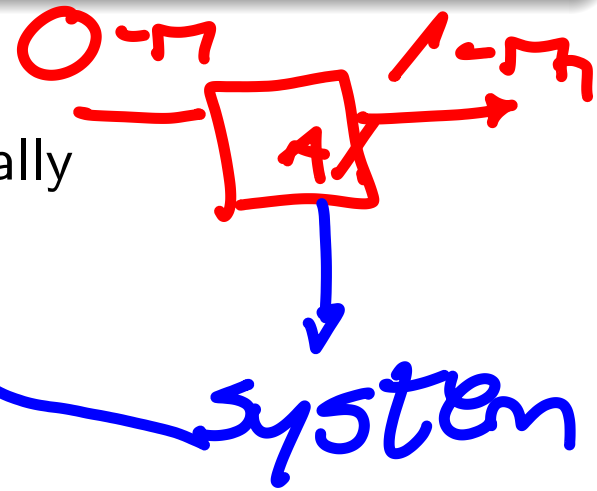

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design

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Algorithm Characteristics

Essential Properties

1. **Finiteness:** Must terminate after finite number of steps
2. **Definiteness:** Each step must be precisely defined
3. **Input:** Zero or more input values
4. **Output:** One or more output values
5. **Effectiveness:** Operations must be basic and feasible

Example

Making Coffee Algorithm:

1. Boil water (definite action)
2. Add coffee grounds (precise amount)
3. Wait 4 minutes (finite time)
4. Pour into cup (effective operation)

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Algorithm vs. Program vs. Process

Algorithm

- Abstract concept
- Language-independent
- Mathematical description
- Design phase

Program

- Concrete implementation
- Language-specific
- Code representation
- Implementation phase

Process

- Runtime execution
- System-dependent
- Active computation
- Execution phase

Example

terminate **Example:** Finding the maximum number

- **Algorithm:** "Compare each element with current maximum"
- **Program:** Python code implementing the comparison
- **Process:** Program running in memory, using CPU cycles

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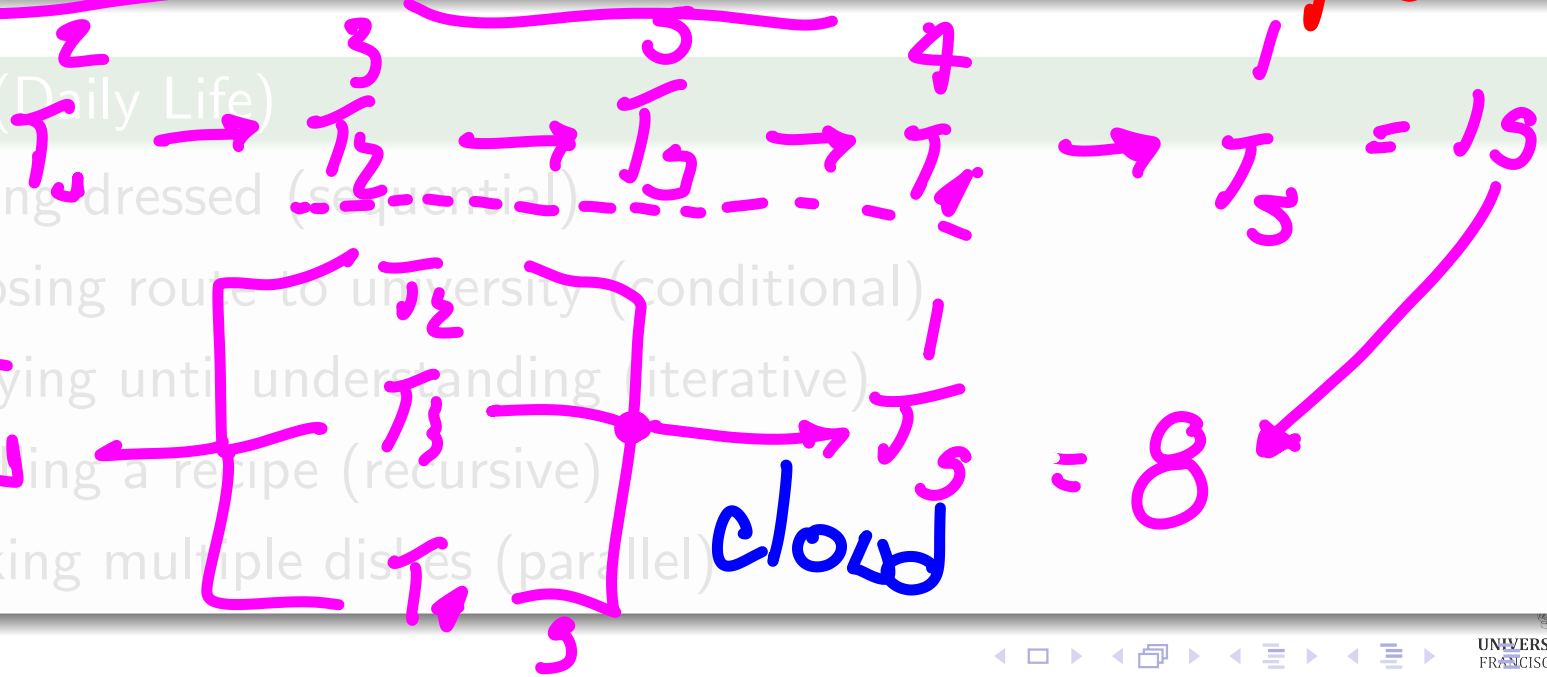
Types of Algorithmic Thinking

Fundamental Approaches

1. **Sequential Thinking:** Step-by-step execution
2. **Conditional Thinking:** Decision-based branching
3. **Iterative Thinking:** Repetitive operations
4. **Recursive Thinking:** Self-referential solutions
5. **Parallel Thinking:** Concurrent execution

Example (Daily Life)

- Getting dressed (sequential)
- Choosing route to university (conditional)
- Studying until understanding (iterative)
- Recalling a recipe (recursive)
- Cooking multiple dishes (parallel)



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Study Case: Euclidean GCD Algorithm

Greatest Common Divisor Problem

Input: Two positive integers a and b

Output: The largest integer that divides both a and b

Algorithm 1 Euclidean GCD Algorithm

Input: Integers a, b where $a \geq b > 0$
while $b \neq 0$ **do**
  $r \leftarrow a \bmod b$
  $a \leftarrow b$
  $b \leftarrow r$
end while
return a

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Study Case: Euclidean GCD Algorithm

Greatest Common Divisor Problem

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Algorithm 2 Euclidean GCD Algorithm

Input: Integers a, b where $a \geq b > 0$

while $b \neq 0$ **do**

$r \leftarrow a \bmod b$

$a \leftarrow b$

$b \leftarrow r$

end while

return a

Exercise Time! [1]

Exercise 1: Square Root Calculation

Design an algorithm to calculate the square root of a positive number using the Babylonian method:

1. Start with an initial guess x_0 (e.g., $S/2$ for number S)
2. Improve the guess: $x_{n+1} = \frac{1}{2}(x_n + \frac{S}{x_n})$
3. Repeat until convergence

Alg Input : S
DIFF = 0.005
x = S/2
DO{
x_next = (1/2)(x + (S/x))
dif_temp = abs(x - x_next)
x = x_next
} while (dif_temp > DIFF)
return x

Exercise 2: Find Maximum in an Array

Write an algorithm that finds the maximum element in an array of integers.
Consider: What if the array is empty? What about duplicate maximums?

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Consider: What if the array is empty? What about duplicate maximums?

INPUT: a → [a0, a1, ..., an]
if length.a == 0:
return "No"
max = a[0]

for i in 1 → n:
if a[i] > max:
max = a[i]
return max

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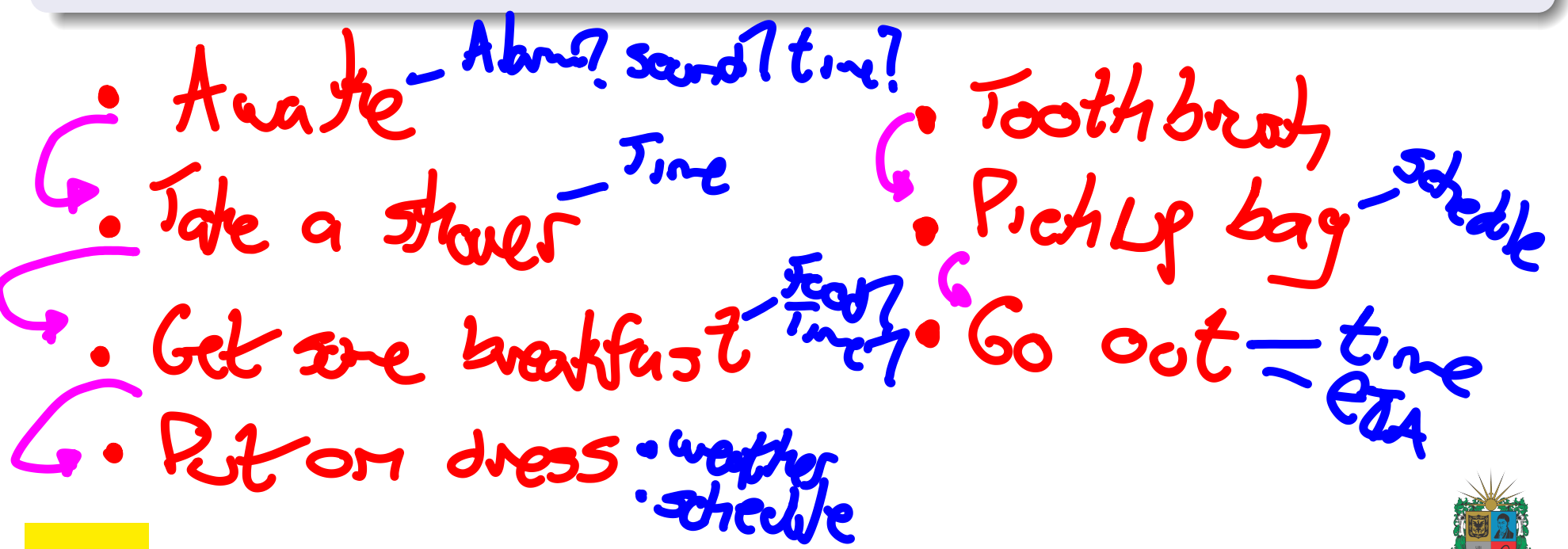
Exercise Time! [2]

Exercise 3: Algorithms in Daily Life

Identify and describe three algorithms you use in your daily life. For each:

- Define clear inputs and outputs
- List the precise steps
- Verify all algorithm characteristics



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Mathematical Notation for Algorithms [I]

Mathematical Notation

Mathematical notation provides a formal way to express algorithms using symbols, formulas, and structures from mathematics.

Why Mathematical Notation?

- **Precision:** Unambiguous specification
- **Universality:** Language-independent
- **Conciseness:** Compact representation
- **Analysis:** Enables formal verification

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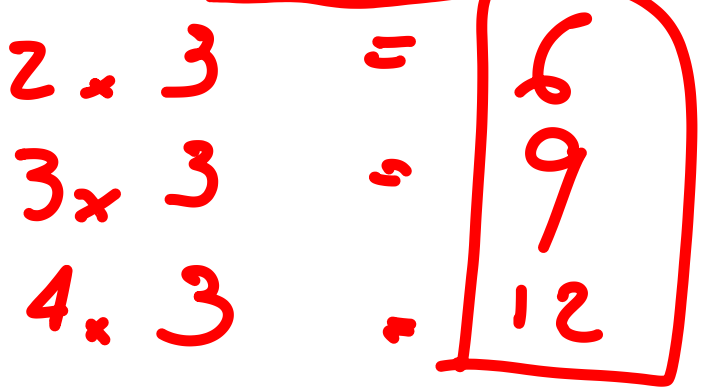

Mathematical Notation for Algorithms [II]

Example

Set Operations:

- $S = \{x \in \mathbb{N} : x \text{ is prime and } x < 20\}$
- $\forall x \in S, P(x)$ (for all elements in S, property P holds)
- $\exists x \in S : Q(x)$ (there exists an element in S with property Q)

Example

Function Definition:

$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$

$$f(a,b) = \begin{cases} a & \text{if } b = 0 \\ f(b, a \bmod b) & \text{otherwise} \end{cases}$$

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Converting Mathematical Formulas to Pseudocode

Mathematical Expression

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Another Example

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

```
1: sum ← 0
2: for i = 1 to n do
3:     sum ← sum + i^2
4: end for
5: formula ← n(n+1)(2n+1)/6
6: if sum = formula then
7:     return true
8: else
9:     return false
10: end if
```

```
1: result ← 1
2: term ← 1
3: n ← 1
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Study Case: Factorial Using Different Notations

Mathematical

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n > 0 \end{cases}$$

$$n! = \prod_{i=1}^{n} i$$

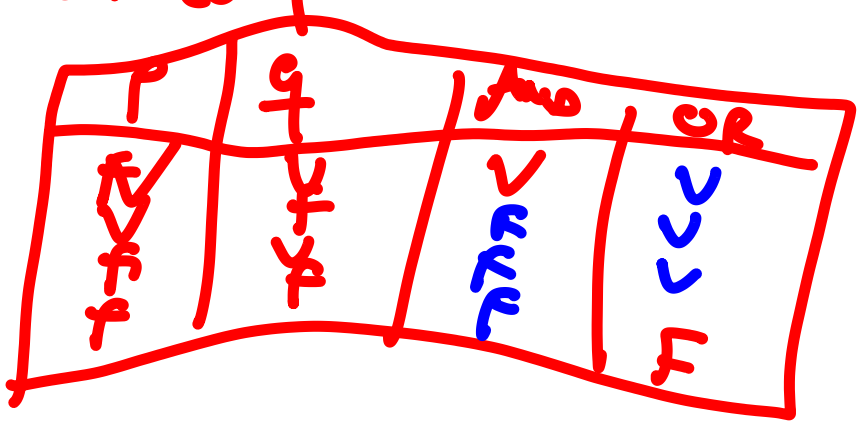
Iterative Pseudocode

1: **function** FACTORIAL(n)
2: $result \leftarrow 1$
3: **for** $i = 1$ to n **do**
4: $\quad result \leftarrow result \times i$
5: **end for**
6: **return** $result$
7: **end function**

Pseudocode Standards and Conventions

Basic Elements

- **Keywords:** `if`, `then`, `else`, `while`, `for`, `return`
- **Assignment:** $x \leftarrow value$ or $x = value$
- **Comparison:** $=, \neq, <, >, \leq, \geq$
- **Logic:** `and`, `or`, `not`
- **Comments:** // Single line, /* Multi-line */



Algorithm Documentation Best Practices

Essential Documentation Elements

1. **Purpose:** What does the algorithm do?
2. **Preconditions:** What must be true before execution?
3. **Postconditions:** What is guaranteed after execution?
4. **Parameters:** Input and output specifications
5. **Complexity:** Time and space requirements
6. **Examples:** Sample inputs and outputs

for → n steps

for
for → n^2 steps

n → steps?

Recorrido

Study Case: Fibonacci Sequence

Problem Definition

The Fibonacci sequence: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$

0 1 1 2 3 5 8 13 21 34 ...

Purpose: Compute the nth Fibonacci number

Preconditions: $n \geq 0$ (non-negative integer)

Postconditions: Returns F_n where $F_0 = 0, F_1 = 1$

Parameters:

- n: position in Fibonacci sequence

Complexity: $O(n)$ time, $O(1)$ space

Example: $\text{FibonacciIterative}(5) = 5$

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Control Structures

Sequential

1: Step 1
2: Step 2
3: Step 3
4: ...

Execution: One after another

Traditional

Conditional

1: **if** condition **then**
2: Action A
3: **else**
4: Action B
5: **end if**

Execution: Based on condition

Iterative

1: **while** condition **do**
2: Action
3: **end while**
4: **for** i = 1 to n **do**
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Execution: Repeated

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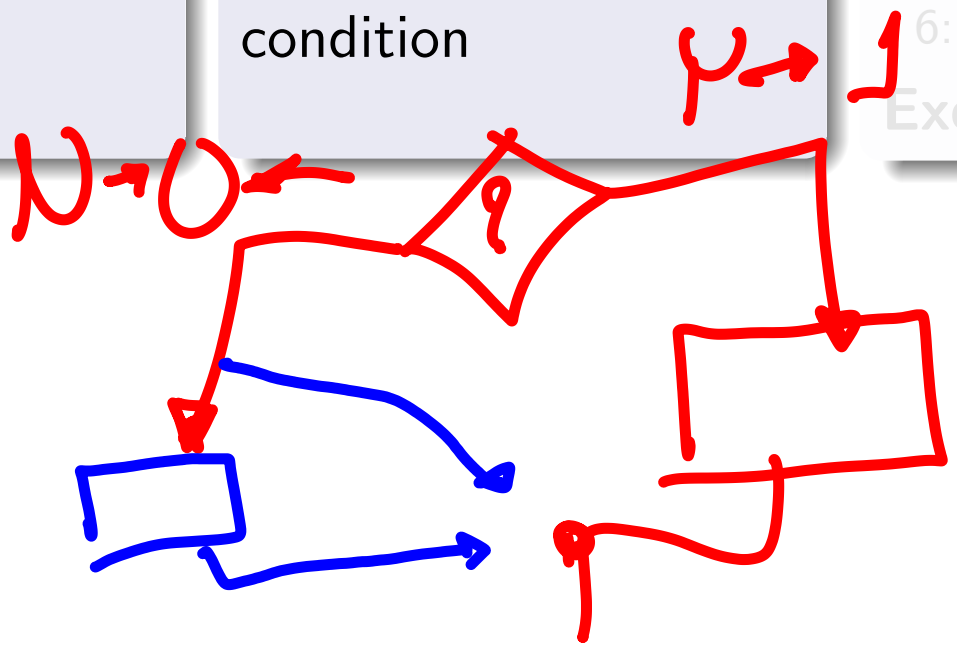

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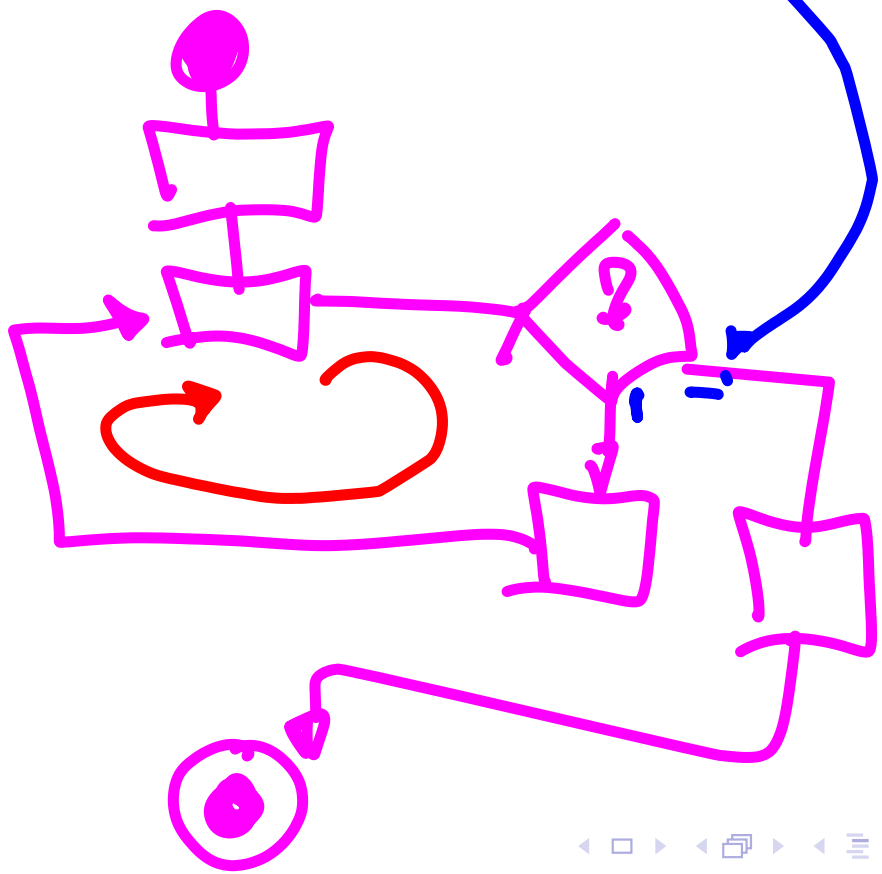
Control Flow Concepts

- **Entry Point:** Where execution begins
- **Exit Point:** Where execution ends
- **Decision Points:** Where flow branches
- **Loop Control:** How iterations are managed



Study Case: Decision-Making Algorithms

Grade Classification Algorithm

Problem: Convert numerical grade to letter grade

Range? 1 - 100

what letters? → A, B, C, D, F

range by letter

A: 100-90

B: 89-80

C: 79-70

D: 69-60

E: less than 60

Decision Tree

- $Score \geq 90? \rightarrow A$
- $Score \geq 80? \rightarrow B$
- $Score \geq 70? \rightarrow C$
- $Score \geq 60? \rightarrow D$
- $Otherwise \rightarrow F$

Key Concepts

- **Mutually exclusive** conditions
- **Order matters** in if-elsif chains
- **Default case** handling

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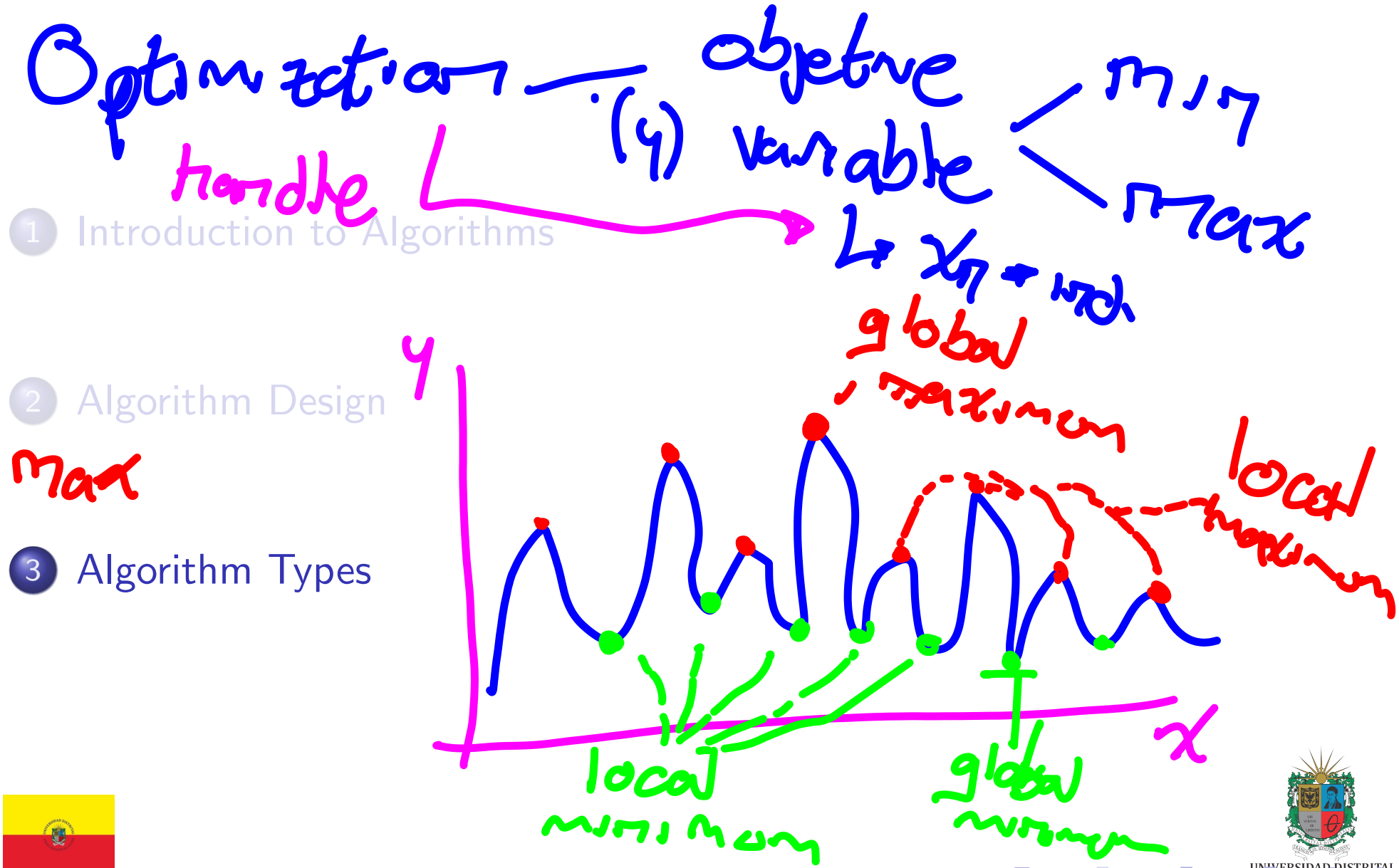

Greedy Algorithms: Philosophy and Approach

Definition

A greedy algorithm makes locally optimal choices at each step, hoping to find a global optimum.

Greedy Strategy

1. **Greedy Choice:** At each step, choose the best available option
2. **Optimal Substructure:** Optimal solution contains optimal solutions to subproblems
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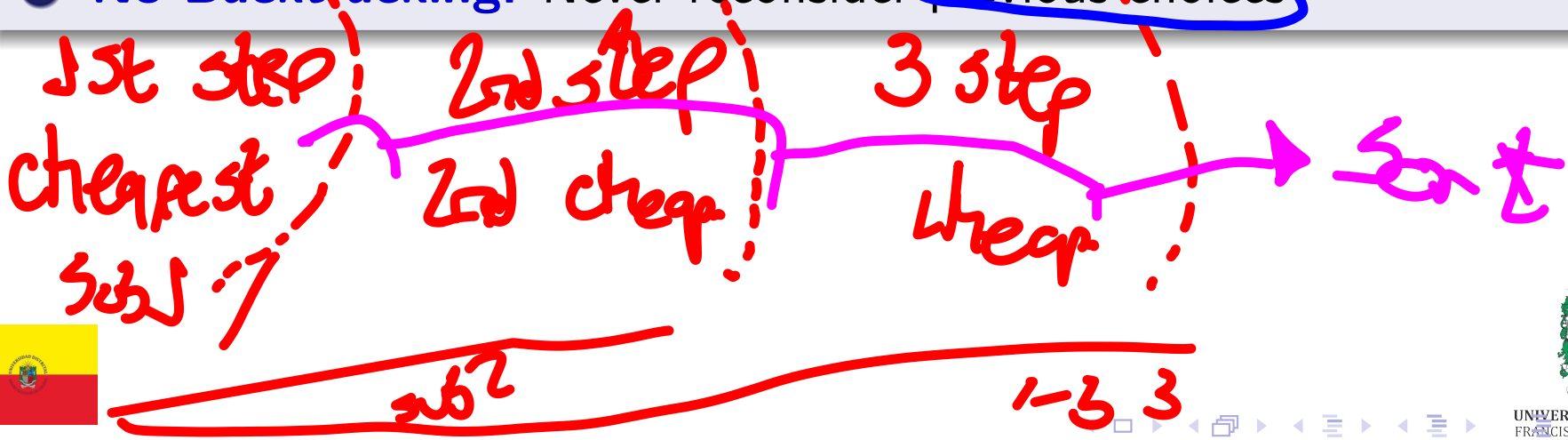


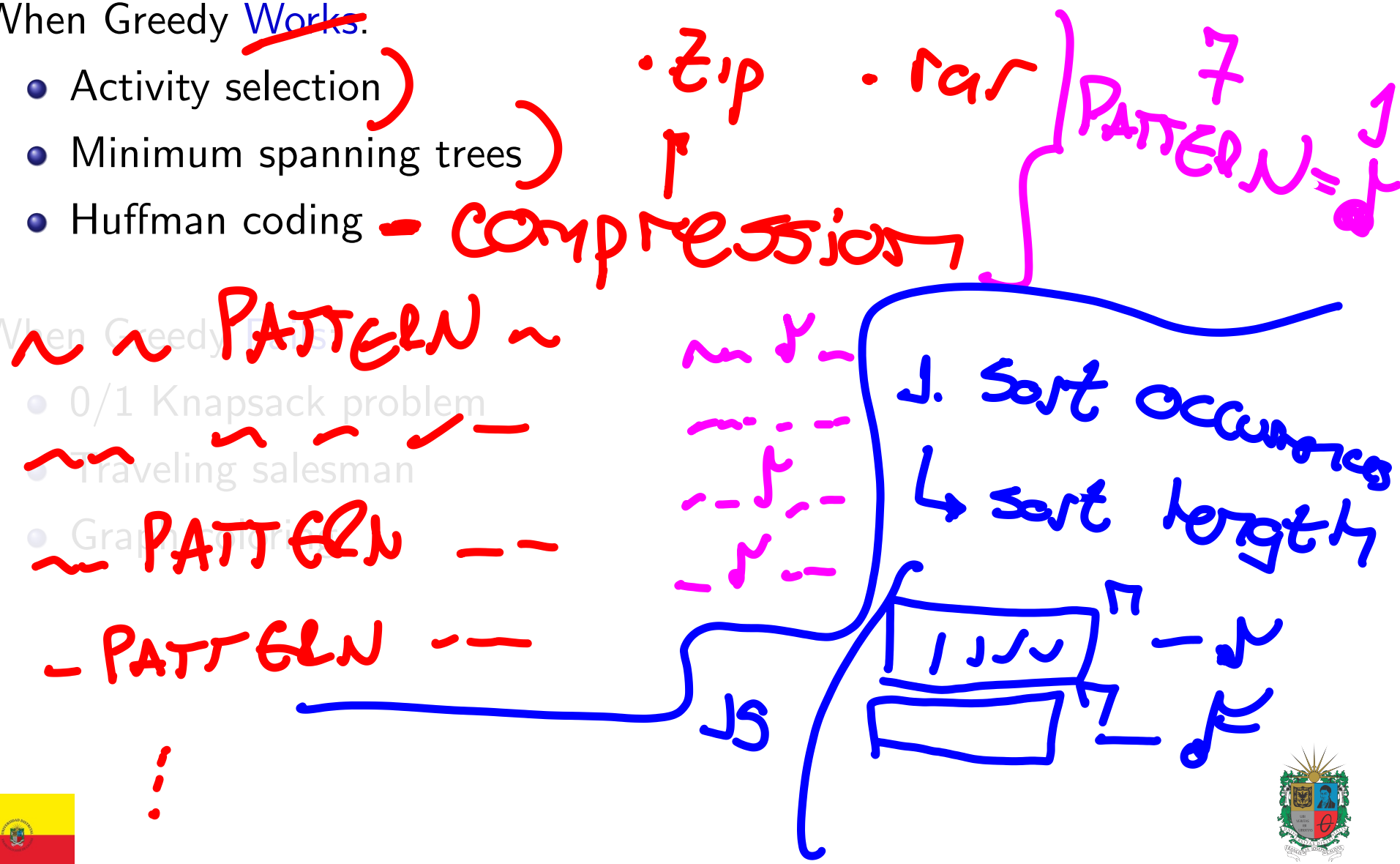
Greedy Algorithms: When to Use Them

When Greedy Works:

- Activity selection
- Minimum spanning trees
- Huffman coding

When Greedy Fails:

- 0/1 Knapsack problem
- Traveling salesman
- Graph coloring



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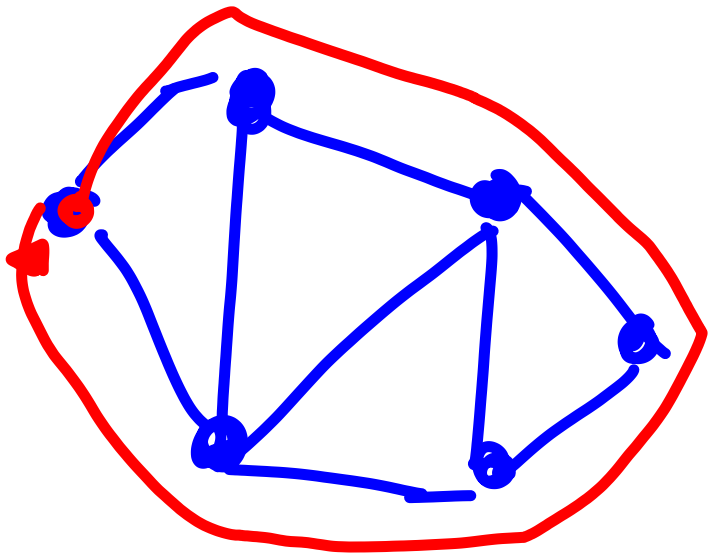
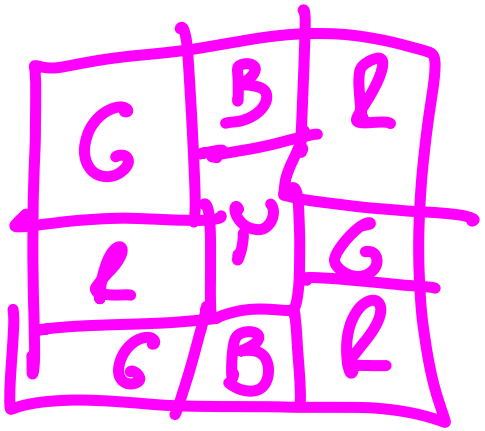
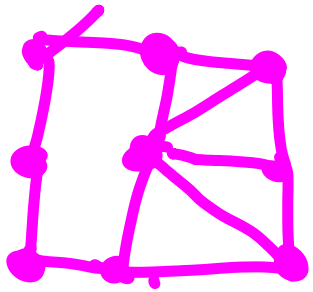




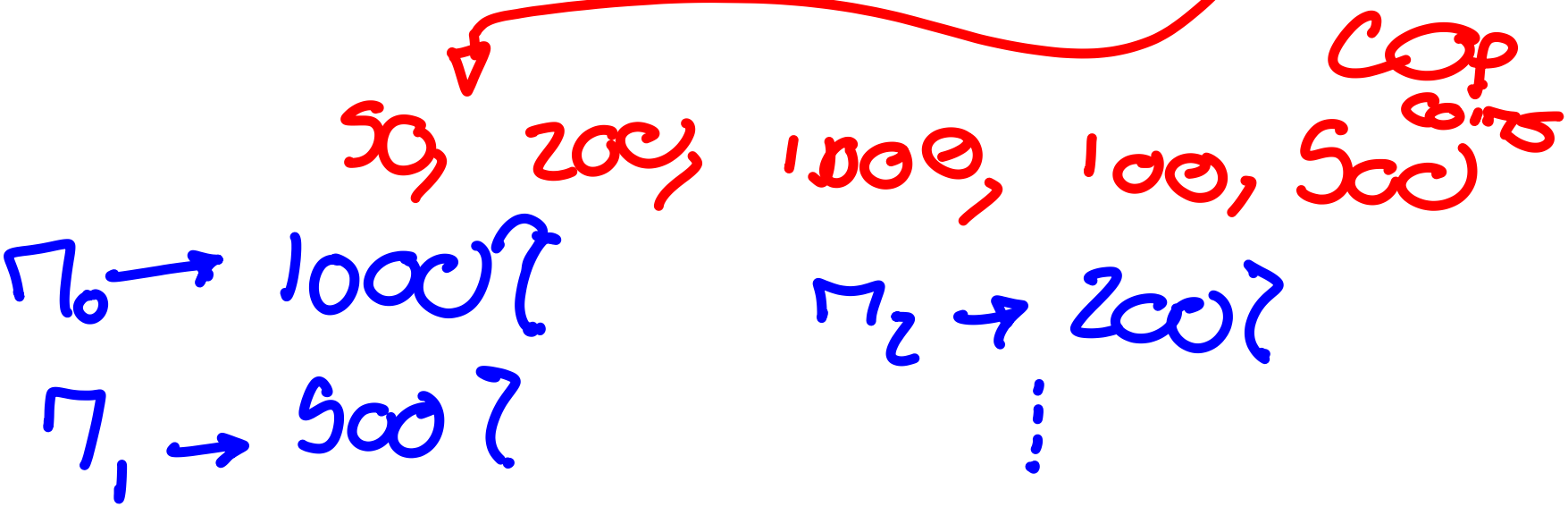


Study Case: Coin Change Problem

Problem Statement

Given: Coin denominations $[d_1, d_2, \ldots, d_k]$ and amount n

Goal: Find minimum number of coins to make amount n



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Study Case: Coin Change Problem [Solution]

Denominations: [25, 10, 5, 1] cents, **Amount:** 67 cents

1: **Algorithm** GreedyCoinChange($denominations, amount$)
2: $result \leftarrow []$
3: **for** each denomination d in descending order **do**
4: **while** $amount \geq d$ **do**
5: $result.append(d)$
6: $amount \leftarrow amount - d$
7: **end while**
8: **end for**
9: **return** $result$

integer
coins_d = (amount / d_i)
amount = amount % d_i

Solution: 67 = 25 + 25 + 10 + 5 + 1 + 1 (6 coins)

42 17 7 2 1 0

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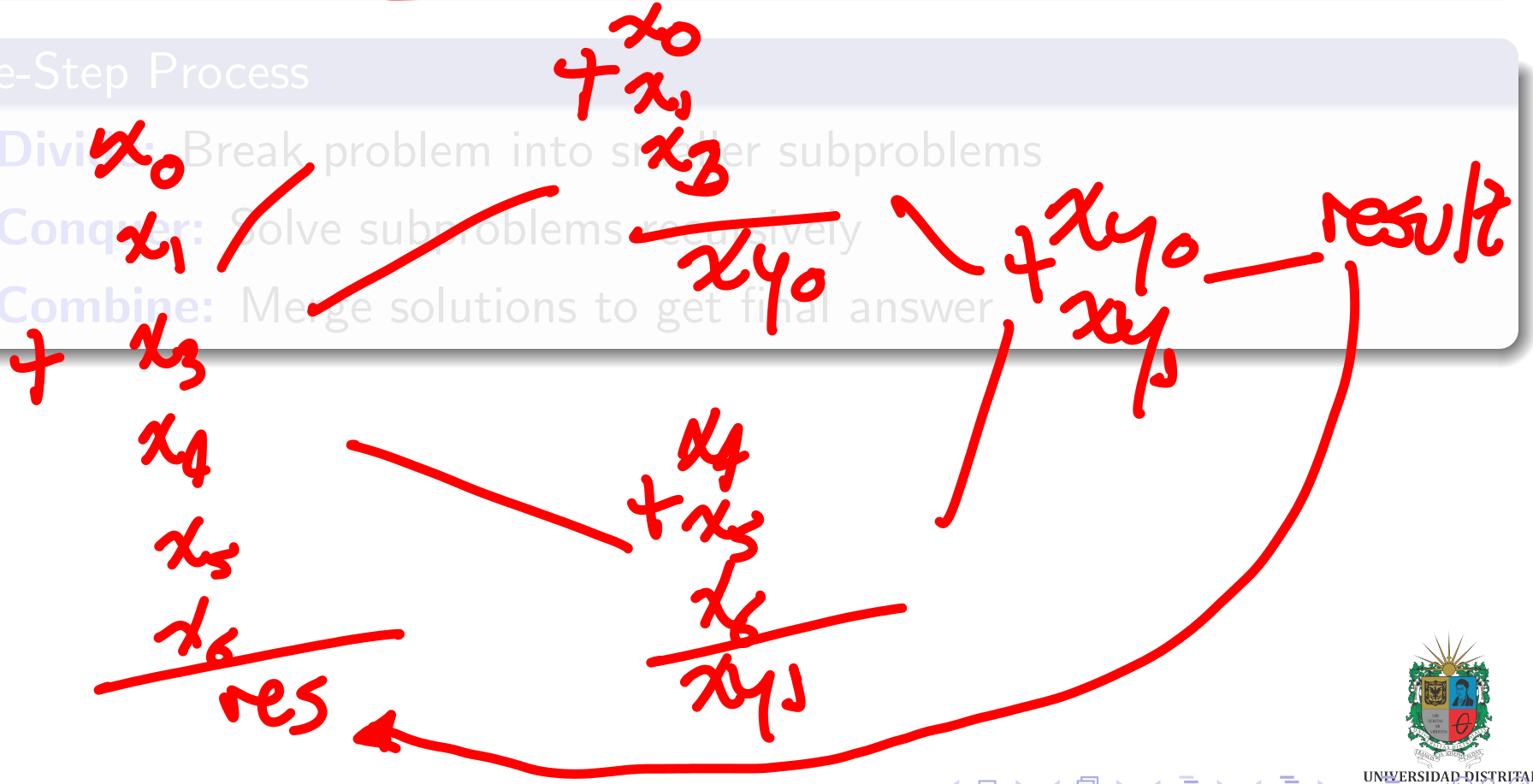
Divide and Conquer: Methodology

Definition

Divide and Conquer breaks a problem into smaller subproblems, solves them recursively, then combines solutions.

Three-Step Process

1. **Divide:** Break problem into smaller subproblems
2. **Conquer:** Solve subproblems recursively
3. **Combine:** Merge solutions to get final answer



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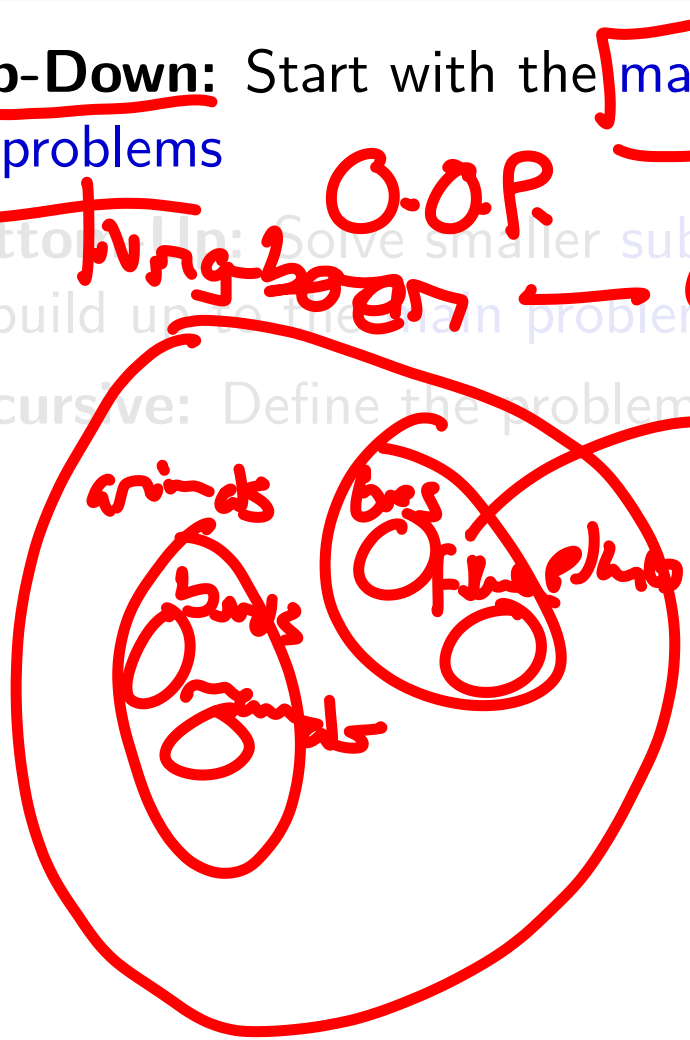
Divide and Conquer: Algorithm Template

```
1:  Algorithm DivideAndConquer(problem)
2:  if problem is small enough then
3:      return SolveDirectly(problem)
4:  else
5:      subproblems ← Divide(problem)
6:      for each subproblem do
7:          solution ← DivideAndConquer(subproblem)
8:      end for
9:      return Combine(solutions)
10: end if
```

Problem Decomposition Strategies

- **Top-Down:** Start with the main problem and break it down into subproblems
- **Bottom-Up:** Solve smaller subproblems first and use their solutions to build up to the main problem
- **Recursive:** Define the problem in terms of smaller instances of itself



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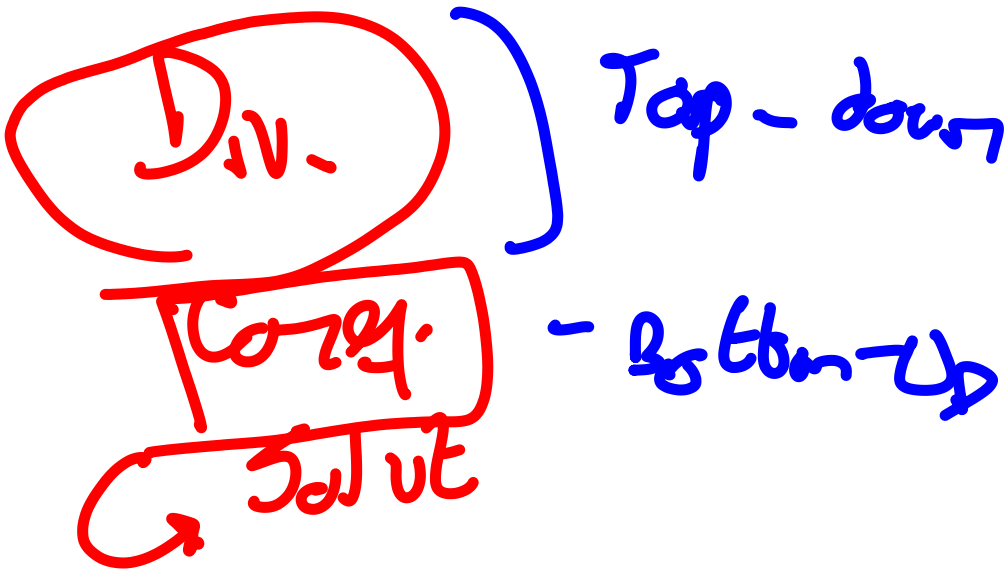

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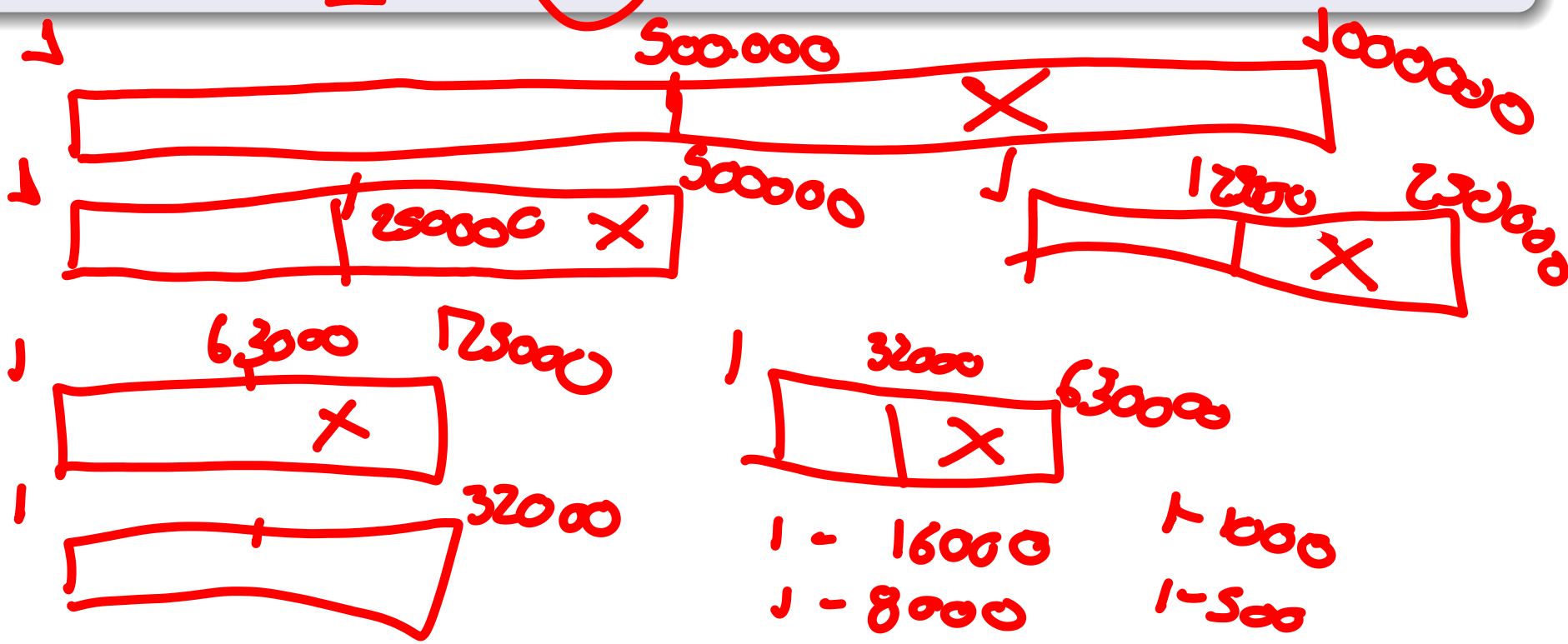
Study Case: Binary Search

Problem

Input: Sorted array $A[1..n]$ and search value x

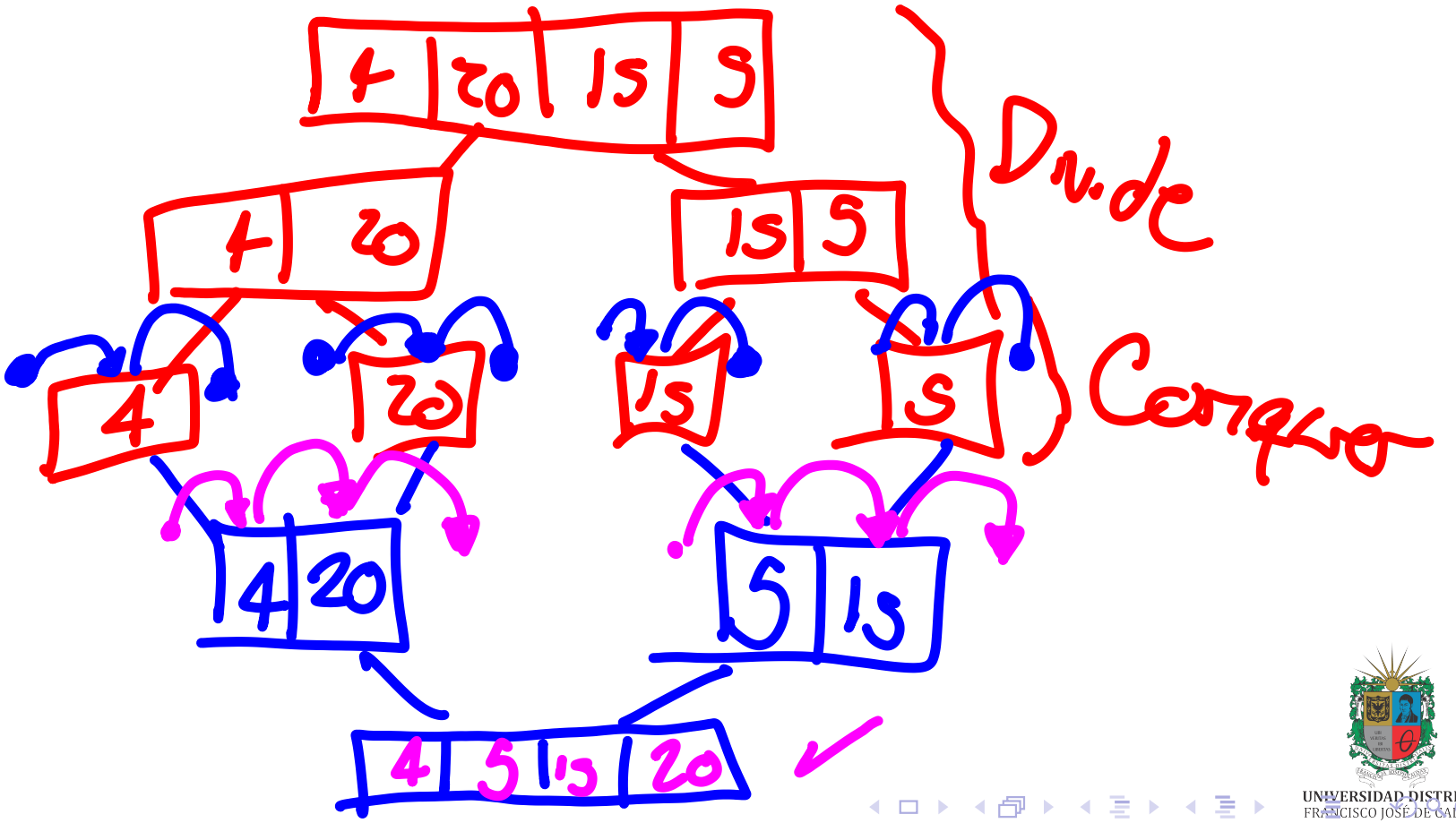
Output: Index of x in A, or -1 if not found



Study Case: Merge Sort Approach

Sorting Problem

Input: Array $A[1..n]$ of comparable elements
Output: Array sorted in ascending order



Demo Time! Clean the House Exercise

Problem

You need to clean your house efficiently. You have different rooms and different cleaning tasks.

Queen - 1087

if $x_1 == x_2$ and $y_1 == y_2$ → 0 no move

else: if ($x_1 == x_2$) or ($y_1 == y_2$) or (abs($x_1 - x_2$) == abs($y_1 - y_2$))

same row / same column / same diagonal

return 1

else:

return 2

Demo Time! Clean the House Exercise

Problem

You need to clean your house efficiently. You have different rooms and different cleaning tasks.

Greedy Strategy: Always clean the dirtiest room first

1. Assess dirt level of all rooms
2. Choose room with highest dirt level
3. Clean that room completely
4. Repeat until all rooms clean

Advantage: Maximum immediate impact

Divide & Conquer Strategy: Split house systematically

1. Divide house into sections (floors/wings)
2. Recursively clean each section
3. Within each room, divide into areas
4. Clean areas systematically

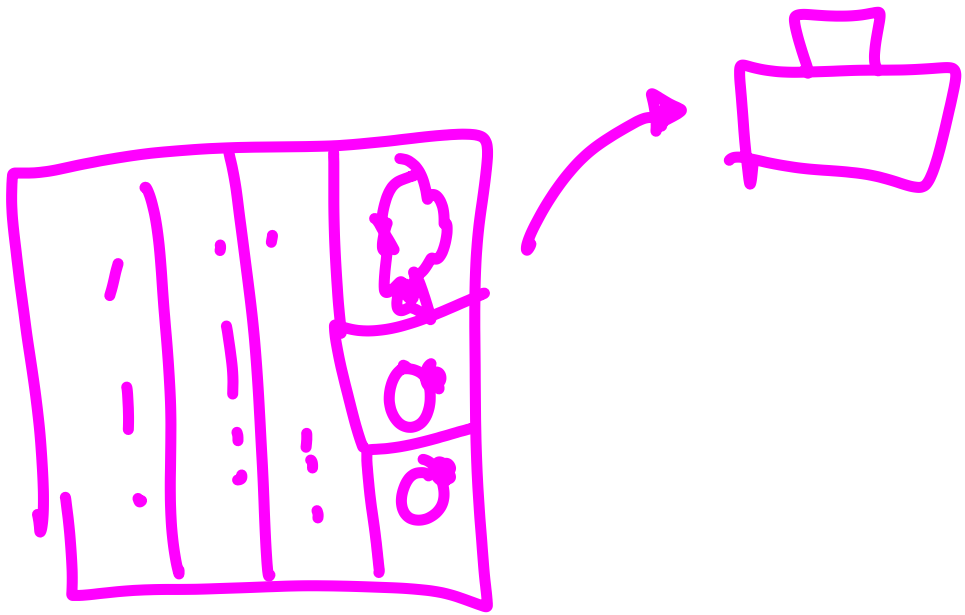
Advantage: Systematic coverage

Exercise: Knapsack Problem

Problem Statement

Given: Knapsack capacity W, items with weights w_i and values v_i

Goal: Maximize total value without exceeding weight capacity



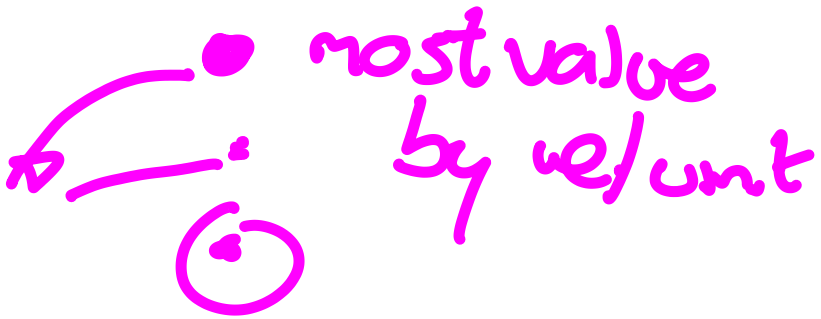
Exercise: Knapsack Problem [Solution]

0/1 Knapsack (Greedy Approach)

Cannot take fractions - either take entire item or leave it

- **Strategy:** Sort by value/weight ratio, take whole items
- **Optimal:** No! (Greedy doesn't guarantee optimal solution)

1: Sort items by v_i/w_i descending
2: **for** each item i **do**
3: **if** $w_i \leq$ remaining capacity **then**
4: Take entire item
5: Update remaining capacity
6: **end if**
7: **end for**



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Brute Force: Exhaustive Search

Definition

Brute force algorithms try all possible solutions until finding the correct one.

$45 \times 44 \times 43 \times 42 \times 41 \times 40$

$= \sim 5 \times 10^9$

Characteristics:

- **Exhaustive:** Examine every possibility
- **Guaranteed:** Always finds optimal solution (if exists)
- **Expensive:** Often exponential time complexity
- **Simple:** Easy to understand and implement

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Brute Force: When to Use It

1. Problem size is small
2. No efficient algorithm is known
3. Correctness is more important than efficiency
4. As baseline for comparing other algorithms
5. When optimization overhead exceeds brute force cost

LLMs
↳ 3 weeks – 20000 - 25000
GPT-4 → 190 M. USD GPU3

Study Case: Password Cracking

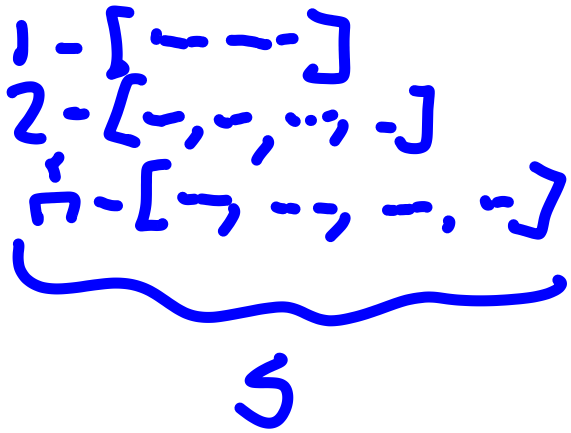
Problem

Given: Encrypted password hash and character set

Goal: Find the original password

Study Case: Password Cracking [Solution]

```
1: Algorithm BruteForcePassword(hash, maxlength)
2: for length = 1 to maxlength do
3:     for each possible string s of given length do
4:         if Hash(s) = hash then
5:             return s // Password found!
6:         end if
7:     end for
8: end for
9: return null // Password not found
```



Character set: [a-z] (26 characters)
Password length: 8 characters
Total attempts: $26^8 \approx 208$ billion
Time estimate: Weeks on single computer!

Study Case: Password Cracking [Solution]

1: **Algorithm** BRUTEFORCEPASSWORD($hash, maxlength$)
2: **for** $length = 1$ to $maxlength$ **do**
3: **for** each possible string s of given $length$ **do**
4: **if** HASH(s) $= hash$ **then**
5: **return** s // *Password found!*
6: **end if**
7: **end for**
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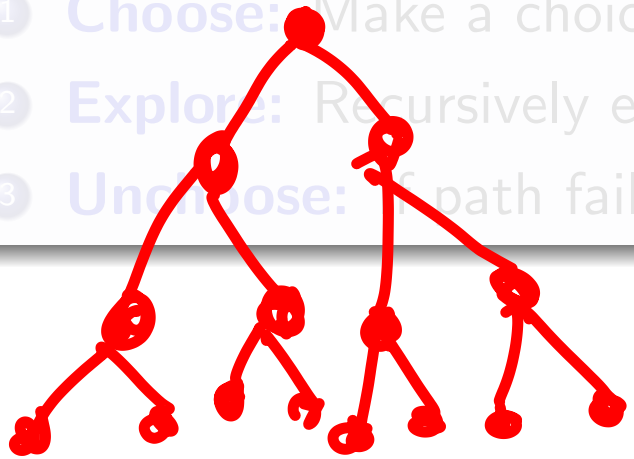
Backtracking: Systematic Trial and Error

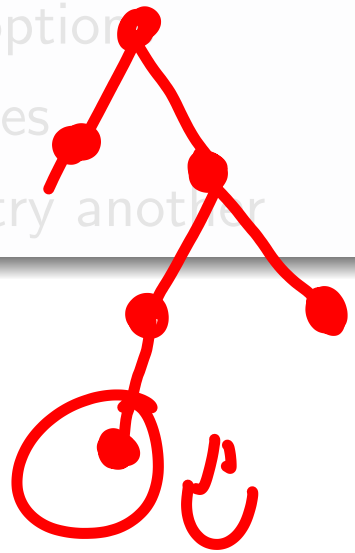
Definition

Backtracking explores solution space systematically, **abandoning partial solutions** that cannot lead to valid solutions.

Backtracking Strategy

1. **Choose:** Make a choice from available options.
2. **Explore:** Recursively explore consequences.
3. **Unchoose:** If path fails, backtrack and try another.





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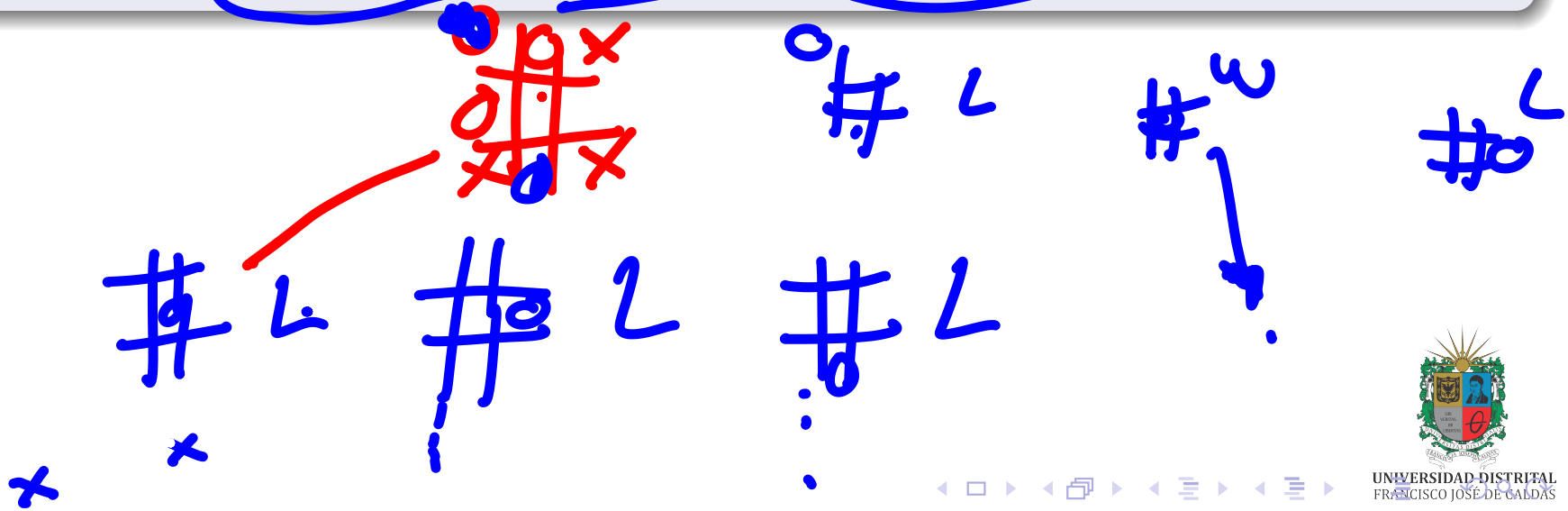


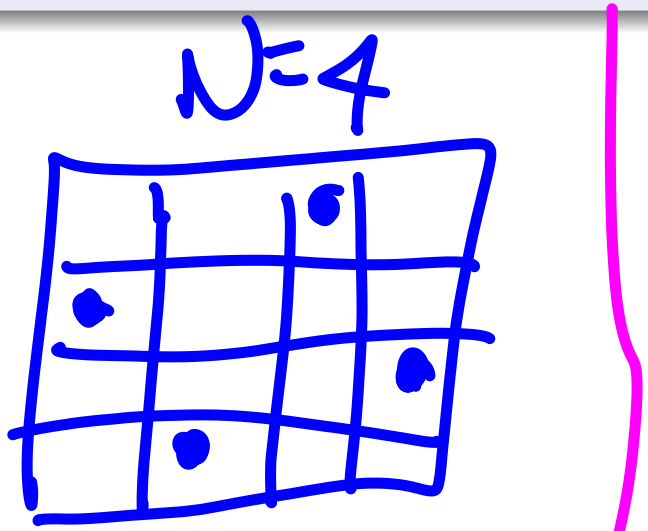
Backtracking: Algorithm

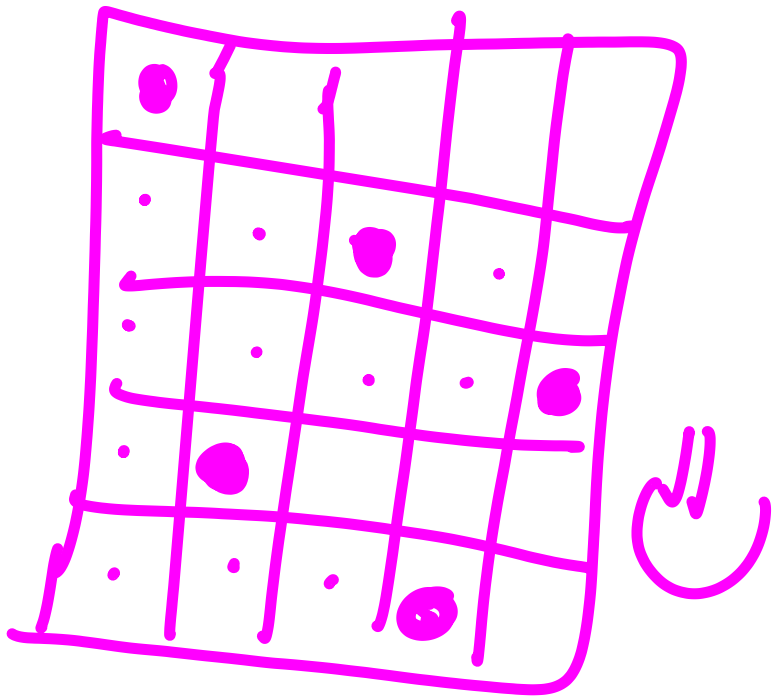
```
1:  Algorithm BACKTRACK(partial_solution)
2:  if ISCOMPLETE(partial_solution) then
3:      return partial_solution
4:  end if
5:  for each possible choice c do
6:      if ISVALID(partial_solution, c) then
7:          partial_solution.add(c)
8:          result ← BACKTRACK(partial_solution)
9:          if result ≠ null then
10:             return result
11:         end if
12:         partial_solution.remove(c) // Backtrack!
13:     end if
14: end for
15: return null
```

Study Case: N-Queens Problem

Problem

Place N queens on an $N \times N$ chessboard such that no two queens attack each other.



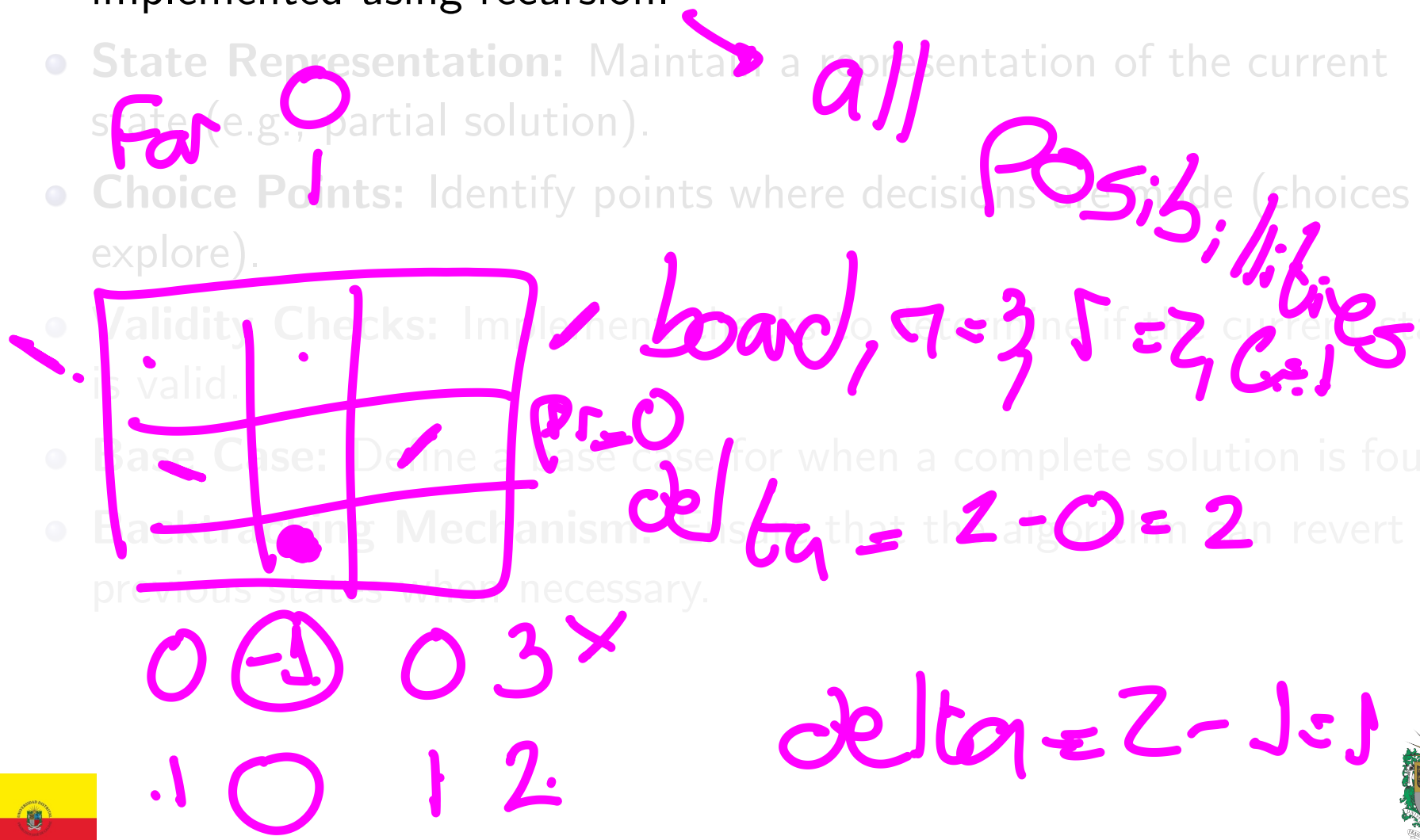


Study Case: N-Queens Problem [Solution]

```
1:  Algorithm SolveNQueens(board, row)
2:  if row = N then
3:      return board // Solution found!
4:  end if
5:  for each column col in 0 to N − 1 do
6:      if IsSafe(board, row, col) then
7:          board[row][col] ← Q // Place queen
8:          result ← SolveNQueens(board, row + 1)
9:          if result ≠ null then
10:             return result
11:         end if
12:         board[row][col] ← . // Remove queen (backtrack)
13:     end if
14: end for
15: return null // No solution found
```

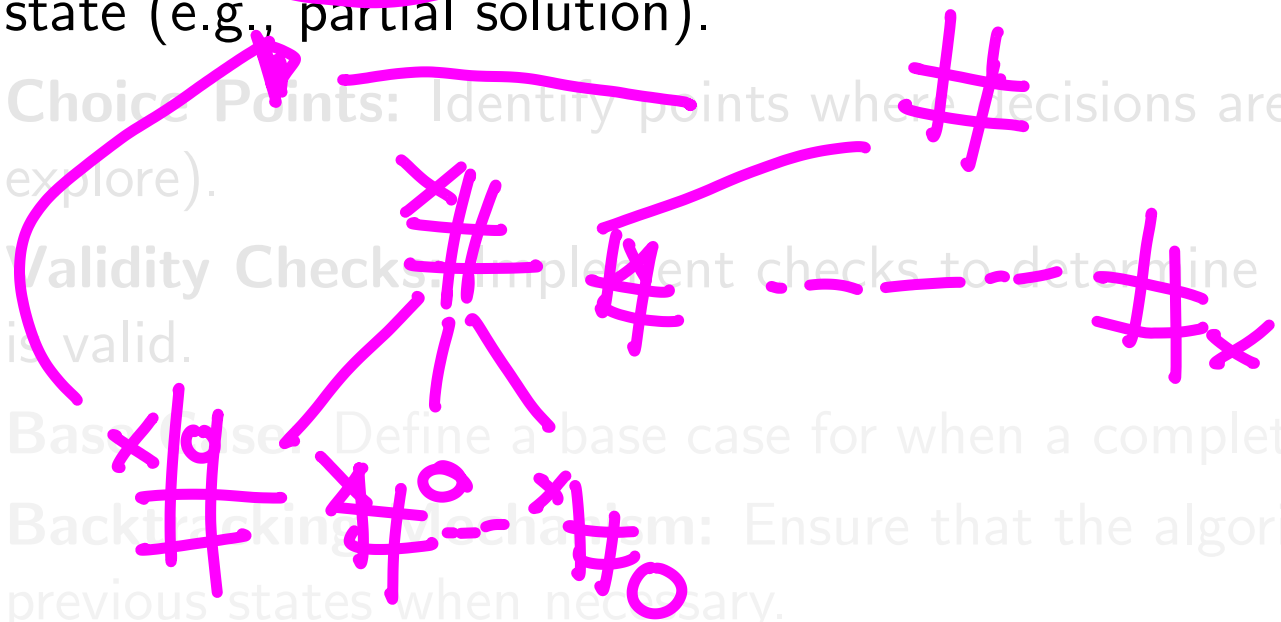
Implementation Patterns for Backtracking

- **Recursive Structure:** Backtracking algorithms are typically implemented using recursion.



Implementation Patterns for Backtracking

- **Recursive Structure:** Backtracking algorithms are typically implemented using recursion.
- **State Representation:** Maintain a representation of the current state (e.g., partial solution).
- **Choice Points:** Identify points where decisions are made (choices to explore).
- **Validity Checks:** Implement checks to determine if the current state is valid.
- **Base Case:** Define a base case for when a complete solution is found.
- **Backtracking Mechanism:** Ensure that the algorithm can revert to previous states when necessary.



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Summary: Algorithm Types

Key Takeaways

- **Greedy:** Make locally optimal choices (works when greedy choice property holds)
- **Divide & Conquer:** Break into subproblems, solve recursively, combine solutions
- **Brute Force:** Try all possibilities (guarantees correctness but expensive)
- **Backtracking:** Systematic search with pruning (intelligent brute force)

Summary: Algorithm Types

Algorithm Selection Guide

- **Small problem size?** → Consider brute force
- **Optimization problem with greedy choice property?** → Try greedy
- **Problem naturally divides into subproblems?** → Use divide & conquer
- **Constraint satisfaction or search problem?** → Apply backtracking

Outline

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Thanks!

Questions?

Repo: *https://github.com/EngAndres/ud-public/tree/main/courses/computer-sciences-i*