

Bio-Model Selection System (BMSS): Models Documentation

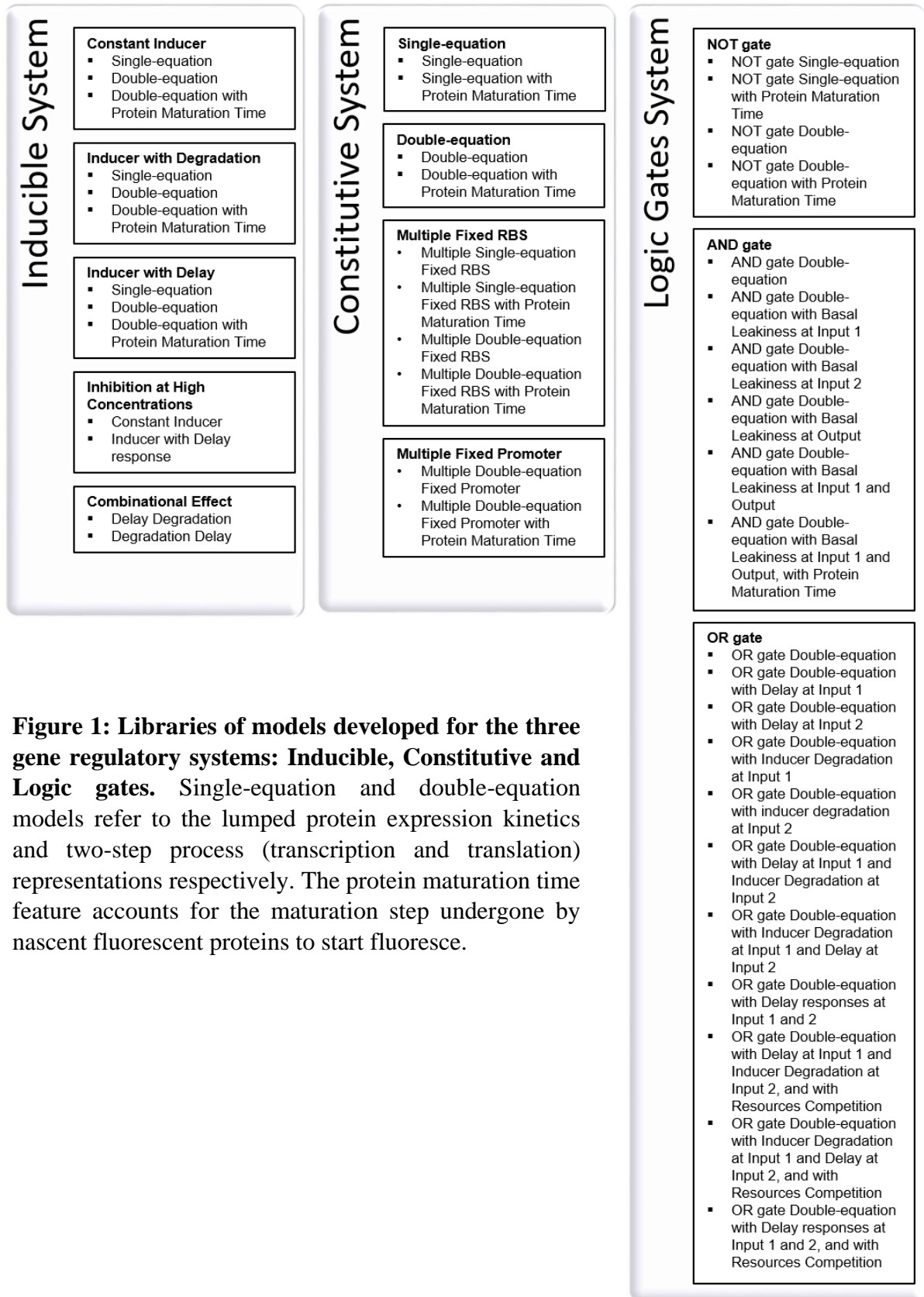


Figure 1: Libraries of models developed for the three gene regulatory systems: Inducible, Constitutive and Logic gates. Single-equation and double-equation models refer to the lumped protein expression kinetics and two-step process (transcription and translation) representations respectively. The protein maturation time feature accounts for the maturation step undergone by nascent fluorescent proteins to start fluoresce.

Table 1: Mathematical Formulations for Inducible Promoter Systems

Model = 'ConstantInducer'
$\frac{d[mRNA]}{dt} = syn_{mRNA} \left(\frac{[Inducer]^n}{[Inducer]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA]$ $\frac{d[Pep]}{dt} = syn_{Pep}[mRNA] - deg_{Pep}[Pep]$
Model = 'ConstantInducerKMat'
$\frac{d[mRNA]}{dt} = syn_{mRNA} \left(\frac{[Inducer]^n}{[Inducer]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA]$ $\frac{d[Pep]}{dt} = syn_{Pep}[mRNA] - K_{mature}[Pep]$ $\frac{d[Pep_m]}{dt} = K_{mature}[Pep] - deg_{Pep}[Pep_m]$
Model = 'DegradationInducer'
$\frac{d[Ind]}{dt} = -deg_{Ind}[Ind]$ $\frac{d[mRNA]}{dt} = syn_{mRNA} \left(\frac{[Ind]^n}{[Ind]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA]$ $\frac{d[Pep]}{dt} = syn_{Pep}[mRNA] - deg_{Pep}[Pep]$
Model = 'DegradationInducerKMat'
$\frac{d[Ind]}{dt} = -deg_{Ind}[Ind]$ $\frac{d[mRNA]}{dt} = syn_{mRNA} \left(\frac{[Ind]^n}{[Ind]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA]$ $\frac{d[Pep]}{dt} = syn_{Pep}[mRNA] - K_{mature}[Pep]$ $\frac{d[Pep_m]}{dt} = K_{mature}[Pep] - deg_{Pep}[Pep_m]$
Model = 'DelayInducer'
$\frac{d[Ind_e]}{dt} = -V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right)$ $\frac{d[Ind_i]}{dt} = V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right)$ $\frac{d[mRNA]}{dt} = syn_{mRNA} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA]$ $\frac{d[Pep]}{dt} = syn_{Pep}[mRNA] - deg_{Pep}[Pep]$
Model = 'DelayInducerKMat'

$$\begin{aligned}
\frac{d[Ind_e]}{dt} &= -V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[Ind_i]}{dt} &= V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[mRNA]}{dt} &= syn_{mRNA} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA] \\
\frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - K_{mature}[Pep] \\
\frac{d[Pep_m]}{dt} &= K_{mature}[Pep] - deg_{Pep}[Pep_m]
\end{aligned}$$

Model = 'SingleConstantInducer'

$$\frac{d[Pep]}{dt} = syn_{Pep} \left(\frac{[Inducer]^n}{[Inducer]^n + K_{ind}^n} \right) - deg_{Pep}[Pep]$$

Model = 'SingleDegradationInducer'

$$\begin{aligned}
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[Pep]}{dt} &= syn_{Pep} \left(\frac{[Ind]^n}{[Ind]^n + K_{ind}^n} \right) - deg_{Pep}[Pep]
\end{aligned}$$

Model = 'SingleDelayInducer'

$$\begin{aligned}
\frac{d[Ind_e]}{dt} &= -V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[Ind_i]}{dt} &= V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[Pep]}{dt} &= syn_{Pep} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) - deg_{Pep}[Pep]
\end{aligned}$$

Model = 'ConstantIndInhibition'

$$\begin{aligned}
\frac{d[mRNA]}{dt} &= K_{Leak} + syn_{mRNA} \left(\frac{[Inducer]^n}{[Inducer]^n + K_{ind}^n} \right) \left(1 - K_{inhmax} \left(\frac{[Inducer]^{n_{inh}}}{[Inducer]^{n_{inh}} + K_{inh}^{n_{inh}}} \right) \right) \\
&\quad - deg_{mRNA}[mRNA] \\
\frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - deg_{Pep}[Pep]
\end{aligned}$$

Model = 'DelayIndInhibition'

$$\begin{aligned}
\frac{d[Ind_e]}{dt} &= -V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[Ind_i]}{dt} &= V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\
\frac{d[mRNA]}{dt} &= K_{Leak} + syn_{mRNA} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) \left(1 - K_{inhmax} \left(\frac{[Ind_e]^{n_{inh}}}{[Ind_e]^{n_{inh}} + K_{inh}^{n_{inh}}} \right) \right) \\
&\quad - deg_{mRNA}[mRNA] \\
\frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - deg_{Pep}[Pep]
\end{aligned}$$

Model = 'DelayDegradationInducer'

$$\begin{aligned}\frac{d[Ind_e]}{dt} &= -V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\ \frac{d[Ind_i]}{dt} &= V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) - deg_{Ind}[Ind_i] \\ \frac{d[mRNA]}{dt} &= syn_{mRNA} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA] \\ \frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - deg_{Pep}[Pep]\end{aligned}$$

Model = 'DegradationDelayInducer'

$$\begin{aligned}\frac{d[Ind_e]}{dt} &= -deg_{Ind}[Ind_e] - V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\ \frac{d[Ind_i]}{dt} &= V_m \left(\frac{[Ind_e]^{n_{trans}}}{[Ind_e]^{n_{trans}} + K_{trans}^{n_{trans}}} \right) \\ \frac{d[mRNA]}{dt} &= syn_{mRNA} \left(\frac{[Ind_i]^n}{[Ind_i]^n + K_{ind}^n} \right) - deg_{mRNA}[mRNA] \\ \frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - deg_{Pep}[Pep]\end{aligned}$$

Table 2: Mathematical Formulations for Constitutive Promoter Systems**Model = 'ConstDouble'**

$$\begin{aligned}\frac{d[mRNA]}{dt} &= syn_{mRNA} - deg_{mRNA}[mRNA] \\ \frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - deg_{Pep}[Pep]\end{aligned}$$

Model = 'ConstDoubleKMat'

$$\begin{aligned}\frac{d[mRNA]}{dt} &= syn_{mRNA} - deg_{mRNA}[mRNA] \\ \frac{d[Pep]}{dt} &= syn_{Pep}[mRNA] - K_{mature}[Pep] \\ \frac{d[Pep_m]}{dt} &= K_{mature}[Pep] - deg_{Pep}[Pep_m]\end{aligned}$$

Model = 'ConstSingle'

$$\frac{d[Pep]}{dt} = syn_{Pep} - deg_{Pep}[Pep]$$

Model = 'ConstSingleKMat'

$$\begin{aligned}\frac{d[Pep]}{dt} &= syn_{Pep} - K_{mature}[Pep] \\ \frac{d[Pep_m]}{dt} &= K_{mature}[Pep] - deg_{Pep}[Pep_m]\end{aligned}$$

Model = 'MultiDoubleFixRBS'

$$\begin{aligned}\frac{d[mRNAX]}{dt} &= syn_{mRNAX} - deg_{mRNA}[mRNAX] \\ \frac{d[PepX]}{dt} &= syn_{Pep}[mRNAX] - deg_{Pep}[PepX] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Model = 'MultiDoubleFixRBSKMat'

$$\begin{aligned}\frac{d[mRNAX]}{dt} &= syn_{mRNAX} - deg_{mRNA}[mRNAX] \\ \frac{d[PepX]}{dt} &= syn_{Pep}[mRNAX] - K_{mature}[PepX] \\ \frac{d[PepX_m]}{dt} &= K_{mature}[PepX] - deg_{Pep}[PepX_m] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Model = 'MultiSingleFixRBS'

$$\begin{aligned}\frac{d[PepX]}{dt} &= syn_{PepX} - deg_{Pep}[PepX] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Model = 'MultiSingleFixRBSKMat'

$$\begin{aligned}\frac{d[PepX]}{dt} &= syn_{PepX} - K_{mature}[PepX] \\ \frac{d[PepX_m]}{dt} &= K_{mature}[PepX] - deg_{Pep}[PepX_m] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Model = 'MultiDoubleFixPromoter'

$$\begin{aligned}\frac{d[mRNAX]}{dt} &= syn_{mRNA} - deg_{mRNA}[mRNAX] \\ \frac{d[PepX]}{dt} &= syn_{PepX}[mRNAX] - deg_{Pep}[PepX] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Model = 'MultiDoubleFixPromoterKMat'

$$\begin{aligned}\frac{d[mRNAX]}{dt} &= syn_{mRNA} - deg_{mRNA}[mRNAX] \\ \frac{d[PepX]}{dt} &= syn_{PepX}[mRNAX] - K_{mature}[PepX] \\ \frac{d[PepX_m]}{dt} &= K_{mature}[PepX] - deg_{Pep}[PepX_m] \\ &for\ X = 1,2, \dots 6\end{aligned}$$

Table 3: Mathematical Formulations for Logic Gate Systems (NOT, AND, and OR gates)

Model = 'NOTgate'
$\frac{d[mRNA1]}{dt} = syn_{mRNA1}(state) - deg_{mRNA}[mRNA1]$ $\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - deg_{Pep}[Pep1]$ $\frac{d[mRNA2]}{dt} = syn_{mRNA2} \left(1 - K_{maxrep} \left(\frac{[Pep1]}{Pep_{max}} \right) \right) - deg_{mRNA}[mRNA2]$ $\frac{d[Pep2]}{dt} = syn_{Pep}[mRNA2] - deg_{Pep}[Pep2]$
Model = 'NOTgateKMat'
$\frac{d[mRNA1]}{dt} = syn_{mRNA1}(state) - deg_{mRNA}[mRNA1]$ $\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - deg_{Pep}[Pep1]$ $\frac{d[mRNA2]}{dt} = syn_{mRNA2} \left(1 - K_{maxrep} \left(\frac{[Pep1]}{Pep_{max}} \right) \right) - deg_{mRNA}[mRNA2]$ $\frac{d[Pep2]}{dt} = syn_{Pep}[mRNA2] - K_{mature}[Pep2]$ $\frac{d[Pep2_m]}{dt} = K_{mature}[Pep2] - deg_{Pep}[Pep2_m]$
Model = 'NOTgateSingle'
$\frac{d[mRNA1]}{dt} = syn_{mRNA}(1 - K_{maxrep}(state)) - deg_{mRNA}[mRNA1]$ $\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - deg_{Pep}[Pep1]$
Model = 'NOTgateSingleKMat'
$\frac{d[mRNA1]}{dt} = syn_{mRNA}(1 - K_{maxrep}(state)) - deg_{mRNA}[mRNA1]$ $\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - K_{mature}[Pep1]$ $\frac{d[Pep1_m]}{dt} = K_{mature}[Pep1] - deg_{Pep}[Pep1_m]$
Model = 'ANDgate'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ANDgateBLeak1'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= K_{Leak} + syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ANDgateBLeak2'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= K_{Leak} + syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ANDgateBLeak3'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= K_{Leak} + syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ANDgateBLeak13'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= K_{Leak1} + syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= K_{Leak} + syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ANDgateBLeak13KMat'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= K_{Leak1} + syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= K_{Leak} + syn_{mRNA3} \left(\frac{[Pep1]}{Pep1_{max}} \right) \left(\frac{[Pep2]}{Pep2_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - K_{mature}[Pep3] \\
\frac{d[Pep3_m]}{dt} &= K_{mature}[mRNA3] - deg_{Pep}[Pep3_m]
\end{aligned}$$

Model = 'ORgate'

$$\begin{aligned}
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2]
\end{aligned}$$

$$\frac{d[mRNA3]}{dt} = syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3]$$

$$\frac{d[Pep3]}{dt} = syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]$$

Model = 'ORgateDelay'

$$\frac{d[Ind_e]}{dt} = - \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e]$$

$$\frac{d[Ind_i]}{dt} = \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e]$$

$$\frac{d[mRNA1]}{dt} = syn_{mRNA1}[Ind_i](state1) - deg_{mRNA}[mRNA1]$$

$$\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - deg_{Pep}[Pep1]$$

$$\frac{d[mRNA2]}{dt} = syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2]$$

$$\frac{d[Pep2]}{dt} = syn_{Pep}[mRNA2] - deg_{Pep}[Pep2]$$

$$\frac{d[mRNA3]}{dt} = syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3]$$

$$\frac{d[Pep3]}{dt} = syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]$$

Model = 'ORgate_Delay'

$$\frac{d[Ind_e]}{dt} = - \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e]$$

$$\frac{d[Ind_i]}{dt} = \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e]$$

$$\frac{d[mRNA1]}{dt} = syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1]$$

$$\frac{d[Pep1]}{dt} = syn_{Pep}[mRNA1] - deg_{Pep}[Pep1]$$

$$\frac{d[mRNA2]}{dt} = syn_{mRNA2}[Ind_i](state2) - deg_{mRNA}[mRNA2]$$

$$\frac{d[Pep2]}{dt} = syn_{Pep}[mRNA2] - deg_{Pep}[Pep2]$$

$$\frac{d[mRNA3]}{dt} = syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3]$$

$$\frac{d[Pep3]}{dt} = syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]$$

Model = 'ORgateDegradation'

$$\begin{aligned}
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}(state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgate_Degradation'

$$\begin{aligned}
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}(state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDelayDegradation'

$$\begin{aligned}
\frac{d[Ind_e]}{dt} &= - \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e] \\
\frac{d[Ind_i]}{dt} &= \left(\frac{[Ind_e]}{[Ind_e] + K_m} \right) [Ind_e] \\
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind_i](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3} \left(\frac{[Pep1] + [Pep2]}{Pep_{max}} \right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDegradationDelay'

$$\begin{aligned}
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[Ind_e]}{dt} &= -\left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[Ind_i]}{dt} &= \left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind_i](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3}\left(\frac{[Pep1] + [Pep2]}{Pep_{max}}\right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDelayDelay'

$$\begin{aligned}
\frac{d[Ind_{e1}]}{dt} &= -\left(\frac{[Ind_{e1}]}{[Ind_{e1}] + K_{m1}}\right)[Ind_{e1}] \\
\frac{d[Ind_{i1}]}{dt} &= \left(\frac{[Ind_{e1}]}{[Ind_{e1}] + K_{m1}}\right)[Ind_{e1}] \\
\frac{d[Ind_{e2}]}{dt} &= -\left(\frac{[Ind_{e2}]}{[Ind_{e2}] + K_{m2}}\right)[Ind_{e2}] \\
\frac{d[Ind_{i2}]}{dt} &= \left(\frac{[Ind_{e2}]}{[Ind_{e2}] + K_{m2}}\right)[Ind_{e2}] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind_{i1}](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind_{i2}](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3}\left(\frac{[Pep1] + [Pep2]}{Pep_{max}}\right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDelayDegradeResCompete'

$$\begin{aligned}
\frac{d[Ind_e]}{dt} &= -\left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[Ind_i]}{dt} &= \left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind_i](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3}\left(\frac{[Pep1] + [Pep2]}{Pep_{max}}\right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep3}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDegradeDelayResCompete'

$$\begin{aligned}
\frac{d[Ind]}{dt} &= -deg_{Ind}[Ind] \\
\frac{d[Ind_e]}{dt} &= -\left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[Ind_i]}{dt} &= \left(\frac{[Ind_e]}{[Ind_e] + K_m}\right)[Ind_e] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind](state1) - deg_{mRNA}[mRNA1] \\
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind_i](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3}\left(\frac{[Pep1] + [Pep2]}{Pep_{max}}\right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep3}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$

Model = 'ORgateDelayDelayResCompete'

$$\begin{aligned}
\frac{d[Ind_{e1}]}{dt} &= -\left(\frac{[Ind_{e1}]}{[Ind_{e1}] + K_{m1}}\right)[Ind_{e1}] \\
\frac{d[Ind_{i1}]}{dt} &= \left(\frac{[Ind_{e1}]}{[Ind_{e1}] + K_{m1}}\right)[Ind_{e1}] \\
\frac{d[Ind_{e2}]}{dt} &= -\left(\frac{[Ind_{e2}]}{[Ind_{e2}] + K_{m2}}\right)[Ind_{e2}] \\
\frac{d[Ind_{i2}]}{dt} &= \left(\frac{[Ind_{e2}]}{[Ind_{e2}] + K_{m2}}\right)[Ind_{e2}] \\
\frac{d[mRNA1]}{dt} &= syn_{mRNA1}[Ind_{i1}](state1) - deg_{mRNA}[mRNA1]
\end{aligned}$$

$$\begin{aligned}
\frac{d[Pep1]}{dt} &= syn_{Pep}[mRNA1] - deg_{Pep}[Pep1] \\
\frac{d[mRNA2]}{dt} &= syn_{mRNA2}[Ind_{i2}](state2) - deg_{mRNA}[mRNA2] \\
\frac{d[Pep2]}{dt} &= syn_{Pep}[mRNA2] - deg_{Pep}[Pep2] \\
\frac{d[mRNA3]}{dt} &= syn_{mRNA3}\left(\frac{[Pep1] + [Pep2]}{Pep_{max}}\right) - deg_{mRNA}[mRNA3] \\
\frac{d[Pep3]}{dt} &= syn_{Pep3}[mRNA3] - deg_{Pep}[Pep3]
\end{aligned}$$