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Fuzzy MFTS Transform

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Abstract:
In this study, a novel fuzzy transform based on the (MFTS) transform is introduced in order to achieve reliable solutions for first-order fuzzy differential equations. The method is based on a detailed explanation of pertinent characteristics and theorems that have been thoroughly examined. To improve comprehension, a number of real-world examples are given that show how to use this transform to obtain exact answers, making it easier to use for addressing fuzzy uncertainty concerns.

Keywords: fuzzy number; strongly generalized differentiable; fuzzy differential equation; fuzzy MFTS transforms; fuzzy first-order differential equation.

1. Introduction:

Fuzzy differential equations have been widely used in several fields in recent decades because of their significant and extensive applicability in a variety of industries. We made this work available, which contains a new method for handling this sort of issue, to remain abreast of the fast advancement and proliferation of fuzzy differential equations. We shall limit our research to the solution of fuzzy differential equations of first order. The concept of fuzzy differential equations was first established by Kandel and Byatt [2] after Chang and Zadeh [1] developed the fuzzy derivative. Abbasbandy and Allahviranloo [3] then gave the numerical solution technique for solving fuzzy differential equations over time. Seikkala [4] described the fuzzy derivative, which is an extension of the Hukuhara derivative. A great introduction to generalized differential is given by Bede and Gal [5. Study this work will develop a new fuzzy transform based on the (MFTS) transform to solve these sorts of problems [6]. However, some scholars have investigated the "fuzzification" of a number of approaches that are often used in the crisp situation and have created fuzzy versions of these techniques, such as fuzzy Laplace and fuzzy Abood (refer to [7,8] and the works referenced therein). In order to estimate expenses, Samer et al. exploited fuzzy systems in the second dimension (system research) [9].

2. Fundamental Preliminaries

For the interest of completeness, the following basic ideas and theorems related to our work in this area are given

(2.1) Definition [10]: By Ʀ, the set of all real numbers is represented as, the mapping ɏ: Ʀ → [0,1] is fuzzy number if it fulfills

1. ɏ is upper semi-continuous.

2. ɏ is fuzzy convex, i. e., ɏ(ɳχ + (1 − ɳ)¥) ≥ min{ɏ(χ), ɏ(¥)},for all χ , ¥ ∈ Ʀ and ɳ ∈ [0,1].

3. ɏ is normal i.e., ∃ χ_0 ∈ Ʀ for which ɏ(χ) = 1.

4. supp(ɏ) = { χ ∈ Ʀ;ɏ(χ) > 0 }, and *cl*(*Supp* (ɏ)) is compact.

Let ϖ be the set of all fuzzy number on $\mathbb{R}$. The η-level set of a fuzzy number $y \in \varpi, 0 \le \eta \le 1$ denoted by $[y]_{\eta}$ is defined as

$$[y]_{\eta} = \begin{cases} \{\chi \in \mathbb{R}, y(\chi) \ge \eta\}, & if\ 0 \le \eta \le 1 \\ cl(Supp\,(y)) & if\ \eta = 0 \end{cases}$$

Done $[y]_{\eta} = [\underline{y}(\eta), \overline{y}(\eta)]$, so the η –level set $[y]_{\eta}$ is a bounded and closed interval for all $\eta \in [0,1]$.

Zadeh's extension principle states that the operation of addition on ϖ is given by

$$(y \oplus \Omega)(\chi) = \sup_{¥ \in \mathbb{R}} \min\{y(¥), \Omega(\chi - ¥)\}, \chi \in \mathbb{R}$$

and a fuzzy number's scalar multiplication is provided by

$$(\rho \odot y)(\chi) = \begin{cases} y(\frac{\chi}{\rho}), & if\ \rho > 0 \\ \hat{0} & if\ \rho = 0 \end{cases} \quad where\ \hat{0} \in \varpi$$

The following characteristics are widely acknowledged to be true at all levels:

$[y \oplus \Omega]_{\eta} = [y]_{\eta} + [\Omega]_{\eta}, \ [\rho \odot y]_{\eta} = \rho[y]_{\eta}$.

(2.2) Definition [11]: A pair that is sorted parametrically is a fuzzy number, $(\underline{y}, \overline{y})$ of functions $\overline{y}(\eta), \underline{y}(\eta)$, $\eta \in [0,1]$, which fulfills:

1. $\underline{y}(\eta)$ is a continuous function with a right function of 0 and a left function of (0,1] that is non-decreasing.

2. $\overline{y}(\eta)$ is a bounded, non-increasing function with 0 continuous right and (0,1] continuous left.

3. $\underline{y}(\eta) \le \overline{y}(\eta), \ \eta \in [0,1]$.

For arbitrary $y = (\underline{y}(\eta), \overline{y}(\eta)), y = (\underline{y}(\eta), \overline{y}(\eta)), 0 \le \eta \le 1$ and $\rho > 0$ we define:

1. Addition $\qquad y \oplus \Omega = (\underline{y}(\eta) + \underline{\Omega}(\eta), \overline{y}(\eta) + \overline{\Omega}(\eta))$.

2. Subtraction $\qquad y \ominus_h \Omega = (\underline{y}(\eta) - \overline{\Omega}(\eta), \overline{y}(\eta) - \underline{\Omega}(\eta))$.

3. Multiplication $\qquad y \odot y =$

$$(\min\{\underline{y}(\eta)\overline{\Omega}(\eta), \underline{y}(\eta)\underline{y}(\eta), \overline{y}(\eta)\overline{\Omega}(\eta), \overline{y}(\eta)\underline{y\Omega}(\eta)\}, \max\{\underline{y}(\eta)\overline{\Omega}(\eta), \underline{y}(\eta)\underline{\Omega}(\eta), \overline{y}(\eta)\overline{\Omega}(\eta), \overline{y}(\eta)\underline{\Omega}(\eta)\}$$

4. Scalar multiplication $\quad \rho \odot y = \begin{cases} (\rho\underline{y}, \rho\overline{y}) & \rho \ge 0, \\ (\rho\overline{y}, \rho\underline{y}) & \rho < 0. \end{cases}$

(2.3) Definition [6]: Let y and Ω are fuzzy numbers, the Hausdorff distance between fuzzy numbers is provided by:

$\mathfrak{H}: \varpi \times \varpi \to [0, +\infty]$

$$\mathfrak{H}(y, \Omega) = \sup_{\eta \in [0,1]} \max\left\{\left|\underline{y}(\eta) - \underline{\Omega}(\eta)\right|, \left|\overline{y}(\eta) - \overline{\Omega}(\eta)\right|\right\},$$

Where $y = \left(\underline{y}(\eta), \overline{y}(\eta)\right), y = \left(\underline{\Omega}(\eta), \overline{\Omega}(\eta)\right) \subset \mathbb{R}$ and following properties are well known:

1. $\mathfrak{H}(y \oplus л, \Omega \oplus y) = \mathfrak{H}(y, \Omega), \forall y, \Omega, л \in \varpi$.
2. $\mathfrak{H}(\rho \odot y, \rho \odot \Omega) = |\rho|\, \mathfrak{H}(y, \Omega), \forall y, \Omega \in \varpi, \rho \in \mathbb{R}$.
3. $\mathfrak{H}(y \oplus \Omega, л \oplus ђ) \le \mathfrak{H}(y, \Omega) + \mathfrak{H}(л, ђ), \forall y, \Omega, л, ђ \in \varpi$.
4. $(\mathfrak{H}, \varpi)$ is a complete metric space.

(2.4) Definition [11]: Let $y: \mathbb{R} \longrightarrow \varpi$ be a function with fuzzy values. Assuming a random fixed point $\chi_0 \in \mathbb{R}$ and $\epsilon > 0$ there exists $\delta > 0$ such that $|\chi - \chi_0| < \delta \Longrightarrow \mathfrak{H}(y(\chi), y(\chi_0)) < \epsilon$, y is said to be a continuous fuzzy-valued function.

(2.5) Definition [12]: A mapping $\mathrm{y}: \mathbb{R} \times \varpi \longrightarrow \varpi$ is referred to as continuous at one point $(\tau_0, \chi_0) \in \mathbb{R} \times \varpi$ supplied for any fixed $n_0 \in [0,1]$ and arbitrary $\varepsilon > 0$ there exists an $\delta(\varepsilon, n)$ such that $\mathfrak{K}([\mathrm{y}(\tau,\chi)]_{n}, [\mathrm{y}(\tau_0,\chi_0)]_{n}) < \varepsilon$ whenever $|\tau - \tau_0| < \delta(\varepsilon, n)$ and $\mathfrak{K}([\chi]_{n}, [\chi_0]_{n}) < \delta(\varepsilon, n)$ for all $\tau \in \mathbb{R}, \chi \in \varpi$

(2.1) Theorem [13]: Assume $\mathrm{y}(x)$ function with fuzzy values on $[p, \infty)$ and it is embodied by $((\underline{\mathrm{y}}(x, n), \overline{\mathrm{y}}(x, n))$. For any fixed $n \in [0,1]$, let $\underline{\mathrm{y}}(x, n)$ and $\overline{\mathrm{y}}(x, n)$ are Riemann-integrals on $[p, q]$. For every $q \geq p$, if there are two positive functions $\underline{\theta}(n)$ and $\overline{\theta}(n)$ such that $\int_p^q \left| \underline{\mathrm{y}}(x, n) \right| dx \leq \underline{\theta}(n)$ and $\int_p^q | \overline{\mathrm{y}}(x, n) | dx \leq \overline{\theta}(n)$, then the fuzzy number is the improper fuzzy Riemann-integrable, and $\mathrm{y}(x)$ is said to be improper fuzzy Riemann-integrable on $[p, \infty]$, i.e.

$$\int_p^{\infty} \mathrm{y}(x)\, dx = \left[\int_p^{\infty} (\underline{\mathrm{y}}(x, n) dx, \int_p^{\infty} \overline{\mathrm{y}}(x, n) dx \right]$$

(2.6) Definition [12]: Assume $\mathrm{y}, \rho \in \varpi$. There is $\pi \in \varpi$ such that $\mathrm{y} = \rho \oplus \pi$ then π is known the H-differential of y and ρ and it is represented by $\mathrm{y} \ominus \rho$. In this paper, the sign " $\ominus$ " always stands for H-difference, and also note that

$\ominus \neq \ominus_h$ *and* $\mathrm{y} \ominus \mathrm{y} \neq \mathrm{y} + (-1)\rho$

(2.7) Definition [14] : A function $\mathrm{y}: (p, q): \longrightarrow \varpi$ and $x_0 \in (p, q)$. We say that y is strongly generalized differentiable at x_0 If such an element exists $(x_0) \in \varpi$, such that:

i. $\forall\ \varrho > 0$ that is adequately little, there are $\mathrm{y}(x_0 + \varrho) \ominus \mathrm{y}(x_0), \mathrm{y}(x_0) \ominus \mathrm{y}(x_0 - \varrho)$,

where $\lim_{\varrho \to 0} \frac{\mathrm{y}(x_0 + \varrho) \ominus \mathrm{y}(x_0)}{\varrho} = \lim_{\varrho \to 0} \frac{\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 - \varrho)}{\varrho} = \mathrm{y}'(x_0)$

or

ii. $\forall\ \varrho > 0$ that is adequately little, there are $\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 + \varrho), \mathrm{y}(x_0 - \varrho) \ominus \mathrm{y}(x_0)$

where $\lim_{\varrho \to 0} \frac{\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 + \varrho)}{-\varrho} = \lim_{\varrho \to 0} \frac{\mathrm{y}(x_0 - \varrho) \ominus \mathrm{y}(x_0)}{-\varrho} = \mathrm{y}'(x_0)$

or

iii. $\forall\ \varrho > 0$ that is adequately little, there are $\mathrm{y}(x_0 + \varrho) \ominus \mathrm{y}(x_0), \mathrm{y}(x_0 - \varrho) \ominus \mathrm{y}(x_0)$

where $\lim_{\varrho \to 0} \frac{\mathrm{y}(x_0 + \varrho) \ominus \mathrm{y}(x_0)}{\varrho} = \lim_{\varrho \to 0} \frac{\mathrm{y}(x_0 - \varrho) \ominus \mathrm{y}(x_0)}{-\varrho} = \mathrm{y}'(x_0)$

or

iv. $\forall\ \varrho > 0$ that is adequately little, there are $\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 + \varrho), \mathrm{y}(x_0) \ominus \mathrm{y}(x_0 - \varrho)$

where $\lim_{\varrho \to 0} \frac{\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 + \varrho)}{-\varrho} = \lim_{\varrho \to 0} \frac{\mathrm{y}(x_0) \ominus \mathrm{y}(x_0 - \varrho)}{\varrho} = \mathrm{y}'(x_0)$.

(2.2) Theorem [15] : Let $\mathrm{y}(x): \mathbb{R} \longrightarrow \varpi$ be a function and represents $\mathrm{y}(x) = ((\underline{\mathrm{y}}(x, n), \overline{\mathrm{y}}(x, n))$ in every instance for $n \in [0,1]$. Then:

1. If $\mathrm{y}(\mathrm{x})$ is differentiable form i, then $(\underline{\mathrm{y}}(\mathrm{x},\mathrm{n}))$ and $\overline{\mathrm{y}}(\mathrm{x},\mathrm{n})$ are differentiable functions and $\mathrm{y}'(\mathrm{x}) = (\underline{\mathrm{y}}'(\mathrm{x},\mathrm{n}), \overline{\mathrm{y}}'(\mathrm{x},\mathrm{n}))$.
2. If $\mathrm{y}(\mathrm{x})$ is differentiable form ii, then $(\underline{\mathrm{y}}(\mathrm{x},\mathrm{n}))$ and $\overline{\mathrm{y}}(\mathrm{x},\mathrm{n})$ are differentiable functions and $\mathrm{y}'(\mathrm{x}) = (\overline{\mathrm{y}}'(\mathrm{x},\mathrm{n}), \underline{\mathrm{y}}'(\mathrm{x},\mathrm{n}))$.

3. Fuzzy (MFTS) Transform:

Fuzzy differential equations and their associated fuzzy beginning and boundary value issues may be resolved using the fuzzy (MFTS) transform technique. Fuzzy (MFTS) transformations do this by reducing the fuzzy differential equation solving challenge to an algebraic one. Operational calculus is a crucial field of applied mathematics that involves moving from calculus operations to algebraic operations on transforms. The fuzzy (MFTS) transform technique is essentially the most significant operational approach for engineers. The fuzzy (MFTS) transform will be defined in this section.

(3.1) (MFTS) Integral Transform [6]: Mhase, Fulari, Tarat and Shaikh introduced a novel integral transform known as (MFTS) integral transform. This Transform is defined for the function $\mathrm{y}(\mathrm{x})$ as:

$\mathbb{T}\,[\mathrm{y}(\mathrm{x})] = \int_0^\infty \mathrm{y}(\mathrm{x})\, e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x} = \mathbb{P}(\mathrm{r})$, Where $\mathrm{x} \geq 0, \mu_1 \leq \mathrm{r} \leq \mu_1$ and in the function y, the variable r is used as a factor to the variable x

(3.2) Definition: Let $\mathrm{y}(\mathrm{x})$ be a fuzzy-valued continuous function. Suppose that $\mathrm{y}(\mathrm{x}) \odot e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}$ is an inappropriate fuzzy Integral at Rimann on $[0,\infty)$, then $\int_0^\infty \mathrm{y}(\mathrm{x}) \odot e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x}$ is being called fuzzy (MFTS) transform and is known as

$$\widehat{\mathbb{T}}[\mathrm{y}(\mathrm{x})] = \int_0^\infty \mathrm{y}(\mathrm{x}) \odot e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x}$$

$$\int_0^\infty \mathrm{y}(\mathrm{x}) \odot e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x} = \left(\int_0^\infty \underline{\mathrm{y}}(\mathrm{x},\mathrm{n})\, e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x}, \int_0^\infty \overline{\mathrm{y}}(\mathrm{x},\mathrm{n})\ d\mathrm{x} \right).$$

Using the definition of classical (MFTS) transform, to get:

$\mathbb{T}\left[\underline{\mathrm{y}}(\mathrm{x},\mathrm{n})\right] = \int_0^\infty \underline{\mathrm{y}}(\mathrm{x},\mathrm{n}) e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x}$ *and* $\mathbb{T}[\overline{\mathrm{y}}(\mathrm{x},\mathrm{n})] = \int_0^\infty \overline{\mathrm{y}}(\mathrm{x},\mathrm{n})\, e^{(\frac{-\mathrm{x}}{\mathrm{r}^2})}\, d\mathrm{x}$, then:

$$\widehat{\mathbb{T}}[\mathrm{y}(\mathrm{x})] = (\mathbb{T}\left[\underline{\mathrm{y}}(\mathrm{x},\mathrm{n})\right], \mathbb{T}[\overline{\mathrm{y}}(\mathrm{x},\mathrm{n})]$$

(3.3) Theorem : Let $\mathrm{y}(\mathrm{x}), ¥(\mathrm{x})$ be continuous fuzzy-valued functions, $\mathbb{d}_1$ and $\mathbb{d}_2$ are constants, then

(1). $\widehat{\mathbb{T}}[\mathbb{d}_1 \odot \mathrm{y}(\mathrm{x})] = \mathbb{d}_1 \odot \widehat{\mathbb{T}}[\mathrm{y}(\mathrm{x})]$.

(2). $\widehat{\mathbb{T}}\left[(\mathbb{d}_1 \odot \mathrm{y}(\mathrm{x})) \oplus (\mathbb{d}_2 \odot ¥(\mathrm{x}))\right] = \left(\mathbb{d}_1 \odot \widehat{\mathbb{T}}[\mathrm{y}(\mathrm{x})]\right) \oplus \left(\mathbb{d}_2 \odot \widehat{\mathbb{T}}[¥(\mathrm{x})]\right)$

Proof

$$\hat{\mathbb{T}}[\mathbb{d}_1 \odot ɣ(x)] = \left(\mathbb{T}\left[\mathbb{d}_1 \underline{ɣ}(x,n)\right], \mathbb{T}[\mathbb{d}_1 \overline{ɣ}(x,n)]\right) = \left(\int_0^{\infty} \mathbb{d}_1\, \underline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx, \int_0^{\infty} \mathbb{d}_1\, \overline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx\right)$$

$$= \left(\mathbb{d}_1 \int_0^{\infty} \underline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx, \mathbb{d}_1 \int_0^{\infty} \overline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx\right)$$

$$= \mathbb{d}_1 \left(\int_0^{\infty} \underline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx, \int_0^{\infty} \overline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)}\, dx\right) = \mathbb{d}_1 \left(\mathbb{T}\left[\underline{ɣ}(x,n)\right], \mathbb{T}[\overline{ɣ}(x,n)]\right)$$

$$= \mathbb{d}_1 \odot \hat{\mathbb{T}}[ɣ(x)]$$

(2). Suppose $ɣ(x) = (\underline{ɣ}(x,n), \overline{ɣ}(x,n)$ and $¥(x) = (\underline{¥}(x,n), \overline{¥}(x,n)$

$$\hat{\mathbb{T}}[(\mathbb{d}_1 \odot ɣ(x)) \oplus (\mathbb{d}_2 \odot ¥(x))] = \left(\mathbb{T}\left[\mathbb{d}_1\, \underline{ɣ}(x,n) + \mathbb{d}_2 \underline{¥}(x,n)\right], \mathbb{T}[\mathbb{d}_1\, \overline{ɣ}(x,n) + \mathbb{d}_2 \overline{¥}(x,n)]\right) =$$

$$\left(\int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)} \left(\mathbb{d}_1\, \underline{ɣ}(x,n) + \mathbb{d}_2 \underline{¥}(x,n)\right) dx, \int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)} \left(\mathbb{d}_1\, \overline{ɣ}(x,n) + \mathbb{d}_2 \overline{¥}(x,n)\right) dx\right)$$

$$= \left(\int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)} \mathbb{d}_1\, \underline{ɣ}(x,n) dx, \int_0^{\infty} \mathbb{d}_1\, \overline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)} dx)\right) + \left(\int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)} \mathbb{d}_2\, \underline{¥}(x,n) dx, \int_0^{\infty} \mathbb{d}_2\, \overline{¥}(x,n)\, e^{\left(\frac{-x}{r^2}\right)} dx)\right)$$

$$= \mathbb{d}_1 \left(\int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)}\, \underline{ɣ}(x,n) dx, \int_0^{\infty} \overline{ɣ}(x,n)\, e^{\left(\frac{-x}{r^2}\right)} dx)\right)$$

$$+ \mathbb{d}_2 \left(\int_0^{\infty} e^{\left(\frac{-x}{r^2}\right)}\, \underline{¥}(x,n) dx, \int_0^{\infty} \overline{¥}(x,n)\, e^{\left(\frac{-x}{r^2}\right)} dx)\right)$$

$$= \mathbb{d}_1 \left(\mathbb{T}\left[\underline{ɣ}(x,n)\right], \mathbb{T}[\overline{ɣ}(x,n)]\right) + \mathbb{d}_2 \left(\mathbb{T}[\underline{¥}(x,n)], \mathbb{T}[\overline{¥}(x,n)]\right)$$

$$= (\mathbb{d}_1 \odot \hat{\mathbb{T}}[ɣ(x)]) \oplus (\mathbb{d}_2 \odot \hat{\mathbb{T}}[¥(x)])$$

4. Fuzzy (MFTS) Transform for First -Order Fuzzy Differential Equation

(4.1) Theorem : Let $ɣ(x)$ is the primitive of $ɣ'(x)$ on $[0, \infty)$ and $ɣ(x)$ is a fuzzy-valued function that is integrable, then:

a. $ɣ(x)$is (i)-differentiable then $\hat{\mathbb{T}}[ɣ'(x)] = \frac{1}{r^2} \odot \hat{\mathbb{T}}[ɣ(x)] \ominus ɣ(0)$.

b. $ɣ(x)$is (ii)-differentiable then $\hat{\mathbb{T}}[ɣ'(x)] = (-ɣ(0)) \ominus \left(-\frac{1}{r^2} \odot \hat{\mathbb{T}}[ɣ(x)]\right)$

Proof (a)

For a fixed, arbitrary $0 \leq n \leq 1$,

$$\frac{1}{s^2}\odot\hat{\mathbb{T}}[y(x)]\ominus y(0)=\left(\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r),\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)]-\overline{y}(0,r)\right)$$

Since

$$\mathbb{T}\left[\underline{y}'(x,r)\right]=\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r),\mathbb{T}[\overline{y}'(x,r)]=\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)]-\overline{y}(0,r).$$

Since $y(x)$ is differentiable of form i using Theorem (2.2):

$$\underline{y'}(x,r)=\underline{y}'(x,r),\overline{y'}(x,r)=\overline{y}'(x,r)$$

$$\mathbb{T}\left[\underline{y'}(x,r)\right]=\mathbb{T}\left[\underline{y}'(x,r)\right]=\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r)$$

$$\mathbb{T}[\overline{y'}(x,r)]=\mathbb{T}[\overline{y}'(x,r)]=\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)]-\overline{y}(0,r)$$

$$\frac{1}{s^2}\odot\hat{\mathbb{T}}[y(x)]\ominus y(0)=\left(\mathbb{T}\left[\underline{y'}(x,r)\right],\mathbb{EF}[\overline{y'}(x,r)]\right)=\hat{\mathbb{T}}[y'(x)]$$

(b)

$$(-y(0))\ominus\left(-\frac{1}{s^2}\odot\hat{\mathbb{T}}[y(x)]\right)=\left(-\overline{y}(0,r)+\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)],-\underline{y}(0,r)+\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]\right)$$

Since

$$\mathbb{T}\left[\underline{y}'(x,r)\right]=\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r),\mathbb{T}[\overline{y}'(x,r)]=\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)]-\overline{y}(0,r).$$

Since $y(x)$ is differentiable of form ii using Theorem (2.2):

$$\underline{y'}(x,r)=\overline{y}'(x,r),\overline{y'}(x,r)=\underline{y}'(x,r)$$

$$\mathbb{T}\left[\underline{y'}(x,r)\right]=\mathbb{T}[\overline{y}'(x,r)]=\frac{1}{s^2}\mathbb{T}[\overline{y}(x,r)]-\overline{y}(0,r)$$

$$\mathbb{T}[\overline{y'}(x,r)]=\mathbb{T}\left[\underline{y}'(x,r)\right]=\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r)$$

$$(-y(0))\ominus\left(-\frac{1}{s^2}\odot\hat{\mathbb{T}}[y(x)]\right)=\left(\mathbb{T}\left[\underline{y'}(x,r)\right],\mathbb{T}[\overline{y'}(x,r)]\right)=\hat{\mathbb{T}}[y'(x)]$$

(4.1) Example: Consider a fuzzy initial value problem:

$y'(x)=y(x),\ \ y(0,r)=(r-1,1-r),0\le r\le 1$.

Solution:

Apply both sides' fuzzy (MFTS) transforms to get

$$\hat{\mathbb{T}}[y'(x)]=\hat{\mathbb{T}}[y(x)]$$

Case (1)

$y(x)$ be (i)-differentiable,

$$\frac{1}{s^2}\odot\hat{\mathbb{T}}[y(x)]\ominus y(0)=\hat{\mathbb{T}}[y'(x)]$$

Using upper and lower functions, to have

$$\frac{1}{s^2}\mathbb{T}\left[\underline{y}(x,r)\right]-\underline{y}(0,r)=\mathbb{T}\left[\underline{y}(x,r)\right]$$

$$\frac{1}{r^2}\mathbb{T}[\overline{y}(x,n)] - \overline{y}(0,n) = \mathbb{T}[\overline{y}(x,n)]$$

$$\left(\frac{1}{r^2}-1\right)\mathbb{T}\left[\underline{y}(x,n)\right] = (n-1)$$

$$\left(\frac{1}{r^2}-1\right)\mathbb{T}[\overline{y}(x,n)] = (1-n)$$

$$\mathbb{T}\left[\underline{y}(x,n)\right] = \frac{r^2}{1-r^2}\,(n-1)$$

$$\mathbb{T}[\overline{y}(x,n)] = \frac{r^2}{1-r^2}(1-n)$$

$$\underline{y}(x,n) = (\mathbb{T})^{-1}\left(\frac{r^2}{1-r^2}(n-1)\right)$$

$$\overline{y}(x,n) = (\mathbb{T})^{-1}\left(\frac{r^2}{1-r^2}(1-n)\right)$$

Using inverse (MFTS) transform

$$\underline{y}(x,n) = (n-1)\,e^{x}, \overline{y}(x,n) = (1-n)\,e^{x}$$

Case (2)

$y(x)$ be (ii)-differentiable,

$$\widehat{\mathbb{T}}[y'(x)] = (-y(0)) \ominus \left(-\frac{1}{r^2}\odot\widehat{\mathbb{T}}[y(x)]\right)$$

Using upper and lower functions, to have

$$\frac{1}{r^2}\mathbb{T}\left[\underline{y}(x,n)\right] - \underline{y}(0,n) = \mathbb{T}[\overline{y}(x,n)]$$

$$\frac{1}{r^2}\mathbb{T}[\overline{y}(x,n)] - \overline{y}(0,n) = \mathbb{T}\left[\underline{y}(x,n)\right]$$

$$\frac{1}{r^2}\mathbb{T}\left[\underline{y}(x,n)\right] = (n-1) + \mathbb{T}[\overline{y}(x,n)]$$

$$\frac{1}{r^2}\mathbb{T}[\overline{y}(x,n)] = (1-n) + \mathbb{T}\left[\underline{y}(x,n)\right]$$

With simple calculation and Using inverse (MFTS) transform obtained the solution of case (2)

$$\underline{y}(x,n) = (n-1)\,e^{-x}, \overline{y}(x,n) = (1-n)\,e^{-x}$$

5. Conclusion:

Using the extremely extended differentiability concept, we have created the fuzzy (MFTS) transform to solve fuzzy initial-value problems for first-order linear fuzzy differential equations. This might lead to solutions whose support fluctuates over time.

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