ME 7120: Project 3

Finite Element Method Applications

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* Methodology

The equation of motion that we are going to solve for the problem is shown below

|  |  |  |
| --- | --- | --- |
|  |  | *(1)* |

where is the mass matrix, is the damping matrix, is the stiffness matrix. , and are displacement, velocity, and acceleration. is the applied load.

1. Undamped system with no loading

First, Let us consider the undamped case with no loading in the system. Reduce the problem from 3D to 2D and apply the boundary condition, the reduced equation of motion is obtained with reduced mass and stiffness matrix shown below

|  |  |  |
| --- | --- | --- |
|  |  | *(2)* |

Perform the Cholesky decomposition on the reduced mass matrix

|  |  |  |
| --- | --- | --- |
|  |  | *(3)* |

Let and pre-multiply to the reduced equation of motion

|  |  |  |
| --- | --- | --- |
|  |  | *(4)* |
|  |  | *(5)* |

where .

Let and submit it into Eq. (5). The eigenvalue problem is obtained below

|  |  |  |
| --- | --- | --- |
|  |  | *(6)* |

Let , where is the eigenvector for each mode.

Let and submit it into Eq. (5), and pre-multiply , we got

|  |  |  |
| --- | --- | --- |
|  |  | *(7)* |

This ends up with the decoupled system

|  |  |  |
| --- | --- | --- |
|  |  | *(8)* |

where

|  |  |  |
| --- | --- | --- |
|  |  | *(9)* |

Then we know

|  |  |  |
| --- | --- | --- |
|  |  | *(10)* |

The mode shape is calculated by the following equation

|  |  |  |
| --- | --- | --- |
|  |  | *(11)* |
|  |  |  |

1. damped system with loading

Let us consider the damped system with loading shown below

|  |  |  |
| --- | --- | --- |
|  |  | *(12)* |

Submit Eq. (10) and pre-multiply , we got

|  |  |  |
| --- | --- | --- |
|  |  | *(13)* |
|  |  | *(14)* |

where

|  |  |  |
| --- | --- | --- |
|  |  | *(15)* |

Then, we can get the matrix by the following equation

|  |  |  |
| --- | --- | --- |
|  |  | *(16)* |
|  |  |  |

1. Transient response by Newmark Method

The Newmark Method can me summarized in the following steps:

Step 1: From the known values of , , find .

Step 2: Select suitable values of , and based on Table 1.

Step 3: Calculate the displacement vector , using the following expression

|  |  |  |
| --- | --- | --- |
|  |  | *(17)* |
|  |  |  |
|  |  |  |

Step 4: Find the acceleration and velocity vectors at time based on the following equations

|  |  |  |
| --- | --- | --- |
|  |  | *(18)* |
|  |  | *(19)* |

Step 5: Repeat

Table 1. Stability and accuracy of different Newmark Methods

