Cantilever Beam Analytical Solution

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June 22, 2016

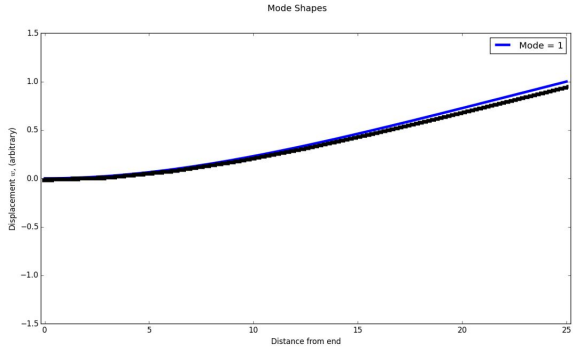
1 Analytic Results

Fourth order PDE for the Euler-Bernoulli was solved to obtain natural frequencies and mode shapes for the clampled-free boundary condition (CANTILEVER BEAM). First 8 natural frequencies are listed in Table 1 below and compared to FE results using ABAQUS.

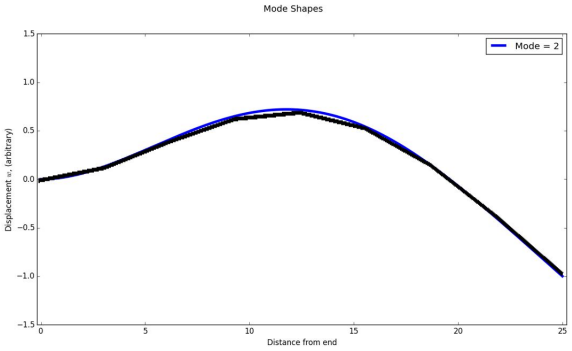
Table 1: Beam Frequencies, ABAQUS

First 8 Natural Frequencies			
Mode	**Analytical Resuls**	**ABAQUS Results**	**% Error**
1	0.0263	0.0261	0.7%
2	0.1647	0.1615	1.9%
3	0.4613	0.4485	2.8%
4	0.9039	0.8749	3.2%
5	1.4942	1.4449	3.3%
6	2.2321	2.1505	3.7%
7	3.1176	2.9235	6.2%
8	4.1507	3.5554	14.3%

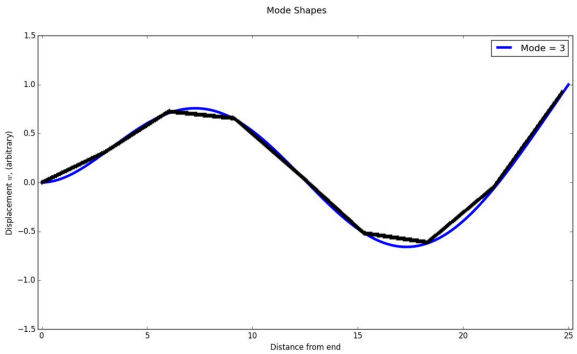
Next section compared the mode shapes predicted using the analytical and FE approaches. For Figures 1 and 2, ABAQUS results with 8 noded beam is used.



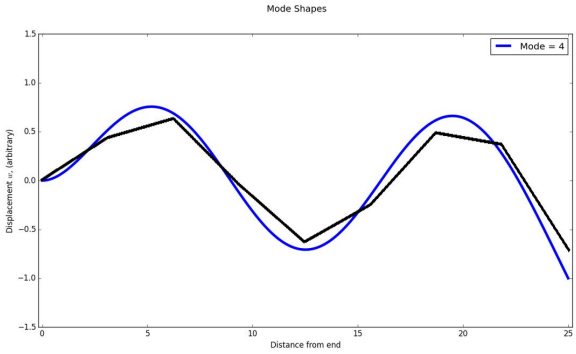
(a) Mode 1 ABAQUS and analytical comparison



(b) Mode 2 ABAQUS and analytical comparison

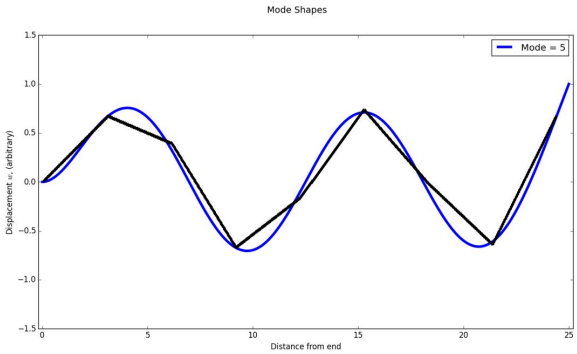


(c) Mode 3 ABAQUS and analytical comparison

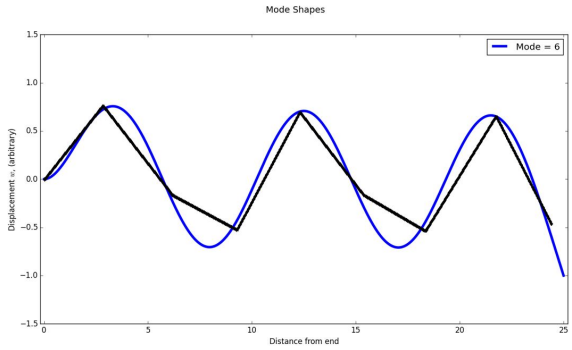


(d) Mode 4 ABAQUS and analytical comparison

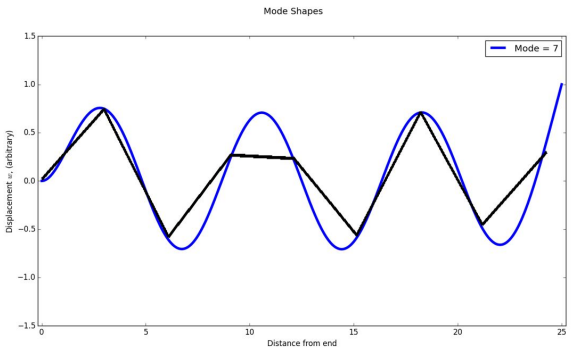
Figure 1: Modes 1 THRU 4 using ABAQUS results



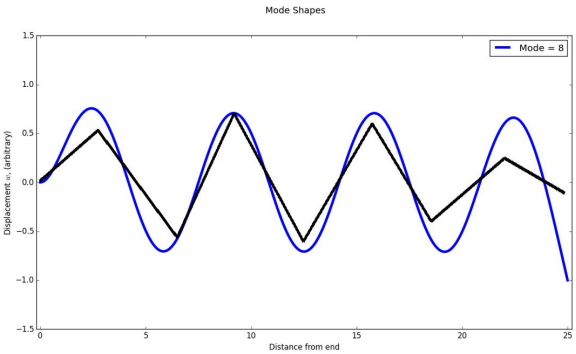
(a) Mode 5 ABAQUS and analytical comparison



(b) Mode 6 ABAQUS and analytical comparison



(c) Mode 7 ABAQUS and analytical comparison



(d) Mode 8 ABAQUS and analytical comparison

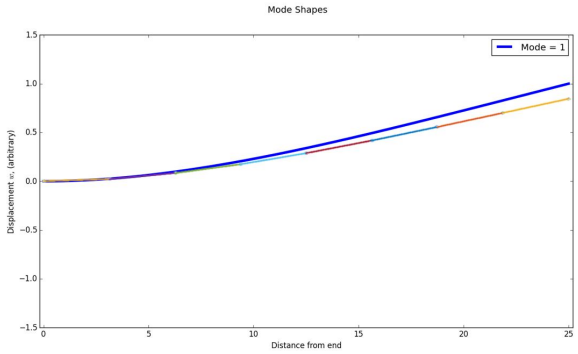
Figure 2: Modes 5 THRU 8 using ABAQUS results

Next section compares analytical results for the 2 noded beam element coded in MATLAB. WFEM model is constructed using 8 beam elements in order to enable comparison with ABAQUS results. Looking at Table 2 below it is evident that predictions of the first 8 natural frequencies using WFEM are more accurate when compared to ABAQUS results. As an example max errors from ABAQUS and WFEM are 14.3% and 2.1% respectively.

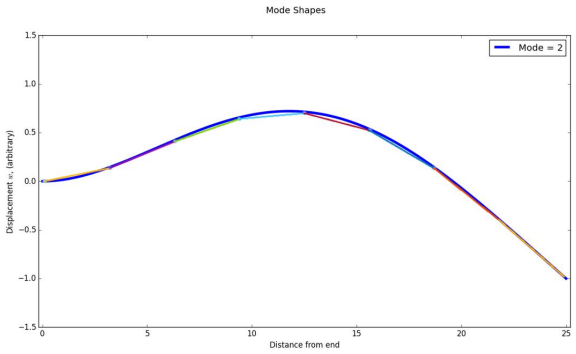
Table 2: Beam Frequencies, WFEM

First 8 Natural Frequencies			
Mode	**Analytical Resuls**	**WFEM Results**	**% Error**
1	0.0263	0.0263	0.0%
2	0.1647	0.1650	0.2%
3	0.4613	0.4620	0.2%
4	0.9039	0.9068	0.3%
5	1.4942	1.5043	0.7%
6	2.2321	2.2610	1.3%
7	3.1176	3.1826	2.1%
8	4.1507	4.2281	1.9%

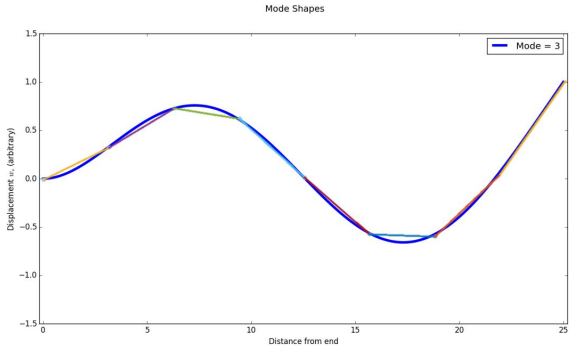
Next, Figures 3 and 4 plot mode shapes predicted by WFEM with and compares them to the analytical results.



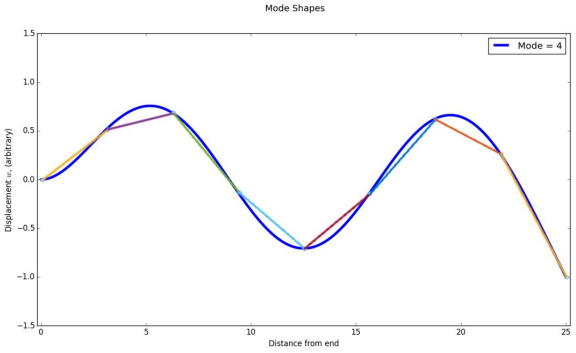
(a) Mode 1 WFEM and analytical comparison



(b) Mode 2 WFEM and analytical comparison

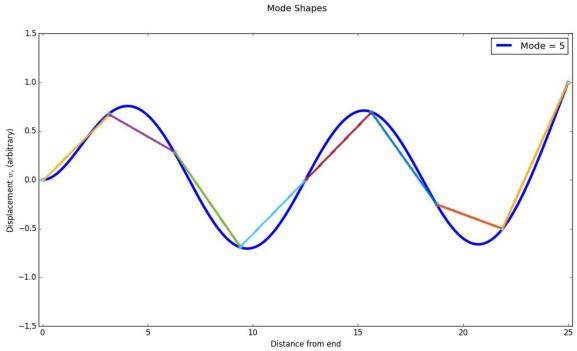


(c) Mode 3 WFEM and analytical comparison

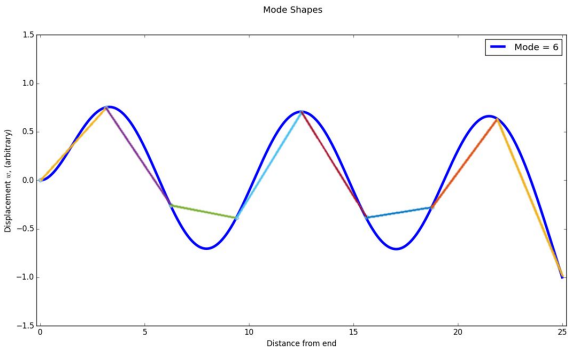


(d) Mode 4 WFEM and analytical comparison

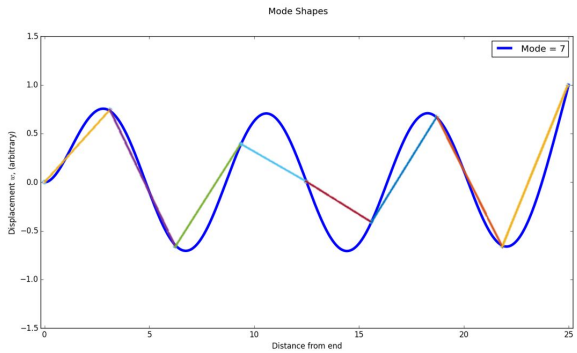
Figure 3: Modes 1 THRU 4 using WFEM results



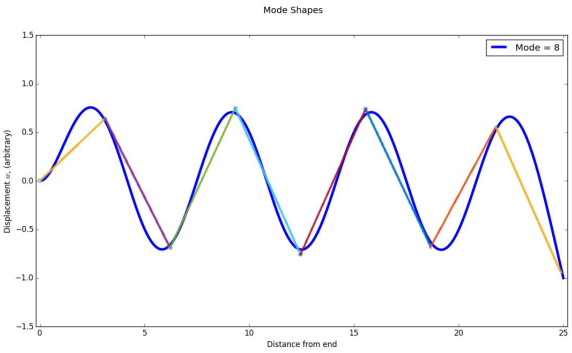
(a) Mode 5 WFEM and analytical comparison



(b) Mode 6 WFEM and analytical comparison



(c) Mode 7 WFEM and analytical comparison



(d) Mode 8 WFEM and analytical comparison

Figure 4: Modes 5 THRU 8 using WFEM results

Finally, max tip deflections are compared between analytical and FE results. All solutions agree well with one another. Max tip deflection values are listed in Table XX. Tip load used in this case in $10N$

Table 3: Max Tip Deflection Values

Max Tip Deflection (m)	
Analytical	19.3
WFEM	19.21
ABAQUS	19.22