Surrogate Modeling for Reliability Based Design

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Outline

- Response Surface Models: Simple Regression Models
  - Linear
  - Nonlinear
- Linear First-order Taylor Series Expansion
  - Linear single point approximation
- Reciprocal First-order Taylor Series Expansion
  - Nonlinear single point approximation
- Kriging
  - Highly nonlinear

Linear Regression Procedure

- Response at a location is estimated by

$$y(x) = \beta_0 + \underbrace{\beta_1}\, f_1(x) + \ldots + \beta_k f_k(x) + \underbrace{\epsilon}$$

- Regression Coefficients
- Error of the model Eq
  - Assumed: normally distributed with mean zero and variance σ_e^2
- In matrix notation for n samples

$$Y = F\hat{\beta} + \epsilon$$

Linear Regression Procedure cont.

- In expanded form we have

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & f_1(x_1) & f_2(x_1) & \cdots & f_k(x_1) \\ 1 & f_1(x_2) & f_2(x_2) & \cdots & f_k(x_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & f_1(x_n) & f_2(x_n) & \cdots & f_k(x_n) \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix}$$

- Where $\hat{\beta}$ is found through general least squares

$$\hat{\beta} = \left(F^T F\right)^{-1} F^T Y$$

Nonlinear Regression Procedure

- Response at a location is estimated by

$$y(x) = \beta_0 p_0(x) + \beta_1 p_1(x) + \cdots + \beta_m p_m(x) + \epsilon$$

- $p_i(x)$ is I = 0, . . ., m and the polynomial is of order m
- Simplest polynomial model is the monomials of
  - x^m i.e. $p_0(x)$ = 1, $p_1(x)$ = x, …, $p_m(x) = x^m$

$$F = \begin{bmatrix} p_0(x_1) & p_1(x_1) & p_2(x_1) & \cdots & p_m(x_1) \\ p_0(x_2) & p_1(x_2) & p_2(x_2) & \cdots & p_m(x_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p_0(x_n) & p_1(x_n) & p_2(x_n) & \cdots & p_m(x_n) \end{bmatrix}$$

Linear Regression Example

A simply supported beam of span L and stiffness EI is loaded with a concentrated load P at the midspan and a uniformly distributed load w along the length of the beam. The maximum deflection at the midspan can be calculated as

$$\delta_{max} = \frac{PL^3}{48EI} + \frac{5}{385}\frac{wL^4}{EI} \qquad (1)$$

Suppose L and I are constant 30ft and 1.33×10^3 in^4, but P and E are normal random variables with means of [50 kip and 29×10^6 lb/in^2] and standard deviations of [10 kip and 1×10^5 lb/in^2] and w is a lognormal variable with a mean of 1 kip/ft and a standard deviation of 0.1 kip/ft.

$$P \sim N(\mu_p, \sigma_p) = N(50x10^3, 10x10^3) \text{ lb}$$
$$w \sim N(\mu_w, \sigma_w) = N(10^3/12\,,\ .1x10^3/12) \text{ lb/in}$$

$$\widehat{\delta_{max}} = \widehat{\beta_0} + P\widehat{\beta_1} + w\widehat{\beta_2}$$

Nonlinear Regression Example

A simply supported beam of span L and stiffness EI is loaded with a concentrated load P at the midspan and a uniformly distributed load w along the length of the beam. The maximum deflection at the midspan can be calculated as

$$\delta_{max} = \frac{PL^3}{48EI} + \frac{5}{385}\frac{wL^4}{EI} \qquad (1)$$

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$$\text{P}\sim N(\mu_p, \sigma_p) = N(50x10^3, 10x10^3) \text{ lb}$$
$$I\sim N(\mu_I, \sigma_I) = N(1.33x10^3, 90) \text{ in\^{}4}$$

$$\widehat{\delta_{max}} = \widehat{\beta_0} + P\widehat{\beta_1} + I\widehat{\beta_2} + PI\widehat{\beta_3}$$

First-order Taylor Series Expansion

$$f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\cdots.$$

- Expansion point is X_1

$$g_L(\mathbf{X}) = g(\mathbf{X_1}) + \sum_{i=1}^{n}(x_i - x_{i,1})\left(\frac{\partial g}{\partial x_i}\right)_{x_1}$$

- Where x_i is the i^{th} component of X
- And $x_{i,1}$ is the i^{th} component of X_1

Linear

$$g_R(\mathbf{X}) = g(\mathbf{X_1}) + \sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i}\right)_{x_1}(x_i - x_{i,1})\left(\frac{x_{i,1}}{x_i}\right)$$

Reciprocal

First-order Taylor Series Expansion

A simply supported beam of span L and stiffness EI is loaded with a concentrated load P at the midspan and a uniformly distributed load w along the length of the beam. The maximum deflection at the midspan can be calculated as

$$\delta_{max} = \frac{PL^3}{48EI} + \frac{5}{385}\frac{wL^4}{EI} \qquad (1)$$

Suppose L and I are constant 30ft and 1.33×10^3 in^4, but P and E are normal random variables with means of [50 kip and 29×10^6 lb/in^2] and standard deviations of [10 kip and 1×10^5 lb/in^2] and w is a lognormal variable with a mean of 1 kip/ft and a standard deviation of 0.1 kip/ft.

$$P \sim N(\mu_p, \sigma_p) = N(50x10^3, 10x10^3) \text{ lb}$$
$$I \sim N(\mu_I, \sigma_I) = N(1.33x10^3, 90) \text{ in^4}$$

$$\widehat{\delta_{max}} = \widehat{\beta_0} + P\widehat{\beta_1} + I\widehat{\beta_2} + PI\widehat{\beta_3}$$