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HW #1, FE Class

Problem 1

Rotate Mass matrix for 2-D rob/beam element by 45^{o}.

- Start with constructing the rotation matrix:

$$\begin{bmatrix} 0.7071 & 0.7071 & 0 \\ -0.7071 & 0.7071 & 0 \\ 0 & 0 & 1.0 \end{bmatrix}$$

Out[6]:
$$\begin{bmatrix} 0.7071 & 0.7071 & 0 & 0 & 0 & 0 \\ -0.7071 & 0.7071 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.7071 & 0.7071 & 0 \\ 0 & 0 & 0 & -0.7071 & 0.7071 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \end{bmatrix}$$

- Define mass matrix:

Out[7]:

$$\begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{13}{35} & \frac{11l}{210} & 0 & \frac{9}{70} & -\frac{13l}{420} \\ 0 & \frac{11l}{210} & \frac{l^2}{105} & 0 & \frac{13l}{420} & -\frac{l^2}{140} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{9}{70} & \frac{13l}{420} & 0 & \frac{13}{35} & -\frac{11l}{210} \\ 0 & -\frac{13l}{420} & -\frac{l^2}{140} & 0 & -\frac{11l}{210} & \frac{l^2}{140} \end{bmatrix}$$

- Next rotate the mass matrix:

Out[15]:

$$\begin{bmatrix} \frac{881A}{2500}L\rho & -\frac{2A}{105}L\rho & -\frac{AL}{27}l\rho & \frac{31A}{210}L\rho & \frac{2A}{105}L\rho & \frac{2189A}{100000}Ll\rho \\ -\frac{2A}{105}L\rho & \frac{881A}{2500}L\rho & \frac{AL}{27}l\rho & \frac{2A}{105}L\rho & \frac{31A}{210}L\rho & -\frac{2189A}{100000}Ll\rho \\ -\frac{AL}{27}l\rho & \frac{AL}{27}l\rho & \frac{AL}{105}l^2\rho & -\frac{2189A}{100000}Ll\rho & \frac{2189A}{100000}Ll\rho & -\frac{AL}{140}l^2\rho \\ \frac{31A}{210}L\rho & \frac{2A}{105}L\rho & -\frac{2189A}{100000}Ll\rho & \frac{881A}{2500}L\rho & -\frac{2A}{105}L\rho & \frac{AL}{27}l\rho \\ \frac{2A}{105}L\rho & \frac{31A}{210}L\rho & \frac{2189A}{100000}Ll\rho & -\frac{2A}{105}L\rho & \frac{881A}{2500}L\rho & -\frac{AL}{27}l\rho \\ \frac{2189A}{100000}Ll\rho & -\frac{2189A}{100000}Ll\rho & -\frac{AL}{140}l^2\rho & \frac{AL}{27}l\rho & -\frac{AL}{27}l\rho & \frac{AL}{140}l^2\rho \end{bmatrix}$$

Problem 2

Write total beam element:

- The beam element will have following deformation properties:
  - Extension along x-axis
  - Torsion about x-axis
  - Vertical displacements along y and z axis
  - Moments about y and z axis

Define components that will fill the stiffness matrix K.

Out[10]:

$$\begin{bmatrix}
\frac{AE}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{AE}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{12E}{l^3} I_{zz} & 0 & 0 & 0 & \frac{6E}{l^2} I_{zz} & 0 & -\frac{12E}{l^3} I_{zz} & 0 & 0 & 0 & \frac{6E}{l^2} I_{zz} \\
0 & 0 & \frac{12E}{l^3} I_{yy} & 0 & -\frac{6E}{l^2} I_{yy} & 0 & 0 & 0 & -\frac{12E}{l^3} I_{yy} & 0 & -\frac{6E}{l^2} I_{yy} & 0 \\
0 & 0 & 0 & \frac{GK}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GK}{l} & 0 & 0 \\
0 & 0 & -\frac{6E}{l^2} I_{yy} & 0 & \frac{4E}{l} I_{yy} & 0 & 0 & 0 & \frac{6E}{l^2} I_{yy} & 0 & \frac{2E}{l} I_{yy} & 0 \\
0 & \frac{6E}{l^2} I_{zz} & 0 & 0 & 0 & \frac{4E}{l} I_{zz} & 0 & -\frac{6E}{l^2} I_{zz} & 0 & 0 & 0 & \frac{2E}{l} I_{zz} \\
-\frac{AE}{l} & 0 & 0 & 0 & 0 & 0 & \frac{AE}{l} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{12E}{l^3} I_{zz} & 0 & 0 & 0 & -\frac{6E}{l^2} I_{zz} & 0 & \frac{12E}{l^3} I_{zz} & 0 & 0 & 0 & -\frac{6E}{l^2} I_{zz} \\
0 & 0 & -\frac{12E}{l^3} I_{yy} & 0 & \frac{6E}{l^2} I_{yy} & 0 & 0 & 0 & \frac{12E}{l^3} I_{yy} & 0 & \frac{6E}{l^2} I_{yy} & 0 \\
0 & 0 & 0 & -\frac{GK}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GK}{l} & 0 & 0 \\
0 & 0 & -\frac{6E}{l^2} I_{yy} & 0 & \frac{2E}{l} I_{yy} & 0 & 0 & 0 & \frac{6E}{l^2} I_{yy} & 0 & \frac{4E}{l} I_{yy} & 0 \\
0 & \frac{6E}{l^2} I_{zz} & 0 & 0 & 0 & \frac{2E}{l} I_{zz} & 0 & -\frac{6E}{l^2} I_{zz} & 0 & 0 & 0 & \frac{4E}{l} I_{zz}
\end{bmatrix}$$