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HW 9

Problem 1a

Write standard LP form for following problem

(a) Find $[x_1, x_2]^T$ that

Maximizes $f = 20x_1 - 6x_2$

Subject to $3x_1 - x_2 \geq 3$

$-4x_1 + 3x_2 = -8$

$x_1, x_2 \geq 0$

In standard LP problem cost function must be minimized. To accomplish this for the above problem need to multiply by -1 to get:
$f = -20x_1 + 6x_2$

For the inequality of type $\geq$ need to add a surplus variable to get following:
$g_1 = 3x_1 - x_2 - s_1 = 3$

For the equality constraint no slack or surplus variable is needed. However, value on right hand side needs to be positive. To do this need to multiply by -1 to get:
$h_1 = 4x_1 - 3x_2 = 8$

In above equation there are 3 variables to solve for $x_1,\ x_2,\ s_1$, which will be denoted as $x_1,\ x_2,\ x_3$
Next the above problem can be put into matrix form.

$x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$

$c = \begin{bmatrix} -20 & 6 & 0 \end{bmatrix}^T$

$b = \begin{bmatrix} 3 & 8 \end{bmatrix}^T$

$A = \begin{bmatrix} 3 & -1 & -1 \\ 4 & -3 & 0 \end{bmatrix}$

Problem 1b

Write standard LP form for following problem

(b) Find $[x_1, x_2, x_3]^T$ that

Minimizes $f = 8x_1 - 3x_2 + 15x_3$

Subject to
$$5x_1 - 1.8x_2 - 3.6x_3 \geq 2$$
$$3x_1 + 6x_2 + 8.2x_3 \geq 5$$
$$1.5x_1 + -4x_2 + 7.5x_3 \geq -4.5$$
$$-x_2 + 5x_3 \geq 1.5$$
$$x_1, x_2 \geq 0$$

Since x_3 is unrestricted it needs to be defined as following:

$$x_3^+ - x_3^-$$

Next apply the definition for x_3 and define the linear programing problem:

Start with the cost function:

$$8x_1 - 3x_2 + 15x_3^+ - 15x_3^-$$

Next define g_1

$$-s_1 + 5x_1 - 1.8x_2 - 3.6x_3^+ + 3.6x_3^- - 2$$

Next define g_2

$$-s_2 + 3x_1 + 6x_2 + 8.2x_3^+ - 8.2x_3^- - 5$$

Next define g_3

$$s_3 - 1.5x_1 + 4x_2 - 7.5x_3^+ + 7.5x_3^- - 4.5$$

Next define g_4

$$-s_4 - x_2 + 5x_3^+ - 5x_3^- - 1.5$$

Now put problem into matrix form:

$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \end{bmatrix}^T$

$c = \begin{bmatrix} 8 & -3 & 15 & -15 & 0 & 0 & 0 & 0 \end{bmatrix}^T$

$b = \begin{bmatrix} 2 & 5 & 4.5 & 1.5 \end{bmatrix}^T$

$$A = \begin{bmatrix} 5 & -1.8 & -3.6 & 3.6 & -1 & 0 & 0 & 0 \\ 3 & 6 & 8.2 & -8.2 & 0 & -1 & 0 & 0 \\ -1.5 & 4 & -7.5 & 7.5 & 0 & 0 & 1 & 0 \\ 0 & -1 & 5 & -5 & 0 & 0 & 0 & -1 \end{bmatrix}$$

Problem 2

Find all basic solutions for following problem

The standard LP form for the maximum profit problem is state below. Find **all 10 basic solutions**; write a Matlab routine to do the Gauss-Jordan elimination. List all of the basic solutions indicating the basic variables, non-basic variables, feasibility and the objective function values. Find that the optimum solution is $\boldsymbol{x}^* = \left[4,12,0,0,\frac{3}{14}\right]^T$ with $f = -8800$.

Find $\boldsymbol{x} \in \mathbb{R}^5$ that

Minimizes $f = \boldsymbol{c}^T\boldsymbol{x}$

Subject to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$

where $\boldsymbol{c} = [-400,-600,0,0,0]^T$, $\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ \frac{1}{28} & \frac{1}{14} & 0 & 1 & 0 \\ \frac{1}{14} & \frac{1}{24} & 0 & 0 & 1 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} 16 \\ 1 \\ 1 \end{bmatrix}$

Once the problem is put into row echelon form the 10 basic solutions can be calculated by setting following sets of variables to zero **(non-basic)** and solving for others.

$x_1, x_2 = 0$
$x_1, x_3 = 0$
$x_1, x_4 = 0$
$x_1, x_5 = 0$
$x_2, x_3 = 0$
$x_2, x_4 = 0$
$x_2, x_5 = 0$
$x_3, x_4 = 0$
$x_3, x_5 = 0$
$x_4, x_5 = 0$

Solutions for $x_1, x_2 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 16.0 \\ 1.0 \\ 1.0 \end{bmatrix}$$

$f = 0$, solution is feasible

Solutions for $x_1, x_3 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 16.0 \\ -0.142857142857143 \\ 0.333333333333333 \end{bmatrix}$$

$f = \; -9600$, solution not feasible since x_4 is negative

Solutions for $x_1, x_4 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_2 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 14.0 \\ 2.0 \\ 0.416666666666667 \end{bmatrix}$$

$f = \; -8400$, solution is feasible

Solutions for $x_1, x_5 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 24.0 \\ -8.0 \\ -0.714285714285714 \end{bmatrix}$$

$f = \; -14400$, solution not feasible since x_3 and x_4 are negative

Solutions for $x_2, x_3 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 16.0 \\ 0.428571428571429 \\ -0.142857142857143 \end{bmatrix}$$

$f = \; -6400$, solution not feasible since x_5 is negative

Solutions for $x_2, x_4 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix} = \begin{bmatrix} 28.0 \\ -12.0 \\ -1.0 \end{bmatrix}$$

$f = \; - 11200$, solution not feasible since x_3 and x_5 are negative

Solutions for $x_2, x_5 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 14.0 \\ 2.0 \\ 0.5 \end{bmatrix}$$

$f = \; - 5600$, solution is feasible

Solutions for $x_3, x_4 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4.0 \\ 12.0 \\ 0.214285714285714 \end{bmatrix}$$

$f = \; - 8800$, solution is feasible

Solutions for $x_3, x_5 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 11.2 \\ 4.8 \\ 0.257142857142857 \end{bmatrix}$$

$f = \; - 7360$, solution is feasible

Solutions for $x_4, x_5 = 0$:

Solution for basic variables become:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8.23529411764706 \\ 9.88235294117647 \\ -2.11764705882353 \end{bmatrix}$$

$f = \ -9223.5$, solution not feasible since x_3 is negative

Optimum solution occurs where x_3 and x_4 are 0. This implies that inequalities g_1 and g_2 are active at the optimum solution, which yields a cost function value of -8800.00

Problem 2 was solved by using Python programming language to put the problem into row echelon form and solving for the 10 cases. Code can be seen below

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```
-#import necessary modules
import numpy as np
from scipy.linalg import lu
import sympy as sp
sp.init_printing(use_latex='mathjax')
from IPython.display import display

-#Define matrix that will be put into reduced row echelon form
a = np.array([[1.0,1.0,1.0,0.0,0.0,16],[1/28,1/14,0.0,1.0,0.0,1.0],[1/14,1/24,0.0,0.0,1.0,1.0]])

-#Perform the row reduction procedure
pl, u = lu(a, permute_l=True)

-#Put results into matrix to be used for further analysis
U = sp.Matrix(u)
display(U)

-#Extract b matrix
b = sp.Matrix(U[:,5])
display(b)

-#Define rows of A matrix and combine
A1 = sp.Matrix(U[0,:-1])

A2 = sp.Matrix([U[1,:-1]])

A3 = sp.Matrix([U[2,:-1]])

A = sp.Matrix([A1, A2, A3])
display(A)

-#Depending on which variables are set to zero the below operation will set corresponding columns
to 0
A.col_del(3)
A.col_del(3)
display(A)

-#Solve system for the basic variabels
x = sp.mpmath.lu_solve(A, b)
Sol = sp.Matrix(x)
print('Solutions')
display(Sol)

-#Function definitions
f=-400Sol[0] - 600Sol[1]

print('Cost function value')
display(f)
```