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EGR 7040 *Design Optimization*
Wright State University, Autumn 2015

Homework # 2 – Due electronically

Solve the following problems (perform calculations), in your own words and to the best of your ability.

Math Preliminaries

Let

$$\boldsymbol{u} = [2,-1,4]^T, \qquad \boldsymbol{v} = [-2,0,1,1]^T, \qquad \boldsymbol{w} = [4,1,-1,3]^T$$

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ -1 & 2 & 4 & 2 \\ 0 & -1 & 4 & -2 \\ 2 & 3 & 1 & 1 \end{bmatrix}$$

1. Vectors and matrices

Calculate the following expressions if they are defined. If they are not defined, state what the issue is.

a) $\|\boldsymbol{u}\|_2 \boldsymbol{u}$

b) $\boldsymbol{A} \cdot \boldsymbol{u}$

c) $\boldsymbol{B} \cdot \boldsymbol{u}$

d) $\boldsymbol{v} \cdot \boldsymbol{u}$

e) $(\boldsymbol{v} \cdot \boldsymbol{w}) \cdot \boldsymbol{v}$

f) $\boldsymbol{A}^2$

g) $\sin(\boldsymbol{u} + \boldsymbol{v})$

h) $\|\boldsymbol{u}\|_2 + \|\boldsymbol{w}\|_2$

2. Norms

Compute $\|\ \|_1, \|\ \|_2, \|\ \|_\infty$ norms of $\boldsymbol{u}$.

References: *http://mathworld.wolfram.com/L1-Norm.html*
http://mathworld.wolfram.com/L2-Norm.html
http://mathworld.wolfram.com/L-Infinity-Norm.html

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3. Vectors Calculus

Let

$$f(\vec{x}) = 2x_1^2 + 3x_2^3 - 2x_3, \qquad g(\vec{x}) = \sin(2x_1) + \exp(-x_2) + x_3^3$$

Calculate the following expressions:

a) $\nabla f, \nabla g$ Gradient

b) $\nabla^2 f, \nabla^2 g$ Laplacian

References: *https://en.wikipedia.org/wiki/Gradient*

https://en.wikipedia.org/wiki/Laplacian

Format:

- Typed (nothing handwritten), short paragraph each questions
- Concise, clear, and complete; in your own words (don't copy from the text)
- Find other source material to develop your understanding of the concepts and ideas (online or other books); include as "References"
- Submit electronically via WSU's "WINGS" on-line class web-site (MS Word or PDF or other readable file format)
- Due date: Friday 18 September 2015 by midnight

It is OK to check your answers using MATLAB or Octave, however do not turn in printouts from MATLAB. Show intermediate steps and be concise, clear, and complete. TA will not grade homework that he cannot read or understand. If you are not comfortable with the calculations, be sure to review the listed references and ask questions in class or instructor office hours.

SOLUTIONS:

1a) Find $\|\boldsymbol{u}\|_2\boldsymbol{u}$:

$$\|\boldsymbol{u}\|_2 = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}$$

Now to get $\|\boldsymbol{u}\|_2\boldsymbol{u}$ :: $\sqrt{21} * [2, -1, 4]^T = \left[2\sqrt{21},\ -\sqrt{21},\ 4\sqrt{21}\right]^T$

1b) Find $\boldsymbol{A} \cdot \boldsymbol{u}$:

Since the inner dimension match this multiplication procedure was possible.

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix} * \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2*2-1*1-1*4 \\ 2*1-4*1+3*4 \\ -2*1-3*1+4*4 \end{bmatrix} = \begin{bmatrix} -1 \\ 10 \\ 11 \end{bmatrix}$$

1c) Find $\boldsymbol{B} \cdot \boldsymbol{u}$:

$$\boldsymbol{B} = \begin{bmatrix} 1 & -1 & 0 & 3 \\ -1 & 2 & 4 & 2 \\ 0 & -1 & 4 & -2 \\ 2 & 3 & 1 & 1 \end{bmatrix}_{\boldsymbol{4x4}} \quad \boldsymbol{u} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}_{\boldsymbol{3x1}}$$

Since the inner dimensions of the two matrices do not match (3≠4) this multiplication is not possible.

1d) Find $\boldsymbol{v} \cdot \boldsymbol{u}$:

$$\boldsymbol{v} = [-2,0,1,1]^T{}_{\boldsymbol{1x4}} \quad \boldsymbol{u} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}_{\boldsymbol{3x1}}$$

Since the inner dimensions of the two vectors do not match (3≠4) this multiplication is not possible.

1e) Find $(\boldsymbol{v} \cdot \boldsymbol{w}) \cdot \boldsymbol{v}$:

Dot product between two vectors will produce a scalar, therefore:

$$(\boldsymbol{v} \cdot \boldsymbol{w}) = [-2,0,1,1]^T * [4,1,-1,3]^T = [-8+0-1+3] = -6$$

Now $-6 * [-2,0,1,1]^T = [12, 0, -6, -6]^T$

1f) Find $\boldsymbol{A}^2$:

$$\boldsymbol{A}^2 = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix} * \begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & 2+4-3 & -2+3-4 \\ 2+4-3 & 1+16+9 & -1+12+12 \\ -2+3-4 & -1+12+12 & 1+9+16 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 3 & -3 \\ 3 & 26 & 23 \\ -3 & 23 & 26 \end{bmatrix}$$

Since $\boldsymbol{A}$ is a square matrix this operation is possible. This matrix is also symmetric.

1g) Find $\sin(\boldsymbol{u} + \boldsymbol{v})$:

$$\boldsymbol{u} = [2, -1, 4]^T; \quad \boldsymbol{v} = [-2, 0, 1, 1]^T{}_{1x4}$$

Since the dimensions of the two vectors do not match the addition operation is not possible. Also addition of two vectors produces another vector, which has no meaning when applied to the sin function.

1h) Find $\|\boldsymbol{u}\|_2 + \|\boldsymbol{w}\|_2$:

$$= \sqrt{2^2 + -1^2 + 4^2} + \sqrt{4^2 + 1^2 + -1^2 + 3^2} = \sqrt{21} + \sqrt{27} = 9.78$$

Even though the two vectors are not the same the dimension, their norms are scalar values, which can be added.

2) Find $\| \quad \|_1, \| \quad \|_2, \| \quad \|_\infty \; of \; \boldsymbol{u}$:

$$\|\boldsymbol{u}\|_1 = (2 + 1 + 4) = 7$$

$$\|\boldsymbol{u}\|_2 = \sqrt{2^2 + -1^2 + 4^2} = \sqrt{21}$$

$$\|\boldsymbol{u}\|_{inf} = max|u_i| = 4$$

3) Find $\nabla \; and \; \nabla^2$ of the following functions:

$$f(\vec{x}) = 2x_1^2 + 3x_2^3 - 2x_3, \qquad g(\vec{x}) = \sin(2x_1) + \exp(-x_2) + x_3^3$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{dx_1} \\ \frac{\partial f}{dx_2} \\ \frac{\partial f}{d3} \end{bmatrix} = \begin{bmatrix} 4x_1 \\ 9x_2^2 \\ -2 \end{bmatrix}; \quad \nabla g = \begin{bmatrix} \frac{\partial g}{dx_1} \\ \frac{\partial g}{dx_2} \\ \frac{\partial g}{d3} \end{bmatrix} = \begin{bmatrix} 2\cos(2x_1) \\ -e^{-x_2} \\ 3x_3^2 \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{dx_1^2} \\ \frac{\partial^2 f}{dx_2^2} \\ \frac{\partial^2 f}{dx_3^2} \end{bmatrix} = \begin{bmatrix} 4 \\ 18x_2 \\ 0 \end{bmatrix}; \ \nabla^2 g = \begin{bmatrix} \frac{\partial^2 g}{dx_1^2} \\ \frac{\partial^2 g}{dx_2^2} \\ \frac{\partial^2 g}{dx_3^2} \end{bmatrix} = \begin{bmatrix} -4\sin(2x_2) \\ e^{-x_2} \\ 6x_3 \end{bmatrix}$$

Taking either the gradient or the Laplacian of the scalar function will result in a vector.