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EGR 7040 Optimization HW#5

Problem 3.17

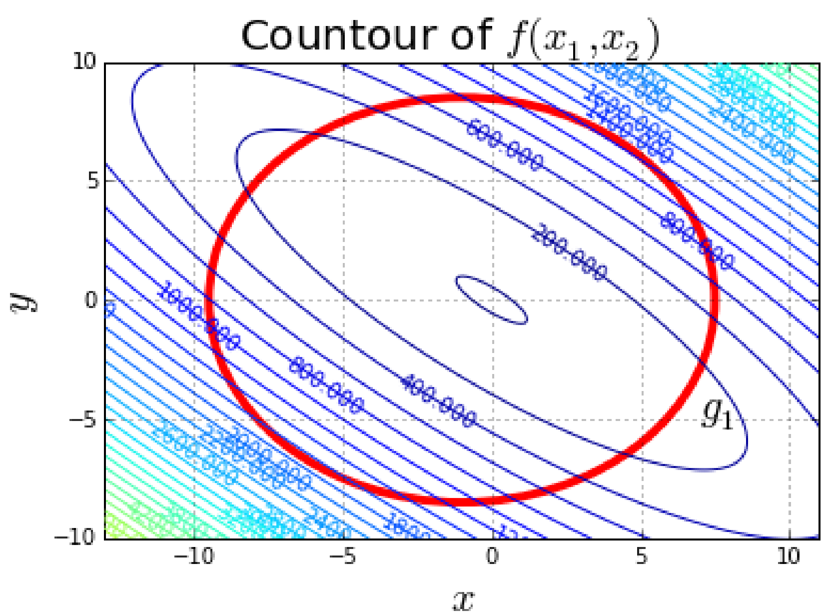
Determine max and min of the below objective function.

$$3.17 \quad f(x, y) = 9x^2 + 13y^2 + 18xy - 4$$
$$\text{subject to } x^2 + y^2 + 2x \geq 16$$

The inequality constraint $x^2 + y^2 + 2x \geq 16$ is the general form of the equation of the circle. It is beneficial to put this equation into center radius form, which is done by completing the square. The revised inequality constraint takes the following form.

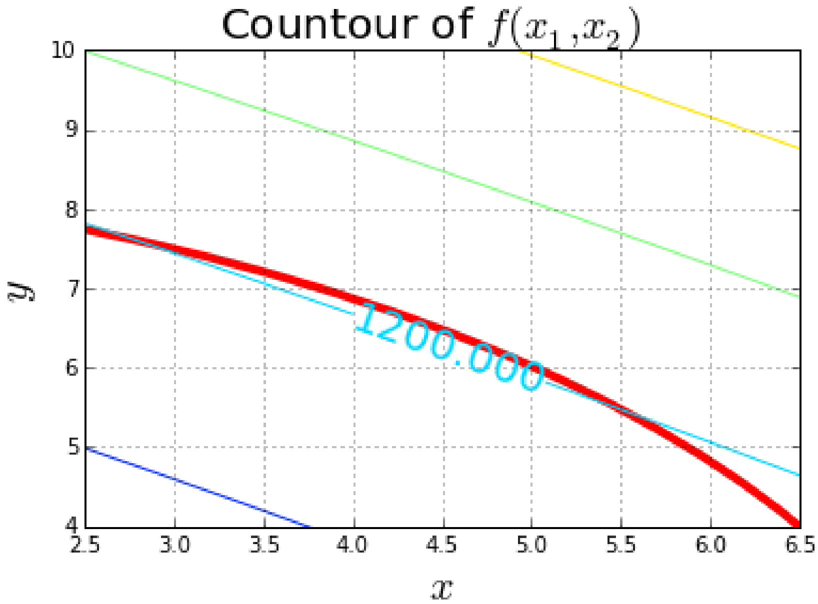
$$(x+1)^2 + y^2 \geq 17$$

This form is used to graph the constraint.



From the above plot it appears that the minimum function value at about 1400.00 while the max function value is infinite. Zooming in onto the min point actually shows that the min value is approximately 1200.00.

Constraint g_1 is active at the minimum point.


Countour of $f(x_1, x_2)$
1200.000
10
9
8
7
6
5
4
2.5
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3.5
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4.5
5.0
5.5
6.0
6.5
x
y

Problem 3.21

3.21 Solve the rectangular beam problem of Exercise 2.17 graphically for the following data: $M = 80$ kN·m, $V = 150$ kN, $\sigma_a = 8$ MPa, and $\tau_a = 3$ MPa.

2.17 A beam of rectangular cross section (Fig. E2-17) is subjected to a maximum bending moment of M and a maximum shear of V. The allowable bending and shearing stresses are σ_a and τ_a, respectively. The bending stress in the beam is calculated as

$$\sigma = \frac{6M}{bd^2}$$

and average shear stress in the beam is calculated as

$$\tau = \frac{3V}{2bd}$$

where d is the depth and b is the width of the beam. It is also desired that the depth of the beam shall not exceed twice its width. Formulate the design problem for minimum cross-sectional area using the following data: $M = 140$ kN·m, $V = 24$ kN, $\sigma_a = 165$ MPa, $\tau_a = 50$ MPa.

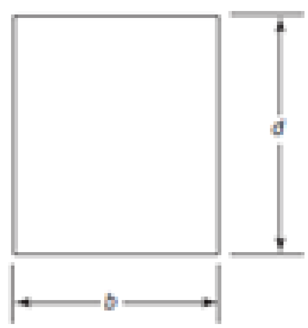


FIGURE E2-17 Cross section of a rectangular beam.

Above problem has following standard form:

Find:

- $\vec{x} = [b = width, d = depth]$

To minimize cross-sec area:

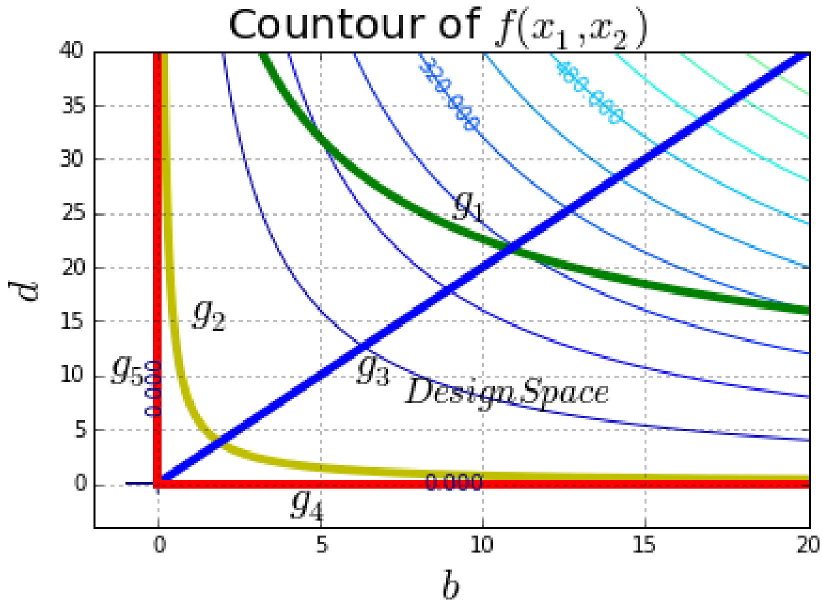
- $A = b * d$

Subject to inequality constraints:

- Bending stress constraint: $\frac{6M}{bd^2} \leq \sigma_a$
- Shear stress constraint: $\frac{3V}{2bd} \leq \tau_a$
- Aspect ratio constraint: $d - 2b \leq 0$

Side constraints:

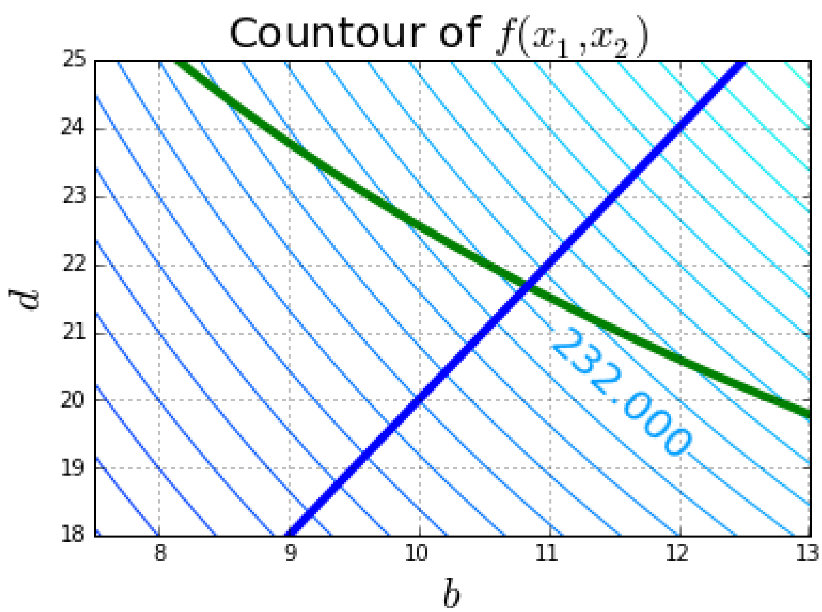
- $b, d \geq 0$



From the above plot it appears that the minimum function value at about 250. Zooming in onto the min point actually shows that the min value is approximately 232 cm^2.

Optimum depth d $\approx$ 21.7
Optimum width b $\approx$ 10.8

Constraints g_1 and g_3 are active at the minimum point.



Problem 3.27

3.27 Formulate and solve the problem of Exercise 2.1 graphically.

2.1 A 100×100 m lot is available to construct a multistory office building. At least $20{,}000\,\text{m}^2$ total floor space is needed. According to a zoning ordinance, the maximum height of the building can be only 21 m, and the area for parking outside the building must be at least 25 percent of the total floor area. It has been decided to fix the height of each story at 3.5 m. The cost of the building in millions of dollars is estimated at $0.6h + 0.001A$, where A is the cross-sectional area of the building per floor and h is the height of the building. Formulate the minimum cost design problem.

Above problem has following standard form:

Find:

- $\vec{x} = [h = height, A = area]$

To minimize:

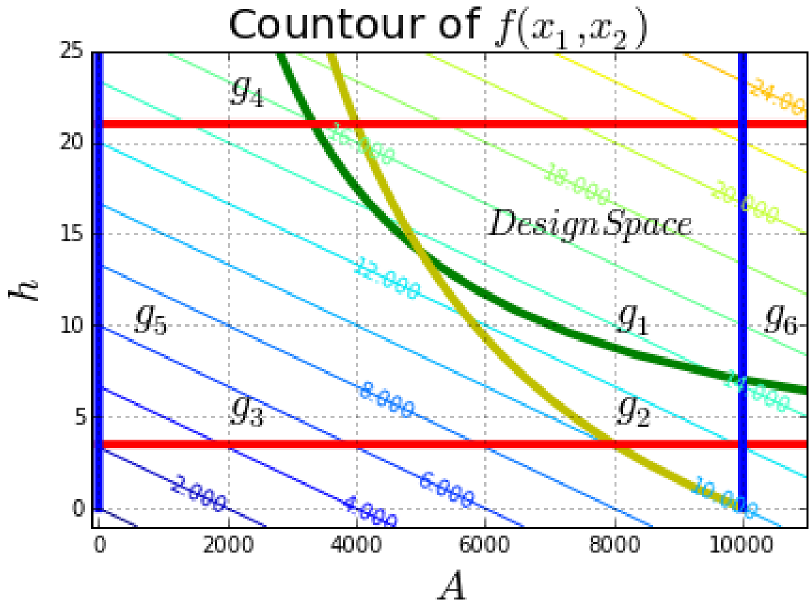
- $Cost = 0.6h + 0.001A$

Subject to inequality constraints:

- g_1 Floor space: $\frac{hA}{3.5} \geq 20,000$
- g_2 Shear stress constraint: $(10,000 - A) \geq 0.25\frac{hA}{3.5}$

Side constraints:

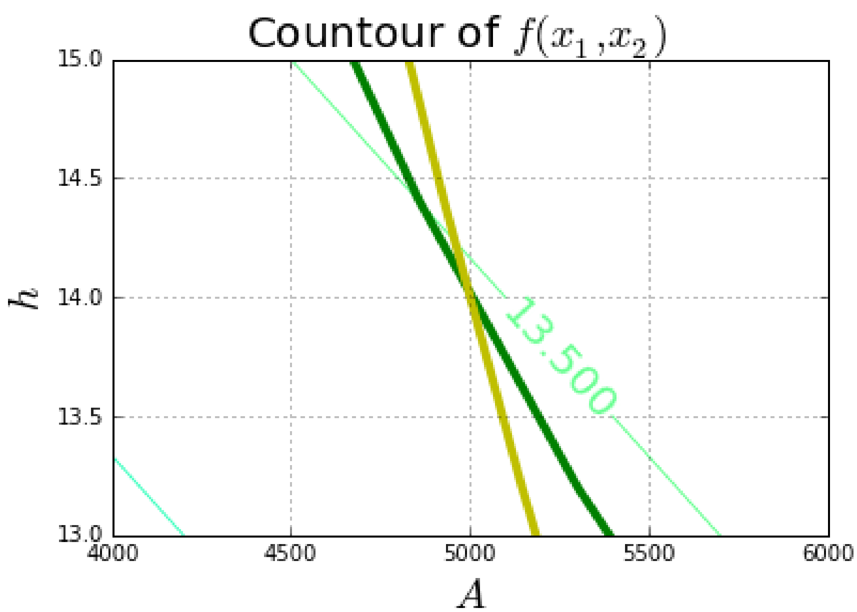
- $3.5 \leq h \leq 21$, m
- $0 \leq A \leq 10,000$, m^2



From the above plot it appears that the min cost is about 14 million. Zooming in onto the min point actually shows that the min cost is approximately 13.5 million.

Optimum building height h $\approx$ 14 m^2
Optimum floor area $\approx$ 5000 m^2

Constraints g_1 and g_2 are active at the minimum point.



Problem 3.31

3.31 Graphically solve the insulated spherical tank design problem formulated in Section 2.3 for the following data: $r = 3.0$ m, $c_1 = \$10{,}000$, $c_2 = \$1000$, $c_3 = \$1$, $c_4 = \$0.1$, $\Delta T = 5$.

EXAMPLE: Formulation with Design Variables Only

Summary of the problem formulation for the design optimization of insulation for a spherical tank formulation in terms of the design variable only is as follows:

Specified data: $r, \Delta T, c_1, c_2, c_3, c_4, t_{\min}$

Design variable: t, m

Cost function: Minimize the life-cycle cooling cost of refrigeration of the spherical tank

$$Cost = at + \frac{b}{t}$$

$$a = 4c_2\pi r^2, \quad b = \frac{(c_3 + 6.14457c_4)}{c_1}(365)(24)(\Delta T)(4\pi r^2)$$

Constraint:

$$t \geq t_{\min}$$

Above problem has following standard form:

Find:

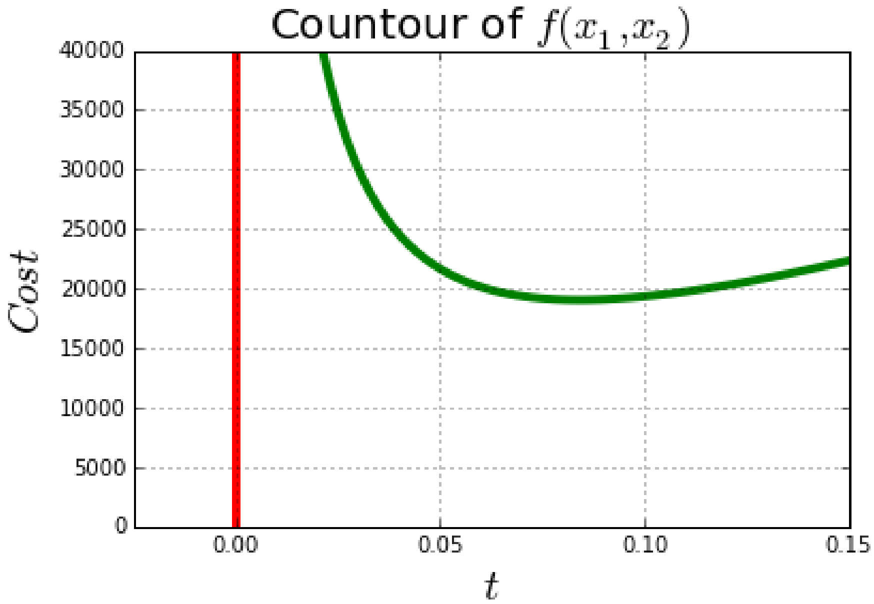
- $\vec{x} = [t = thickness]$

To minimize cost:

- $Cost = at + \frac{b}{t}$
  - where: $a = 4c_2\pi r^2$ and $b = \frac{c_3+6.14457c_4}{c_1}$

Side constraints:

- $t \geq t_{min}$



From the above plot it appears that the min cost is about 20,000.

Optimum thickness t $\approx$ 0.080 m

No contraints are active for this point.

Problem 3.37

3.37 Formulate the problem of Exercise 2.3 and solve it using the graphical method.

2.3 Design a beer mug, shown in Fig. E2-3, to hold as much beer as possible. The height and radius of the mug should be not more than 20 cm. The mug must be at least 5 cm in radius. The surface area of the sides must not be greater than 900 cm^2 (ignore the area of the bottom of the mug and ignore the mug handle—see figure). Formulate the optimum design problem.

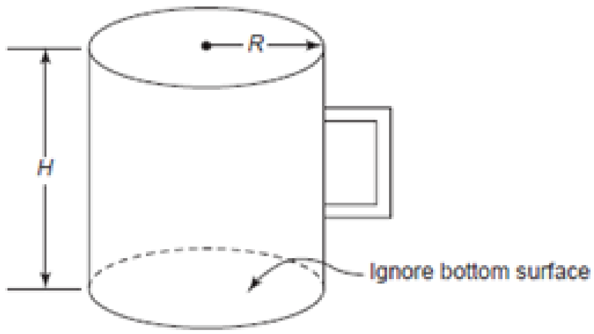


FIGURE E2-3 Beer mug.

Above problem has following standard form:

Find:

- $\vec{x} = [h = height, r = radius]$

To maximize:

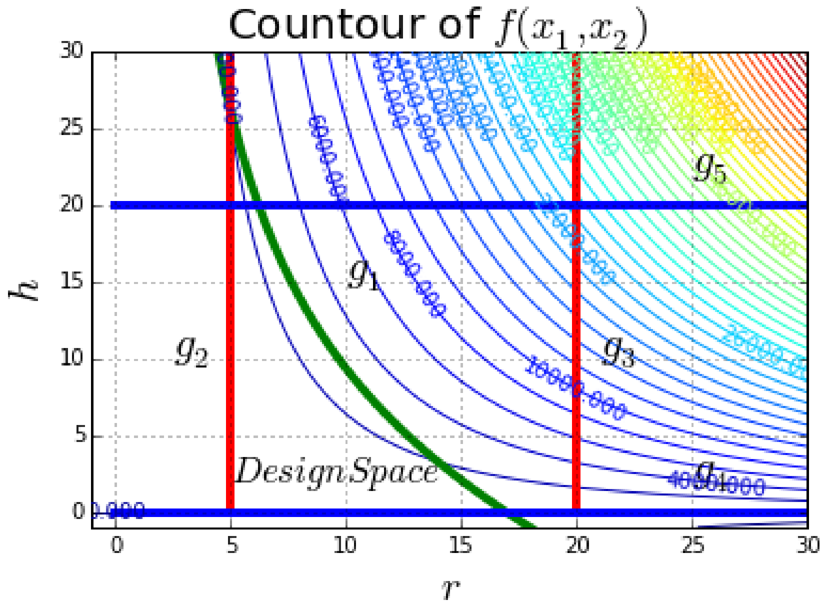
- $Volume = \pi r^2 h$

Subject to inequality constraints:

- g_1 Surface area: $2\pi rh + \pi r^2 - 900 \leq 0$

Side constraints:

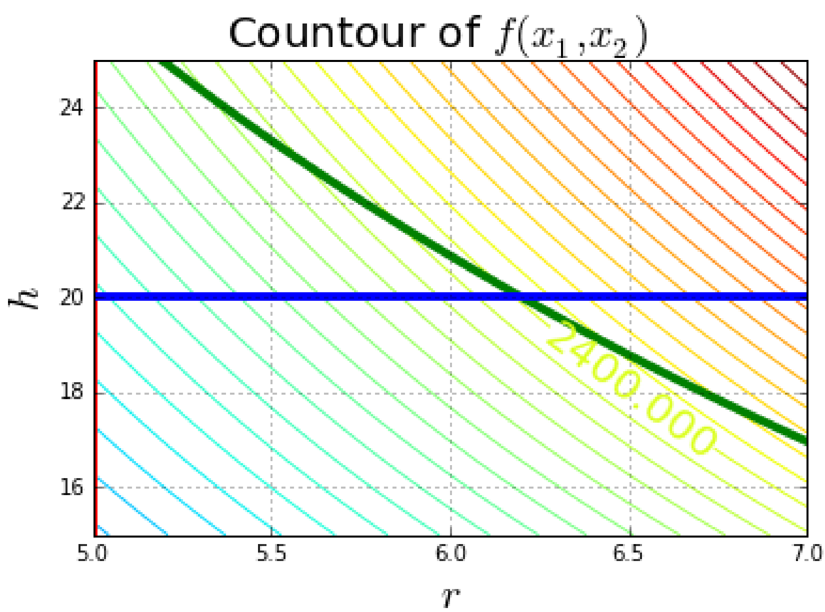
- $5 \leq r \leq 20$, cm
- $0 \leq h \leq 20$, cm



From the above plot it appears that the max volume is about 2000 cm^3. Zooming in onto the max point actually shows that the max volume is approximately 2400 cm^3.

Optimum height h $\approx$ 20 cm
Optimum radius r $\approx$ 6.2 cm

Constraints g_1 and g_5 are active at the minimum point.



Problem 3.38

3.38 Formulate the problem of Exercise 2.4 and solve it using the graphical method.

2.4 A company is redesigning its parallel flow heat exchanger of length l to increase its heat transfer. An end view of the unit is shown in Fig. E2-4. There are certain limitations on the design problem. The smallest available conducting tube has a radius of 0.5 cm and all tubes must be of the same size. Further, the total cross-sectional area of all the tubes cannot exceed 2000 cm^2 to ensure adequate space inside the outer shell. Formulate the problem to determine the number of tubes and the radius of each tube to maximize the surface area of the tubes in the exchanger.

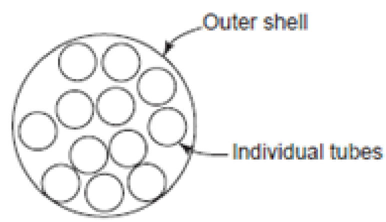


FIGURE E2-4 Cross section of heat exchanger.

Above problem has following standard form:

Find:

- $\vec{x} = [n = number\ of\ tubes, r = radius]$

To maximize:

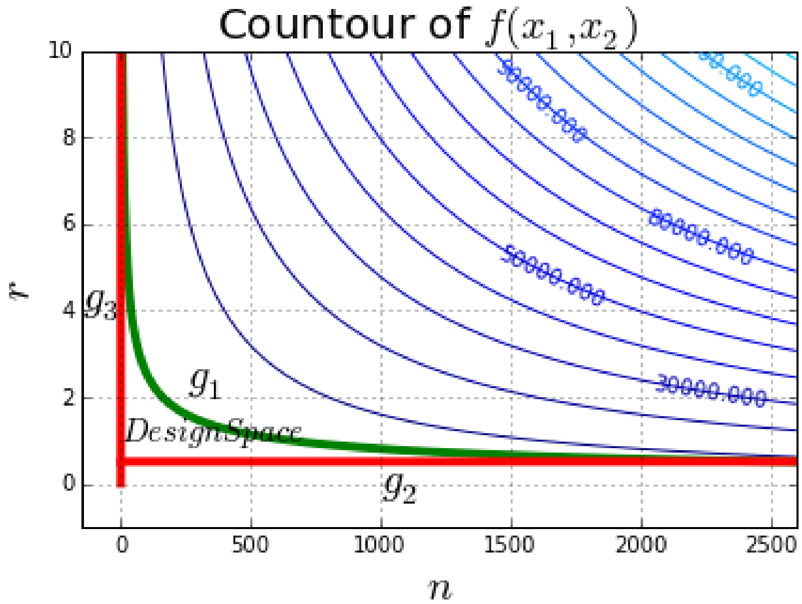
- $Area = 2\pi r l n$

Subject to inequality constraints:

- g_1 Min radius: $r - 0.5 \geq 0$
- g_2 Max X-sec Area: $\pi r^2 n - 2000 \leq 0$

Side constraints:

- $n \geq 0$, cm

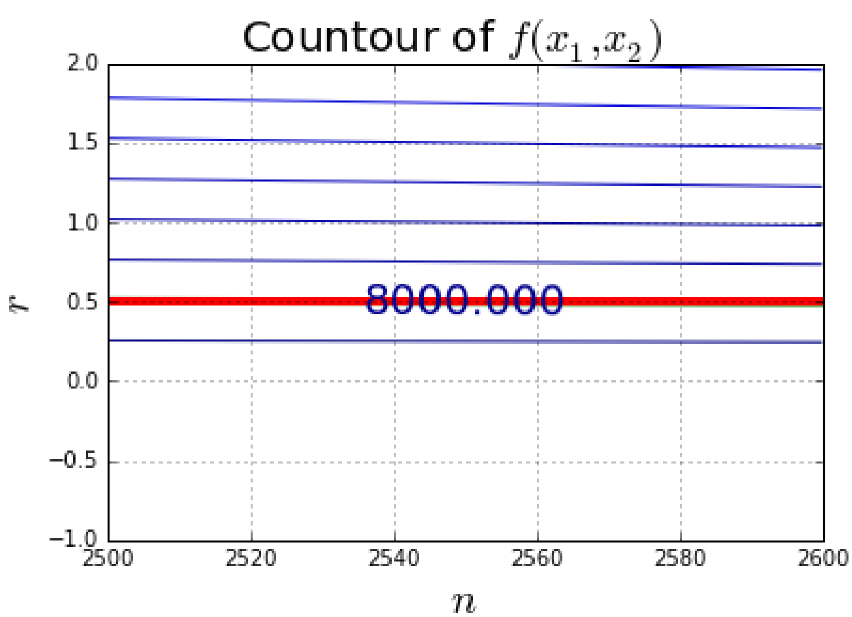


From the graph above it appears that the max surface area will occur at the smallest possible radius of tube. Based on constraint g_1 the max amount of tubes at $r = 0.5$ is 2546. Graph below zooms into this area to determine what surface area will be at $r = 0.5$ and $n = 2546$.

From the graph below we get the following optimum values:

- Surface Area = 8000 cm^2
- Radius = 0.5 cm
- Number of tubes = 2546

Constraints g_1 and g_2 are active



Problem 3.45

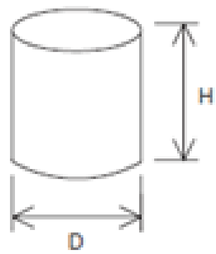
3.45 Solve the can design problem formulated in Section 2.2 using the graphical approac

Step 1: Project/Problem Statement The purpose of this project is to design a can to hol *at least* 400 ml of liquid, as well as to meet other design requirements (1 ml = 1 cm^3). Th cans will be produced in the billions so it is desirable to minimize manufacturing costs. Sinc cost can be directlv related to the surface area of the sheet metal. it is reasonable t minimize the amount of sheet metal required to fabricate the can. Fabrication, handling, aes thetics, and shipping considerations impose the following restrictions on the size of the can the diameter should be no more than 8 cm and no less than 3.5 cm, whereas the height shoul be no more than 18 cm and no less than 8 cm.

Step 2: Data and Information Collection Given in the project statement.

Step 3: Identification/Definition of Design Variables The two design variables are defined as

D = diameter of the can, cm
H = height of the can, cm



Step 4: Identification of a Criterion to Be Optimized The design objective is to minimize the total surface area S of the sheet metal for the three parts of the cylindrical can: the surface area of the cylinder (circumference × height) and the surface area of the two ends. Therefore, the optimization criterion or *cost function* (the total area of sheet metal), is written as

$$S = \pi DH + \frac{\pi}{2}D^2, cm^2 \tag{a}$$

Step 5: Identification of Constraints The first constraint is that the can must hold *at least* 400 cm^3 of fluid, which is written as

$$\frac{\pi}{4}D^2 H \geq 400, cm^3 \tag{b}$$

If it had been stated that the "can must hold 400 ml of fluid," then the preceding volume constraint would be an equality. The other constraints on the size of the can are:

$$\begin{aligned} &3.5 \leq D \leq 8, cm \\ &8 \leq H \leq 18, cm \end{aligned} \tag{c}$$

Above problem has following standard form:

Find:

- $\vec{x} = [d = diameter, h = height]$

To minimize:

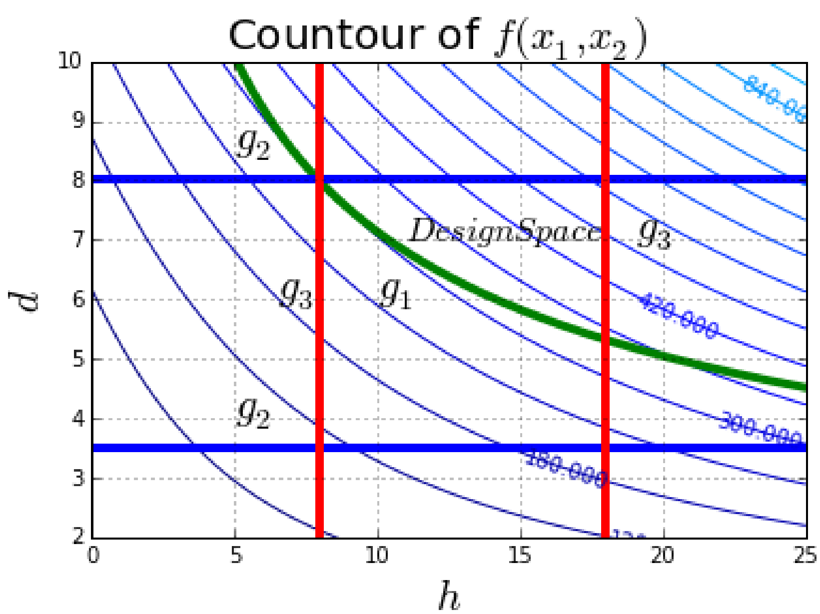
- $Area = \pi dh + 2(\frac{\pi}{4}d^2)$

Subject to inequality constraints:

- g_1 required volume: $\frac{\pi}{4}d^2h - 400 \geq 0$

Side constraints:

- $3.5 \leq d \leq 8$, cm
- $8 \leq h \leq 18$, cm

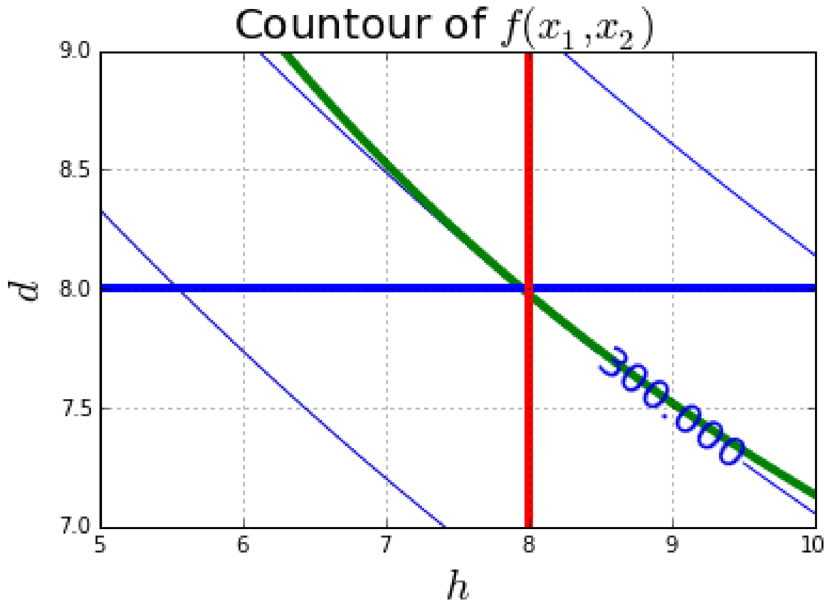


From the graph above it appears that the min surface area is approximatelly 300 cm^2. Graph below zooms in at the subject area to confirm the result.

From the graph below we get the following optimum values:

- Surface Area = 300 cm^2
- Diameter = 8.0 cm
- Height = 8.0 cm

Constraints g_1, g_2, and g_3 are active


Countour of $f(x_1, x_2)$
9.0
8.5
8.0
7.5
7.0
d
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