HW#8

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Problem 7.1

3.34*

Design a hollow torsion rod shown in Fig.E3.34 to satisfy the following requirements (created by J.M. Trummel):

1. The calculated shear stress, τ , shall not exceed the allowable shear stress τ_a under the normal operation torque T_o (N·m).
2. The calculated angle of twist, θ, shall not exceed the allowable twist, θ_a (radians).
3. The member shall not buckle under a short duration torque of T_{max} (N·m).

Requirements for the rod and material properties are given in Table E3.34(A) and E3.34(B) (select a material for one rod). Use the following design variables:
x_1 = outside diameter of the shaft; x_2 = ratio of inside/outside diameter, d_i/d_o

Using graphical optimization, determine the inside and outside diameters for a minimum mass rod to meet the above design requirements. Compare the hollow rod with an equivalent solid rod ($d_i/d_o = 0$). Use consistent set of units (e.g. Newtons and millimeters) and let the minimum and maximum values for design variables be given as

$$0.02 \le d_o \le 0.5\ \text{m}, \quad 0.60 \le \frac{d_i}{d_o} \le 0.999$$

STANDARD FORM

$$f = \left(7.85 \times 10^{-6}\right)(\pi/4)(500)\,x_1^2\left(1 - x_2^2\right) = \left(3.08269 \times 10^{-3}\right)x_1^2\left(1 - x_2^2\right)$$

$$g_1 = \left(16 \times 10^7/\pi\right)\Big/x_1^3\left(1 - x_2^4\right) - 275 = 5.093 \times 10^7\Big/x_1^3\left(1 - x_2^4\right) - 275 \le 0$$

$$g_2 = \frac{32\left(10^7\right)(500)}{\left(8.0 \times 10^4\right)\pi}\Big/x_1^4\left(1 - x_2^4\right) - \pi/90 = 6.36619 \times 10^5\Big/x_1^4\left(1 - x_2^4\right) - 3.49066 \times 10^{-2} \le 0$$

$$g_3 = \frac{-\left(2.1 \times 10^5\right)}{12\sqrt{2}\left(1 - 0.3^2\right)0.75}\left(x_1^3\left(1 - x_2\right)2.5\right) + \left(2.0 \times 10^7\right)$$

$$= 2.0 \times 10^7 - \left(4.17246 \times 10^4\right)\left(x_1^3\right)\left(1 - x_2\right)^{2.5} \le 0$$

$$g_4 = 20 - x_1 \le 0; \quad g_5 = x_1 - 500 \le 0; \quad g_6 = 0.6 - x_2 \le 0; \quad g_7 = x_2 - 0.999 \le 0$$

First point of the discussion will be the results using numerical optimization algorithm called "fmincon", which is a function found in matlab optimization toolbox.

Function "fmincon" is used for constrained optimization problems and results for problem 3.34 can be found below.

First, function outputs for cost function and optimum design variable values is given. This is followed by the convergnece plot for the numerical solver.

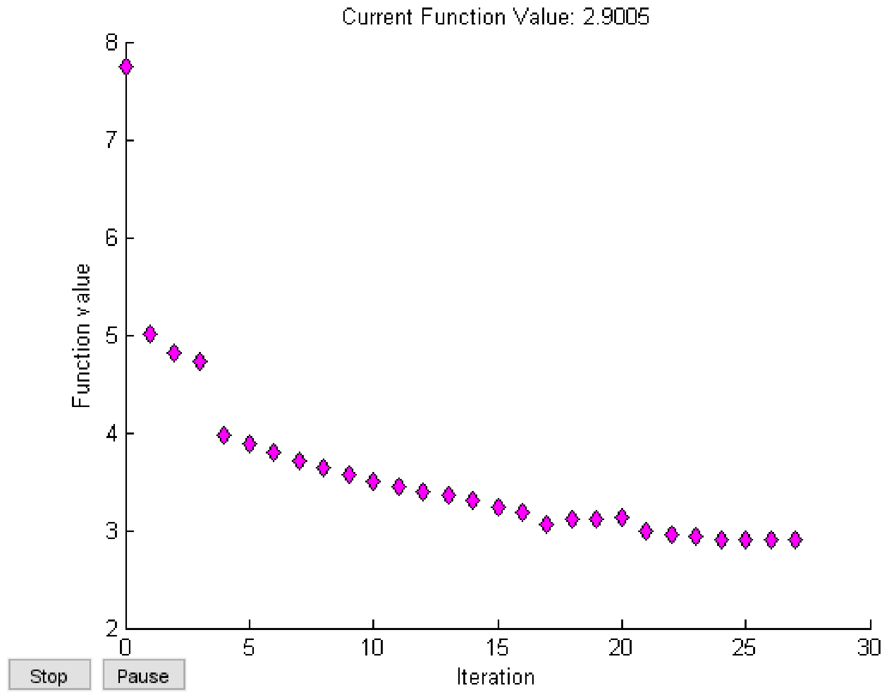
```
>> Problem3_34

Optimization completed: The relative first-order optimality measure, 6.407935e-09,
is less than options.TolFun = 1.000000e-06, and the relative maximum constraint
violation, 0.000000e+00, is less than options.TolCon = 1.000000e-08.

Optimization Metric                                          Options
relative first-order optimality =    6.41e-09       TolFun =    1e-06 (default)
relative max(constraint violation) =    0.00e+00    TolCon =    1e-08 (selected)


x =

  102.9857
    0.9546
```

Optimum design variables

```
FunVal =

    2.9005
```

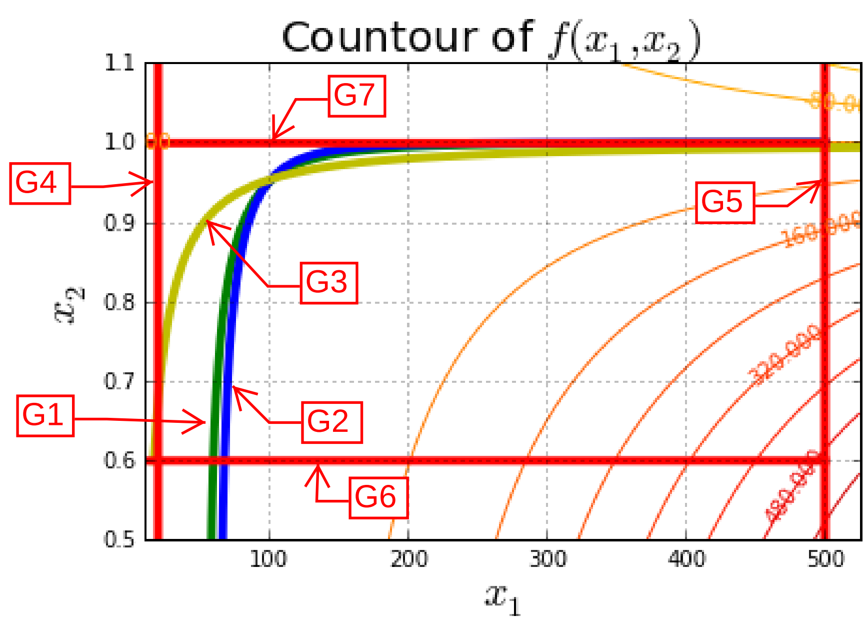
Optimum function value

```
ExitFlag =

     1


Output =

         iterations: 29
          funcCount: 146
    constrviolation: 0
           stepsize: 7.7740e-08
          algorithm: 'interior-point'
      firstorderopt: 4.0000e-07
       cgiterations: 32
            message: 'Local minimum found that satisfies the constraints.

Optimization completed because the ob...'
```

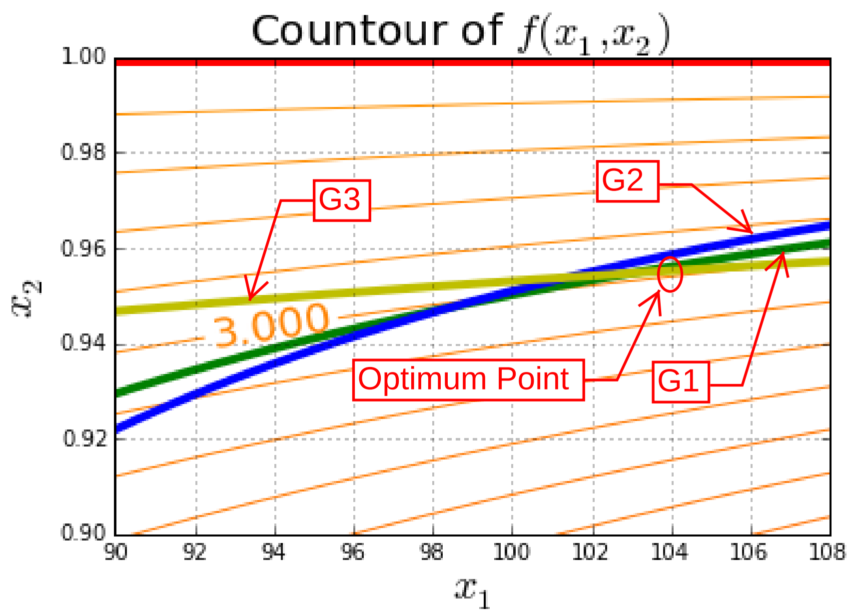


Next, numerical solution is confirmed using the graphical method, which agrees quite well.

Graphical representation of the solution



Zoom in view



Looking at the plot above, optimum solution is around 3.0, which agrees with the numerical solution of 2.900. From the plot it can be seen that constraints g1 and g3 are active.

Problem 7.5

3.51*

Design of a water tower support column. As a member of the ABC consulting Engineers you have been asked to design a cantilever cylindrical support column of minimum mass for a new water tank. The tank itself has already been designed in the tear-drop shape shown in Fig. E3.51. The height of the base of the tank (H), the diameter of the tank (D), and wind pressure on the tank (w) are given as $H = 30$ m, $D = 10$ m, and $w = 700$ N/m^2. Formulate the design optimization problem and solve it graphically. (created by G.Baenziger).

In addition to designing for combined axial and bending stresses and buckling, several limitations have been placed on the design. The support column must have an inside diameter of at least 0.70 m (d_i) to allow for piping and ladder access to the interior of the tank. To prevent local buckling of the column walls the diameter/thickness ratio (d_0/t) shall not be greater than 92. The large mass of water and steel makes deflections critical as they add to the bending moment. The deflection effects as well as an assumed construction eccentricity (e) of 10 cm must be accounted for in the design process. Deflection at C.G. of the tank should not be greater than Δ .

Limits on the inner radius and wall thickness are $0.35 \le R \le 2.0$ m and $1.0 \le t \le 20$ cm.

STANDARD FORM

$$f = (\gamma_s/g)(2\pi Rt)H = (0.08/9.81)(2\pi Rt)(3000) = 153.71Rt$$

$$f_a/\sigma_a + f_b/\sigma_b - 1 \le 0, \quad \text{or} \quad \frac{(6.075\times10^6)/Rt}{1.50190(R^2+t^2/4)}$$

$$+\frac{\left\{\left[8.6744\times10^7/(R^3t+Rt^3/4)\right]+\left[2.7212\times10^{14}/(R^3t+Rt^3/4)^2\right]\right\}(2R+t)}{1.65\times10^4} - 1 \le 0;$$

$$g_1 = \frac{4.04488\times10^6}{(R^3t+Rt^3/4)} + \frac{5257.21(2R+t)}{(R^3t+Rt^3/4)} + \frac{(1.6492\times10^{10})(2R+t)}{(R^3t+Rt^3/4)^2} - 1 \le 0;$$

$$g_2 = 70 - d_i = 70 - 2R + t \le 0; \quad g_3 = d_o/t - 92 = (2R+t)/t - 92 = 2R/t - 91 \le 0;$$

$$g_4 = \delta - \Delta = 1.4072\times10^8/\left[\pi(R^3t+Rt^3/4)\right] - 20 = 4.479\times10^7/(R^3t+Rt^3/4) - 20 \le 0;$$

$$g_5 = R - 0.5t - 250 \le 0; \quad g_6 = 35 - R + 0.5t \le 0; \quad g_7 = t - 40 \le 0; \quad g_8 = 1 - t \le 0$$

First point of the discussion will be the results using numerical optimization algorithm called "fmincon", which is a function found in matlab optimization toolbox.

Function "fmincon" is used for constrained optimization problems and results for problem 3.51 can be found below.

First, function outputs for cost function and optimum design variable values is given. This is followed by the convergnece plot for the numerical solver.

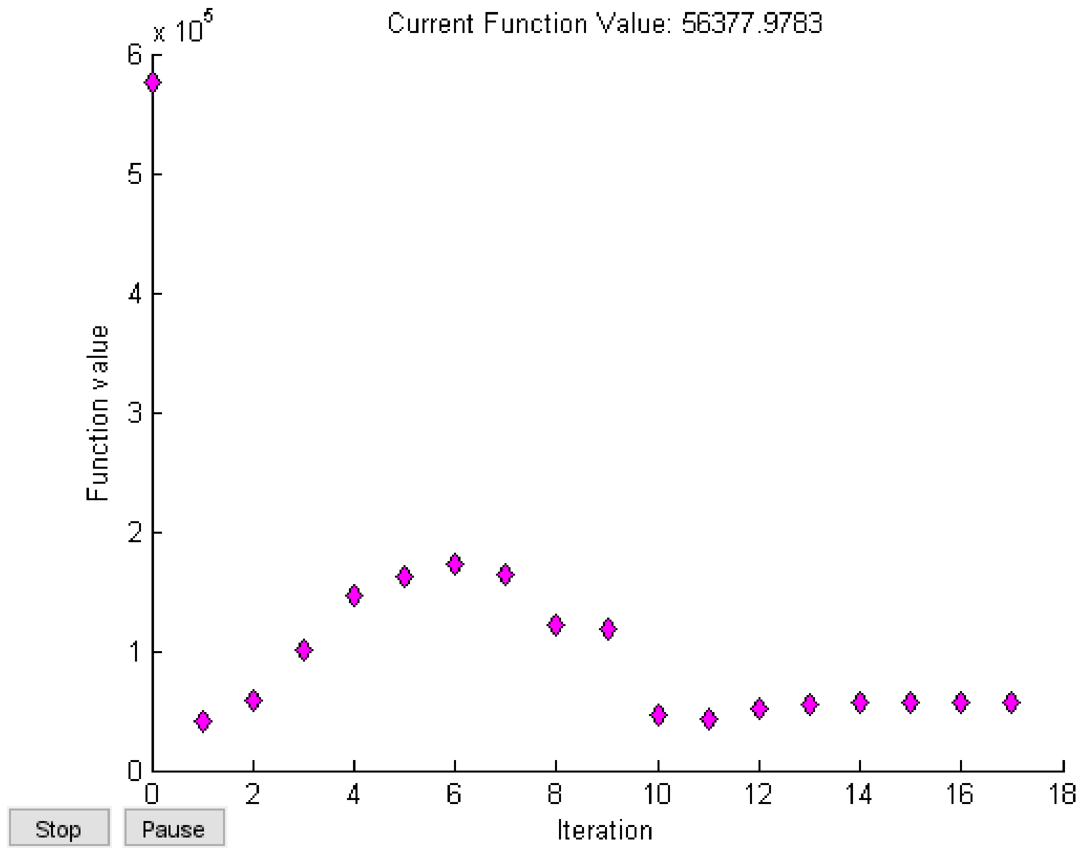
```
Optimization Metric                                            Options
relative first-order optimality =  8.26e-008              TolFun =  1e-006 (default)
max(constraint violation) =  0.00e+000                    TolCon =  1e-008 (selected)


x =

  129.1842
    2.8392
```

Optimum design variable values

```
FunVal =

  5.6378e+004
```

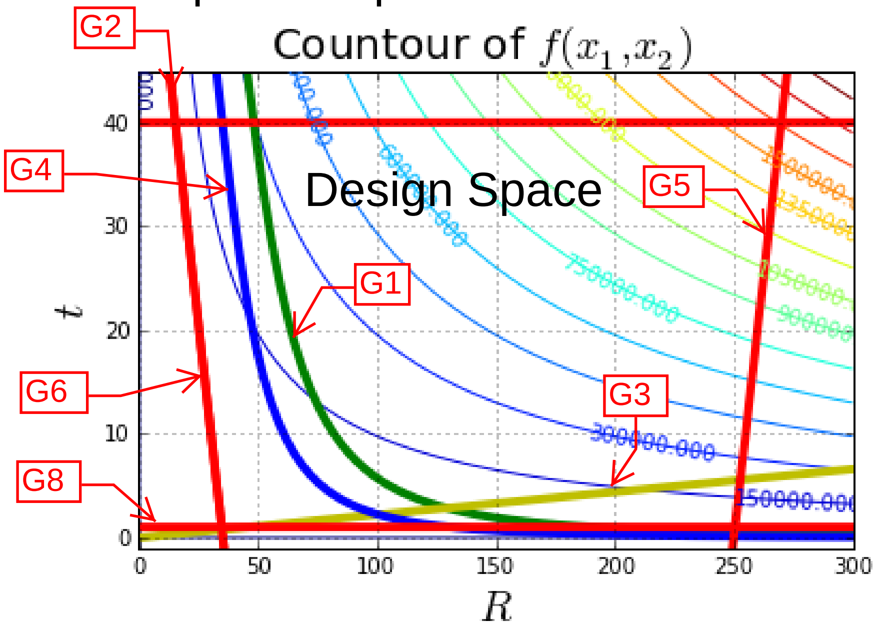
Optimum function value

```
ExitFlag =

     1


Output =

         iterations: 17
          funcCount: 20
    constrviolation: 0
           stepsize: 1.6944e-005
          algorithm: 'interior-point'
      firstorderopt: 0.0016
       cgiterations: 0
            message: [1x793 char]
```

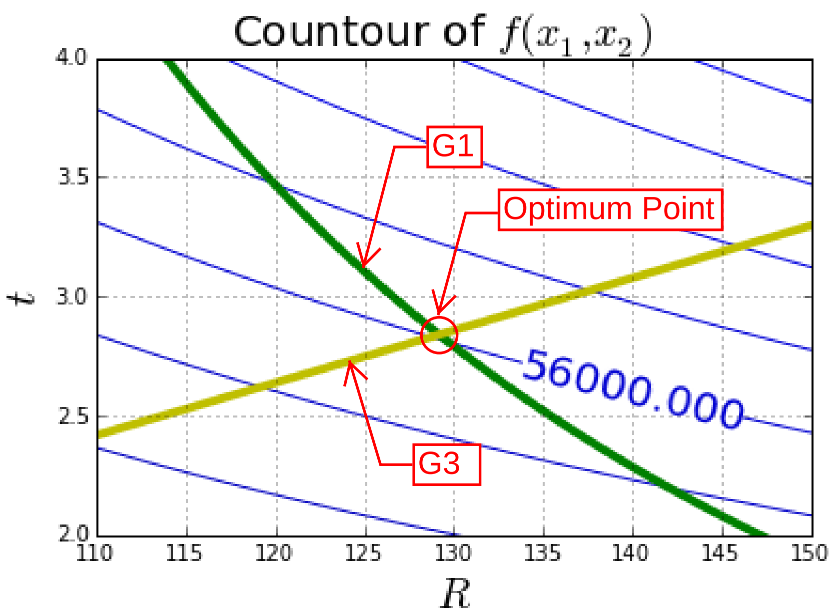


Next, numerical solution is confirmed using the graphical method, which agrees quite well.

Graphical representation of solution



Zoom in view



Looking at the plot above, optimum solution is around 56,000, which agrees with the numerical solution of 57,000. From the plot it can be seen that constraints g1 and g3 are active.

Problem 7.8

3.54*

Design of a tripod. Design a minimum mass tripod of height H to support a vertical load $W = 60$ kN. The tripod bas is an equilateral triangle with sides $B = 1200$ mm. The struts have a solid circular cross section of diameter D (Fig. E3.54).

The axial stress in the struts must not exceed the allowable stress in compression, and axial load in the strut P must not exceed the critical buckling load P_{cr} divided by a safety factor FS = 2. Use consistent units of Newtons and centimeters. The minimum and maximum values for design variables are $0.5 \le H \le 5$m and $0.5 \le D \le 50$ cm. Material properties and other relationship are given below:

STANDARD FORM

$$f = 3(2.8\times10^{-3})(\pi D^2/4)(H^2+120^2/3)^{\frac{1}{2}} = (6.59734\times10^{-3})D^2(H^2+4800)^{\frac{1}{2}}$$

$$g_1 = (2.546475\times10^{4})(H^2+4800)^{\frac{1}{2}}\big/D^2H - 1.5\times10^{4} \le 0;$$

$$g_2 = \frac{(2.0\times10^{4})(H^2+4800)^{\frac{1}{2}}}{H} - (1.816774\times10^{6})D^4/(H^2+4800) \le 0;$$

$$g_3 = H - 500 \le 0;\quad g_4 = 50 - H \le 0;\quad g_5 = D - 50 \le 0;\quad g_6 = 0.5 - D \le 0$$

First point of the discussion will be the results using numerical optimization algorithm called "fmincon", which is a function found in matlab optimization toolbox.

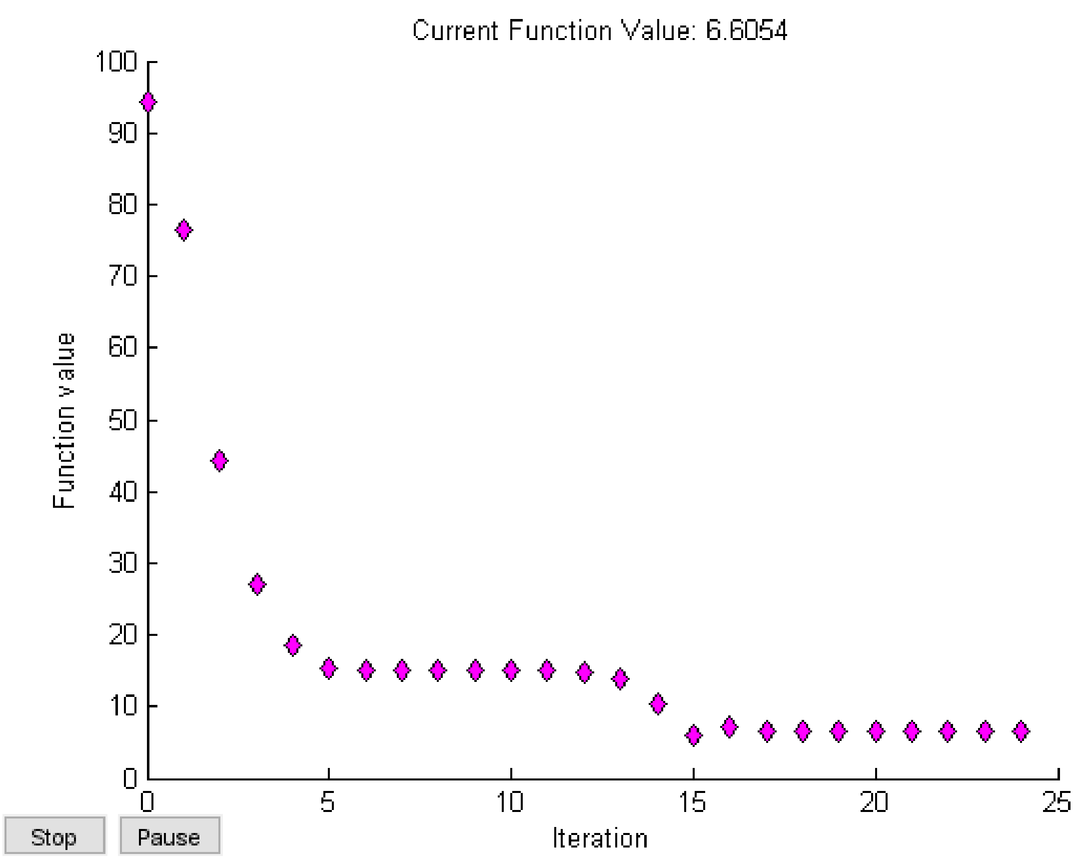
Function "fmincon" is used for constrained optimization problems and results for problem 3.54 can be found below.

First, function outputs for cost function and optimum design variable values is given. This is followed by the convergnece plot for the numerical solver.

```
Optimization Metric                                              Options
relative first-order optimality =  1.04e-007             TolFun =  1e-006 (default)
max(constraint violation) =  0.00e+000                   TolCon =  1e-008 (selected)


x =

   50.0000
    3.4232


FunVal =

    6.6054


ExitFlag =

     1


Output =

         iterations: 24
          funcCount: 25
    constrviolation: 0
           stepsize: 4.6428e-008
          algorithm: 'interior-point'
      firstorderopt: 4.0000e-007
       cgiterations: 0
            message: [1x793 char]
```

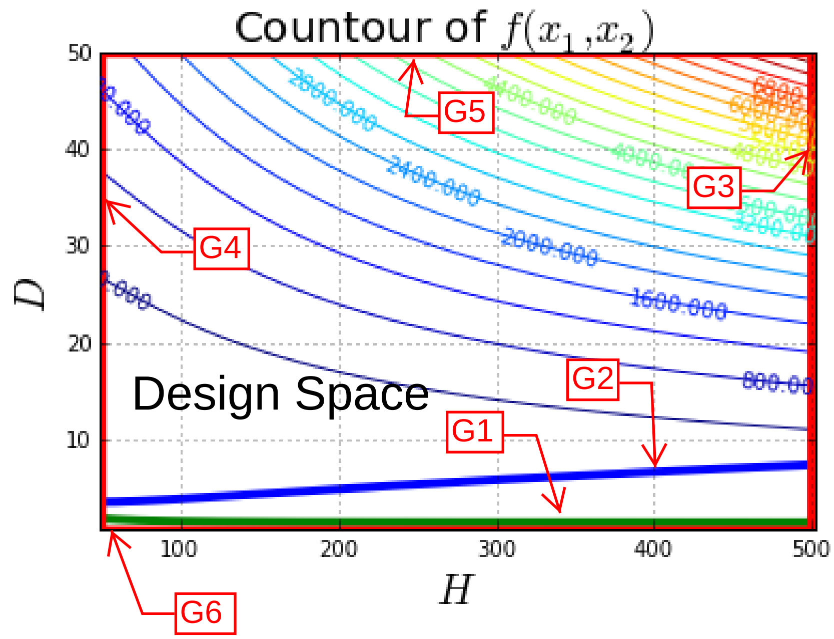
Design variables optimum values

Optimum function value

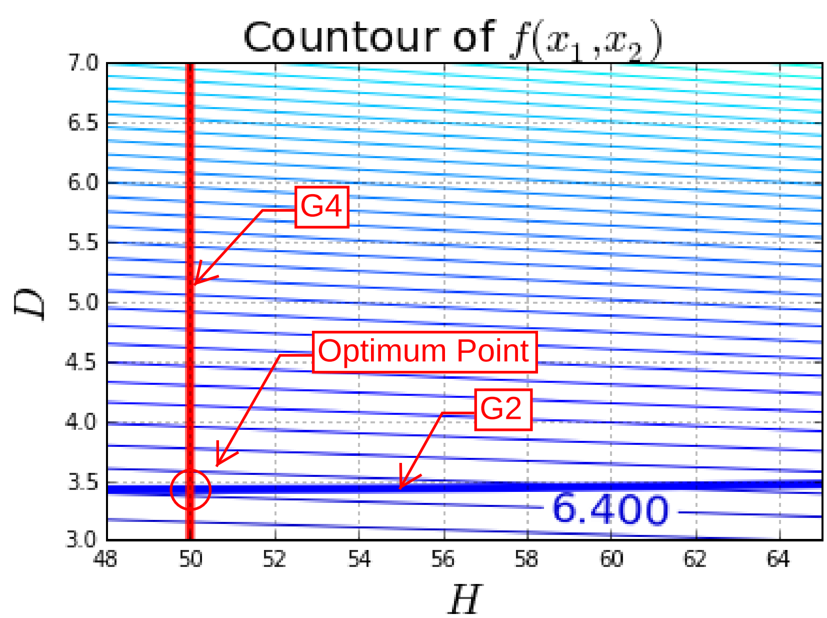


Next, numerical solution is confirmed using the graphical method, which agrees quite well.

Graphical representation of the solution



Zoom in view



Looking at the plot above, optimum solution is around 6.400, which agrees with the numerical solution of 6.600. From the plot it can be seen that constraints g2 and g4 are active.

Additional Problems:

Problem 1

1. Solve the following constrained optimization problem using MATLAB. Use the following as your starting points: (a) [1; 1] and (b) [1; 6]

   Find {x_1, x_2} to minimize $f(\boldsymbol{x}) = x_1^2 - 5x_2$

   Subject to:

   $$x_1 + x_2 \leq 5$$

   $$-\infty \leq x_1 \leq 10$$

   Answer the following questions:

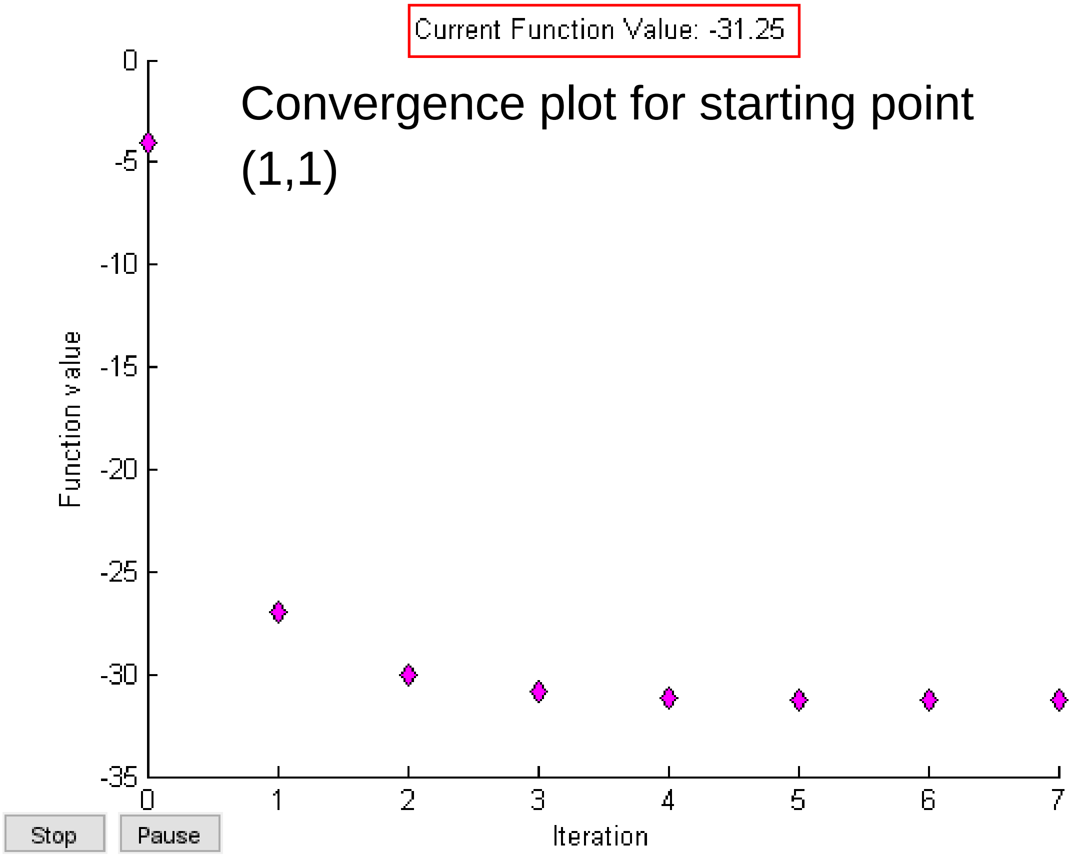
   (a) Solve the above problem using MATLAB. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(\boldsymbol{x})$ for both starting points.

   (b) Does the minimum objective function value change when the starting point is changed?

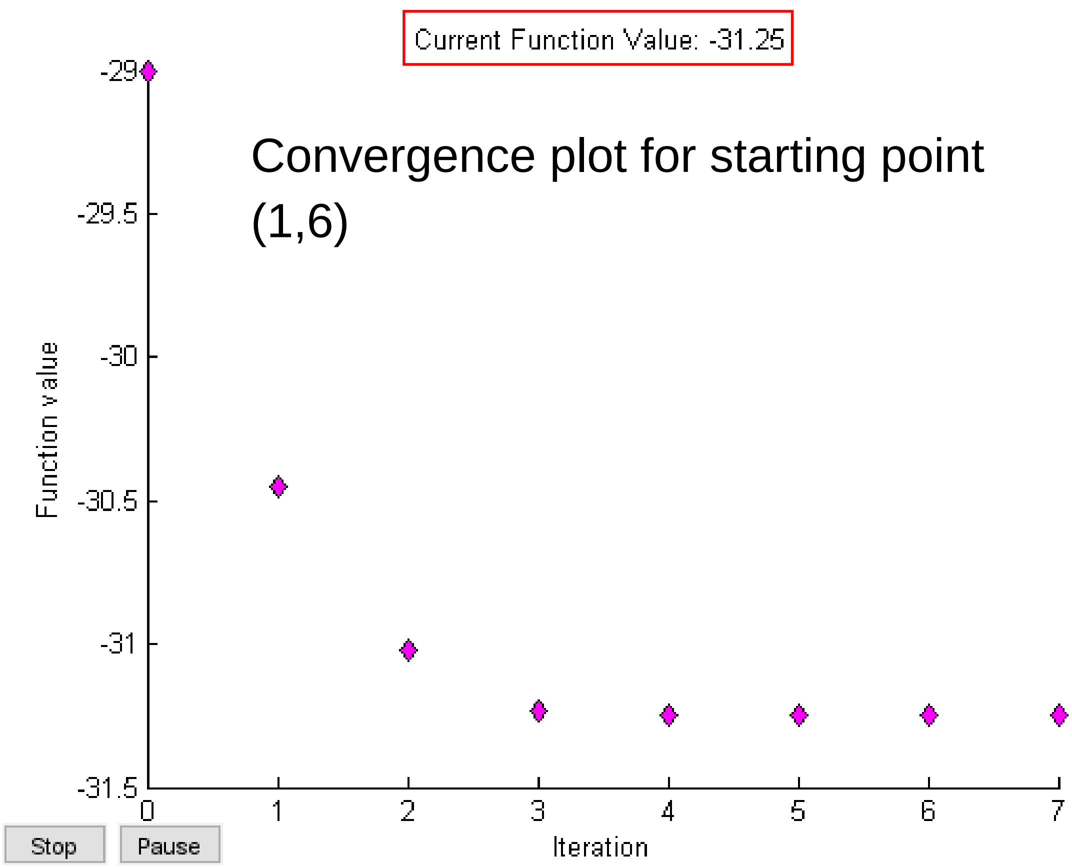
   (c) What is the effect of the starting point on the optimum value of x_2?

   (d) Explain in your own words any interesting features of this problem in view of the above questions.

Plots for starting points (1,1) and (1,6) are plotted below



Convergence plot for starting point (1,1)



Convergence plot for starting point (1,6)

- a) For both starting points stated above the solution converges to the same optimum solution of -31.25.
- b) The optimum solution does not seem to change using different starting points.
- c) There appears to be no effect on x_2. In both cases optimum point for x_2 = 7.5.
- d) The initial starting value of the cost function is changed when using different starting points. In both cases the solution converged in 7 iterations.

Below are the iteration details for the MATLAB solution

Optimum solution for starting point (1,1) and (1,6)

```
x =

   -2.5000
    7.5000


FunVal =

  -31.2500


ExitFlag =

     1


Output =

         iterations: 7
          funcCount: 8
    constrviolation: 0
           stepsize: 3.1415e-005
          algorithm: 'interior-point'
      firstorderopt: 4.0001e-007
       cgiterations: 0
            message: [1x793 char]
```

Problem 2

2. Consider the following problem:

   Find $\{x_1, x_2\}$ to minimize $f(\boldsymbol{x}) = x_1^2 + 10x_2^2 - 3x_1x_2$

   Subject to:

   $$2x_1 + x_2 \geq 4$$
   $$x_1 + x_2 \geq -5$$
   $$-5 \leq x_1, x_2 \leq +5$$

   (a) Solve the above problem using MATLAB. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(\boldsymbol{x})$.

   (b) Solve the above problem by removing the first constraint $2x_1 + x_2 \geq 4$. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(\boldsymbol{x})$.

   (c) Now, solve the same problem by removing all the constraints. Report the optimum value of x_1 and x_2, and the corresponding minimum value of $f(\boldsymbol{x})$.

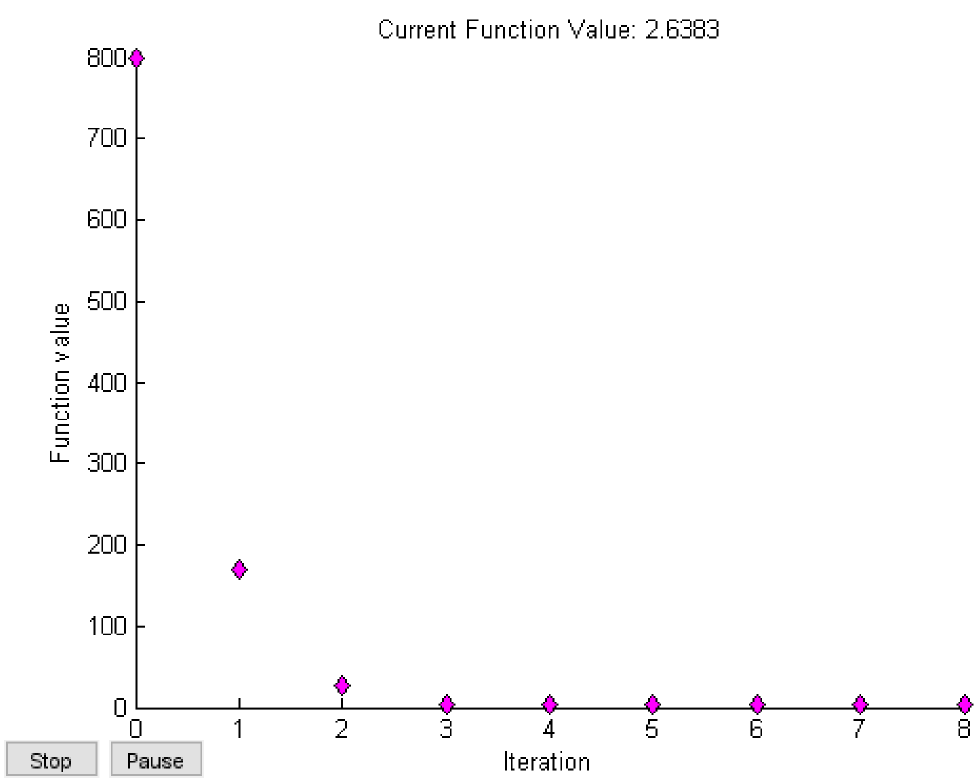
   (d) Create a contour plot showing the contours of the objective function. Show on it the locations of the optima obtained in Parts (a) – (c).

- a) Solve above problems and report optimal values for x_1, x_2, and cost function.

From data below optimum values are as follows:

- x_1 = 1.8298
- x_2 = 0.3404
- Function Value = 2.6383

```
x =

    1.8298
    0.3404


FunVal =

    2.6383


ExitFlag =

     1


Output =

         iterations: 8
          funcCount: 10
    constrviolation: 0
           stepsize: 1.1828e-004
          algorithm: 'interior-point'
      firstorderopt: 2.5908e-006
       cgiterations: 0
            message: [1x793 char]
```

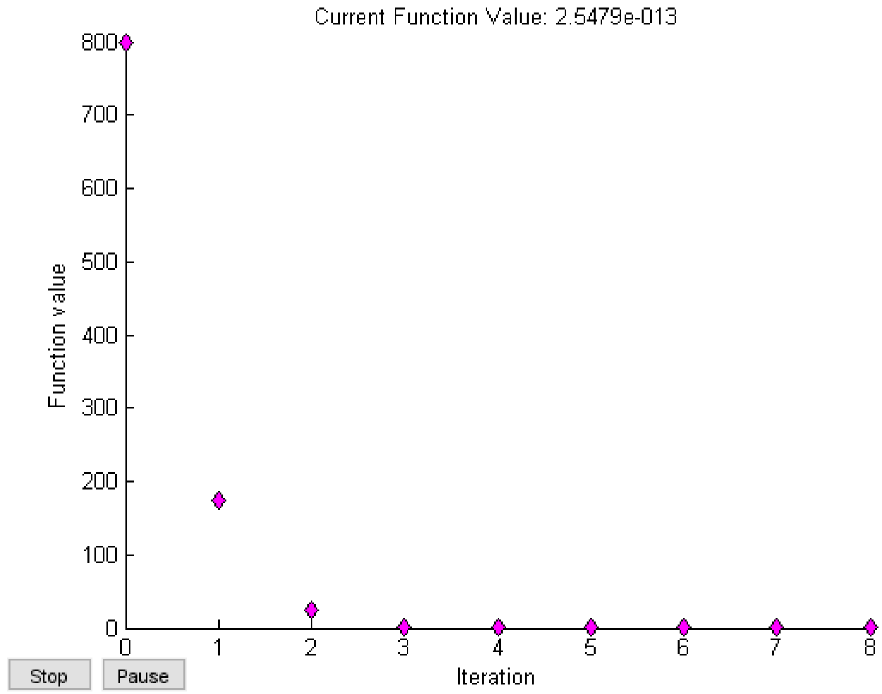


- b) Solve problem by removing g_1 constrain and report new optimum values.

With g_1 constraint removed the optimum values are:

- x_1 = 0
- x_2 = 0
- Function Values = 0

```
x =

  1.0e-006 *

    0.5258
    0.1425


FunVal =

  2.5479e-013


ExitFlag =

     1


Output =

         iterations: 8
          funcCount: 10
    constrviolation: 0
           stepsize: 5.1061e-005
          algorithm: 'interior-point'
      firstorderopt: 6.8082e-007
       cgiterations: 0
            message: [1x793 char]
```

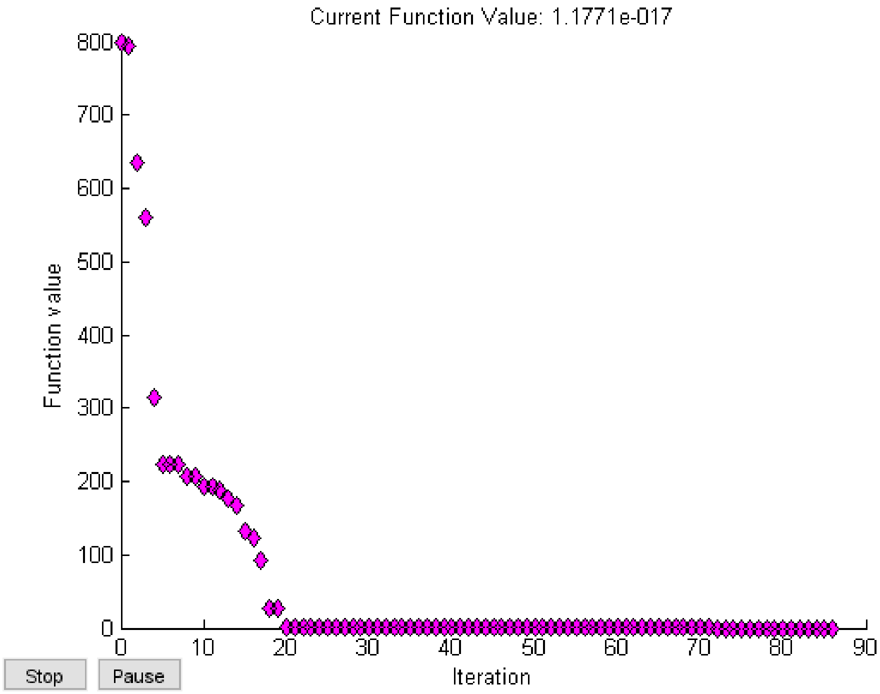


- c) Solve the problem unconstrained.For this case function "fminsearch" was used.

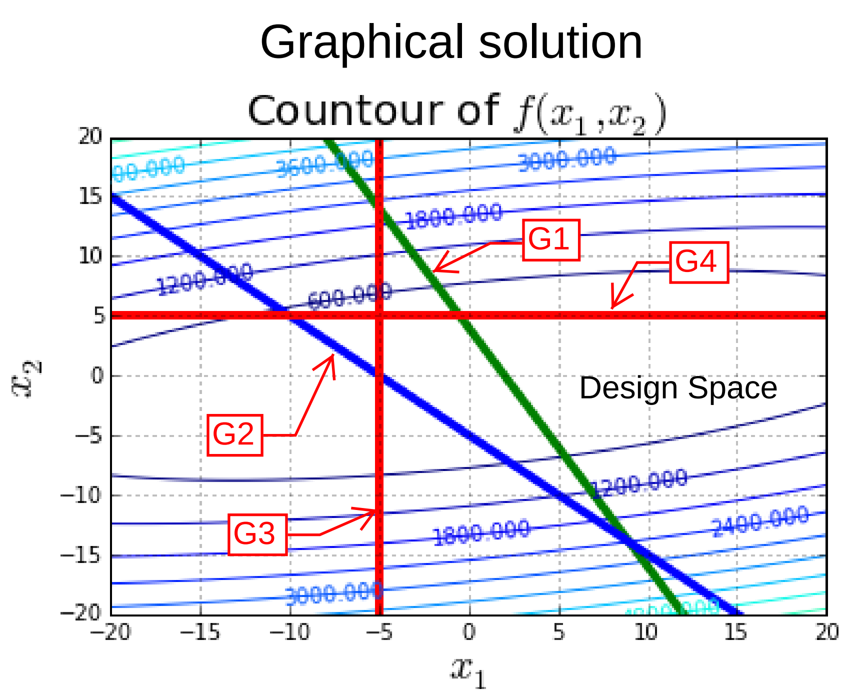
Optimum values are:

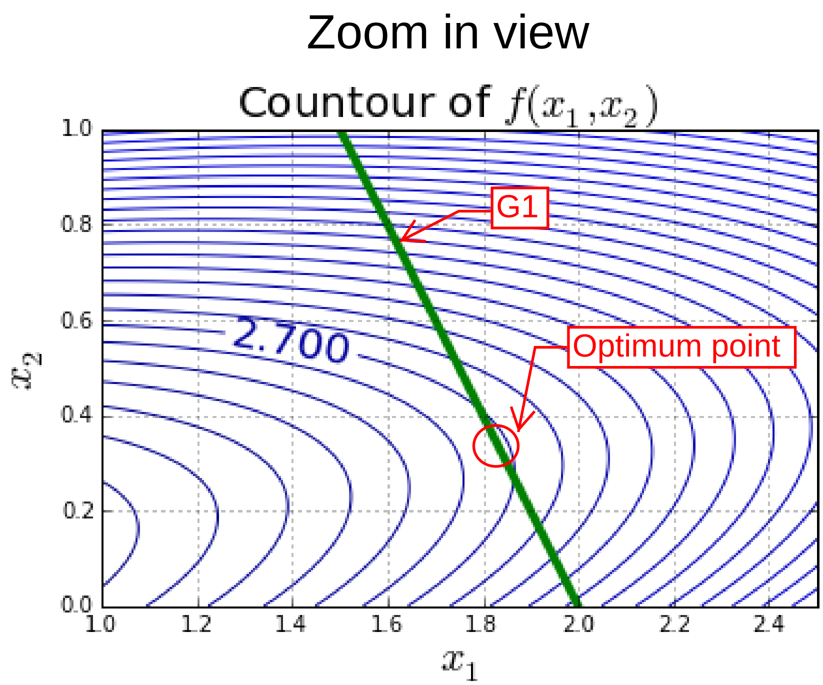
- x_1 = 0
- x_2 = 0
- Function Value = 0

```
x =

  1.0e-008 *

    0.2930
    0.1155


FunVal =

  1.1771e-017


ExitFlag =

     1


Output =

    iterations: 86
     funcCount: 167
     algorithm: 'Nelder-Mead simplex direct search'
       message: [1x196 char]
```



Graphical Solution to problem 2 (part d)





Graphical solution shows that g_1 costraint is active at the optimum point. Cost function optimum point from the graph is very close the numerical soution from part **a** (i.e. 2.638).

From the above graphs it is clear that if g_1 is not active (**part b**) the optimum soution will be 0. This will also be the case for the unconstrained version of this problem (**part c**).

Problem 3

3. The following is a linear programming problem.

Find $\{x_1, x_2\}$ to minimize $f(\boldsymbol{x}) = 20x_1 + 64x_2$

Subject to:

$$25x_1 + 70x_2 \geq 2{,}100$$
$$0 \leq x_1 \leq 70$$
$$0 \leq x_2 \leq 50$$

(a) Solve the above linear optimization problem using the `linprog` command in MATLAB.

(b) Solve the above problem using the `fmincon` command in MATLAB. Compare the results with the results obtained in (a).

- a) Solve above problem using "linprog" function.

MATLAB output can be seen below:

```
x =

   70.0000
    5.0000


FunVal =

  1.7200e+003


ExitFlag =

     1


Output =

         iterations: 2
    constrviolation: 3.5527e-015
          algorithm: 'medium-scale: active-set'
       cgiterations: []
            message: 'Optimization terminated.'
```

Solution using "LINPROG"

- b) Solve problem using "fmincon" and compare results:

MATLAB output for "fmincon" can be seen below:

```
x =

   70.0000
    5.0000


FunVal =

  1.7200e+003


ExitFlag =

     1


Output =

         iterations: 7
          funcCount: 8
    constrviolation: 0
           stepsize: 2.8664e-004
          algorithm: 'interior-point'
      firstorderopt: 4.9997e-006
       cgiterations: 0
            message: [1x793 char]
```

Solution using "FMINCON"

In summary both algorithms give same results. Function "linprog" is more efficient since it took only 2 iterations to converge to the optimum solution. Function "fmincon" on the other hand took 7 iterations to converge to optimum point.

For more complex problems it is crucial to select the approproate solver in order to minimize run time.