# Random Field // (Random process)

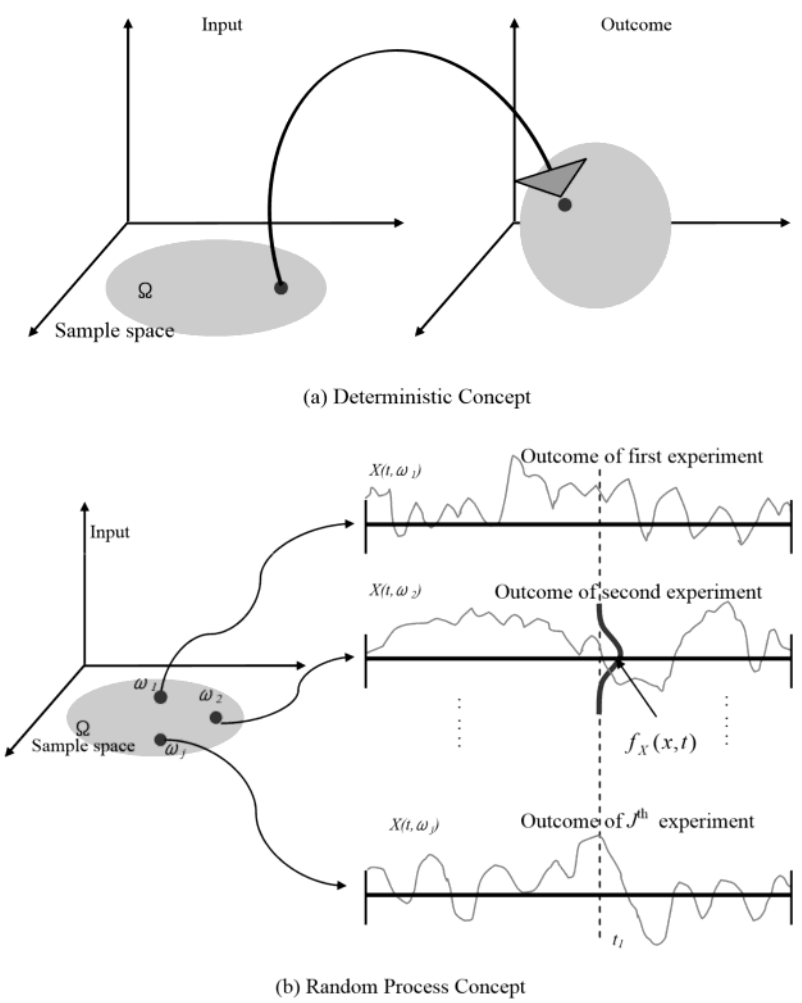



**Figure 2.11.** Deterministic and Random Process Concepts

- Random Fluctuations over space and time domain
- Fluctuations due to one or more random variables over the space of interest
  (Ex, Young's modulus over a plate)

The mean of the random process $X(t)$ is

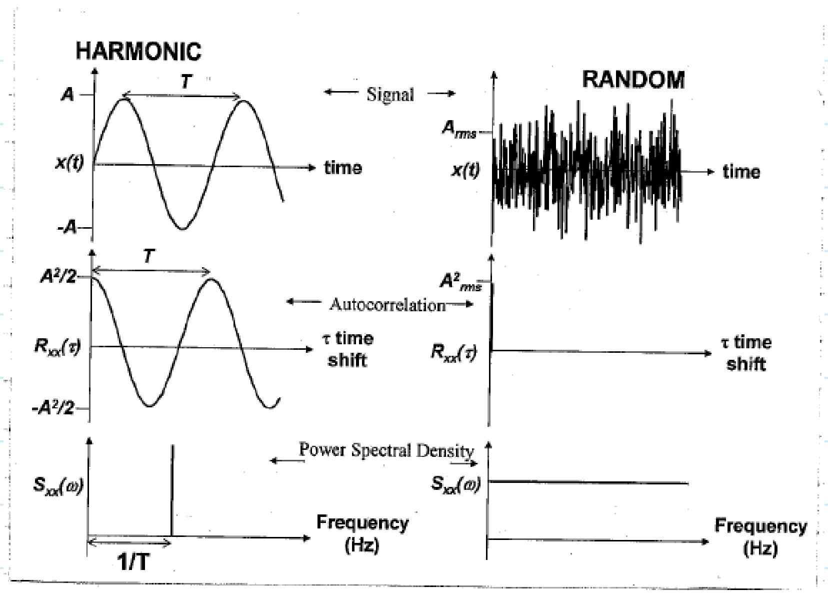
$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x \; f_X(x,t)dx \tag{2.70}$$

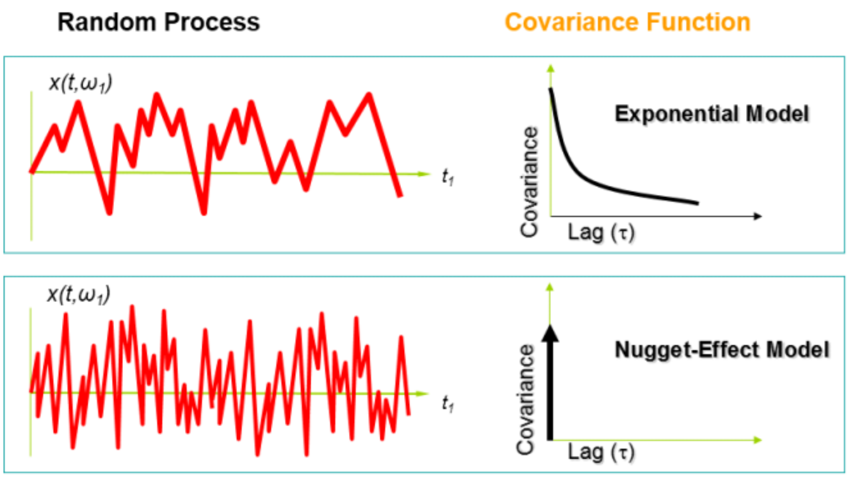
the *autocovariance* is

$$\begin{aligned} C_{XX}(t_1,t_2) &= E[(X(t_1)-\mu_X(t_1))(X(t_2)-\mu_X(t_2))] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x_1-\mu_X(t_1))(x_2-\mu_X(t_2))f_{XX}(x_1,x_2;t_1,t_2)dx_1dx_2 \end{aligned} \tag{2.71}$$

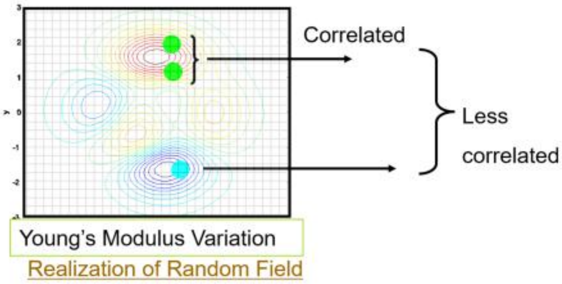
and the *autocorrelation* is

$$\begin{aligned} R_{XX}(t_1,t_2) &= E[X(t_1)X(t_2)] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x_1x_2f_{XX}(x_1,x_2;t_1,t_2)dx_1dx_2 \end{aligned} \tag{2.72}$$



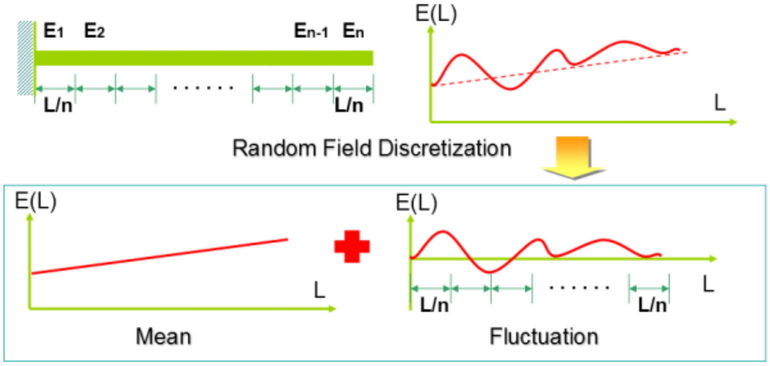



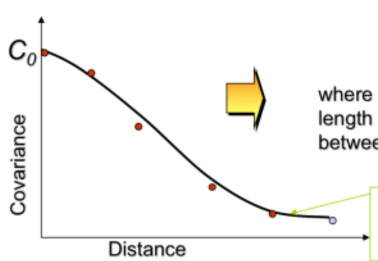
- Random field - a mathematical model of fluctuations




- Spatial variability is characterized by:
  - Distribution type
  - Mean
  - Correlation between two locations

Cantilever Beam (Young's modulus fluctuates over the length)




Gaussian covariance

$$C_{ij} = C_0 \exp\left[-\left(\frac{d_{ij}}{cL}\right)^2\right]$$

where $C_0$ is variance, $cL$ is a correlation length of the field and $d_{ij}$ is the distance between two points.

**Correlation Length (spatial range) : distance at which the covariance drops by 95% of $C_0$**

Random field discretization involves the discretization of its covariance function

**Discretization of Covariance Function**


Increasing the number of elements facilitates an accurate approximation of the actual covariance

Covariance functions

- Exponential Model

$$C_{ij} = C_0 \exp\left[ - \left| \frac{X_{ij}}{l} \right| \right]$$

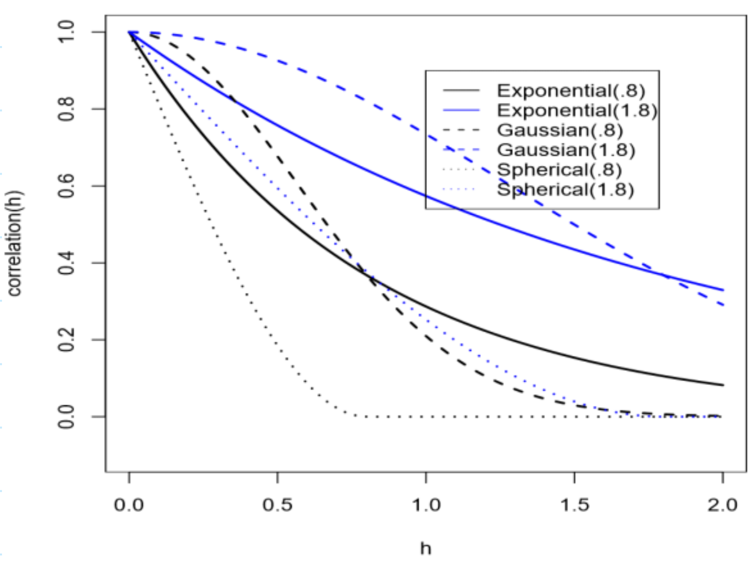
- Gaussian Model

– Gaussian Model

$$C_{ij} = C_0 \exp\left[ -\left( \frac{|X_{ij}|}{\ell} \right)^2 \right]$$

– Nugget-effect Model

$$C_{ij} = \begin{cases} C_0 & \text{for } |X_{ij}| = 0 \\ 0 & \text{otherwise} \end{cases}$$




(*) Correlation length