WSU Mechanical Engineering

ME 4210: Mechanical Vibration Lab, Section 4

Lab 4 Modal Analysis of a Cruise Missile Wing

Daniel Clark

Assigned: April 10, 2014

Due: April 25, 2014

Instructor: Dr. Ha-Rok Bae

TA: James Davidson

Objective (1):

The purpose of this lab report is to preform modal analysis for the first 5 modes of a cruise missile wing located in the basement of Russ Engineering Center. Data collected using the Bobcat data acquisition system will be imported into MatLab where the requested plots and tables can be created and interoperated.

Theory (3):

The theory behind determining the mass matrix M, the damping matrix C and the stiffness matrix K is as follows. The mass matrix M is found by the following equation:

$$M = (S^{-1})^T I \, (S^{-1}) \tag{1}$$

The damping matrix C is found by the following equation:

$$C = (S^{-1})^T (2\zeta\Lambda^{\frac{1}{2}}) \, (S^{-1}) \tag{2}$$

The Stiffness matrix K is found by the following equation:

$$K = (S^{-1})^T \Lambda \, (S^{-1}) \tag{3}$$

Experimental Procedure (5):

The initial set up and the data collection of all of the cases is the exact same as Labs 1 and 2, except for the beam being replaced by a wing, please refer to them for the process. The positions of the hit locations also vary along the length as well as the width of the wing.

After the data is exported from the BobCat software and imported into MatLab where the plots and required results are generated. The MatLab code is found in the appendix 1 of this report in published form. All of the plots and requested data can be found here as well as the results and discussion section.

Results and Discussion (10):

The required results include: Tabulated experimental parameters from mdofcf(), model parameters and plots. The Tabulated experimental parameters include: first five damping ratios, first five natural frequencies and the first five mode shapes. This can all be found below in Table 1.

Table 1: Tabulated experimental parameters from mdofcf().

Mode 1	1	2	3	4	5
Damping Ratio	0.0006	0.0006	0.0088	0.0002	0.0048
Natural Frequency	13.5448	51.7398	126.1874	167.7924	226.9669
Mode Shape Vector	65.995	-364.4539	526.6165	-791.5805	673.2007
	62.1385	-343.6708	464.255	671.5555	758.09
	54.6218	-4.6934	-415.7119	653.8541	-885.2481
	0.4473	12.2814	30.6479	52.2696	-175.4648
	18.7344	152.1476	-89.1278	-214.1911	56.9976

The model parameters include: spectral matrix, matrix of mode shapes, damping ratio matrix, mass matrix, damping matrix and stiffness matrix. This can all be found below in Figure 1.

```
spectral_matrix =

   1.0e+06 *

    0.0072         0         0         0         0
         0    0.1057         0         0         0
         0         0    0.6286         0         0
         0         0         0    1.1115         0
         0         0         0         0    2.0337

mode_shapes =

   65.9950 -364.4539  526.6165 -791.5805  673.2007
   62.1385 -343.6708  464.2550  671.5555  758.0900
   54.6218   -4.6934 -415.7119  653.8541 -885.2481
    0.4473   12.2814   30.6479   52.2696 -175.4648
   18.7344  152.1476  -89.1278 -214.1911   56.9976

damping_ratio_matrix =

    0.0006         0         0         0         0
         0    0.0006         0         0         0
         0         0    0.0088         0         0
         0         0         0    0.0002         0
         0         0         0         0    0.0048


M =

[  0.000013323355, 0.0000096935909, 0.000018956313,  0.00001359767, 0.000045333988]
[ 0.0000096935909,  0.000012796413, 0.000015329259, 0.000030929316, 0.000055431135]
[  0.000018956313,  0.000015329259, 0.000029556962,  0.00001471689, 0.000071834369]
[   0.00001359767,  0.000030929316,  0.00001471689,  0.00017432502,  0.00013180607]
[  0.000045333988,  0.000055431135, 0.000071834369,  0.00013180607,   0.0002578252]


C =

[   0.0000029484244, -0.00000067233748,  0.0000032290442, 0.000013072502,  0.0000014063091]
[ -0.00000067233748,     0.00001016463,  -0.00010778018, 0.000071300172,   0.000017391334]
[   0.0000032290442,   -0.000010778018,  0.000022519665,  -0.0001296531, -0.0000086160209]
[    0.000013072502,    0.000071300172,   -0.0001296531,   0.0013153983,   0.000099364705]
[   0.0000014063091,    0.000017391334, -0.0000086160209, 0.000099364705,   0.000047330812]

K =

[  0.83790727, -0.71686628,  0.41923519, 0.90827394, -0.51416332]
[ -0.71686628,   1.2729187, -0.68187081,   2.178297,   1.5245555]
[  0.41923519, -0.68187081,   1.3121257, -5.1175675, -0.91974864]
[  0.90827394,    2.178297, -5.1175675,  80.914621,   4.9499216]
[ -0.51416332,   1.5245555, -0.91974864,  4.9499216,   5.2133448]
```

Figure 1: Model parameters generated using MatLab.

The requested plots include: frequency response for all cases on the same graph, time response of the hammer for all cases on the same graph, power spectrum density for all cases plotted on the same graph and the coherence for case number 2. The plots are located blow as well, figures 2 through 5 respectfully. Case number 2 was selected for the coherence plot because it coherence plot is the only one which trends or gets close to 1.

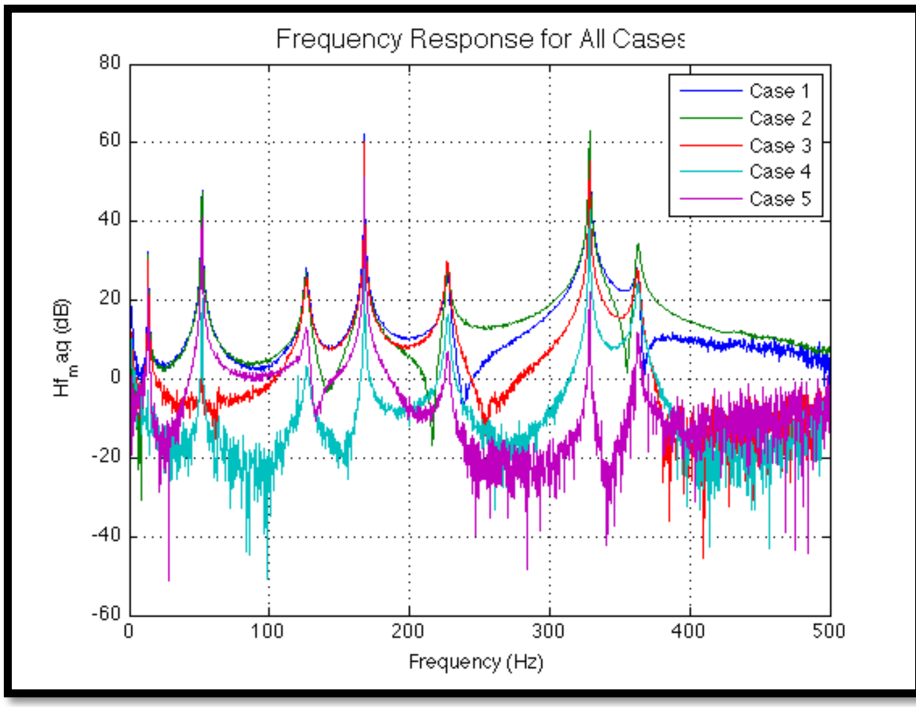

Figure 2: Frequency Response for all cases on a single plot. It is easy to see that there are distinct and repetitive natural frequencies that occur in all of the cases.

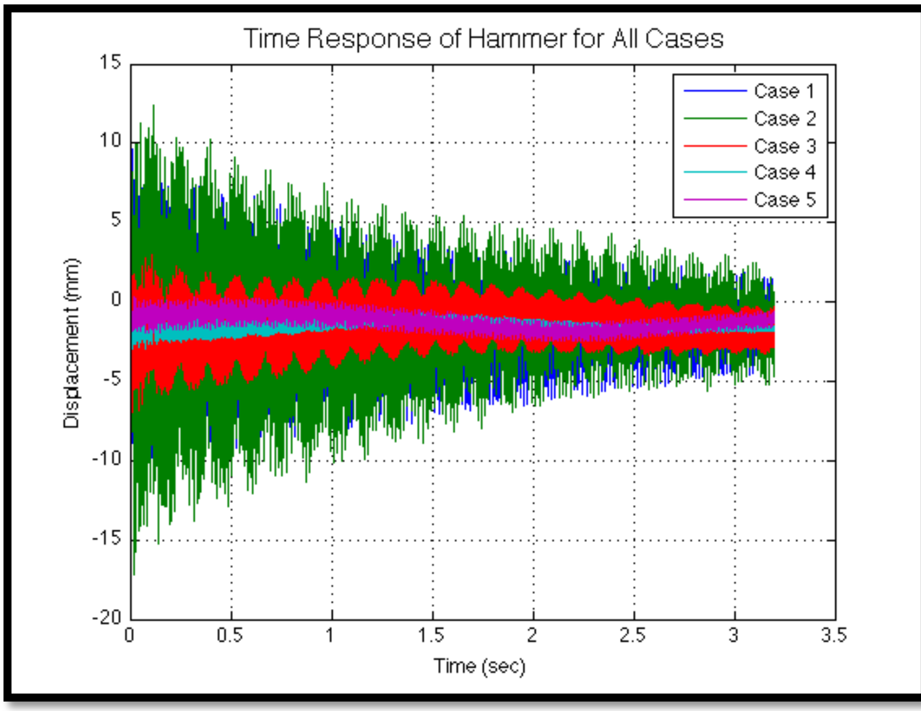

Figure 3: Time response of the hammer for all the cases on a single plot.

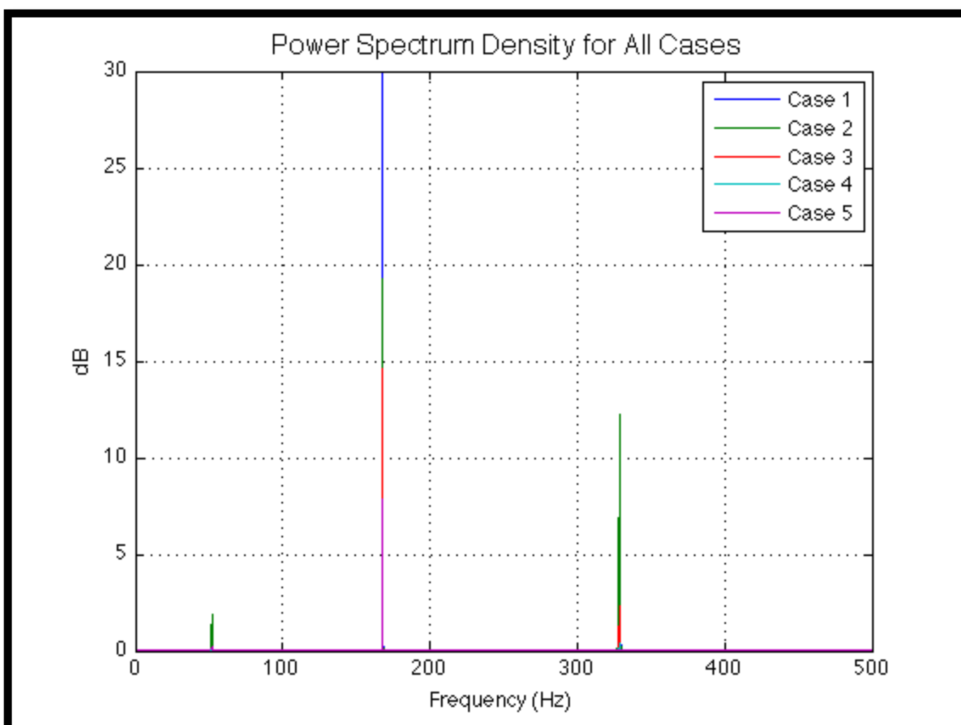

Figure 4: Power spectrum density for all cases on a single plot.

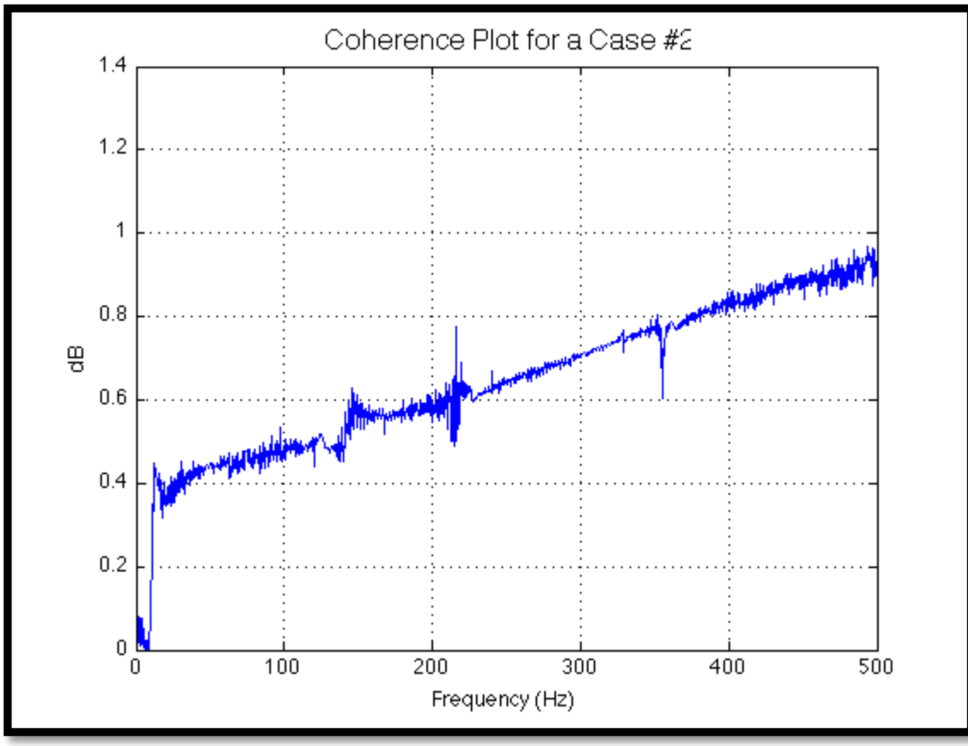

Figure 5: Coherence plot of the data for case 2.

Now, the answers to the 2 questions found on the final page of the report:

1. The data collected can be considered to be poor quality because none of the coherence plots were oriented around 1. However, all of the peaks for the frequency response are aligned so conclusions can still be made.
2. All of the frequency response functions are used to find each of the natural frequencies and damping ratios because MDOFCF takes them all as an input matrix TR.

Conclusion (10):

In conclusion, all of the required computed matrices were found and are close to what is expected for the missile wing response. However, the coherence data shows that this experiment may be invalid. But, this method for analyzing multiple degree of freedom systems has proven far easier than doing the work by hand.

Appendix (5):

MatLab code starts on the next page.

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Lab #4 % Student: Daniel Clark 1
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Lab #4 % Student: Daniel Clark

```
clear all                                        % Instructor: Dr. Ha-Rok Bae
close all                                        % Class:              ME-4210
clc
format short
```

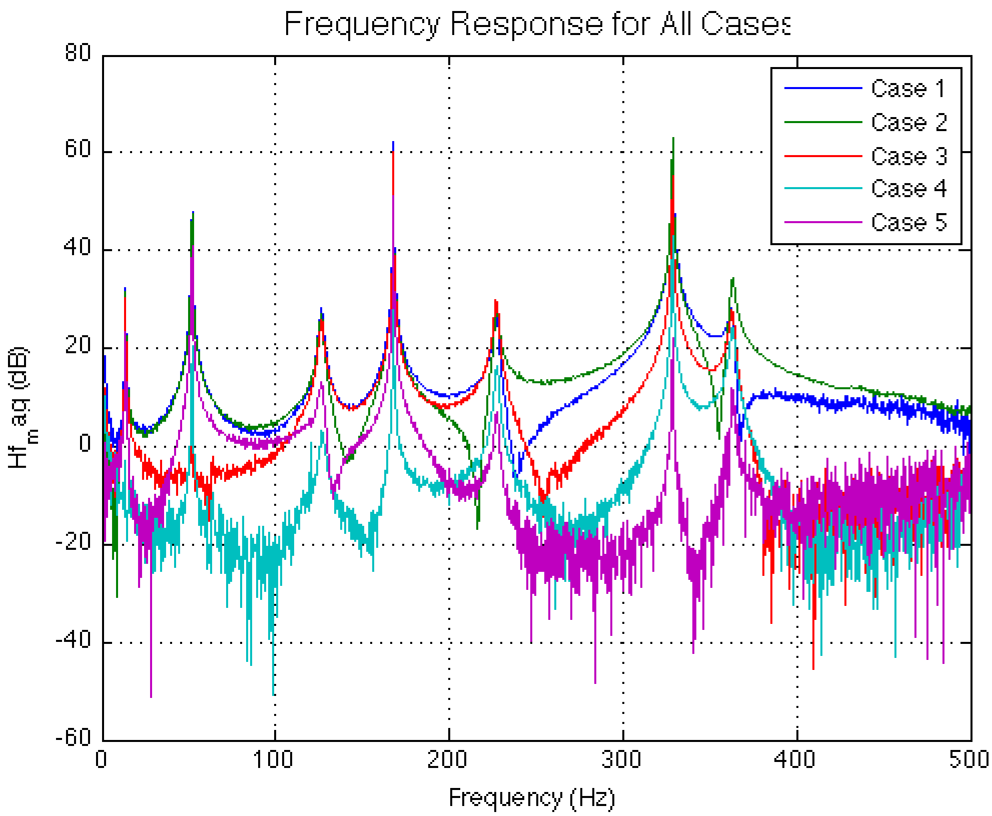
Using Metric units

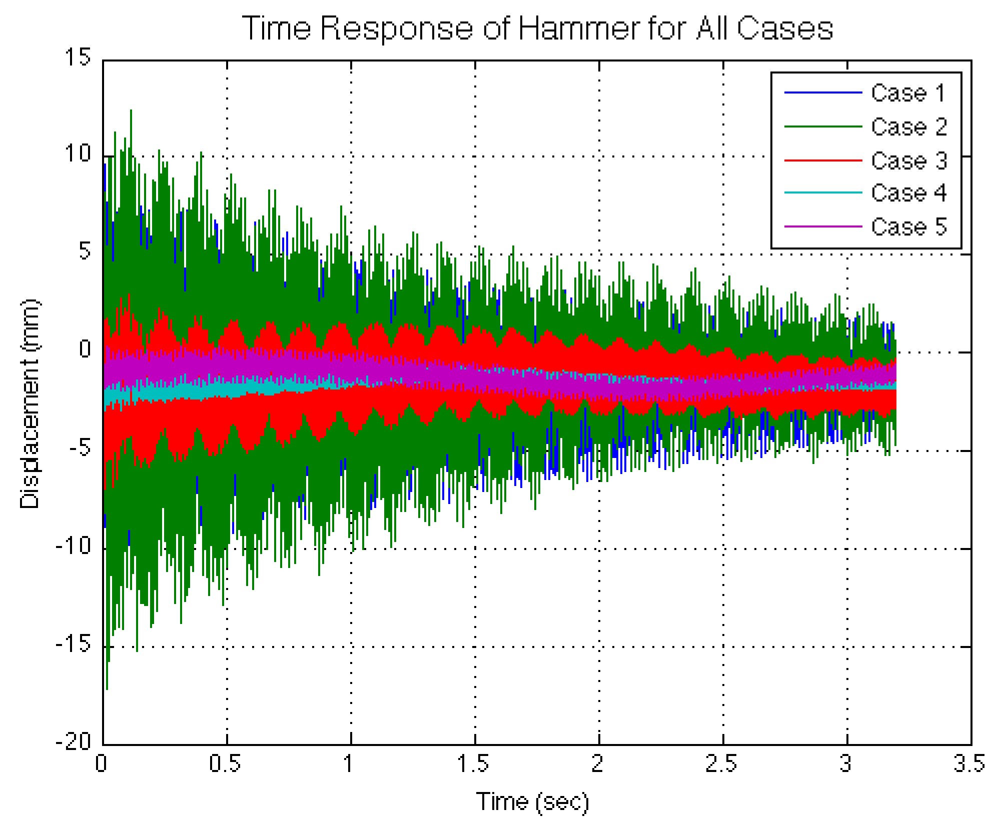
```
titlesize = 13;                      % Perfered title size
fontsize = titlesize-3;              % Prefered axis label size
Z = zeros(5,5);
spectral_matrix = zeros(5,5);
mode_shapes = zeros(5,5);
U = zeros(5,5);

% loading all useful information from data files
load Lab4_Case1.mat

h1 = Hf_chan_2;
hf_1_db = 20*log10(abs(h1));
Freq_1 = Freq_domain;
t_d_1 = Time_domain;
t_c_1 = Time_chan_2;
psd_1 = PSD_chan_2;

load Lab4_Case2.mat

h2 = Hf_chan_2;
hf_2_db = 20*log10(abs(h2));
Freq_2 = Freq_domain;
t_d_2 = Time_domain;
t_c_2 = Time_chan_2;
psd_2 = PSD_chan_2;

load Lab4_Case3.mat

h3 = Hf_chan_2;
hf_3_db = 20*log10(abs(h3));
Freq_3 = Freq_domain;
t_d_3 = Time_domain;
t_c_3 = Time_chan_2;
psd_3 = PSD_chan_2;

load Lab4_Case4.mat

h4 = Hf_chan_2;
```

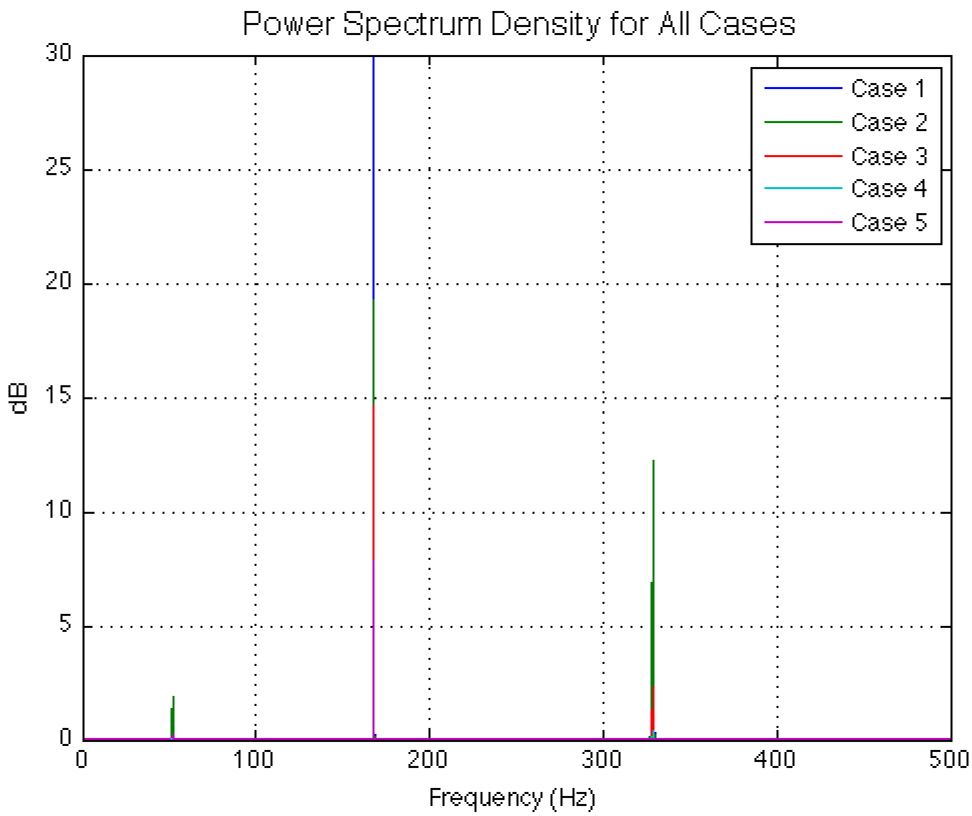
```
hf_4_db = 20*log10(abs(h4));
Freq_4 = Freq_domain;
t_d_4 = Time_domain;
t_c_4 = Time_chan_2;
psd_4 = PSD_chan_2;

load Lab4_Case5.mat

h5 = Hf_chan_2;
hf_5_db = 20*log10(abs(h5));
Freq_5 = Freq_domain;
t_d_5 = Time_domain;
t_c_5 = Time_chan_2;
psd_5 = PSD_chan_2;
```

Plots

```
figure

% frequency response for all cases plotted on same graph
plot(Freq_1,hf_1_db,Freq_2,hf_2_db,Freq_3,hf_3_db,Freq_4,hf_4_db,...
    Freq_5,hf_5_db)
xlabel(' Frequency (Hz) ','FontSize',fontsize)
ylabel(' Hf_mag (dB) ','FontSize',fontsize)
title(' Frequency Response for All Cases ','FontSize',titlesize)
legend('Case 1','Case 2','Case 3','Case 4','Case 5')
grid

figure

% time response of the hammer for all cases plotted on the same graph
plot(t_d_1,t_c_1,t_d_2,t_c_2,t_d_3,t_c_3,t_d_4,t_c_4,t_d_5,t_c_5)
xlabel(' Time (sec) ','FontSize',fontsize)
ylabel(' Displacement (mm) ','FontSize',fontsize)
title(' Time Response of Hammer for All Cases ','FontSize',titlesize)
legend('Case 1','Case 2','Case 3','Case 4','Case 5')
grid

figure

% Power Spectrum Density for All Cases plotted on the same graph
plot(Freq_1,psd_1,Freq_2,psd_2,Freq_3,psd_3,Freq_4,psd_4,Freq_5,psd_5)
xlabel(' Frequency (Hz) ','FontSize',fontsize)
ylabel(' dB ','FontSize',fontsize)
title(' Power Spectrum Density for All Cases ','FontSize',titlesize)
legend('Case 1','Case 2','Case 3','Case 4','Case 5')
grid
```


Frequency Response for All Cases
Case 1
Case 2
Case 3
Case 4
Case 5
Frequency (Hz)


Time Response of Hammer for All Cases
Case 1
Case 2
Case 3
Case 4
Case 5
Displacement (mm)
Time (sec)



Determining M C and K

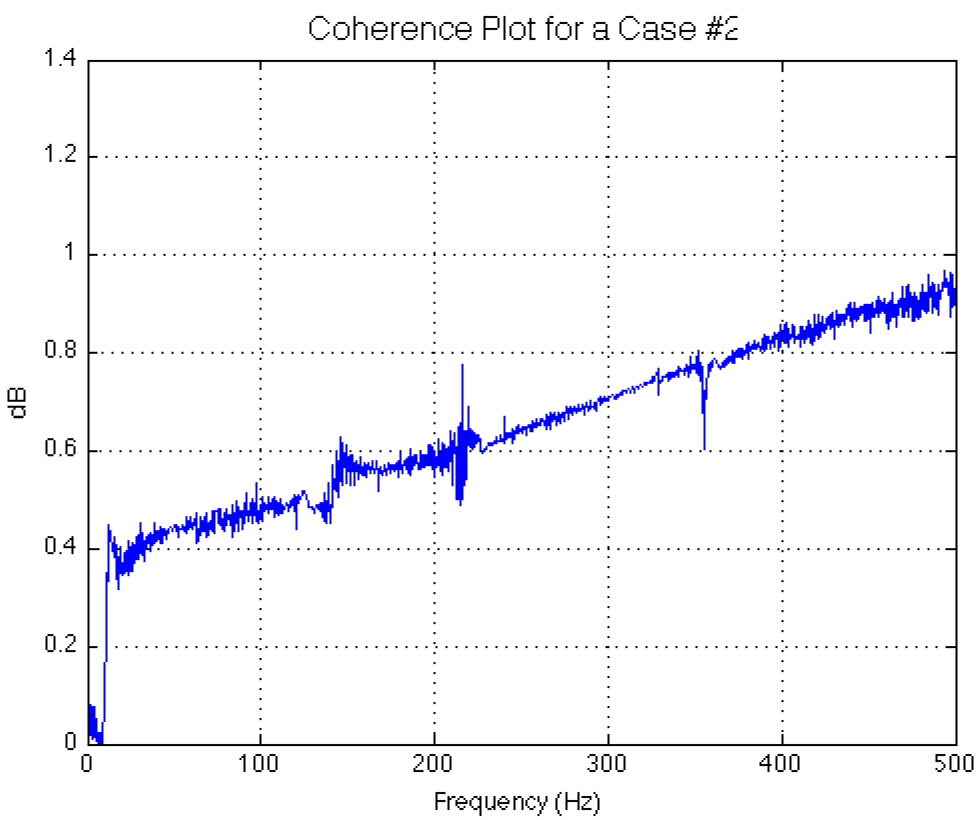
```
TF = [h1,h2,h3,h4,h5];

[Z(1,1),nf(1),U(:,1)] = mdofcf(Freq_1,TF,11.25,16.88);        %Peak 1
[Z(2,2),nf(2),U(:,2)] = mdofcf(Freq_2,TF,45,55);        %Peak 2
[Z(3,3),nf(3),U(:,3)] = mdofcf(Freq_3,TF,120,130);      %Peak 3
[Z(4,4),nf(4),U(:,4)] = mdofcf(Freq_4,TF,160,170);      %Peak 4
[Z(5,5),nf(5),U(:,5)] = mdofcf(Freq_5,TF,222,233);      %Peak 5

damping_ratio_matrix = Z
nf
U = real(U)

for int=1:5
    spectral_matrix(int, int) = (2*pi*(nf(int)))^2;
    mode_shapes(:,int) = real(U(:,int));
end

spectral_matrix
mode_shapes

M = vpa(mode_shapes'\eye(5)/mode_shapes,8)
C = vpa(mode_shapes'\(2*Z*sqrt(spectral_matrix))/mode_shapes,8)
K = vpa(mode_shapes'\spectral_matrix/mode_shapes,8)


Warning: Rank deficient, rank = 4,  tol =    6.7128e+00.
Warning: Rank deficient, rank = 2,  tol =    6.6846e-05.
Warning: Rank deficient, rank = 4,  tol =    4.1411e+01.
```

```
Warning: Rank deficient, rank = 2,  tol =    6.6846e-05.
Warning: Rank deficient, rank = 4,  tol =    7.2056e+01.
Warning: Rank deficient, rank = 2,  tol =    6.6846e-05.
Warning: Rank deficient, rank = 4,  tol =    1.6171e+02.
Warning: Rank deficient, rank = 2,  tol =    7.8982e-05.

damping_ratio_matrix =

    0.0006         0         0         0         0
         0    0.0006         0         0         0
         0         0    0.0088         0         0
         0         0         0    0.0002         0
         0         0         0         0    0.0048


nf =

   13.5448   51.7398  126.1874  167.7924  226.9669


U =

   65.9950 -364.4539  526.6165 -791.5805  673.2007
   62.1385 -343.6708  464.2550  671.5555  758.0900
   54.6218   -4.6934 -415.7119  653.8541 -885.2481
    0.4473   12.2814   30.6479   52.2696 -175.4648
   18.7344  152.1476  -89.1278 -214.1911   56.9976


spectral_matrix =

   1.0e+06 *

    0.0072         0         0         0         0
         0    0.1057         0         0         0
         0         0    0.6286         0         0
         0         0         0    1.1115         0
         0         0         0         0    2.0337


mode_shapes =

   65.9950 -364.4539  526.6165 -791.5805  673.2007
   62.1385 -343.6708  464.2550  671.5555  758.0900
   54.6218   -4.6934 -415.7119  653.8541 -885.2481
    0.4473   12.2814   30.6479   52.2696 -175.4648
   18.7344  152.1476  -89.1278 -214.1911   56.9976


M =

[   0.000013323355, 0.0000096935909, 0.000018956313,  0.00001359767, 0.000045333
[  0.0000096935909,  0.000012796413, 0.000015329259, 0.000030929316, 0.000055431
[   0.000018956313,  0.000015329259, 0.000029556962,  0.00001471689, 0.000071834
[    0.00001359767,  0.000030929316,  0.00001471689,  0.00017432502,  0.00013180
[   0.000045333988,  0.000055431135, 0.000071834369,  0.00013180607,   0.0002578


C =

[    0.0000029484244, -0.00000067233748,  0.0000032290442, 0.000013072502,  0.00
[ -0.00000067233748,     0.00001016463,  -0.000010778018, 0.000071300172,   0.0
[    0.0000032290442,   -0.000010778018,   0.000022519665,  -0.0001296531, -0.00
[     0.000013072502,    0.000071300172,    -0.0001296531,   0.0013153983,   0.0
```

```
[    0.0000014063091,     0.000017391334, -0.0000086160209, 0.000099364705,    0.0

K =

[   0.83790727, -0.71686628,   0.41923519, 0.90827394, -0.51416332]
[ -0.71686628,    1.2729187, -0.68187081,    2.178297,    1.5245555]
[   0.41923519, -0.68187081,    1.3121257, -5.1175675, -0.91974864]
[   0.90827394,     2.178297,   -5.1175675,   80.914621,    4.9499216]
[ -0.51416332,    1.5245555, -0.91974864,    4.9499216,    5.2133448]
```

Coherence Plot for a single case

```
clear all
load Lab4_Case2.mat

figure
plot(Freq_domain,Hf_coh_chan_2)
xlabel(' Frequency (Hz) ','FontSize',10)
ylabel(' dB ','FontSize',10)
title(' Coherence Plot for a Case #2 ','FontSize',13)
grid
```



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