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EGR7040 Optimization HW#3

Example Problem	**Objective Function**	**Design Variables**	**Constraints**
2.1 Cantelever Beam	$$f(w,t) = A = 4t(w-t), mm^2$$ Cost function to be minimized.	w=width, mm t=wall thickness, mm	Bending Stress: $$\frac{PLw}{2I} - \sigma_a \leq 0$$ Shear Stress: $$\frac{PQ}{2It} - \tau_a \leq 0$$ Deflection: $$\frac{PL^3}{3EI} - q_a \leq 0$$ Width-Thickness Restriction: $$w - 8t \leq 0$$ Dimension Restriction: $$60 - w \leq 0$$ $$w - 300 \leq 0$$ $$3 - t \leq 0$$ $$t - 15 \leq 0$$
2.2 Cylindrical Can	$$S = \pi DH + 2(\frac{\pi}{4}D^2), cm^2$$ Cost function to be minimized.	D=diameter of can, cm H=height of the can, cm	Volume Contraint: $$\frac{\pi}{4}D^2H \geq 400$$ Side Constraints: $$3.5 \leq D \leq 8$$ $$8 \leq H \leq 18$$
	$$Cost = c_2At + c_3G + 6.14c_4G$$ where:		

2.3 Spherical Tank	$A = 4\pi r^2$ $G = \frac{(365)(24)(\triangle T)A}{c_1 t}$ c_1=thermal resistivity c_2=insulation cost c_3=cost of refrigiration equipment c_4=annual cost of running refrigiration equipment	t = insulation thickness	$t \geq t_{min}$
2.4 Sawmill	$Cost = 240x_1 + 205x_2 + 172x_3 + 180x_4$ Goal is to minimize this function.	x_1=number of logs from forest 1 to mill A x_2=number of logs from forest 2 to mill A x_3=number of logs from forest 1 to mill B x_4=number of logs from forest 2 to mill B	Capacity Constraints: $x_1 + x_2 \leq 240$ $x_3 + x_4 \leq 300$ $x_1 + x_3 \leq 200$ $x_2 + x_4 \leq 200$ Logs needed: $x_1 + x_2 + x_3 + x_4 \geq 300$ Side Constraints: $x_i \geq 0;\ for\ i = 1\ to\ 4$
2.5 Two-bar Bracket	Cost function to be minimized. $Mass = \rho l(A_1 + A_2)$ Where: $A_1 = \frac{\pi}{4}(x_3^2 - x_4^2)$ $A_2 = \frac{\pi}{4}(x_5^2 - x_6^2)$ $l = \sqrt{x_1^2 + (0.5x_2)^2}$	x_1=height h of the bracket x_2=span s of bracket x_3=outer diamenter of bar 1 x_4=inner diameter of bar 1 x_5=outer diameter of bar 2 x_6=inner diameter of bar 2	Bar Stresses: $-\sigma_1 \leq \sigma_a$ $\sigma_1 \leq \sigma_a$ $-\sigma_2 \leq \sigma_a$ $\sigma_2 \leq \sigma_a$ where $F1 = -0.5Wl[\frac{sin\theta}{x_1} + \frac{2cos\theta}{x_2}]$ $F2 = -0.5Wl[\frac{sin\theta}{x_1} - \frac{2cos\theta}{x_2}]$ $\sigma_1 = \frac{F_1}{A_1}$ $\sigma_2 = \frac{F_2}{A_2}$ Design variable limits: $x_{iL} \leq x_i \leq x_{iU}$ for $i = 1\ to\ 6$
2.6 Cabinet	$Cost = 3.5x_1 + 3.0x_2 + 6.0x_3 + 4.8x_4 + 1.8x_5 + 3.0x_6$	x_1 = # of C_1 to be bolted. x_2 = # of C_1 to be riveted. x_3 = # of C_2 to be bolted.	Required # of C_1, C_2, and C_3: $x_1 + x_2 = 8x100$ $x_3 + x_4 = 5x100$ $x_5 + x_6 = 15x100$ Bolting $\&$ riveting capacities:

Design	Cost function to be minimized.	x_4 = # of C_2 to be riveted. x_5 = # of C_3 to be bolted. x_6 = # of C_3 to be riveted.	$5x_1 + 6x_3 + 3x_5 \leq 6000$ $5x_2 + 6x_4 + 3x_6 \leq 8000$ Side Constraints: $x_i \geq 0$; for $i = 1\ to\ 6$
2.7 Tubular Column	Cost function to be minimized: $Mass = 2\rho l\pi Rt$	R = mean radius t = wall thickness	Stress: $\frac{P}{2\pi Rt} \leq \sigma_a$ Buckling: $P \leq \frac{\pi^3 ER^3 t}{4l^2}$ Side Constraints: $R_{min} \leq R \leq Rmax$ $t_{min} \leq t \leq t_{max}$
2.8 Cylindrical Tank	Cost function to be minimized. $f = c(2\pi R^2 + 2\pi RH)$ where: c = cost per unit area.	R=radius H=height	Volume: $\pi R^2 H = V$ Side constraints: $R_{min} \leq R \leq R_{max}$ $H_{min} \leq H \leq H_{max}$
2.9 Coil Springs	Minimize spring mass: $Mass = \frac{1}{4}(N+Q)\pi^2 Dd^2\rho$ where: Q = # of inactive coils ρ = density Required Equations: $P = K\delta$ $K = \frac{d^4 G}{8D^3 N}$ $\tau = \frac{8kPD}{\pi d^3}$ $k = \frac{4D-d}{4(D-d)} + \frac{0.615d}{D}$ $\omega = \frac{d}{2\pi ND^2}\sqrt{\frac{G}{2\rho}}$	d=wire diameter D=mean coil diameter N=number of active coils	Deflection constraint $\frac{P}{K} \geq \triangle$ Shear Stress $\tau \leq \tau_a$ Frequency $\omega \geq \omega_a$ Diameter $D + d \leq D_o$ side constraints $d_{min} \leq d \leq d_{max}$ $D_{min} \leq D \leq D_{max}$ $N_{min} \leq N \leq N_{max}$

Problem 2:

2. A refinery has two crude oils: (1) Crude A costs \$120/barrel (bbl) and 20,000 bbl are available. (2) Crude B costs \$150/bbl and 30,000 bbl are available. The company manufactures gasoline and lube oil from its crudes. Yield and sale price per barrel and markets are shown in Table E2.2. How much crude oil should the company use to maximize its profit? Formulate the optimum design problem.

2) Data Collection

- Refinery buys two types of crude oil:
  - Crude A: costs $\$120.0$ per barrel and 20,000 are available.
  - Crude B: costs $\$150.0$ per barrel and 30,000 are available.

- Company manufactures gasoline and lube oil from the purchased crudes. The yield, sales price, and markets are outlined in table below:

Product	Yield from A	Yield from B	Sales price (bbl)	Market (bbl)
Gas	0.6	0.8	$\$200$	20,000
Lube Oil	0.4	0.2	$\$450$	10,000

Optimization Goal: How much oil should the company use to maximize their profit?

Profit = Sales - Cost

3-4) Define variables and objective function:

First define cost:

$Cost = 120x_1 + 150x_2$
Where:
$x_1 = crude\ A$
$x_2 = crude\ B$

Next define sales:

$Sales = gas\ from\ A + gas\ from\ B + lube\ from\ A + lube\ from\ B$
From this we have the other necessary variables:
$x_3 = gas\ from\ A$
$x_4 = gas\ from\ B$
$x_5 = lube\ from\ A$
$x_6 = lube\ from\ B$

Plugging in the values:
$Sales = 0.6(200)x_3 + 0.8(200)x_4 + 0.4(450)x_5 + 0.2(450)x_6$
$\Downarrow$
$Sales = 120x_3 + 160x_4 + 180x_5 + 90x_6$

Finally defining profit :

$$Profit = [120x_3 + 160x_4 + 180x_5 + 90x_6] - [120x_1 + 150x_2]$$

Summary of variables (6 total):
$x_1 = crude\ A$
$x_2 = crude\ B$
$x_3 = gas\ from\ A$
$x_4 = gas\ from\ B$
$x_5 = lube\ from\ A$
$x_6 = lube\ from\ B$

5) Constraints:

- Inequality constraints:
  - $x_1 \leq 20,000 \implies x_1 - 20,000 \leq 0$
  - $x_2 \leq 30,000 \implies x_2 - 30,000 \leq 0$
- Equality constraints:
  - $x_3 + x_4 = 20,000 \implies x_3 + x_4 - 20,000 = 0$, want to ensure all barrels are sold.
  - $x_5 + x_6 = 10,000 \implies x_5 + x_6 - 10,000 = 0$, want to ensure all barrels are sold.
  - $x_1 = 0.6x_3 + 0.4x_5$, define x_1 in terms of x_3 and x_5
  - $x_2 = 0.8x_4 + 0.2x_6$, define x_2 in terms of x_4 and x_6
- Side constraints:
  - $x_i \geq 0\ for\ i = 1\ to\ 6$

Standard form:

- Find:
  - $\vec{x} = [x_1, x_2, x_3, x_4, x_5, x_6]^T$
- That maximizes the objective function:
  - $Profit = [120x_3 + 160x_4 + 180x_5 + 90x_6] - [120x_1 + 150x_2]$
- Such that:
  - $g_1(\vec{x}) :\ x_1 - 20,000 \leq 0$, Limit on Crude A
  - $g_2(\vec{x}) :\ x_2 - 30,000 \leq 0$, Limit on Crude B
  - $h_1(\vec{x}) :\ x_3 + x_4 - 20,000 = 0$, Market limit on Gas
  - $h_2(\vec{x}) :\ x_5 + x_6 - 10,000 = 0$, Market limit on Lube

- $h_3(\vec{x}) : \ x_1 = 0.6x_3 + 0.4x_5$, define x_1 in terms of x_3 and x_5.
- $h_4(\vec{x}) : \ x_2 = 0.8x_4 + 0.2x_6$, define x_2 in terms of x_4 and x_6.
- $x_i \geq 0 \ for \ i = 1 \ to \ 6$, Side Constraints

Problem 5

5. Proposals for a parking ramp have been defeated, so we plan to build a parking lot in the downtown urban renewal section. The cost of land is $200W + 100D$, where W is the width along the street and D is the depth of the lot in meters. The available width along the street is 100 m, while the maximum depth available is 200 m. We want the size of the lot to be at least 10,000 m^2. To avoid unsightliness, the city requires that the longer dimension of any lot be no more than twice the shorter dimension. Formulate the minimum-cost design problem.

2) Data Collection

Plan is to build a new parking lot in downtown area. Cost is defined by $Cost = 200W + 100D$ where W is width along street and D is depth of lot in meters. Following limits are set on the proposed lot.

- Max available width (W) along street is 100m.
- Max avaliable depth (D) is 200m.
- Lot area must be at least 10,000m^2.
- City requires longer dimension to be no more than 2 times the shorter dimension.

★ Goal is to formulate the minimum cost design.

3) Design Variables:

- W = width along street
- D = depth

4) Optimization function:

- $Cost = 200W + 100D$, minimize this function

5)Constraints:

- Inequality constraints:
  - $W - 100 \leq 0$
  - $D - 200 \leq 0$
  - $(W * D) - 10,000 \geq 0$
  - $D - 2W \leq 0$
- Side constraints:
  - $W \geq 0$
  - $D \geq 0$

Standard form:

- Find:
  - $\vec{x} = [W, D]^T$
- That minimizes the objective function:
  - $Cost = 200W + 100D$
- Such that:
  - $g_1(\vec{x}) :\ W - 100 \leq 0$, limit on width
  - $g_2(\vec{x}) :\ D - 200 \leq 0$, limit on depth
  - $g_3(\vec{x}) :\ (W * D) - 10,000 \geq 0$, limit on parking lot area
  - $g_4(\vec{x}) :\ D - 2W \leq 0$, limit on aspect ratio
  - $W \geq 0$, side constraint
  - $D \geq 0$, side constraint

Problem 8

8. Enterprising chemical engineering students have set up a still in a bathtub. They can produce 225 bottles of pure alcohol each week. They bottle two products from alcohol: (1) wine, at 20 proof, and (2) whiskey, at 80 proof. Recall that pure alcohol is 200 proof. They have an unlimited supply of water, but can only obtain 800 empty bottles per week because of stiff competition. The weekly supply of sugar is enough for either 600 bottles of wine or 1200 bottles of whiskey. They make a \$1.00 profit on each bottle of wine and a \$2.00 profit on each bottle of whiskey. They can sell whatever they produce. How many bottles of wine and whiskey should they produce each week to maximize profit? Formulate the design optimization problem (created by D. Levy).

2) Data collection

Chemical engineering students produce 225 bottles of pure alcohol (200 proof) in their bathtub still per week. They manufacture two products from the alcohol.

- Vine, which is 20 proof. **Meaning that each bottle of pure alcohol can make 10 bottles of vine.**
- Wiskey, which is 80 proof. **Meaning that each bottle of pure alcohol can make 2.5 bottles of wiskey.**

They make a profit of $\$1.00$ for each bottle of vine sold and $\$2.00$ for each bottle of wiskey sold.

Additional requiremets are listed below:

- Only 800 empty bottles can be procured per week.
- There is enough sugar each week for either 600 bottles of vine or 1200 bottles of wiskey.

★ How many bottles of vine and wiskey should they make each week to maximize their profits?

3) Design variables:

- V = number of vine bottles sold
- W = number of wiskey bottles sold

4) Optimization function:

- $Profit = 2W + V$; maximize this function

5) Constraints:

- Inequality constraints:
  - $V + W - 800 \leq 0$, limitation of empty bottles per week.
  - $\frac{2V}{W} \leq \frac{600}{1200}$ which gives $\frac{2V}{W} \leq \frac{1}{2}$, constraint on sugar limit. Each vine bottle takes 2

times as much sugar as wiskey.

- Equality constraint:
  - $\frac{1}{10}V + \frac{1}{2.5}W - 225 = 0$, we want to ensure that all of the alcohol produced is sold every week.
- Side constraints:
  - $V \geq 0$
  - $W > 0$

Standard form:

- Find:
  - $\vec{x} = [W, V]^T$
- That maximizes the objective function:
  - $Profit = 2W + V$
- Such that:
  - $g_1(\vec{x}) : \ V + W - 800 \leq 0$, limit on available bottles
  - $g_2(\vec{x}) : \ \frac{2V}{W} - \frac{1}{2} \leq 0$, limit on available sugar
  - $h_1(\vec{x}) : \ \frac{1}{10}V + \frac{1}{2.5}W - 225 = 0$, amount of alcohol available per month
  - $V \geq 0$, side constraint
  - $W > 0$, side constraint, do not wish to have a case where $W = 0$ since it may cause contraint $g_2(\vec{x})$ to be undefined.

Problem 15

15. **Transportation problem**. A company has m manufacturing facilities. The facility at the ith location has capacity to produce b_i units of an item. The product should be shipped to n distribution centers. The distribution center at the jth location requires at least a_j units of the item to satisfy demand. The cost of shipping an item from the ith plant to the jth distribution center is c_{ij}. Formulate a minimum-cost transportation system to meet each of the distribution center's demands without exceeding the capacity of any manufacturing facility.

2) Data collection:

- Company has m manufacturing facilities. Each facility at the i^{th} location has the capacity to produce b_i units of an item.
- This product is shipped to n distribution centers.Each distribution center at the j^{th} location reqiures at least a_j units.
- Cost of shipping from the i^{th} manufacturing location to the j^{th} distribution center is c_{ij} per item.
- **Summary of data collection:**
  - There is a vector b_i ranging from $i = 1 \; to \; m$ that contains production capacity of each manufacturing center.
  - There is a vector a_j ranging from $j = 1 \; to \; n$ that contains product quantity minimums for each distribution center.
  - The is a $m \; by \; n$ martix c_{ij} that contains shipping cost data per item from each manufacturing center i to each distribution center j.

  ★ **Formulate the minimum cost transportation system to meet each of the distribution centers demands and not exceed the capacity of any manufacturing facility.**

3) Design variables:

- Design variable in this case will be quantity of product shipped from each manufacturing plant to each distribution center j.
  - x_i^j = quantity of product shipped from each manufacturing facility to each distribution center j.

4) Objective function:

- Goal is to minimize shipping cost, which can be defined as follows:
  - $Cost_j = c_{ij}x_i^j$, goal is to find the optimum vector x_i^j, which will minimize cost for each distribution center j. Since $Cost_j$ is a vector quantity we will track first norm of $Cost_j$ and minimize that (i.e. minimize $||Cost_j||_1$).

5) Constraints:

- Inequality contraints:
  - $\sum_{j=1}^{n} x_i^j \leq b_i \implies \sum_{j=1}^{n} x_i^j - b_i \leq 0$, ensure demand does not exceed available supply.
  - $a_j \geq \sum_{i=1}^{m} x_i^j \implies a_j - \sum_{i=1}^{m} x_i^j \geq 0$, This inequality ensures that each distribution facility receives minimum required amount of product.
- Side constraints:
  - $x_i \geq 0$, make sure that product shipped from each manufacturing facility is a positive number.

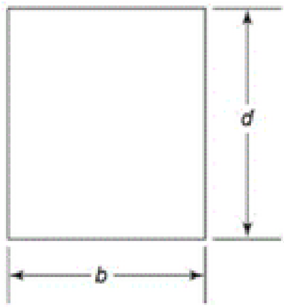
Standard form:

- Find:
  - $\vec{x}^j = [x_1, x_2, x_3 \longrightarrow x_i]^T$
- That minimizes the objective function:
  - $||Cost_j||_1$, where $Cost_j = c_{ij} x_i^j$
- Such that:
  - $g_1(\vec{x}):\ \sum_{j=1}^{n} x_i^j - b_i \leq 0$, limit on available product from each manufacturing facility.
  - $g_2(\vec{x}):\ a_j - \sum_{i=1}^{m} x_i^j \geq 0$, required to meet minimum demand from each distribution facility.
  - $x_i^j \geq 0$, side constraints

Problem 17

17. A beam of rectangular cross-section (Figure E2.17) is subjected to a maximum bending moment of M and a maximum shear of V. The allowable bending and shearing stresses are σ_a and τ_a, respectively. The bending stress in the beam is calculated as

$$\sigma = \frac{6M}{bd^2}$$



The average shear stress in the beam is calculated as $\tau = 3V/2bd$ where d is the depth and b is the width of the beam. It is also desirable to have the beam depth not exceed twice its width. Formulate the design problem for minimum cross-sectional area using this data: M=140 kN•m, V=24 kN, σ_a=165 MPa, τ_a=50 MPa.

Figure E2.17 – Cross section of rectangular beam.

2) Data collection:

- Beam of rectangular cross section is subject to max bending moment (M) and max shear force (v).
- Allowable bending stress is σ_a.
- Allowable shear stress is τ_a .
- Bending stress is calculated by following expression: $\sigma = \frac{6M}{bd^2}$
- Shear stress is calculated by following expression: $\tau = \frac{3V}{2bd}$.
- Here $d = depth$ and $b = width$ of the beam.

★ Formulate the design problem for minimum cross section area using following data:

- $M = 140\ kN \cdot m, V = 24\ kN, \sigma_a = 165\ MPa, \tau_a = 50\ MPa$

3) Design variables:

- d = depth
- b = width

4) Objective function:

- $A = b * d$, this function is to be minimized.

5) Constraints:

- Inequality constraints:
  - $\sigma \leq \sigma_a \Longrightarrow \sigma - \sigma_a \leq 0 \Longrightarrow \frac{6(140)}{bd^2} - 165 \leq 0$, bending stress requirement.
  - $\tau \leq \tau_a \Longrightarrow \tau - \tau_a \leq 0 \Longrightarrow \frac{3(24)}{2bd} - 50 \leq 0$, shear stress requirement.

    - $d \leq 2b \implies d - 2b \leq 0$, aspect ratio constraint
- Side constraints:
    - $b > 0$
    - $d > 0$

Standard form:

- Find:
    - $\vec{x} = [b, d]^T$
- That minimizes the objective function:
    - $A = b * d$, cross-sectional area
- Such that:
    - $g_1(\vec{x}) : \frac{6(140)}{bd^2} - 165 \leq 0$, bending stress requirement
    - $g_2(\vec{x}) : \frac{3(24)}{2bd} - 50 \leq 0$, shear stress requirement
    - $g_3(\vec{x}) : d - 2b \leq 0$, aspect ratio constraint
    - $b > 0 \ \& \ d > 0$, side constraints