Assignment # 2

Section : 1 الاسم : بيتر عاطف فتحي

B.N. : 18 , ID : 9202395

• Problem 2.16 :

a) it's noticed that there are (D+1) degrees of freedom (weights)

so we got → $d_{VC} = D+1$

also, for $\underline{D=1}$: $h_c(X) = \text{sign}\left(\sum_{i=0}^{1} c_i X^i\right)$ (2D)

• 2 weights → $d_{VC} = 2$

Break Point = 3

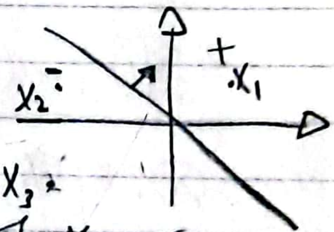
→ To get the (D+1) weights we need D+1 Points to solve this linear system

$\underline{\text{For } N=1}$: $h_c(X_1) = \text{sign}\left(\sum_{i=0}^{1} X_1^i c_i\right)$

so we can get all possible dichotomies

$\underline{\text{For } N=2}$: we can get 2^2 dichotomies

$\underline{\text{For } N=3}$: by adding third Point,



The new Point will get either the sign of X_1 or the sign of X_2 & that happens because there is no bias term so that the streight line must Pass through the origin so there is a distribution for dataset of size 2 that get 2^2 dichotomies but there is no distribution for dataset of size $\underline{3}$ that gives 2^3 dichotomies with a line Passes through the origin so the break Point = 3

so $d_{VC} = 3-1 = 2$

∴ There are (D+1) Points shattered by H

& There are (D+2) Points shattered by H

Problem 2.24 8-

a) $E_{in}(g) = \sum_{i=1}^{2} (f(x_i) - h(x_i))^2$

$= \sum_{i=1}^{2} (x_i^2 - (ax_i + b))^2$

$\frac{\partial E_{in}}{\partial a} = -2 \sum_{i=1}^{2} (x_i^2 - (ax_i + b)) (-x_i)$

$= 2 \sum_{i=1}^{2} x_i (x_i^2 - (ax_i + b))$

$= 2 [x_1 (x_1^2 - (ax_1 + b)) + x_2 (x_2^2 - (ax_2 + b))] = 0$ ①

$\frac{\partial E_{in}}{\partial b} = -2 \sum_{i=1}^{2} (x_i^2 - (ax_i + b)) (-1)$

$= 2 \sum_{i=1}^{2} (x_i^2 - (ax_i + b)) =$

$= 2 [(x_1^2 - (ax_1 + b)) + (x_2^2 - (ax_2 + b))] = 0$ ②

From ① & ② :

$x_1^2 - ax_1 - b = 0$

$x_2^2 - ax_2 - b = 0$

$\therefore a = x_1 + x_2$, $b = -x_1 x_2$

$\therefore g^D(x) = (x_1 + x_2) x - x_1 x_2$

$\therefore \bar{g}(x) = E_D(g^D(x)) = E_D[x_1 x + x_2 x - x_1 x_2]$

$= E_D[x_1 x] + E[x_2 x] - E[x_1 x_2]$

$= E_D[x_1] x + E[x_2] x - \underbrace{E[x_1] E[x_2]}$ ↳ due to independence

$\therefore$ The data is uniformally distributed

$\therefore E_D(x) = 0$

$\therefore \bar{g}(x) = 0$

d) Variance $= E_x[E_D[g^D(x) - \bar{g}(x)^2]]$

$= E_x[E_D((x_1+x_2)x - x_1 x_2)^2]$

$= E_x[E_D((x_1+x_2)^2 x^2 - 2x_1 x_2 (x_1+x_2)x + x_1^2 x_2^2)]$

$= E_x[x^2 E_D[(x_1+x_2)^2] + E_D[x_1^2 x_2^2] - 2x E_D[x_1 x_2 (x_1+x_2)]]$

$E(x) = 0$ $\qquad$ $E(x^3) = 0$

$E(x^2) = 1/3$ $\qquad$ $E(x^4) = 1/5$

$\therefore$ Variance $= E_x[x^2(\frac{1}{3}+\frac{1}{3}) + (\frac{1}{3} * \frac{1}{3})]$

$= E_x[x^2 * \frac{2}{3} + \frac{1}{9}] = \frac{1}{3} * \frac{2}{3} + \frac{1}{9} = \frac{3}{9} = 1/3$

Bias $= E_x[(\bar{g}(x) - f(x))^2] = E_x[(0 - x^2)^2]$

$= E_x[x^4] = \frac{1}{5}$

$\therefore$ E_{out} = Bias + Variance $= \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$