Session: 1
BN: 18

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$\boxed{1.6}$

a) • $\mu = 0.05$

$P(V=0) = (1-0.05)^{10} = 0.598$

• $\mu = 0.5 \rightarrow P(V=0) = (1-0.5)^{10} = 9.76 * 10^{-4}$

• $\mu = 0.8 \rightarrow P(V=0) = (1-0.8)^{10} = 1.024 \times 10^{-7}$

b) • let X be a DRV that representing the number of samples that contain no red balls $(V=0)$

• let y be the event that represents having at least one sample that doesn't contain red balls $(V=0)$

so we want to get $P(X \geq 1) = P(y)$

$\because X$ is a Binomial R.V.

$\therefore P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{n}{x} p^x (1-p)^{n-x}$

$= 1 - \binom{n}{0} p^0 (1-p)^n = 1 - (1-p)^n = P(y)$

• For $\mu = 0.05$,

$p = 0.598 \rightarrow P(y) = 1 - (1-0.598)^{1000} = 1 - 0 = 1$

• For $\mu = 0.5$, $p = 9.76 * 10^{-4}$

$P(y) = 1 - (1 - 9.76 * 10^{-4})^{1000} = 0.6236$

• For $\mu = 0.8$, $p = 1.024 * 10^{-7}$

$\therefore P(y) = 1 - (1 - 1.024 \times 10^{-7})^{1000} = 1.023 \times 10^{-4}$

(c) repeat (b) for $n = 10^6$

• For $\mu = 0.05 \rightarrow p = 0.598$

$P(y) = 1 - (1 - 0.598)^{10^6} = 1$

• For $\mu = 0.5 \rightarrow p = 9.76 \times 10^{-4}$

$P(y) = 1 - (1 - 9.76 \times 10^{-4})^{1000000} = 1$

• For $\mu = 0.8 \rightarrow p = 1.024 * 10^{-7}$

$P(y) = 1 - (1 - 1.024 * 10^{-7})^{1000000} = 0.09733$

[2.5] Prove by induction that:

$\sum_{i=0}^{D} \binom{N}{i} \leq N^{D} + 1$, hence $m_{H}(N) \leq N^{d_{VC}} + 1$

Sol: [1] Base Case:

- For $N = 1$, $D = 0$:

$\sum_{i=0}^{0} \binom{N}{i} = \binom{1}{0} = 1$ (LHS) $\rightarrow$ ①

$N^{D} + 1 = N^{0} + 1 = 2 \rightarrow$ ② (RHS)

$\therefore$ LHS $\leq$ RHS

- For $N = 1$, $D = 1$:

LHS $= \binom{1}{0} + \binom{1}{1} = 1 + 1 = 2$

RHS $= 1^{1} + 1 = 2$

$\therefore$ LHS $\leq$ RHS

[2] Check for N:

assume that $\sum_{i=0}^{D} \binom{N}{i} \leq N^{D} + 1$

- we want to prove: $\sum_{i=0}^{D} \binom{N+1}{i} \leq (N+1)^{D} + 1$

$\because \binom{N}{i-1} + \binom{N}{i} = \binom{N+1}{i}$

$\therefore \sum_{i=0}^{D} \binom{N+1}{i} = \sum_{i=0}^{D} \left[\binom{N}{i} + \binom{N}{i-1} \right]$

$= \sum_{i=0}^{D} \binom{N}{i} + \sum_{j=0}^{D-1} \binom{N}{j}$

$\leq (N^{D} + 1) + (N^{D-1} + 1)$

$\leq (N+1)^{D} + 1$ $\rightarrow$ because $(N+1)^{D} = N^{D} + DN^{D-1} + \dots + 1$

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$\because m_{H}(N) \leq \sum_{i=0}^{d_{VC}} \binom{N}{i}$

$\because \sum_{i=0}^{d_{VC}} \binom{N}{i} \leq N^{d_{VC}} + 1$

$\therefore m_{H}(N) \leq N^{d_{VC}} + 1$ #

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