· Faculty of Engineering
Computer Department
Communications (ELC 325B) – Spring 2023

Assignment 3

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1. Part One

1.1 Gram-Schmidt Orthogonalization

$$E_{s1} = \int_{-\infty}^{\infty} (S_1(t))^2 \, dt = S_1 \, . \, S_1 = 1$$

$$\emptyset_1(t) = \frac{S_1(t)}{\sqrt{E_{s1}}} = S_1(t)$$

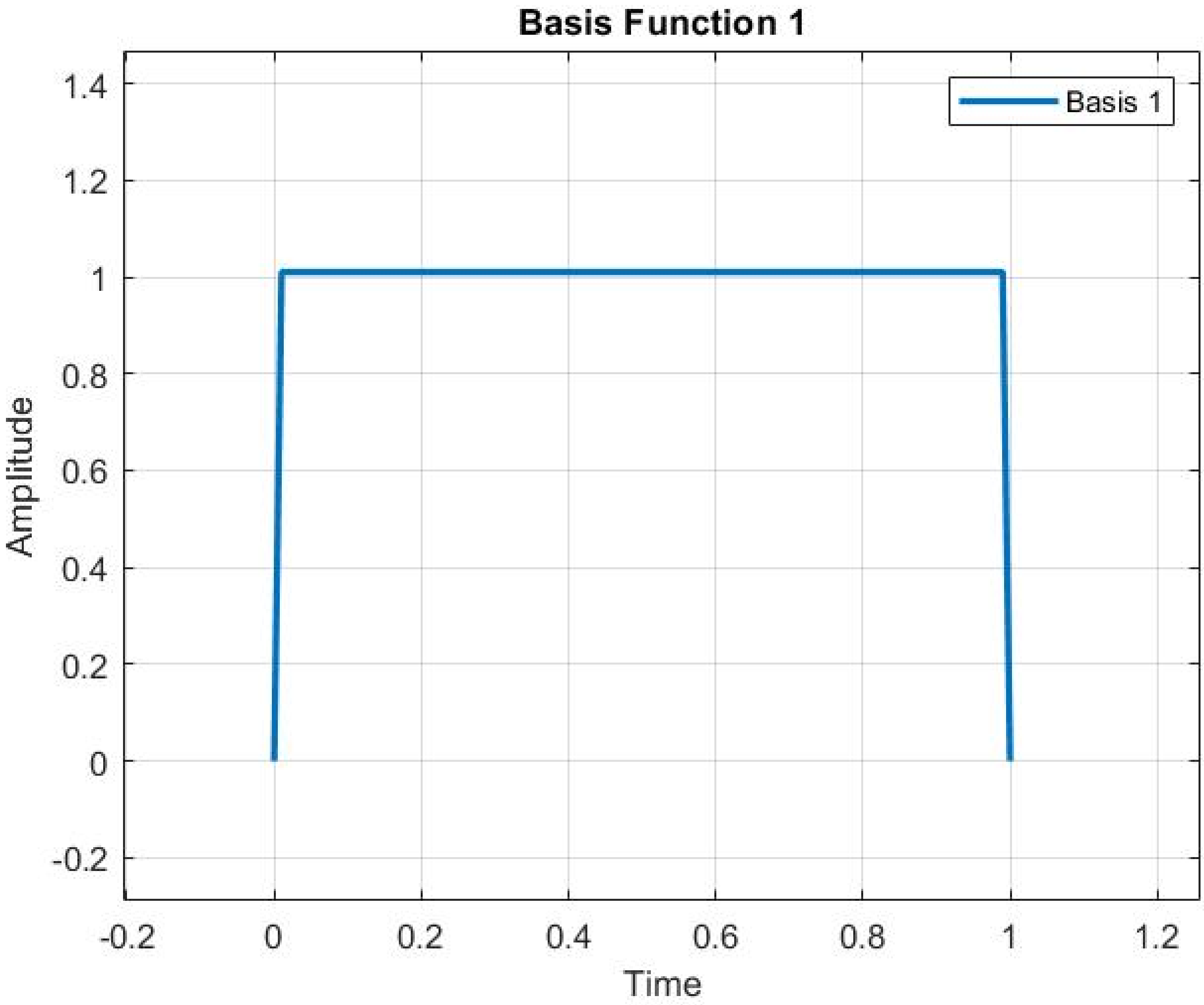


Figure 1 Φ1 VS time after using the GM_Bases function

$$E_{s2} = \int_{-\infty}^{\infty} \left(S_2(t)\right)^2 dt = \int_{0}^{0.75} \left(S_2(t)\right)^2 dt + \int_{0.75}^{1} \left(S_2(t)\right)^2 dt = 0.75 + 0.25 = 1$$

$$s_{21} = \int_{-\infty}^{\infty} S_2(t) * \text{Ø}_1(t)\, dt = \int_{0}^{0.75} 1 * 1\, dt + \int_{0.75}^{1} -1 * 1\, dt = 0.75 - 0.25 = 0.5$$

$$\text{Ø}_2(t) = \frac{S_2(t) - s_{21} * \text{Ø}_1(t)}{\sqrt{E_{s2}}}$$

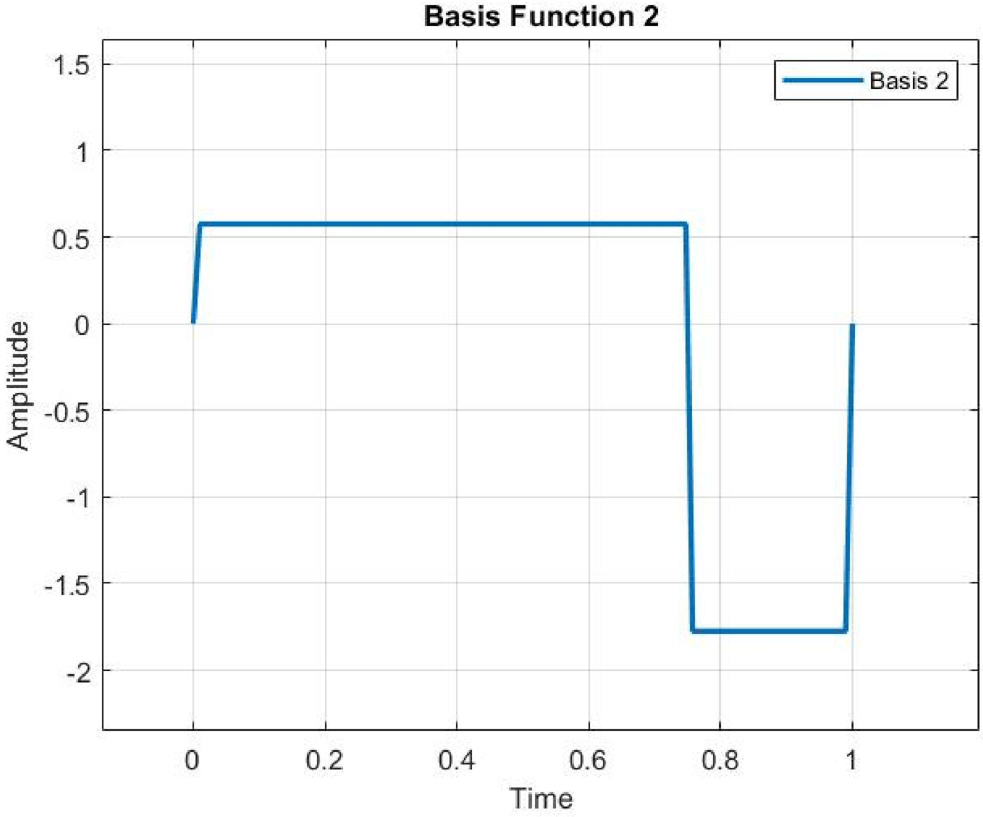


Figure 2 Φ2 VS time after using the GM_Bases function.

Solution verification:

$$\int_{-\infty}^{\infty} (\emptyset_1(t))^2 \, dt = 1$$

$$\int_{-\infty}^{\infty} (\emptyset_2(t))^2 \, dt = \int_{0}^{0.75} \left(\frac{\sqrt{3}}{3}\right)^2 \, dt + \int_{0.75}^{1} \left(\sqrt{3}\right)^2 \, dt = 1$$

$$\int_{-\infty}^{\infty} \emptyset_1(t) * \emptyset_2(t) \, dt = \int_{0}^{0.75} \frac{\sqrt{3}}{3} * 1 \, dt + \int_{0.75}^{1} \sqrt{3} * -1 \, dt = 0$$

So, the bases are orthogonal.

1.2 Signal Space Representation

Here we represent the signals using the base functions.

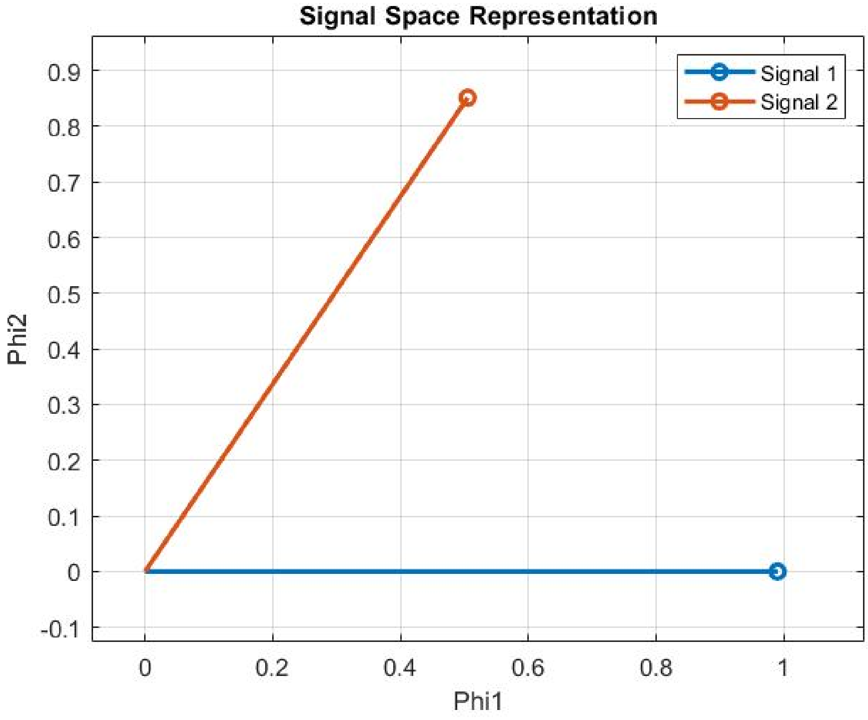


Figure 3 Signal Space representation of signals s1, s2

1.3 Signal Space Representation with adding AWGN

-the expected real points will be solid and the received will be hollow

Case 1: $10\,log(E/\sigma^2)\; = \;10\,dB$

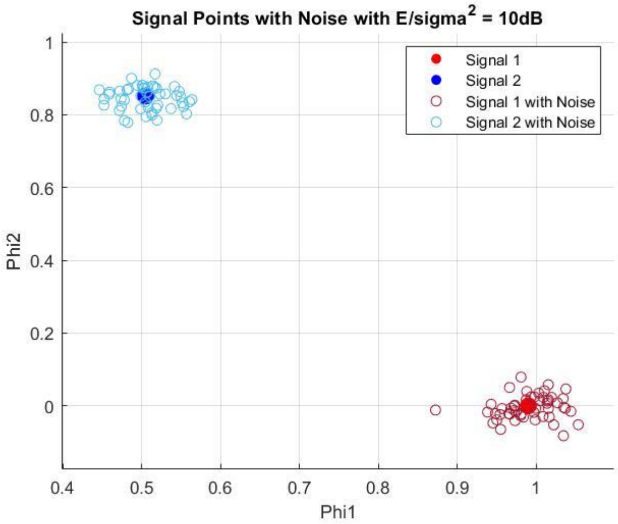


Figure 4 Signal Space representation of signals s1, s2 with E/σ¬2 =10dB

Case 2: $10\,log(E/\sigma^2)\; = \;0\,dB$

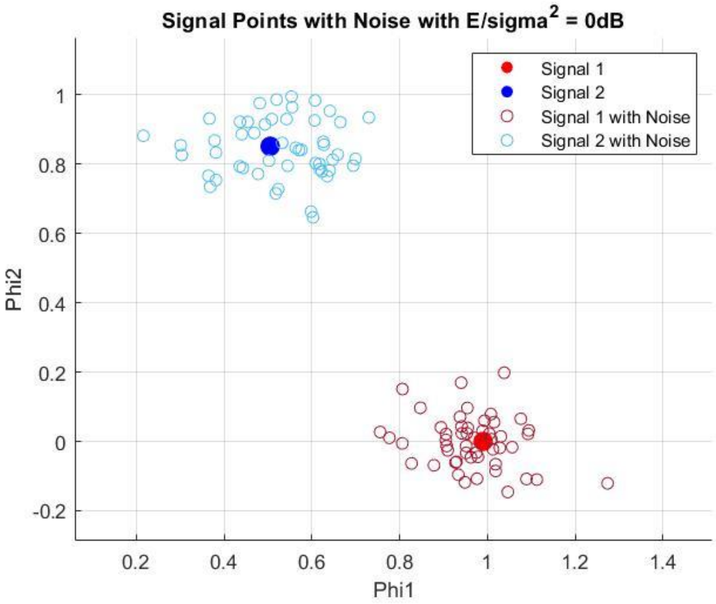


Figure 5 Signal Space representation of signals s1,s2 with E/σ¬2 =0dB

Case 3: $\mathbf{10\, log(E/\sigma^2) = -5\, dB}$

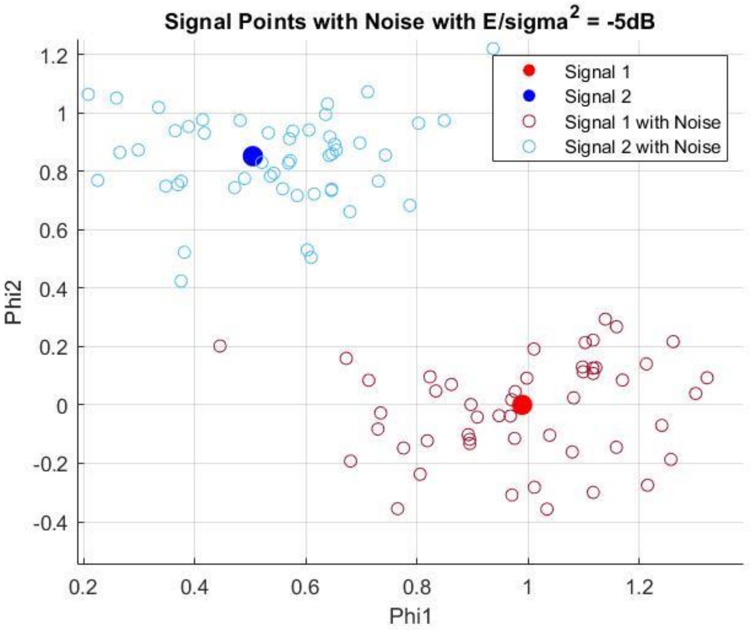


Figure 6 Signal Space representation of signals s1, s2 with E/σ¬2 =-5dB

1.4 Noise Effect on Signal Space

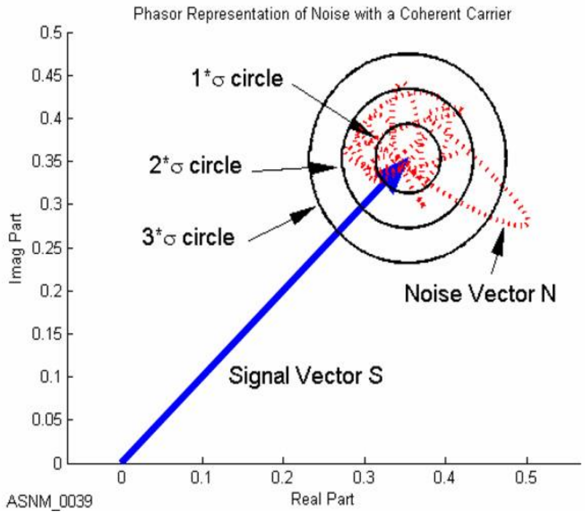
How does the noise affect the signal space?

Receiver will get $X(t) = S_i(t) + w(t)$

Then we find $y_j = \int_0^T X(t) * Ø_j(t)\, dt$

$$y_j = \int_0^T S_i(t) * Ø_j(t)\, dt + \int_0^T w(t) * Ø_j(t)\, dt$$

The receiver will get a signal y_{ij} whose space vector near to the space vector of S_{ij} "the space vector of the sent pulse."



Does the noise effect increase or decrease with increasing σ^2?

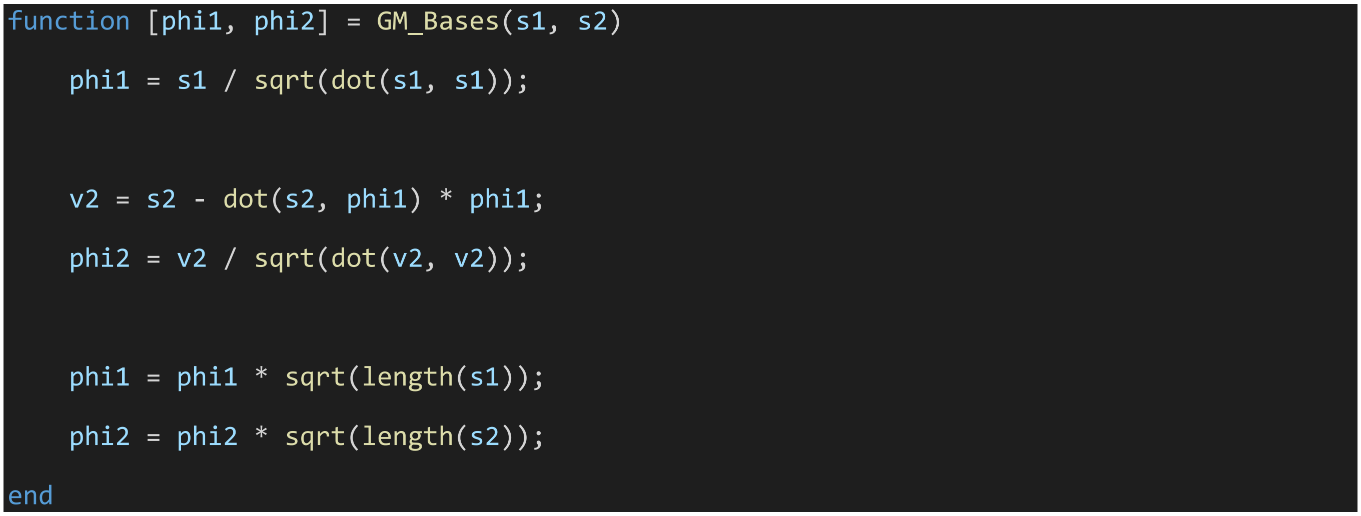
It’s noticed that with the increase of the variance of the noise, the signal to noise ratio decreases then the probability of error increases “the uncertainty increases” as observed from the following image.

References

1. Matlab documentations for rectangularPulse function
2. Wikipedia

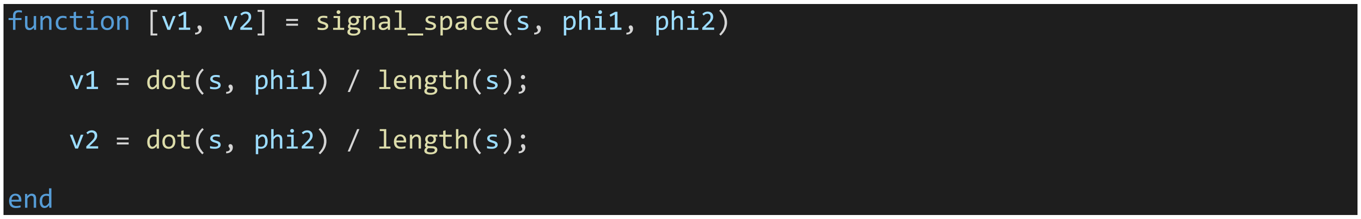
Appendix A: Codes for Part One:

A.1 Code for Gram-Schmidt Orthogonalization



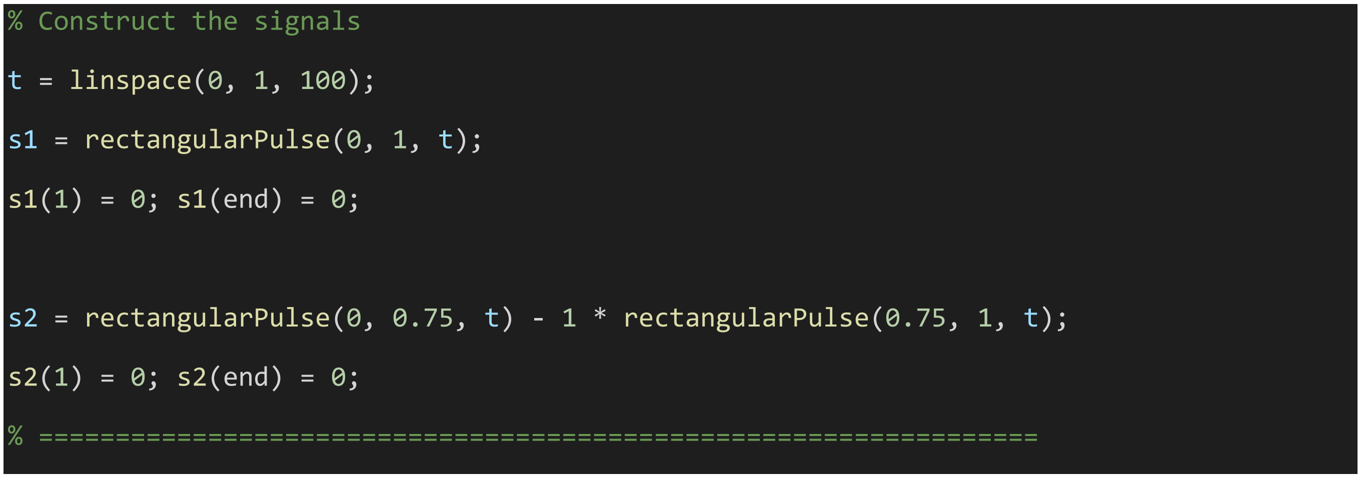
```
function [phi1, phi2] = GM_Bases(s1, s2)
    phi1 = s1 / sqrt(dot(s1, s1));

    v2 = s2 - dot(s2, phi1) * phi1;
    phi2 = v2 / sqrt(dot(v2, v2));

    phi1 = phi1 * sqrt(length(s1));
    phi2 = phi2 * sqrt(length(s2));
end
```

A.2 Code for Signal Space representation



```
function [v1, v2] = signal_space(s, phi1, phi2)
    v1 = dot(s, phi1) / length(s);
    v2 = dot(s, phi2) / length(s);
end
```

A.3 Code for plotting the bases functions



```
% Construct the signals
t = linspace(0, 1, 100);
s1 = rectangularPulse(0, 1, t);
s1(1) = 0; s1(end) = 0;

s2 = rectangularPulse(0, 0.75, t) - 1 * rectangularPulse(0.75, 1, t);
s2(1) = 0; s2(end) = 0;
% =================================================================
% ==============================================================
% REQUIREMENTS 1:
[phi1, phi2] = GM_Bases(s1, s2);

% Plot the signals
figure('Name', 'Basis Functions', 'NumberTitle', 'off');
plot(t, phi1, 'LineWidth', 2);
legend('Basis 1');
xlabel('Time');
ylabel('Amplitude');
title('Basis Function 1');
grid on;

figure('Name', 'Basis Functions', 'NumberTitle', 'off');
plot(t, phi2, 'LineWidth', 2);
legend('Basis 2');
xlabel('Time');
ylabel('Amplitude');
title('Basis Function 2');
grid on;
```

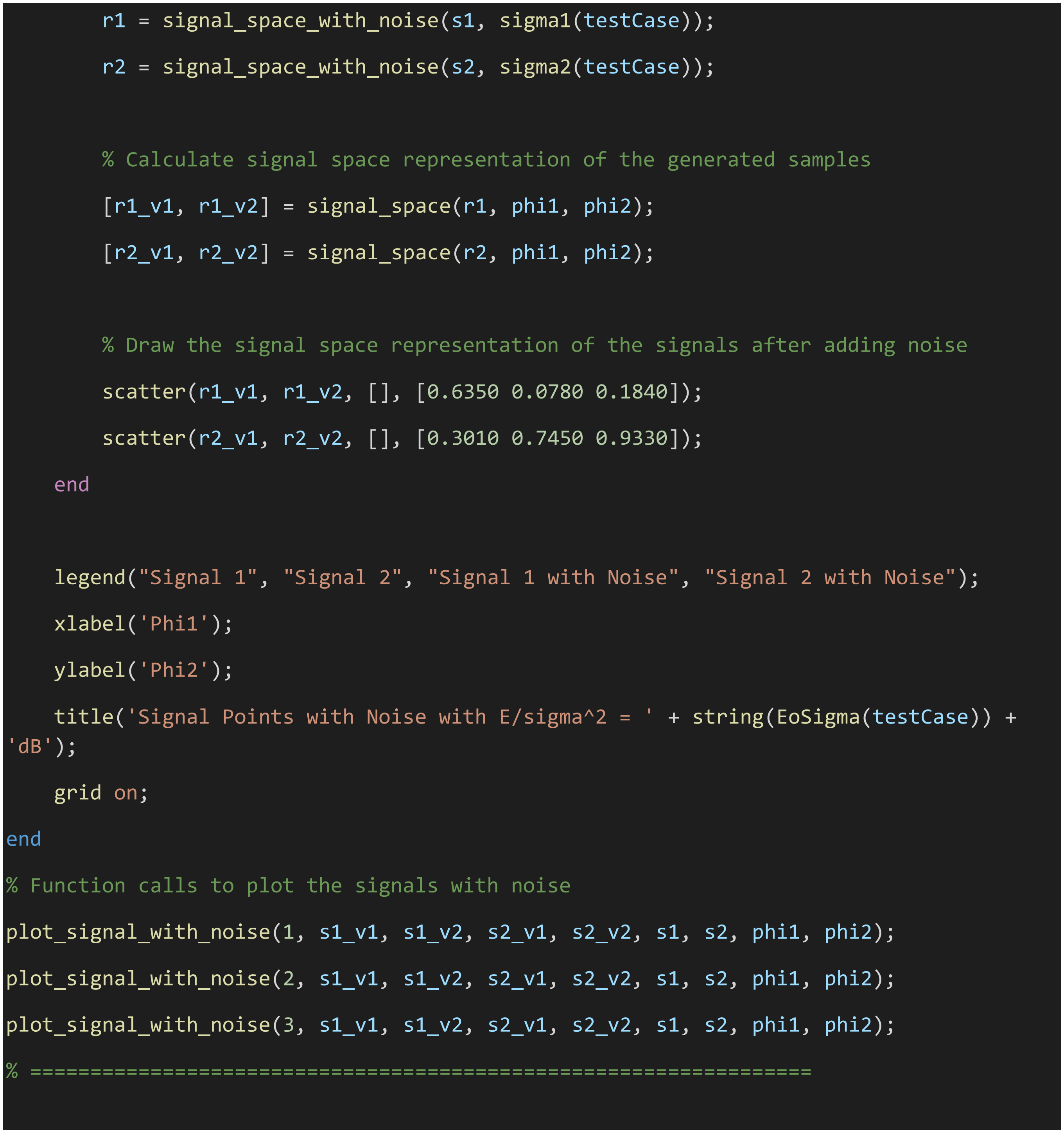
A.4 Code for plotting the Signal space Representations

```
[s1_v1, s1_v2] = signal_space(s1, phi1, phi2);
[s2_v1, s2_v2] = signal_space(s2, phi1, phi2);


% Plot the signal
```

```
figure('Name', 'Signal Space Representation', 'NumberTitle', 'off');

plot([0 s1_v1], [0 s1_v2], '-o', 'MarkerIndices', [2 2], 'LineWidth', 2);

hold on;

plot([0 s2_v1], [0 s2_v2], '-o', 'MarkerIndices', [2 2], 'LineWidth', 2);

legend('Signal 1', 'Signal 2');

xlabel('Phi1');

ylabel('Phi2');

title('Signal Space Representation');

grid on;
```

A.5 Code for effect of noise on the Signal space Representations

```
function plot_signal_with_noise(testCase, s1_v1, s1_v2, s2_v1, s2_v2, s1, s2, phi1,
phi2)

    % Draw the signal space representation of the signals before adding noise

    figure('Name', 'Signal Points with Noise', 'NumberTitle', 'off');

    scatter(s1_v1, s1_v2, 100, 'r', 'filled');

    hold on;

    scatter(s2_v1, s2_v2, 100, 'b', 'filled');


    % E / sigma^2 list in dB

    EoSigma = [-5, 0, 10];

    Es1 = sqrt(dot(s1, s1)) / sqrt(length(s1));

    Es2 = sqrt(dot(s2, s2)) / sqrt(length(s2));


    sigma1 = Es1 ./ db2mag(EoSigma);

    sigma2 = Es2 ./ db2mag(EoSigma);


    for i = 1 : 50
```



```
        r1 = signal_space_with_noise(s1, sigma1(testCase));
        r2 = signal_space_with_noise(s2, sigma2(testCase));

        % Calculate signal space representation of the generated samples
        [r1_v1, r1_v2] = signal_space(r1, phi1, phi2);
        [r2_v1, r2_v2] = signal_space(r2, phi1, phi2);

        % Draw the signal space representation of the signals after adding noise
        scatter(r1_v1, r1_v2, [], [0.6350 0.0780 0.1840]);
        scatter(r2_v1, r2_v2, [], [0.3010 0.7450 0.9330]);
    end

    legend("Signal 1", "Signal 2", "Signal 1 with Noise", "Signal 2 with Noise");
    xlabel('Phi1');
    ylabel('Phi2');
    title('Signal Points with Noise with E/sigma^2 = ' + string(EoSigma(testCase)) + 'dB');
    grid on;
end
% Function calls to plot the signals with noise
plot_signal_with_noise(1, s1_v1, s1_v2, s2_v1, s2_v2, s1, s2, phi1, phi2);
plot_signal_with_noise(2, s1_v1, s1_v2, s2_v1, s2_v2, s1, s2, phi1, phi2);
plot_signal_with_noise(3, s1_v1, s1_v2, s2_v1, s2_v2, s1, s2, phi1, phi2);
% ==================================================================
```