20/OCT/2020

Rosenbluth gain calculation.

EM waves:

$$k = \frac{\omega}{c}\left(1 - n_e/n_c\right)^{1/2} = \frac{1}{c}\left(\omega^2 - \omega_{pe}^2\right)^{1/2}$$

$$\vec{V}_g = \frac{c^2 \vec{k}}{\omega}$$

Electron plasma waves

$$k = \frac{\omega}{V_{Te}}\left(1 - n_e/n_c\right)^{1/2} = \frac{1}{V_{Te}}\left(\omega^2 - \omega_{pe}^2\right)^{1/2}$$

$$\vec{V}_g = \frac{V_{Te}^2 \vec{k}}{\omega_{pe}}$$

Now *locally* we can determine the direction of the density gradient. For what follows, we'll call this the z direction.

The phase mismatch $\vec{K} \equiv \vec{k}_0 - \vec{k}_1 - \vec{k}_2 = 0$ is zero at the matching point.

$$\boxed{\vec{k}_0 = \vec{k}_1 + \vec{k}_2}$$

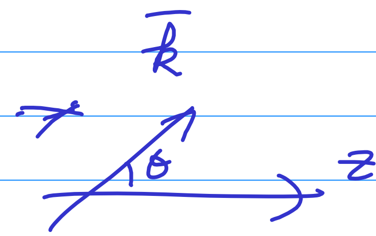
$$K(\vec{x}^*) = 0$$

In our local approximation, the density and hence the wave numbers only change in the z direction.

$$K_z(x^* + \delta\vec{x}) = \overbrace{\frac{\partial}{\partial z}\left(k_{0z} - \underset{\substack{\uparrow \\ \text{EM.}}}{k_{1z}} - \underset{\substack{\uparrow \\ \text{EPW}}}{k_{2z}}\right)}^{\equiv K'} \delta z$$

[Assuming we're not too close to the critical density, the EPW's wave number will change the most rapidly (we can fix this up later).]

$$K' \simeq -\frac{\partial}{\partial z} k_{2z}$$



$$k_{2z} = k_2 \cos\theta$$

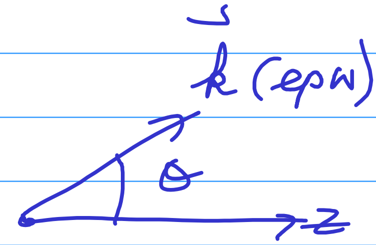
$$K' \simeq -\cos\theta \frac{\partial}{\partial z} k$$

$$= -\cos\theta \frac{\partial}{\partial z}\left[\frac{1}{v_{Te}}\left(\omega^2 - \omega_{pe}^2\right)^{1/2}\right]$$

$$= \cancel{-}\cos\theta \frac{1}{\cancel{2} v_{Te}^{2}} \frac{v_{Te}}{\left(\omega^2 - \omega_{pe}^2\right)^{1/2}}\left[\cancel{-2}\,\omega_{pe} \frac{\partial \omega_{pe}}{\partial z}\right]$$

$$K' = \cos\theta \frac{\omega_{pe}}{v_{Te}^2} \overset{1/|k|}{\frac{v_{Te}}{\left(\omega^2 - \omega_{pe}^2\right)^{1/2}}} \frac{\partial \omega_{pe}}{\partial z}$$

$$k' = \cos\theta \; \overset{1/|v_g|}{\frac{\omega_{pe}}{|k|\, v_{T_e}^2}} \; \frac{\partial \omega_{pe}}{\partial z}$$



$$k' = \frac{\cos\theta}{|v_g|} \frac{\partial \omega_{pe}}{\partial z}$$ — if $\theta = 90°$, we should include contrib. from light wave.

Rosenbluth gain is:

$$G = \frac{2\pi \gamma_0^2}{|v_{g_1} v_{g_2} k'|}$$ EPW.

From the physical picture of convective loss as an equivalent to damping, the group velocities that occur in the above should be the z components.

We said that the plasma wave propagates at the angle θ, so

$$(v_{g_2})_z = v_{g_2} \cos\theta$$

$$|v_{g_2} k'| \longrightarrow |\cancel{v_g}|\,|\cos\theta| \cdot \frac{|\cos\theta|}{|\cancel{v_g}|} \frac{\partial \omega_{pe}}{\partial z}$$

$$= \cos^2\theta \, \frac{\partial \omega_{pe}}{\partial z}$$

The light wave will, in general, propagate at a different angle (ϕ).

$$V_{g1} \rightarrow \frac{c^2 k}{\omega} \cos\phi$$

$$= \underbrace{c\left(1-\frac{n_e}{n_c}\right)^{1/2}}_{V_{light}} \cos\phi .$$

$V_{light} \cos\phi$

Recall the expression for γ_0^2:

EPW wave vector

$$\gamma_0 = \frac{k V_{osc}}{4}\left[\frac{\omega_{pe}^2}{\omega_{pe}(\omega_0-\omega_{pe})}\right]^{1/2}$$

$$= \frac{k V_{osc}}{4}\left[\frac{1}{(n_c/n_e)^{1/2}-1}\right]^{1/2}$$

Just need to square

$$\gamma_0 = \frac{k V_{osc}}{4}\left[\frac{(n_e/n_c)^{1/2}}{1-(n_e/n_c)^{1/2}}\right]^{1/2}$$

$V_{osc} \simeq 8.095 \times 10^{8} \sqrt{I_{15}}\, \lambda_{0,\mu m}$ (cm/sec). ← n_c is for incident light

$$2\pi \gamma_0^2 = 2\pi \frac{k^2 v_{osc}^2}{16} \left[\frac{(n_e/n_c)^{1/2}}{1-(n_e/n_c)^{1/2}} \right]$$

$$= \frac{\pi}{8} k^2 v_{osc}^2 \left[\frac{(n_e/n_c)^{1/2}}{1-(n_e/n_c)^{1/2}} \right]$$

Let's write $\frac{\partial \omega_{pe}}{\partial z}$ in terms of n_e:

$$\frac{\partial \omega_{pe}}{\partial z} = \frac{1}{2\omega_{pe}} \frac{\partial \omega_{pe}^2}{\partial z}$$

$$= \frac{1}{2\cancel{\omega_{pe}}} \cdot \frac{\omega_{pe}^{\cancel{2}}}{n_e} \frac{\partial n_e}{\partial z}$$

$$= \frac{\omega_{pe}}{2} \frac{1}{n_e} \frac{\partial n_e}{\partial z}$$

$$= \frac{\partial}{\partial z} \log_e n_e$$

→ evaluate using $\frac{\partial n_e}{\partial R'}$ $\frac{\partial n_e}{\partial z}$ that we know in code.

Notice that $\left.\frac{1}{n_e} \frac{\partial n_e}{\partial z}\right|_{x^*}$ defines a local scale length

$$\frac{1}{L} = \frac{1}{n_e} \frac{\partial n_e}{\partial z}$$

So, $$\frac{\partial \omega_{pe}}{\partial z} = \frac{\omega_{pe}}{2L}$$

Using this, we get the Rosenbluth gain:

This is the formula

I'll put it in a nicer form at the end.

$$\boxed{G = \frac{\frac{\pi}{8} k^2 V_{osc}^2 \left[\frac{(n_e/n_c)}{1-(n_e/n_c)^{1/2}}\right]^{1/2}}{V_{light} \cos\phi \cos^2\theta \, \frac{\omega_{pe}}{2L}}}$$

[Which diverges when any of the decay waves are normal to the gradient. We should catch this and assign a "marker" gain (e.g. 999). In reality, this can signify absolute instability.]

To check the above, look at low density and backscatter $\phi, \phi = 0$

$$k \to 2k_0 \leftarrow \text{in the plasma}$$

$$G = \frac{\pi}{8} \cdot \frac{4 k_0^2 V_{osc}^2 (n_e/n_c)^{1/2}}{c\, \omega_{pe}/(2L)}$$

threshold.

$$G = \frac{\cancel{\pi}\, k_0^2 V_{osc}^2 (\cancel{\omega_{pe}}/\omega_0)}{2 c\, \cancel{\omega_{pe}}/L} \geq 1 \; \cancel{2\pi}$$

$$\frac{V_{os}^2}{c} \geq \frac{2}{L} \frac{\omega_0}{k_0^2} = \frac{2}{L} \frac{c\, \cancel{k_0}}{k_0^{\cancel{2}}}$$

$$\boxed{\frac{V_{osc}^2}{c^2} \geq \frac{2}{k_0 L}}$$

which is the expression in Kruer's book for the "convective threshold"!

So, it looks ok.

Plug in some numbers:

$$\frac{V_{osc}}{c} \simeq \frac{8.095}{3} \times 10^{-2} \sqrt{I_{15}}\, \lambda_{0,\mu m}$$

$$= 2.70 \times 10^{-2} \sqrt{I_{15}}\, \lambda_{0,\mu m}$$

$$\left(\frac{V_{osc}}{c}\right)^2 = 7.28 \times 10^{-4}\, I_{15}\, \lambda_{0,\mu m}^2$$

For $\lambda = 0.351$ um, this gives

$$\left(\frac{V_{osc}}{c}\right)^2 = 0.897 \times 10^{-4}\ I_{15}$$

$$= 8.97 \times 10^{-5}\ I_{15}$$

$$k_0 = \frac{2\pi}{0.351}\ \mu m^{-1}$$

$$k_0 = 17.9\ \mu m^{-1}$$

We have:

$$8.97 \times 10^{-5}\ I_{15} > \frac{2}{(17.9)\,L}$$

$$L > \frac{2}{(8.97\times 10^{-5})\cdot(17.9)\ I_{15}}\ \mu m$$

$$\boxed{\begin{aligned} L &\gtrsim \frac{2\times 623}{I_{15}}\ \mu m \\ &\gtrsim 1250/I_{15}\ \mu m \end{aligned}}$$

Nicer form of the gain for implementation

$$G = \frac{\frac{\pi}{8} k^2 V_{osc}^2 \dfrac{\overbrace{(n_e/n_c)^{1/2}}^{= \omega_{pe}/\omega_0}}{1-(n_e/n_c)^{1/2}}}{c\,(1-n_e/n_c)^{1/2} \cos\phi \cos^2\theta \dfrac{\omega_{pe}}{2L}}$$

$\alpha(n_e/n_c) = \cancel{\left[\dfrac{1}{1-(n_e/n_c)^{1/2}}\right]} \cdot \left[\dfrac{1}{(1-n_e/n_c)^{1/2}}\right]$

$$G = \frac{\overset{4}{\cancel{\frac{\pi}{8}}} k^2 V_{osc}^2 \cancel{\omega}_{pe}/\omega_0\, \alpha}{c \cos\phi \cos^2\theta \dfrac{\cancel{\omega}_{pe}}{\cancel{2}L}}$$

$$G = L \frac{\pi}{4} \frac{k^2 V_{osc}^2}{\omega_0 c} \frac{\alpha}{\cos\phi \cos^2\theta}$$

$\beta = \cos\phi \cos^2\theta$

$$G = \frac{\pi}{4} L \left(\frac{V_{osc}^2}{c^2}\right) \frac{c k^2}{\omega_0} \frac{\alpha(n_e/n_c)}{\beta(\theta,\varphi)}.$$

dimless EPW wave vector

$$\boxed{G = \frac{\pi}{4} \left(\frac{\omega_0 L}{c}\right) \left(\frac{V_{osc}}{c}\right)^2 \left(\frac{c^2 k^2}{\omega_0^2}\right) \frac{\alpha(n_e/n_c)}{\beta(\theta,\varphi)}}$$

dimensionless scale length

Recall:

Compute from rayGd $d\text{Log}Ne\ dz$
$d\text{Log}Ne\ dn$

$$L = \left[\frac{1}{n_e}\frac{\partial n_e}{\partial z}\right]^{-1}$$

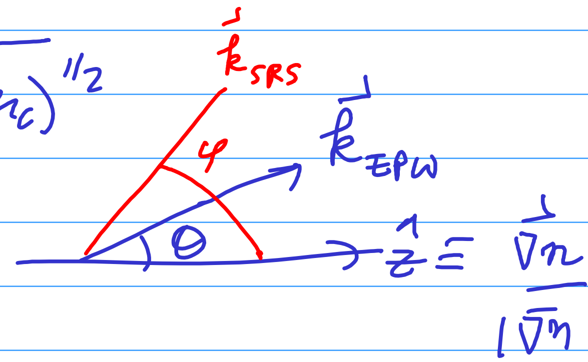
know   know

$$\left(\frac{V_{osc}}{c}\right)^2 = 8.97\times10^{-5}\ I_{15}\left(\frac{\lambda_0^2}{351\,nm^2}\right)$$

use interpon Traj

$$\alpha = \frac{1}{\left(1-n_e/n_c\right)^{1/2}\left(1-n_e/n_c\right)^{1/2}}$$

$$\beta = \cos\varphi\cos^2\theta$$



must compute after calculating the direction of $\vec{\nabla} n$.

$$\cos\theta = \hat{k}\cdot\hat{\nabla n}$$

$$\cos\varphi = \hat{k}_s\cdot\hat{\nabla n}.$$

So it looks like we have everything that we need. since k is known.