

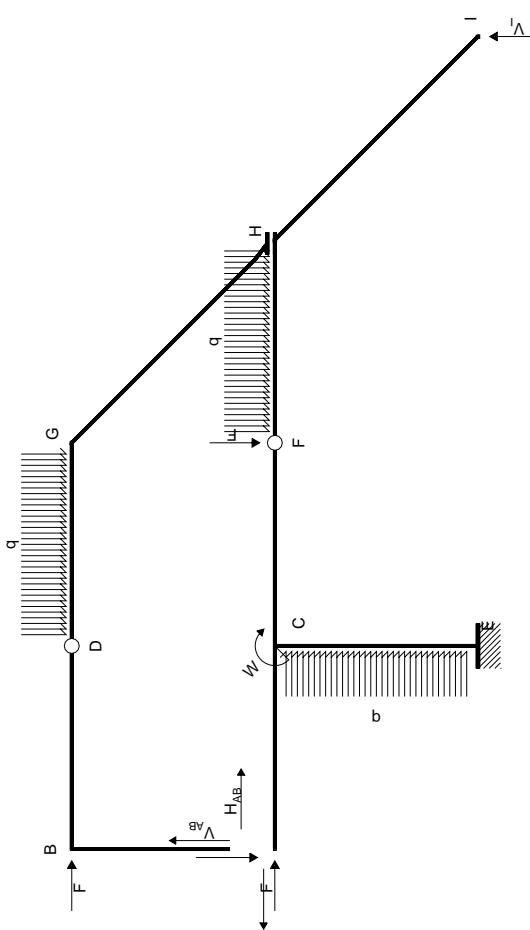
Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
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## EQUILIBRIO Nome:

## Struttura Isostatica.001

## REAZIONI Nome:

## Struttura Isostatica.001



### EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FH HG HI GD DB BA

$$2V_{AB} - 2V_{AB}b = Fb$$

Traslazione orizzontale: aste HG GD DB BA

$$H_{AB} - F = 0$$

Rotazione intorno a D: aste DB BA  
 $H_{AB}b - V_{AB}b = 0$

### Matrice di equilibrio

$$\begin{bmatrix} V_B & H_{AB} & V_{AB} \end{bmatrix} = \begin{bmatrix} Fb & W & qb^2 \end{bmatrix}$$

$$\begin{bmatrix} \varphi_{FC} \\ u_{HG} \\ \varphi_{DB} \end{bmatrix} = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

### Soluzione del sistema

$$\begin{bmatrix} V_B \\ H_{AB} \\ V_{AB} \end{bmatrix} = \begin{bmatrix} Fb \\ -1/2 \\ 0 \end{bmatrix}$$

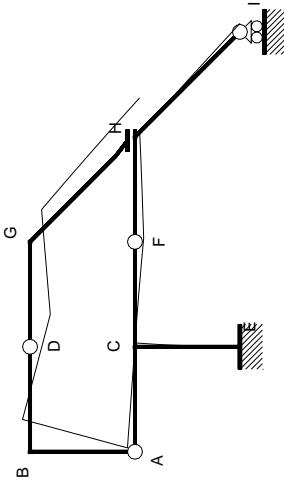
$$\begin{bmatrix} \varphi_{FC} \\ u_{HG} \\ \varphi_{DB} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Fb \\ 0 \\ 0 \end{bmatrix}$$

## DEFORMATA E AZIONI INTERNE Nome:

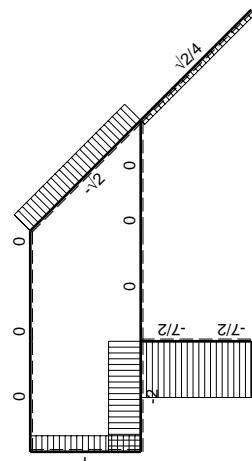
## Struttura Isostatica.001

## PROCEDIMENTO E RISULTATI Nome:

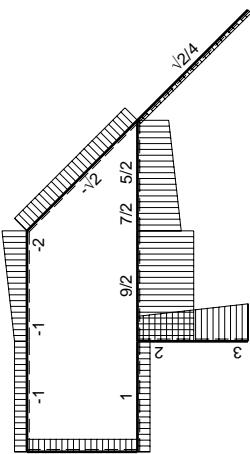
## Struttura Isostatica.001



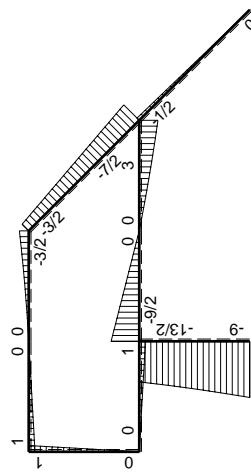
$\rightarrow 80 Fb^3/EJ$



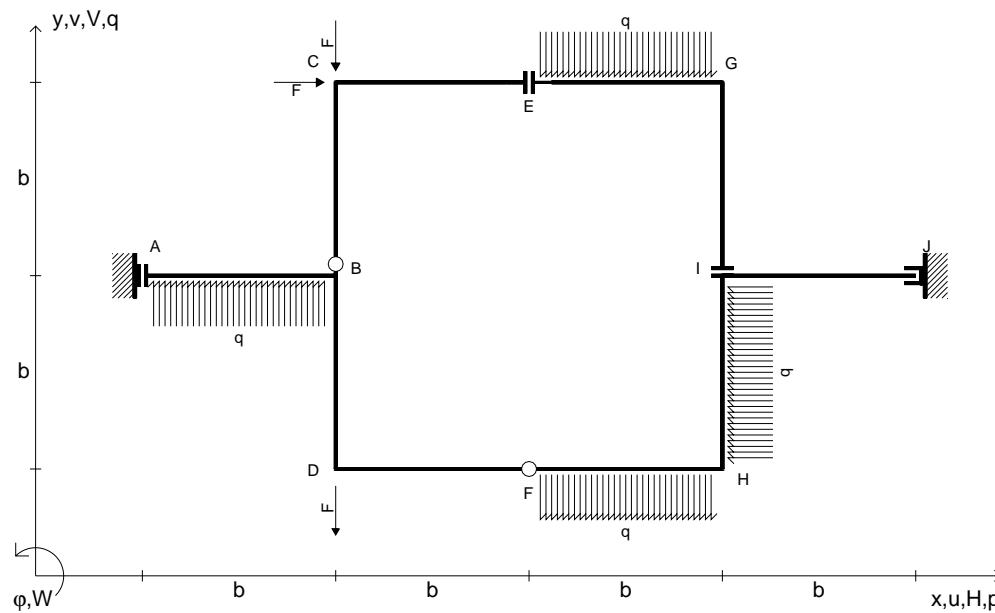
$\rightarrow F$



$\uparrow F$



$\zeta \left( \begin{array}{c} + \\ - \end{array} \right) F_b$



$$\begin{array}{llllll}
 H_C = F & q_{AB} = q = F/b & p_{HI} = -q = -F/b & EJ_{BD} = EJ & EJ_{EG} = EJ & EJ_{HI} = EJ \\
 V_C = -F & q_{EG} = -q = -F/b & EJ_{AB} = EJ & EJ_{CE} = EJ & EJ_{FH} = EJ & EJ_{IJ} = EJ \\
 V_D = -F & q_{FH} = -q = -F/b & EJ_{BC} = EJ & EJ_{DF} = EJ & EJ_{GI} = EJ &
 \end{array}$$

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Tracciare i diagrammi quotati delle azioni interne nelle aste.

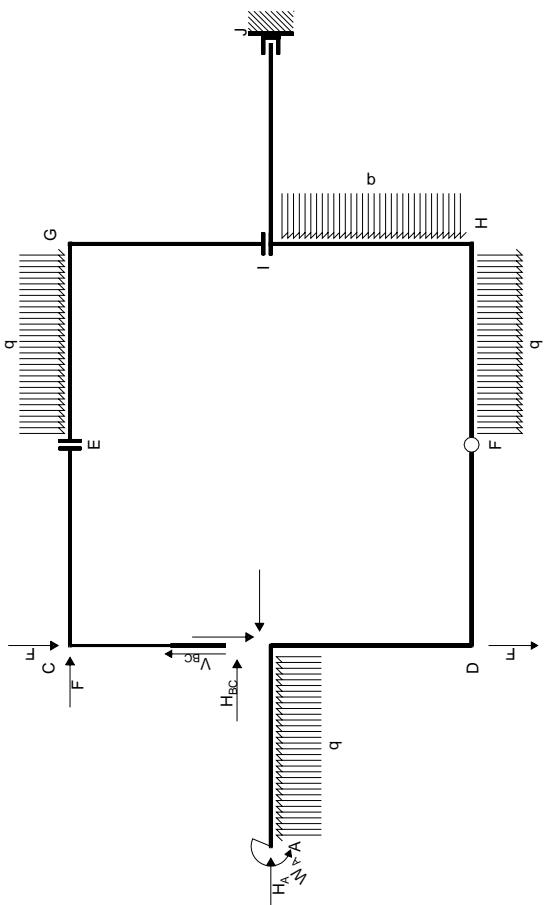
$J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

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09.02.16

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09.02.16



EQUAZIONI DI EQUILIBRIO

Traslazione orizzontale globale

Traslazione orizzontale: asse  $\text{IG}_A$

$$H_{BC} = -F$$

Traslazione verticale: aste EC CB

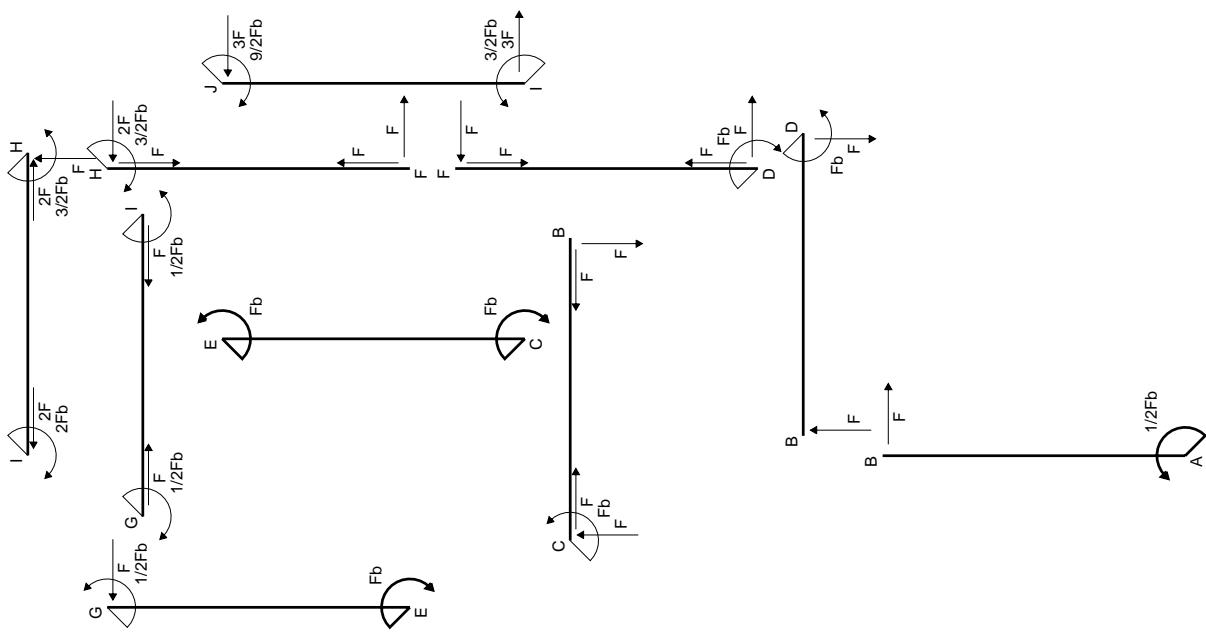
$$V_{BC} = F$$

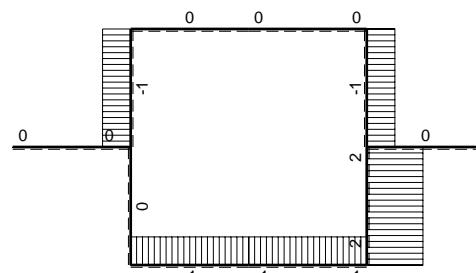
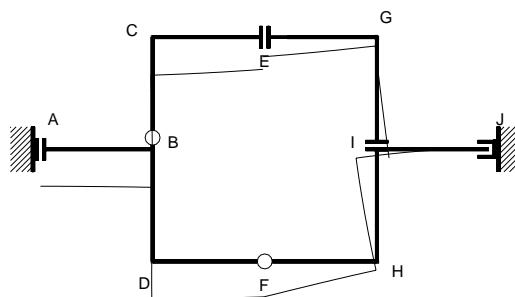
Rotazione intorno a F: aste FD DB BA

$$-H \cdot h + W_1 + H_{ab} + V_{ab} = -Fb + 3/2 q b^2$$

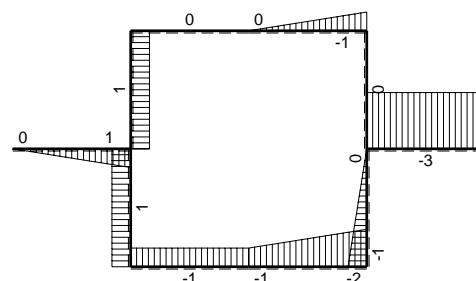
$$\text{Matrice di equilibrio} \quad \begin{bmatrix} H_{AB} & W_A & H_{BC}b & V_{BC}b \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} Fb & qb^2 \\ -1 & 1 \\ -1 & 0 \\ 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\begin{array}{l} \text{Soluzione del sistema} \\ \left[ \begin{array}{c} F_b \\ q b^2 \end{array} \right] = \left[ \begin{array}{cc} -1 & 1 \\ -1 & 0 \\ 1 & 0 \\ -2 & 5/2 \end{array} \right] \left[ \begin{array}{c} H_A b \\ H_{B C} b \\ V_{B C} b \\ W_A \end{array} \right] \end{array}$$

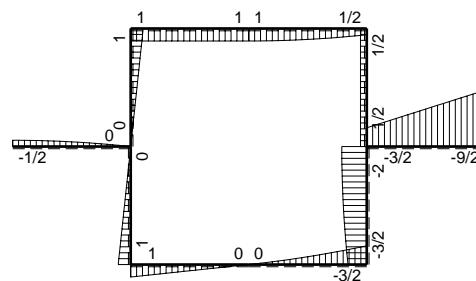




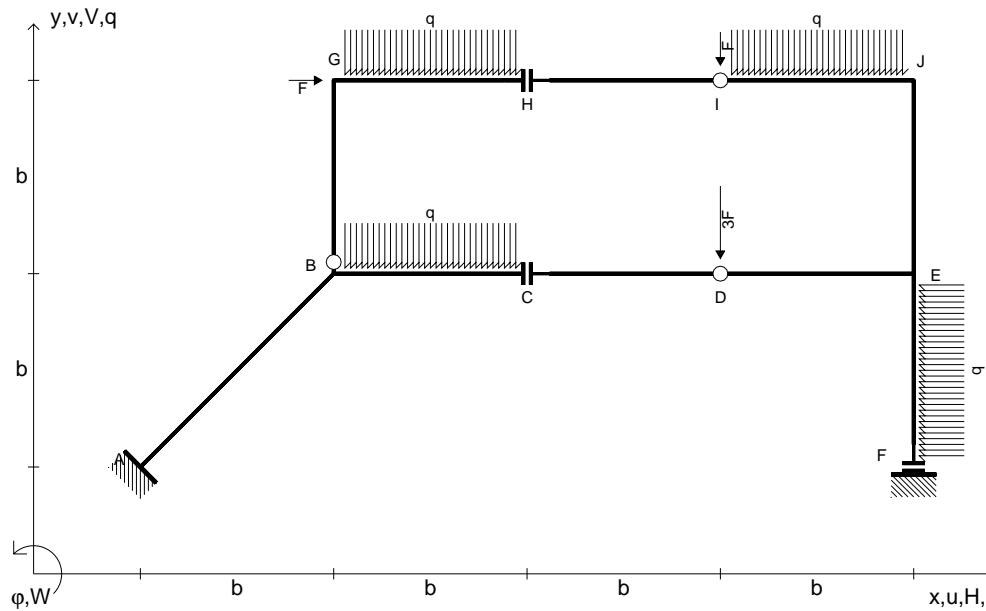
$\leftarrow \boxed{+} \rightarrow F$



$\uparrow \boxed{+} \downarrow F$



$\zeta \boxed{+} \zeta Fb$



$$\begin{array}{lllll}
 H_G = F & q_{IJ} = -q = -F/b & EJ_{BC} = EJ & EJ_{BG} = EJ & EJ_{JE} = EJ \\
 V_I = -F & q_{GH} = -q = -F/b & EJ_{CD} = EJ & EJ_{GH} = EJ & \\
 V_D = -3F & p_{EF} = -q = -F/b & EJ_{DE} = EJ & EJ_{HI} = EJ & \\
 q_{BC} = -q = -F/b & EJ_{AB} = EJ & EJ_{EF} = EJ & EJ_{IJ} = EJ &
 \end{array}$$

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Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi quotati delle azioni interne nelle aste.

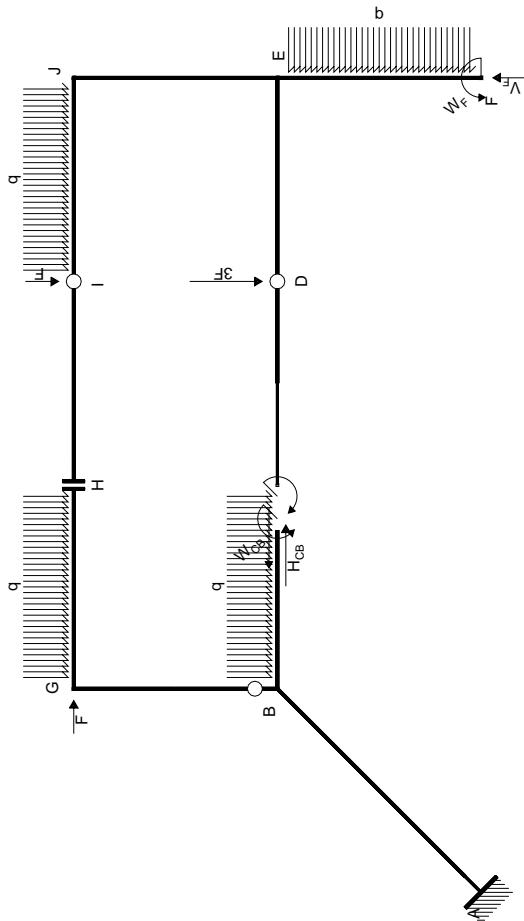
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27.06.16

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27.06.16



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a B: aste BG

$$3V_F b + W_F - W_{CB} = 9Fb + 7/2qb$$

Traslazione verticale:  $V_E = 4F + qb$

Rotazione intorno a  $\vec{a}$ : aste |  $\cdot$  | E ED EE DC

$$V_F b + W_F - H_{CB} b - W_{CB} = 2qb^2$$

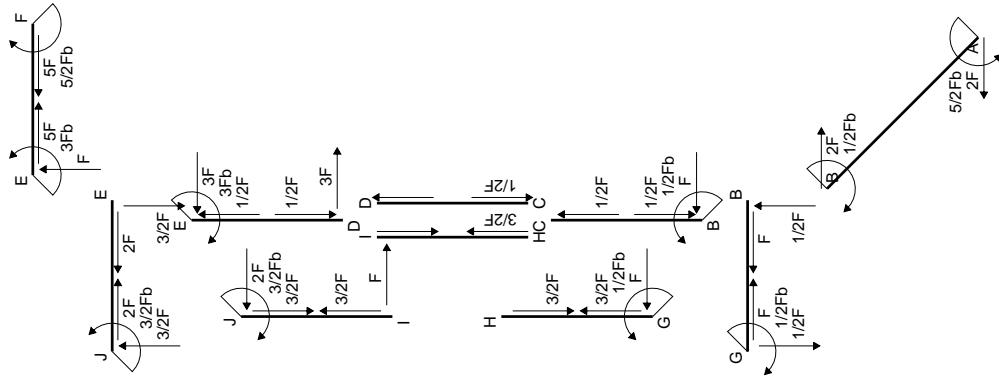
Rotazione intorno a D: aste DC

$$-W_{CB} = 0$$

Matrice di equilibrio

$$\text{Matrice d'équilibrio} \quad \begin{bmatrix} V_{pb} & W_F & W_{cb} & W_{cb} \\ \Phi_{BG} & 3 & 1 & 0 \\ V_{HG} & 1 & 0 & 0 \\ \Phi_{IH} & 1 & 1 & -1 \\ \Phi_{OC} & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} F_B & qB^2 \\ 9 & 7/2 \\ 4 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$\text{Soluzione del sistema} \quad \begin{bmatrix} F_b \\ q b^2 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -3 & 1/2 \\ 1 & -1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_F b \\ W_F \\ H_{CB} b \\ W_{CR} \end{bmatrix}$$

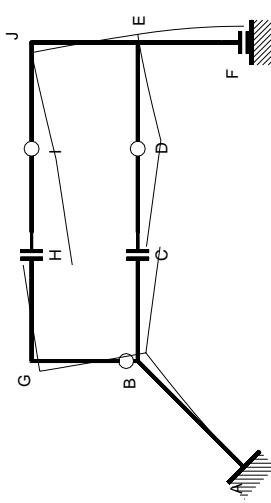


## DEFORMATA E AZIONI INTERNE Nome:

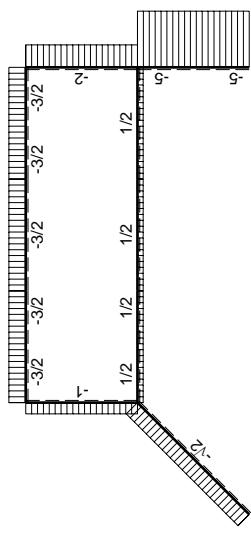
## Struttura Isostatica.001

## PROCEDIMENTO E RISULTATI Nome:

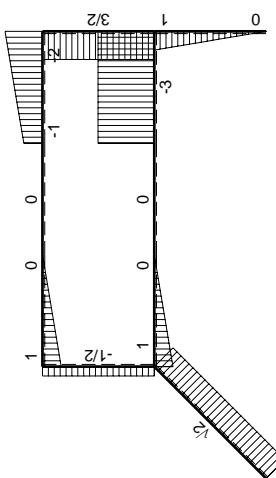
## Struttura Isostatica.001



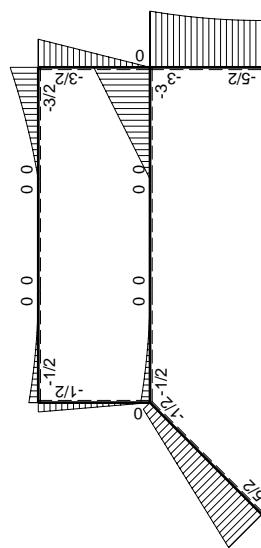
$\rightarrow -12 Fb^3/EJ$



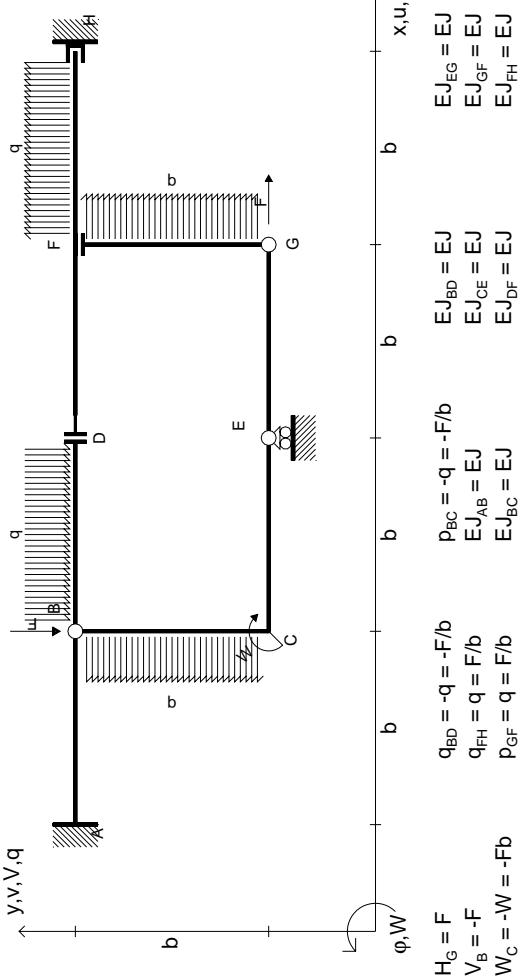
$\leftarrow \boxed{+} \rightarrow F$



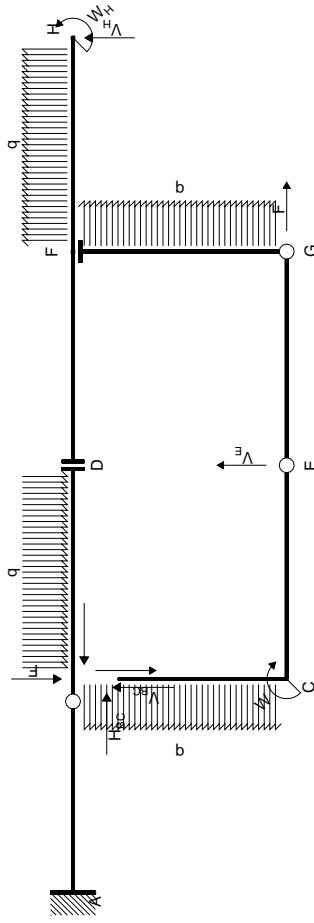
$\uparrow \boxed{+} \downarrow F$



$\zeta \boxed{+} F_b$



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## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a B: asta BD DF FG FH GE EC CB

$$V_E b + 3V_H b + W_H = -Fb + W - 2qb^2$$

Traslazione verticale: asta DF FG FH GE EC CB

$$V_E + V_H + V_{BC} = -qb$$

Traslazione orizzontale: asta FG GE EC CB

$$H_{BC} = -F$$

Rotazione intorno a G: asta GE EC CB

$$-V_E b - H_{BC} b - 2V_{BC} b = W - 1/2qb^2$$

Rotazione intorno a E: asta EC CB

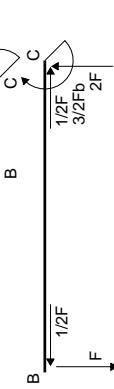
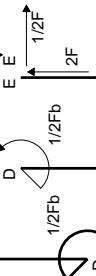
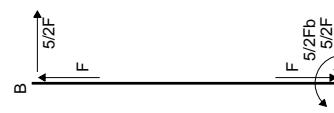
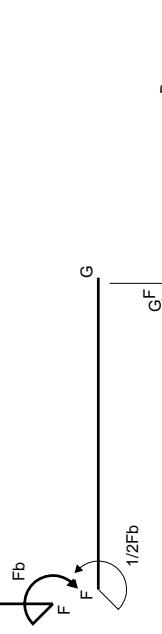
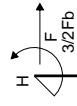
$$-H_{BC} b - V_{BC} b = W - 1/2qb^2$$

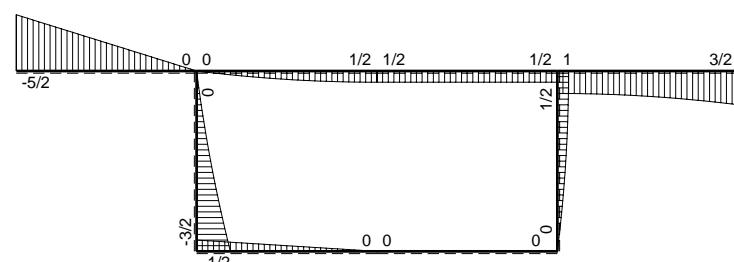
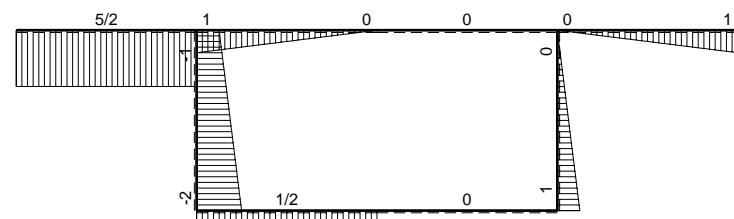
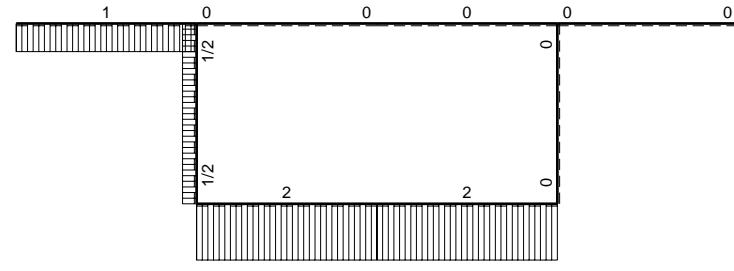
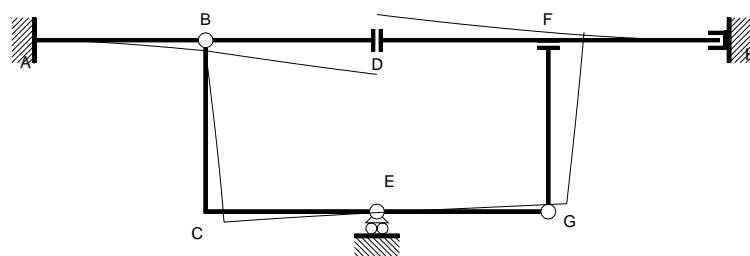
$$\text{Matrice di equilibrio}$$

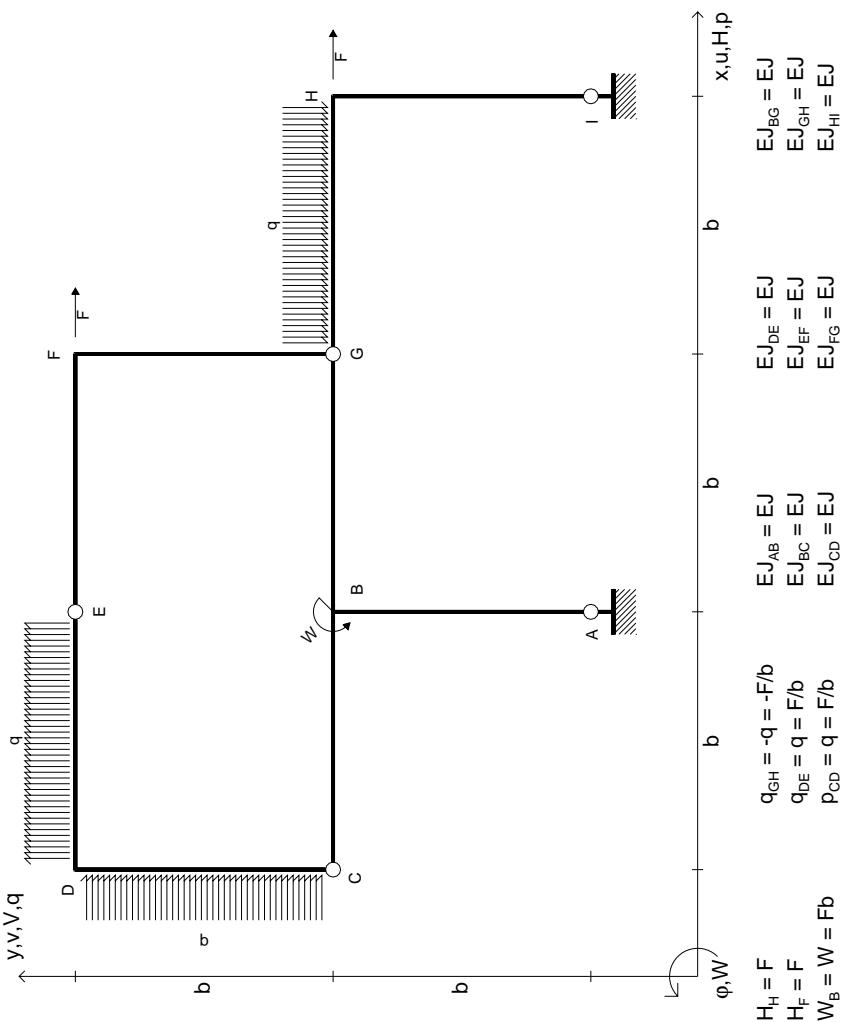
$$\begin{bmatrix} V_E b & V_H b & W_H & H_{BC} b & V_{BC} b \end{bmatrix} \begin{bmatrix} F_b \\ W \\ qb^2 \\ -1 & 1 & -2 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 0 & 3/2 \\ 1 & -1 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$$

$$\text{Soluzione del sistema}$$

$$\begin{bmatrix} V_E b \\ V_H b \\ H_{BC} b \\ W_H \\ V_{BC} b \end{bmatrix} = \begin{bmatrix} -1 & 1 & -1/2 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 0 & 3/2 \\ 1 & -1 & 1/2 \end{bmatrix}$$



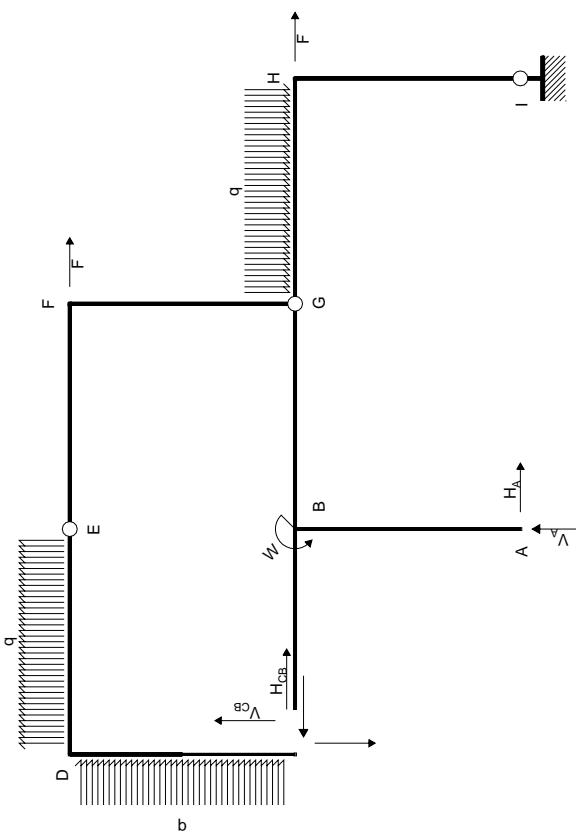




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## EQUILIBRIO Nome:

## Struttura Isostatica.001



### EQUAZIONI DI EQUILIBRIO

Rotazione globale intorno a I

$$-2V_A b = 3Fb - W + 7/2qb^2$$

Rotazione intorno a G: asta GF FE ED DC

$$2V_B b = Fb + 2qb^2$$

Rotazione intorno a G: asta GB BA BC

$$H_A b - V_A b - 2V_B b = -W$$

Rotazione intorno a E: asta ED DC

$$-H_{CB} b + V_{CB} b = 0$$

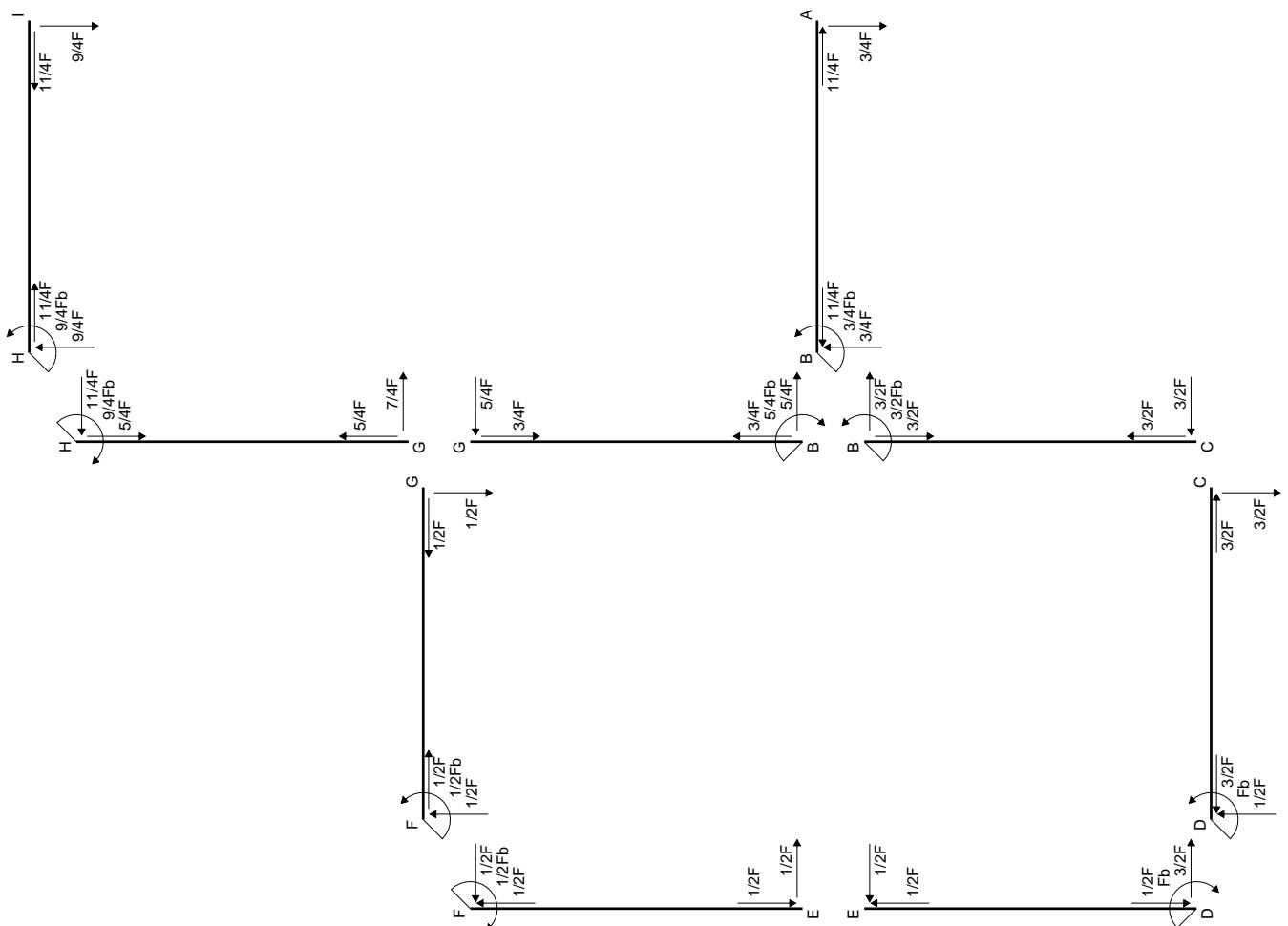
### Matrice di equilibrio

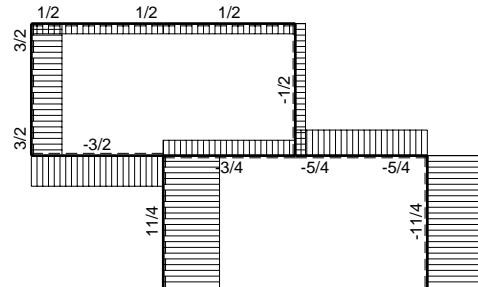
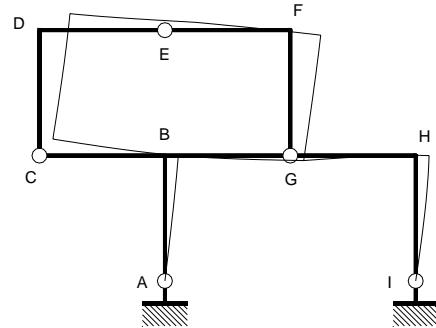
$$\Phi_1 \begin{bmatrix} H_A & V_A & H_{CB} & V_{CB} \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & -1 & 0 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} Fb & W & qb^2 \\ 3 & -1 & 7/2 \\ 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Soluzione del sistema} \\ \begin{bmatrix} V_A \\ V_B \\ V_{CB} \\ H_A \\ H_{CB} \end{bmatrix} = \begin{bmatrix} Fb & W & qb^2 \\ -3/2 & 1/2 & -7/4 \\ 1/2 & 0 & 1 \\ -1/2 & -1/2 & 1/4 \\ 1/2 & 0 & 1 \end{bmatrix}$$

## REAZIONI Nome:

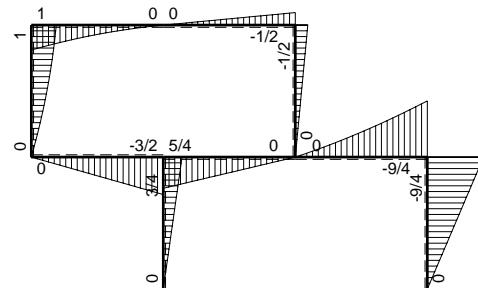
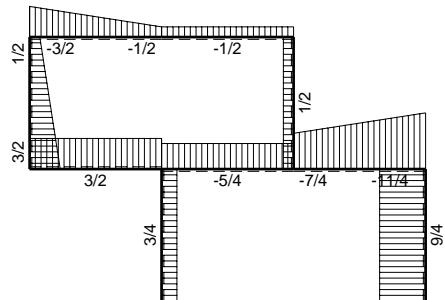
## Struttura Isostatica.001





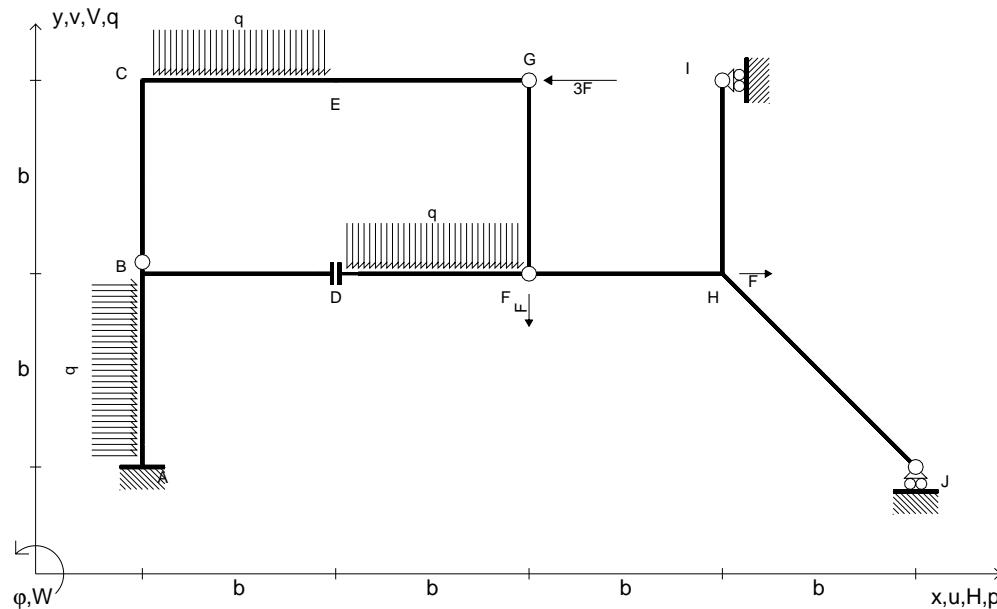
$\boxed{+}$   $6 Fb^3/EJ$

$\leftarrow \boxed{+} \rightarrow F$



$\uparrow \boxed{+} \downarrow F$

$\curvearrowleft \boxed{+} \curvearrowright Fb$



$$\begin{array}{llllll}
 H_H = F & p_{AB} = q = F/b & EJ_{AB} = EJ & EJ_{CE} = EJ & EJ_{GF} = EJ & EJ_{HJ} = EJ \\
 H_G = -3F & q_{DF} = -q = -F/b & EJ_{BC} = EJ & EJ_{DF} = EJ & EJ_{FH} = EJ & \\
 V_F = -F & q_{CE} = -q = -F/b & EJ_{BD} = EJ & EJ_{EG} = EJ & EJ_{HI} = EJ &
 \end{array}$$

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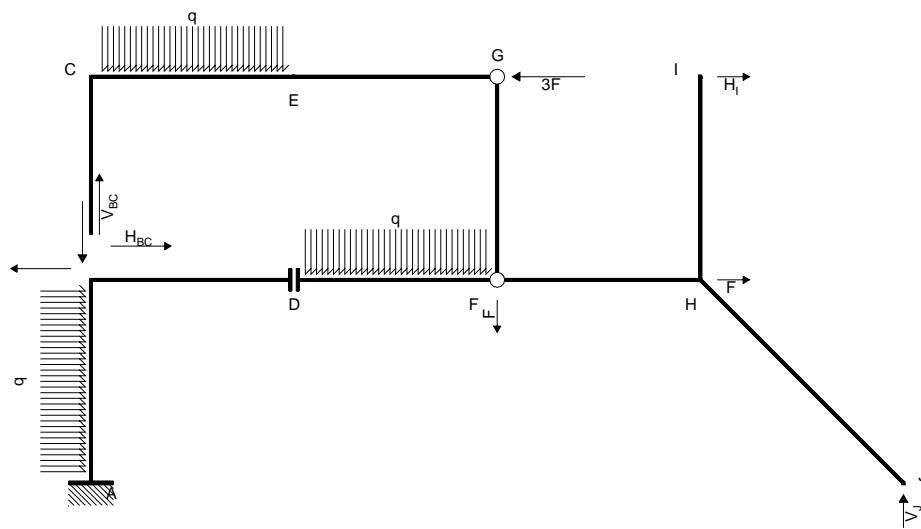
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19.11.15

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19.11.15



## EQUAZIONI DI EQUILIBRIO

Traslazione verticale: aste DF FG FH GE HI HJ EC CB

$$V_J + V_{BC} = F + 2qb$$

Rotazione intorno a F: aste FG FH GE HI HJ EC CB

$$-H_I b + 2V_J b - 2V_{BC} b = -3Fb - 3/2qb^2$$

Rotazione intorno a F: aste FG GE EC CB

$$-2V_{BC} b = -3Fb - 3/2qb^2$$

Rotazione intorno a G: aste GE EC CB

$$H_{BC} b - 2V_{BC} b = -3/2qb^2$$

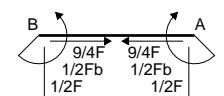
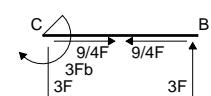
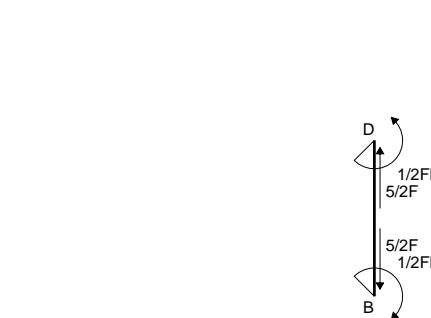
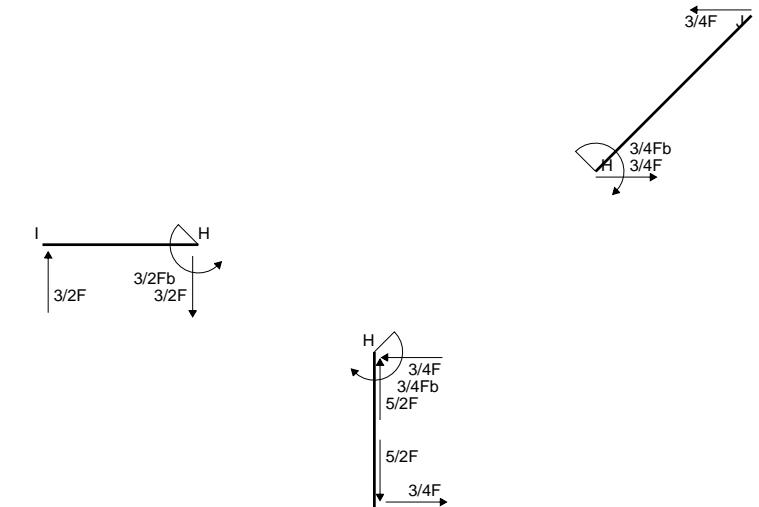
## Matrice di equilibrio

$$\begin{bmatrix} H_I b & V_J b & H_{BC} b & V_{BC} b \end{bmatrix} = \begin{bmatrix} Fb & qb^2 \end{bmatrix}$$

$$\begin{bmatrix} V_{DB} \\ \Phi_{FD} \\ \Phi_{FG} \\ \Phi_{GE} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 2 & 0 & -2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Fb & qb^2 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_J b \\ H_I b \\ V_{BC} b \\ H_{BC} b \end{bmatrix} = \begin{bmatrix} Fb & qb^2 \\ -1/2 & 5/4 \\ -1 & 5/2 \\ 3/2 & 3/4 \\ 3 & 0 \end{bmatrix}$$

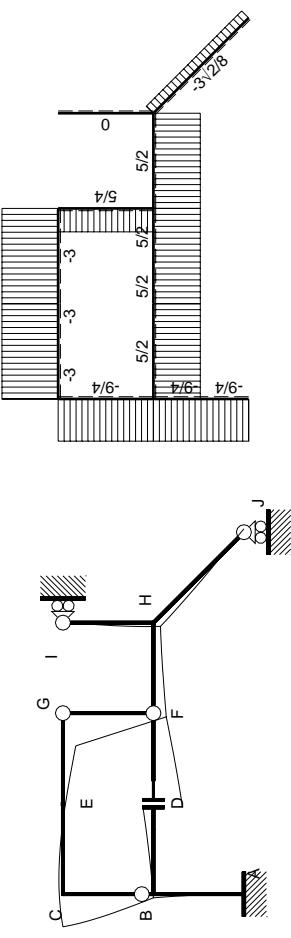


## DEFORMATA E AZIONI INTERNE Nome:

## Struttura Isostatica.001

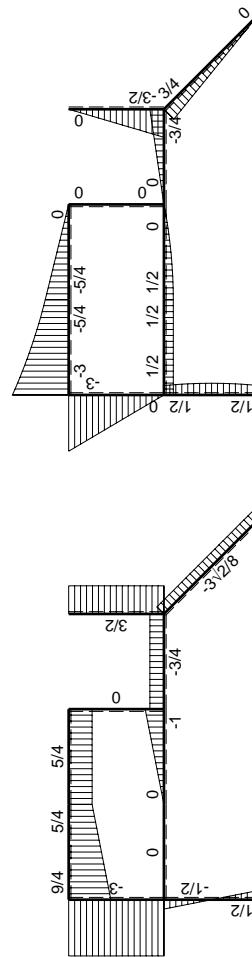
## PROCEDIMENTO E RISULTATI Nome:

## Struttura Isostatica.001



$\rightarrow + \leftarrow -$

$Fb^3/EJ$

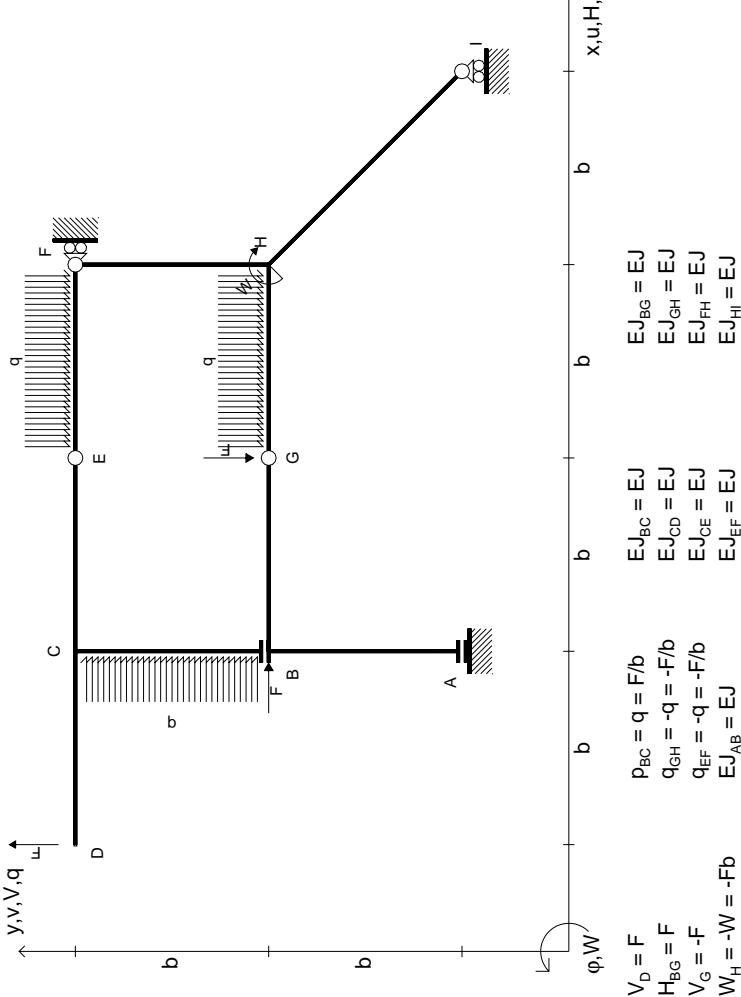


$\left[ \begin{array}{c} + \\ - \end{array} \right]_F$   
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19.11.15

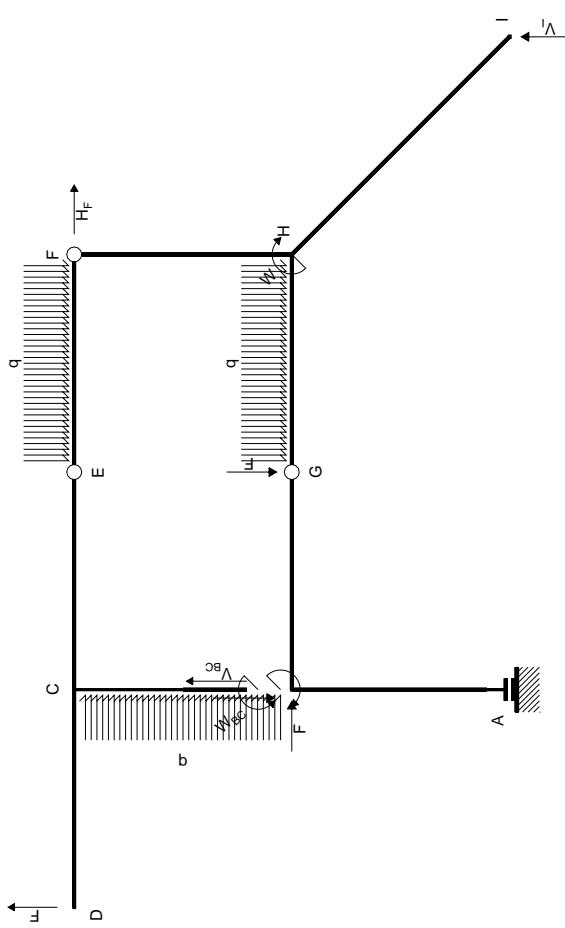
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19.11.15



$$\begin{aligned}
 V_B &= F & EJ_{BC} &= EJ \\
 H_{BG} &= F & EJ_{CD} &= EJ \\
 V_G &= -F & EJ_{CE} &= EJ \\
 W_H &= -W = -Fb & EJ_{EF} &= EJ \\
 EJ_{AB} &= EJ & EJ_{FG} &= EJ \\
 EJ_{EF} &= EJ & EJ_{FH} &= EJ \\
 EJ_{FH} &= EJ & EJ_{HI} &= EJ
 \end{aligned}$$

Carichi e deformazioni date hanno verso efficace in disegno.  
 Calcolare reazioni vincolari della struttura e delle aste.  
 Tracciare i diagrammi quotati delle azioni interne nelle aste.  
 $J_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta  $YZ$  con origine in  $Y$ .  
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## EQUAZIONI DI EQUILIBRIO

Traslazione orizzontale globale

$H_F = -F - qb$

Rotazione intorno a G: asta GH HF HI FE EC CB CD

$-H_F b + 2V_F b - V_{BC} b + W_{BC} = 2Fb + W + 3/2qb^2$

Rotazione intorno a F: asta FE EC CB CD

$-2V_{BC} b + W_{BC} = 3Fb - qb^2$

Rotazione intorno a E: asta EC CB CD

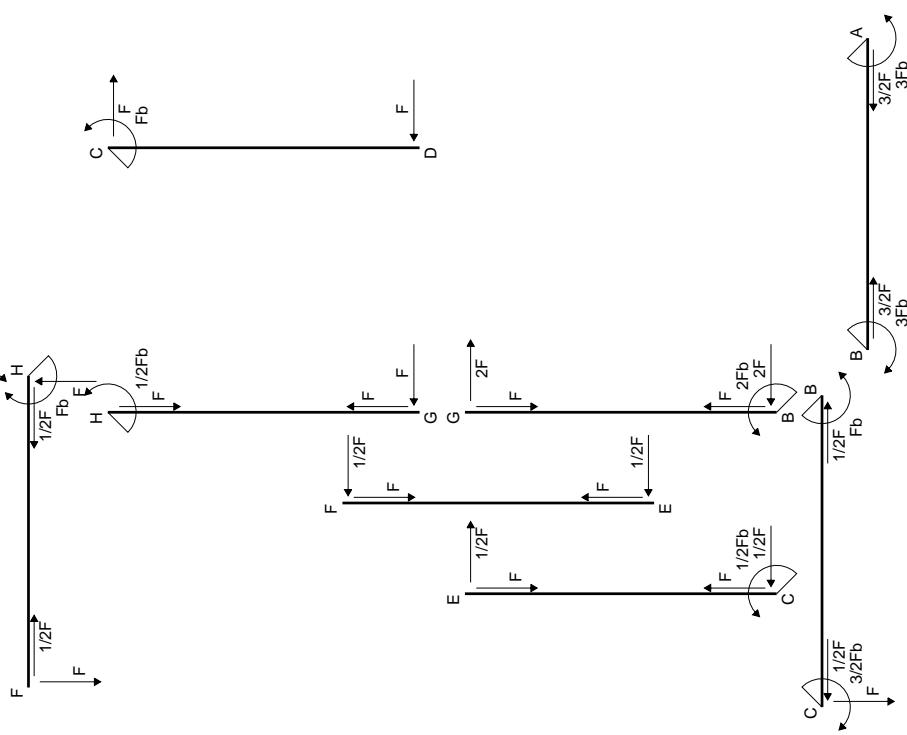
$-V_{BC} b + W_{BC} = 2Fb - 1/2qb^2$

Matrice di equilibrio

$$\begin{bmatrix} H_F b & V_F b & V_{BC} b & W_{BC} \end{bmatrix} \begin{bmatrix} Fb \\ W \\ qb^2 \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 2 & 1 & 3/2 \\ 3 & 0 & -1 \\ 2 & 0 & -1/2 \end{bmatrix}$$

Soluzione del sistema

$$\begin{bmatrix} H_F b \\ V_F b \\ V_{BC} b \\ W_{BC} \end{bmatrix} = \begin{bmatrix} Fb & W & qb^2 \\ -1 & 0 & -1 \\ -1/2 & 1/2 & 1/2 \\ -1 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

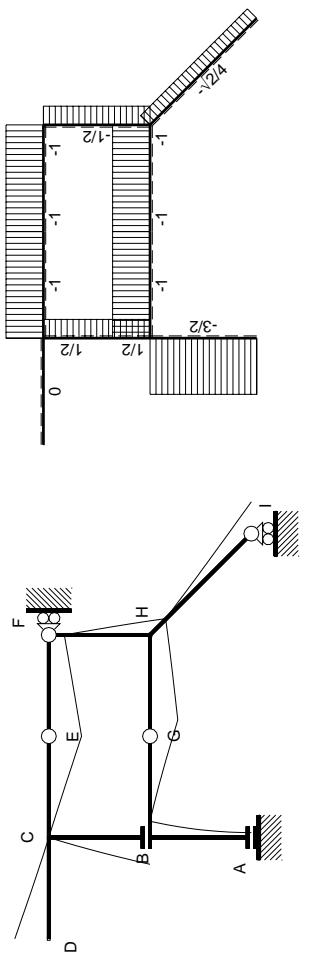


## DEFORMATA E AZIONI INTERNE Nome:

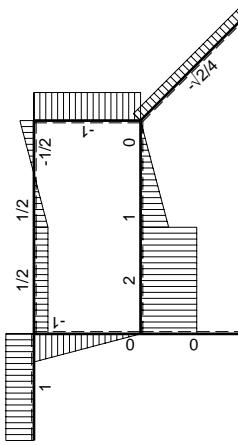
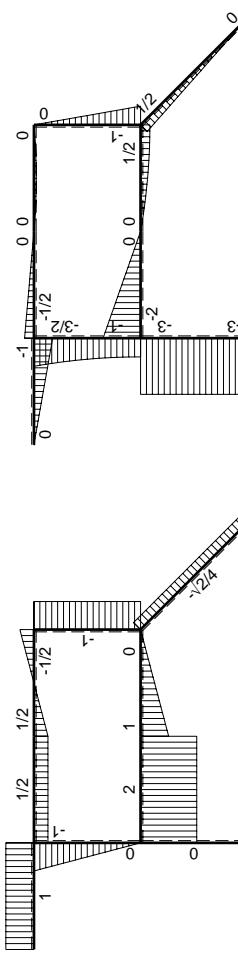
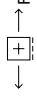
## Struttura Isostatica.001

## PROCEDIMENTO E RISULTATI Nome:

## Struttura Isostatica.001



$\rightarrow -10 \text{ } F b^3/EJ$



$\uparrow \left[ + \right] F$

$\curvearrowleft \left[ + \right] F_b$

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25.01.16

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25.01.16