FASURA: A Scheme for Quasi-Static Massive MIMO Unsourced Random Access Channels

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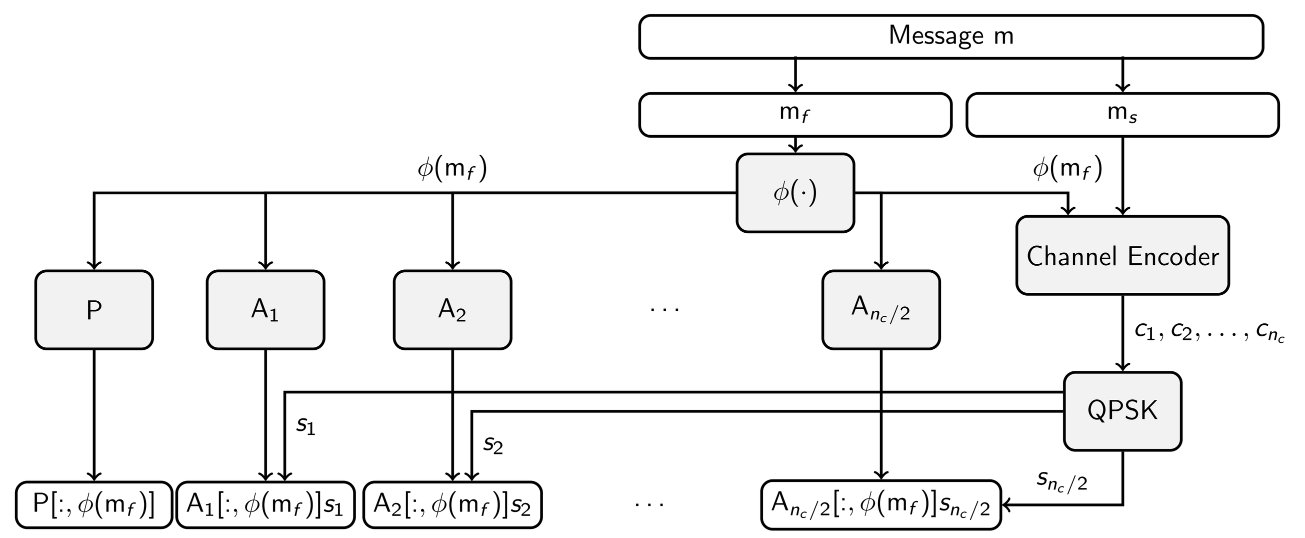
Problem Statement

- Fading Spread Unsourced Random Access (FASURA) is a scheme for unsourced random access (URA) channel
- URA models aim to capture the sporadic transmission of IoT devices
- Consider the massive MIMO URA problem on a quasi-static Rayleigh fading channel
- Probability of error is defined as the sum of the probability of missed detection P_{md} and probability of false alarm P_{fa}, i.e. $\mathrm{P_e} = \mathrm{P}_{\mathrm{md}} + \mathrm{P}_{\mathrm{fa}}$
- Several schemes exist with varying levels of complexity and probability of error
  - Pilot-based: Based on MMV-AMP and polar codes (Fengler et al. 2022)
  - Orthogonal Pilots: Slot based scheme with interesting pilot design (Ahmadi et al. 2022)

Goal

Construct a communication scheme, that minimizes the energy per bit to noise power spectral density ratio E_b/N_0 while also satisfying the constraint $\mathrm{P_e} \le \varepsilon$

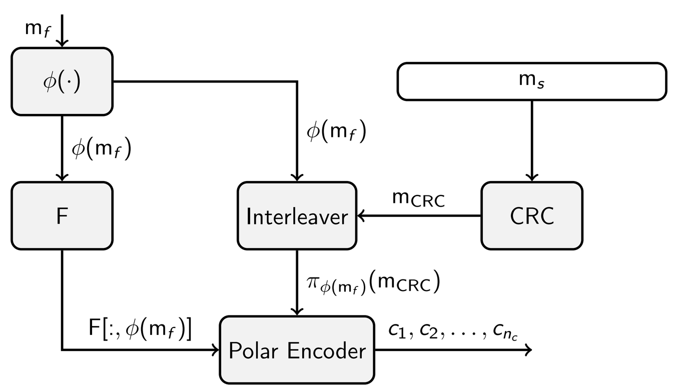
FASURA: Encoder



- Splits the message into two parts
- Computes the decimal representation of $\mathbf{m}_f$, i.e. $\phi(\mathbf{m}_f)$
- Picks the $\phi(\mathbf{m}_f)$th row of $\mathbf{P}, \mathbf{A}_1, \ldots, \mathbf{A}_{n_c/2}$
- Encodes $\mathbf{m}_s$, and modulate the coded bits, $c_1, c_2, \ldots, c_{n_c}$
- Spreads the QPSK symbols $\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_{n_c/2}$, and form the channel input
- Channel input

$$\mathbf{x} = \left[\mathbf{P}^{\intercal}[:, \phi(\mathbf{m}_f)]\ s_1\mathbf{A}_1^{\intercal}[:, \phi(\mathbf{m}_f)]\ s_2\mathbf{A}_2^{\intercal}[:, \phi(\mathbf{m}_f)]\ \ldots\ s_{n_c/2}\mathbf{A}_{n_c/2}^{\intercal}[:, \phi(\mathbf{m}_f)]\right]^{\intercal} = \left[\mathbf{p}^{\intercal}(\mathbf{m}_f)\ \mathbf{q}^{\intercal}(\mathbf{m}_f, \mathbf{m}_s)\right]^{\intercal}$$

Channel Encoder



- Uses the decimal representation of $\mathbf{m}_f$ to select values for the frozen positions and an interleaver
- Encodes the second part of the message, $\mathbf{m}_s$, using a CRC encoder
- Permutes the CRC codeword using the pre-selected interleaver $\pi_{\phi(\mathbf{m}_f)}(\cdot)$
- Frozen values, $\mathbf{F}[:, \phi(\mathbf{m}_f)]$, and the permute CRC codeword act as an input to the Polar Encoder
- The codeword bits, $c_1, c_2, \ldots, c_{n_c}$, are passed to the QPSK modulator

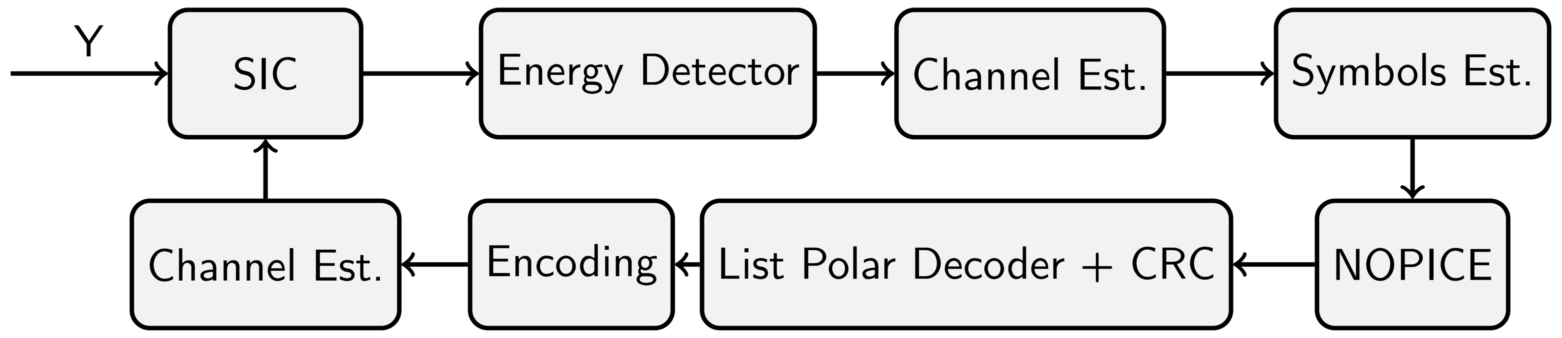
Massive MIMO URA Channel

- Consider a network with K active devices with single antenna
- Access point is equipped with M antennas
- The signal at the receiver is

$$\begin{bmatrix}\mathbf{Y}_p\\ \mathbf{Y}_q\end{bmatrix} = \begin{bmatrix}\mathbf{P}_a\\ \mathbf{Q}_a\end{bmatrix}\mathbf{H} + \begin{bmatrix}\mathbf{Z}_p\\ \mathbf{Z}_q\end{bmatrix}, \quad \mathbf{Y} = \mathbf{X}\mathbf{H} + \mathbf{Z}$$

where $\mathbf{Y} := \begin{bmatrix}\mathbf{Y}_p\\ \mathbf{Y}_q\end{bmatrix} \in \mathbb{C}^{n \times M}$, $\mathbf{H} \in \mathbb{C}^{K \times M}$ and $\mathbf{Z} := \begin{bmatrix}\mathbf{Z}_p\\ \mathbf{Z}_q\end{bmatrix} \in \mathbb{C}^{n \times M}$ is AWGN

FASURA: Decoder



1. Energy Detector: Compute the statistic

$$\lambda_j = \|\mathbf{P}^*[:, j]\mathbf{Y}_p\|^2 + \sum_{t=1}^{n_c/2} \|\mathbf{A}_t^*[:, j]\mathbf{Y}_q[\mathbf{n}_t, :]\|^2, \forall j \in [J], \text{ where } \mathbf{n}_t = [(t-1)L + 1 : tL]$$

   - Output the indices correspond to the largest K values

$$\hat{\mathcal{K}} = \text{argmax}(\{\lambda_1, \lambda_2, \ldots, \lambda_J\}, K)$$

2. Channel Estimation

$$\hat{\mathbf{H}} = \left(\sigma_z^2\mathbf{I}_K + \hat{\mathbf{P}}^*\hat{\mathbf{P}}\right)^{-1}\hat{\mathbf{P}}^*\mathbf{Y}_p, \text{ where } \hat{\mathbf{P}} = \mathbf{P}[:, \hat{\mathcal{K}}]$$

3. Symbol Estimation
   - The received signal of length L, for each symbol QPSK symbol, is given by

$$\begin{aligned}\mathbf{Y}_q[\mathbf{n}_t, m] &= \mathbf{A}_t\,\mathrm{diag}(\mathbf{r}_t)\mathbf{H}[:, m] + \mathbf{Z}[\mathbf{n}_t, m]\\ &= \mathbf{A}_t\,\mathrm{diag}\left(\mathbf{H}[:, m]\right)\mathbf{r}_t + \mathbf{Z}[\mathbf{n}_t, m],\end{aligned}$$

   where $\mathbf{r}_t = (s_{t,1}, s_{t,2}, \ldots, s_{t,K})$ are the symbols of the users at time t
   - Stacking the columns of $\mathbf{Y}_q$

$$\underbrace{\begin{bmatrix}\mathbf{Y}_q[\mathbf{n}_t, 1]\\ \vdots\\ \mathbf{Y}_q[\mathbf{n}_t, M]\end{bmatrix}}_{\mathrm{y}_t \in \mathbb{C}^{LM \times 1}} = \underbrace{\begin{bmatrix}\mathbf{A}_t\,\mathrm{diag}\left(\mathbf{H}[:, 1]\right)\\ \vdots\\ \mathbf{A}_t\,\mathrm{diag}\left(\mathbf{H}[:, M]\right)\end{bmatrix}}_{\mathrm{B}_t \in \mathbb{C}^{LM \times K}} \underbrace{\mathbf{r}_t}_{K \times 1} + \underbrace{\begin{bmatrix}\mathbf{Z}[\mathbf{n}_t, 1]\\ \vdots\\ \mathbf{Z}[\mathbf{n}_t, M]\end{bmatrix}}_{\mathrm{z}_t \in \mathbb{C}^{LM \times 1}}$$

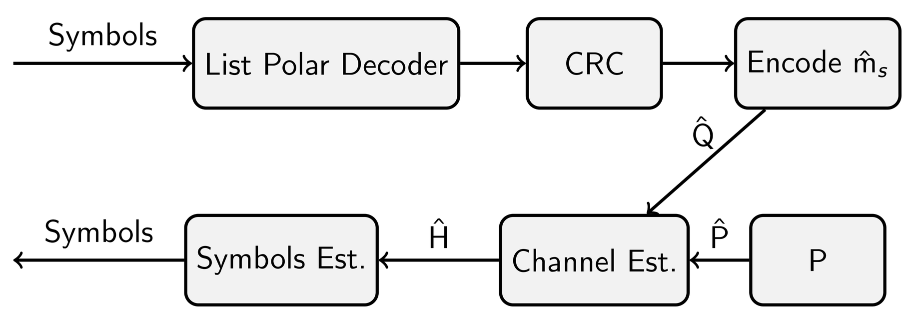
   - As a consequence, we can apply an LMMSE estimator to the vectorized received signal

$$\mathrm{y}_t = \mathrm{B}_t\mathbf{r}_t + \mathrm{z}_t$$

   - The estimated symbols of the active users at time t are given by

$$\hat{\mathbf{r}}_t = \left(\sigma_z^2\mathrm{I}_K + \hat{\mathrm{B}}_t^*\hat{\mathrm{B}}_t\right)^{-1}\hat{\mathrm{B}}_t^*\mathrm{y}_t, \ \forall \quad t = 1, 2, \ldots, n_c/2$$

4. NOPICE (Noisy Pilot Channel Estimation)
   - **Purpose**: Improve the symbols estimates
   - Use the recovered symbols as pilots and re-estimate the channel
   - Then re-estimate the QPSK symbols



5. Channel Decoder
   - Includes list-polar decoder and a CRC step
6. Channel Estimation
   - Use the pilot part and the data part to estimate the channel from the received signal,

$$\mathbf{Y}[:, m] = \hat{\mathbf{X}}\mathbf{H}[:, m] + \mathbf{Z}[:, m], \quad m = 1, 2, \ldots, M$$

   - LMMSE filter to estimate $\mathbf{H}$

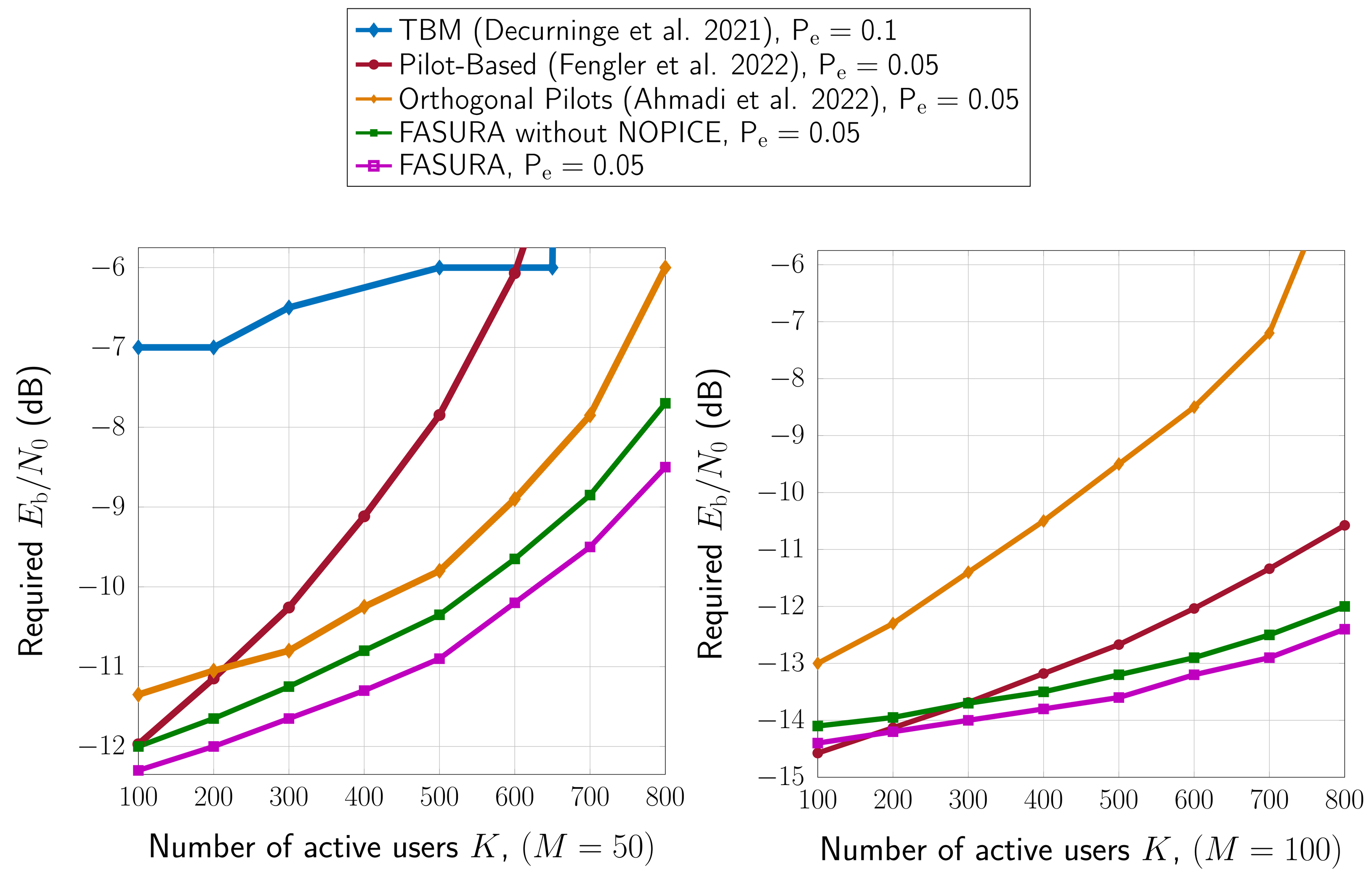
$$\mathbf{W}_2 = \hat{\mathbf{X}}^*\left(\hat{\mathbf{X}}^*\hat{\mathbf{X}} + \sigma_z^2\mathbf{I}_K\right)^{-1}$$

7. SIC (Successive Interference Cancellation)
   - Subtract the interference of the decoded users and proceed to the next round

$$\mathbf{Y}_{t+1} = \mathbf{Y}_t - \sum_{i \in \hat{\mathcal{K}}} \hat{\mathbf{x}}_i\hat{\mathbf{h}}_i^{\intercal}$$

8. Repeat steps 1-7 until the receiver is not able to decode a new user

Numerical Results



E_b/N_0 as a function of the number of users, and number of antennas, $M = 50$ and $M = 100$

Selected References

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Paper

Source Code