PolarAir: A Compressed Sensing Scheme for Over-the-Air Federated Learning

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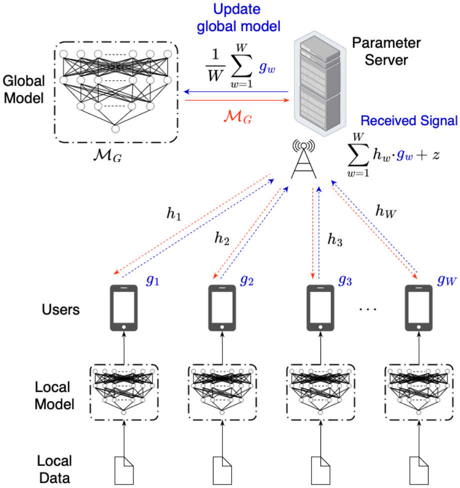
Introduction: Over-the-Air Federated Learning

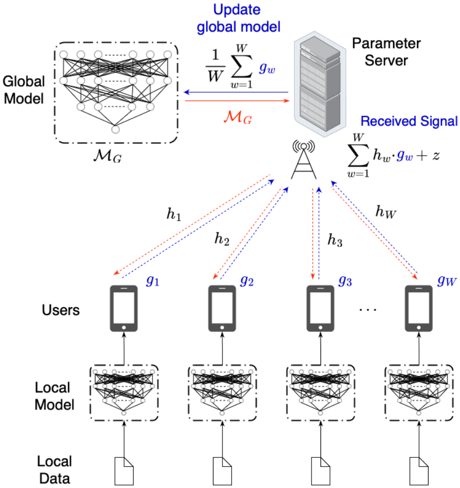
- Joint training of ML models with distributed data
- Exploit the additive nature of the wireless medium
- Uplink naturally computes sum of gradients



$$y = \sum_{w=1}^{W} h_w \, g_w + z$$

- No fading (AWGN) - $h_w = 1$
- SISO - precompensate for h_w

Introduction: Over-the-Air Federated Learning



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- No fading (AWGN) - $h_i = 1$
- SISO - precompensate for h_i

- No. of channel uses = dimension of g_w = no. of parameters

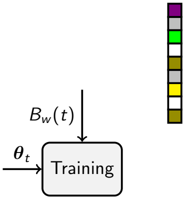
Problem Statement

- Train a ResNet Network over an AWGN channel
- Fix the Test Accuracy
- Minimize the number of channel uses needed to achieve it

Main Objective

- Design Compressed Sensing Algorithm
- Examine its behavior during the training

Framework for Over-the-air Federated Learning[1]



- The access point broadcasts the current model, $\boldsymbol{\theta}_t$, and workers use the available batch, $B_w(t)$ to compute the gradient $\mathrm{g}_w(\boldsymbol{\theta}_t)$
- Add the error from the previous sparsification step, $\mathrm{g}_w^{e.a.}(\boldsymbol{\theta}_t) = \mathrm{g}_w(\boldsymbol{\theta}_t) + \Delta_w(t-1)$
- Compute the error $\Delta_w(t) = \mathrm{g}_w^{a.e.}(\boldsymbol{\theta}_t) - \mathrm{g}_w^{K}(\boldsymbol{\theta}_t)$
- Sparsify $\mathrm{g}_w^{a.e.}(\boldsymbol{\theta}_t)$
- Compress the sparsified vector
- Scale the compressed version to satisfy the power constraint

[1] Proposed by M. M. Amiri and D. Gündüz's 2019 ISIT paper

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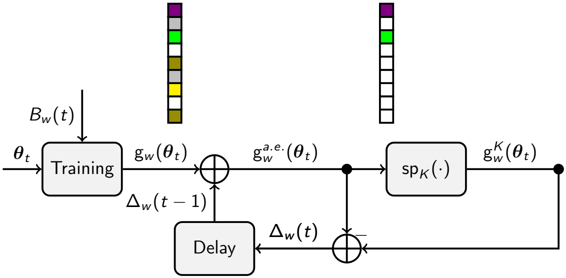

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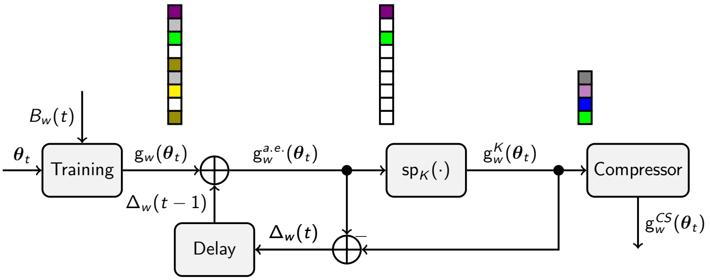

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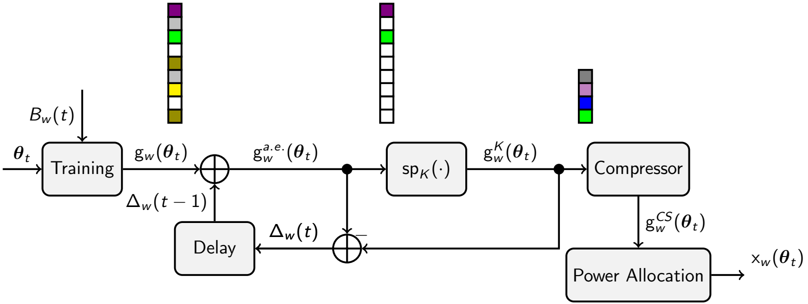

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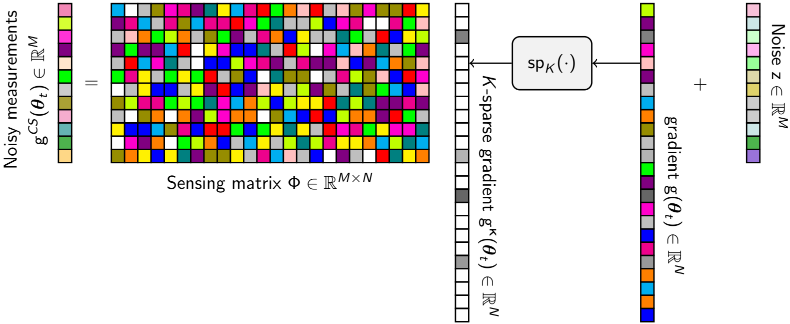
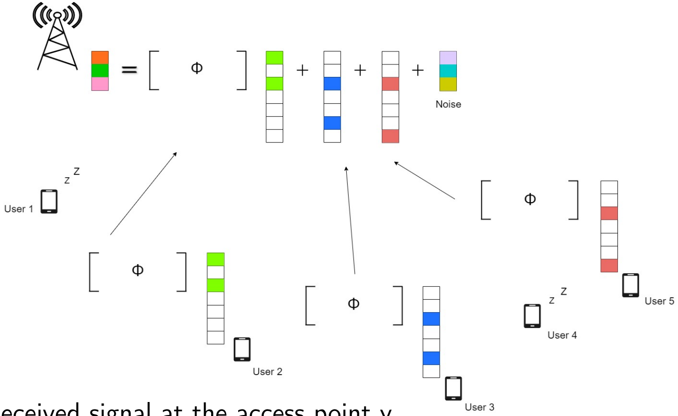
Compressed Sensing



- Gradient (model parameter updates) is K-sparse
- Each worker transmits $g^{CS}(\boldsymbol{\theta}_t) = \Phi g^K_w(\boldsymbol{\theta}_t)$

Over-the-air Federated Learning with Compressed Sensing



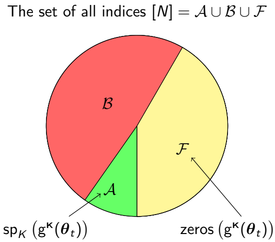
- Received signal at the access point y

$$\mathrm{y}(t) = \sum_{w=1}^{W} \Phi(t)\mathrm{g}_w^{\mathsf{K}}(\boldsymbol{\theta}_t) + \mathrm{z}(t) = \Phi(t)\sum_{w=1}^{W} \mathrm{g}_w^{\mathsf{K}}(\boldsymbol{\theta}_t) + \mathrm{z}(t)$$
$$= \Phi(t)\mathrm{g}^{\mathsf{K}}(\boldsymbol{\theta}_t) + \mathrm{z}(t)$$

- The channel "computes" the aggregated gradient

Recovery of aggregated gradient

- Sum of the W, K-sparse vectors has $K^{'} \in [K : KW]$ non-zero entries
- We choose to recover top-K entries from $\mathrm{y}(t) = \Phi(t)\mathrm{g}^{\mathsf{K}}(\boldsymbol{\theta}_t) + \mathrm{z}(t)$



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False Alarm Set $= \mathcal{B} \cup \mathcal{F}$

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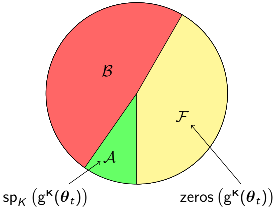
The set of all indices $[N] = \mathcal{A} \cup \mathcal{B} \cup \mathcal{F}$



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- Goal - estimate indices in $\mathcal{A}$ and corresponding values

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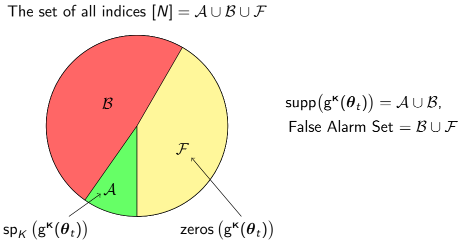

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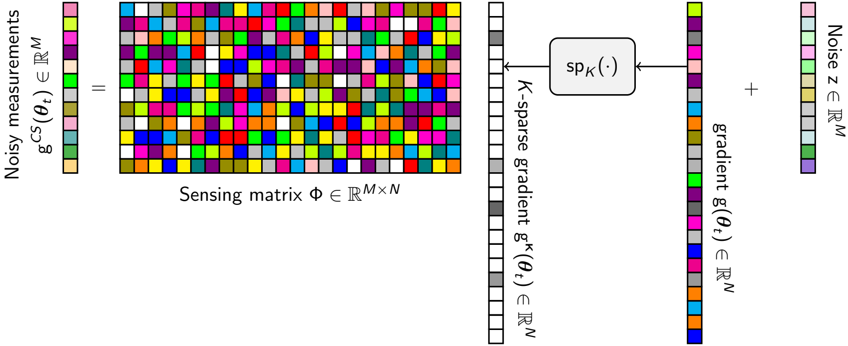
$$\mathrm{y}(t) = \underbrace{\Phi(t)\mathrm{g}_{\mathcal{A}}}_{\text{top-}K\text{ entries}} + \underbrace{\Phi(t)\mathrm{g}_{\mathcal{B}}}_{\text{other non-zero}} + \mathrm{z}(t)$$

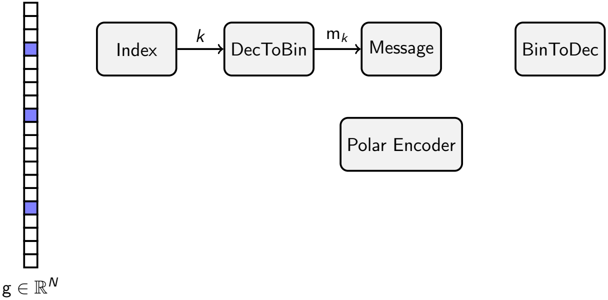
- What we want is task-specific compression

Limitations of current schemes



- Many go-to recovery algorithms have complexity $\mathcal{O}(MN)$
- State-of-the-art machine learning models can have very large N
- Our goal - construct a structured Φ and a recovery algorithm with lower complexity

PolarAir: Encoding the indices



- Convert non-zero index k to a binary string m_k.
- Divide m_k into two parts: $\mathrm{m}_{k,f}, \mathrm{m}_{k,s}$.
- $\mathrm{m}_{k,s}$ is encoded and the coded bits are modulated using BPSK.
- Based on $\mathrm{m}_{k,s}$, a spreading sequence is chosen from the Master set A, where $\mathrm{A} = \{\mathrm{a}_1, \mathrm{a}_2, \ldots, \mathrm{a}_{2^{\mathrm{len}(\mathbf{m}_{k,s})}}\} \in \mathbb{R}^{L \times 2^{\mathrm{len}(\mathbf{m}_{k,s})}}$.
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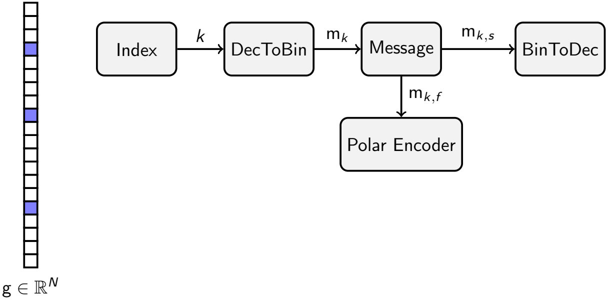

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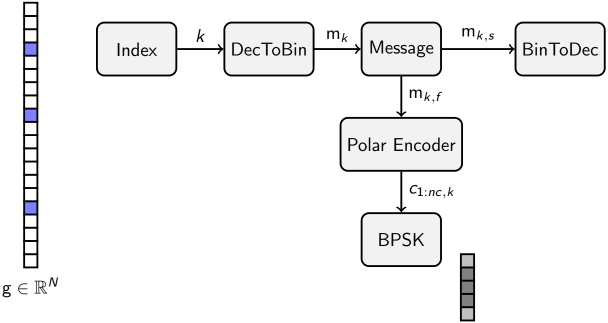

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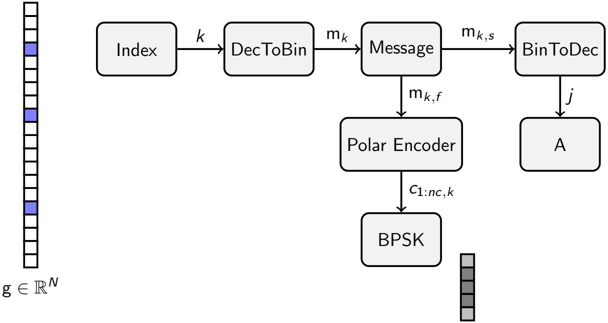

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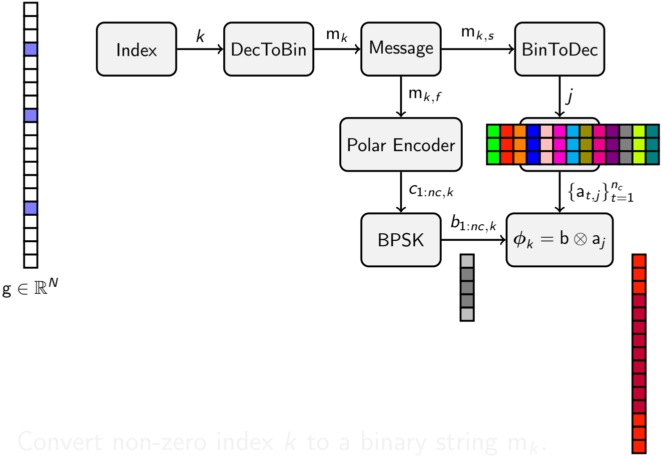

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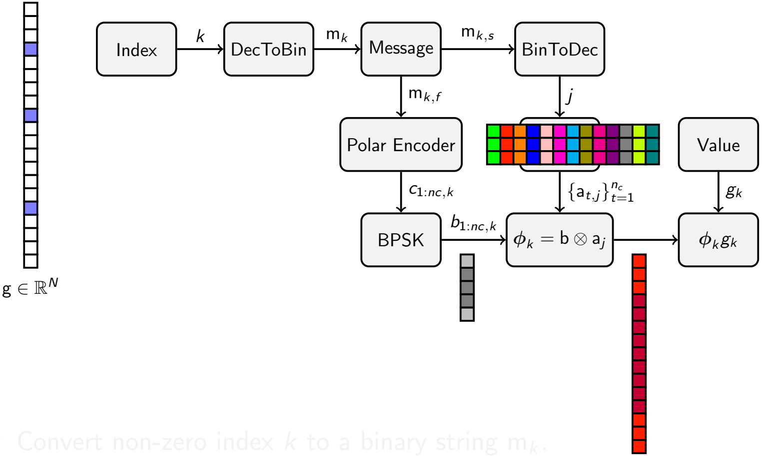

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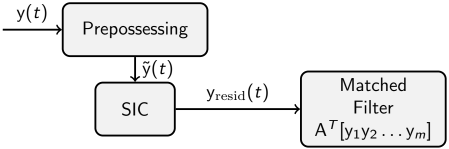

- Matched Filter: Estimate the symbols
- Energy Detector: Recover active sequence
- Unknown sign of gradient, use two Polar Decoders
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- Least Squares: Estimate the values of the gradient

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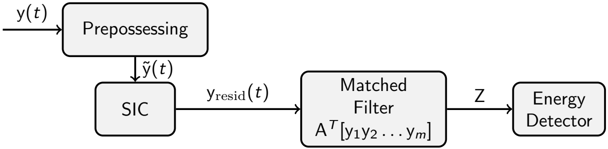

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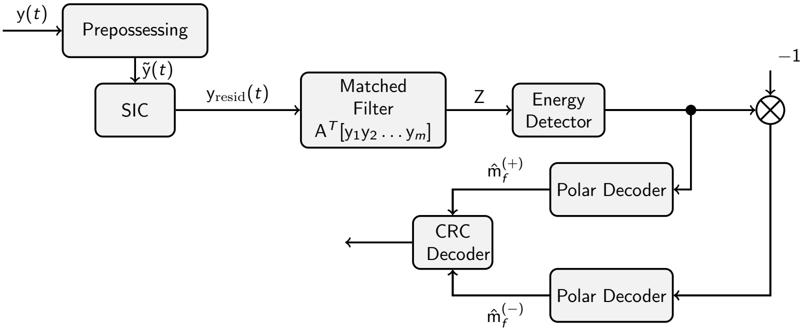

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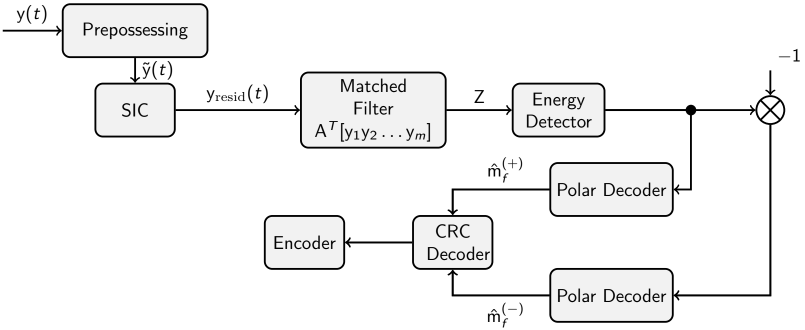

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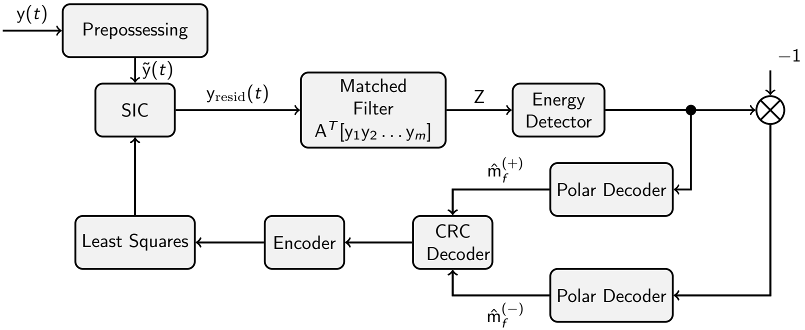

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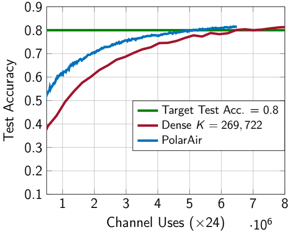
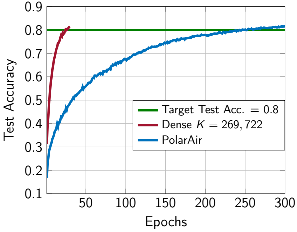
Results - Test Accuracy





- CIFAR-10 dataset, classification task, ResNet model
- $N = 269722$, we sparsified to $K = 270$, $W = 8$ users
- $\approx$ 30% less channel uses
- Much better compared to Dense (Genie)

Results - Probability of Missed Detection & False Alarm

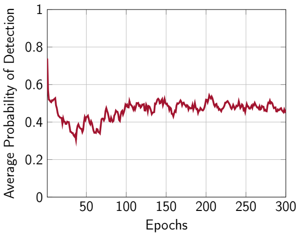
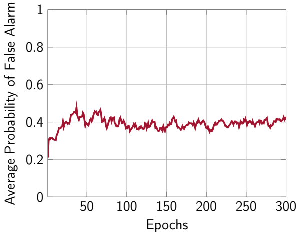


Figure: Average probability of Detection.



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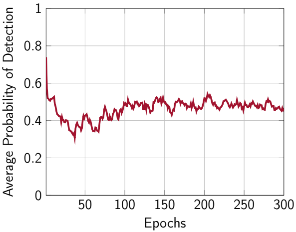

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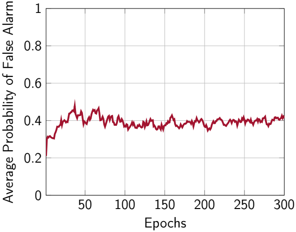

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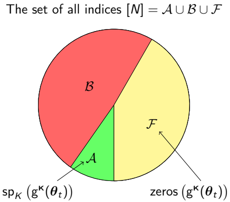

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Results - behavior of the sum of sparse gradients

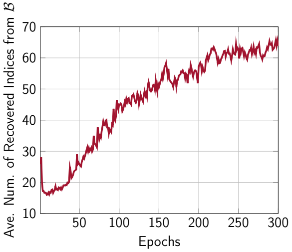


Figure: Average Number of Recovered Indices from $\mathcal{B}$.

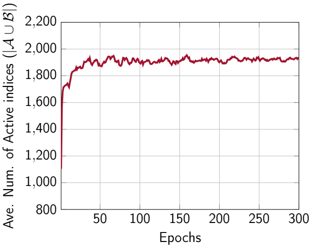
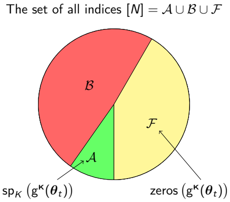


Figure: Average number of active indices as a function of epochs.



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Conclusion

- PolarAir is a method to compress Deep Learning Models for OTAFL
- The complexity is $\mathcal{O}(K^3 + K^2 \log N)$
- $\approx$ 30% less channel uses compared to naive approach

Figure: Group's WebPage

Figure: Presentation

Figure: Source Code