# Leveraging Structural Properties of Tree Coding in AMP for the Unsourced MAC

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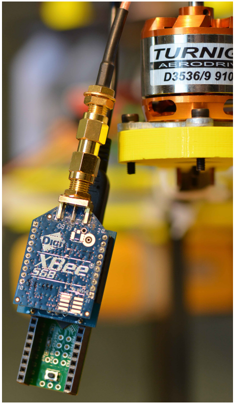
# Current Wireless Landscape

## Current and Future Trends

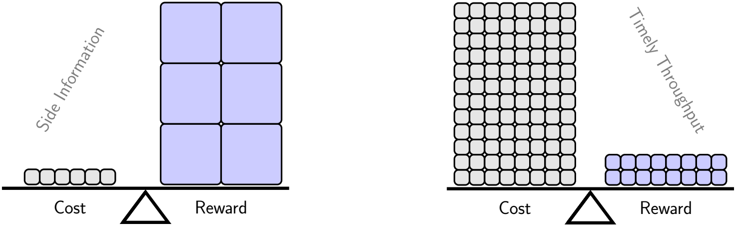
- **Growth and Market Penetration**:
  Number of connected wireless devices exceeds world population
- **Screen Quality**:
  Screens are near boundary of visual acuity (less than 2 inches)
- **Content-Rich Apps**:
  Video watching & gaming are prevalent (4 hours per day)

**What's Next?**



# Emerging Machine-Driven Traffic Characteristics

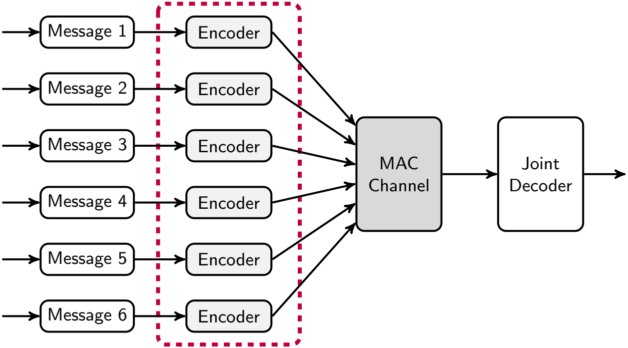
Anticipated traffic characteristics **invalidate acquisition-estimation-scheduling** paradigm!




## New Reality

- Must address **sporadic** nature of machine-driven communications
- Transfer of **small payloads** without ability to amortize cost of acquiring channel and buffer states over long connections
- Preclude use of opportunistic scheduling

# Uncoordinated and Unsourced MAC




## Without Personalized Feedback

- All devices employ same encoder
- No explicit knowledge of identities
- Need only return unordered list

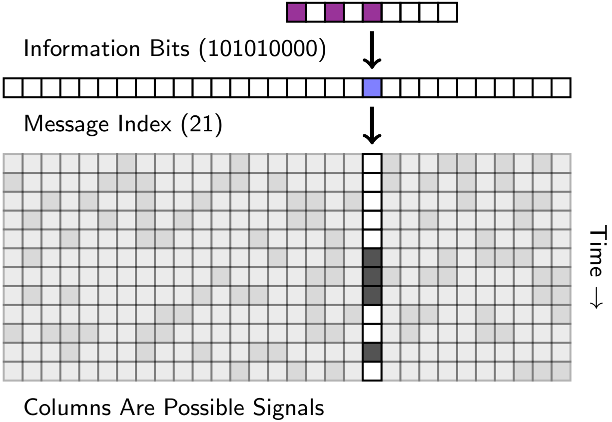
## Model

$$\mathbf{y} = \sum_{i \in \mathbf{S}_{\mathrm{a}}} \mathbf{A}\mathbf{s}_i + \mathbf{n}$$

where $\mathbf{s}_i = f(\mathbf{w}_i)$ is codeword, only depends on message

Y. Polyanskiy. *A Perspective on Massive Random-Access.* ISIT, 2017

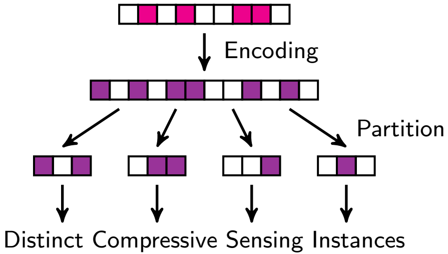
# UMAC – Compressed Sensing Interpretation




- Bit sequence $\mathbf{w}_i \in \{0,1\}^B$ converted to index $\mathbf{s}_i$ in $[0, 2^B - 1]$
- Stack codewords into $N \times 2^B$ *sensing* matrix with $B \approx 128$
- Message index determines transmitted codeword

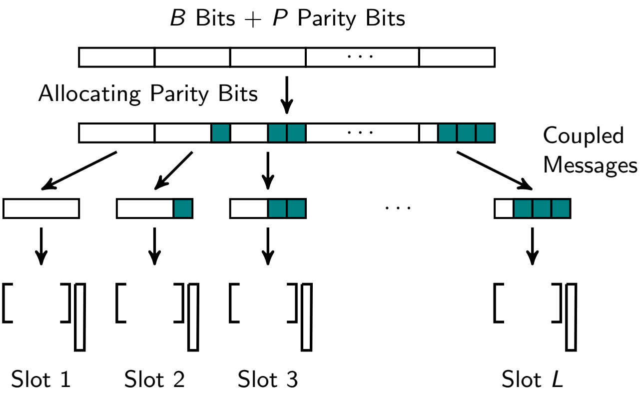
# Quest for Low-Complexity Unsourced MAC

## Idea 1: Divide and Conquer Information Bits




- Split problem into sub-components suitable for CS framework
- Get lists of sub-packets, one list for every slot
- Stitch pieces of one packet together using error correction

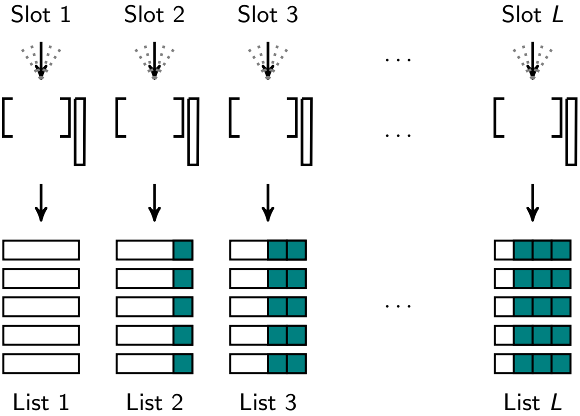
# Coded Compressive Sensing – Device Perspective




- Collection of $L$ CS matrices and 1-sparse vectors
- Each CS generated signal is sent in specific time slot

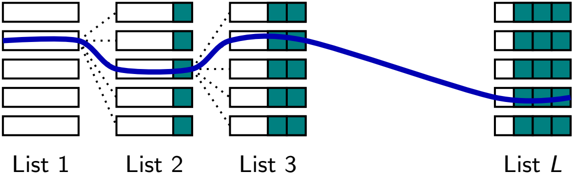
V. Amalladinne, A. Vem, D. Soma, K. R. Narayanan, J.-F. Chamberland. *Coupled Compressive Sensing Scheme for Unsourced Multiple Access*. ICASSP 2018

# Coded Compressive Sensing – Multiple Access



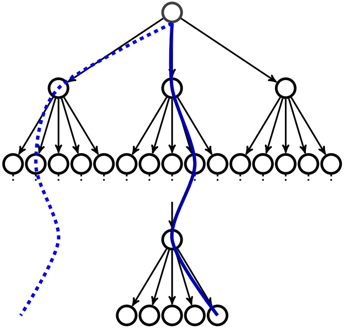

- $L$ instances of CS problem, each solved with non-negative LS
- Produces $L$ lists of $K$ decoded sub-packets (with parity)
- Must piece sub-packets together using tree decoder

# Coded Compressive Sensing – Stitching Process




## Tree Decoding Principles

- Every parity is linear combination of bits in preceding blocks
- Late parity bits offer better performance
- Early parity bits decrease decoding complexity



# Extending CCS Framework

## SPARCs for Unsourced Random Access

Alexander Fengler, Peter Jung, Giuseppe Caire

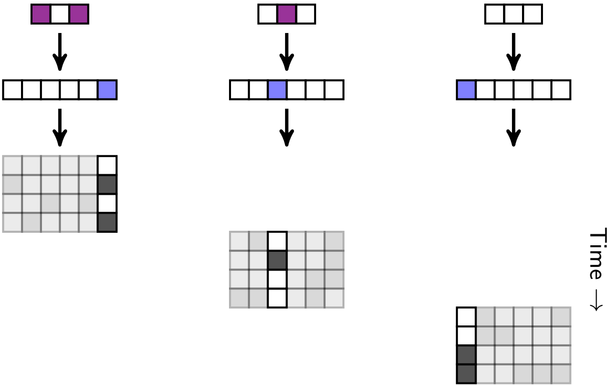
*(Submitted on 18 Jan 2019)*

This paper studies the optimal achievable performance of compressed sensing based unsourced random-access communication over the real AWGN channel. "Unsourced" means, that every user employs the same codebook. This paradigm, recently introduced by Polyanskiy, is a natural consequence of a very large number of potential users of which only a finite number is active in each time slot. The idea behind compressed sensing based schemes is that each user encodes his message into a sparse binary vector and compresses it into a real or complex valued vector using a random linear mapping. When each user employs the same matrix this creates an effective binary inner multiple-access channel. To reduce the complexity to an acceptable level the messages have to be split into blocks. An outer code is used to assign the symbols to individual messages. This division into sparse blocks is analogous to the construction of sparse regression codes (SPARCs), a novel type of channel codes, and we can use concepts from SPARCs to design efficient random-access codes. We analyze the asymptotically optimal performance of the inner code using the recently rigorized replica symmetric formula for the free energy which is achievable with the approximate message passing (AMP) decoder with spatial coupling. An upper bound on the achievable rates of the outer code is derived by classical Shannon theory. Together this establishes a framework to analyse the trade-off between SNR, complexity and achievable rates in the asymptotic infinite blocklength limit. Finite blocklength simulations show that the combination of AMP decoding, with suitable approximations, together with an outer code recently proposed by Amalladinne et. al. outperforms state of the art methods in terms of required energy-per-bit at lower decoding complexity.

Comments: 16 pages, 7 Figures
Subjects: **Information Theory (cs.IT)**
Cite as: **arXiv:1901.06234 [cs.IT]**
(or **arXiv:1901.06234v1 [cs.IT]** for this version)

- Alexander Fengler, Peter Jung, Giuseppe Caire on arXiv
- Connection between CCS indexing and sparse regression codes
- Circumvent slotting under CCS and dispersion effects

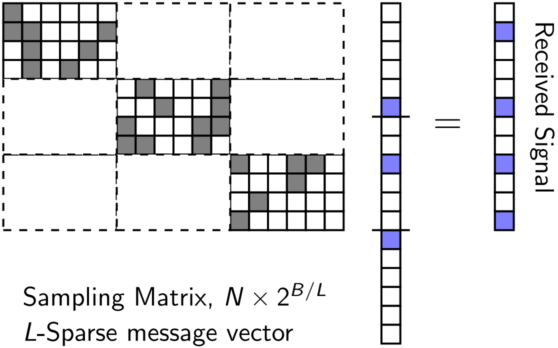
# UMAC – CCS Revisited




Columns Are Possible Signals

- Bit sequence split into $L$ fragments
- Each bit + parity block converted to index in $[1, 2^{M/L}]$
- Stack sub-codewords into $(N/L) \times 2^{M/L}$ sensing matrices

# UMAC – CCS Unified CS Analogy



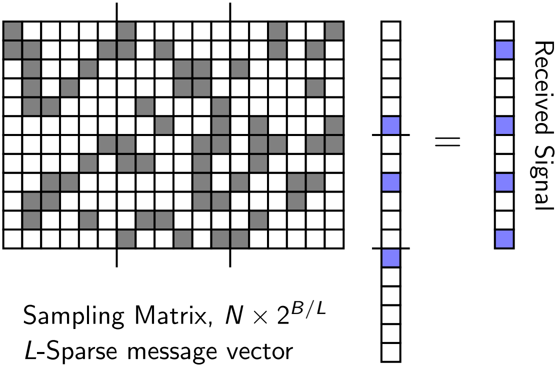

Sampling Matrix, $N \times 2^{B/L}$

$L$-Sparse message vector

- Initial non-linear indexing step
- Index vector is block sparse
- Connection to sparse regression codes

C. Rush, A. Greig, R. Venkataramanan. *Capacity-Achieving Sparse Superposition Codes via Approximate Message Passing Decoding*. IEEE IT Trans 2017

# CCS-AMP




- Complexity management comes from dimensionality reduction
- Use full sensing matrix on sparse regression codes
- Decode inner code with low-complexity AMP
- Decode outer code with tree decoding

A. Fengler, P. Jung, and G. Caire. *SPARCs and AMP for Unsourced Random Access*. ISIT 2019

# Approximate Message Passing Algorithm

## Governing Equations

- AMP algorithm iterates through

$$\mathbf{z}^{(t)} = \mathbf{y} - \mathbf{A}\mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right) + \underbrace{\frac{\mathbf{z}^{(t-1)}}{n}\,\mathrm{div}\,\mathbf{D}\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Onsager correction}}$$

$$\mathbf{r}^{(t+1)} = \mathbf{A}^{\mathrm{T}}\mathbf{z}^{(t)} + \mathbf{D}\,\underbrace{\eta_t\left(\mathbf{r}^{(t)}\right)}_{\text{Denoiser}}$$

Initial conditions $\mathbf{z}^{(0)} = \mathbf{0}$ and $\eta_0\left(\mathbf{r}^{(0)}\right) = \mathbf{0}$

- Application falls within framework for non-separable functions

## Tasks

- Define denoiser
- Derive correction term

R. Berthier, A. Montanari, and P.-M. Nguyen. *State Evolution for Approximate Message Passing with Non-Separable Functions*. arXiv 2017

# Marginal Posterior Mean Estimate (PME)

## Proposed Denoiser (Fengler, Jung, and Caire)

- State estimate based on Gaussian model

$$\hat{s}^{\mathrm{OR}}(q, r, \tau) = \mathbb{E}\left[ s \middle| \sqrt{P_\ell} s + \tau\zeta = r \right]$$
$$= \frac{q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2}\right)}{(1-q)\exp\left(-\frac{r^2}{2\tau^2}\right) + q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2}\right)}$$

  with prior $q = K/m$ fixed
- $\boldsymbol{\eta}_t\left(\mathbf{r}^{(t)}\right)$ is aggregate of PME values
- $\tau_t$ is obtained from state evolution or $\tau_t^2 = \|\mathbf{z}^{(t)}\|^2/n$

**Performance is quite good!**

# Marginal PME Revisited

## Enhanced CCS-AMP

- Can one use tree structure to inform AMP denoiser?
- Idea: Propagate beliefs through $q$ within PME exisiting framework

$$\hat{s}^{\mathrm{OR}}(q, r, \tau) = \mathbb{E}\left[ s \middle| \sqrt{P_\ell} s + \tau \zeta = r \right]$$
$$= \frac{q \exp\left( - \frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}{(1-q) \exp\left( - \frac{r^2}{2\tau^2} \right) + q \exp\left( - \frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}$$

but leverage extrinsic information to compute $q = \Pr(s = 1)$

- Proposed denoiser becomes

$$(\boldsymbol{\eta}_t(\mathbf{r}))_k = \hat{s}^{\mathrm{OR}}\left( (\mathbf{q}(\mathbf{r}))_k , (\mathbf{r})_k , \tau_t \right)$$

where $(\cdot)_k$ is $k$th component

# Updated CCS-AMP Equations

- Onsager correction from divergence of $\boldsymbol{\eta}_t(\mathbf{r})$

$$\frac{1}{n} \operatorname{div} \mathbf{D}\boldsymbol{\eta}_t\left(\mathbf{r}\right) = \frac{1}{n\tau_t^2}\left(KP - \left\|\mathbf{D}\boldsymbol{\eta}_t\left(\mathbf{r}\right)\right\|^2\right)$$

- Robust to tree dynamics
- Simplified AMP equations

$$\begin{aligned} \mathbf{z}^{(t)} &= \mathbf{y} - \mathbf{A}\mathbf{D}\mathbf{s}^{(t)} + \frac{\mathbf{z}^{(t-1)}}{n\tau_t^2}\left(KP - \left\|\mathbf{D}\mathbf{s}^{(t)}\right\|^2\right) \\ \mathbf{s}^{(t+1)} &= \boldsymbol{\eta}_{t+1}\left(\mathbf{A}^{\mathrm{T}}\mathbf{z}^{(t)} + \mathbf{D}\mathbf{s}^{(t)}\right) \end{aligned}$$

with $\left(\boldsymbol{\eta}_t\left(\mathbf{r}\right)\right)_k = \hat{s}^{\mathrm{OR}}\left(\left(\mathbf{q}\left(\mathbf{r}\right)\right)_k, \left(\mathbf{r}\right)_k, \tau_t\right)$

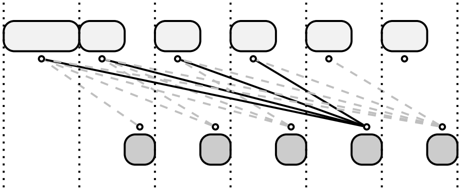
## Tasks

1. Devise a suitable tree code
2. Compute $\mathbf{q}\left(\mathbf{r}\right)$ from tree code

# Redesigning Outer Code

## Properties of Original Tree Code

- Aimed at stitching message fragments together
- Works on short lists of $K$ fragments
- Parities allocated to control growth and complexity



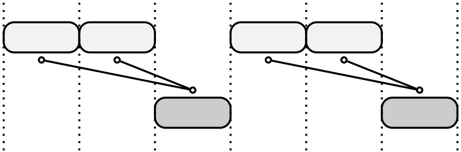
## Challenges to Integrate into AMP

1. Must compute beliefs for all possible $2^v$ fragments
2. Must provide pertinent information to AMP
3. Should maintain ability to stitch outer code

# Redesigning Outer Code

## Solutions to Integrate into AMP

- Parity bits are generated over Abelian group amenable to Hadamard transform (original) or FFT (modified)
- Discrimination power proportional to # parities



## New Design Strategy

1. Information sections with parity bits interspersed in-between
2. Parity over two blocks (triadic dependencies)
3. Multiplicative effect across concentrated sections

# Redesigning Outer Code

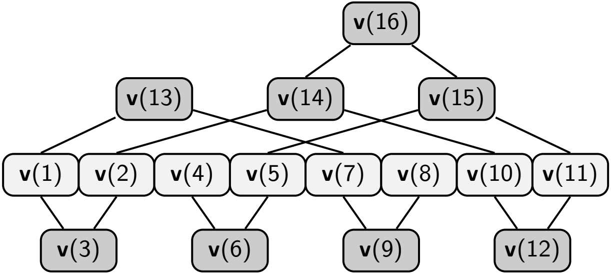
- Circular convolution structure

$$\text{Extrinsic Info } (\mathbf{q}(\ell))_k \propto \sum_{\{g_j\},\sum_j g_j \equiv k} \left(\prod_j \mathcal{L}_j(\hat{\mathbf{s}}(j))\right)$$

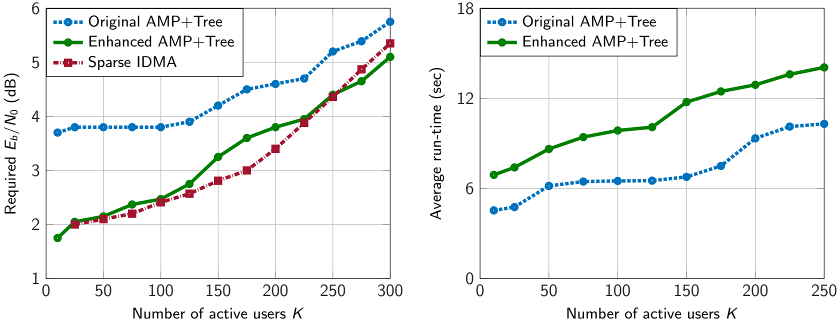
where $\hat{\mathbf{s}}(j) \in \mathbf{G}_{j,\ell}^{-1}(g_j)$.

- Fast transform techniques

$$\text{Extrinsic Info Vector } \mathbf{q}(\ell) \propto \mathrm{FFT}^{-1}\left(\prod_j \mathrm{FFT}\left(\boldsymbol{\mathcal{L}}_{j,\ell}\right)\right)$$



# Preliminary Performance Enhanced CCS




- Overall performance improves significantly with enhanced CCS-AMP decoding
- Computational complexity is approximately maintained
- Reparametrization may offer additional gains in performance?

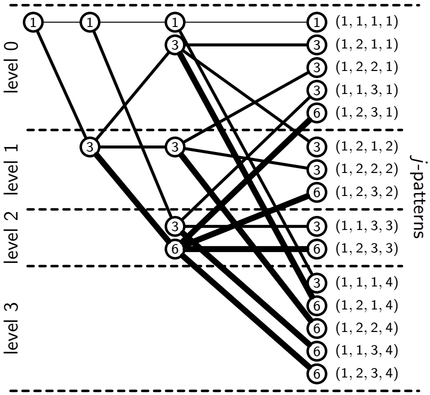
# Discussion – Unsourced Multiple Access Channel

## Summary

- Introduced new framework for CCS-AMP and unsourced multiple access
- There are close connections between compressive sensing, graph-based codes, and UMAC
- Many theoretical and practical challenges/opportunities exist

**Questions?**

# Thank You!

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