Velocity analysis

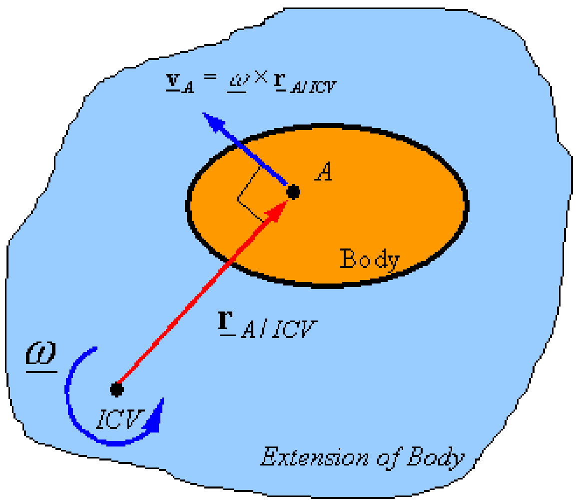
Instantaneous Center Method – ICM

- To determine the velocities of different points on links of a mechanism for a given input motion.
- Determination of the motion characteristics of links in a mechanism is required for the force analysis
- Velocities of links and of points of mechanism can be determined by different methods.
  1. Instantaneous center method.
  2. Relative velocity method or Velocity Vector Diagram.

Instantaneous centre (IC)

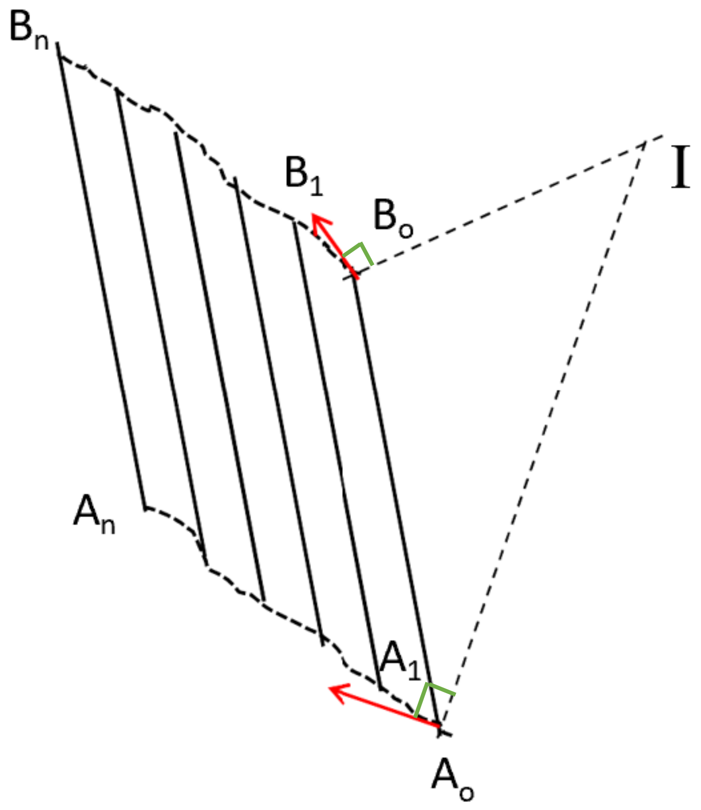
A body can have a motion of rotation as well as translation simultaneously.

This combined motion of a body may be assumed to be a motion of pure rotation about some centre known as instantaneous centre of rotation.



Instantaneous centre (IC) method

- A link AB has moved from A_oB_o to A_nB_n. Its motion can be considered to have taken place in a number of infinitesimally small steps as shown in the figure.
- Its motion from A_oB_o to A_1B_1 can be considered as a pure rotation about a centre I.
- The location of the centre can be obtained from direction of the velocities of the ends of the link as shown. Such centre is called Instantaneous centre of rotation.
- The location of the centre changes from instant to instant for different positions of the link and locus of these centres is called centrode.



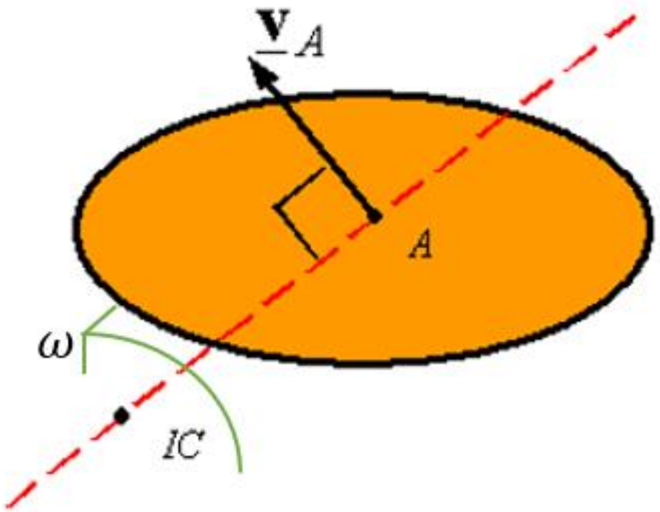
Locating Instantaneous centre

- For a point on a rigid body

Instantaneous centre of a point in a rigid body is located on a line drawn perpendicular to the direction of velocity.

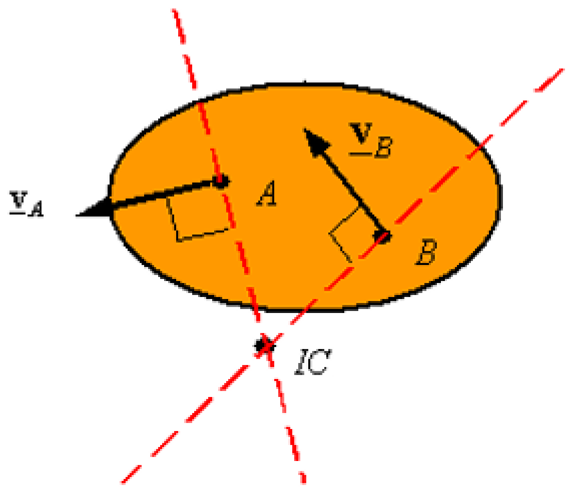
$$V_A = r\omega = AIC \times \omega$$

$$\omega = \frac{V_A}{AIC}$$



- For a rigid body

Given the velocity of points A and B on a rigid body, the instantaneous center is located by drawing a line perpendicular to V_A and passing through A, and by drawing a line perpendicular to V_B and passing through B. The intersection of these two line is the IC of the rigid body.



The lines intersect at one point

The IC is at infinity, and the angular velocity is zero.

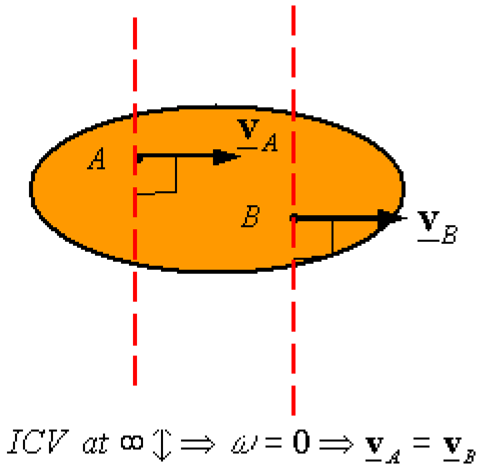
The body is in pure translation and the velocity of the two points must be the same.

$$V_A = r\omega = AIC \times \omega$$

$$\omega = \frac{V_A}{AIC}$$

$$IC\ at\ infinity\ \Rightarrow AIC = \infty$$

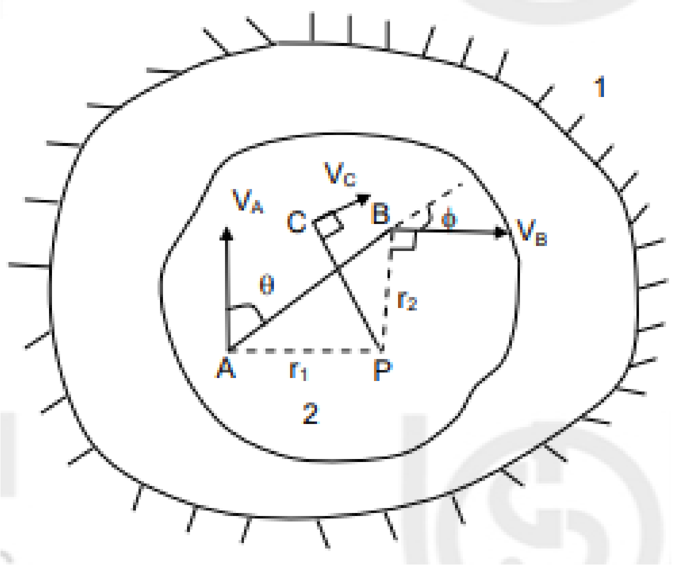
$$\omega = \frac{V_A}{\infty} = 0$$



The lines are parallel
(they intersect at infinity)

Velocity analysis

- As shown in Figure a rigid body 2 has plane motion with respect to fixed body 1.
- A and B are arbitrary points on body 2.
- The velocities of points A and B are shown in the Figure by V_A and V_B respectively.
- The perpendiculars to the directions of velocities V_A and V_B are drawn at A and B.
- These perpendiculars meet at point P.
- Let AP and BP be r_1 and r_2 respectively.



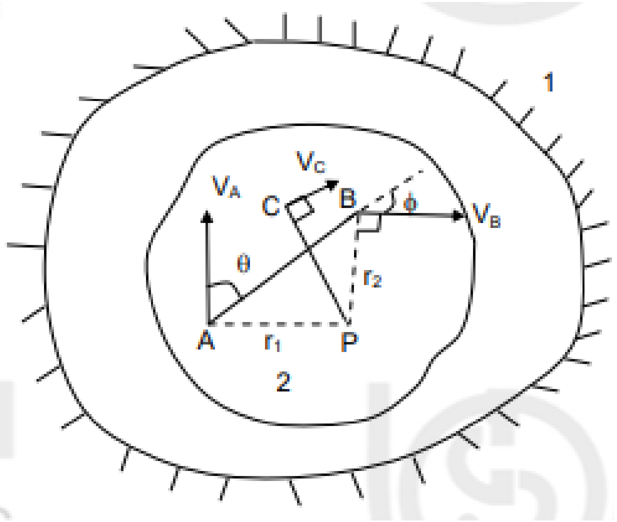
The velocity of point A along AB = $V_A \cos\theta$

The velocity of point B along AB = $V_B \cos\varphi$

Body 2 is rigid, distance between points A and B does not change.

This is only possible when velocity of B relative to A, component along AB, V_{BA} is zero.

$$V_{BA} = V_{B,\overrightarrow{AB}} - V_{A,\overrightarrow{AB}}$$



$$V_{BA} = V_B \cos\phi - V_A \cos\theta$$

$$V_B \cos\phi - V_A \cos\theta = 0$$

$$V_B \cos\phi = V_A \cos\theta$$

$$\frac{V_A}{V_B} = \frac{\cos\phi}{\cos\theta} \qquad \boxed{1}$$

From triangle ABP

$$\frac{AP}{\sin(90-\phi)} = \frac{BP}{\sin(90-\theta)}$$

$$\frac{r_1}{\cos\phi} = \frac{r_2}{\cos\theta}$$

$$\frac{r_1}{r_2} = \frac{\cos\phi}{\cos\theta} \qquad \boxed{2}$$

From 1 & 2

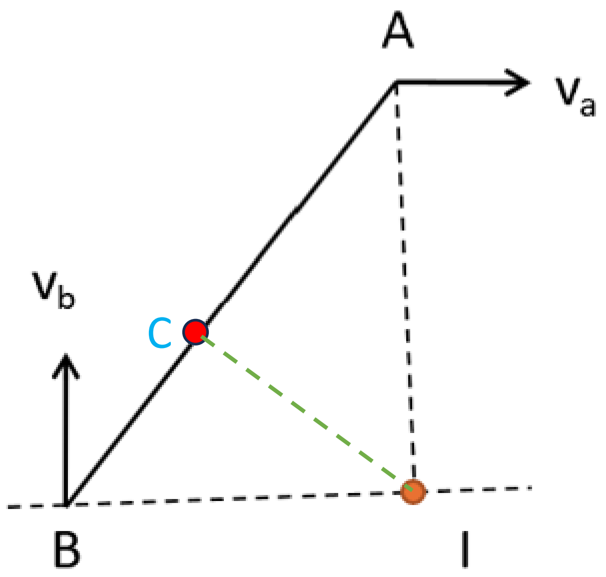
$$\frac{V_A}{V_B} = \frac{\cos\phi}{\cos\theta} = \frac{r_1}{r_2} \Rightarrow \frac{r_1}{r_2} = \frac{V_A}{V_B}$$

$$\frac{V_A}{r_1} = \frac{V_B}{r_2} = Angular\ velocity\ of\ body\ 2, \omega$$

ω= angular velocity of AB at the given instant.

Velocity of end A = $\omega \times \; IA$

Velocity of end B = $\omega \times IB$



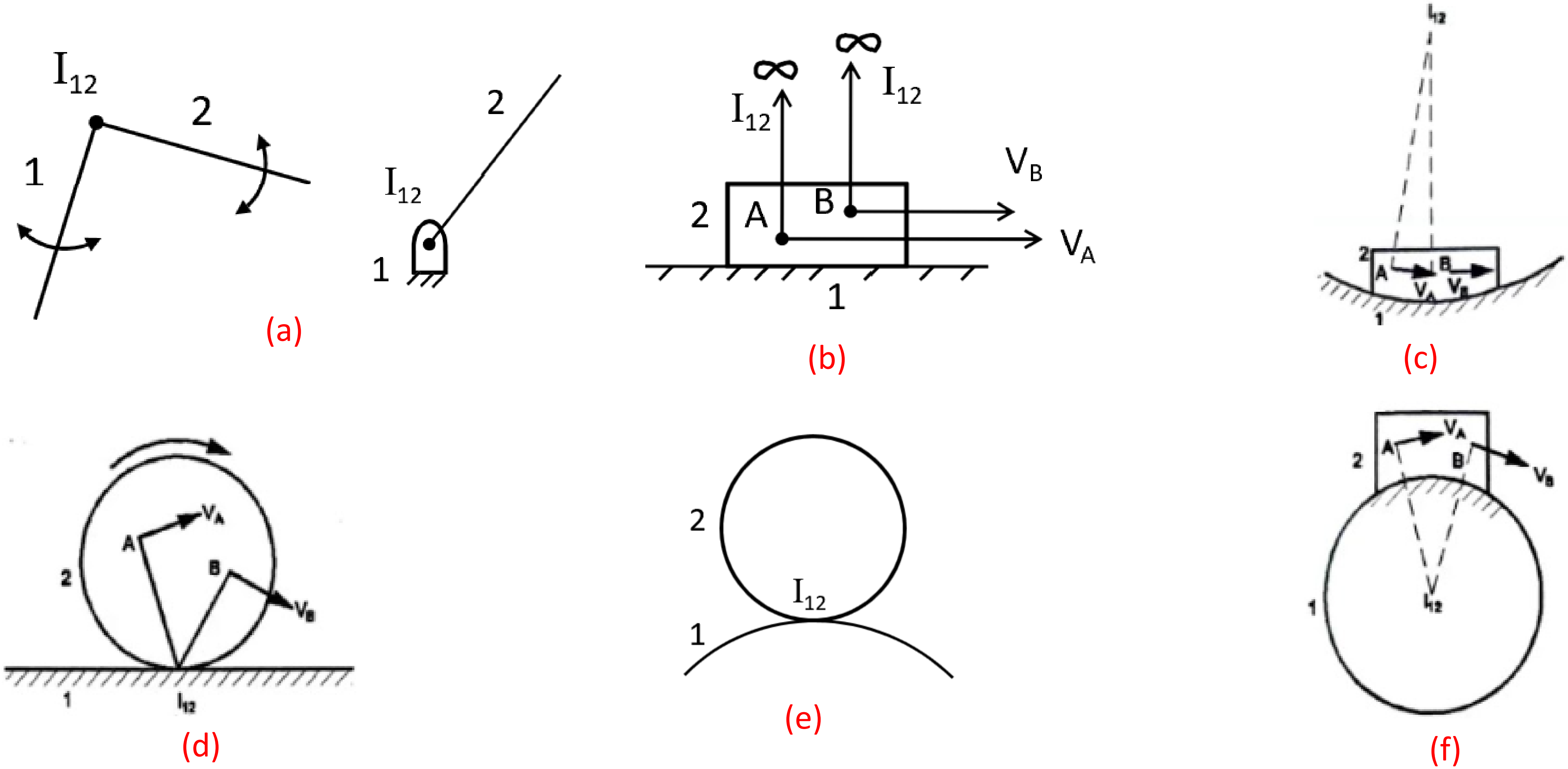
Velocity of C*, where* C is any point on the link.

$$V_C = \omega \times IC$$

The angular velocities of various links can be determined easily using the equation below.

$$\frac{v_a}{IA} = \frac{v_b}{IB} = \frac{v_{ab}}{AB} = \omega = \frac{v_C}{IC}$$

Instantaneous centres (ICs) of different moving links

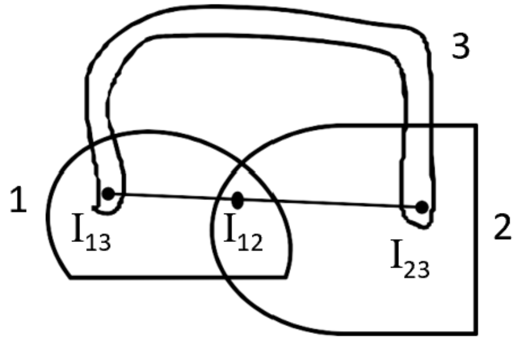


Arnold Kennedy's theorem (Line of three centres theorem)

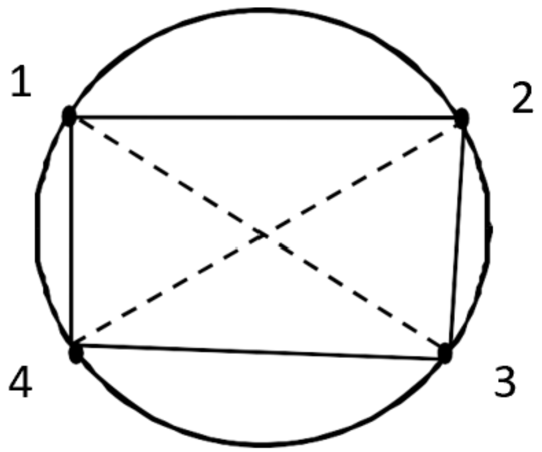
It states that:

If three bodies have relative motion with respect to each other, their relative instantaneous centres lie on a straight line.

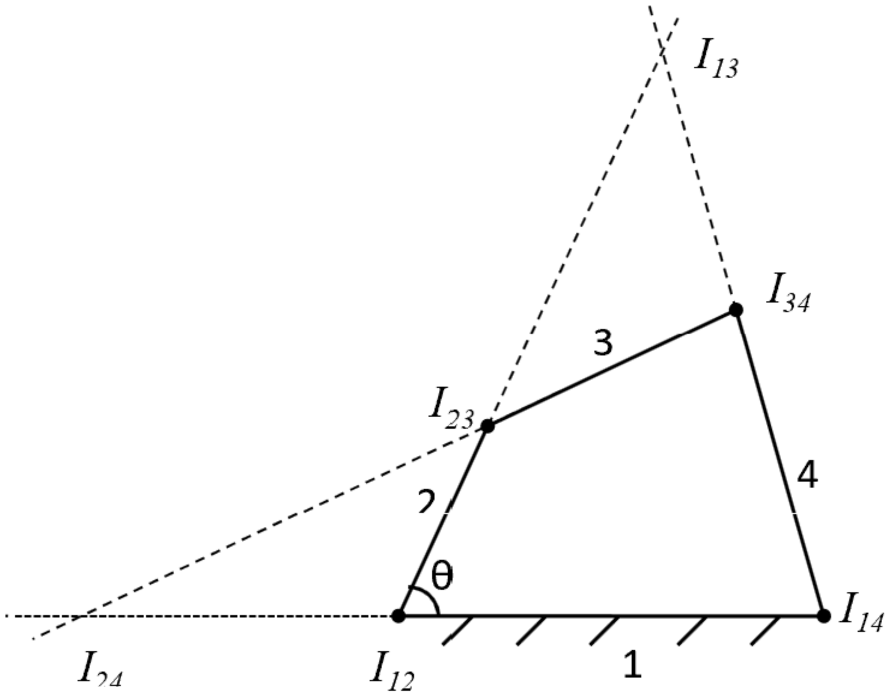
Three bodies 1, 2 and 3 are having relative motion with respect to each other. The instantaneous centres of I_{12}, I_{23} and I_{13} lie on a straight line.



Types of instantaneous centres (ICs)



Circle diagram



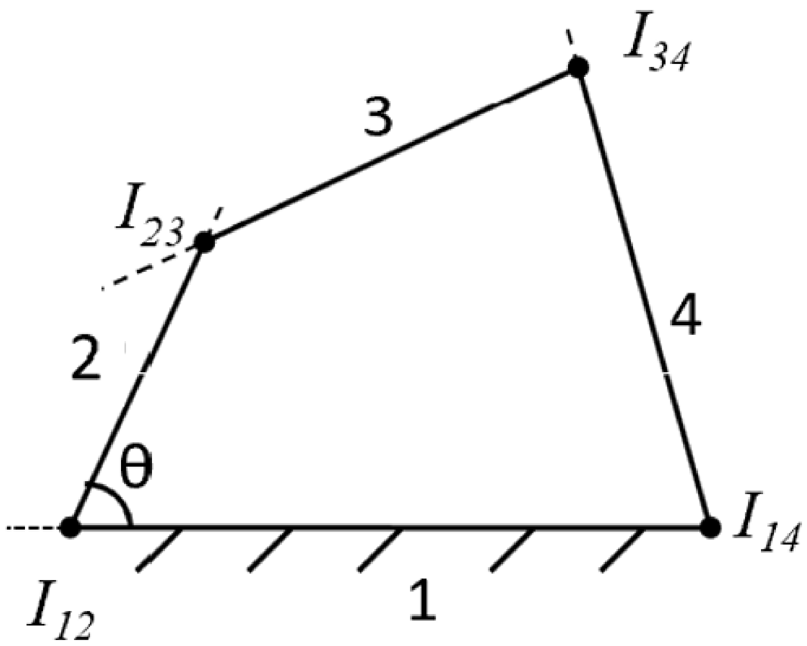
- I_{12} , I_{14} : Fixed type
- I_{23} , I_{34} : Permanent type
- I_{24} , I_{13} : Neither fixed nor permanent

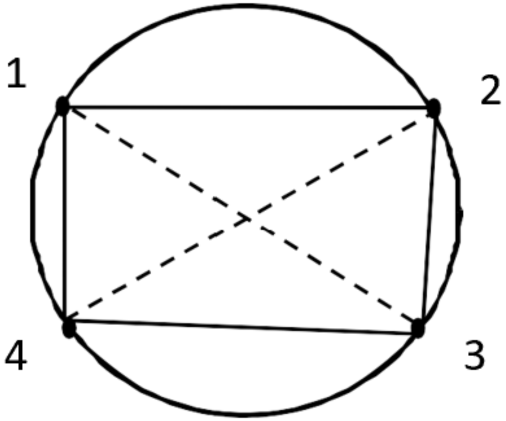
Number of instantaneous centres, N

Each pair of links has one instantaneous centre.

The total number of combinations of n links taken two at a time is

$$N = \frac{n(n-1)}{2}$$





Circle diagram

Fixed ICs

I_{12} - IC of links 1 & 2 – at the joint of 1 & 2

I_{14} - IC of links 1 & 4 – at the joint of 1 & 4

Permanent ICs

I_{23} - IC of links 2 & 3 – at the joint of 2 & 3

I_{34} - IC of links 3 & 4 – at the joint of 3 & 4

$$N = \frac{n(n-1)}{2}$$
$$= \frac{4 \times 3}{2}$$
$$= 6$$

To locate the other 2 ICs, Kennedy's theorem will be used.

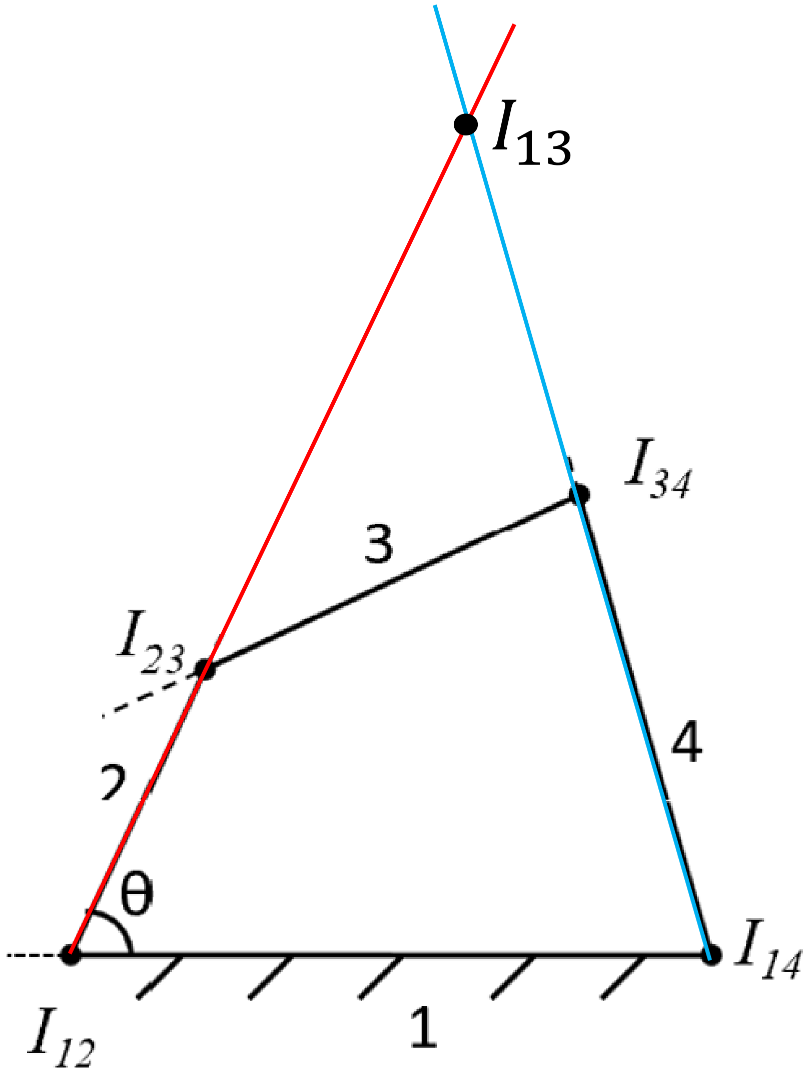
I_{13} - IC of links 1 & 3

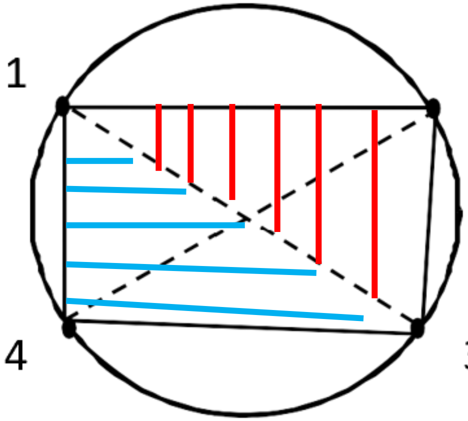
Links 1, 2 & 3 have relative motion with respect to each other.

According to Kennedy's theorem I_{12}, I_{23} & I_{13} lie on a straight line.

Links 1, 3 & 4 have relative motion with respect to each other.

According to Kennedy's theorem I_{14}, I_{34} & I_{13} lie on a straight line.





Circle diagram

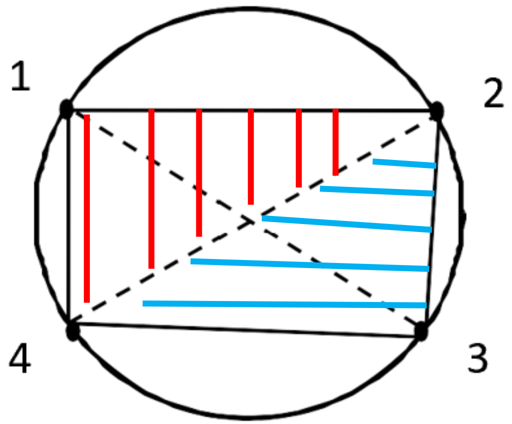
I_{24} - IC of links 2 & 4

Links 1, 2 & 4 have relative motion with respect to each other.

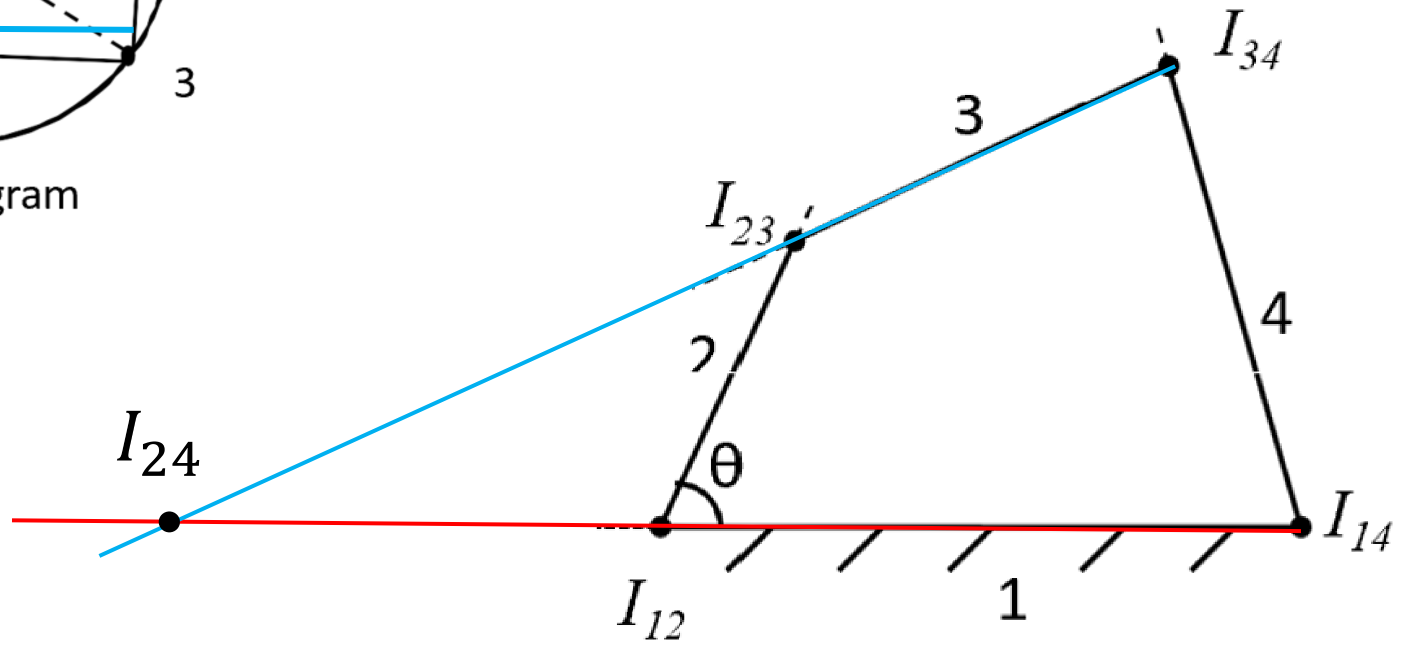
According to Kennedy's theorem I_{12}, I_{14} & I_{24} lie on a straight line.

Links 2, 3 & 4 have relative motion with respect to each other.

According to Kennedy's theorem I_{23}, I_{34} & I_{24} lie on a straight line.

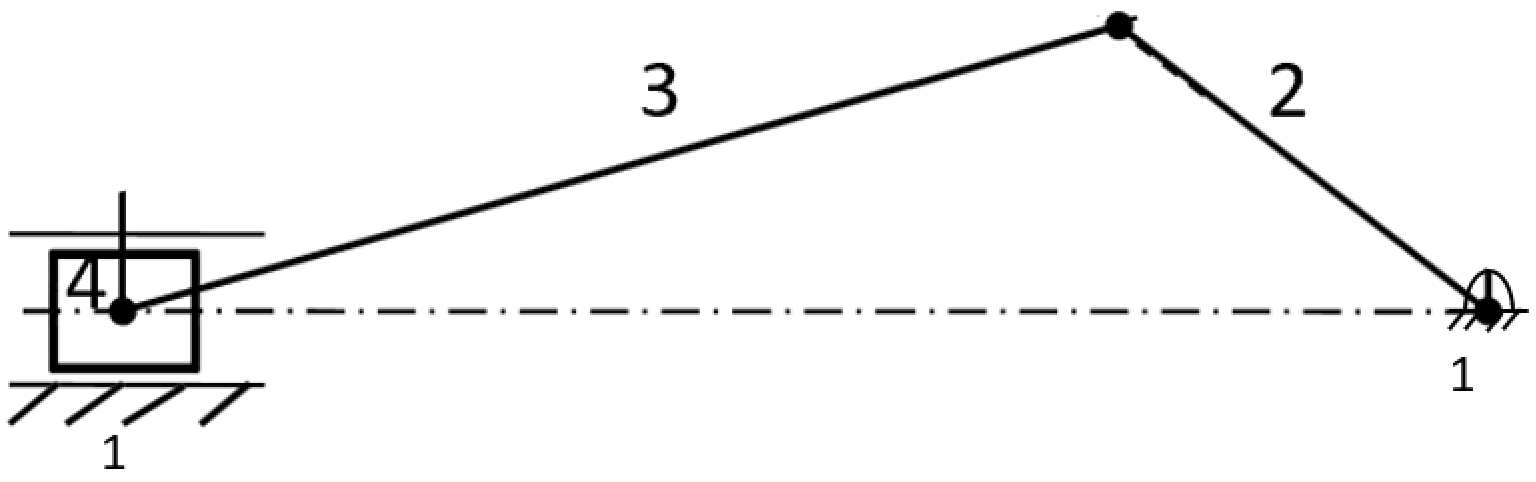


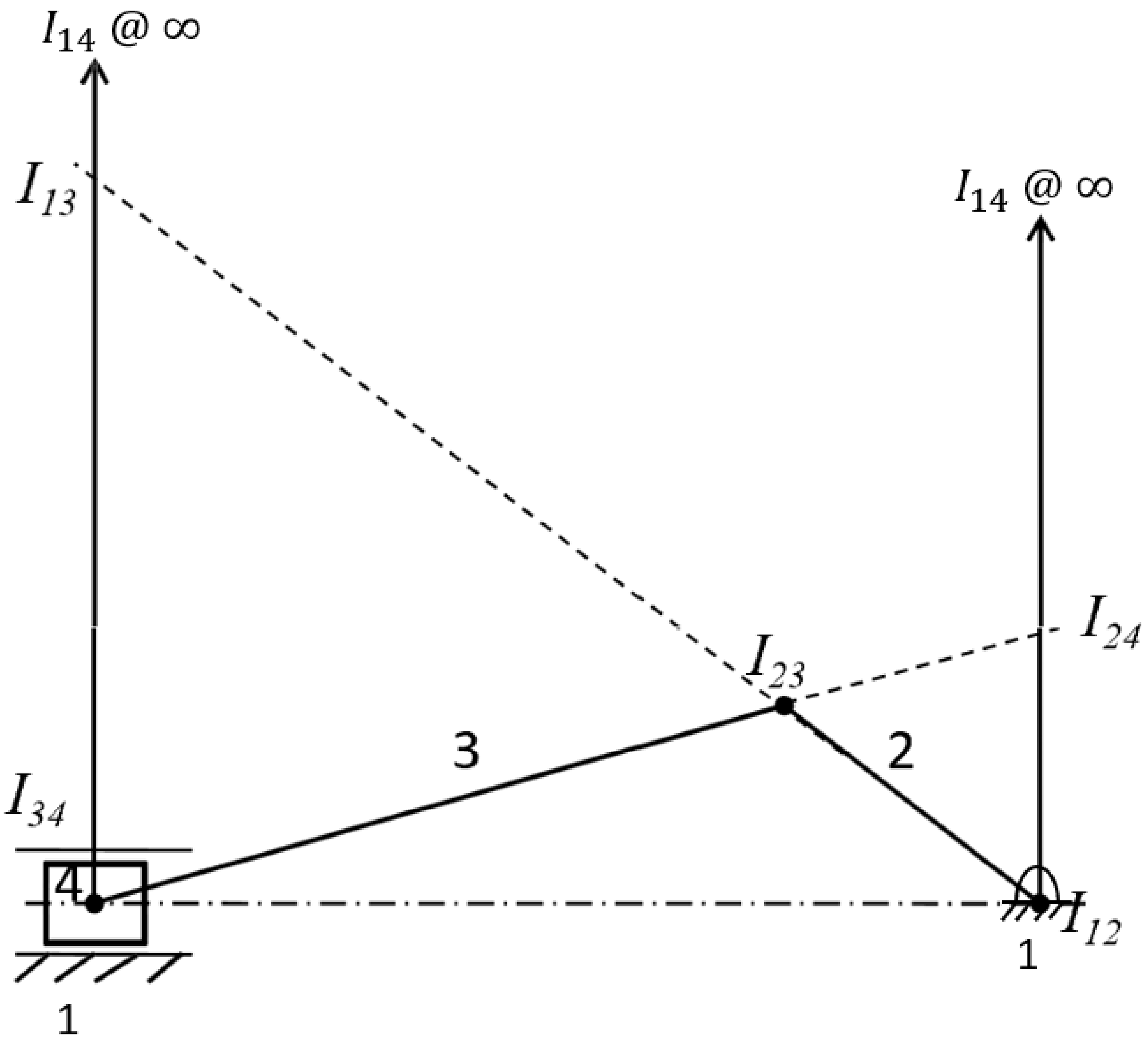
Circle diagram



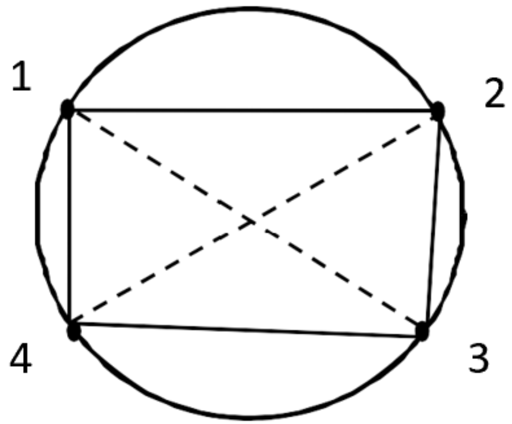
Slider-crank chain

Locate all the instantaneous centers in the mechanism.





$$N = \frac{n(n-1)}{2} = \frac{4 \times 3}{2} = 6$$



Circle diagram

Example 1

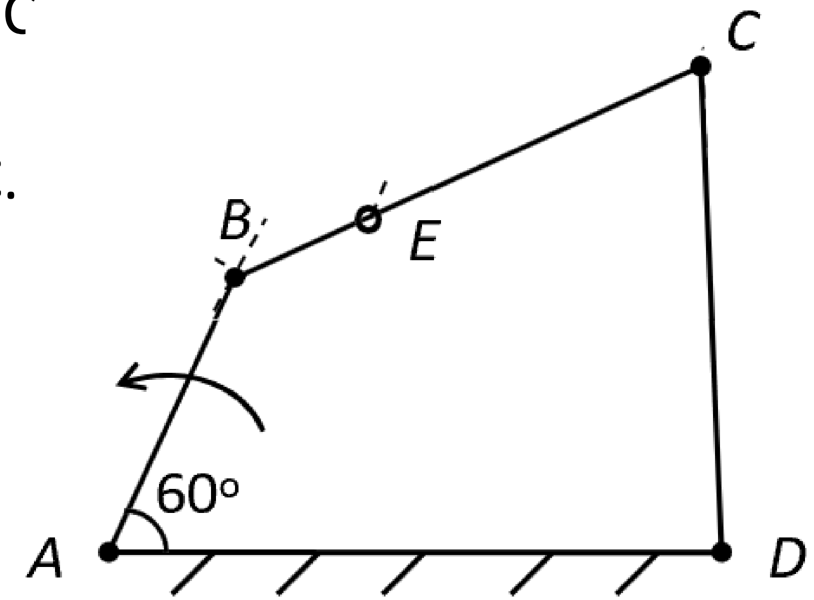
The link AB rotates with angular velocity of π rad/s. Given AB = 0.5 m, BC = 1.5 m, = 1 m, DA = 1.75 m, BE = 0.5 m.

i. Draw the space diagram
ii. Locate all the instantaneous centers

Using instantaneous center method, determine

iii. the angular velocity of BC
iv. velocity of C
v. velocity of point E on BC.

Scale
Space diagram 1:20



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Scale 1:20

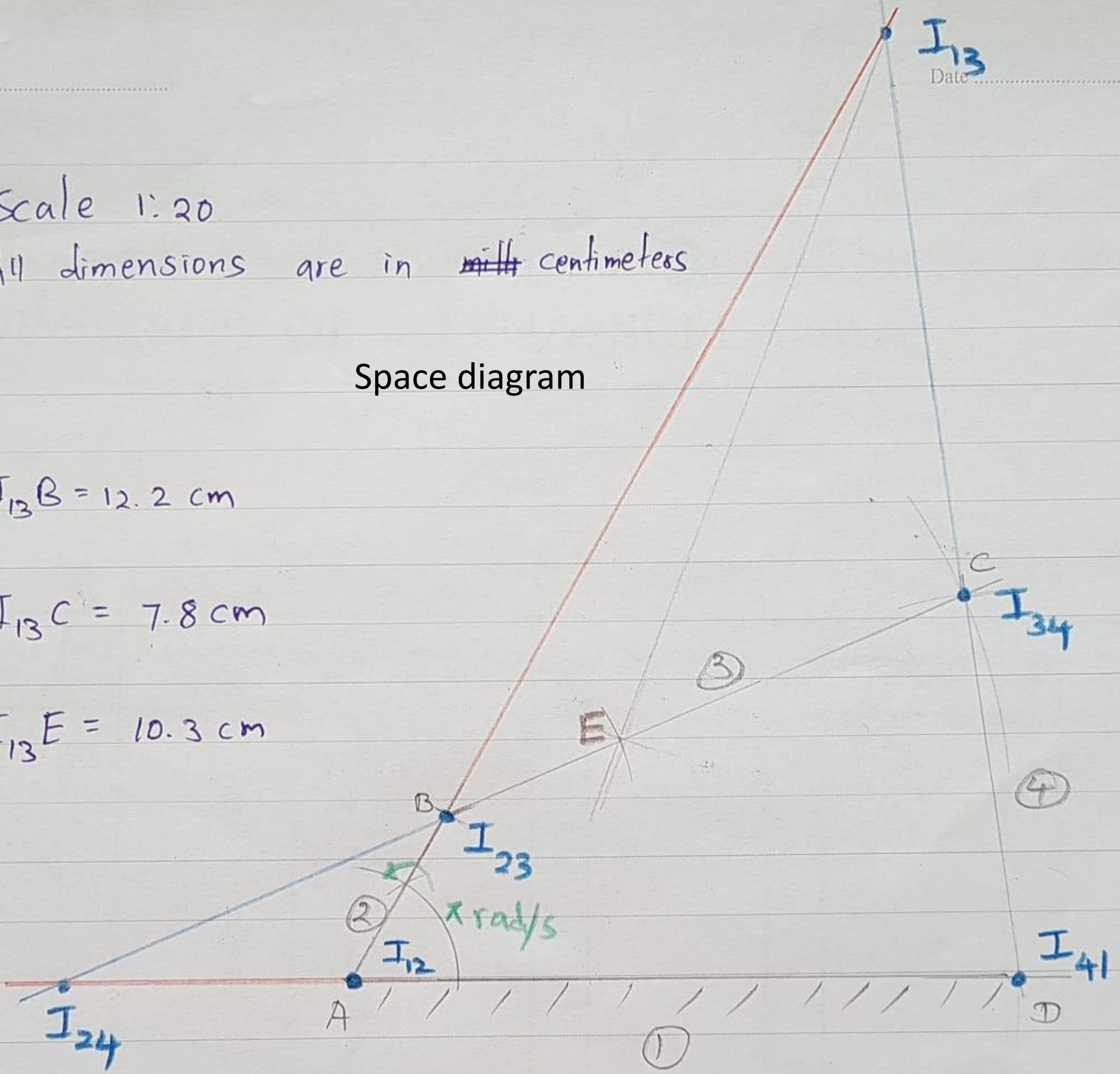
All dimensions are in ~~milli~~ centimeters

Space diagram

$I_{13}B = 12.2$ cm

$I_{13}C = 7.8$ cm

$I_{13}E = 10.3$ cm



1) Angular velocity of BC

$$V_b = AB \cdot \omega_{ab}$$
$$= 0.5 \times \pi$$
$$= 1.57 \text{ m/s}$$

$$V_b = I_{13}B \cdot \omega_{bc}$$
$$1.57 = 2.44 \cdot \omega_{bc}$$
$$\omega_{bc} = \frac{1.57}{2.44} = 0.643 \text{ rad/s}$$

2) Velocity of C

$$V_c = I_{13}C \times \omega_{bc}$$
$$= 1.56 \times 0.643$$
$$= 1.003 \text{ m/s}$$

3) Velocity of point E $\quad V_E = I_{13}E \times \omega_{bc} = 2.06 \times 0.643$

$$= 1.32 \text{ m/s}$$

Example 2

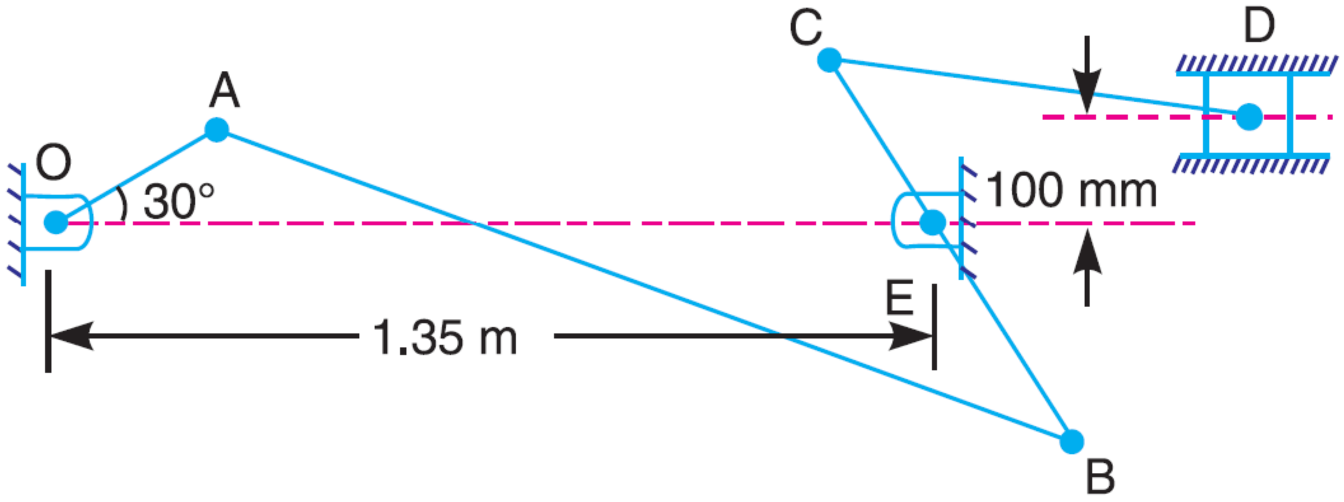
A mechanism, as shown in Figure, has the following dimensions:

OA = 200 mm; AB = 1500 mm; BC = 600 mm; CD = 500 mm and BE = 400 mm.

i. Draw the space diagram
ii. Locate all the instantaneous centers

If crank OA rotates uniformly at 120 rev/min in clockwise direction, find

iii. the velocity of B, C and D
iv. the angular velocity of the links AB, BC and CD.



1. Total number of ICs, $N = \frac{n(n-1)}{2} = \frac{6\times 5}{2} = 15$
2. Locate fixed and permanent ICs.
3. Prepare a Book keeping table

$\boldsymbol{I_{12}}$	$\boldsymbol{I_{13}}$	$\boldsymbol{I_{14}}$	$\boldsymbol{I_{15}}$	$\boldsymbol{I_{16}}$
$\boldsymbol{I_{23}}$	$\boldsymbol{I_{24}}$	$\boldsymbol{I_{25}}$	$\boldsymbol{I_{26}}$	
$\boldsymbol{I_{34}}$	$\boldsymbol{I_{35}}$	$\boldsymbol{I_{36}}$		
$\boldsymbol{I_{45}}$	$\boldsymbol{I_{46}}$			
$\boldsymbol{I_{56}}$				