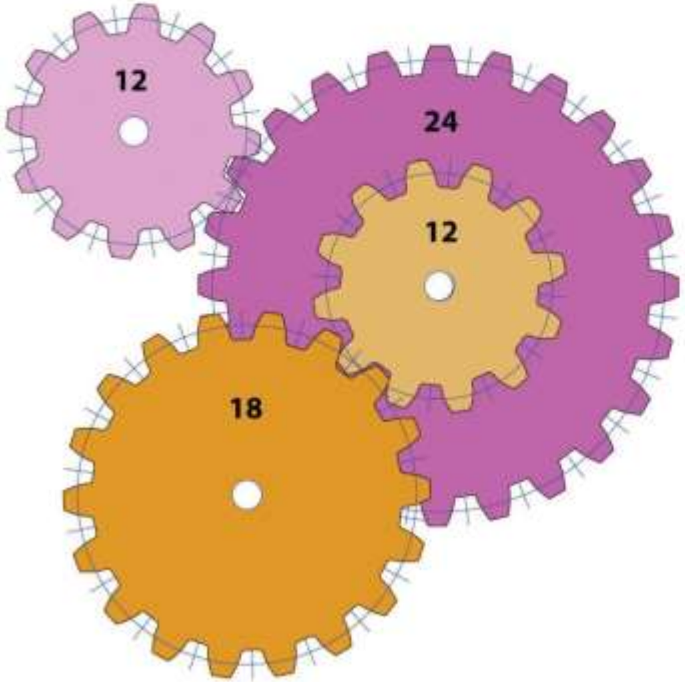
Gear Trains

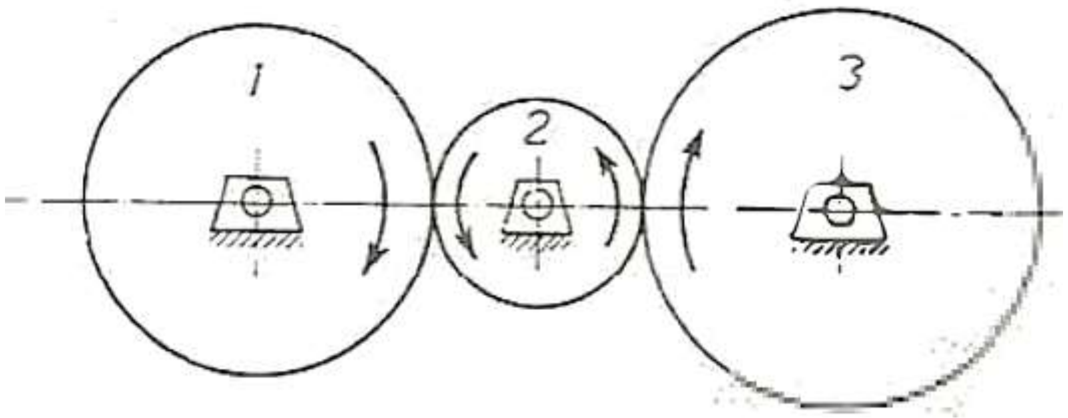
Any combination of two or more gears employed to mesh with each other to transmit power from one shaft to another and such combination of gears is called a gear train or train of toothed wheels.



Train value

The ratio of the speed of driven gear wheel to the speed of driver gear wheel is defined as the train value.

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}}$$

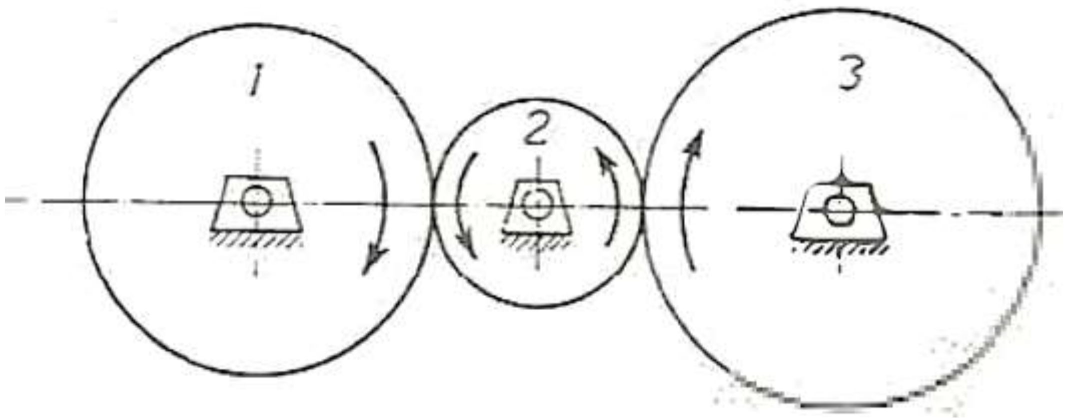


$$Train\ value\ = \frac{N_3}{N_1} = \frac{T_1}{T_3}$$

Speed ratio

The ratio of the speed of the driver wheel to the speed of the driven wheel is called the speed value.

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}}$$



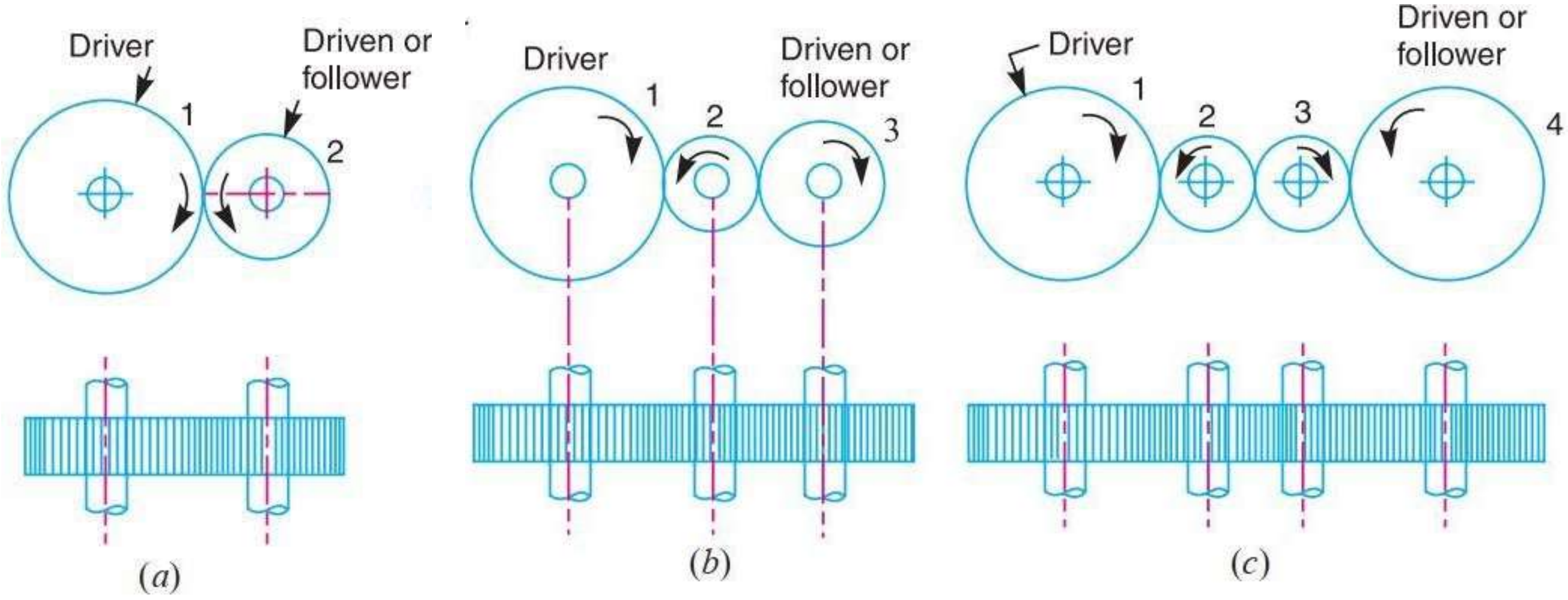
$$Speed\ ratio\ = \frac{N_1}{N_3}$$

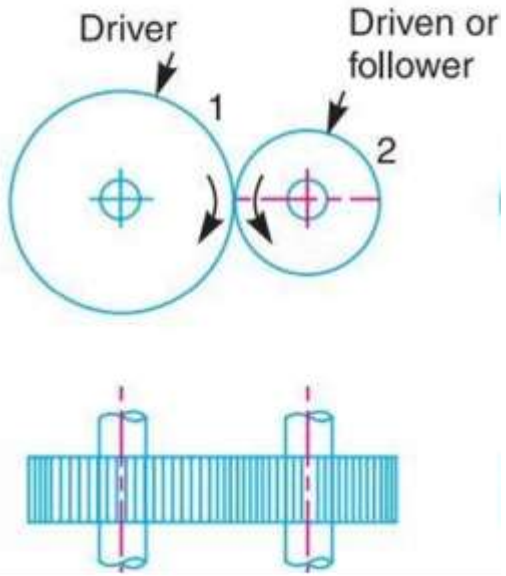
Types of Gear Trains

- Simple gear train
- Compound gear train
- Reverted gear train
- Epicyclic gear train

Simple Gear Train

If there is only one gear on each shaft as shown in the figure, it is known as ***simple gear train***.





N_1 = Speed of gear 1(or driver) in r.p.m.,

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.,

T_1 = Number of teeth on gear 1, and

T_2 = Number of teeth on gear 2.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.,

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

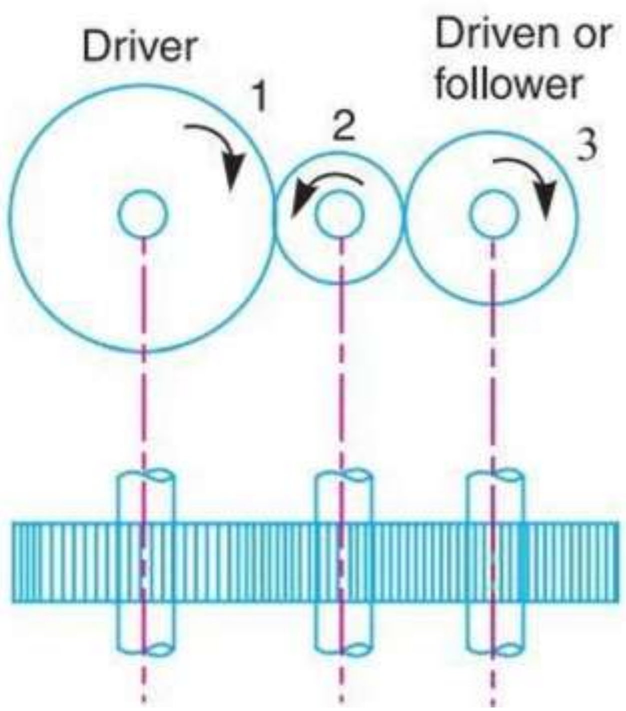
T_3 = Number of teeth on driven or follower.

Apply speed ratio to gear 1 and 2

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Apply speed ratio to gear 2 and 3

$$\frac{N_2}{N_3} = \frac{T_3}{T_2}$$



$$\frac{N_1}{N_3} = \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

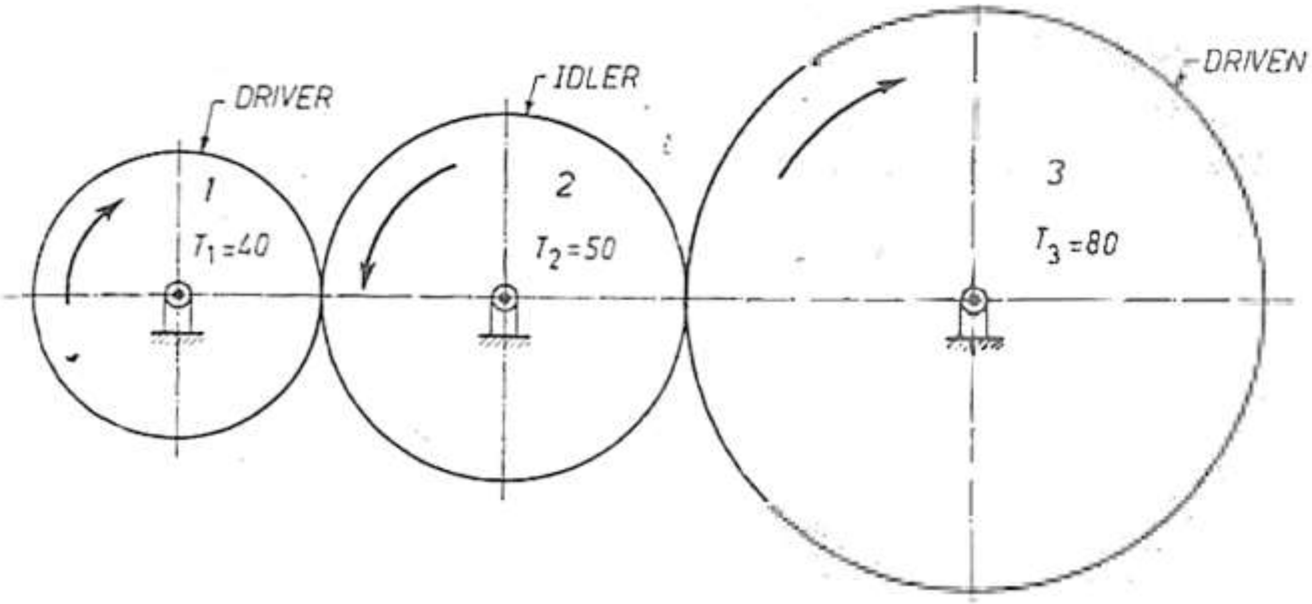
- The speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears.
- These intermediate gears are called ***idle gears*,** as they do not effect the speed ratio or train value of the system.
- The purpose of using idle gears are,
  - ✓To connect gears where a large centre distance is required
  - ✓To obtain the desired direction of motion of the driven gear (*i*.*e*. clockwise or anticlockwise).

Example 1

A simple gear train comprises three gears, each fixed on to separate shaft. The gear 1 is the driver and gear 3 is the driven gear. The gear 1 is running at 1000 rpm. The number of teeth on gears 1, 2 and 3 are 40, 50 and 80 respectively. Determine;

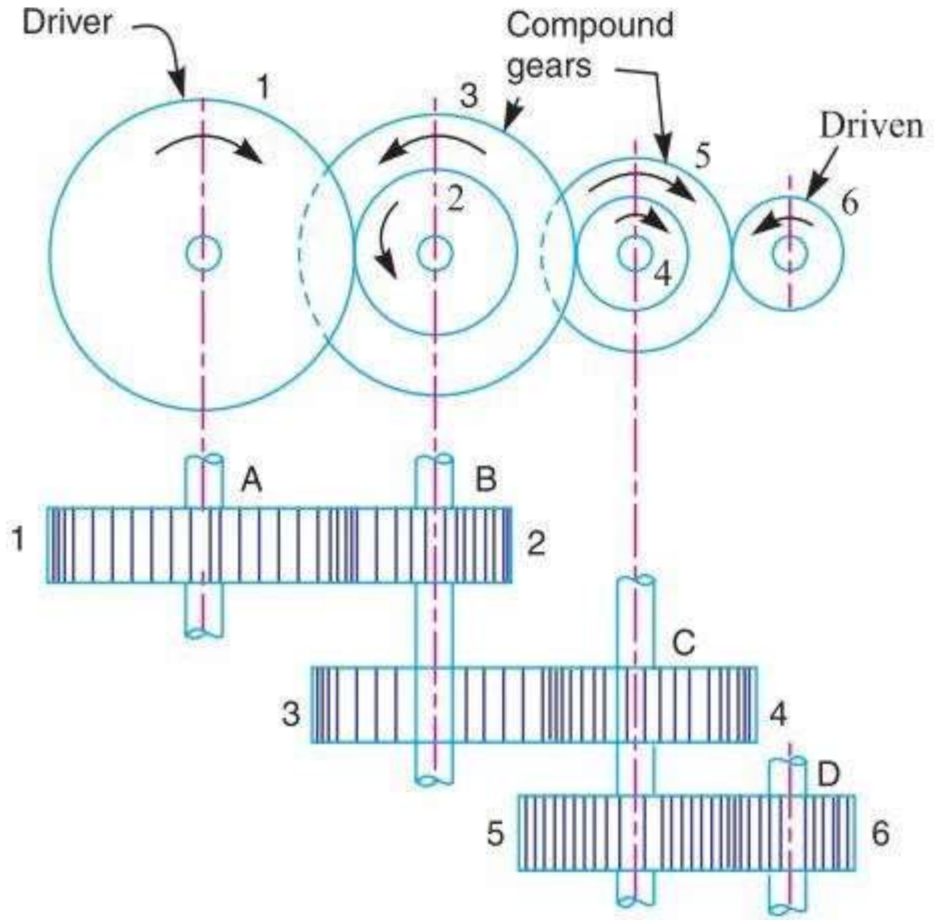
i. The speed ratio of the gear train

ii. The speed of rotation of the driven wheel and its direction



Compound Gear Trains

If there is more than one gear at least on one shaft these types of gear trains are called as ***Compound Gear trains***.



N_1 = Speed of driving gear 1,
T_1 = Number of teeth on driving gear 1,
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and
$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Apply speed ratio equation to gear 1 and 2

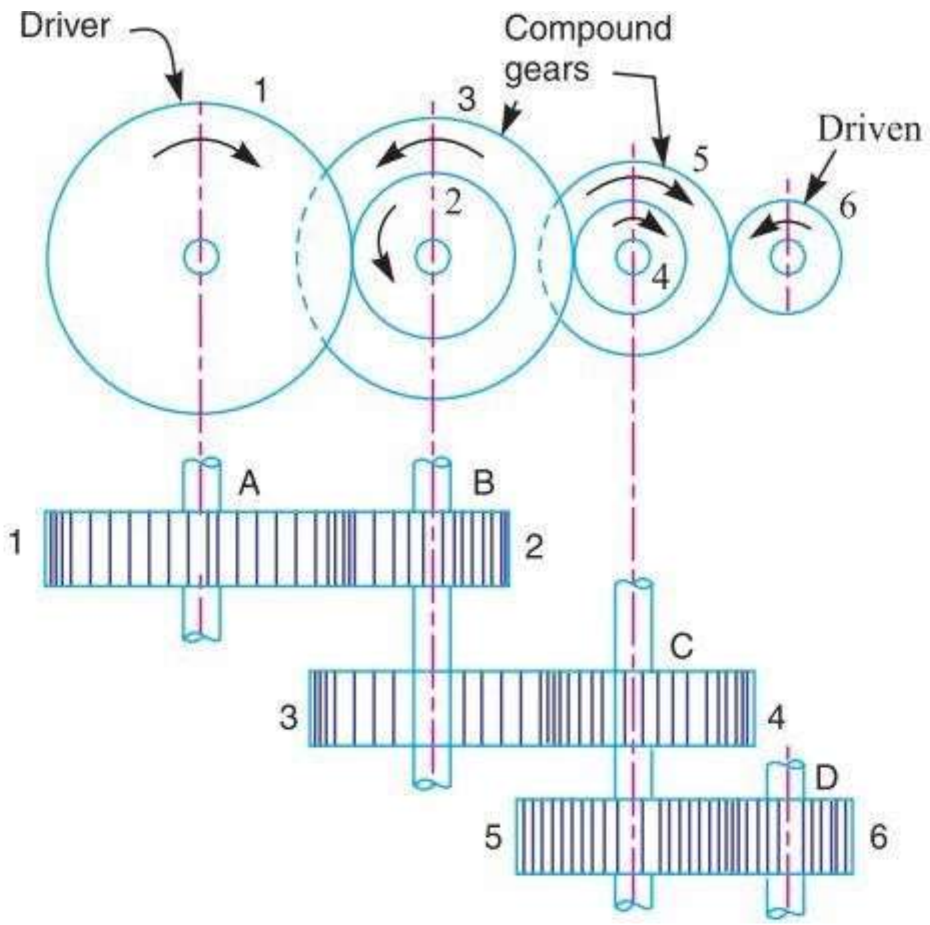
$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Apply speed ratio equation to gear 3 and 4

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}$$

Apply speed ratio equation to gear 5 and 6

$$\frac{N_5}{N_6} = \frac{T_6}{T_5}$$



$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Since the gears 2 and 3 are on the same shaft their speeds N_2 and N_3 are equal. Similarly, for the gear 4 and 5, N_4 and N_5 are equal.

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

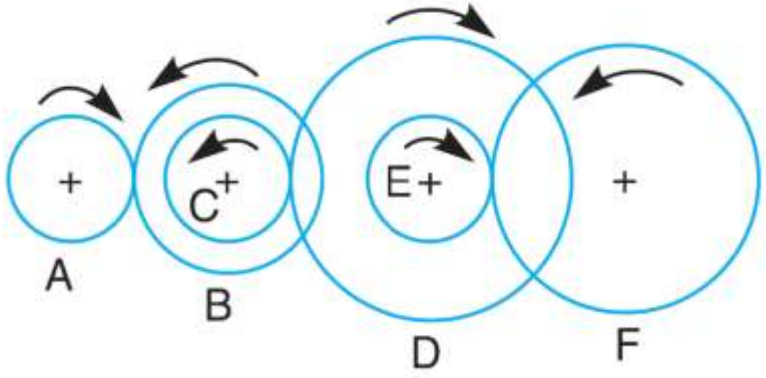
Example 2

The gearing of a machine tool is shown in Figure below. The motor shaft is connected to gear A and rotates at 975 rev/min. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft.

What is the speed of gear F ?

The number of teeth on each gear are as given below.

Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65



Drivers	Drivens
A	B
C	D
E	F

Design of Spur Gears

The spur gears are to be designed for the given velocity ratio and distance between the center of their shafts.

x = Distance between the centres of two shafts,

N_1 = Speed of the driver,

T_1 = Number of teeth on the driver,

d_1 = Pitch circle diameter of the driver,

N_2, T_2 and d_2 = Corresponding values for the driven or follower, and

p_c = Circular pitch.

The distance between the two shafts is

$$x = \frac{d_1 + d_2}{2}$$

Speed ratio or the velocity ratio,

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

From the above equation, the values of d_1 and d_2 and the circular pitch can be found.

Example 3

Two parallel shafts, about 600 mm apart are to be connected by spur gears. One shaft is to run at 360 rev/min and other at 120 rev/min. If the circular pitch is to be 25 mm, determine;

i. Number of teeth in each gears

ii. Actual distance between the two shafts

<u>Useful equations</u>

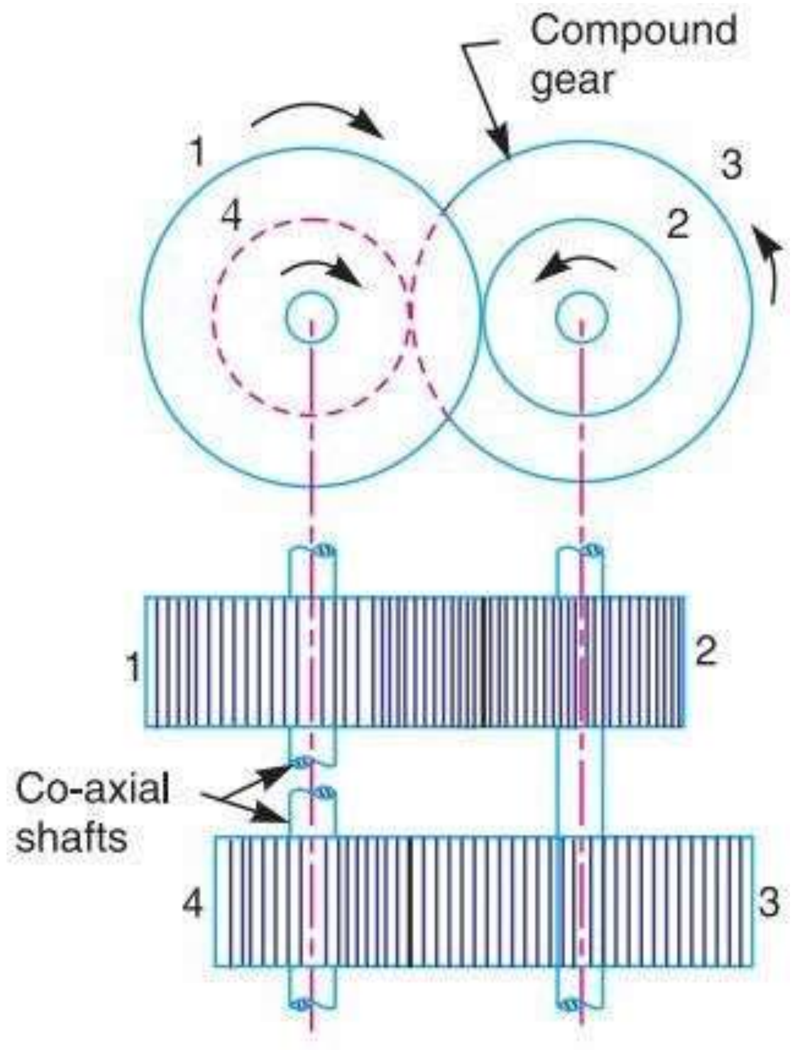
$$m = \frac{D}{T}$$

$$p_c = \frac{\pi D}{T} = \pi m$$

Reverted Gear Train

When the axes of the first gear and the last gear are co-axial, then the gear train is known as ***reverted gear train.***

Used in speed reducers, clocks and machine tools.



T_1 = Number of teeth on gear 1,

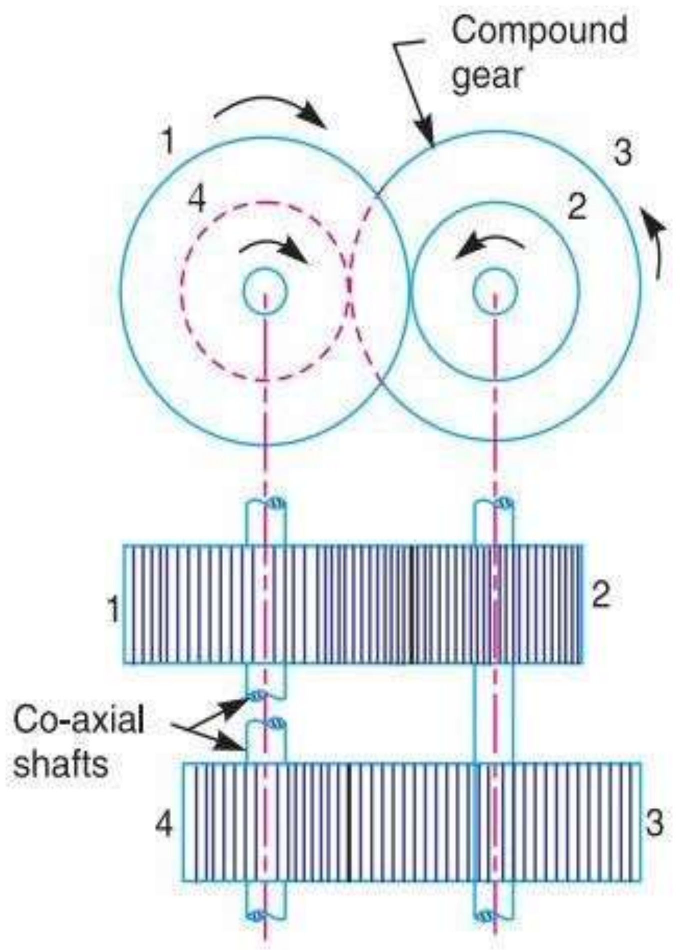
r_1 = Pitch circle radius of gear 1,

N_1 = Speed of gear 1 in r.p.m.

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears,

N_2, N_3, N_4 = Speed of respective gears in r.p.m.



Center distance between the shafts

$$r_1 + r_2 = r_3 + r_4$$

Assume modules of all gears to be same

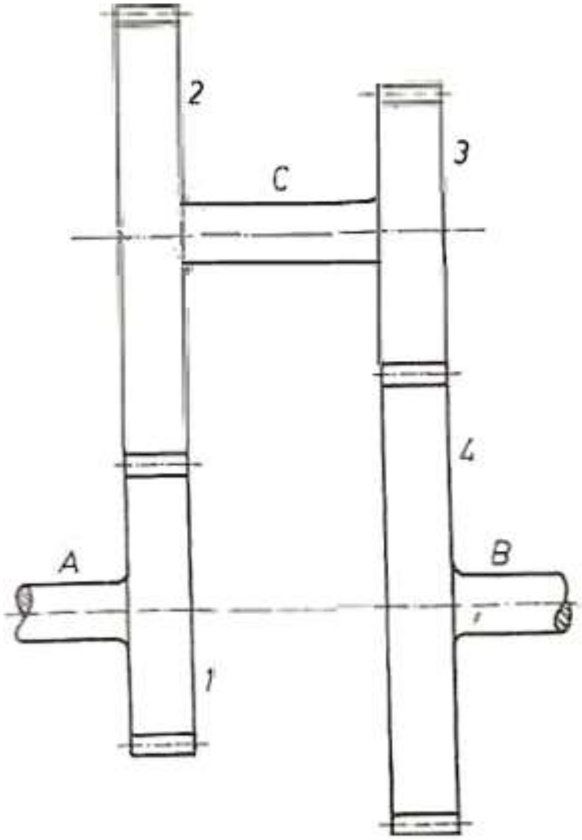
$$T_1 + T_2 = T_3 + T_4$$

Speed ratio $$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Example 4

Input shaft A and the output shaft B are coincident and are geared together through a parallel counter shaft C. The wheels connecting shafts A and C have a module of 2 mm and those connecting C and B have a module of 3.5 mm. Speed of output shaft has to be less than 1/10 that of shaft A. if the two pinions have each 24 teeth, determine;

i. Suitable teeth for other gear wheels
ii. Actual speed ratio
iii. The corresponding distance of shaft C from A.



Useful equations

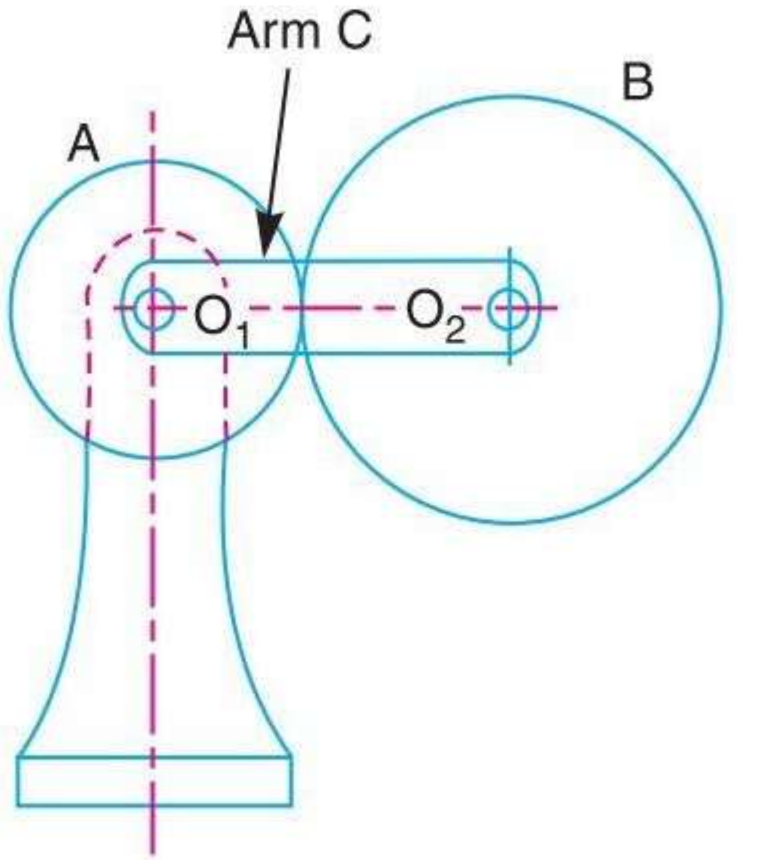
$$m = \frac{D}{T}$$

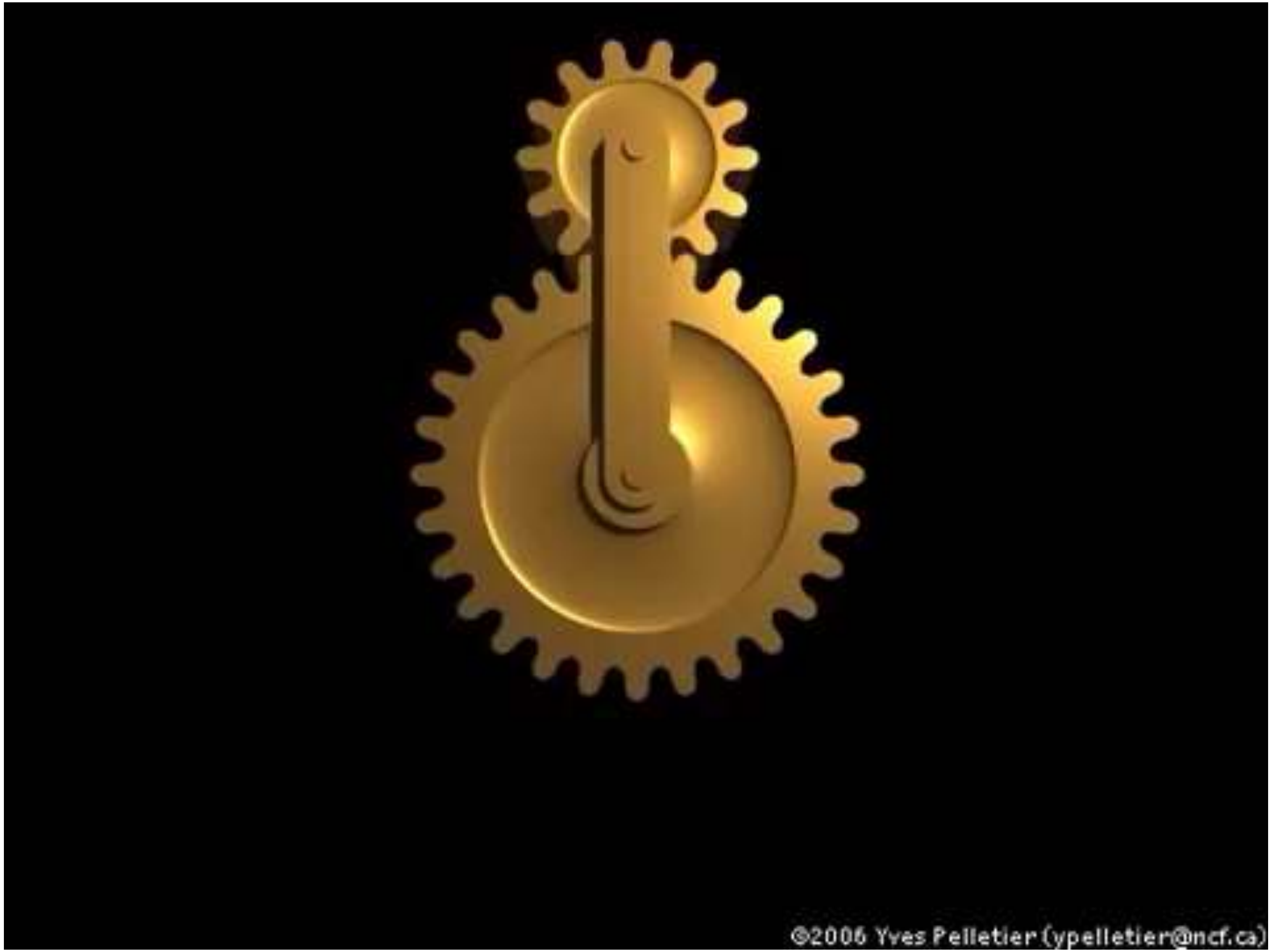
$$p_c = \frac{\pi D}{T} = \pi m$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

Epicyclic gear train

If the axes of the shafts, the gears are mounted, may move relative to a fixed axis is known as ***Simple epicyclic gear*** train.

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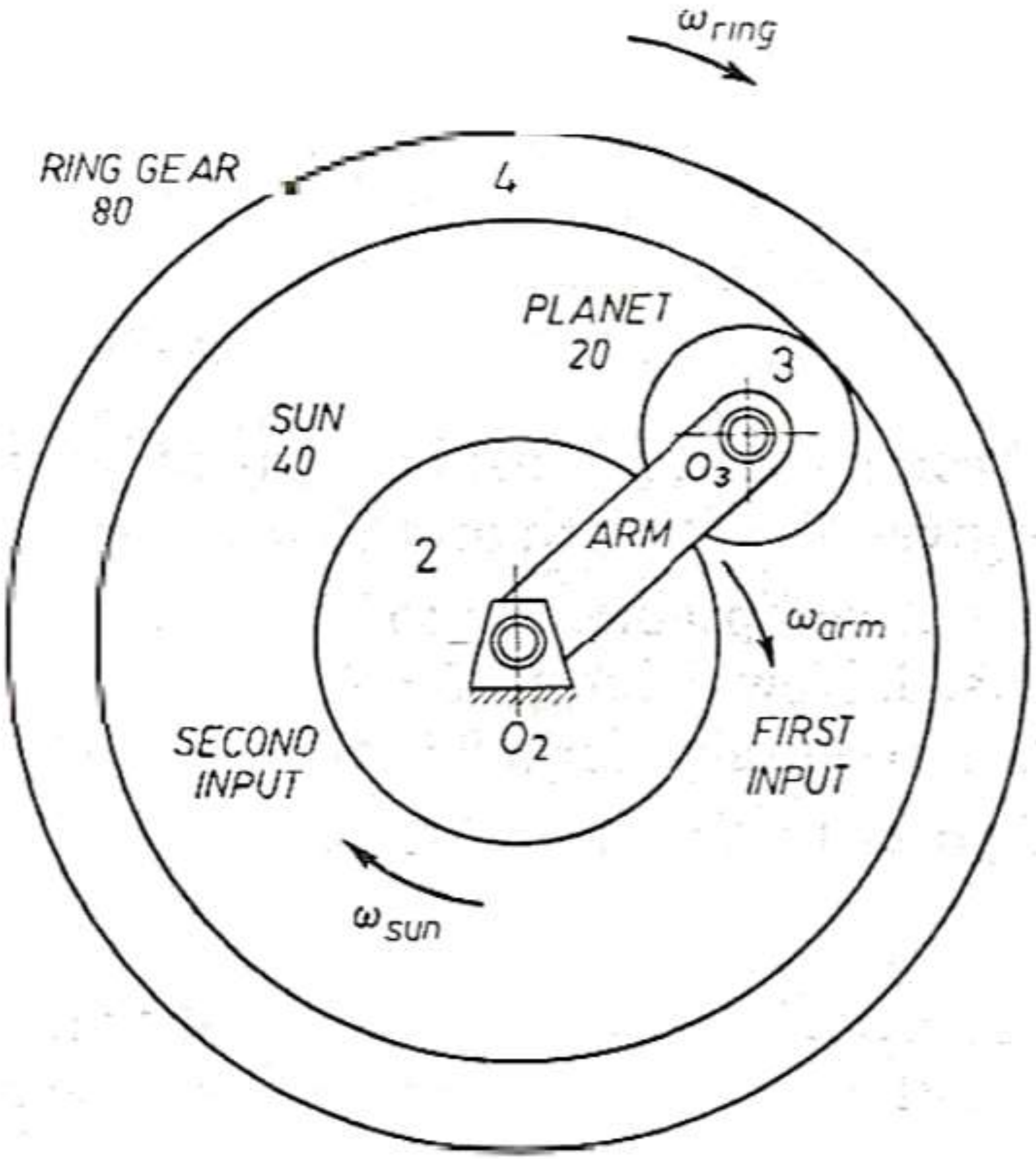
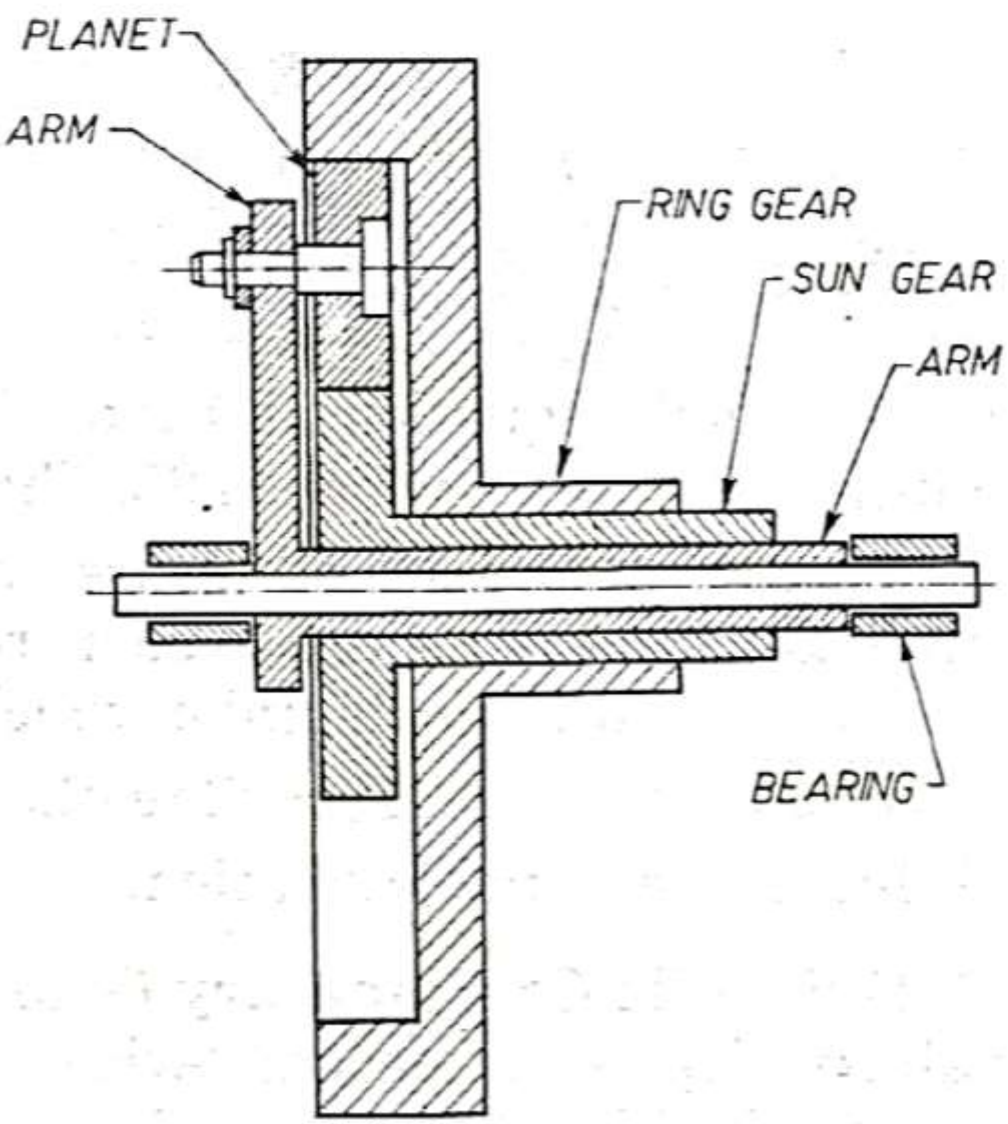
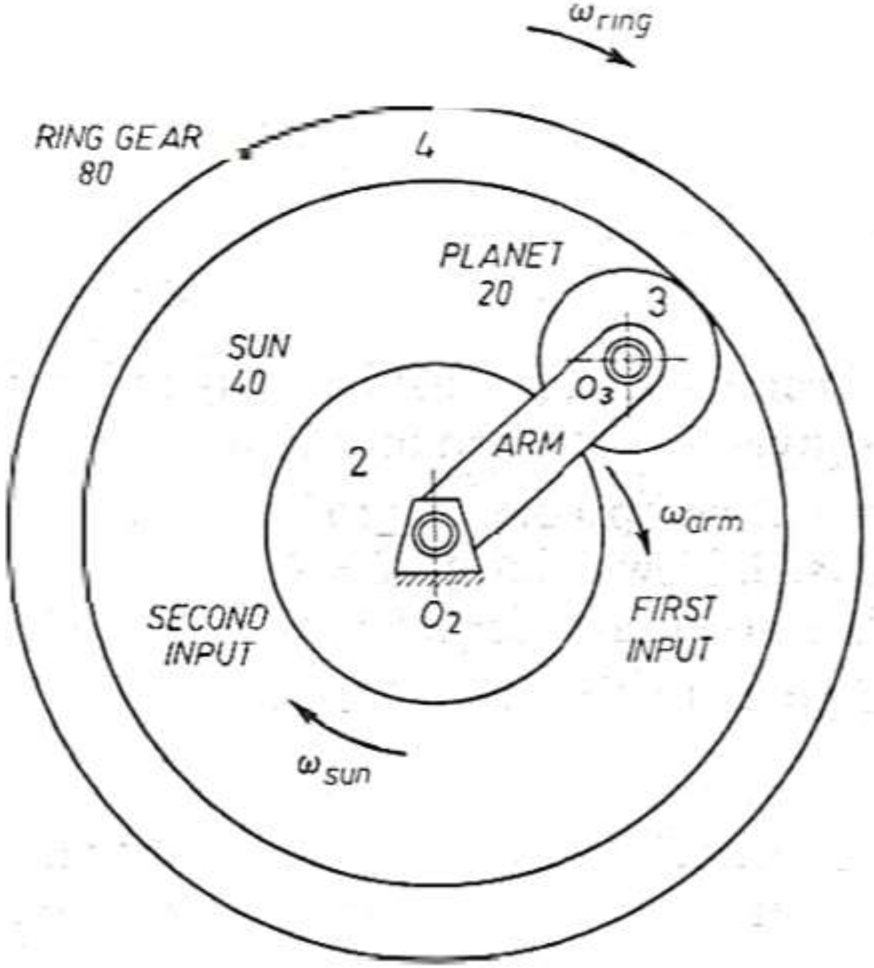


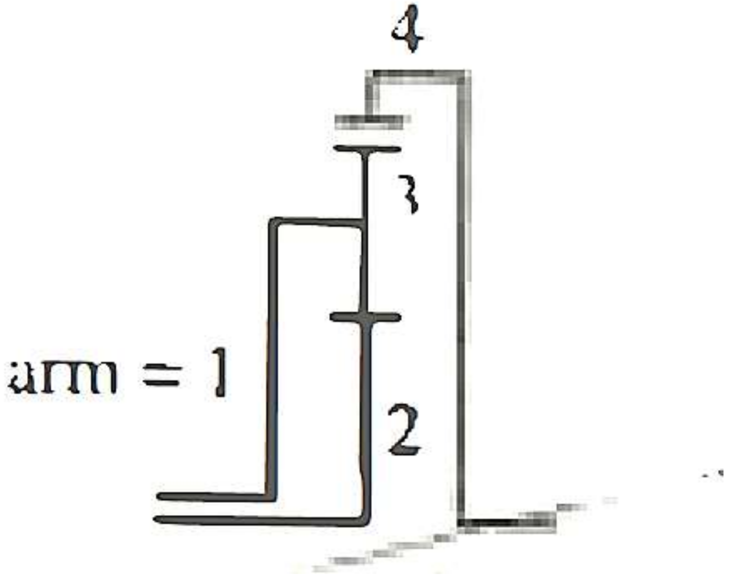
ω_{ring}
RING GEAR
80
4
PLANET
20
3
SUN
40
O_3
ARM
2
ω_{arm}
SECOND
INPUT
O_2
FIRST
INPUT
ω_{sun}



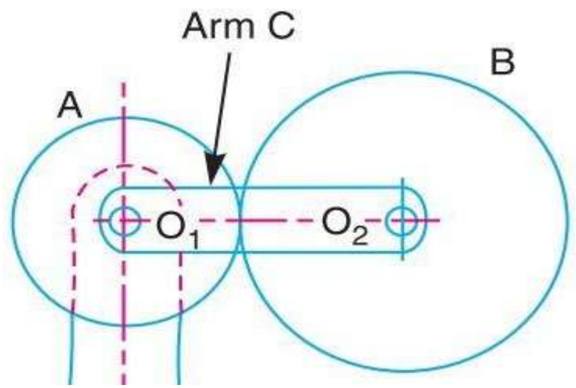
PLANET
ARM
RING GEAR
SUN GEAR
ARM
BEARING

Schematic diagram of epicyclic gear train





Velocity Ratio of Epicyclic Gear Train



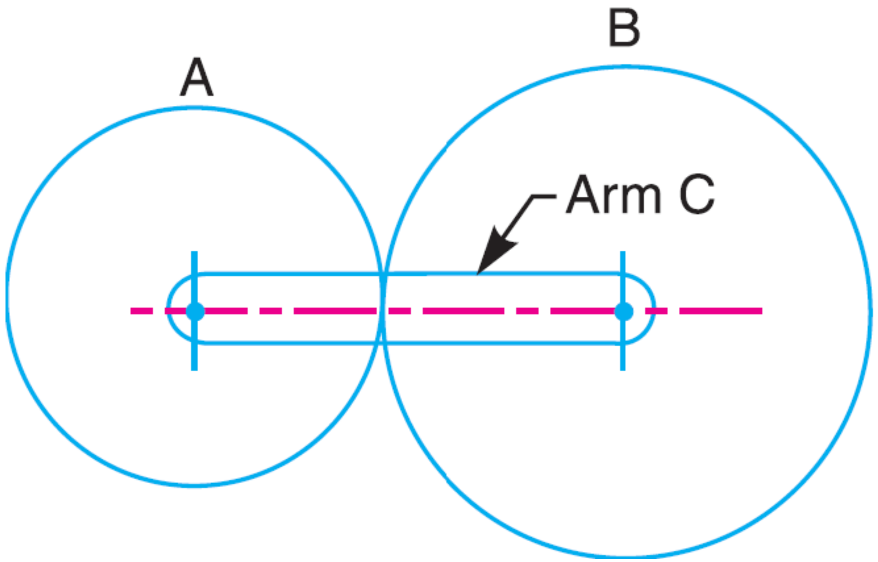
Tabular method

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Example 5

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rev/min in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B.

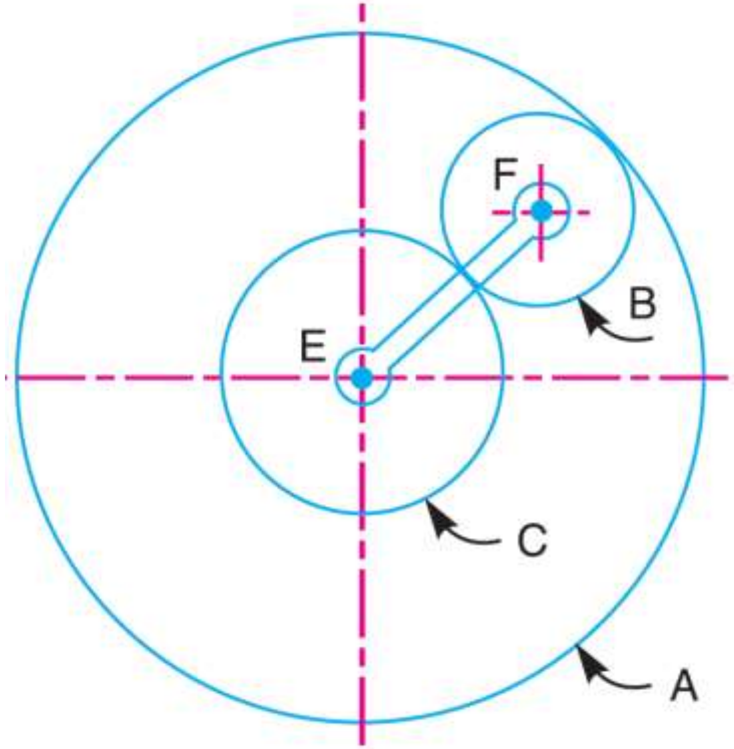
If the gear A instead of being fixed, makes 300 rev/min in the clockwise direction, what will be the speed of gear B ?



Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear *A* rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Example 6

An epicyclic gear consists of three gears A, B and C as shown. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 rev/min in counter clockwise direction. If the gear A is fixed, determine the speed of gears B and C.



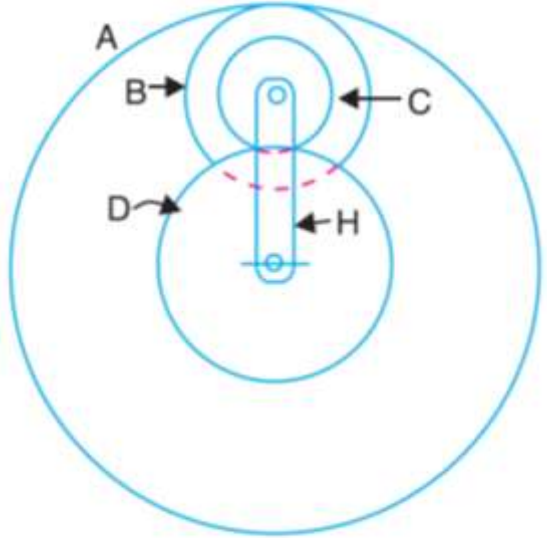
Step No.	Conditions of motion	Revolutions of elements			
		Arm EF	Gear C	Gear B	Gear A
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

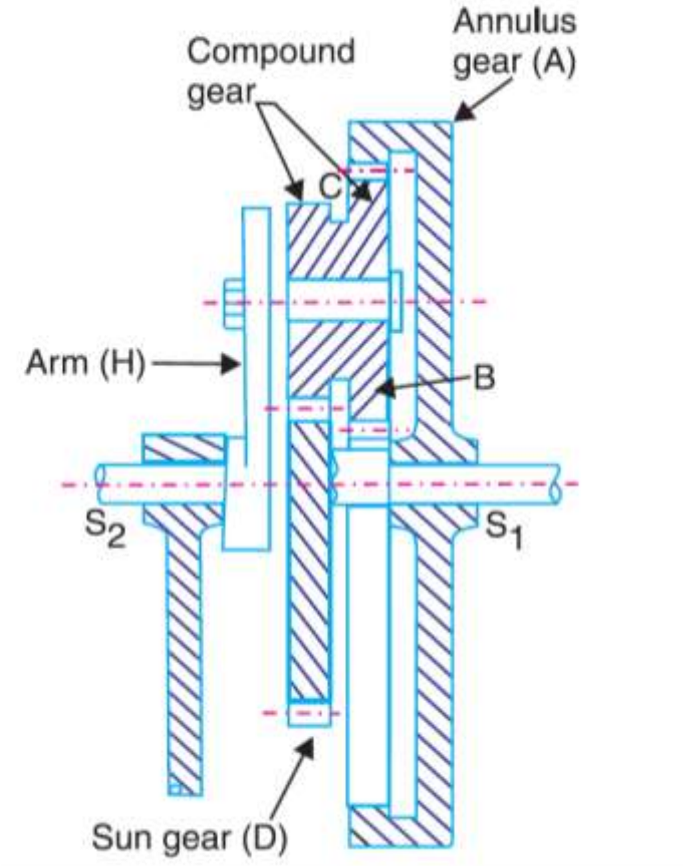
Compound Epicyclic Gear Train

It consists of two co-axial shafts, an annulus gear *A* which is fixed, the compound gear (or planet gear) *B-C*, the sun gear *D* and the arm *H*.

The sun gear is co-axial with the annulus gear and the arm but independent of them.

When the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive.



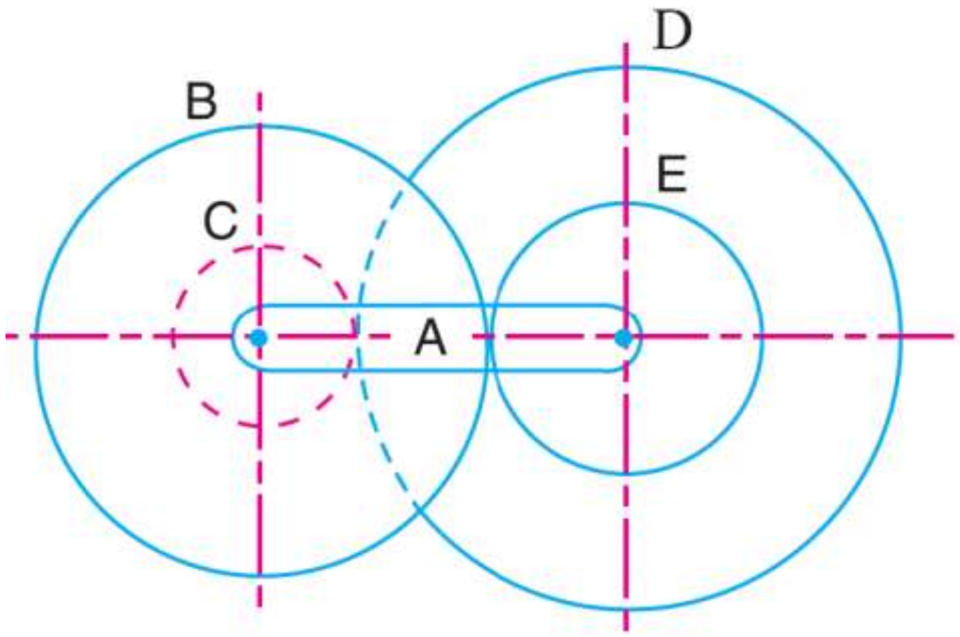


Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound gear B-C	Gear A
1.	Arm fixed-gear D rotates through + 1 revolution	0	+ 1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-gear D rotates through + x revolutions	0	$+x$	$-x \times \frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_D}{T_C}$	$y - x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

Example 7

In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. All the gears have the same module and the number of teeth on gears B, C and D are 75, 30 and 90 respectively.

Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 rev/min clockwise.

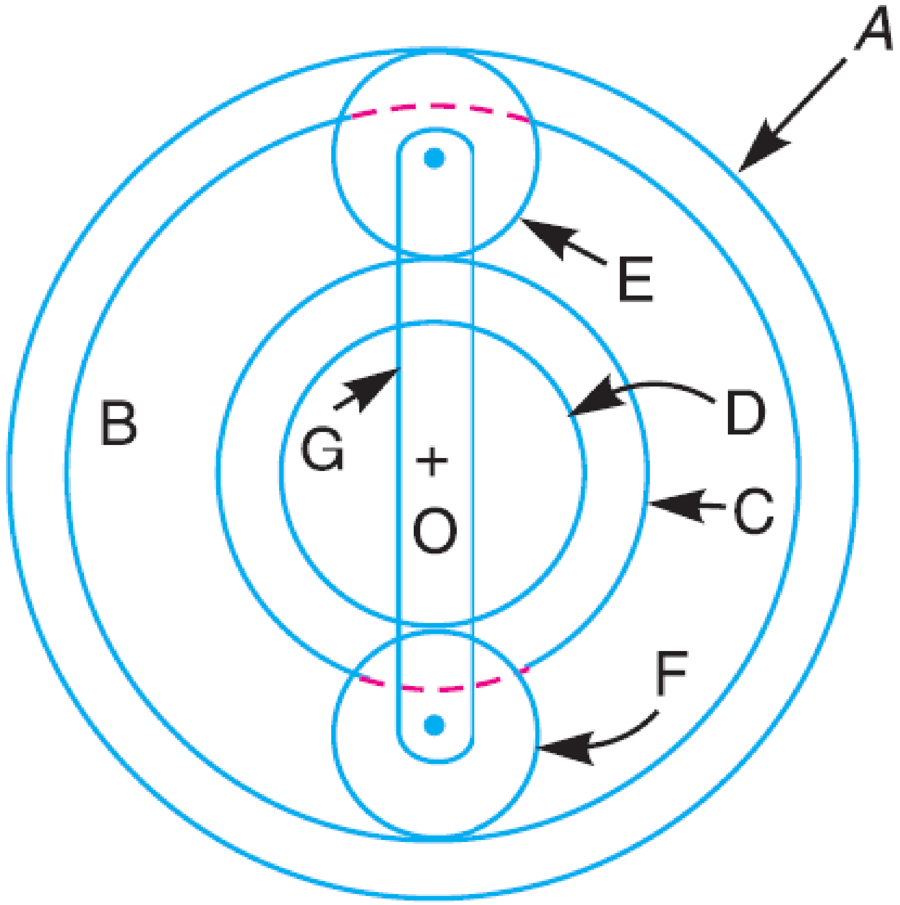


Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D*-*E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D*-*E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Example 8

In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : TC = 28; TD = 26; TE = TF = 18.

1. *Sketch the arrangement*
2. *Find the number of teeth on A and B*
3. *If the arm G makes 100 rev/min clockwise and A is fixed, find the speed of B*
4. *If the arm G makes 100 rev/min clockwise and wheel A makes 10 rev/min counter clockwise ; find the speed of wheel B.*


A
E
B
G
D
+
O
C
F

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed- wheel A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	$+1$	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2.	Arm fixed-wheel A rotates through $+x$ revolutions	0	$+x$	$+x\times\frac{T_A}{T_E}$	$-x\times\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_A}{T_E}$	$y-x\times\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Question 1

A compound train consists of six gears. The number of teeth on the gears are as follows :

Gear:	A	B	C	D	E	F
No. of teeth:	60	40	50	25	30	24

The gears B and C are on one shaft while the gears D and E are on another shaft. The gear A drives gear B, gear C drives gear D and gear E drives gear F.

If the gear A transmits 1.5 kW at 100 rev/min and the gear train has an efficiency of 80%, find the torque on gear F.

Question 2

In a reverted gear train, two shafts A and B are in the same straight line and are geared together through an intermediate parallel shaft C. The gears connecting the shafts A and C have a module of 2 mm and those connecting the shafts C and B have a module of 4.5 mm. The speed of shaft A is to be about but greater than 12 times the speed of shaft B, and the ratio at each reduction is same.

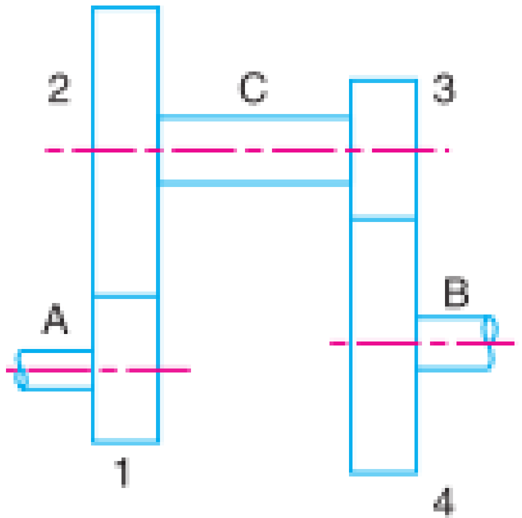
1. Find suitable number of teeth for gears. The number of teeth of each gear is to be a minimum but not less than 16.
2. Find the exact velocity ratio and the distance of shaft C from A and B.

<u>Useful equations</u>

$$m = \frac{D}{T}$$

$$p_c = \frac{\pi D}{T} = \pi m$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$



Question 3

In an epicyclic gear train, the wheel C is keyed to the shaft B and wheel F is keyed to shaft A. The wheels D and E rotate together on a pin fixed to the arm G. The number of teeth on wheels C, D, E and F are 35, 65, 32 and 68 respectively.

If the shaft A rotates at 60 rev/min and the shaft B rotates at 28 rev/min in the opposite direction, find the speed and direction of rotation of arm G.

(Assume the anticlockwise rotation as positive and clockwise as negative.)