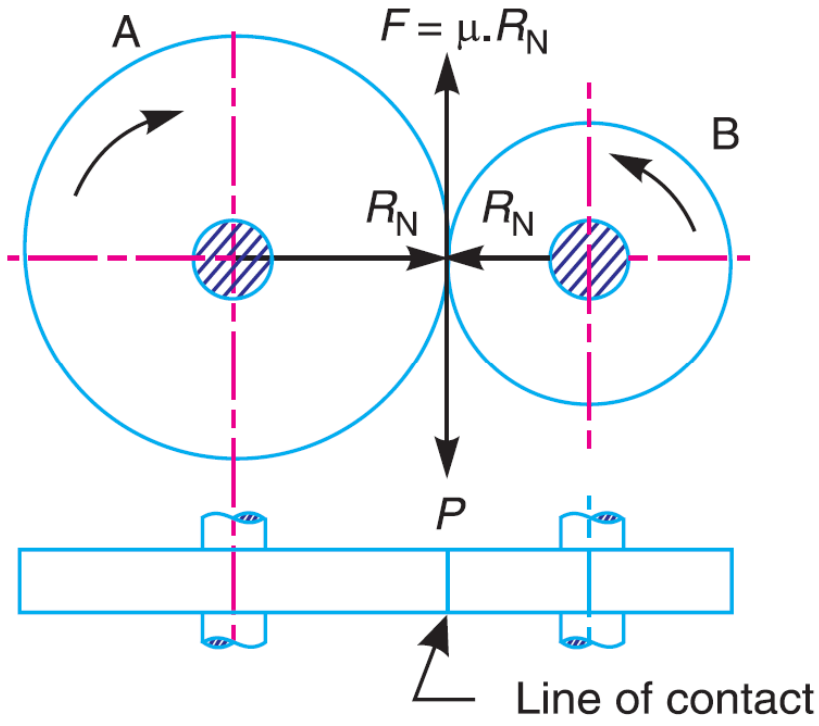
Gears

Friction wheels

- Consider two plain circular wheels *A* and *B* mounted on shafts, having sufficient rough surfaces and pressing against each other.
- The wheel *A* is keyed to the rotating shaft and the wheel *B* to the shaft, to be rotated.
- When the wheel *A* is rotated by a rotating shaft, it will rotate the wheel *B* in the opposite direction.

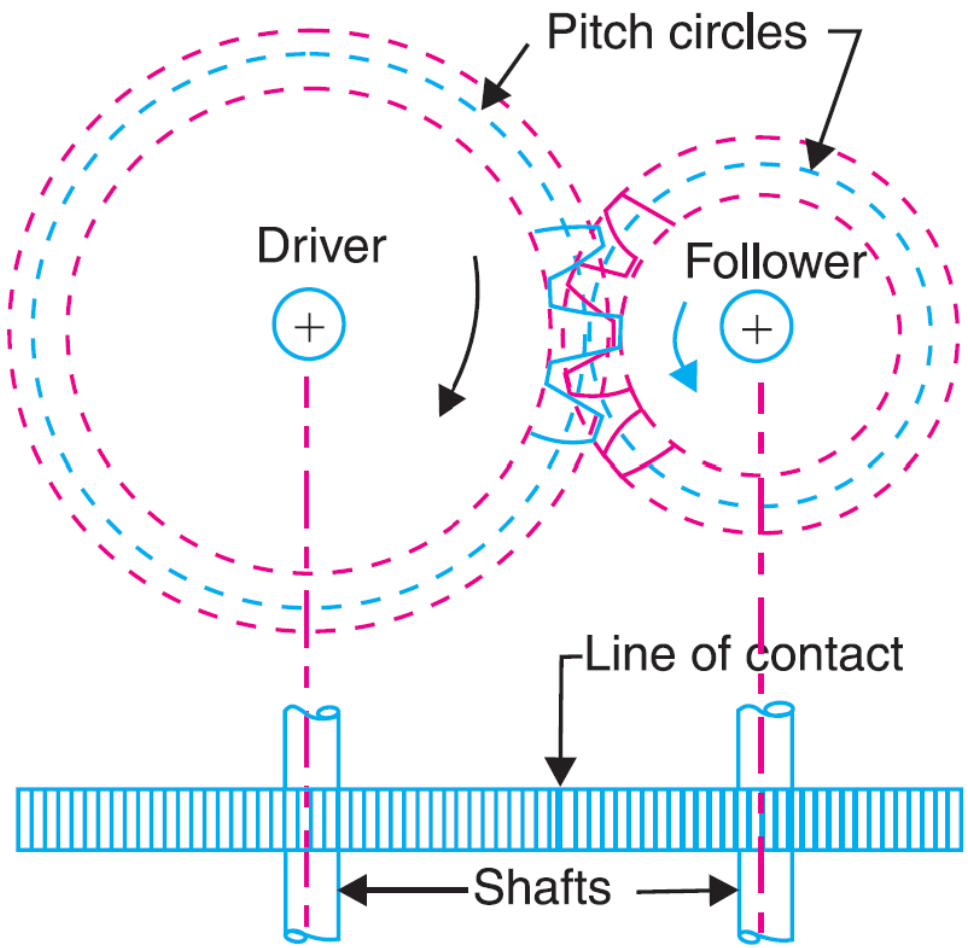


- Wheel *B* will be rotated (by wheel *A*) until the tangential force exerted by wheel *A* does not exceed the maximum frictional resistance between the two wheels.

- When the tangential force (*P*) exceeds the frictional resistance (*F*), slipping will take place between the two wheels.

- To avoid the slipping, a number of projections (called teeth) are provided on the periphery of the wheel *A*, which will fit into the corresponding recesses on the periphery of the wheel *B*.

- A friction wheel with the teeth cut on it is ***toothed wheel*** **or** ***gear.***


Pitch circles
Driver
Follower
Line of contact
Shafts

Advantages of gear drive

- Transmits exact velocity ratio.
- Transmit large power.
- High efficiency.
- Long durability.
- Compact layout.

Disadvantages of Gear Drive

- Manufacture of gears require special tools and equipment.
- Error in cutting teeth may cause vibrations and noise during operation.

Classification of toothed wheels

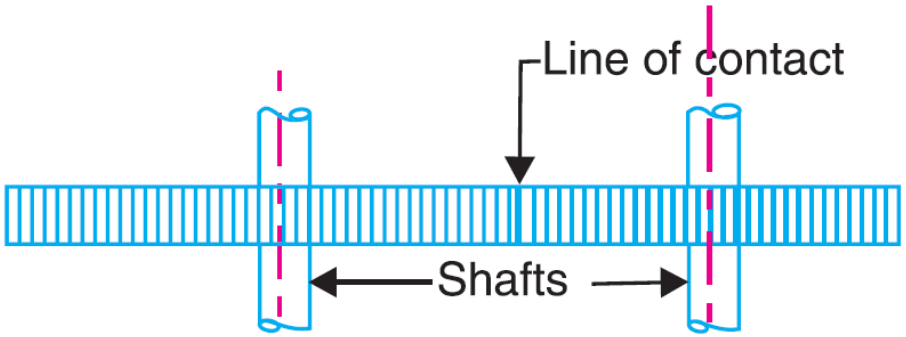
1. ***According to the position of axes of the shafts***

The axes of the two shafts between which the motion is to be transmitted, will be

   i. Parallel
   ii. Intersecting
   iii. Non-intersecting and non-parallel.

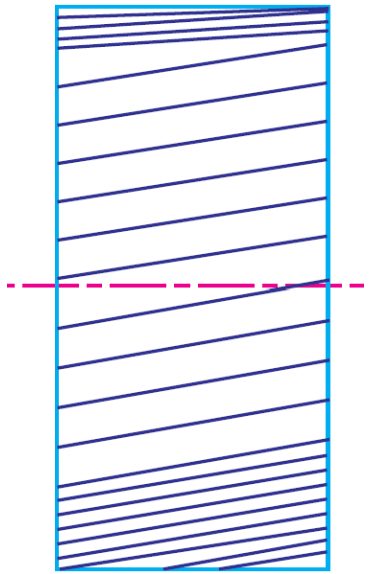
Parallel axes

- The two parallel and co-planar shafts are connected by the gears.
- These gears are called ***spur gears*** and the arrangement is known as ***spur gearing***.
- These gears have teeth parallel to the axis of the wheel.

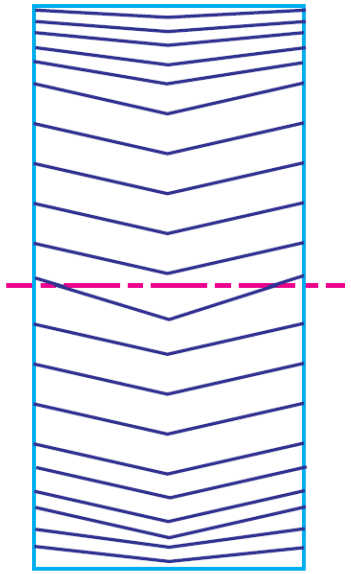


Spur gear

- When the teeth are inclined to the axis, the spur gearing is known as ***helical gearing***.
- The double helical gears are known as ***herringbone gears***.
- A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to parallel shafts and having a line contact.

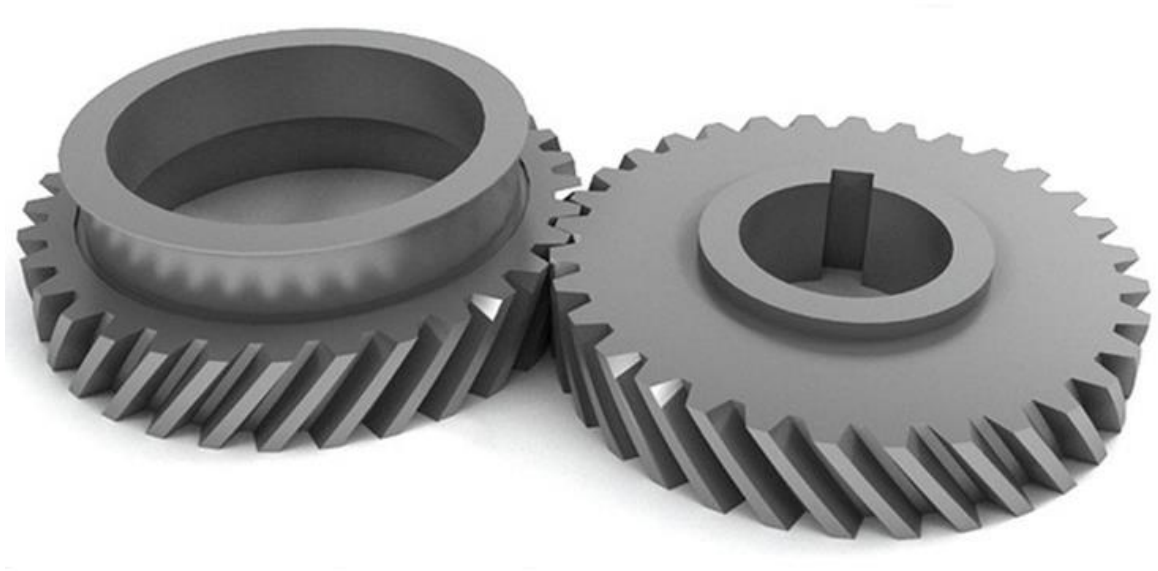


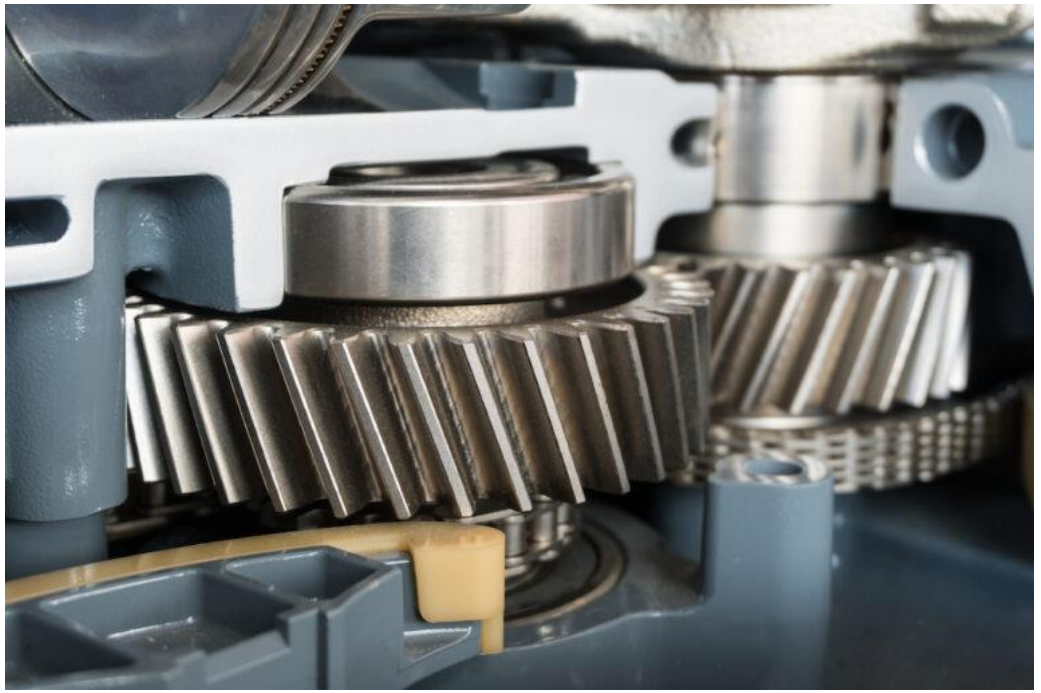
Single helical gear



Double helical gear

Single helical gear

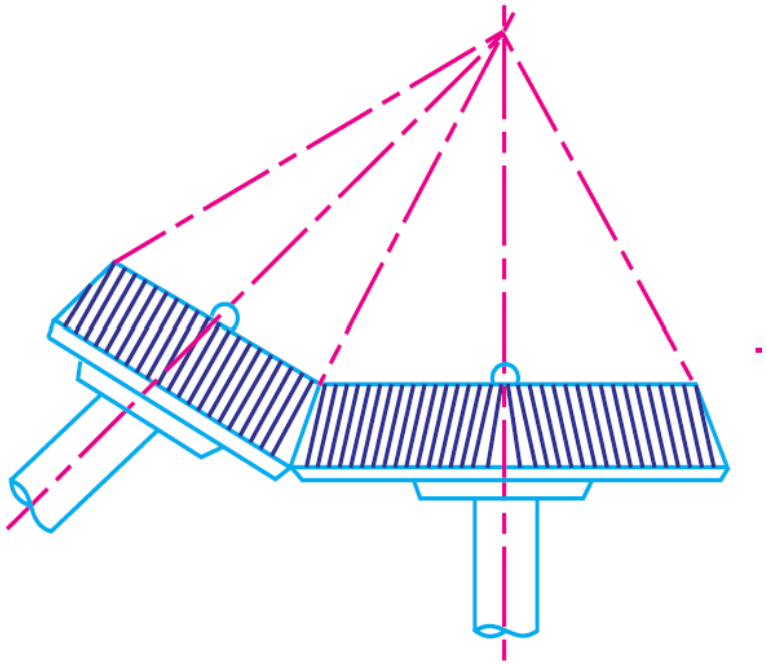




Double helical gear

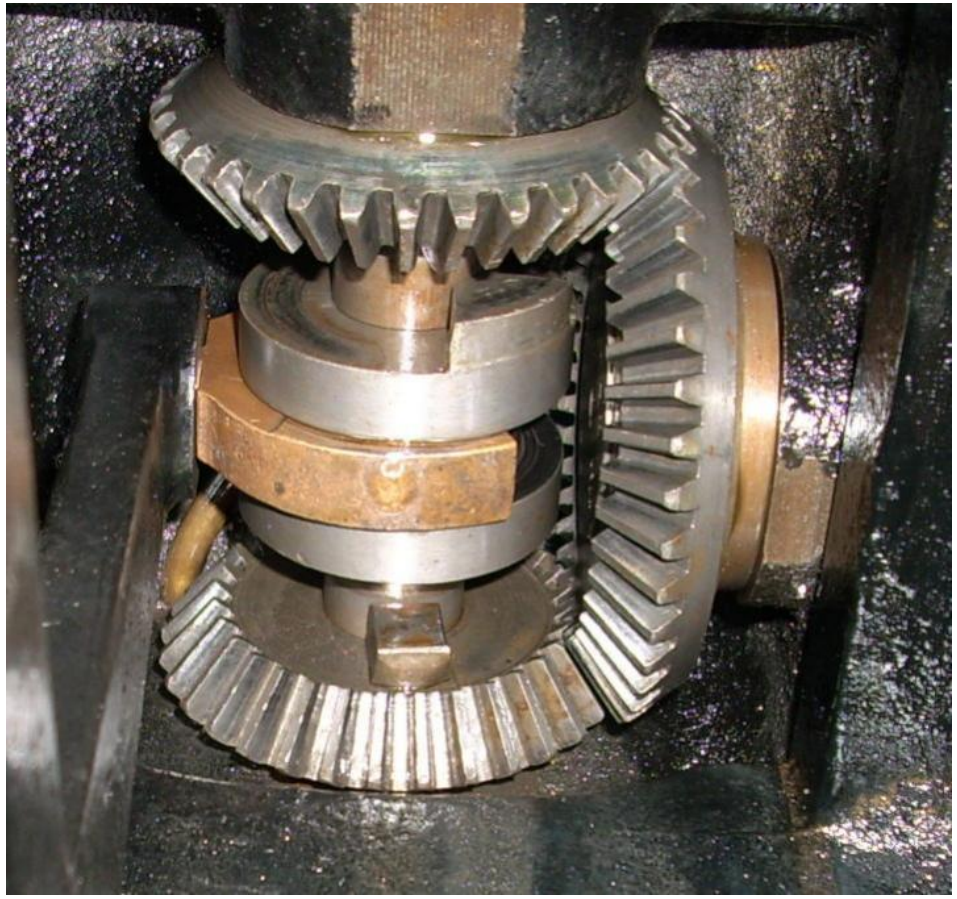
Intersecting axes

- The two non-parallel but intersecting coplanar shafts are connected by gears.
- These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing***.
- The bevel gears also have their teeth inclined to the face of the bevel, known as ***helical bevel gears***.



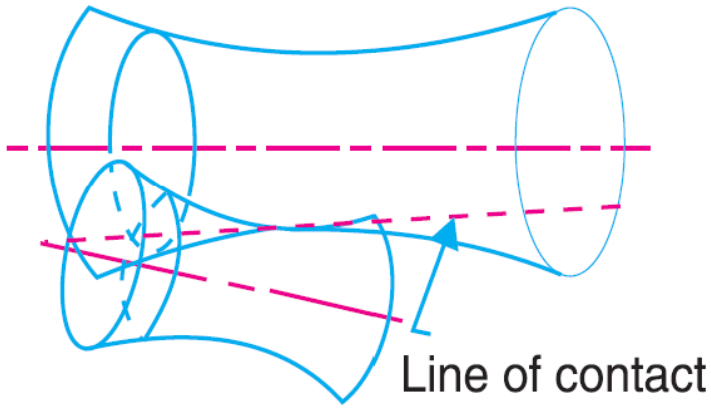
Industrial Applications of Bevel Gears - Bevel Gears (India) Pvt. Ltd. (bevelgearsindia.com)

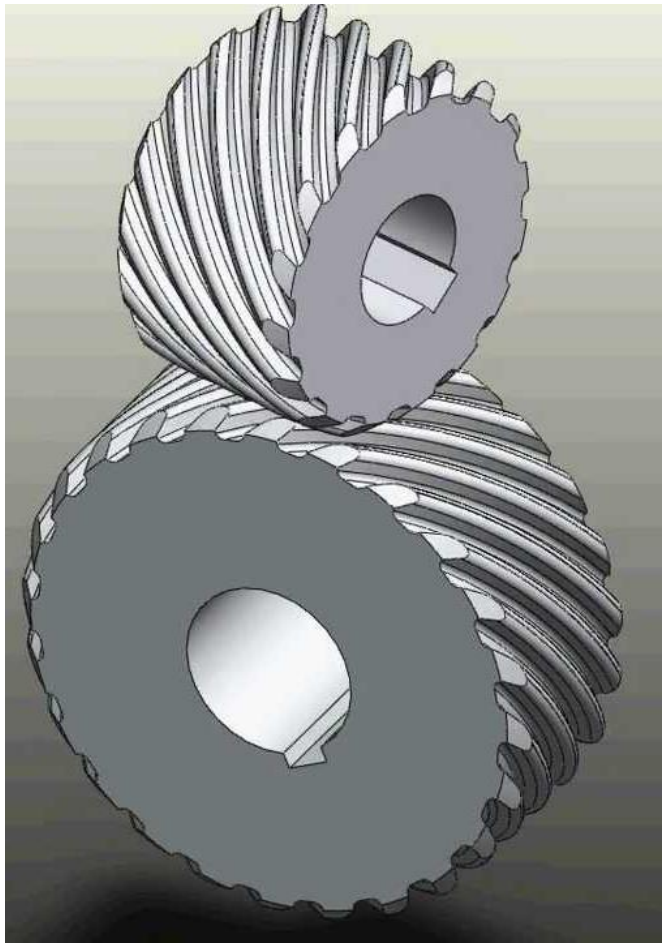
Bevel gear



Non-intersecting and non-parallel axes

- The two non-intersecting and non-parallel, non-coplanar shafts are connected by gears.
- These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing***.
- This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids***.





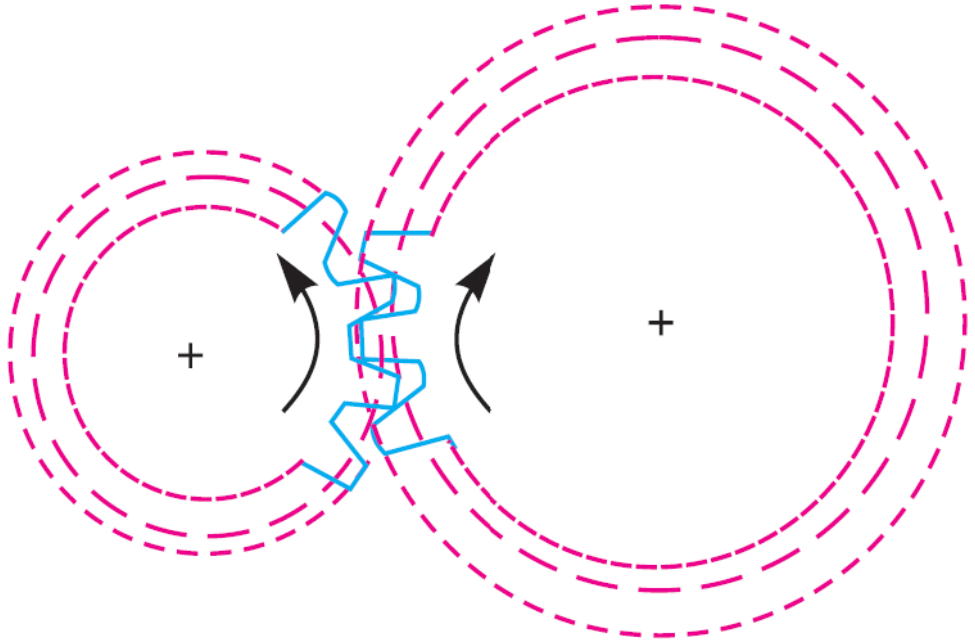
2. ***According to the type of gearing***
    i. External gearing
    ii. Internal gearing
    iii. Rack and pinion

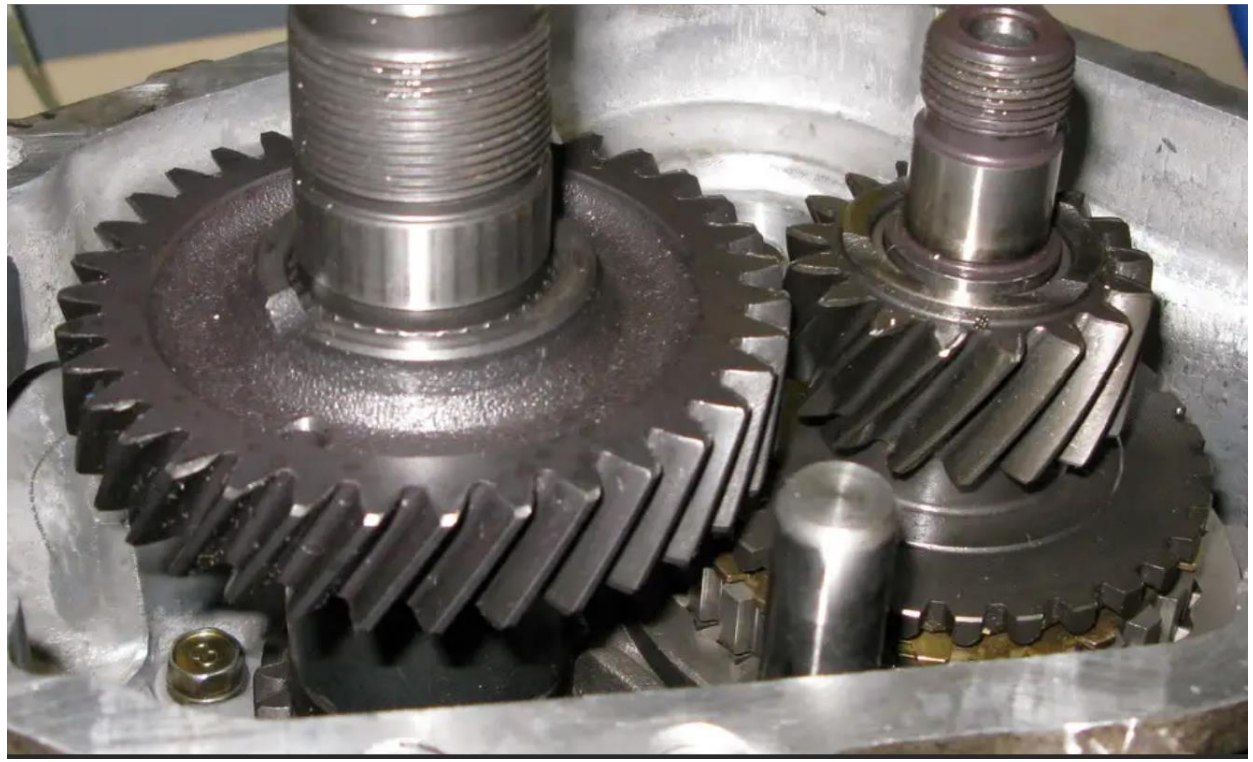
External gearing

The gears of the two shafts mesh externally with each other.

The larger of these two wheels is called ***spur wheel*** and the smaller wheel is called **pinion.**

In an external gearing, the motion of the two wheels is always in ***opposite direction.***



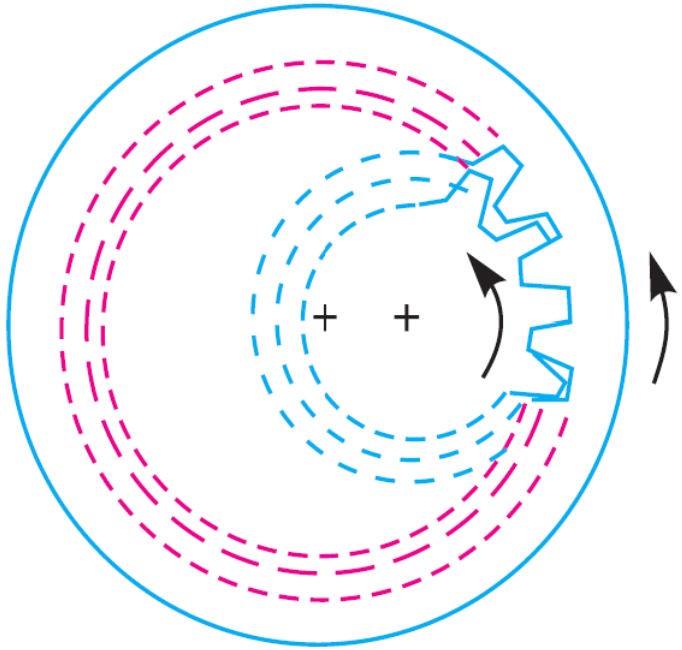


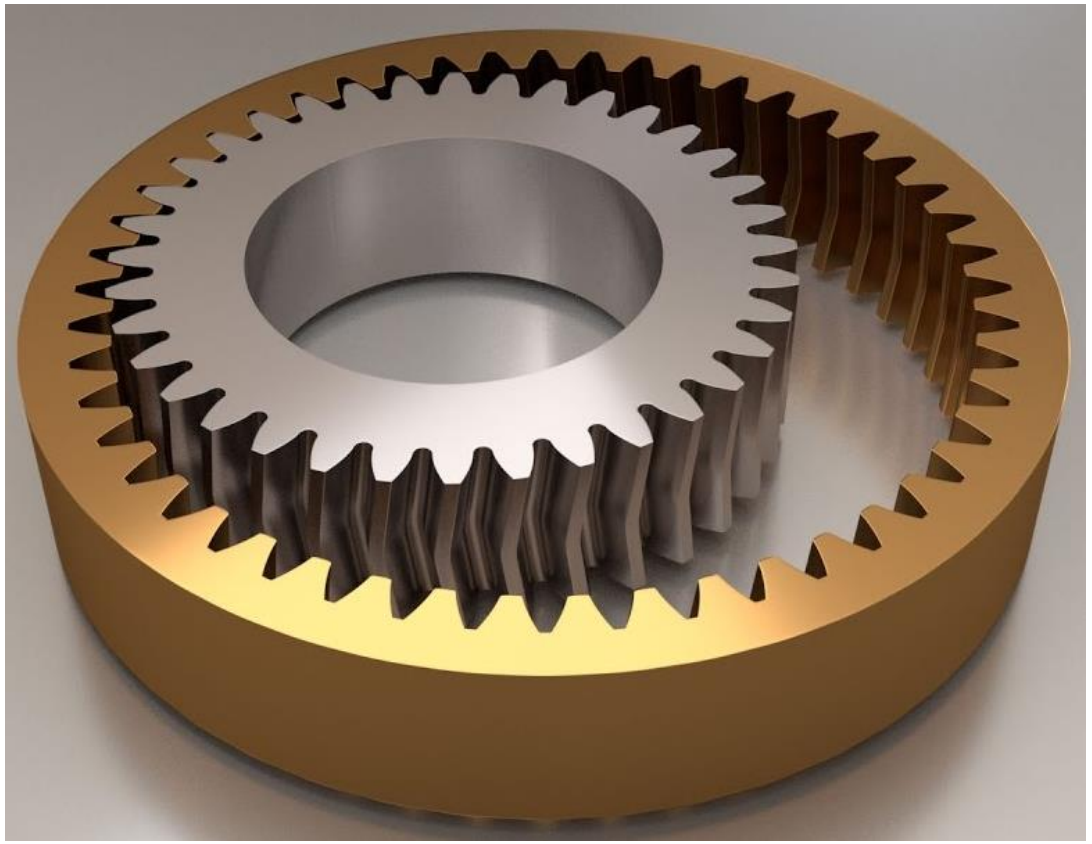
Internal gearing

The gears of the two shafts mesh ***internally*** with each other.

The larger of these two wheels is called ***annular wheel*** and the smaller wheel is called ***pinion***.

In an internal gearing, the motion of the two wheels is always ***in same direction.***



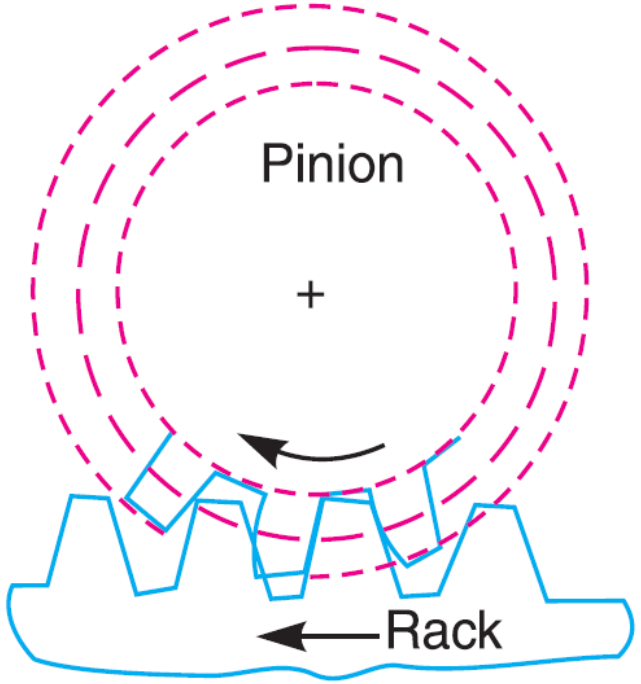


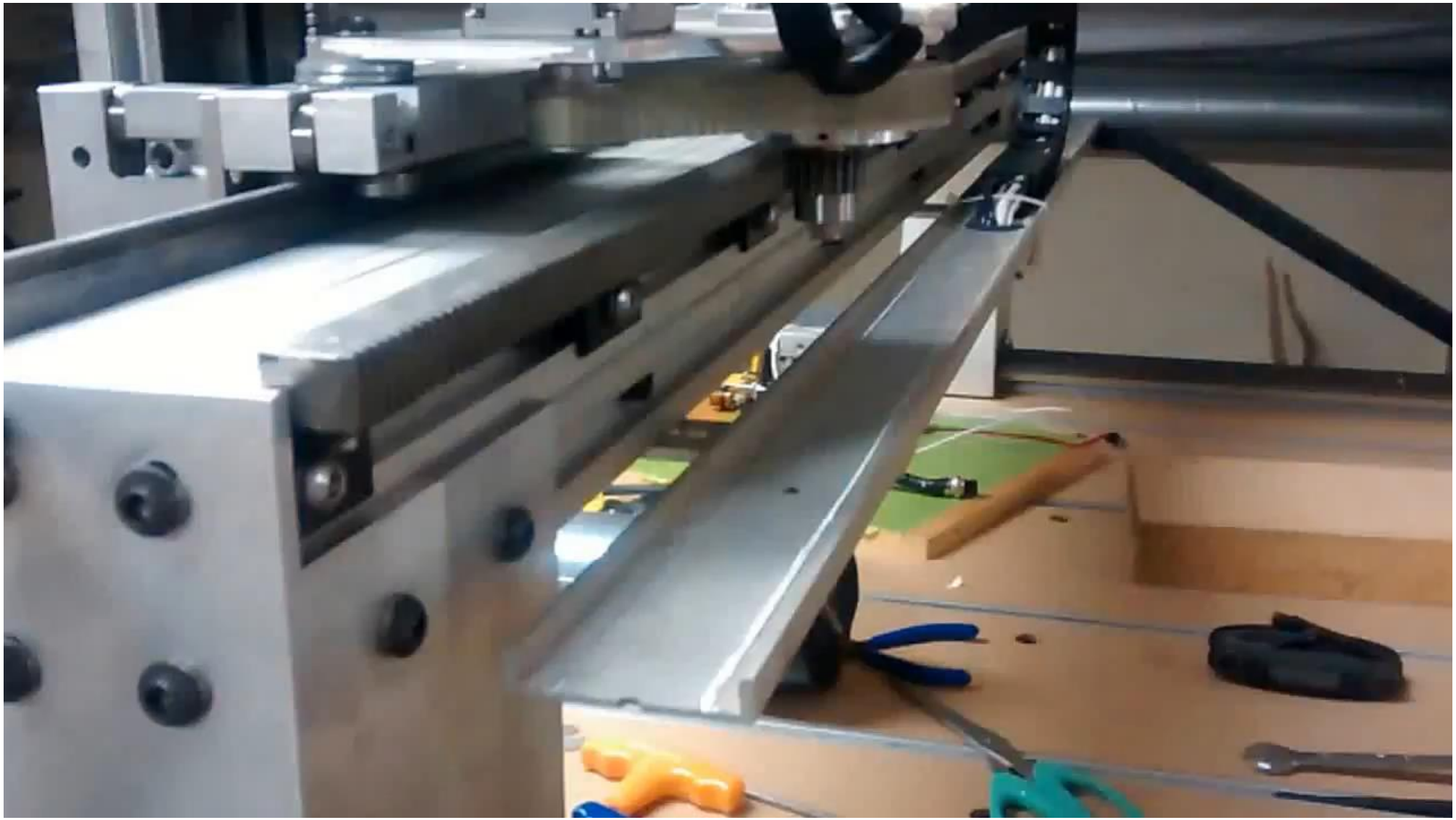
Rack and pinion

The gear of a shaft meshes externally and internally with the gears in a straight line.

Such type of gear is called ***rack and pinion***. The straight line gear is called **rack** and the circular wheel is called **pinion**.

Using rack and pinion, we can convert linear motion into rotary motion and ***vice-versa***.



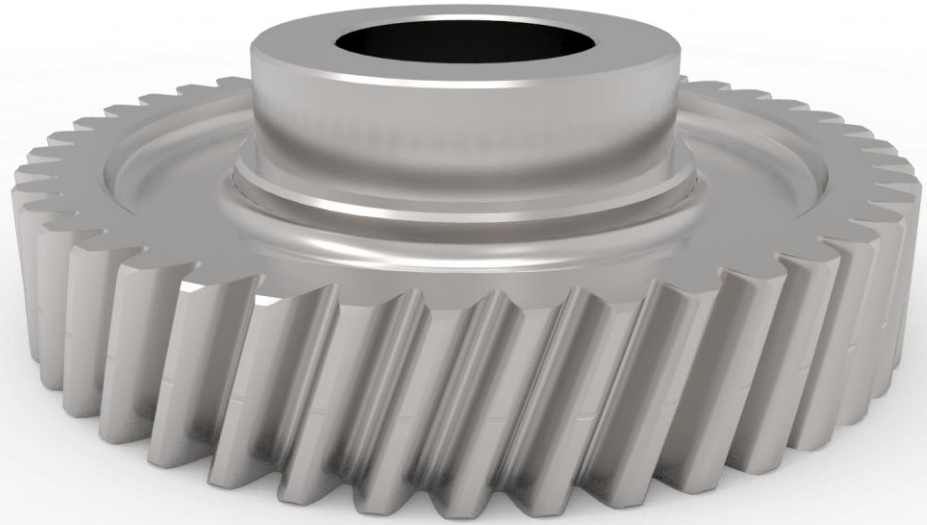


3. ***According to position of teeth on the gear surface***
    i. Straight
    ii. Inclined
    iii. Curved

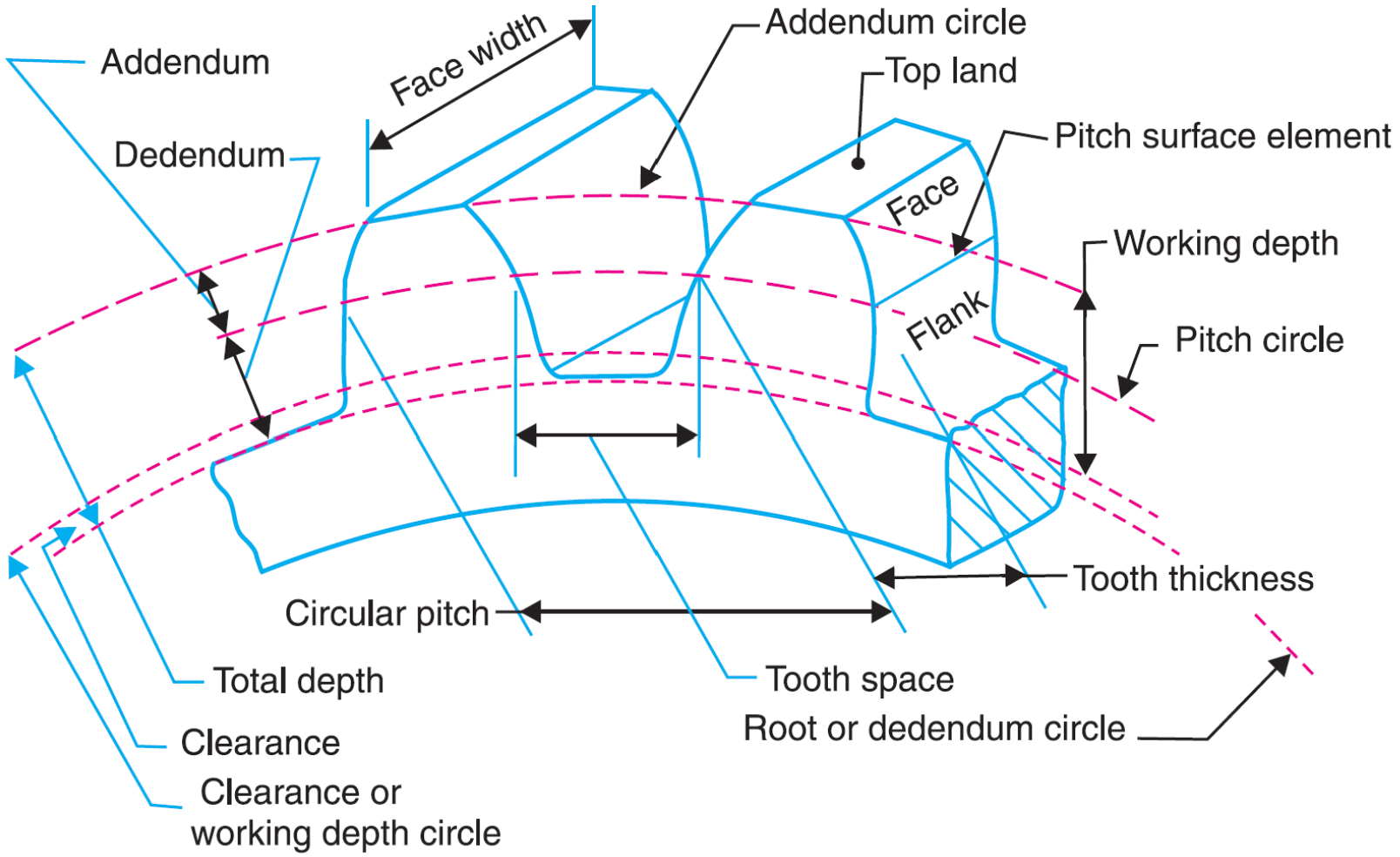
Spur gears have straight teeth.

Helical gears have their teeth inclined to the wheel rim.

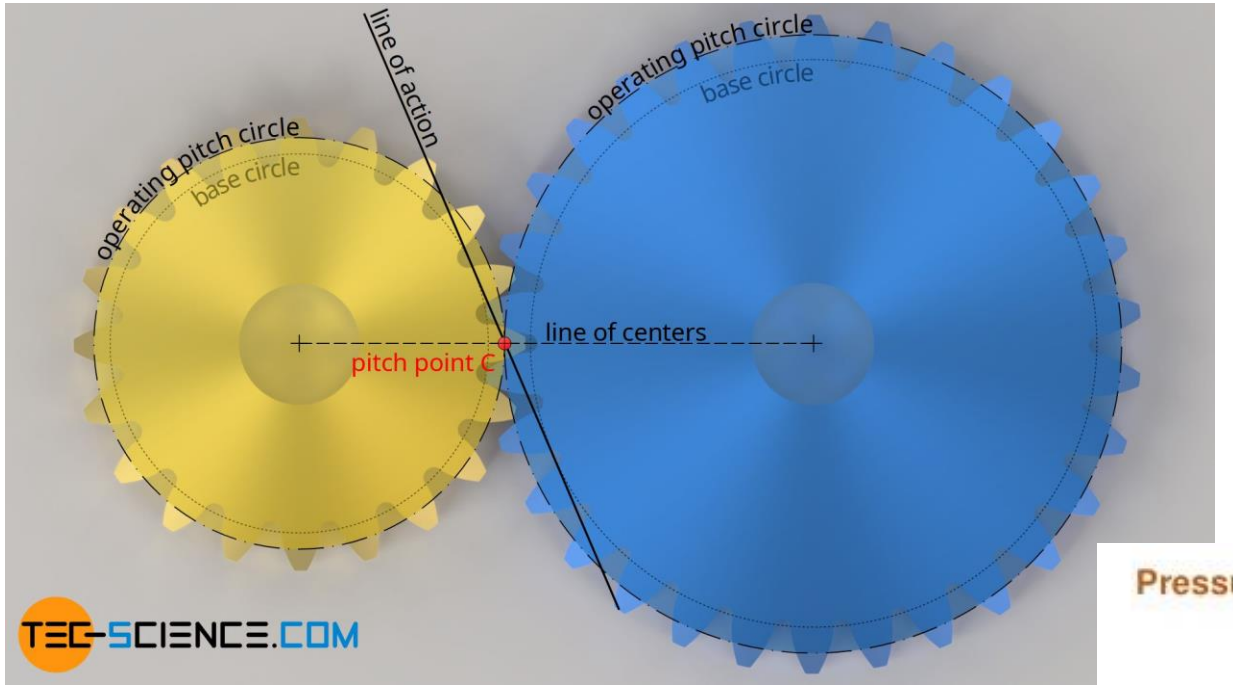
Spiral gears have the teeth curved over the rim surface.



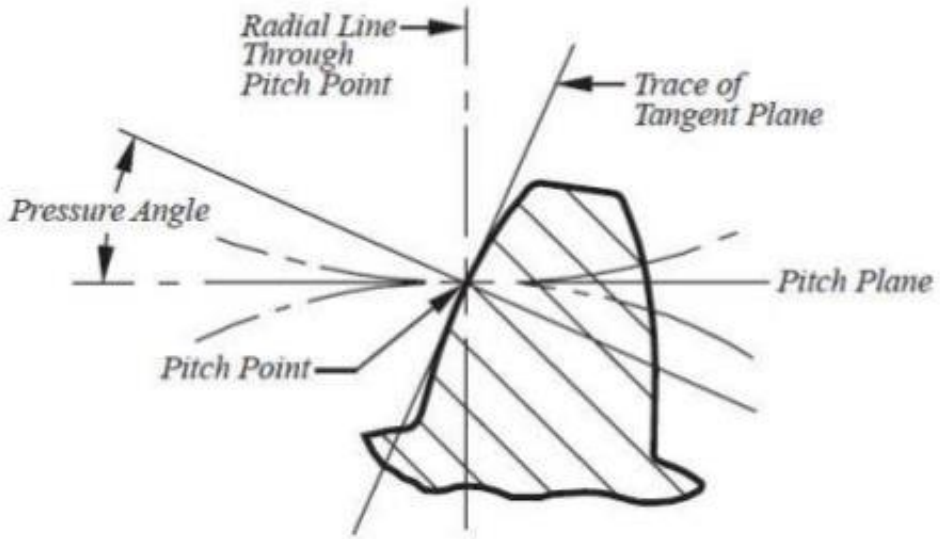
Terms used in gears



- ***Pitch circle -*** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.
- ***Pitch circle diameter -*** It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.
- ***Pitch point -*** It is a common point of contact between two pitch circles.
- ***Pitch surface -*** It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.
- ***Pressure angle or angle of obliquity -*** It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ.



Pressure angle (ϕ) / Angle of Obliquity

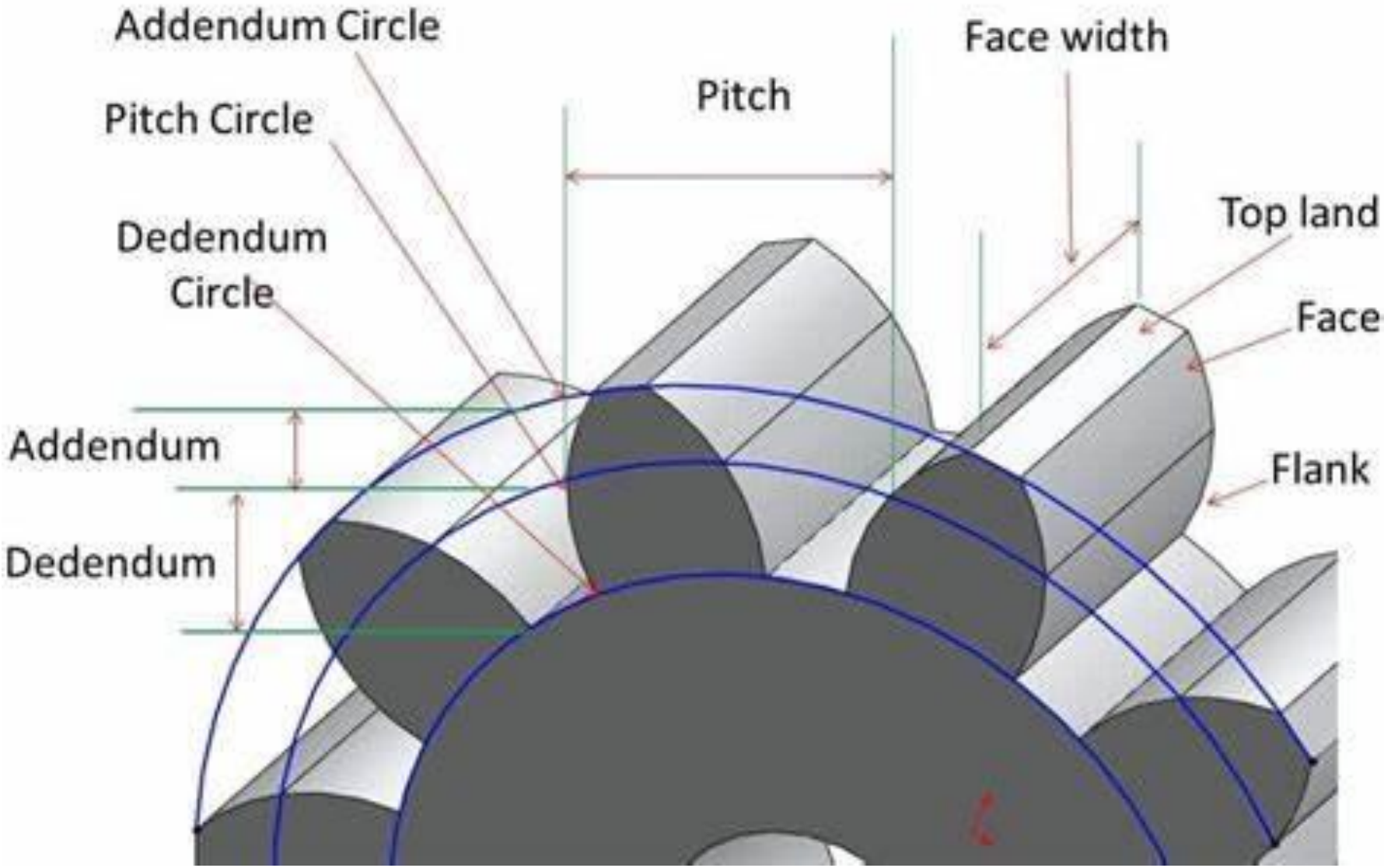


- ***Addendum -*** It is the radial distance of a tooth from the pitch circle to the top of the tooth.
- ***Dedendum -*** It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
- ***Addendum circle -*** It is the circle drawn through the top of the teeth and is concentric with the pitch circle.
- ***Dedendum circle -*** It is the circle drawn through the bottom of the teeth. It is also called root circle.

$$\text{Root circle diameter} = \text{Pitch circle diameter} \times \cos \phi$$

Where,

ϕ - pressure angle


Addendum Circle
Pitch Circle
Dedendum Circle
Addendum
Dedendum
Pitch
Face width
Top land
Face
Flank

- ***Circular pitch -*** It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.

$$p_c = \frac{\pi D}{T}$$

Where,

D - Diameter of the pitch circle

T - Number of teeth on the wheel

The two gears will mesh together correctly, if the two wheels have the same circular pitch.

If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for correct meshing,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

- ***Diametral pitch -*** It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by p_d .

$$p_d = \frac{T}{D} = \frac{\pi}{p_c}$$

Where,

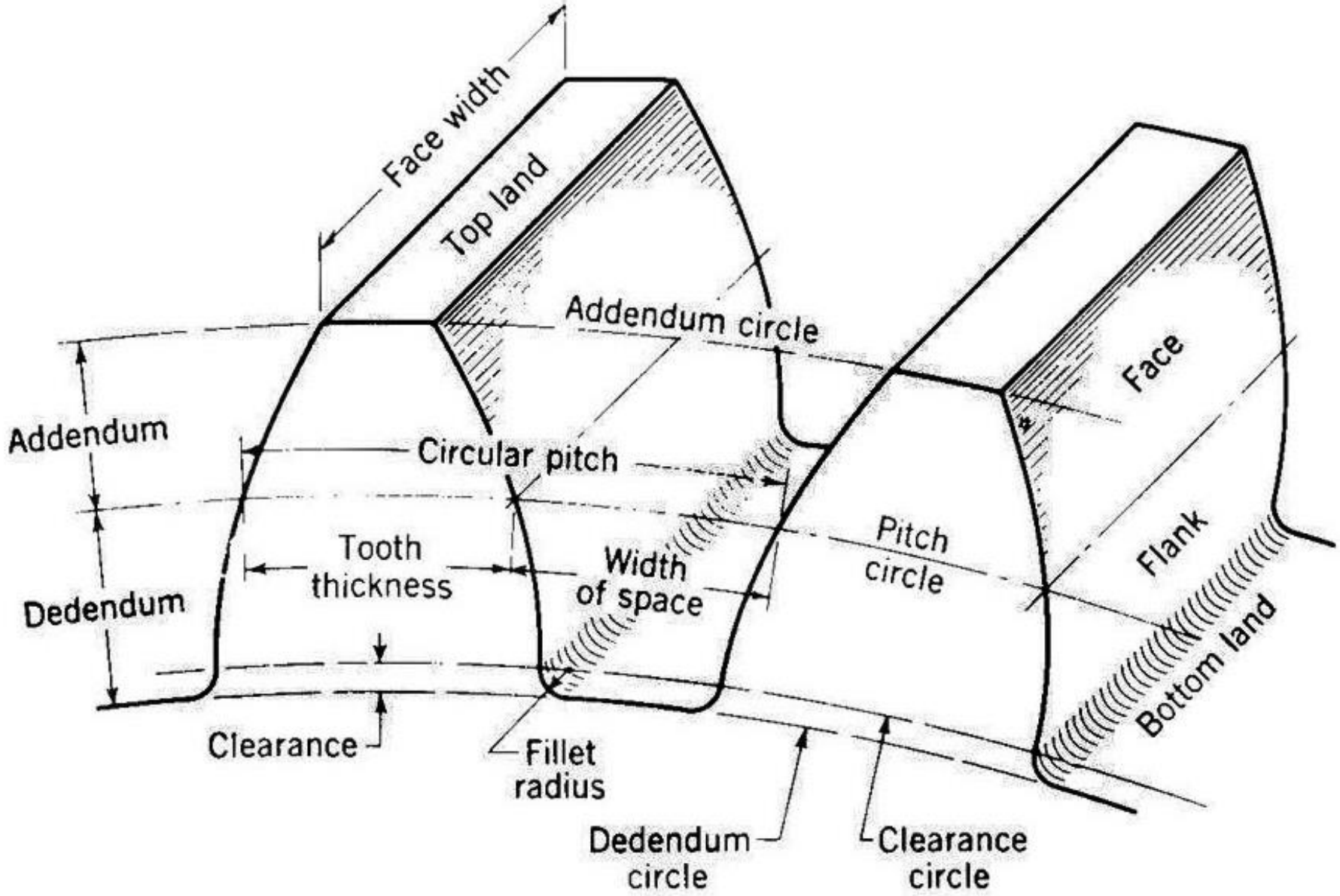
D - Pitch circle diameter

T - Number of teeth

- ***Module -*** It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

$$m = \frac{D}{T}$$

- ***Clearance -*** It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.
- ***Total depth***. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.
- ***Working depth -*** It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.
- ***Tooth thickness -*** It is the width of the tooth measured along the pitch circle.
- ***Tooth space -*** It is the width of space between the two adjacent teeth measured along the pitch circle.


Face width
Top land
Addendum circle
Face
Addendum
Circular pitch
Pitch
circle
Flank
Tooth
thickness
Width
of space
Dedendum
Bottom land
Clearance
Fillet
radius
Dedendum
circle
Clearance
circle

- ***Backlash -*** It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle.

Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

- ***Face of tooth -*** It is the surface of the gear tooth above the pitch surface.
- ***Flank of tooth -*** It is the surface of the gear tooth below the pitch surface.
- ***Top land -*** It is the surface of the top of the tooth.
- ***Face width -*** It is the width of the gear tooth measured parallel to its axis.
- ***Profile -*** It is the curve formed by the face and flank of the tooth.

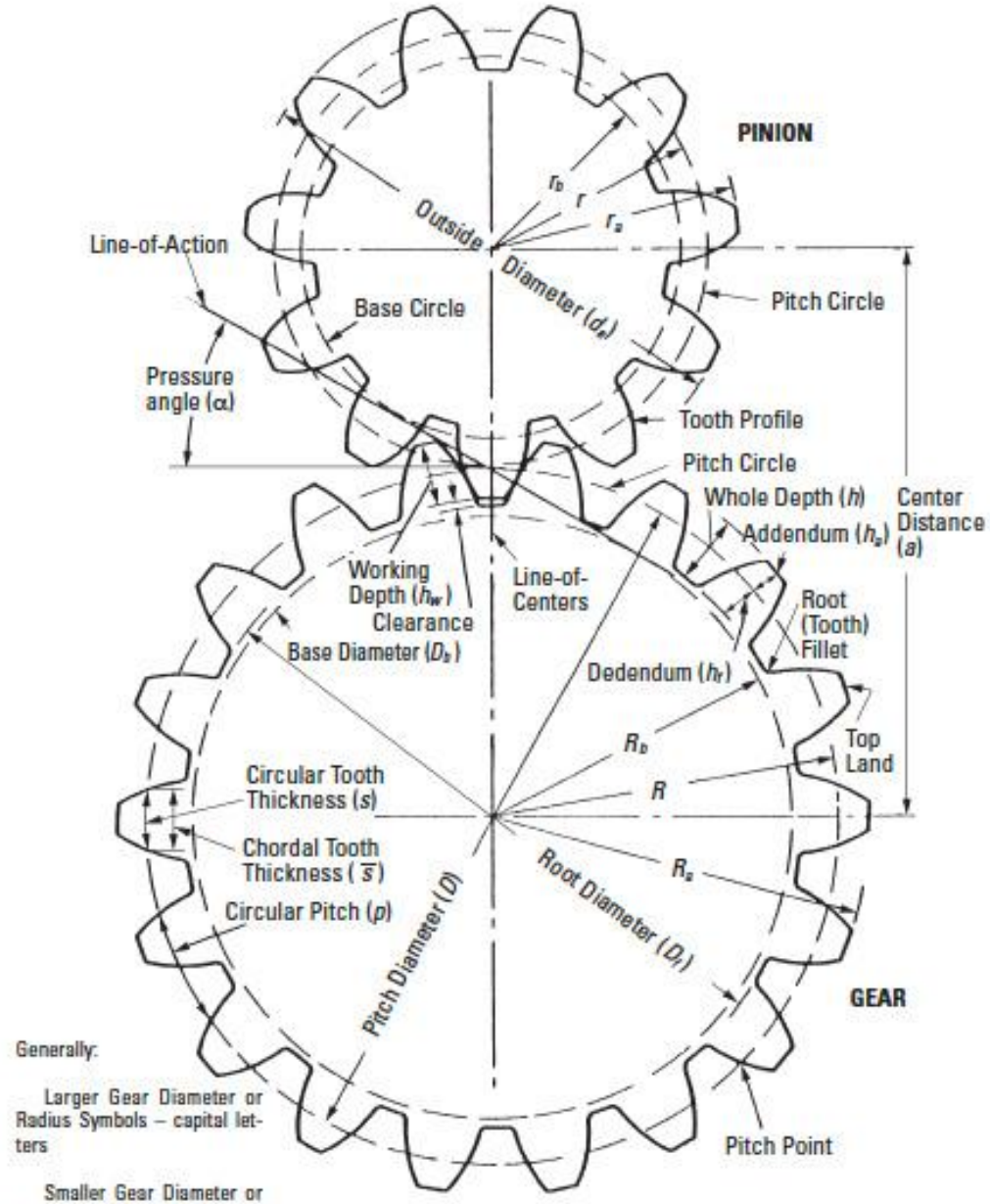
- ***Fillet radius -*** It is the radius that connects the root circle to the profile of the tooth.
- ***Path of contact -*** It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.
- ***Length of the path of contact -*** It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

- ***Arc of contact***. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts,

(*a*) *Arc of approach*. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess*. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e*. number of pairs of teeth in contact.



Generally:

Larger Gear Diameter or Radius Symbols – capital letters

Smaller Gear Diameter or Radius Symbols – lower case letters

Gear Materials

Material used for the manufacture of gears depends upon the strength and service conditions like wear and noise.

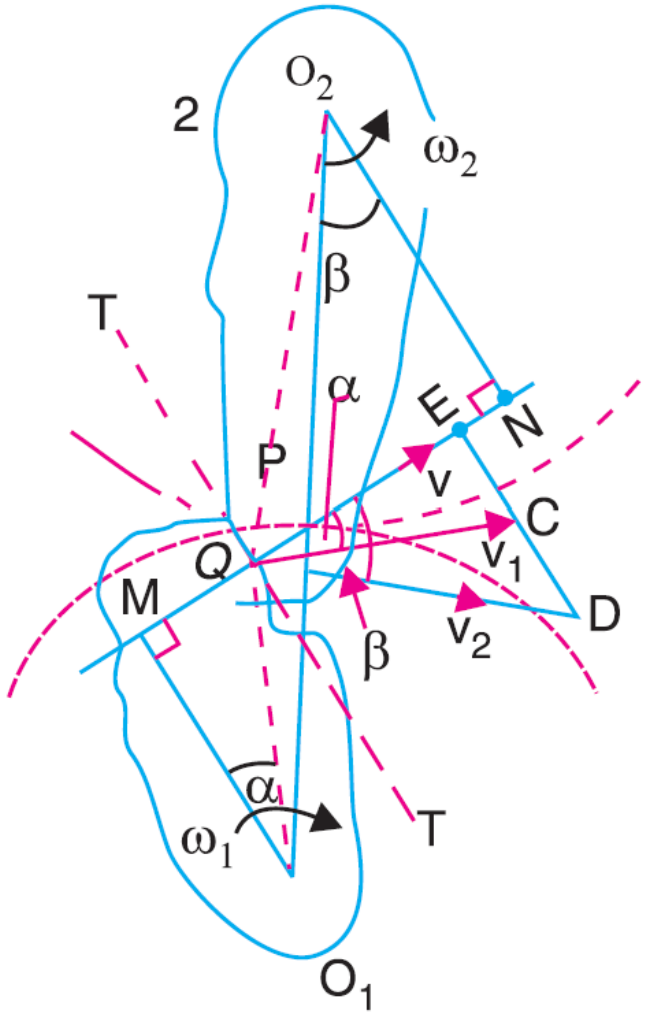
Cast iron is widely used to manufacture gears due to

- Good wearing properties
- Excellent machinability
- Easy to produce complicated shapes by casting method

Steel is used for high strength gears.

Law of Gearing

- Two wheels with centers O_1 and O_2 in contact is shown.
- The two teeth is in contact at point *Q.*
- *T-T is* the common tangent and *MN is* the common normal at the point of contact *Q.*
- v_1 and v_2 are the velocities of the point *Q* on the wheels 1 and 2 in the direction of QC and QD respectively.



For the teeth to remain in contact, the components of these velocities along the common normal *MN* must be equal.

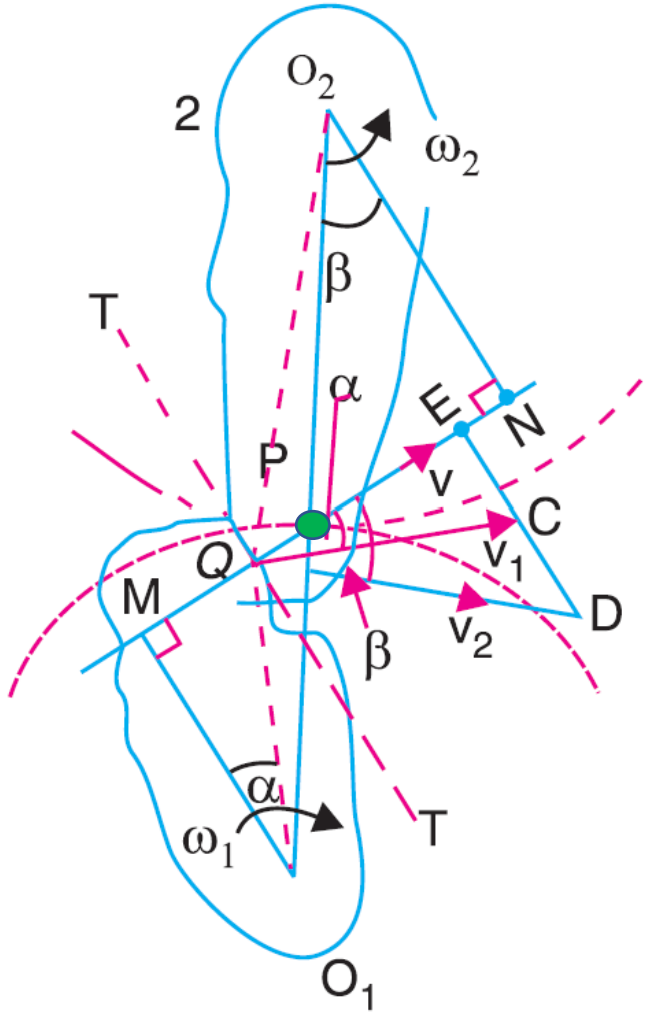
$$v_1 \cos \alpha = v_2 \cos \beta$$

$$(\omega_1 \times O_1 Q) \cos \alpha = (\omega_2 \times O_2 Q) \cos \beta$$

$$(\omega_1 \times O_1 Q) \frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q) \frac{O_2 N}{O_2 Q}$$

$$\omega_1 \times O_1 M = \omega_2 \times O_2 N$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} \qquad \text{——— 1}$$



From similar triangles O_1MP and O_2NP,

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} \qquad \text{——— 2}$$

From 1 & 2 $\dfrac{\omega_1}{\omega_2} = \dfrac{O_2 N}{O_1 M} = \dfrac{O_2 P}{O_1 P}$

P is the intersection of the common normal at the point of contact *Q* and the line of centers.

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

Angular velocity ratio is inversely proportional to the ratio of the distances of the point *P* from the centers O_1 and O_2.

Law of Gearing: ***The common normal at the point of contact between a pair of teeth must always pass through the pitch point*.**

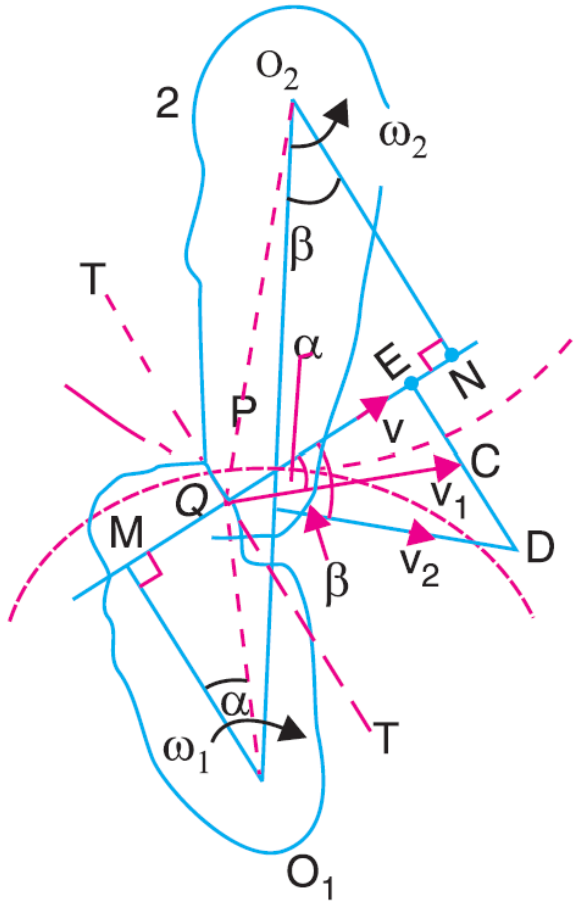
Wheels 1 and 2 having teeth T_1 and T_2 and pitch circle diameters D_1 and D_2.

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$$

Velocity of Sliding of Teeth

The velocity of sliding is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.

$$v_S = (\omega_1 + \omega_2)\,QP$$



Form of teeth in gears

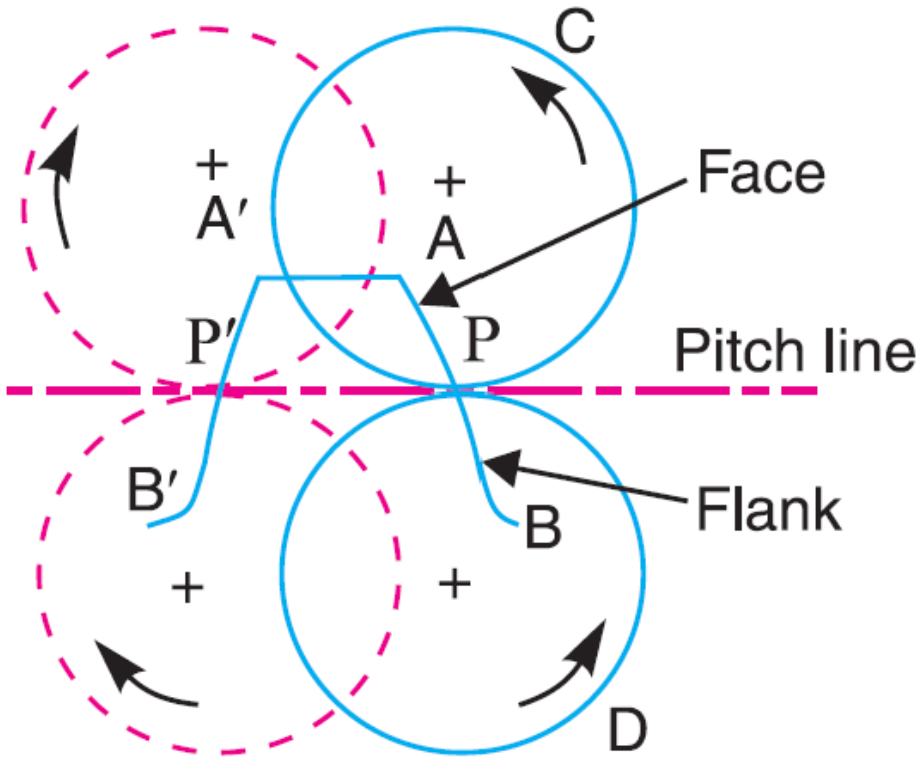
1. Cycloidal teeth

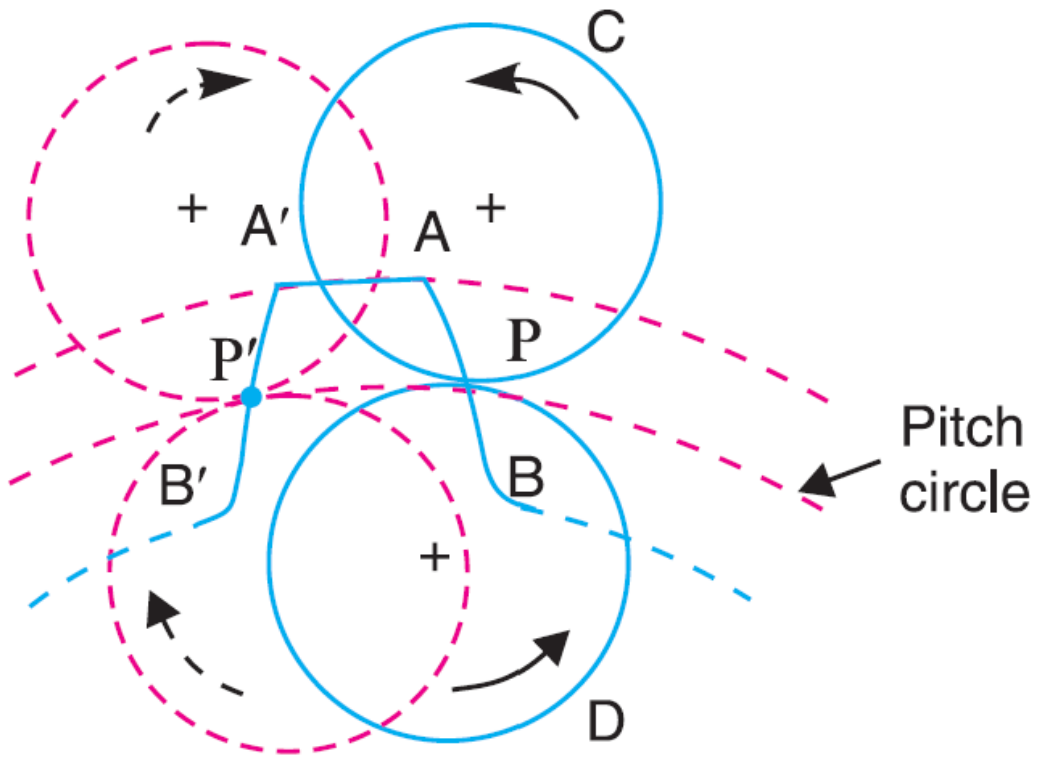
Cycloid - curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line.

Epicycloid - curve traced by a point on the circumference of a circle which rolls without slipping on the outside of a fixed circle.

Hypocycloid - curve traced by a point on the circumference of a circle which rolls without slipping on the inside of a fixed circle.

Construction of cycloidal teeth of a gear.

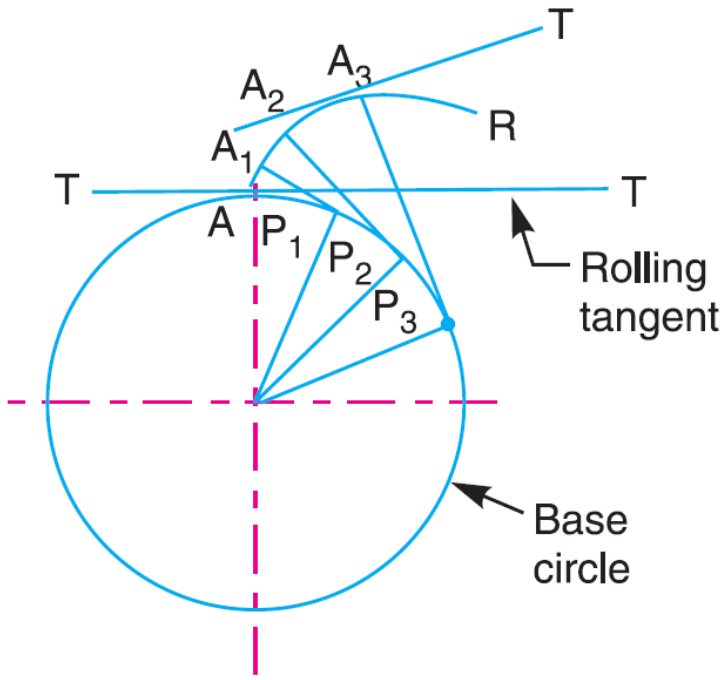




2. Involute Teeth

An involute of a circle is a plane curve generated by a point on a taut string which in unwrapped from a reel as shown. In toothed wheels, the circle is known as base circle.

Normal at any point of an involute is a tangent to the circle.



- During the power transmission maximum tooth pressure (neglecting friction at the teeth) is exerted along the common normal through the pitch point.
- This force is resolved into tangential and radial or normal components. These components act along and at right angles to the common tangent to the pitch circles.

F is maximum tooth pressure.

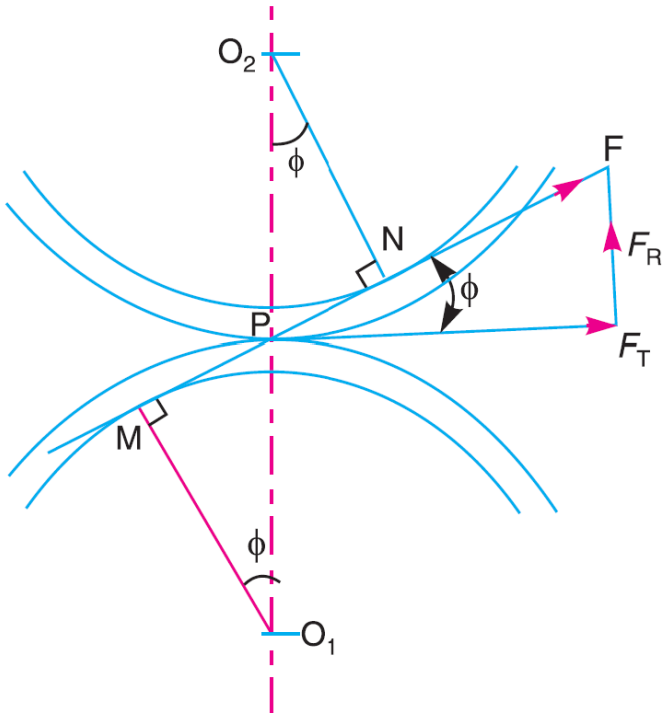
Tangential force,

$$F_T = F cos\ \phi$$

Radial or normal force,

$$F_R = F sin\ \phi$$

ϕ – Pressure angle



Torque exerted on the gear shaft

$$\tau = F_T \times r$$

Where,

r - the pitch circle radius of the gear.

Question

A single reduction gear of 120 kW with a pinion 250 mm pitch circle diameter and speed 650 rev/min is supported in bearings on either side. Calculate the total load due to the power transmitted, the pressure angle being 20°.

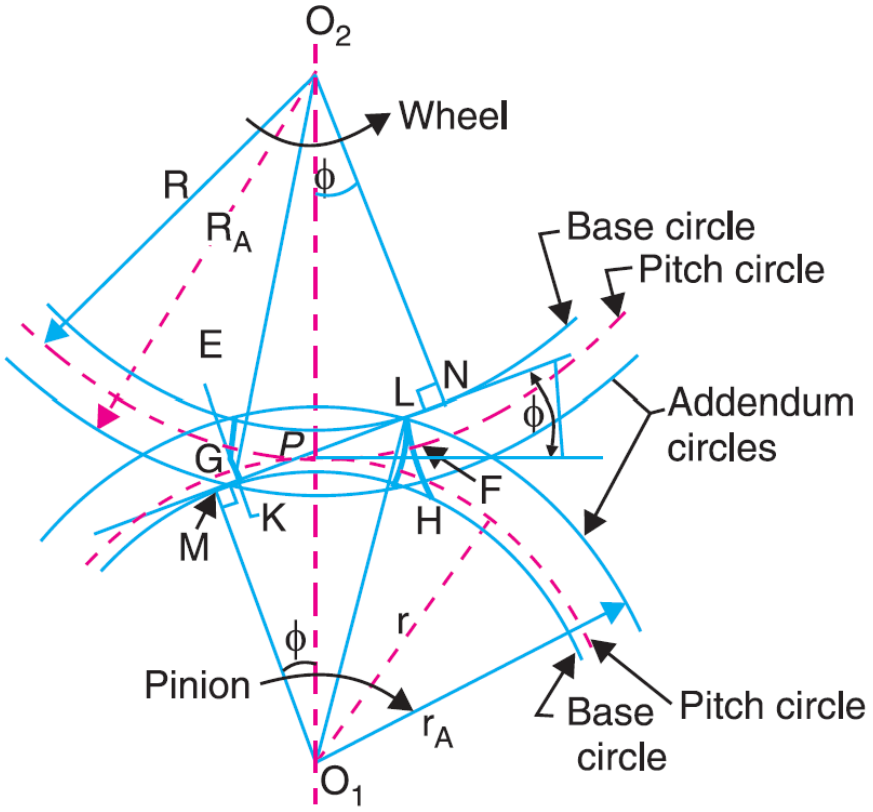
Length of Path of Contact

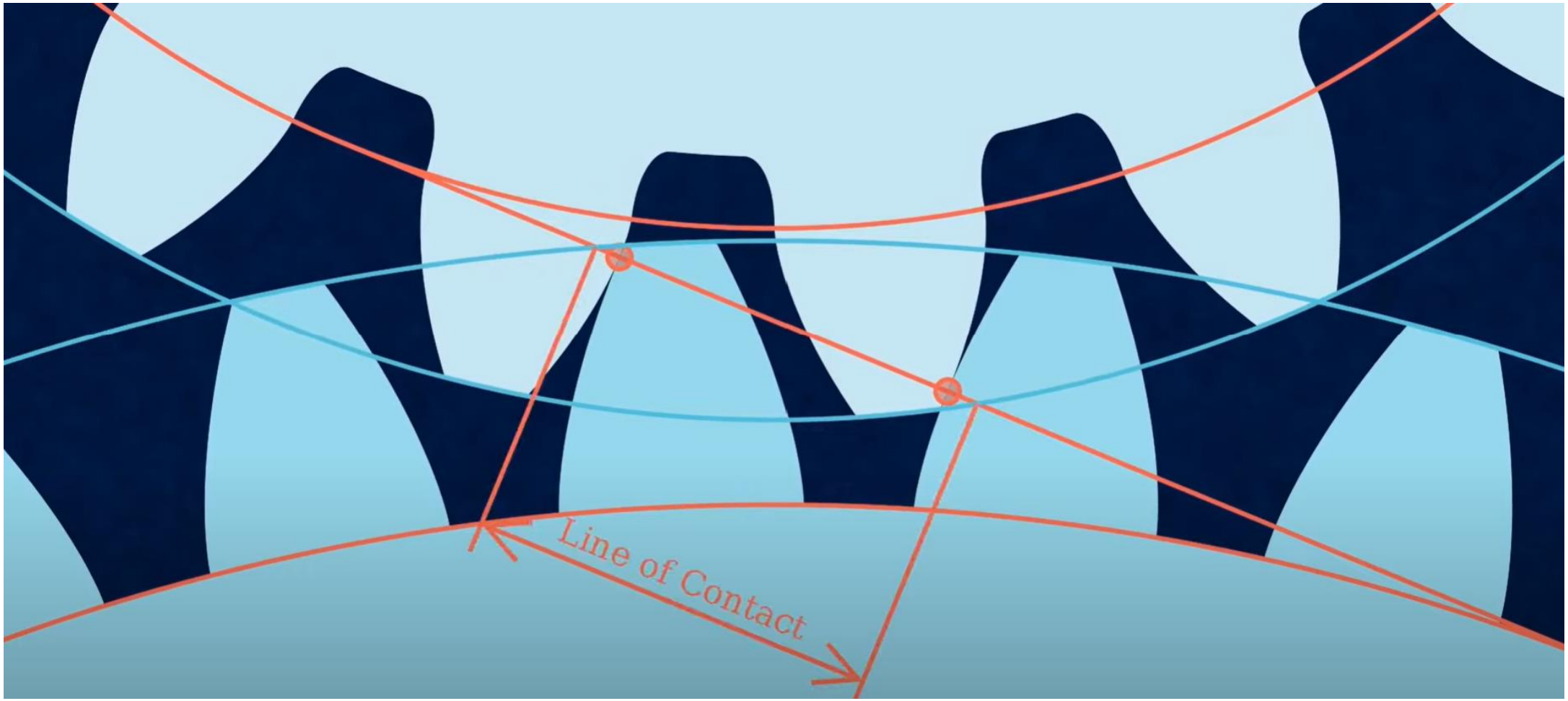
- Pinion is driving the wheel as shown.
- When the pinion rotates in clockwise direction, the contact between a pair of involute teeth begins at *K* and ends at *L*.

  K is the intersection of the addendum circle of wheel and the common normal.

  L is the intersection of the addendum circle of pinion and common normal.

- *MN* is the common normal at the point of contacts.




Line of Contact

Length of path of contact (*KL*) is the length of common normal cutoff by the addendum circles of the wheel and the pinion.

$$KL = KP + PL$$

KP - ***Path of approach***

PL – ***Path of recess***

$r_A = O_1L$ – Radius of addendum circle of pinion

$R_A = O_2K$ – Radius of addendum circle of wheel

$r = O_1P$ – Radius of pitch circle of pinion

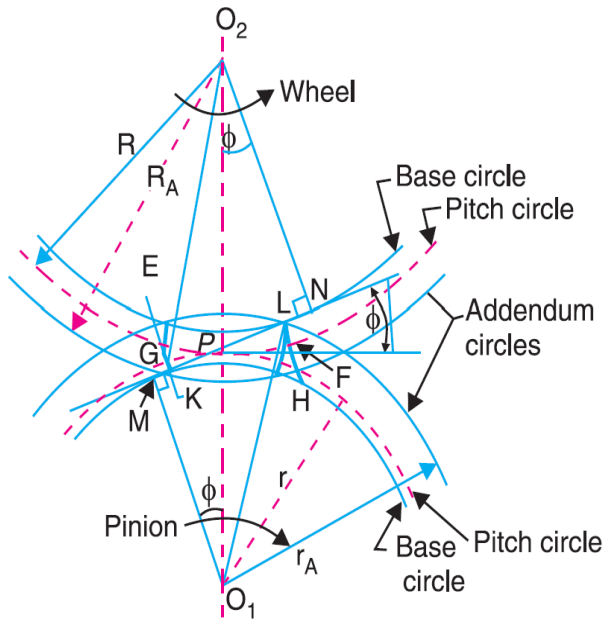
$R = O_2P$ – Radius of pitch circle of wheel

Radius of the base circle of pinion

$$O_1M = O_1P\cos\phi = r\cos\phi$$

Radius of the base circle of wheel

$$O_2N = O_2P\cos\phi = R\cos\phi$$



Right angled triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

Length of path of approach

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

Right angled triangle O_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

Length of path of recess

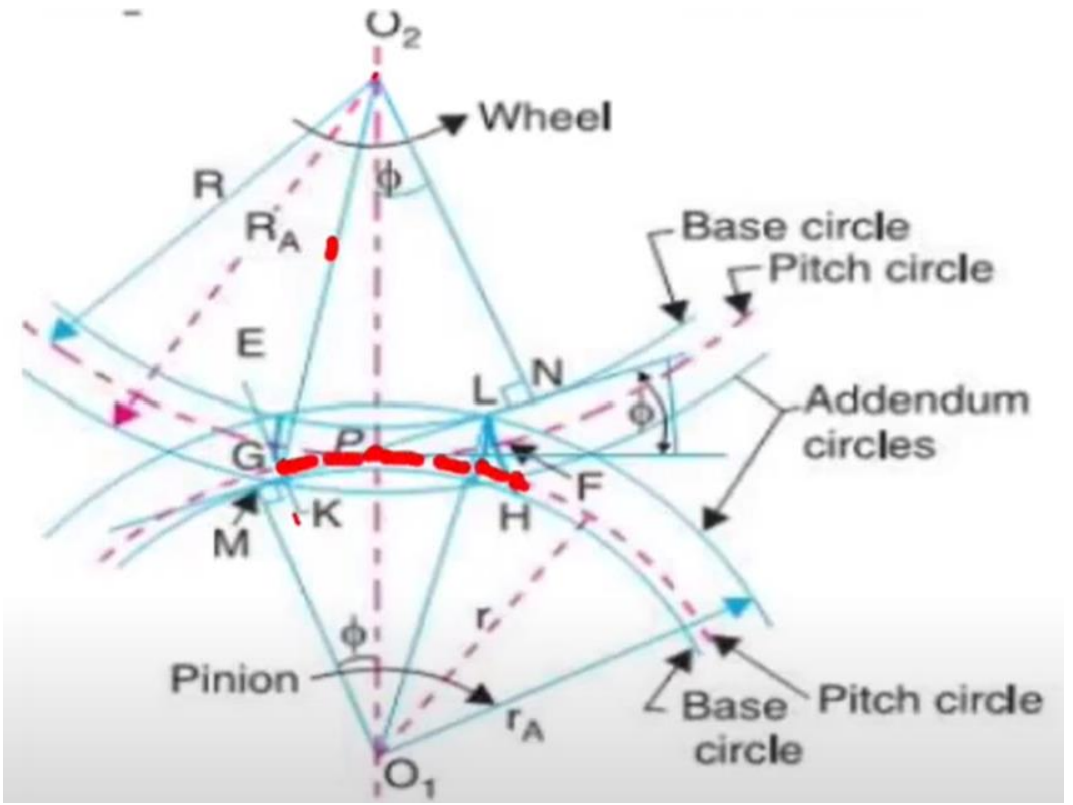
$$PL = ML - MP = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

Length of path of contact

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

Length of Arc of Contact

- Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.
- Arc of contact is *GPH*.



G is the intersection of the normal to MN at K and pitch circle of pinion.

H is the intersection of the normal to MN at L and pitch circle of pinion.

$$GPH = \frac{KL}{\cos\phi} = \frac{\text{Length of path of contact}}{\cos\phi}$$

$$GPH = GP + PH$$

GP - ***Arc of approach***

$$GP = \frac{KP}{\cos\phi} = \frac{\text{Length of path of approach}}{\cos\phi}$$

PH - ***Arc of recess***

$$PH = \frac{PL}{\cos\phi} = \frac{\text{Length of path of recess}}{\cos\phi}$$

Contact Ratio

Contact ratio is the number of pairs of teeth in contact.

$$Contact\ ratio = \frac{Length\ of\ arc\ of\ contact}{Circular\ pitch}$$

- The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.
- Larger the contact ratio, more quietly the gears will operate.

The number of teeth on each of the two equal spur gears in mesh are 40. The teeth have 20° involute profile and the module is 6 mm. If the arc of contact is 1.75 times the circular pitch, find the addendum.

Useful equations

$$m = \frac{D}{T}$$

$$p_c = \frac{\pi D}{T} = \pi m$$

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2 \cos^2\phi} - (R + r)\sin\phi$$

$$\text{Length of the arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

A pinion having 30 teeth drives a gear having 80 teeth. The profile of the gears is involute with 20° pressure angle, 12 mm module and 10 mm addendum. Find the length of path of contact, arc of contact and the contact ratio.

Useful equations

$$m = \frac{D}{T}$$

$$p_c = \frac{\pi D}{T} = \pi m$$

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2\phi} + \sqrt{(r_A)^2 - r^2 \cos^2\phi} - (R + r)\sin\phi$$

$$\text{Length of the arc of contact} = \frac{\text{Length of path of contact}}{\cos\phi}$$

$$Contact\ ratio = \frac{Length\ of\ arc\ of\ contact}{Circular\ pitch}$$