

Construction of Cam Profile for a Radial Cam

Example 1

A cam is to give the following motion to a *knife-edged follower* :

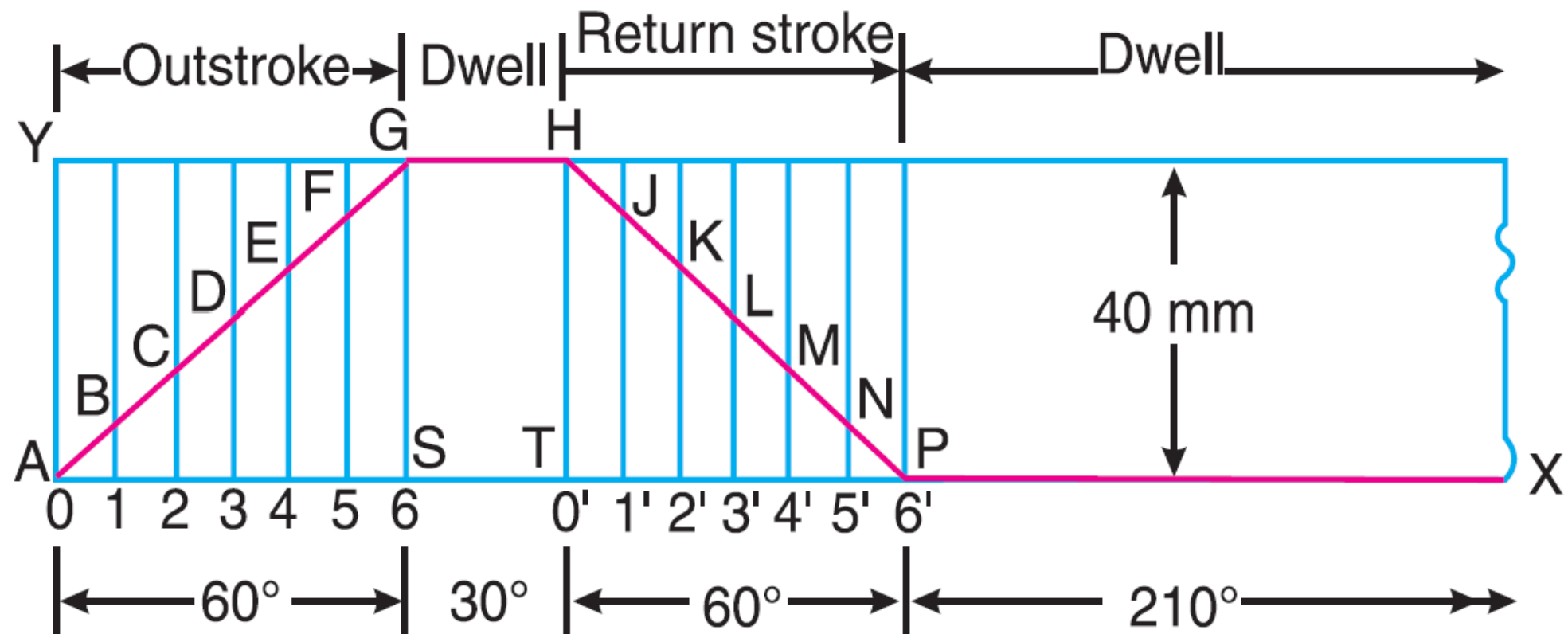
1. *Outstroke during 60° of cam rotation*
2. *Dwell for the next 30° of cam rotation*
3. *Return stroke during next 60° of cam rotation*
4. *Dwell for the remaining 210° of cam rotation.*

*The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with *uniform velocity* during both the outstroke and return strokes.*

Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

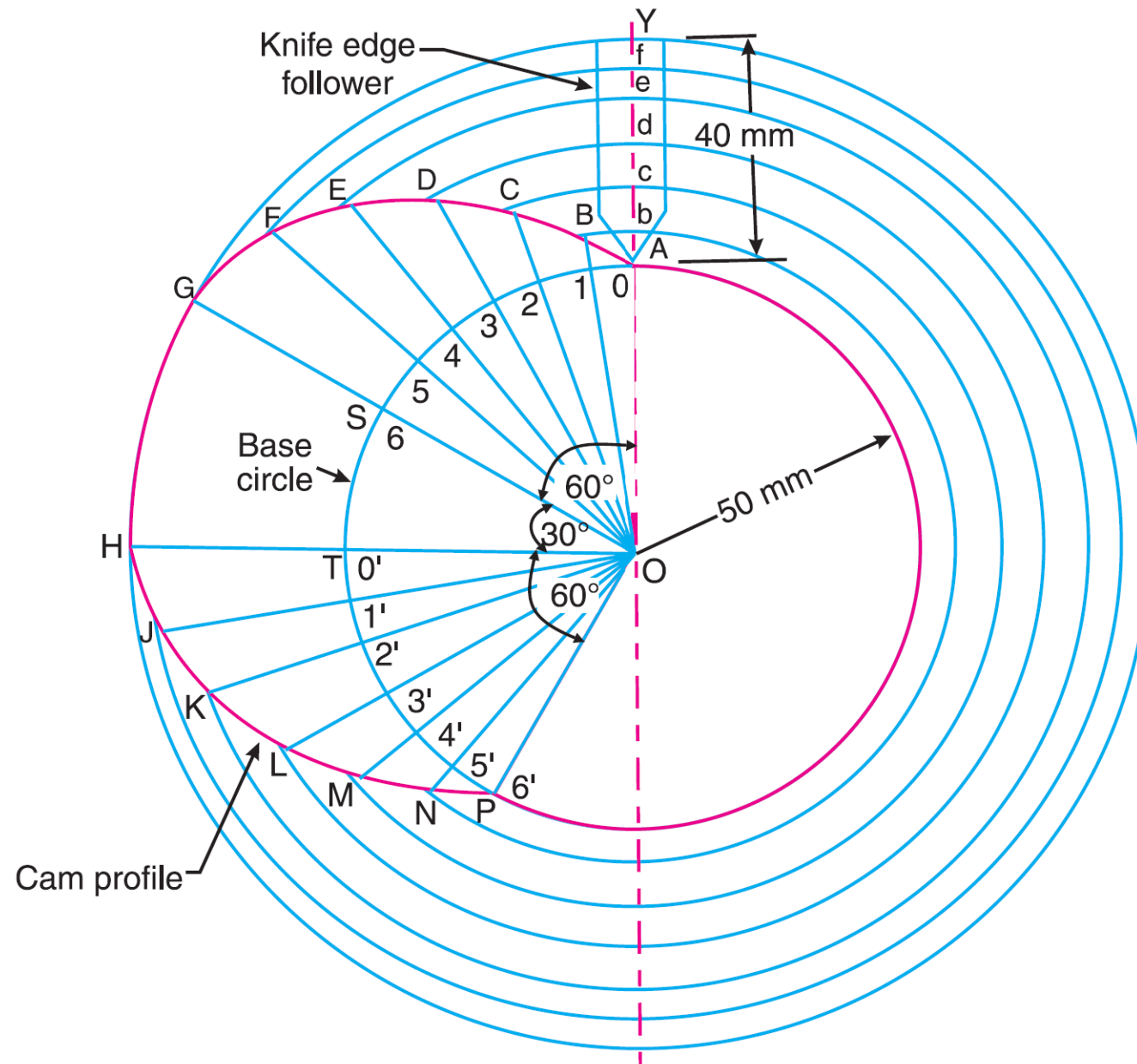
Displacement diagram of the follower.

1. Draw a horizontal line $AX = 360^\circ$ to some suitable scale. On this line, mark $AS = 60^\circ$ to represent outstroke of the follower, $ST = 30^\circ$ to represent dwell, $TP = 60^\circ$ to represent return stroke and $PX = 210^\circ$ to represent dwell.
2. Draw vertical line AY equal to the stroke of the follower (*i.e.* 40 mm) and complete the rectangle.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join AG and HP .
5. The complete displacement diagram is shown by $AGHPX$.



Profile of the cam when the axis of follower passes through the axis of cam shaft

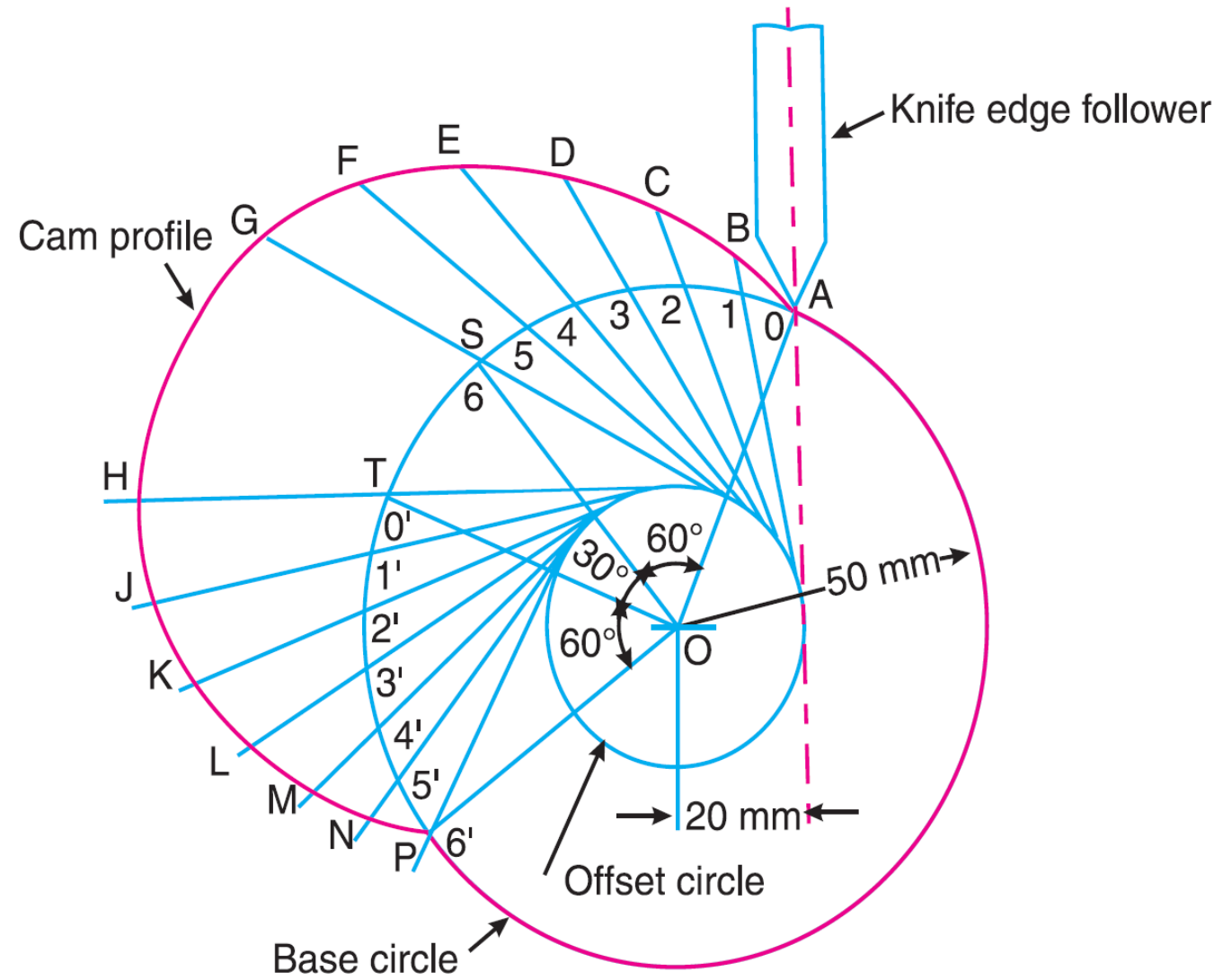
1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with O as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point A .
3. From OA , mark angle $AOS = 60^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent return stroke.
4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3 ...etc. and $O', 1', 2', 3', \dots$ etc. with centre O and produce beyond the base circle.
6. Set off $1B, 2C, 3D \dots$ etc. and $O' H, 1' J \dots$ etc. from the displacement diagram.
7. Join the points $A, B, C, \dots M, N, P$ with a smooth curve. The curve $AGHPA$ is the complete profile of the cam.



Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with O as centre.
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at A .
3. Join AO and draw an offset circle of radius 20 mm with centre O .
4. From OA , mark angle $AOS = 60^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
6. From the points 1, 2, 3 ... etc. and 0',1', 2',3' ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle.
7. Set off 1*B*, 2*C*, 3*D* ... etc. and 0' *H*,1' *J* ... etc. from the displacement diagram.
8. Join the points *A*, *B*, *C* ...*M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.



Example 2

A cam is to be designed for a *knife edge follower* with the following data:

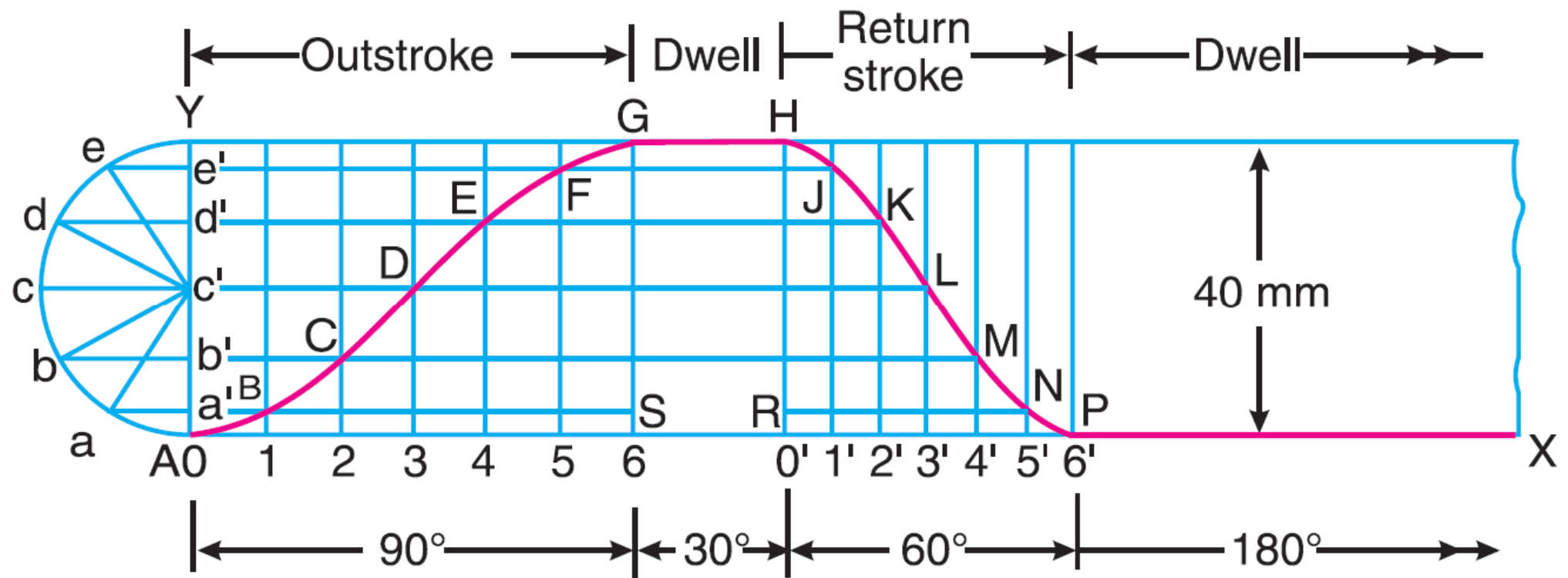
1. Cam lift = 40 mm during 90° of cam rotation with *simple harmonic motion*.
2. Dwell for the next 30° .
3. During the next 60° of cam rotation, the follower returns to its original position with *simple harmonic motion*.
4. Dwell during the remaining 180° .

Draw the profile of the cam when **(a)** the line of stroke of the follower passes through the axis of the cam shaft, and **(b)** the line of stroke is offset 20 mm from the axis of the cam shaft.

The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 rev/min.

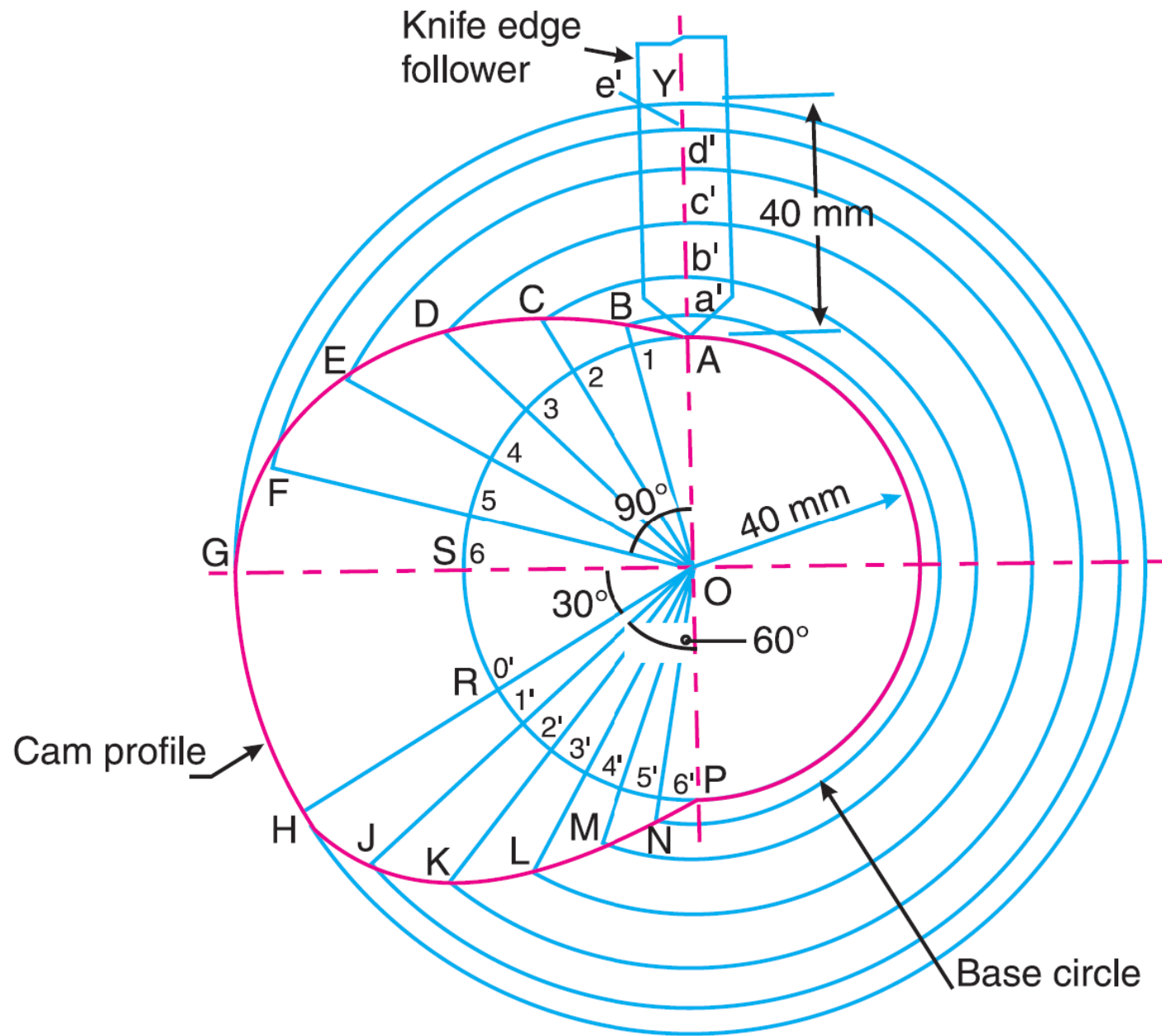
Displacement diagram of the follower.

- 1) Draw horizontal line $AX = 360^\circ$ to some suitable scale. On this line, mark $AS = 90^\circ$ to represent out stroke ; $SR = 30^\circ$ to represent dwell ; $RP = 60^\circ$ to represent return stroke and $PX = 180^\circ$ to represent dwell.
- 2) Draw vertical line $AY = 40$ mm to represent the cam lift or stroke of the follower and complete the rectangle.
- 3) Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
- 4) Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.
- 5) From points $a, b, c \dots$ etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and $0', 1', 2' \dots$ etc. at $B, C, D \dots M, N, P$.
- 6) Join the points $A, B, C \dots$ etc. with a smooth curve. This is the required displacement diagram.



Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft

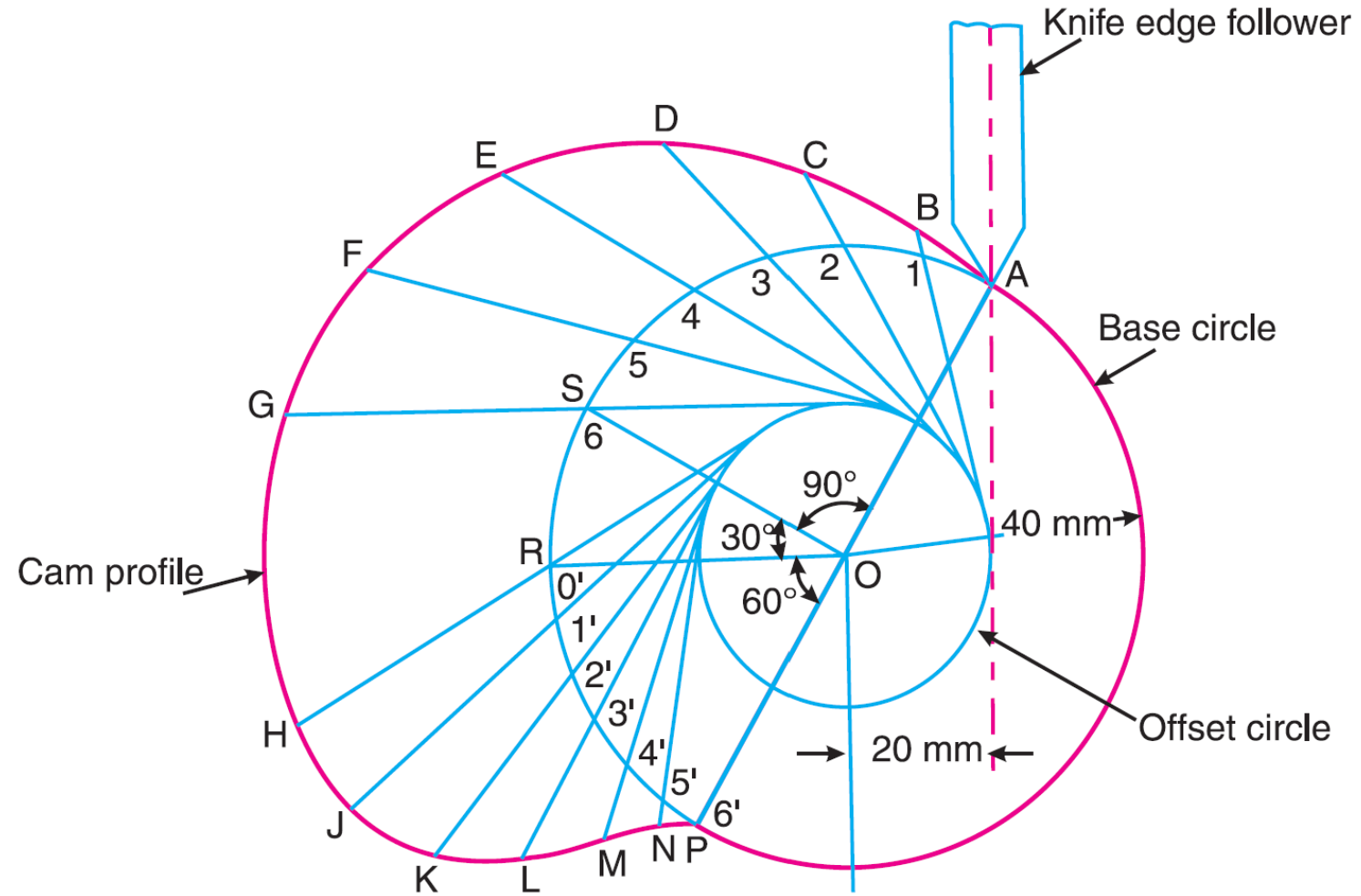
1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 40 mm) with O as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point A .
3. From OA , mark angle $AOS = 90^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent return stroke.
4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3 ...etc. and $O', 1', 2', 3', \dots$ etc. with centre O and produce beyond the base circle.
6. Set off $1B, 2C, 3D \dots$ etc. and $O' H, 1' J \dots$ etc. from the displacement diagram.
7. Join the points $A, B, C, \dots M, N, P$ with a smooth curve. The curve $AGHPA$ is the complete profile of the cam.



Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 40 mm) with O as centre.
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at A .
3. Join AO and draw an offset circle of radius 20 mm with centre O .
4. From OA , mark angle $AOS = 90^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
6. From the points 1, 2, 3 ... etc. and 0',1', 2',3' ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle.
7. Set off 1*B*, 2*C*, 3*D* ... etc. and 0' *H*,1' *J* ... etc. from the displacement diagram.
8. Join the points *A*, *B*, *C* ...*M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.



Maximum velocity and acceleration of follower in simple harmonic motion

Time required for the out stroke of the follower

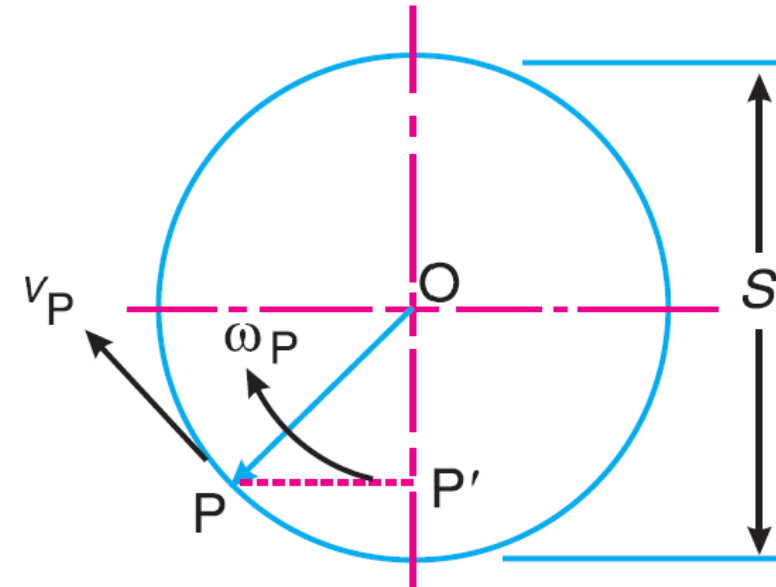
$$t_o = \frac{\theta_o}{\omega}$$

Peripheral speed of the point P'

$$v_p = \frac{\pi S}{2} \times \frac{1}{t_o} = \frac{\pi S}{2} \times \frac{\omega}{\theta_o}$$

Maximum velocity of the follower on the outstroke

$$v_o = v_p = \frac{\pi \omega S}{2\theta_o}$$



Centripetal acceleration of the point P

$$a_p = \frac{v_p^2}{OP} = \left(\frac{\pi \omega S}{2\theta_o} \right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 S}{2\theta_o^2}$$

Maximum acceleration of the follower on the outstroke

$$a_o = a_p = \frac{\pi^2 \omega^2 S}{2\theta_o^2}$$

Maximum velocity of the follower on the return stroke

$$v_R = \frac{\pi \omega S}{2\theta_R}$$

Maximum acceleration of the follower on the return stroke

$$a_R = \frac{\pi^2 \omega^2 S}{2\theta_R^2}$$

θ_o and θ_R are in **radians**

Angular velocity of the cam

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

Maximum velocity of the follower during its ascent

$$v_O = \frac{\pi \omega .S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s}$$

Maximum velocity of the follower during its descent

$$v_R = \frac{\pi \omega .S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s}$$

Maximum acceleration of the follower during its ascent

$$a_O = \frac{\pi^2 \omega^2 .S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2$$

Maximum acceleration of the follower during its descent

$$a_R = \frac{\pi^2 \omega^2 .S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2$$

Example 3

A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a **knife edge follower** the motion as described below:

1. To move outwards through 40 mm during 100° rotation of the cam
2. To dwell for next 80°
3. To return to its starting position during next 90°
4. To dwell for the rest period of a revolution i.e. 90° .

Draw the profile of the cam (i) when the line of stroke of the follower passes through the centre of the cam shaft, and (ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with **uniform acceleration and uniform retardation**.

Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 rev/min.

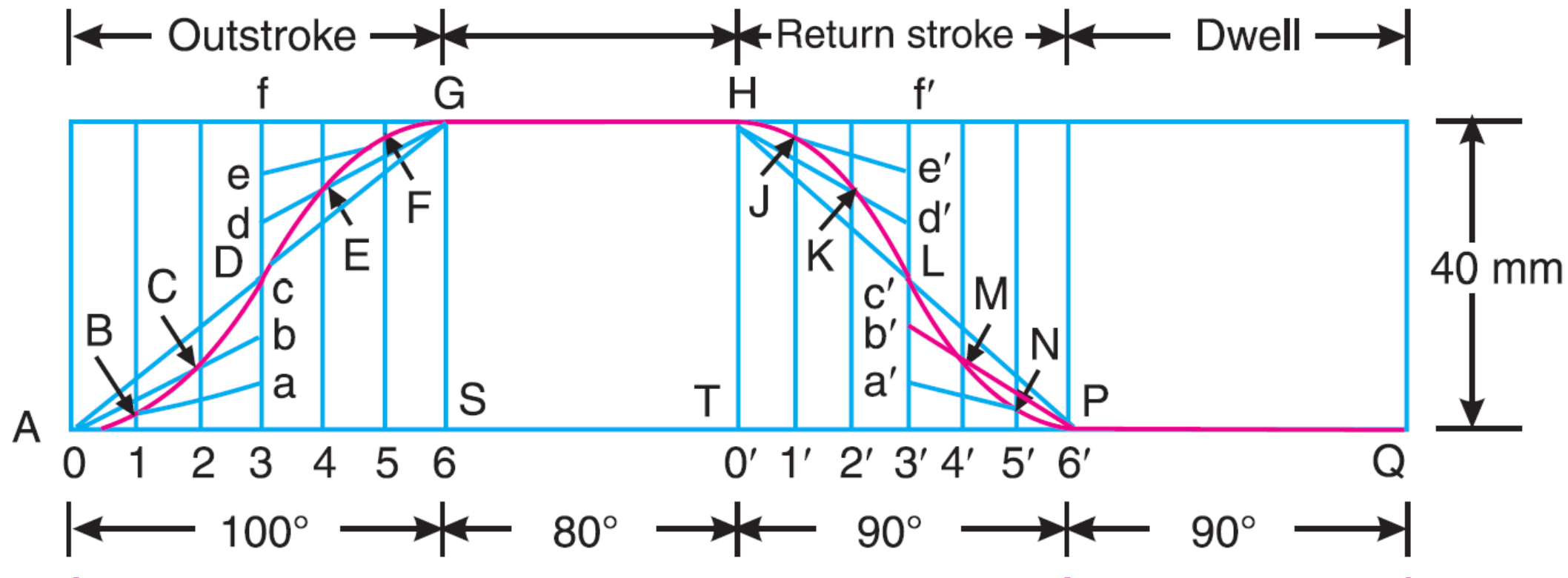
Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam.

Displacement diagram of the follower.

- 1) Draw a horizontal line $ASTPQ$ such that AS , angular displacement of the cam during outward stroke (100°) to some suitable scale. The line ST , the dwell period of 80° . The line TP , angular displacement of the cam during return stroke (90°) and the line PQ the dwell period of 90° after return stroke.
- 2) Divide AS and TP into any number of equal even parts (say six).
- 3) Draw vertical lines through points 0, 1, 2, 3 etc. and equal to the lift of the valve *i.e.* 40 mm.
- 4) Divide the vertical lines 3- f and 3'- f' into six equal parts as shown by points $a, b, c \dots$ and $a', b', c' \dots$

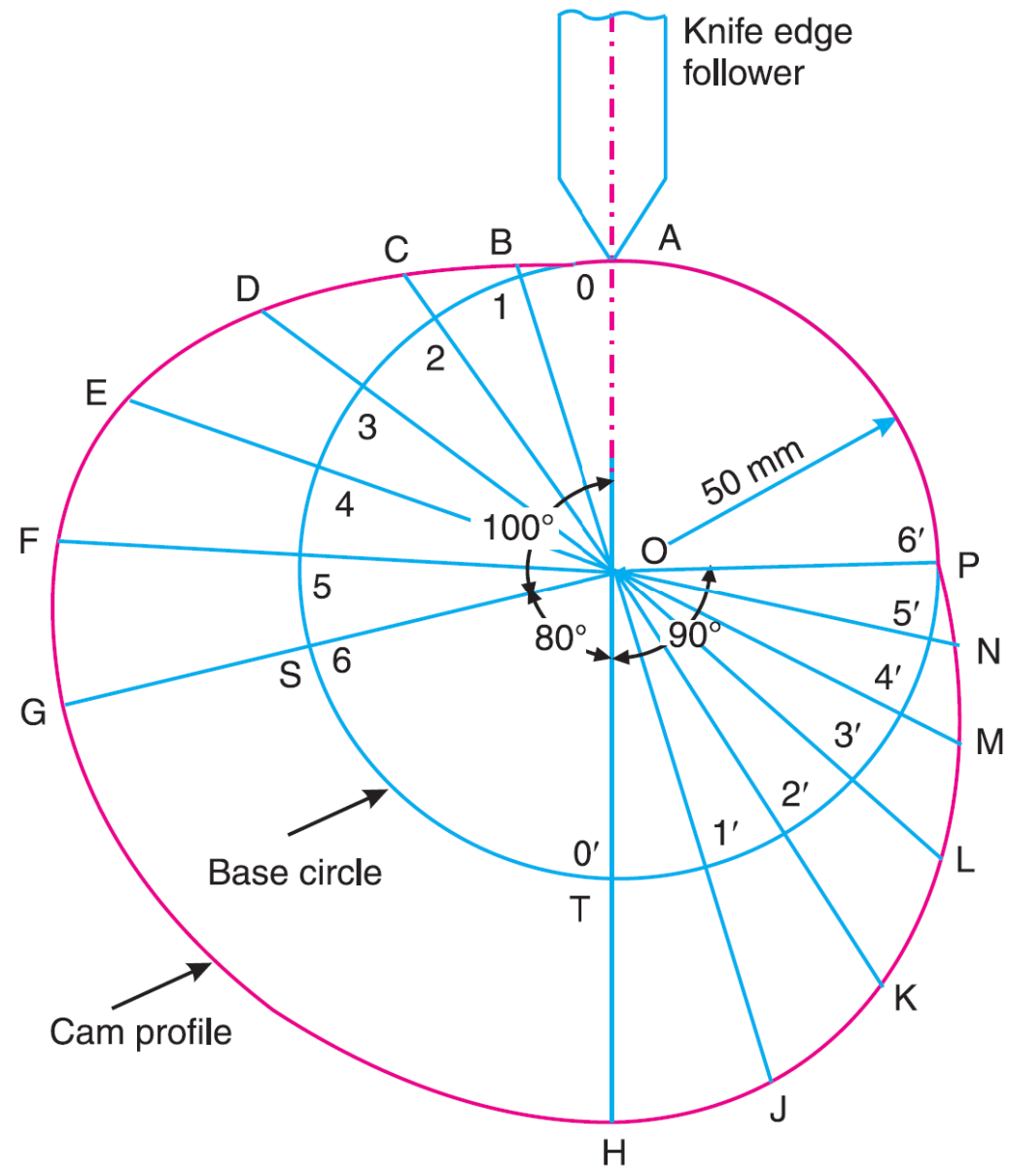
Since the follower moves with equal uniform acceleration and uniform retardation, therefore the displacement diagram of the outward and return stroke consists of a double parabola.

- 5) Join Aa , Ab and Ac intersecting the vertical lines through 1, 2 and 3 at B , C and D respectively.
- 6) Join the points B , C and D with a smooth curve. This is the required parabola for the half outstroke of the valve. Similarly the other curves may be drawn.
- 7) The curve $A B C \dots N P Q$ is the required displacement diagram.



Profile of the cam when the line of stroke of the follower passes through the centre of the cam shaft

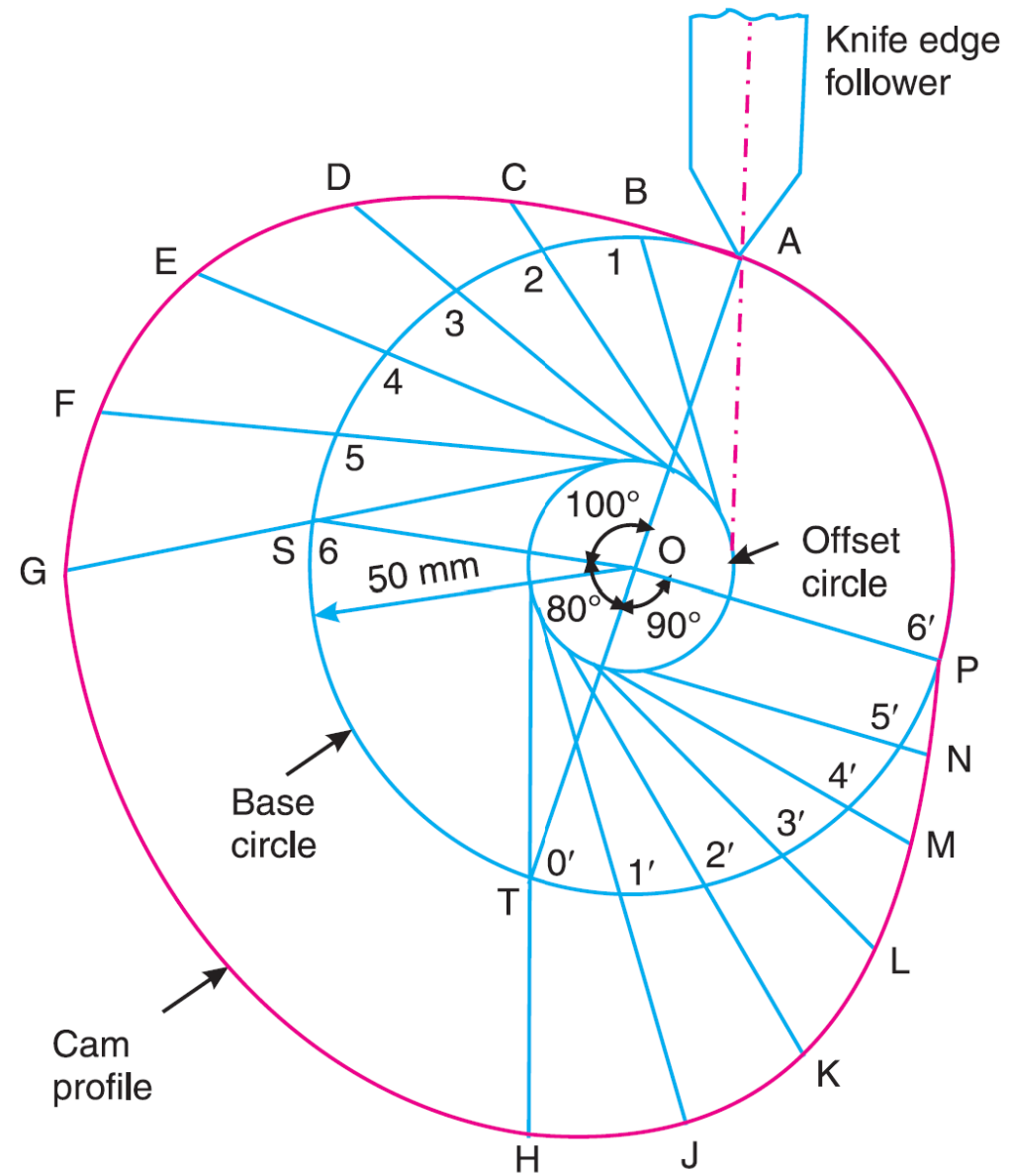
- 1) Draw a base circle with centre O and radius 50 mm (equal to minimum radius of the cam).
- 2) Divide the base circle such that angle $AOS = 100^\circ$; angle $SOT = 80^\circ$ and angle $TOP = 90^\circ$.
- 3) Divide angles AOS and TOP into the same number of equal even parts as in displacement diagram (*i.e.* six parts).
- 4) Join the points 1, 2, 3 . . . and $1'$, $2'$, $3'$, . . . with centre O and produce these lines beyond the base circle.
- 5) From points 1, 2, 3 . . . and $1'$, $2'$, $3'$, . . . mark the displacements 1B, 2C, 3D . . . etc. as measured from the displacement diagram.
- 6) Join the points A, B, C . . . M, N, P with a smooth curve. This is the required profile of the cam.



Profile of the cam when the line of stroke of the follower is offset by 15 mm

1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with O as centre.
2. Draw the axis of the follower at a distance of 15 mm from the axis of the cam, which intersects the base circle at A .
3. Join AO and draw an offset circle of radius 15 mm with centre O .
4. From OA , mark angle $AOS = 100^\circ$ to represent outstroke, angle $SOT = 80^\circ$ to represent dwell and angle $TOP = 90^\circ$ to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle *AOS* and angle *TOP*) into the same number of equal even parts as in displacement diagram.
6. From the points 1, 2, 3 ... etc. and 0', 1', 2', 3' ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle.
7. Set off 1*B*, 2*C*, 3*D* ... etc. and 0' *H*, 1' *J* ... etc. from the displacement diagram.
8. Join the points *A*, *B*, *C* ... *M*, *N*, *P* with a smooth curve. The curve *AGHPA* is the complete profile of the cam.



Maximum velocity and acceleration of follower in uniform acceleration and retardation

Time required for the follower during outstroke

$$t_O = \frac{\theta_O}{\omega}$$

Mean velocity of the follower during outstroke = $\frac{S}{t_O}$

Maximum velocity of the follower during outstroke

$$v_O = \frac{2S}{t_O} = \frac{2\omega.S}{\theta_O}$$

Maximum velocity of the follower during return stroke

$$v_R = \frac{2\omega.S}{\theta_R}$$

Maximum acceleration of the follower during outstroke

$$a_O = \frac{v_O}{t_O / 2} = \frac{2 \times 2 \omega . S}{t_O . \theta_O} = \frac{4 \omega^2 . S}{(\theta_O)^2}$$

Maximum acceleration of the follower during return stroke

$$a_R = \frac{4 \omega^2 . S}{(\theta_R)^2}$$

θ_O and θ_R are in **radians**

Angular velocity of the cam

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 900}{60} = 94.26 \text{ rad/s}$$

Maximum velocity of the follower during its ascent

$$v_O = \frac{2\omega.S}{\theta_O} = \frac{2 \times 94.26 \times 0.04}{1.745} = 4.32 \text{ m/s}$$

Maximum velocity of the follower during its descent

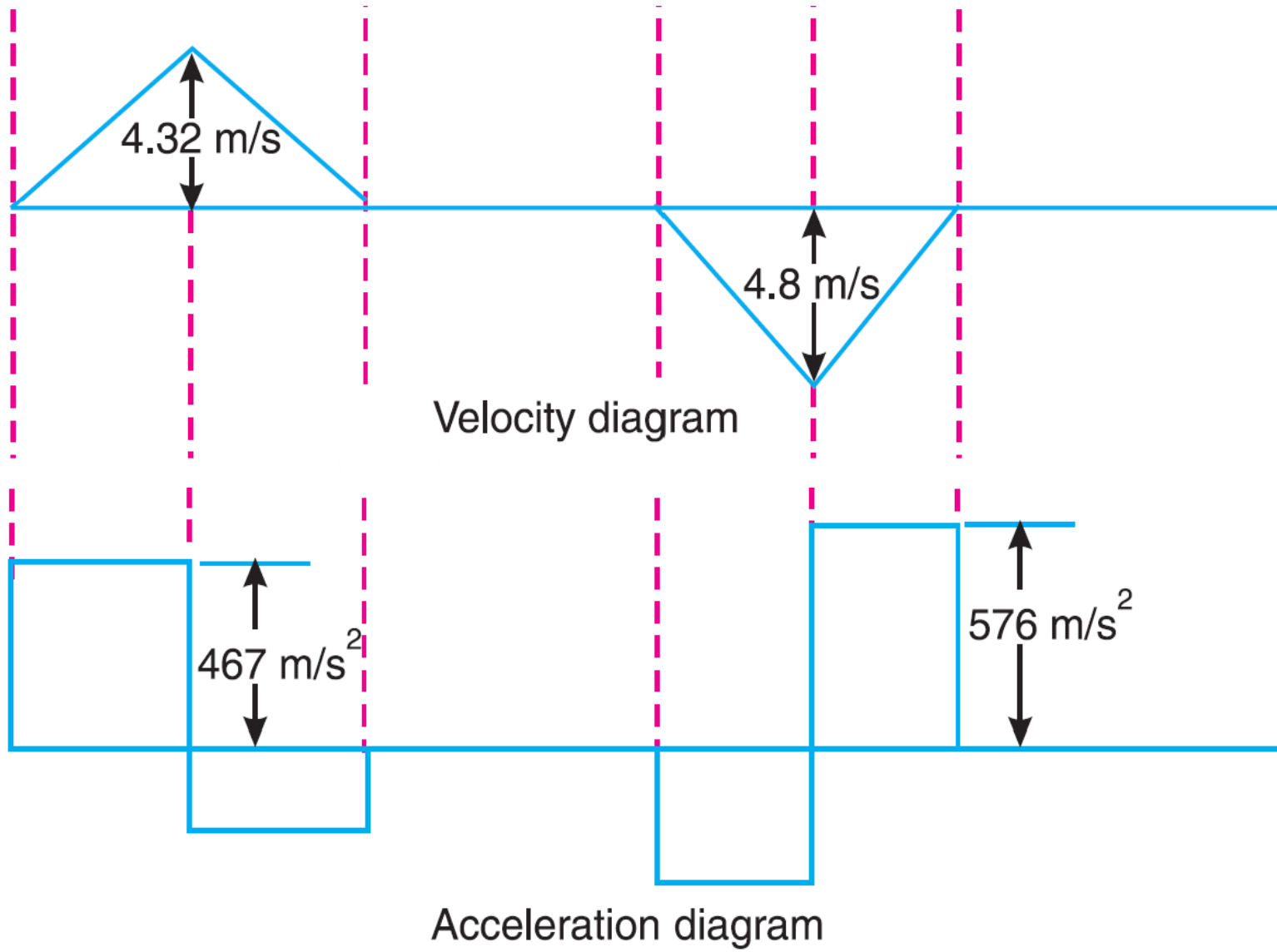
$$v_R = \frac{2\omega.S}{\theta_R} = \frac{2 \times 94.26 \times 0.04}{1.571} = 4.8 \text{ m/s}$$

Maximum acceleration of the follower during its ascent

$$a_O = \frac{4\omega^2.S}{(\theta_O)^2} = \frac{4(94.26)^2 0.04}{(1.745)^2} = 467 \text{ m/s}^2$$

Maximum acceleration of the follower during its descent

$$a_R = \frac{4\omega^2.S}{(\theta_R)^2} = \frac{4(94.26)^2 0.04}{(1.571)^2} = 576 \text{ m/s}^2$$



Example 4

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a **roller follower**, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30° ;
3. To lower the valve during next 60°
4. To keep the valve closed during rest of the revolution i.e. 150°

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

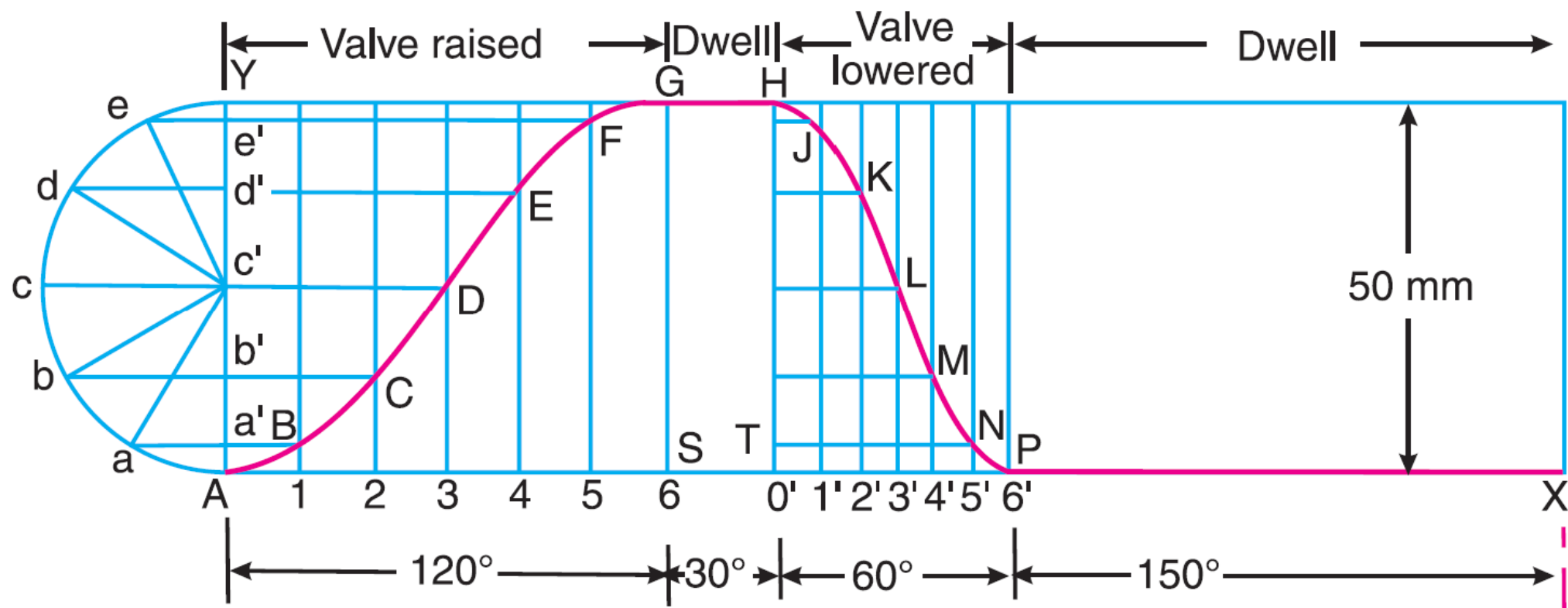
Draw the profile of the cam when **(a)** the line of stroke of the valve rod passes through the axis of the cam shaft, and **(b)** the line of the stroke is offset 15 mm from the axis of the cam shaft.

The displacement of the valve, while being raised and lowered, is to take place with **simple harmonic motion**. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 rev/min.

Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

Displacement diagram of the follower.

- 1) Draw horizontal line $AX = 360^\circ$ to some suitable scale. On this line, mark $AS = 120^\circ$ to represent out stroke ; $SR = 30^\circ$ to represent dwell ; $RP = 60^\circ$ to represent return stroke and $PX = 150^\circ$ to represent dwell.
- 2) Draw vertical line $AY = 50$ mm to represent the cam lift or stroke of the follower and complete the rectangle.
- 3) Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
- 4) Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.
- 5) From points $a, b, c \dots$ etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and $0', 1', 2' \dots$ etc. at $B, C, D \dots M, N, P$.
- 6) Join the points $A, B, C \dots$ etc. with a smooth curve. This is the required displacement diagram.

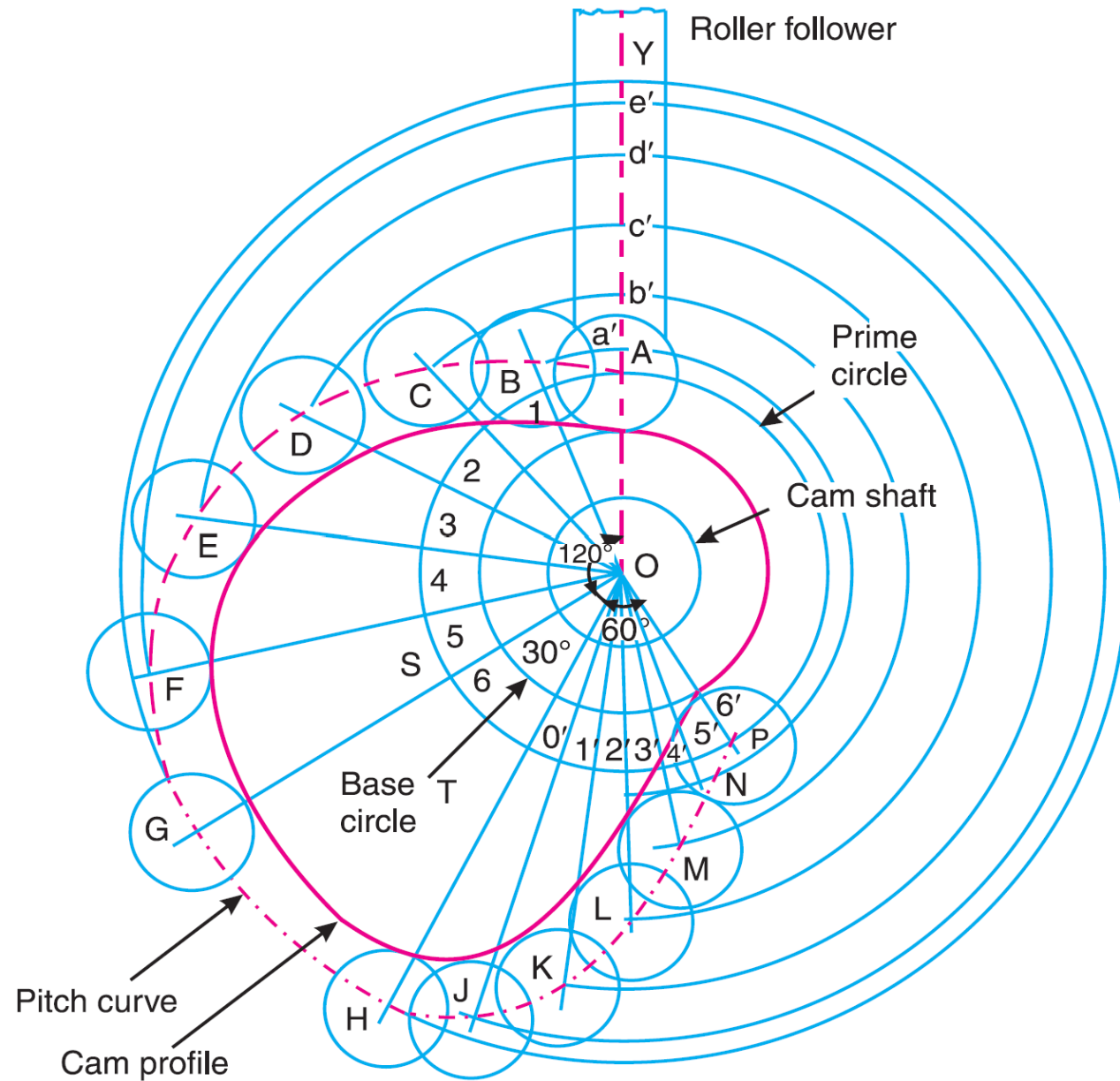


Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft

- 1) Draw a base circle with centre O and radius equal to the minimum radius of the cam (25 mm).
- 2) Draw a prime circle with centre O and radius, $OA = \text{Min. radius of cam} + \text{radius of roller}$ (35 mm)
- 3) Draw angle $AOS = 120^\circ$ to represent raising or out stroke of the valve, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent lowering or return stroke of the valve.

Prime circle - It is the smallest circle that can be drawn from the centre of the cam and tangent to the pitch curve.

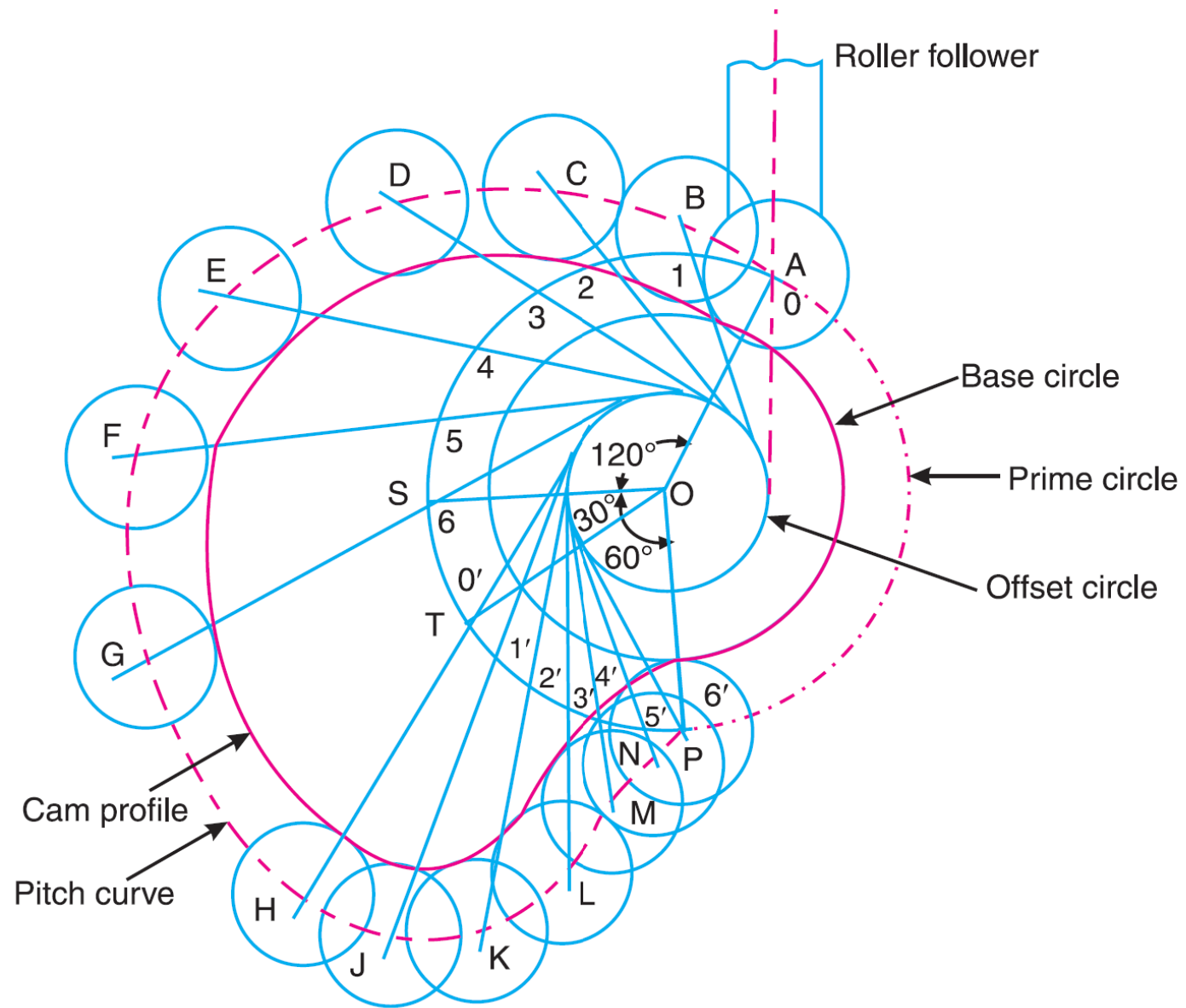
- 4) Divide the angular displacements of the cam during raising and lowering of the valve (*i.e.* angle *AOS* and *TOP*) into the same number of equal even parts as in displacement diagram.
- 5) Join the points 1, 2, 3, etc. with the centre *O* and produce the lines beyond prime circle.
- 6) Set off 1*B*, 2*C*, 3*D* etc. equal to the displacements from displacement diagram.
- 7) Join the points *A*, *B*, *C* ... *N*, *P*, *A*. The curve drawn through these points is known as ***pitch curve***.
- 8) From the points *A*, *B*, *C* ... *N*, *P*, draw circles of radius equal to the radius of the roller.
- 9) Join the bottoms of the circles with a smooth curve. This is the required profile of the cam



Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

- 1) Draw a base circle with centre O and radius equal to 25 mm.
- 2) Draw a prime circle with centre O and radius $OA = 35$ mm.
- 3) Draw an off-set circle with centre O and radius equal to 15 mm.
- 4) Join OA . From OA draw the angular displacements of cam *i.e.* draw angle $AOS = 120^\circ$, angle $SOT = 30^\circ$ and angle $TOP = 60^\circ$.
- 5) Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (*i.e.* six parts) as in displacement diagram.

- 6) From points 1, 2, 3 etc. and $0'$, $1'$, $3'$, ...etc. on the prime circle, draw tangents to the offset circle.
- 7) Set off $1B$, $2C$, $3D$... etc. equal to displacements as measured from displacement diagram.
- 8) By joining the points A , B , C ... M , N , P , with a smooth curve, we get a ***pitch curve***.
- 9) Now A , B , C ...etc. as centre, draw circles with radius equal to the radius of roller.
- 10) Join the bottoms of the circles with a smooth curve. This is the required profile of the cam.



Angular velocity of the cam

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/s}$$

Maximum velocity of the follower during its ascent

$$v_O = \frac{\pi \omega S}{2\theta_O} = \frac{\pi \times 10.47 \times 0.05}{2 \times 2.1} = 0.39 \text{ m/s}$$

Maximum velocity of the follower during its descent

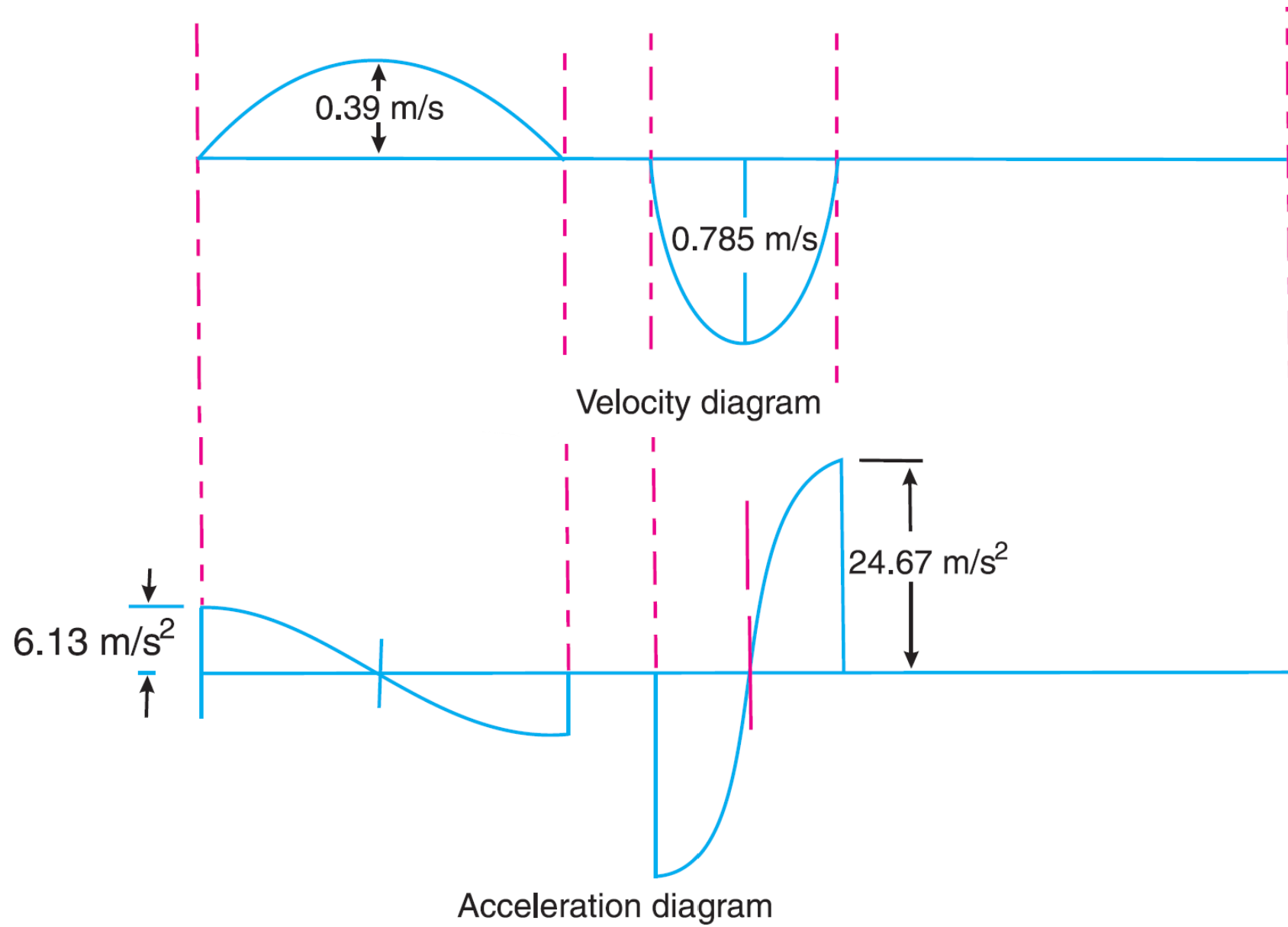
$$v_R = \frac{\pi \omega S}{2\theta_R} = \frac{\pi \times 10.47 \times 0.05}{2 \times 1.047} = 0.785 \text{ m/s}$$

Maximum acceleration of the follower during its ascent

$$a_O = \frac{\pi^2 \omega^2 S}{2(\theta_O)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(2.1)^2} = 6.13 \text{ m/s}^2$$

Maximum acceleration of the follower during its descent

$$a_R = \frac{\pi^2 \omega^2 S}{2(\theta_R)^2} = \frac{\pi^2 (10.47)^2 0.05}{2(1.047)^2} = 24.67 \text{ m/s}^2$$



Example 5

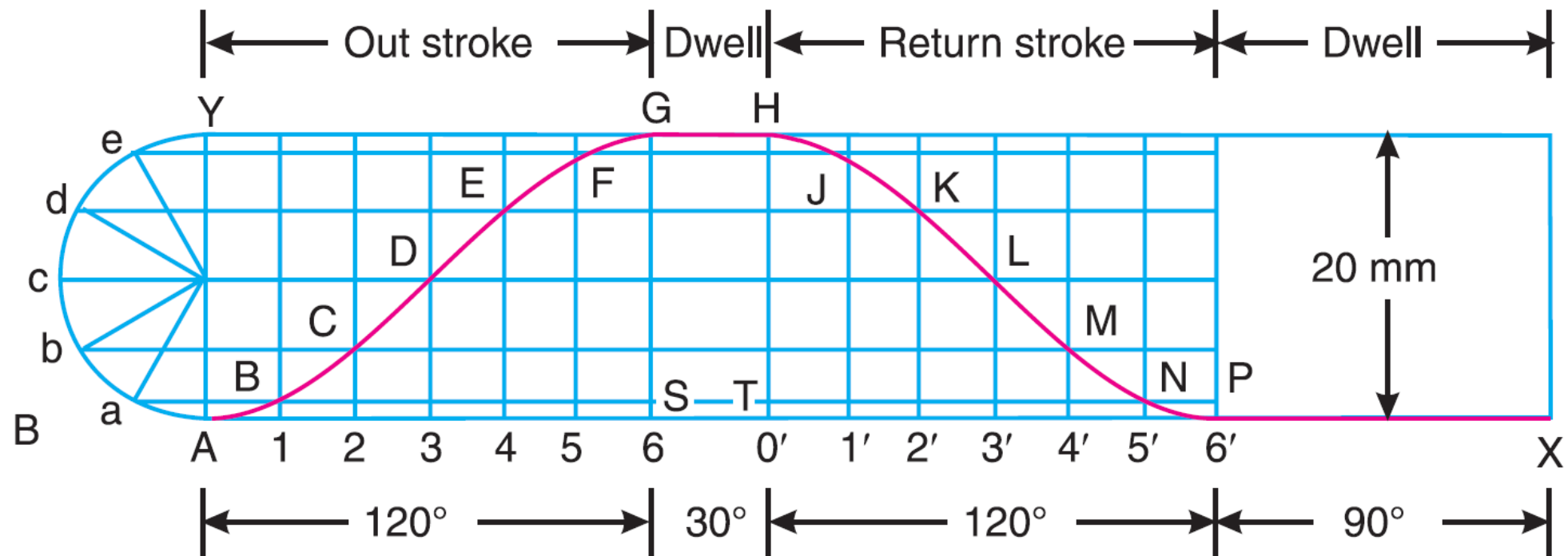
A cam drives a *flat reciprocating follower* in the following manner:

1. During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with *simple harmonic motion*.
2. The follower dwells during next 30° of cam rotation.
3. During next 120° of cam rotation, the follower moves inwards with *simple harmonic motion*.
4. The follower dwells for the next 90° of cam rotation.

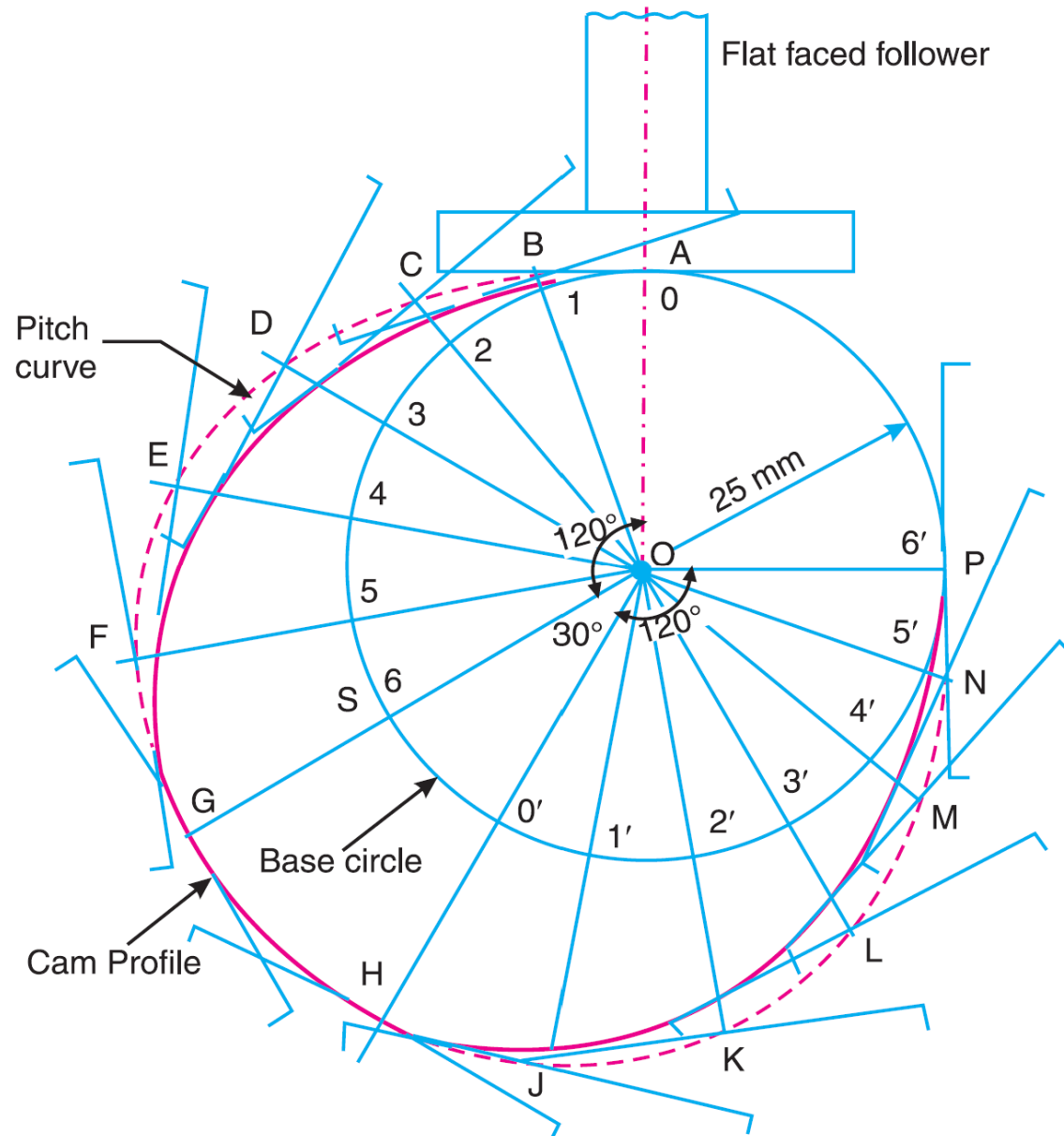
The minimum radius of the cam is 25 mm. Draw the profile of the cam.

Displacement diagram of the follower.

- 1) Draw horizontal line $AX = 360^\circ$ to some suitable scale. On this line, mark $AS = 120^\circ$ to represent out stroke ; $SR = 30^\circ$ to represent dwell ; $RP = 120^\circ$ to represent return stroke and $PX = 90^\circ$ to represent dwell.
- 2) Draw vertical line $AY = 20$ mm to represent the cam lift or stroke of the follower and complete the rectangle.
- 3) Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
- 4) Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.
- 5) From points $a, b, c \dots$ etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and $0', 1', 2' \dots$ etc. at $B, C, D \dots M, N, P$.
- 6) Join the points $A, B, C \dots$ etc. with a smooth curve. This is the required displacement diagram.



- 1) Draw a base circle with centre O and radius OA equal to the minimum radius of the cam (25 mm).
- 2) Draw angle $AOS = 120^\circ$ to represent the outward stroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 120^\circ$ to represent inward stroke.
- 3) Divide the angular displacement during outward stroke and inward stroke (*i.e.* angles AOS and TOP) into the same number of equal even parts as in the displacement diagram.
- 4) Join the points 1, 2, 3 . . . etc. with centre O and produce beyond the base circle.
- 5) From points 1, 2, 3 . . . etc., set off $1B$, $2C$, $3D$. . . etc. equal to the distances measured from the displacement diagram.
- 6) Now at points B , C , D . . . M , N , P , draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.
- 7) The curve drawn tangentially to the flat side of the follower is the required profile of the cam.



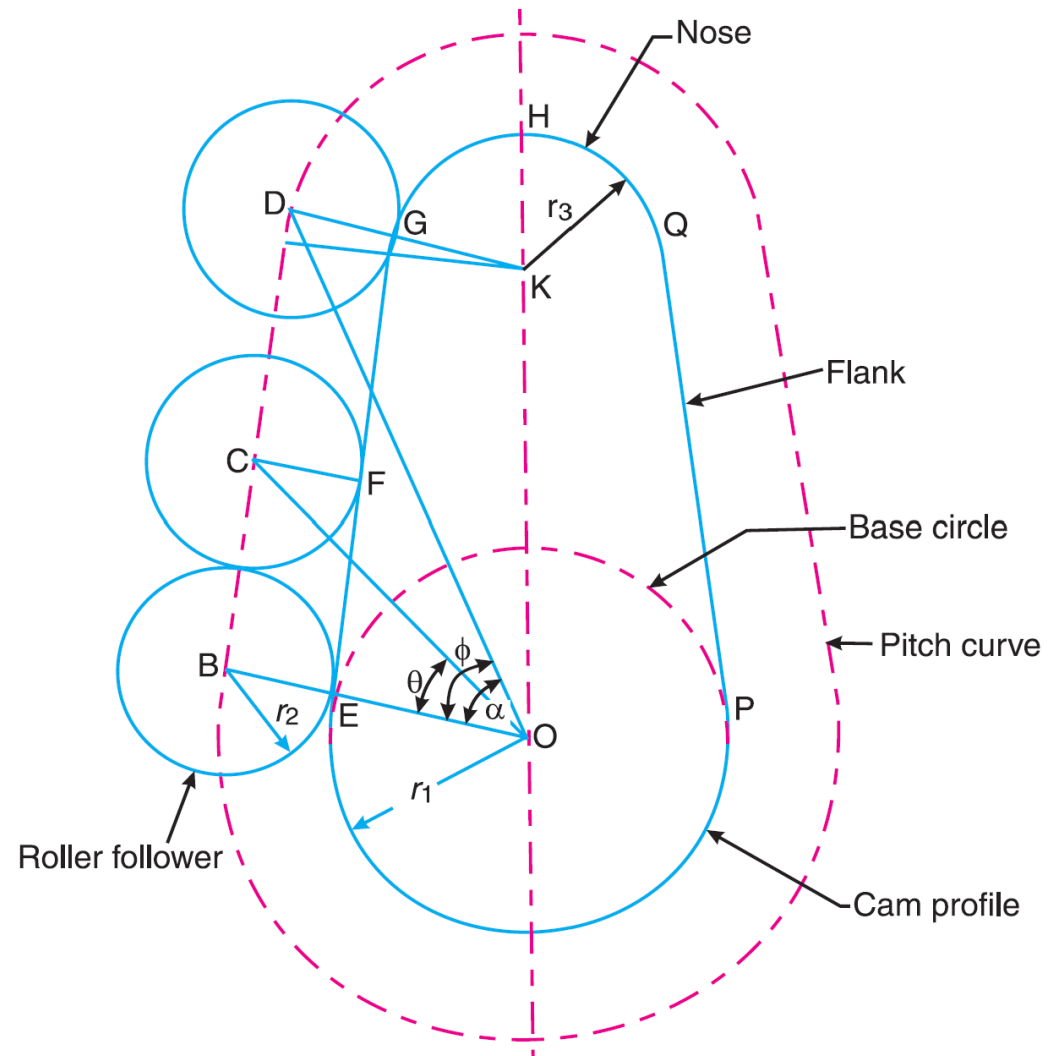
Cams with Specified Contours

Cams with specified contours (cam profiles consisting of circular arcs and straight lines are preferred) are assumed and then motion of the follower is determined.

Tangent Cam

When the flanks of the cam are straight and tangential to the base circle and nose circle. These cams are usually symmetrical about the centre line of the cam shaft.

Application: operating the inlet and exhaust valves of internal combustion engines



Circular Arc Cam

When the flanks of the cam connecting the base circle and nose are of convex circular arcs.

O and Q are the centres of cam and nose, EF and GH are two circular flanks whose centres lie at P and P' .

