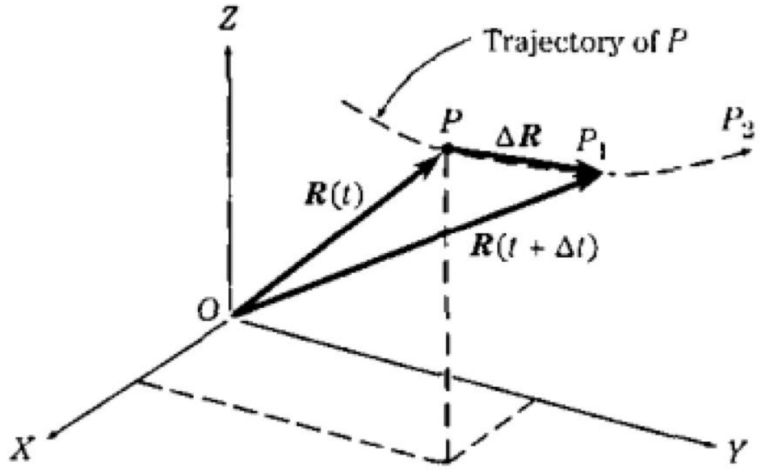
Rigid body dynamics

- Dynamics is the branch of classical mechanics concerned with the study of forces and their effects on motion.
- Kinematics: the study of the motion.
- Concerns with the time behavior of position, velocity and acceleration of systems and components of systems.
- No reference is made to the forces that cause the motion or the forces that are generated as a consequence of the motion.

Position, velocity and acceleration

Position, Velocity and Acceleration of a body are defined only with respect to a specified reference frame.



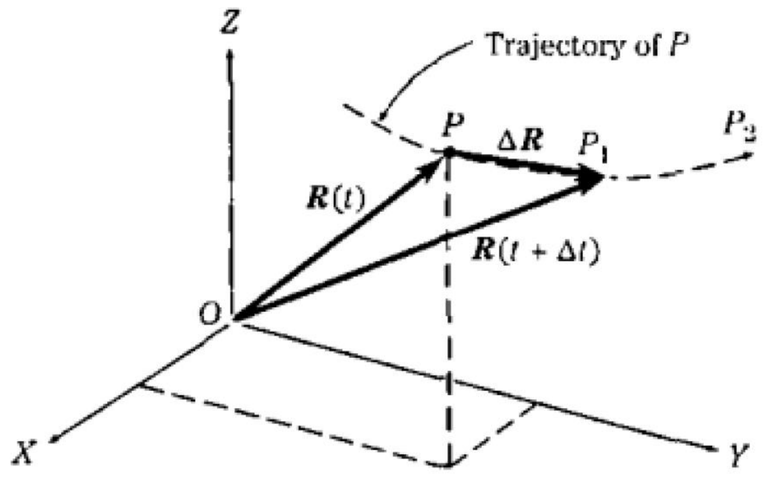
OXYZ is a right handed coordinate system.

OXYZ is a rigid rectangular reference frame and point P moves with respect to the reference frame.

Position vector

The location of a point with respect to any specified reference frame is defined as the position vector in that reference frame.

- Position vector of OP $\boldsymbol{R} = \boldsymbol{R}(t)$
- Displacement vector $\Delta\boldsymbol{R} = PP_1$
- Velocity vector $\boldsymbol{v} = v(t) = \frac{d\boldsymbol{R}}{dt}$
- Acceleration vector $\boldsymbol{a} = a(t) = \frac{d^2\boldsymbol{R}}{dt^2}$



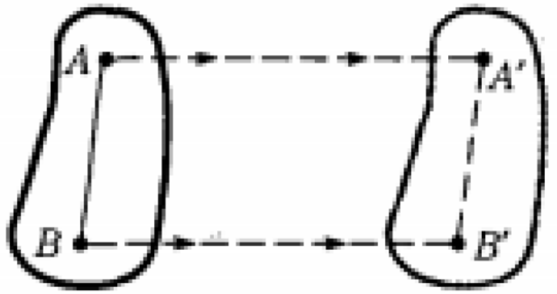
OXYZ is the fixed reference frame.

Types of plane motion of a rigid body

- **Translation** is any motion in which every line in the body remains parallel to its past and future positions at all times.

  No rotation of any line in the body at any time.

In **Rectilinear translation**, all points in the body move in parallel straight lines.



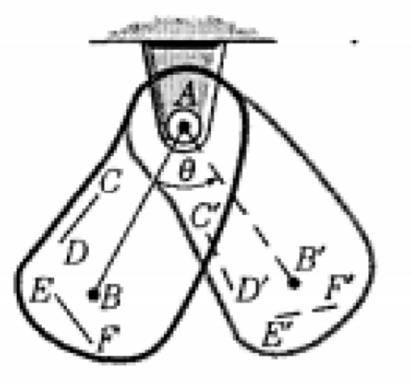
- In **curvilinear translation**, all points move along congruent curves.

- **Rotation** about a fixed axis is the angular motion about that axis.

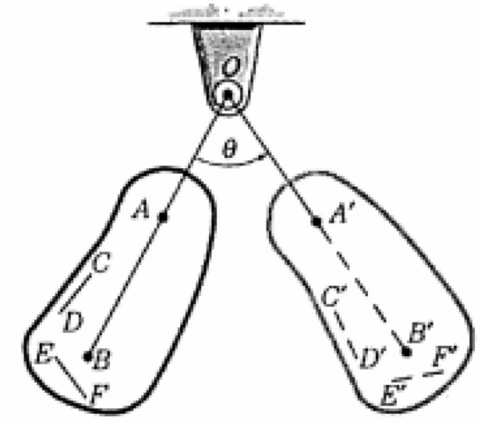
All points in the body move in circular paths about the axis of rotation.

All lines in the body that lie in the plane of the motion rotate through the same angle during the same time.

- If the axis of rotation passes through the body, points on the axis do not move.



- If the axis of rotation does not pass through the body, all points of the body move.

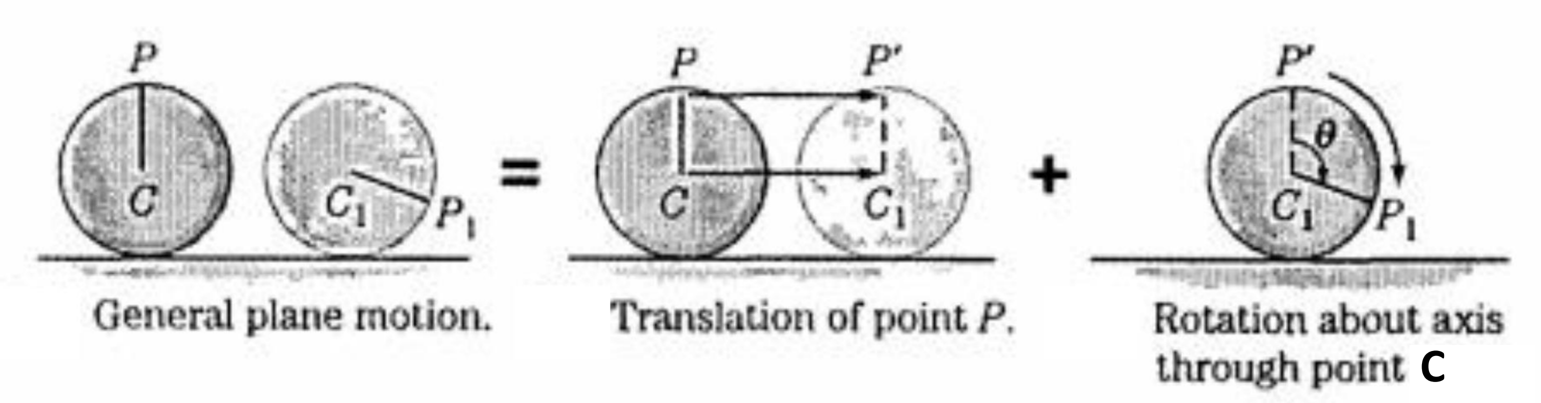


General plane motion

Combination of translation and rotational motion.

This motion is expressed by Chasles' theorem.

General motion of a rigid body is equivalent to a translation of some point and a rotation about an axis passing through that point.



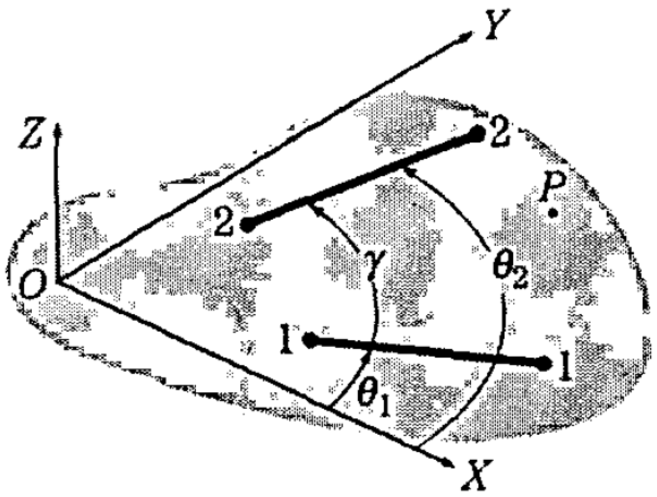
The general motion of CP to C_1P_1

Translation of point C to point C_1.

Rotation about an axis passing through point C.

Angular displacement, Angular velocity and Angular acceleration

A rigid body is moving parallel to the XY plane as it rotates about the fixed-axis OZ.



OXYZ is a fixed reference frame.

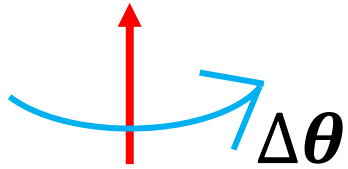
$$\theta_2 = \theta_1 + \gamma$$

The body undergoes an angular displacement $\Delta\boldsymbol{\theta}$ as it rotates about the axis OZ.

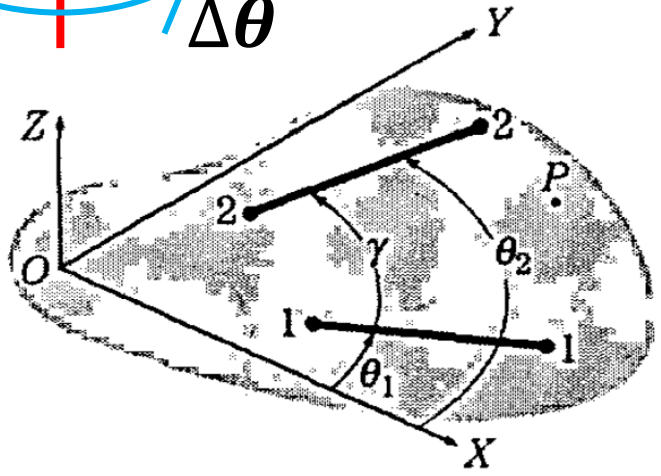
Directed line segment

- Magnitude of the rotation – length
- Sense – right hand rule

when the axis OZ is grasped with the right hand so that the fingers curl in the direction of the rotation $\Delta\boldsymbol{\theta}$, the right hand thump points in the sense of the directed line segment.



Once the axis of rotation has been chosen, the magnitude and the direction of the rotation determine the directed line segment $\Delta\boldsymbol{\theta}$.



The body undergoes an angular displacement $\Delta\boldsymbol{\theta}$ as it rotates about the axis OZ.

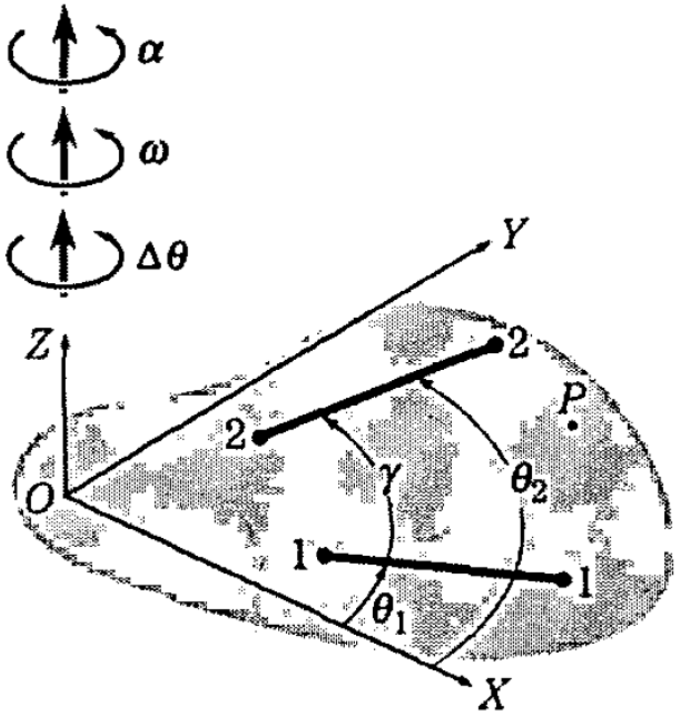
During a finite time interval Δt

$$\Delta\boldsymbol{\theta}_1 = \Delta\boldsymbol{\theta}_2 = \Delta\boldsymbol{\theta}$$

The first and second time derivatives give,

Angular velocity $$\boldsymbol{\omega} = \frac{d\boldsymbol{\theta}}{dt}$$

Angular acceleration $$\boldsymbol{\alpha} = \frac{d\boldsymbol{\omega}}{dt} = \frac{d^2\boldsymbol{\theta}}{dt^2}$$



Directed line segment for the angular velocity $\boldsymbol{\omega}$.

The line of action of $\boldsymbol{\omega}$ is called the *instantaneous axis of rotation* of the body with respect to the reference frame.

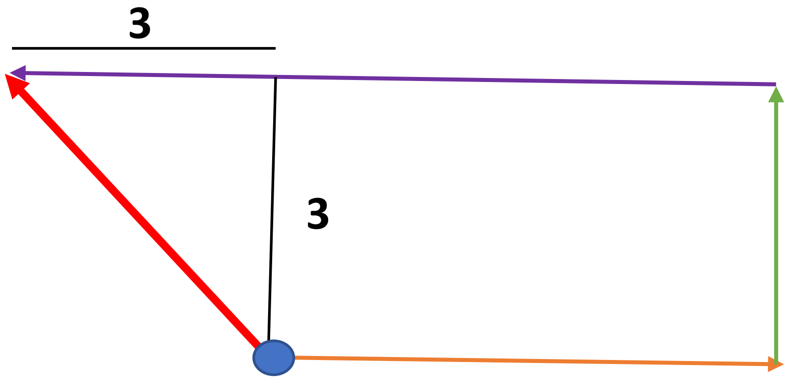
The length of the directed line segment of $\boldsymbol{\omega}$ is called the *angular speed* and is defined as the magnitude $|\boldsymbol{\omega}|$

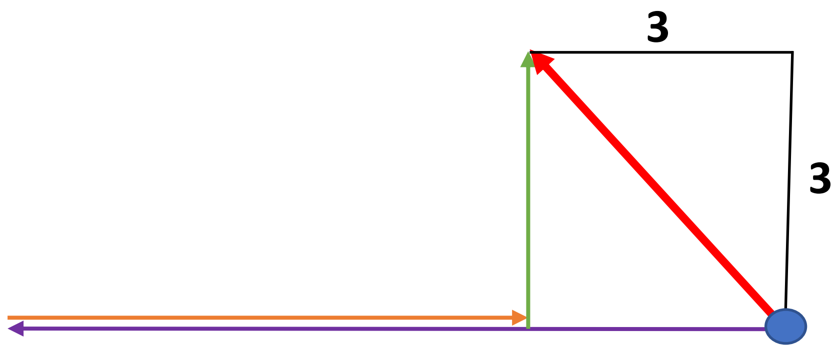
Finite rotations for Linear displacements

If a point P undergoes many linear displacements in succession, the resulting total linear displacement does <u>not depend on the order of displacement</u>.

East(5), North(3), West(8)

West(8), East(5), North(3)



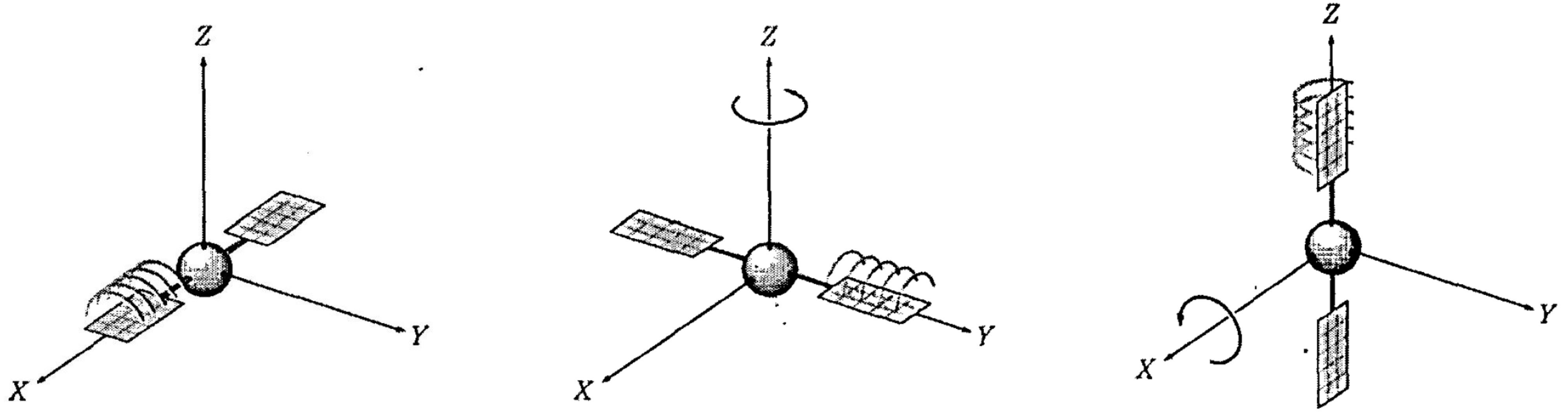


Finite rotations for Angular displacements

When a finite body undergoes several finite angular displacements about different axes in succession the result <u>depends on the order</u> in which the individual angular displacements are taken.

Finite rotations

A satellite undergoing successive 90° rotations about Z then about X axes.



A satellite undergoing successive 90° rotations about X then about Z axes.