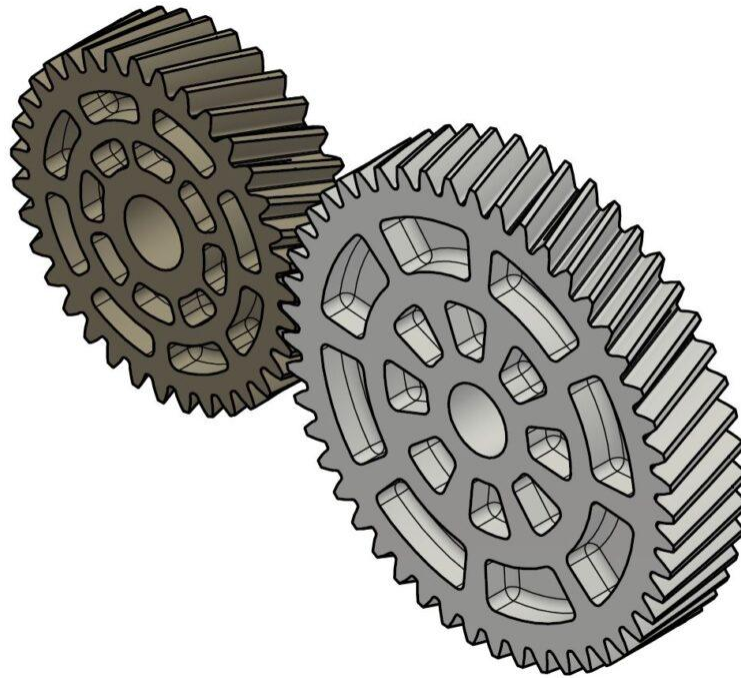


Kinematics and Dynamics

Day 04 | MP 3010



Gears
Gear Trains

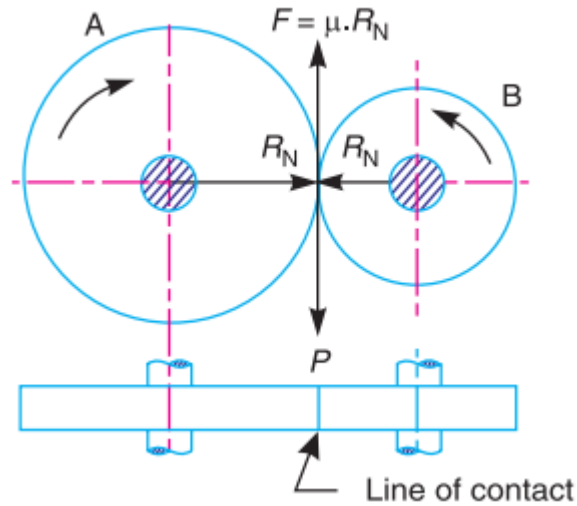
Gears

Section 1

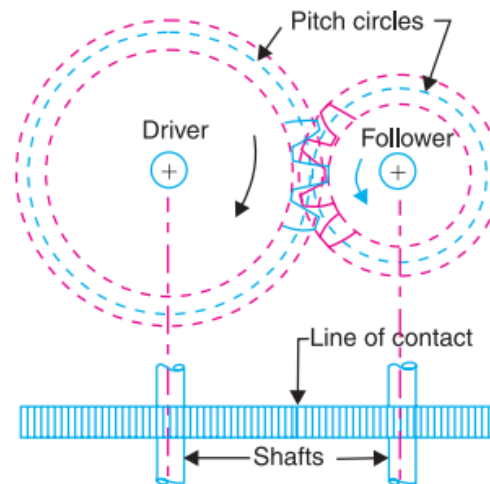


Types of Gears

Friction Wheels



Toothed Wheels



Advantages and Disadvantages of Gear Drive

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

Classification of toothed gears

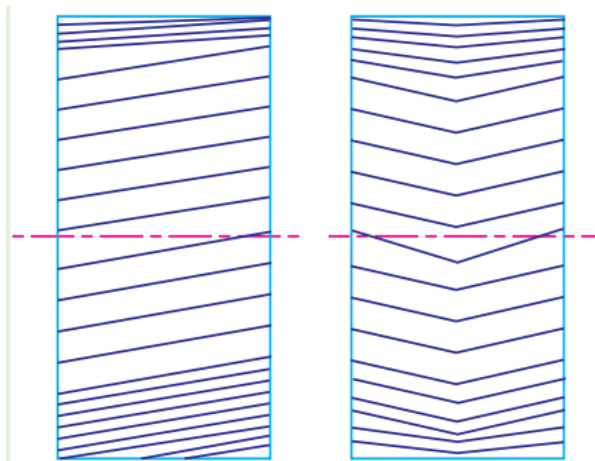
1. According to the position of axes of the shafts.

The axes of the two shafts between which the motion is to be transmitted, may be;

(a) *Parallel*

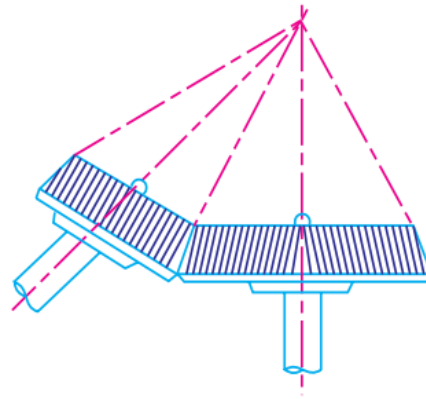
(b) *Intersecting*

(c) *Non-intersecting and non-parallel.*



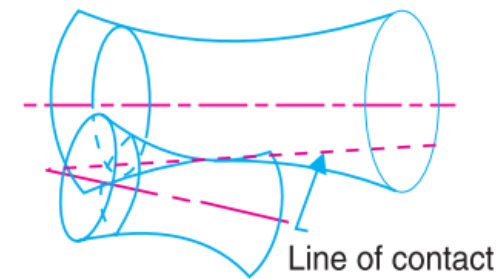
(a) Single helical gear. (b) Double helical gear.

Parallel



(c) Bevel gear.

Intersecting



(d) Spiral gear.

Non -intersecting and non parallel

Classification of toothed gears

2. According to the peripheral velocity of the gears. The gears, according to the peripheral velocity of the gears may be classified as :

(a) Low velocity

(b) Medium velocity

(c) High velocity.

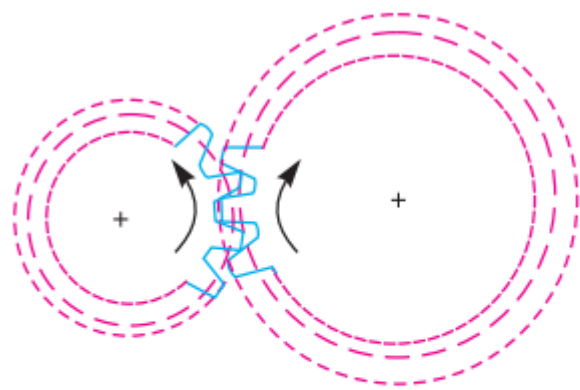
Classification of toothed gears

3. According to the type of gearing. The gears, according to the type of gearing may be classified as :

(a) External gearing

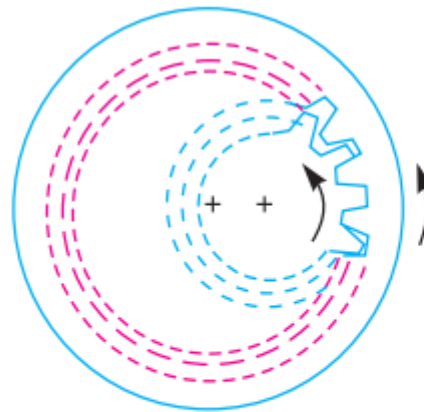
(b) Internal gearing

(c) Rack and pinion



(a) External gearing.

Fig. 12.3



(b) Internal gearing.

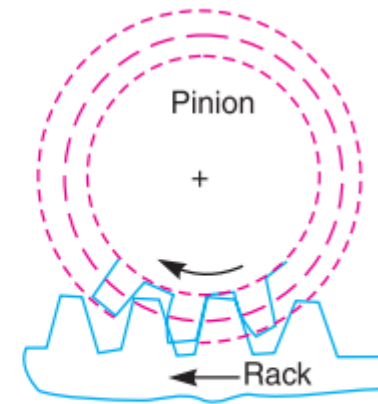


Fig. 12.4. Rack and pinion.

Classification of toothed gears

4. According to position of teeth on the gear surface. The teeth on the gear surface may be

(a) straight

(b) Inclined

(c) curved.

Terms Used in Gears

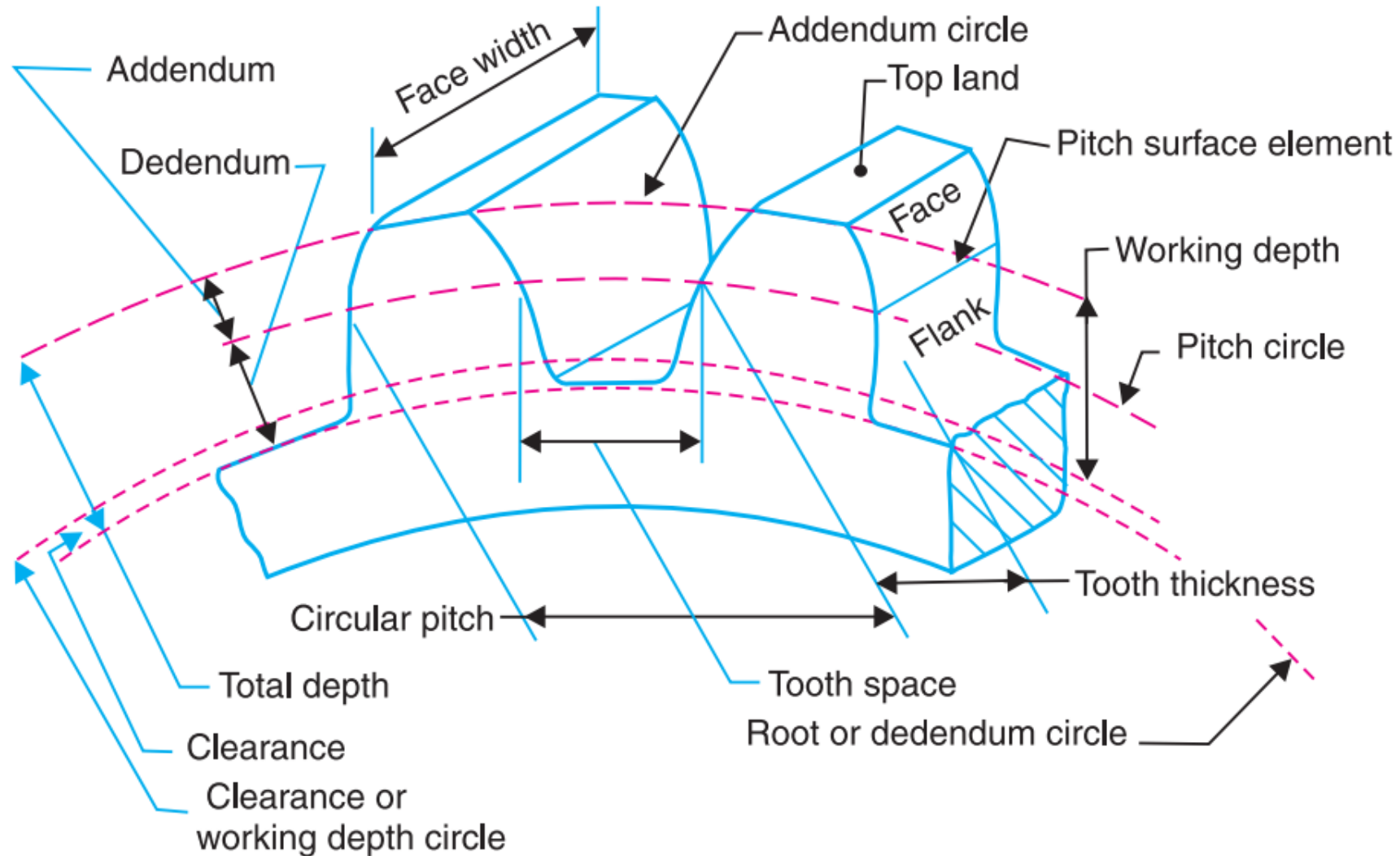


Fig. 12.5. Terms used in gears.

1. Pitch circle

It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter.

It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.

3. Pitch point.

It is a common point of contact between two pitch circles.

4. Pitch surface.

It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ . The standard pressure angles are 14.5° and 20° .

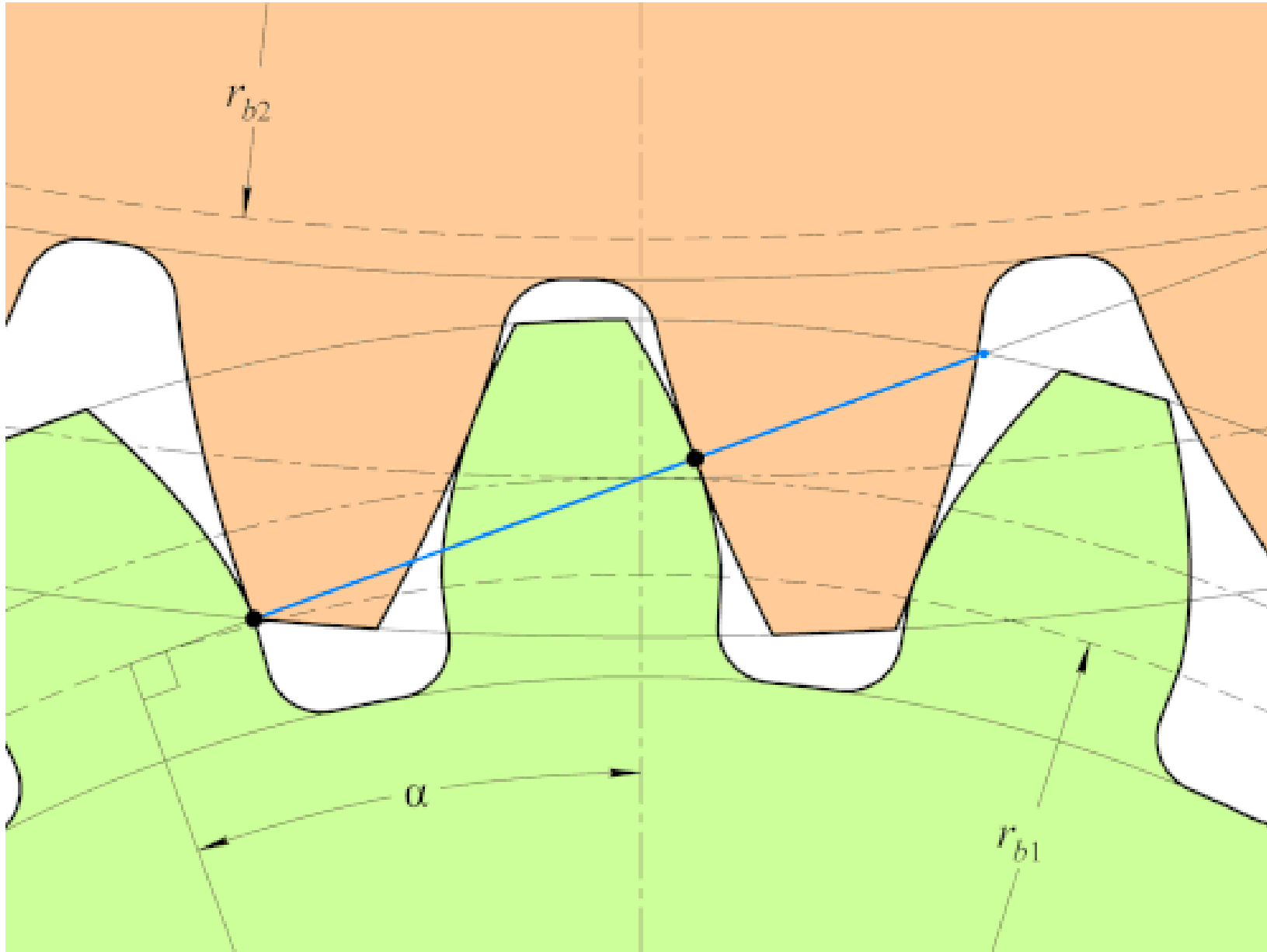
6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter $\times \cos \phi$, where ϕ is the pressure angle.



Circular Pitch - p_c

It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

Mathematically,

$$\text{Circular pitch, } p_c = \pi D/T$$

Where;

D = Diameter of the pitch circle

T = Number of teeth on the wheel.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Diametral Pitch

It is the ratio of number of teeth to the pitch circle diameter in millimeters. It is denoted by p_d .

Mathematically,

$$\text{Diametral pitch, } p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \dots \left(\because p_c = \frac{\pi D}{T} \right)$$

where

T = Number of teeth, and

D = Pitch circle diameter.

Module

It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.

Mathematically,

$$\text{Module, } m = D / T$$

Law of Gearing

Let $QED\Delta$ and $QEC\Delta$;

$$QE = v_1 \cos \alpha$$

$$QE = v_2 \cos \beta$$

$$v_2 \cos \beta = v_1 \cos \alpha$$

$$v_1 = O_1Q \times \omega_1$$

$$v_2 = O_2Q \times \omega_2$$

$$(O_1Q \times \omega_1) \cos \alpha = (O_2Q \times \omega_2) \cos \beta$$

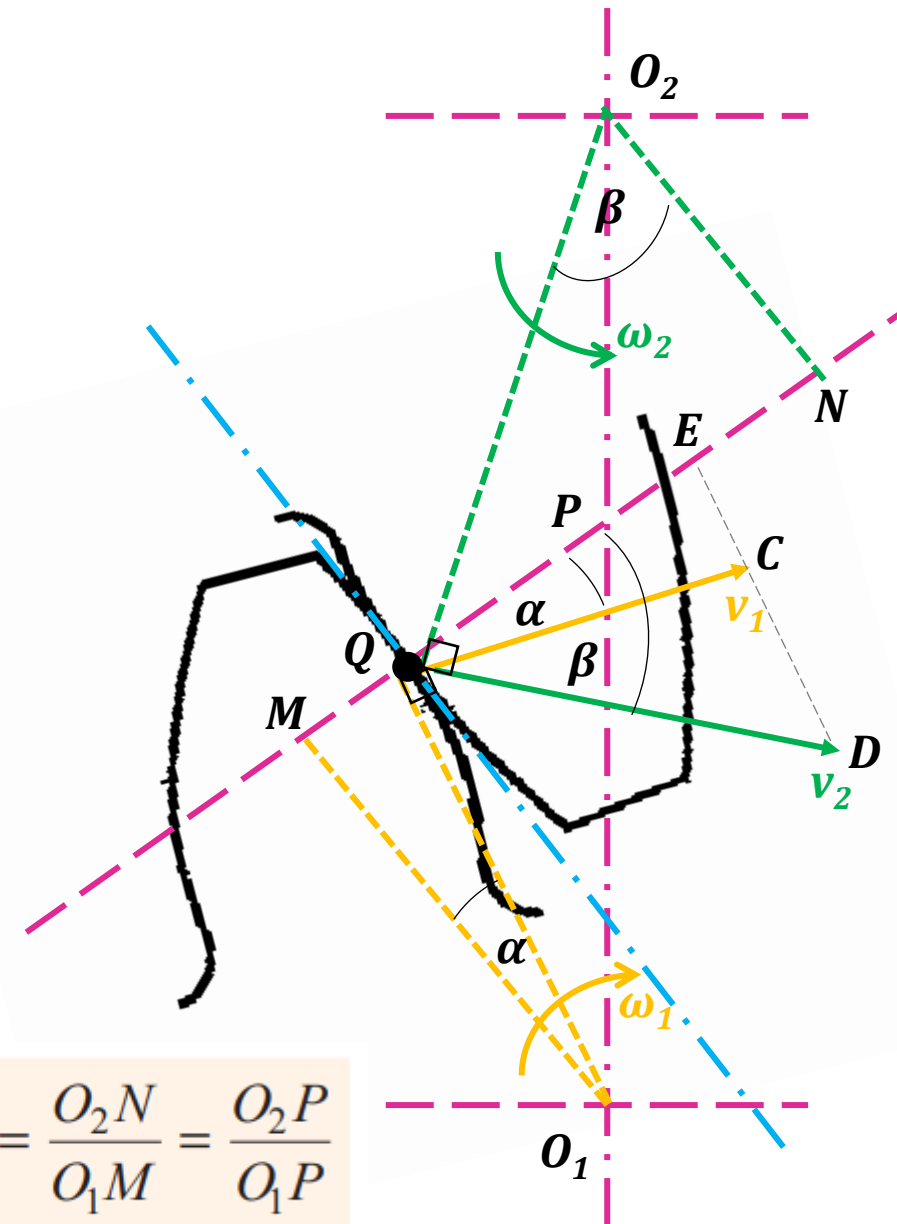
$$(O_1Q \times \omega_1) \times \frac{O_1M}{O_1Q} = (O_2Q \times \omega_2) \times \frac{O_2N}{O_2Q}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

Also, from similar triangles
 O_1MP and O_2NP ,

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$



Law of Gearing

The common normal at the point of contact between a pair of teeth must always pass through the pitch point.

Wheels 1 and 2 having teeth T_1 and T_2 and pitch circle diameters D_1 and D_2 .

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{D_2}{D_1} = \frac{T_2}{T_1}}$$

Velocity of Sliding of Teeth

For equal addendum gears;

$$v_{\text{sliding}} = (\omega_1 + \omega_2) \cdot a \cdot \sin \phi$$

Velocity of Sliding of Teeth

At engagement: $v_{\text{sliding}} = \omega_1 \cdot r_{a1} - \omega_2 \cdot r'_2$

At disengagement: $v_{\text{sliding}} = \omega_1 \cdot r'_1 - \omega_2 \cdot r_{a2}$

Term	Meaning
ω_1	Angular velocity of driver gear
ω_2	Angular velocity of driven gear
r_{a1}	Addendum radius of driver gear
r_{a2}	Addendum radius of driven gear
r'_1, r'_2	Radius to point of contact (projection on pitch circle)
v_{sliding}	Relative velocity of tooth surfaces at point of contact

Form of teeth in gears

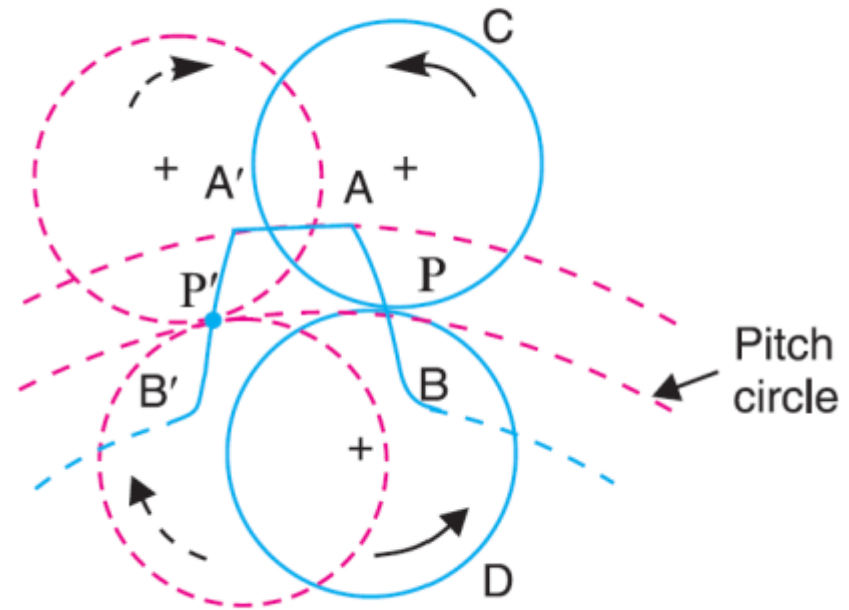
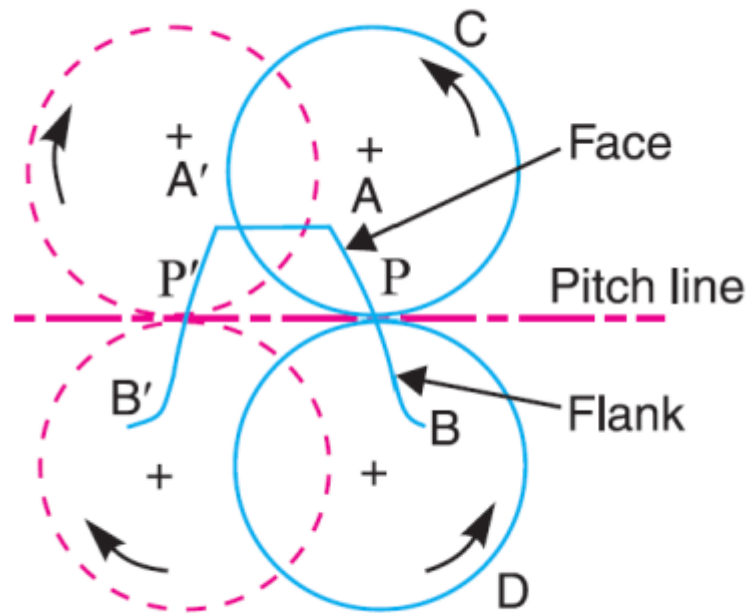
Cycloidal teeth

Cycloid - curve traced by a point on the circumference of a circle which rolls without slipping on a fixed straight line.

Epicycloid - curve traced by a point on the circumference of a circle which rolls without slipping on the outside of a fixed circle.

Hypocycloid - curve traced by a point on the circumference of a circle which rolls without slipping on the inside of a fixed circle.

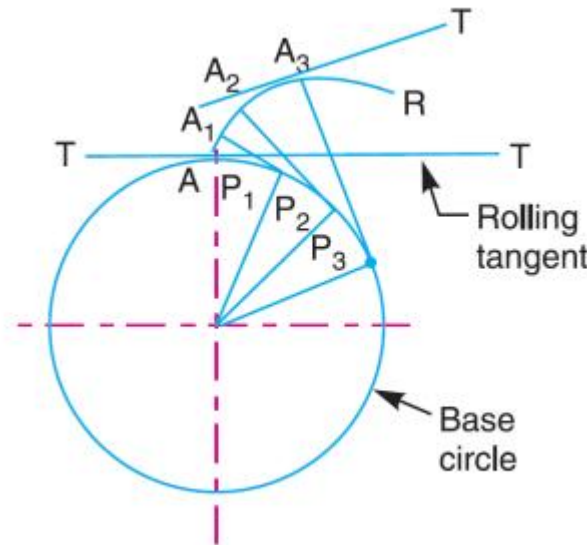
Construction of cycloidal teeth of a gear.



Involute Teeth

An involute of a circle is a plane curve generated by a point on a taut string which is unwrapped from a reel as shown. In toothed wheels, the circle is known as base circle.

Normal at any point of an involute is a tangent to the circle.



Torque exerted on the gear shaft

F is maximum tooth pressure.

Tangential force,

$$F_T = F \cos \phi$$

Radial or normal force,

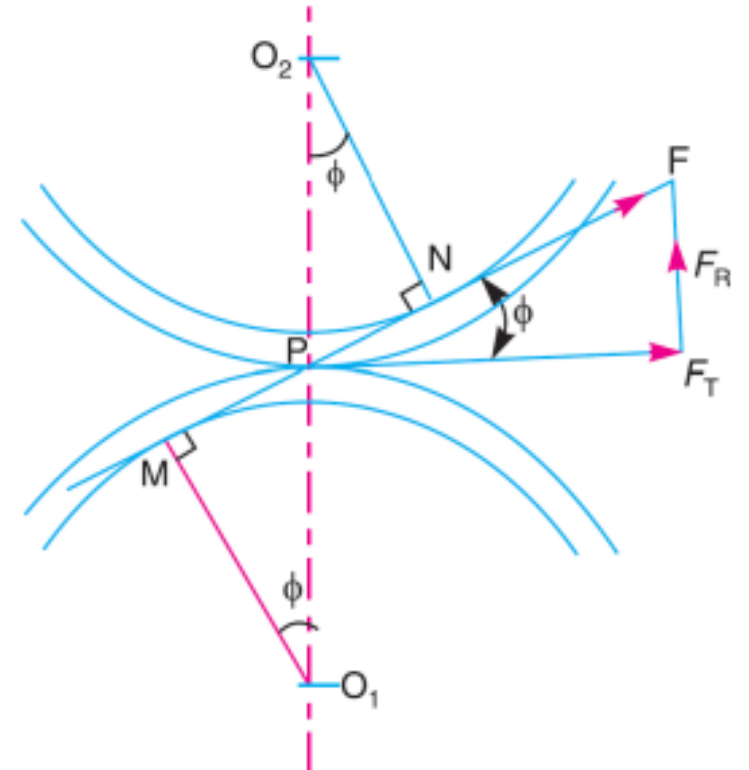
$$F_R = F \sin \phi$$

ϕ – Pressure angle

Torque exerted on the gear shaft;

$$\tau = F_T \times r$$

Where, r - the pitch circle radius of the gear.



Length of Path of Contact

Length of path of contact (KL) is the length of common normal cutoff by the addendum circles of the wheel and the pinion.

$$\mathbf{KL = KP + PL}$$

KP - Path of approach

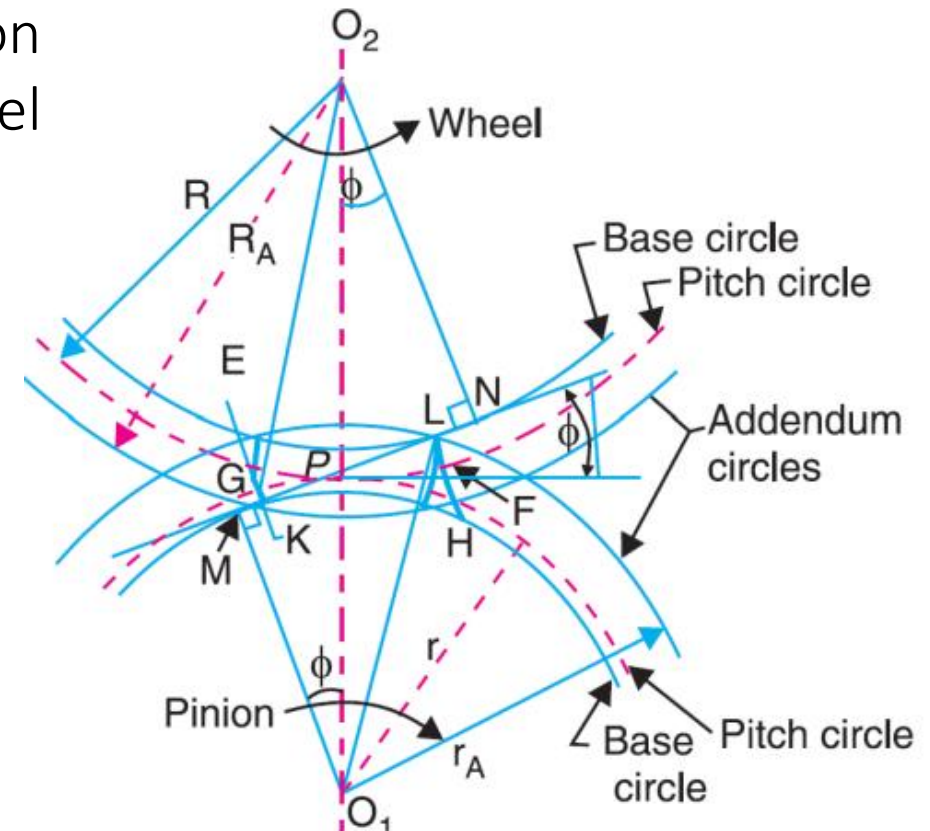
PL – Path of recess

$r_A = O_1L$ – Radius of addendum circle of pinion

$R_A = O_2K$ – Radius of addendum circle of wheel

$r = O_1P$ – Radius of pitch circle of pinion

$R = O_2P$ – Radius of pitch circle of wheel



Radius of the base circle of pinion

$$O_1M = O_1P \cos \phi = r \cos \phi$$

Radius of the base circle of wheel

$$O_2N = O_2P \cos \phi = R \cos \phi$$

Right angled triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2 \cos^2 \phi}$$

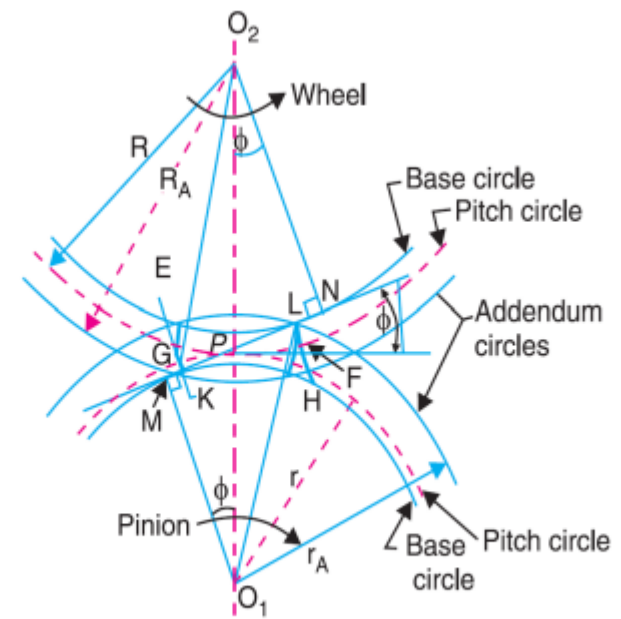
$$PN = O_2P \sin \phi = R \sin \phi$$

Length of path of approach

$$KP = KN - PN = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

Length of path of contact

$$KL = KP + PL = \sqrt{(R_A)^2 - R^2 \cos^2 \phi} + \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - (R + r) \sin \phi$$



Right angled triangle O_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_A)^2 - r^2 \cos^2 \phi}$$

$$MP = O_1P \sin \phi = r \sin \phi$$

Length of path of recess

$$PL = ML - MP = \sqrt{(r_A)^2 - r^2 \cos^2 \phi} - r \sin \phi$$

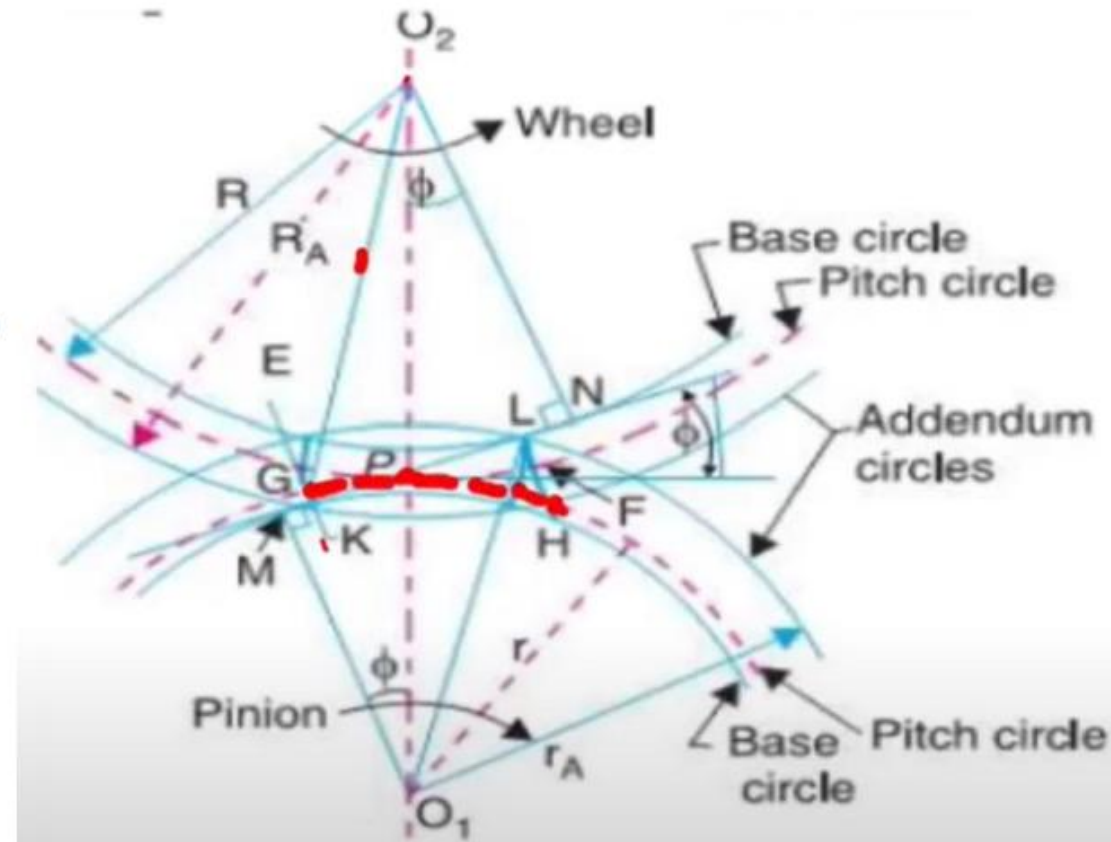
Length of Arc of Contact

- Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.
- Arc of contact is GPH .

G is the intersection of the normal to MN at K and pitch circle of pinion.

H is the intersection of the normal to MN at L and pitch circle of pinion.

$$GPH = \frac{KL}{\cos \phi} = \frac{\text{Length of path of contact}}{\cos \phi}$$



$$GPH = GP + PH$$

GP - *Arc of approach*

$$GP = \frac{KP}{\cos \phi} = \frac{\text{Length of path of approach}}{\cos \phi}$$

PH - *Arc of recess*

$$PH = \frac{PL}{\cos \phi} = \frac{\text{Length of path of recess}}{\cos \phi}$$

Contact Ratio

Contact ratio is the number of pairs of teeth in contact.

$$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

- The theoretical minimum value for the contact ratio is one, that is there must always be at least one pair of teeth in contact for continuous action.
- Larger the contact ratio, more quietly the gears will operate.

Questions

1. The pitch circle diameter of the smaller of the two spur wheels which mesh externally and have involute teeth is 100 mm. The number of teeth are 16 and 32. The pressure angle is 20° and the addendum is 0.32 of the circular pitch. Find the length of the path of contact of the pair of teeth.

[Ans. 29.36 mm]

2. A pair of gears, having 40 and 30 teeth respectively are of 25° involute form. The addendum length is 5 mm and the module pitch is 2.5 mm. If the smaller wheel is the driver and rotates at 1500 r.p.m., find the velocity of sliding at the point of engagement and at the point of disengagement.

[Ans. 2.8 m/s ; 2.66 m/s]

3. Two gears of module 4mm have 24 and 33 teeth. The pressure angle is 20° and each gear has a standard addendum of one module. Find the length of arc of contact and the maximum velocity of sliding if the pinion rotates at 120 r.p.m.

[Ans. 20.58 mm ; 0.2147 m/s]

4. The number of teeth in gears 1 and 2 are 60 and 40 ; module = 3 mm ; pressure angle = 20° and addendum = 0.318 of the circular pitch. Determine the velocity of sliding when the contact is at the tip of the teeth of gear 2 and the gear 2 rotates at 800 r.p.m.

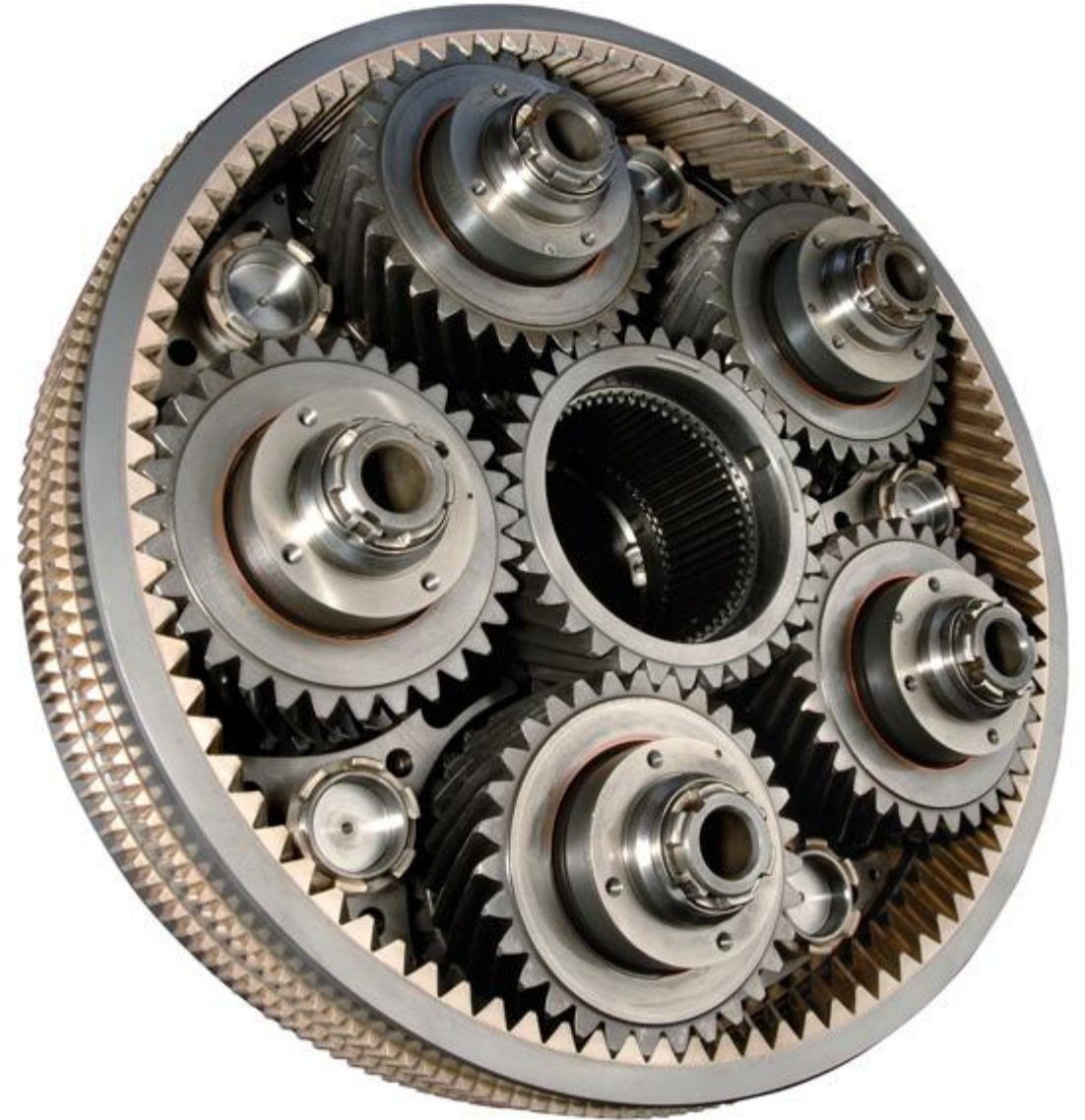
[Ans. 1.06 m/s]

5. Two spur gears of 24 teeth and 36 teeth of 8 mm module and 20° pressure angle are in mesh. Addendum of each gear is 7.5 mm. The teeth are of involute form. Determine : 1. the angle through which the pinion turns while any pair of teeth are in contact, and 2. the velocity of sliding between the teeth when the contact on the pinion is at a radius of 102 mm. The speed of the pinion is 450 r.p.m.

[Ans. 20.36° , 1.16 m/s]

Gear Trains

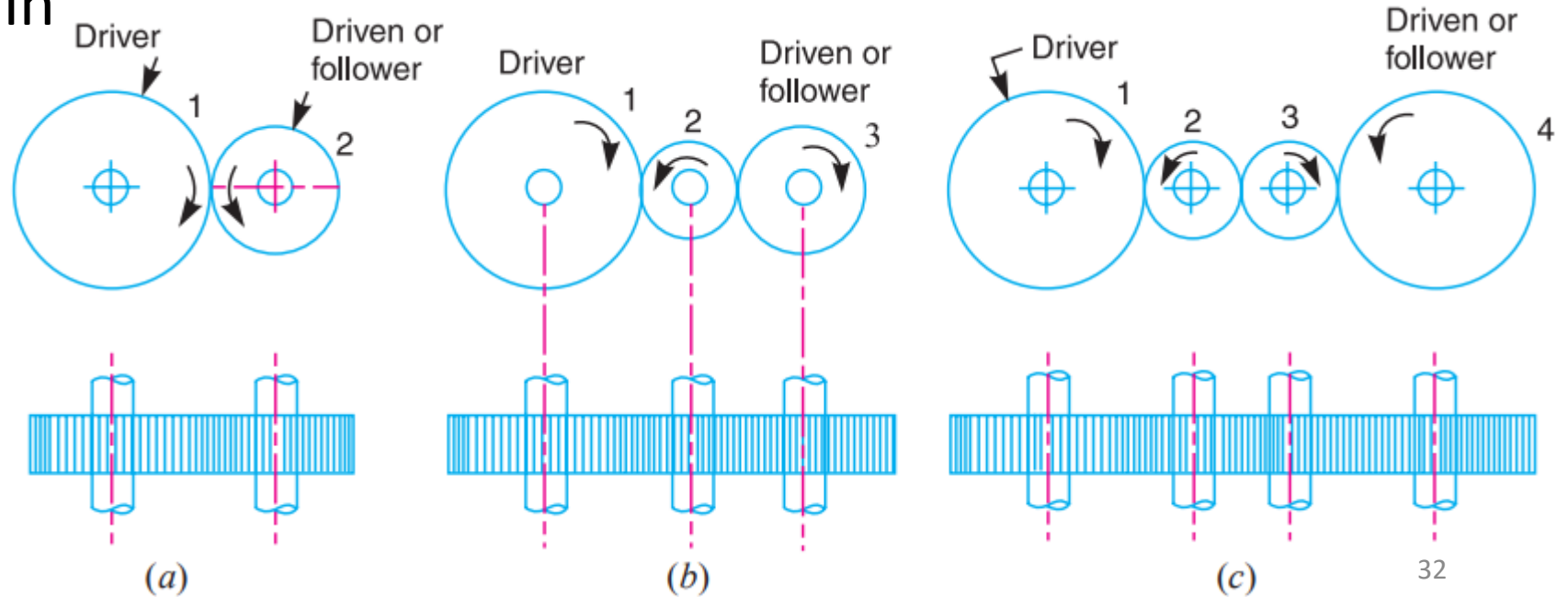
Section 2



Types of Gear Trains

Following are the different types of gear trains, depending upon the arrangement of wheels :

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear



Let;

N_1 = Speed of gear 1 (or driver) in r.p.m.

N_2 = Speed of gear 2 (or driven or follower) in r.p.m.

T_1 = Number of teeth on gear 1

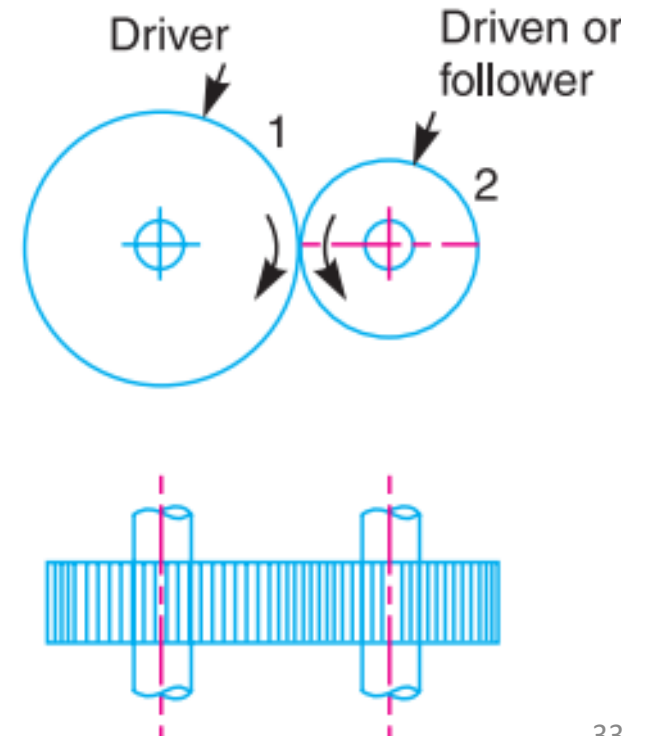
T_2 = Number of teeth on gear 2.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

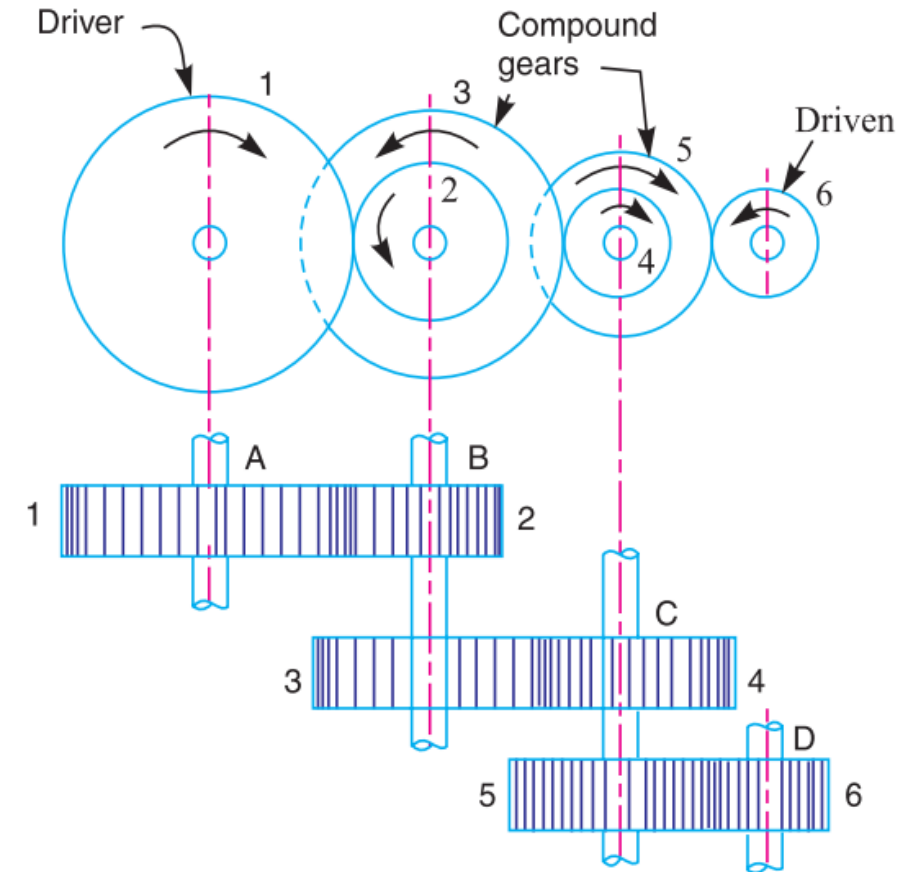


Compound Gear Trains

$$\begin{aligned}\text{Speed ratio} &= \frac{\text{Speed of the first driver}}{\text{Speed of the last driven or follower}} \\ &= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the driven}}\end{aligned}$$

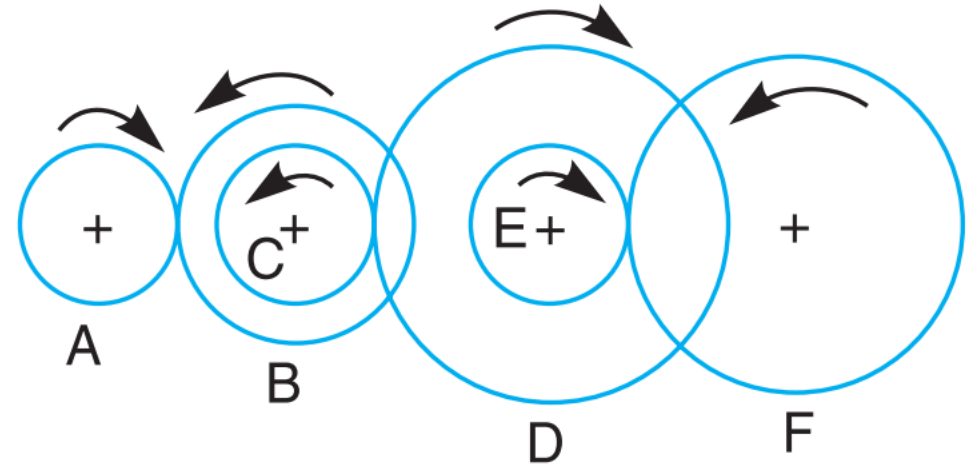
$$\begin{aligned}\text{Train value} &= \frac{\text{Speed of the last driven or follower}}{\text{Speed of the first driver}} \\ &= \frac{\text{Product of the number of teeth on the drivers}}{\text{Product of the number of teeth on the driven}}\end{aligned}$$

$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of no. of teeth on drivers}}{\text{Product of no. of teeth on driven}}$$



Example

The gearing of a machine tool is shown in Figure. The motor shaft is connected to gear A and rotates at 975 r.p.m. The gear wheels B, C, D and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F ? The number of teeth on each gear are as given below :



Gear	A	B	C	D	E	F
No. of teeth	20	50	25	75	26	65

$$\text{Speed Ratio} = \frac{N_A}{N_F} = \frac{\text{Product of teeth on driven}}{\text{Product of teeth on drivers}}$$

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E}$$

$$\frac{975}{N_F} = \frac{50 \times 75 \times 6}{20 \times 25 \times 26}$$

$$\frac{975}{N_F} = 18.75$$

$$N_F = 52 \text{ r. p. m.}$$

<i>Driver</i>	<i>Driven</i>
A	B
C	D
E	F

<i>Gear</i>	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>No. of teeth</i>	20	50	25	75	26	65

Design of Spur Gears

The spur gears are to be designed for the given velocity ratio and distance between the center of their shafts.

Let

- x = Distance between the centres of two shafts,
- N_1 = Speed of the driver,
- T_1 = Number of teeth on the driver,
- d_1 = Pitch circle diameter of the driver,
- N_2 , T_2 and d_2 = Corresponding values for the driven or follower, and
- p_c = Circular pitch.

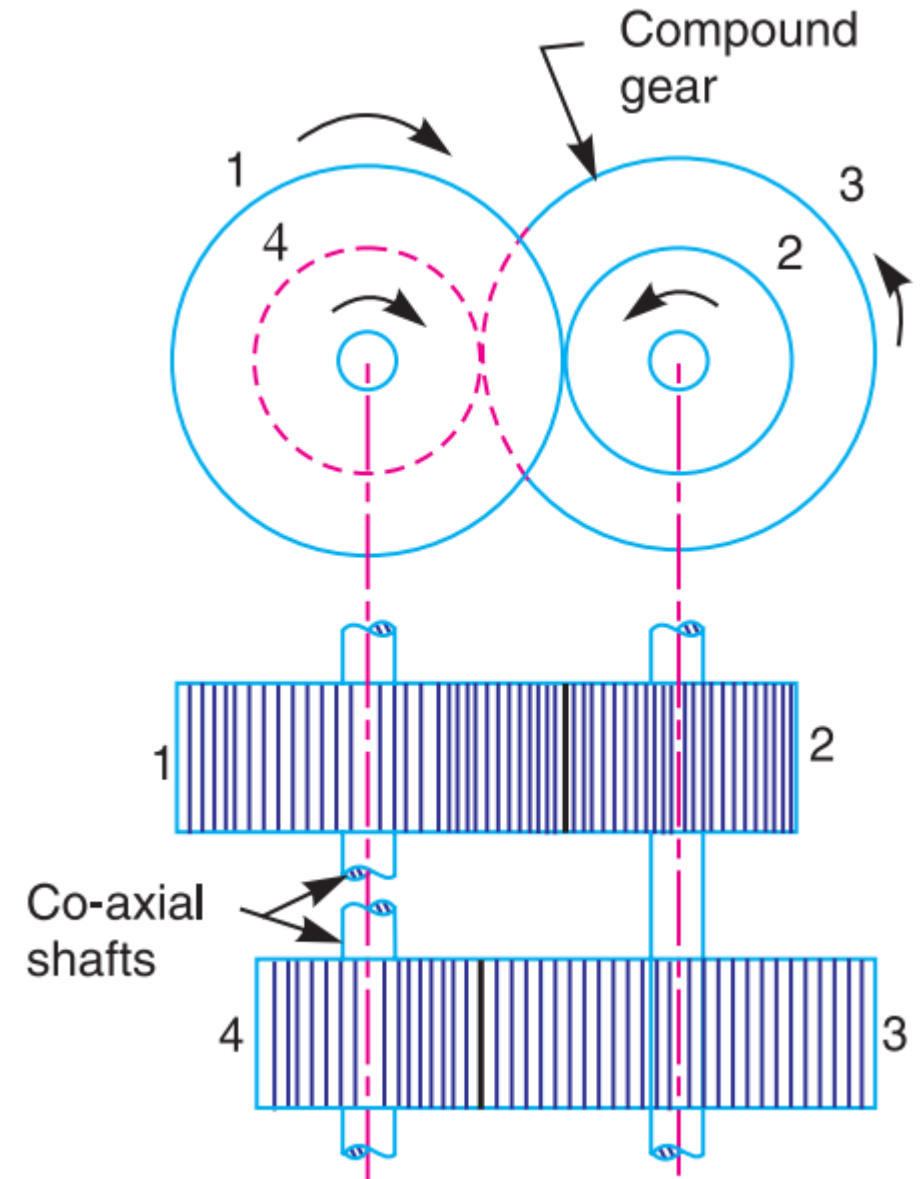
$$x = \frac{d_1 + d_2}{2}$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

Reverted Gear Train

When the axes of the first gear and the last gear are co-axial, then the gear train is known as ***reverted gear train***.

Used in speed reducers, clocks and machine tools.



T_1 = Number of teeth on gear 1,

r_1 = Pitch circle radius of gear 1,

N_1 = Speed of gear 1 in r.p.m.

T_2, T_3, T_4 = Number of teeth on respective gears,

r_2, r_3, r_4 = Pitch circle radii of respective gears,

N_2, N_3, N_4 = Speed of respective gears in r.p.m.

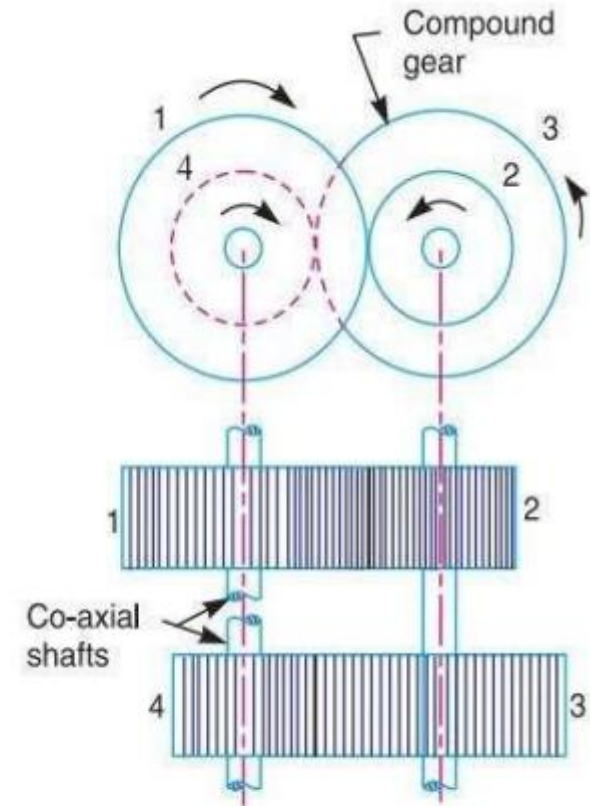
Center distance between the shafts

$$r_1 + r_2 = r_3 + r_4$$

Assume modules of all gears to be same

$$T_1 + T_2 = T_3 + T_4$$

Speed ratio $\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$



Example

Input shaft A and the output shaft B are coincident and are geared together through a parallel counter shaft C. The wheels connecting shafts A and C have a module of 2 mm and those connecting C and B have a module of 3.5 mm. Speed of output shaft has to be less than 1/10 that of shaft A. if the two pinions have each 24 teeth, determine;

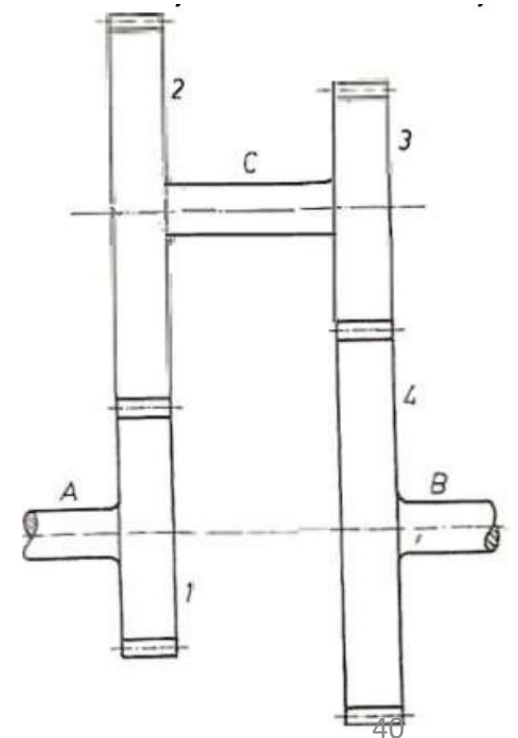
- Suitable teeth for other gear wheels
- Actual speed ratio
- The corresponding distance of shaft C from A.

Useful equations

$$m = \frac{D}{T}$$

$$p_c = \frac{\pi D}{T} = \pi m$$

$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivers}}{\text{Product of number of teeth on driven}}$$



- i) *Module of gears between A and C: $m_1=2$ mm*
Module of gears between C and B: $m_2=3.5$ mm
Number of teeth on both pinions (on A and C): $T_1=T_3=24$
Required speed ratio: Output speed $< 1/10$ of input speed

Let:

Gear 1: on shaft A (driver), pinion, $T_1=24$, $m_1=2$

Gear 2: on shaft C (driven by A), wheel, T_2 , $m_1=2$

Gear 3: on shaft C (driver for B), pinion, $T_3=24$, $m_2=3.5$

Gear 4: on shaft B (driven), wheel, T_4 , $m_2=3.5$

Gear Pair: 1 (driver) to 2(driven) and 3(driver) to 4(driven)

$$\text{Speed Ratio} = \frac{N_A}{N_F} = \frac{\text{Product of teeth on driven}}{\text{Product of teeth on drivers}}$$

$$\text{Speed Ratio} = \frac{T_2 T_4}{T_1 T_3} > 10$$

$$\frac{T_2 T_4}{24 \times 24} > 10$$

$$T_2 T_4 > 5760 \rightarrow \textcircled{1}$$

We know that circular pitch,

$$p_c = \frac{2\pi r}{T} = \pi m \quad \text{or} \quad r = \frac{m.T}{2}, \text{ where } m \text{ is the module.}$$

$$\therefore r_1 = \frac{m.T_1}{2} ; r_2 = \frac{m.T_2}{2} ; r_3 = \frac{m.T_3}{2} ; r_4 = \frac{m.T_4}{2}$$

$$r_1 + r_2 = r_3 + r_4$$

$$\frac{m_1 T_1}{2} + \frac{m_1 T_2}{2} = \frac{m_2 T_3}{2} + \frac{m_2 T_4}{2}$$

$$2(24 + T_2) = 3.5(24 + T_4)$$

$$48 + 2 T_2 = 84 + 3.5 T_4$$

$$2 T_2 - 3.5 T_4 = 36 \rightarrow \textcircled{2}$$

$$T_2 = \frac{5761}{T_4} \rightarrow \text{from equation 1}$$

$$T_2 = 110$$

$$T_4 = 53$$

ii)

$$\text{Speed Ratio} = \frac{\text{Product of teeth on driven}}{\text{Product of teeth on drivers}}$$

$$\text{Speed Ratio} = \frac{53 \times 110}{24 \times 24}$$

$$\text{Actual Speed Ratio} = \mathbf{10.1215}$$

iii)

$$d = r_1 + r_2$$

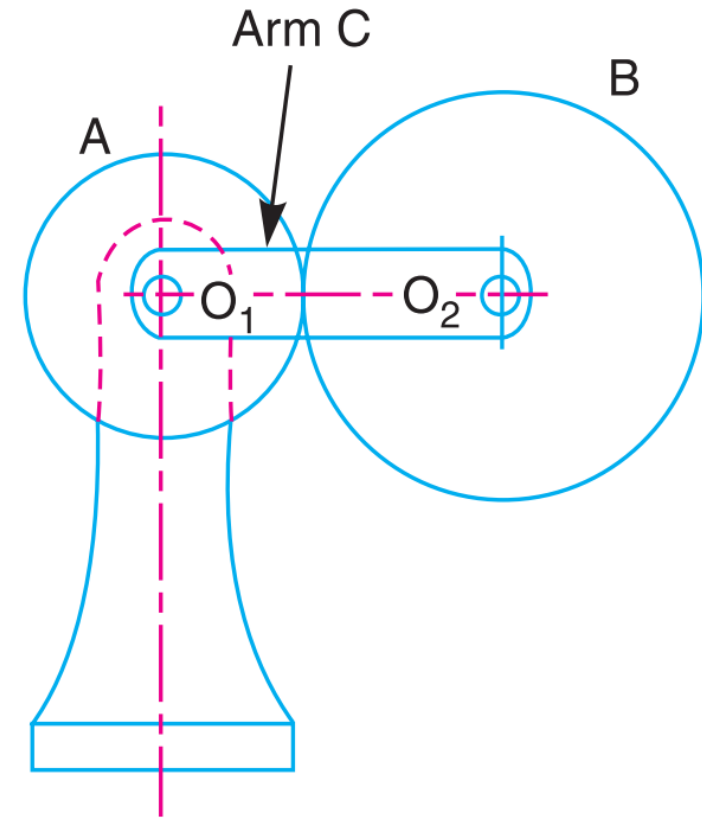
$$d = \frac{m_1 T_1}{2} + \frac{m_1 T_2}{2}$$

$$d = T_1 + T_2$$

$$d = 110 + 24 = \mathbf{134\text{mm}}$$

Epicyclic Gear Trains

If the axes of the shafts, the gears are mounted, may move relative to a fixed axis is known as **Simple epicyclic gear train**.

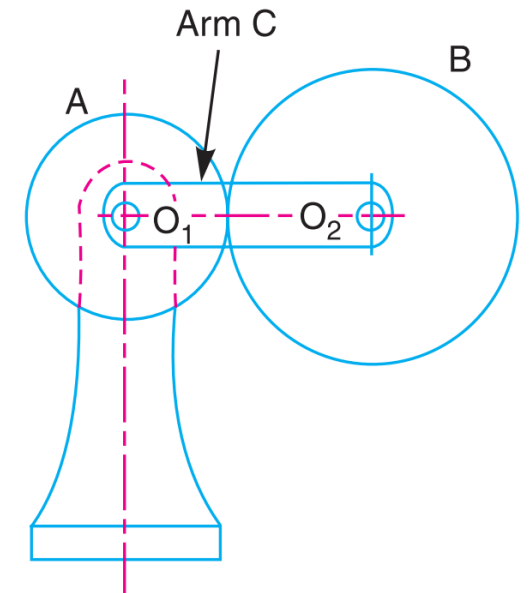


Velocity Ratios of Epicyclic Gear Train

Tabular Method

Consider an epicyclic gear train as shown in Figure.

Let T_A = Number of teeth on gear A, and T_B = Number of teeth on gear B.

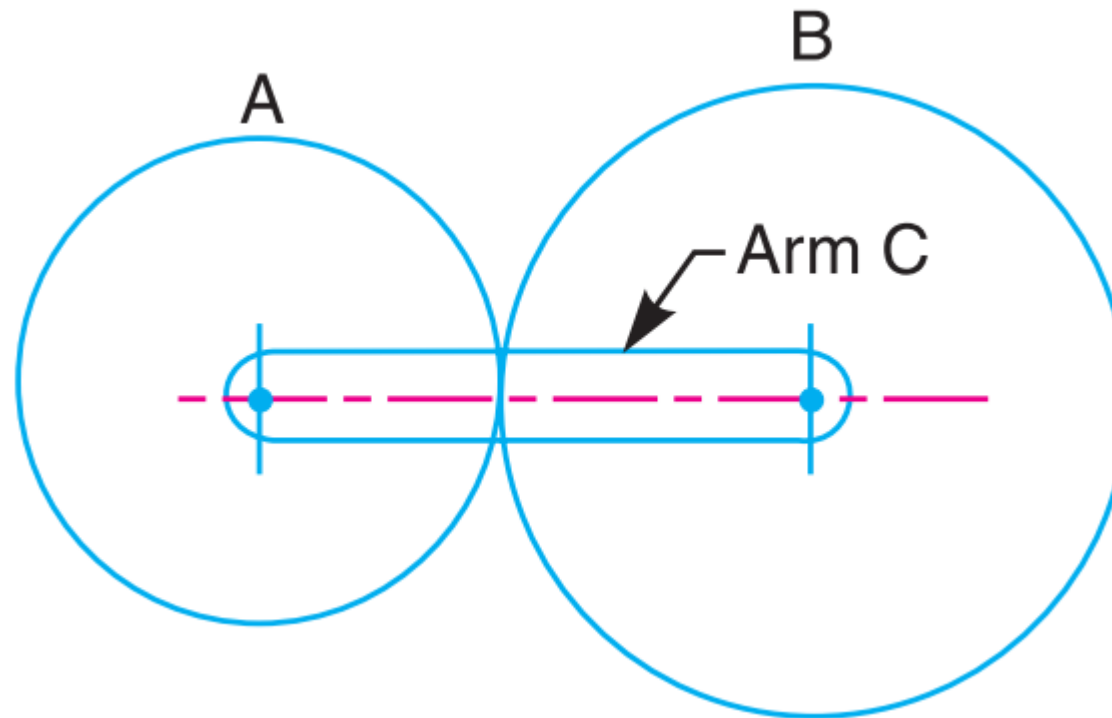


Steps for tabular method:

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution <i>i.e.</i> 1 rev. anticlockwise	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	+ x	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	+ y	+ y	+ y
4.	Total motion	+ y	x + y	$y - x \times \frac{T_A}{T_B}$

Governing Equations

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the center of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?



Tabular Method

Step No.	Condition of Motion	Revolution of elements		
		Gear A	Gear B	Arm C
1	Arm fixed, +1 for Gear A	+1	$-\frac{T_A}{T_B}$	0
2	Arm fixed, +x for Gear A	+x	$-x \frac{T_A}{T_B}$	0
3	+y for all Gears	+y	+y	+y
4	Total motion	x+y	$y - x \frac{T_A}{T_B}$	+y

If the Gear A is fixed, $x+y = 0$;

$$T_A = 36 ; T_B = 45 ;$$

$$N_C = 150 \text{ r.p.m. (anticlockwise)}$$

$$\text{So, } y = 150 \text{ r.p.m.}$$

$$x = -150 \text{ r.p.m.}$$

$$\begin{aligned}\text{Thus, speed of Gear B} &= y - x \frac{T_A}{T_B} \\ &= 150 + 150 \times \frac{36}{45} \\ &= \mathbf{+270 \text{ r.p.m. (Anticlockwise)}}$$

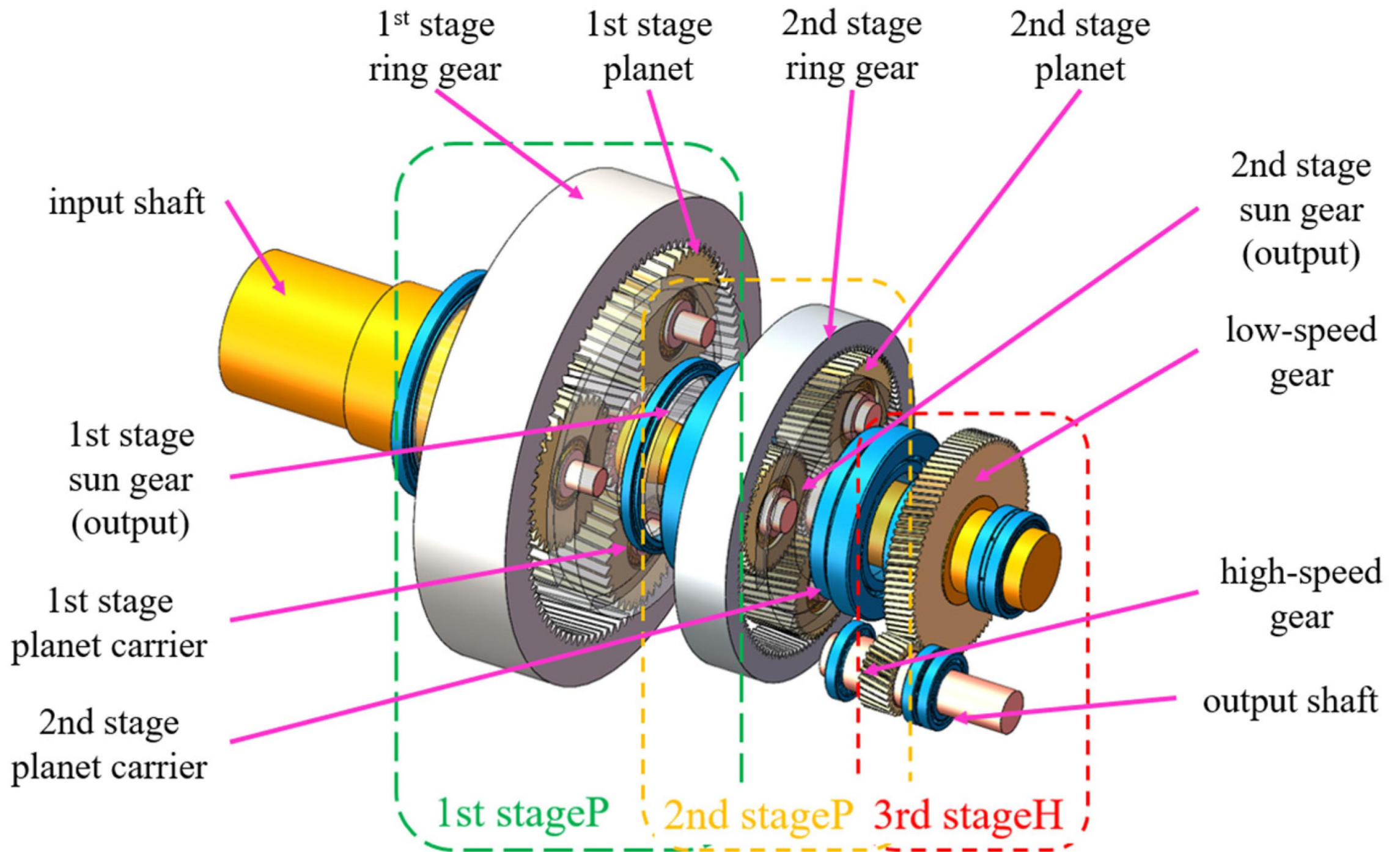
Speed of gear B when gear A makes 300 r.p.m. clockwise;

$$x+y = -300$$

$$y = 150$$

$$x = -450$$

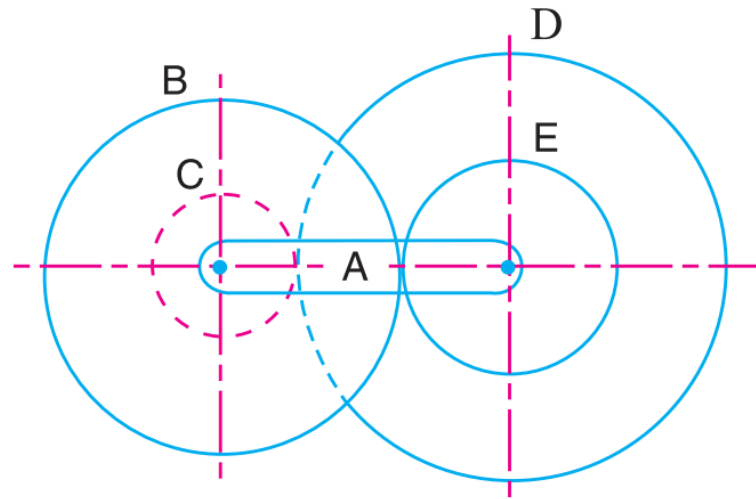
$$\begin{aligned}\text{Thus, speed of Gear B} &= y - x \frac{T_A}{T_B} \\ &= 150 + 450 \times \frac{36}{45} \\ &= \mathbf{+510 \text{ r.p.m. (Anticlockwise)}}$$



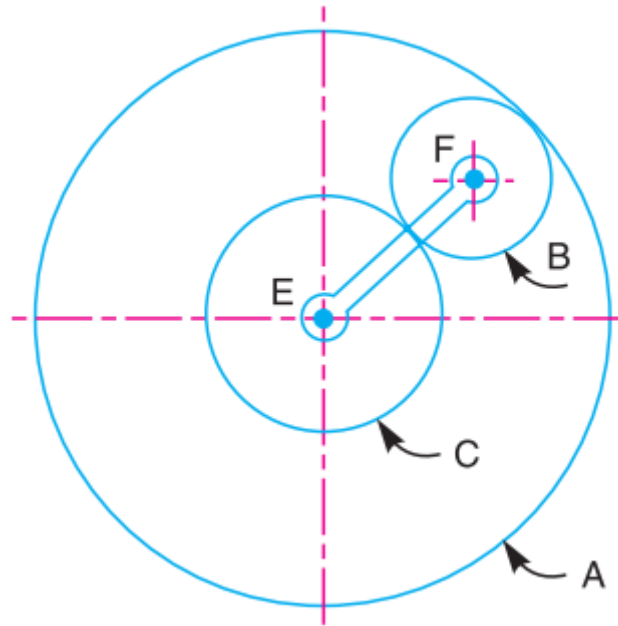


Practice Questions

In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.

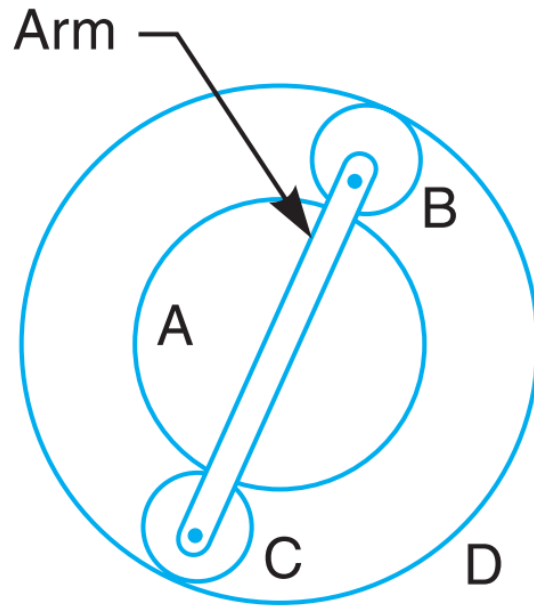


An epicyclic gear consists of three gears A, B and C as shown in Figure. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.



An epicyclic train of gears is arranged as shown in Figure. How many revolutions does the arm, to which the pinions B and C are attached, make :

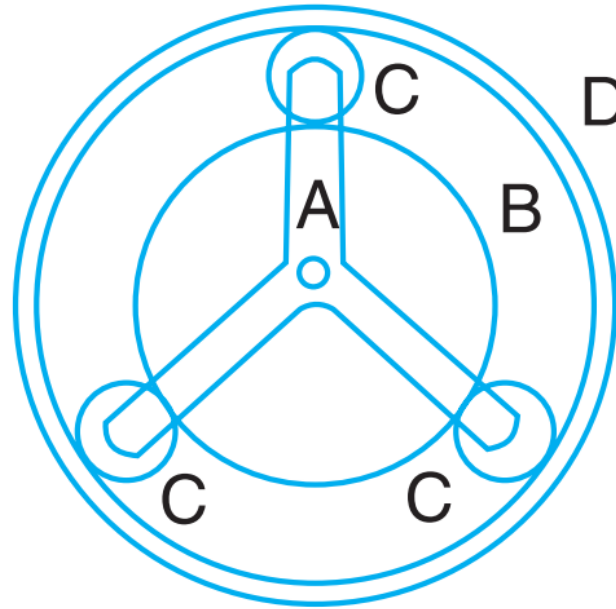
- 1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*
 - 2. when A makes one revolution clockwise and D is stationary ?*
- The number of teeth on the gears A and D are 40 and 90 respectively.*



In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$.

- 1. Sketch the arrangement*
- 2. Find the number of teeth on A and B*
- 3. If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B*
- 4. If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise ; find the speed of wheel B.*

In an epicyclic gear of the 'sun and planet' type shown in Figure, the pitch circle diameter of the internally toothed ring is to be 224 mm and the module 4 mm. When the ring D is stationary, the spider A, which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sunwheel B for every five revolutions of the driving spindle carrying the sunwheel B. Determine suitable numbers of teeth for all the wheels



Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 r.p.m., find the speed of shaft B.

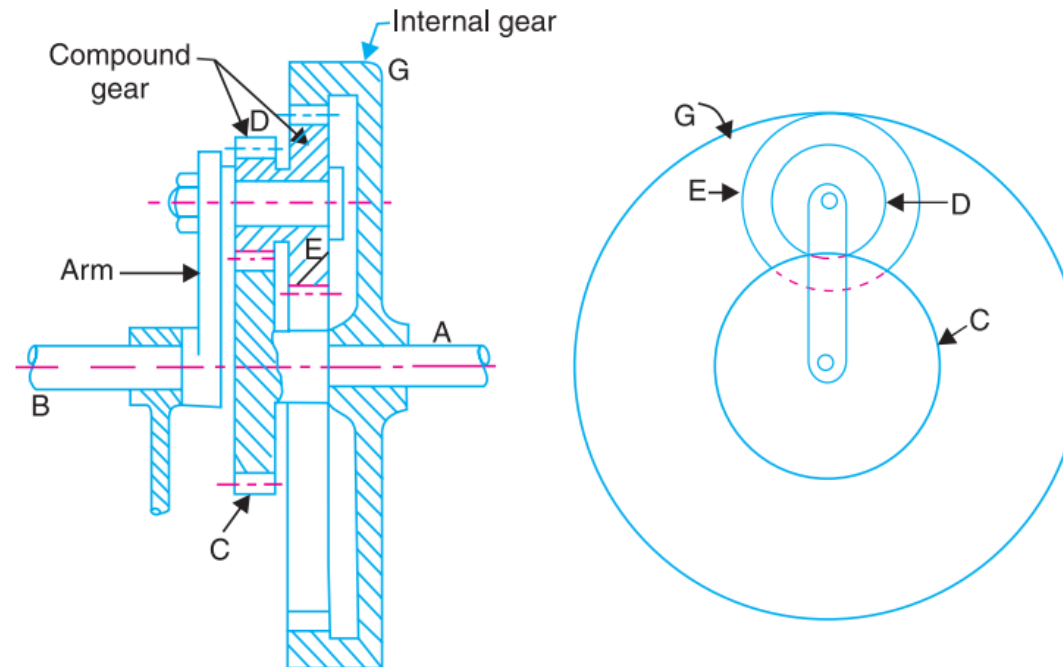
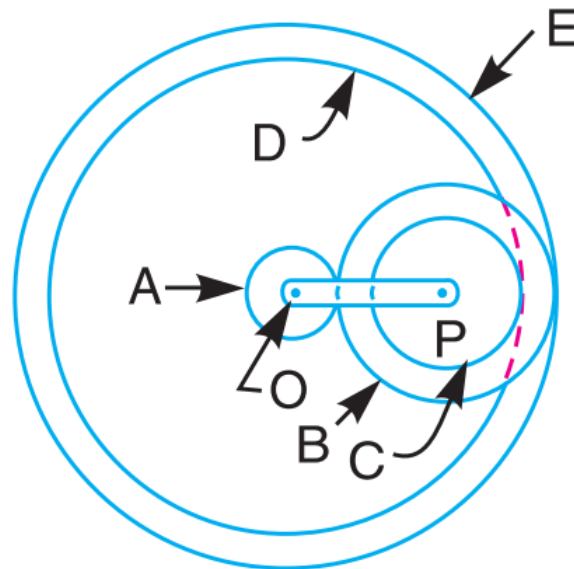


Figure shows diagrammatically a compound epicyclic gear train. Wheels A , D and E are free to rotate independently on spindle O, while B and C are compound and rotate together on spindle P, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, B has 30 teeth and C has 14 teeth cut externally. Find the number of teeth on wheels D and E which are cut internally. If the wheel A is driven clockwise at 1 r.p.s. while D is driven counter clockwise at 5 r.p.s., determine the magnitude and direction of the angular velocities of arm OP and wheel E.



An internal wheel B with 80 teeth is keyed to a shaft F. A fixed internal wheel C with 82 teeth is concentric with B. A compound wheel D-E gears with the two internal wheels; D has 28 teeth and gears with C while E gears with B. The compound wheels revolve freely on a pin which projects from a disc keyed to a shaft A co-axial with F. If the wheels have the same pitch and the shaft A makes 800 r.p.m., what is the speed of the shaft F ? Sketch the arrangement.

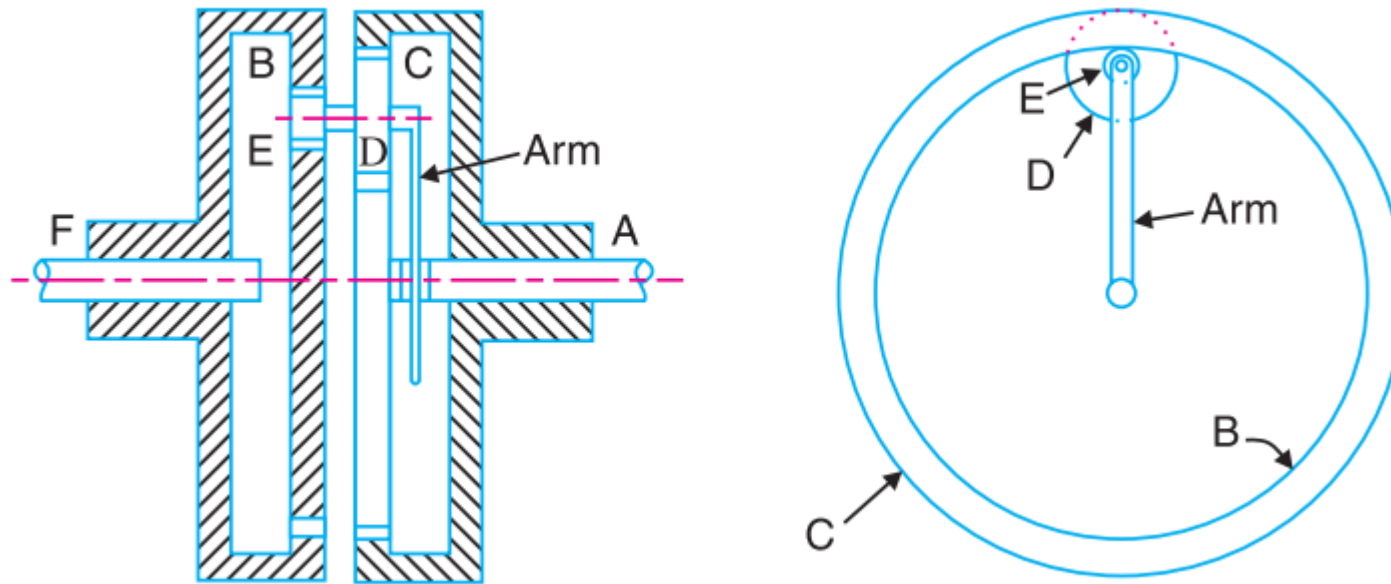
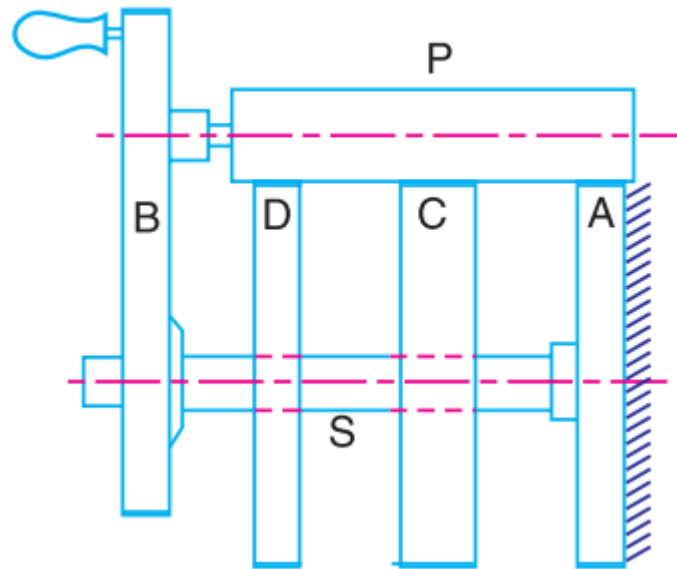


Figure shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is, therefore, stationary. The arm B and gears C and D are free to rotate on the shaft S. Gears A, C and D have 100, 101 and 99 teeth respectively. The planet gear P has 20 teeth. The pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of the arm B.



In a gear train, as shown in Fig. 13.23, gear B is connected to the input shaft and gear F is connected to the output shaft. The arm A carrying the compound wheels D and E, turns freely on the output shaft. If the input speed is 1000 r.p.m. counter-clockwise when seen from the right, determine the speed of the output shaft under the following conditions : 1. When gear C is fixed, and 2. when gear C is rotated at 10 r.p.m. counter clockwise.

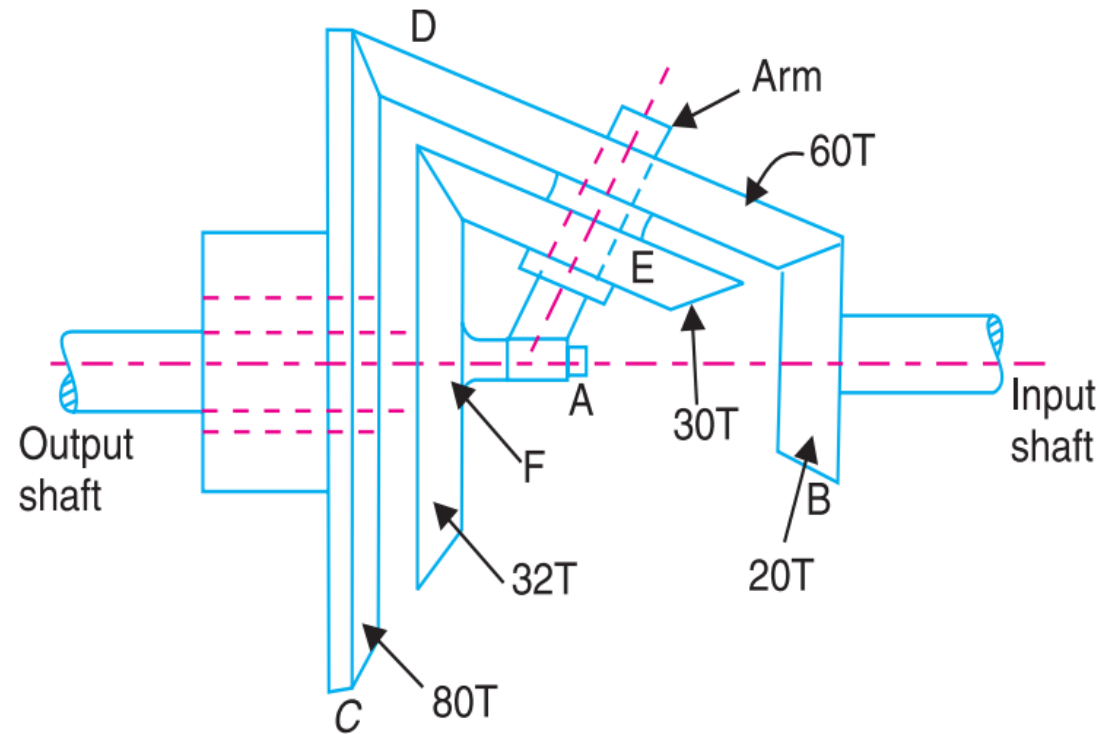
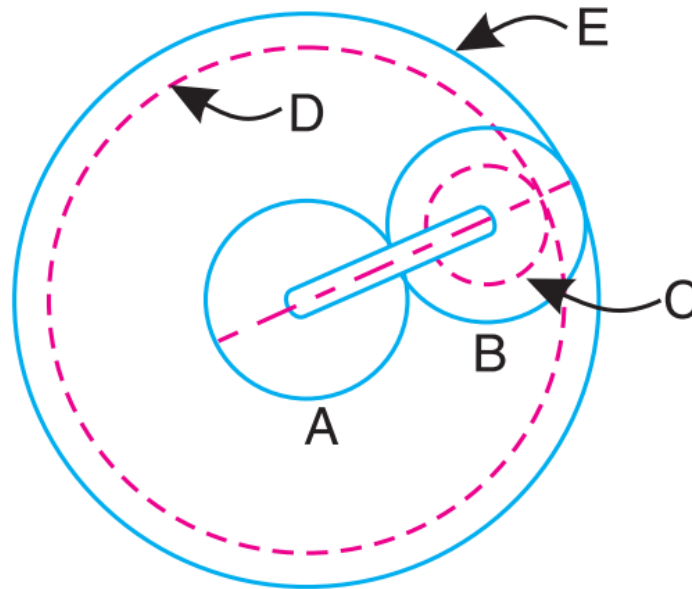
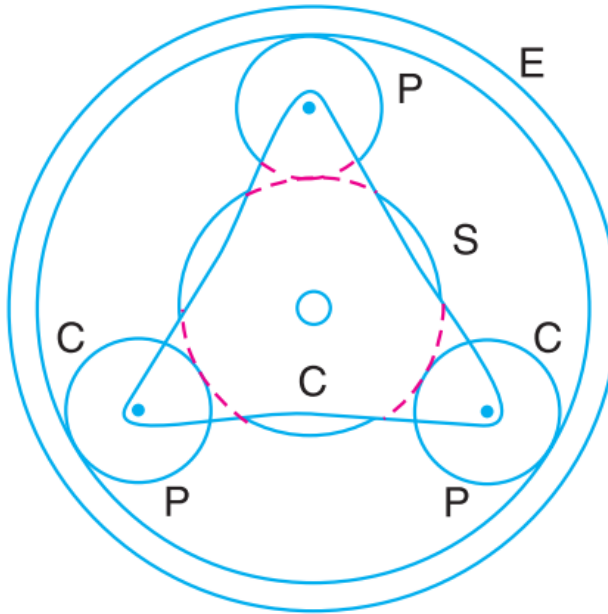


Fig. 13.26 shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.

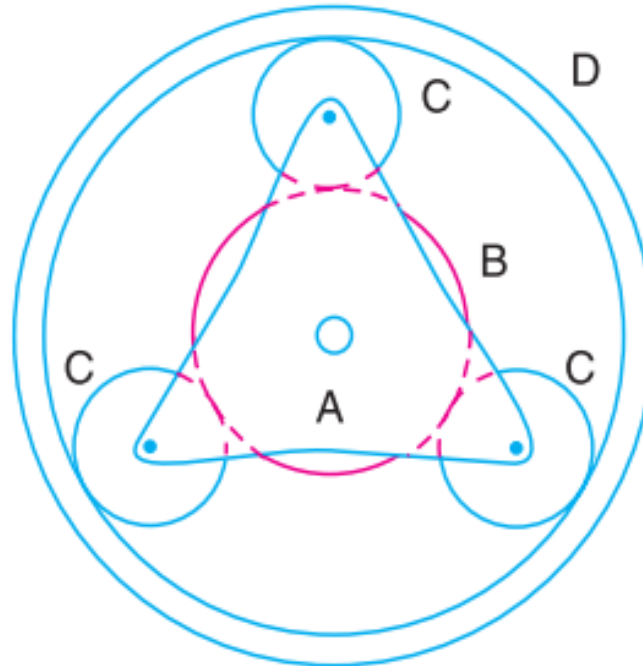


An epicyclic gear train consists of a sun wheel S , a stationary internal gear E and three identical planet wheels P carried on a star-shaped planet carrier C . The size of different toothed wheels are such that the planet carrier C rotates at $1/5$ th of the speed of the sunwheel S . The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N-m. Determine : 1. number of teeth on different wheels of the train, and 2. torque necessary to keep the internal gear stationary



In an epicyclic gear train of the 'sun and planet type' as shown in Fig. 13.41, the pitch circle diameter of the internally toothed ring D is to be 216 mm and the module 4 mm. When the ring D is stationary, the spider A, which carries three planet wheels C of equal size, is to make one revolution in the same sense as the sun wheel B for every five revolutions of the driving spindle carrying the sunwheel B. Determine suitable number of teeth for all the wheels and the exact diameter of pitch circle of the ring.

[Ans. $T_B = 14$, $T_C = 21$, $T_D = 56$; 224 mm]



References: Theory of Machines by Khurmi