Kinematics and Dynamics

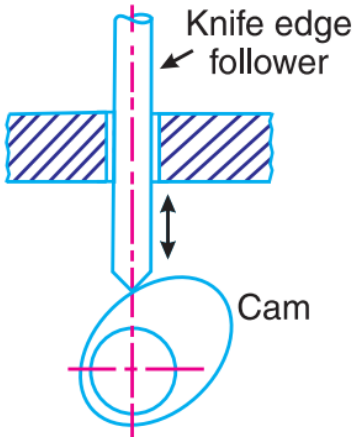
Day 05 | MP 3010

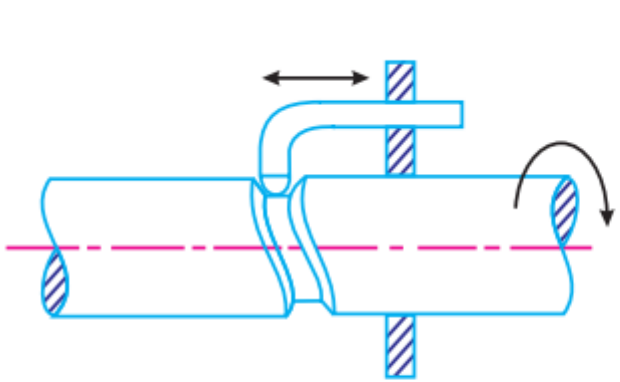
Cam and Follower
Development of Cam Profile

Cams

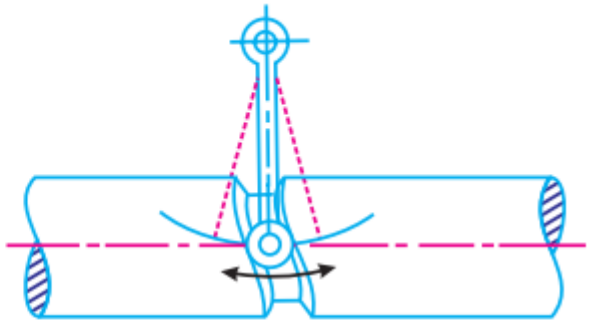
Section 1

Classification of Cams





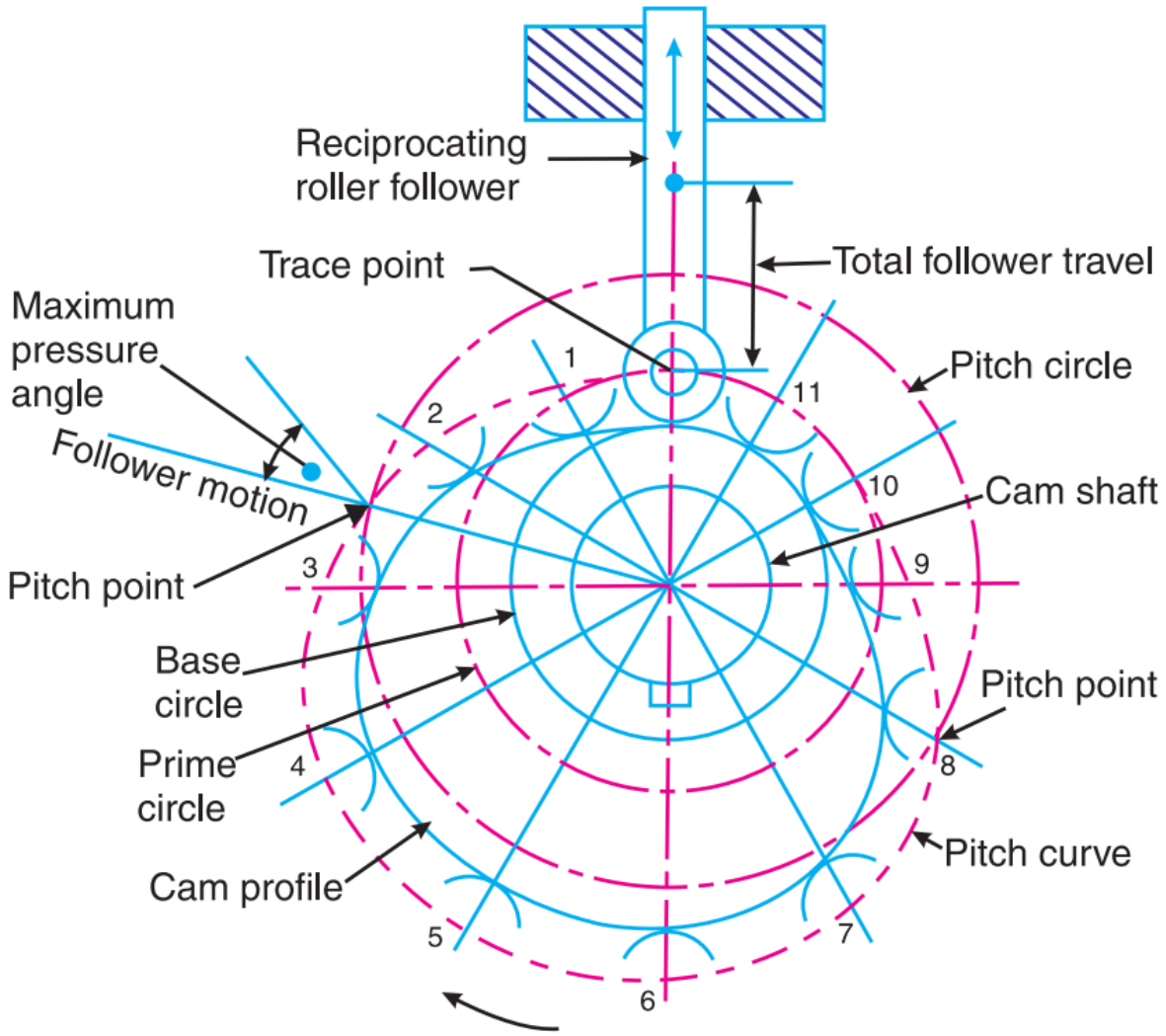
(*a*) Cylindrical cam with reciprocating follower.



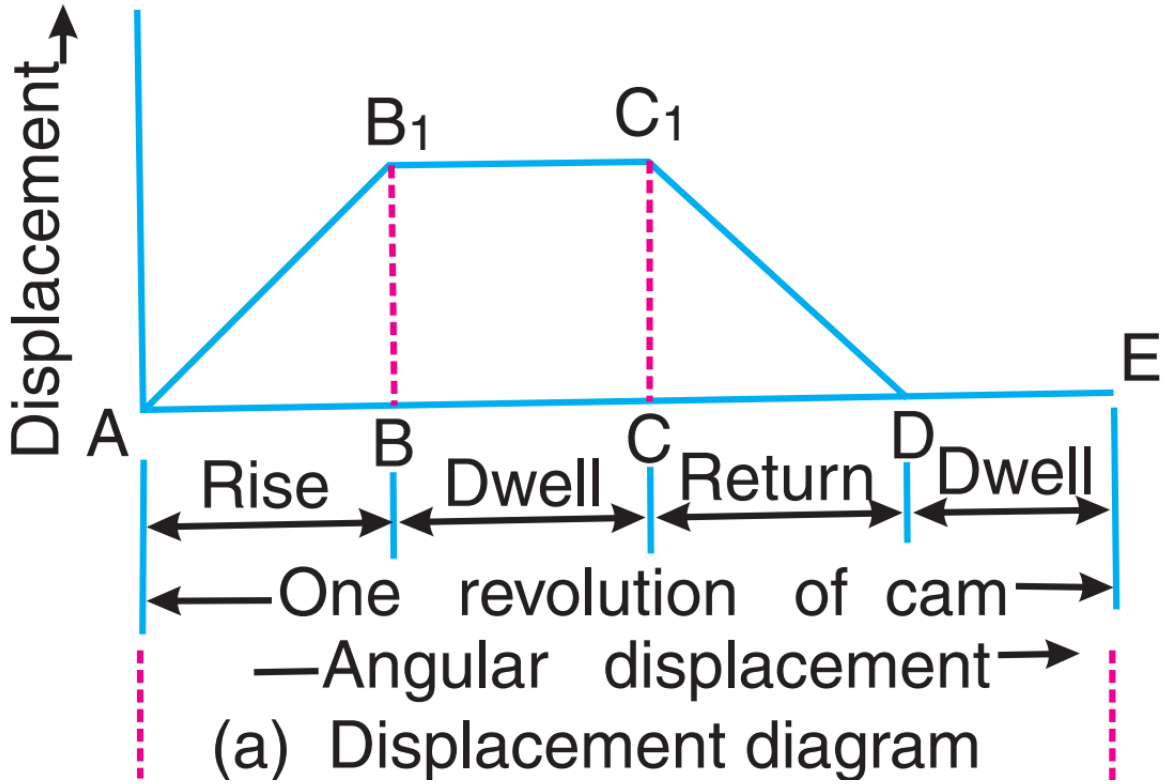
(*b*) Cylindrical cam with oscillating follower.

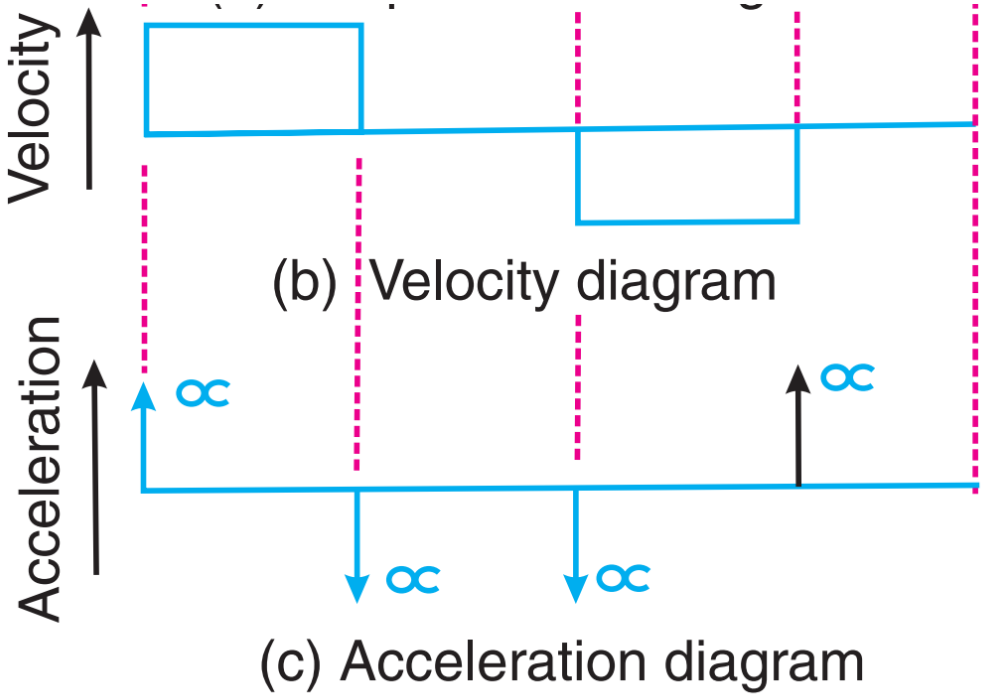
Fig. 20.2. Cylindrical cam.

Terms

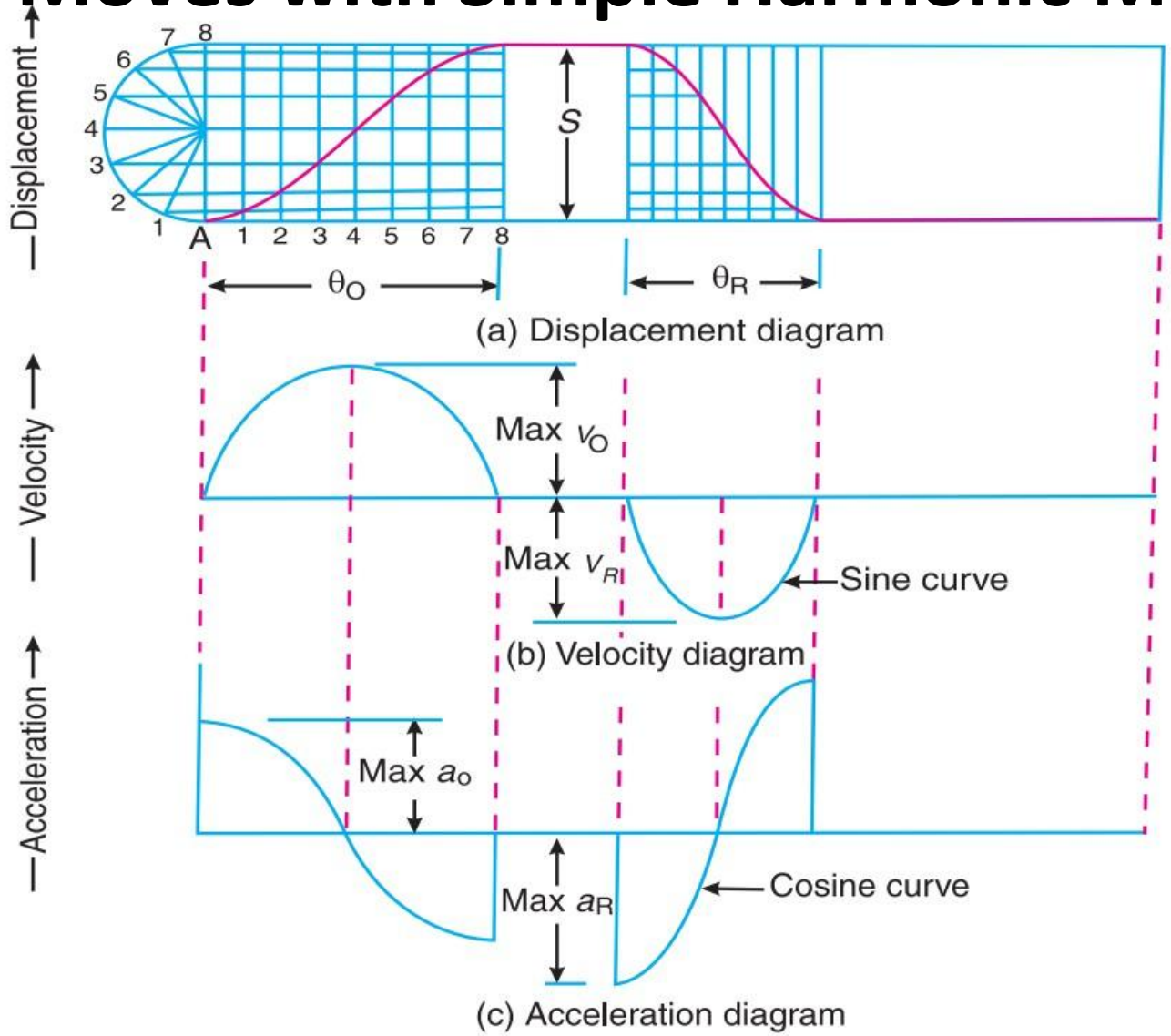

Reciprocating roller follower
Trace point
Total follower travel
Maximum pressure angle
Follower motion
Pitch circle
Cam shaft
Pitch point
Base circle
Prime circle
Pitch point
Cam profile
Pitch curve
1
2
3
4
5
6
7
8
9
10
11

Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Uniform Acceleration and Retardation





Displacement, Velocity and Acceleration Diagrams when the Follower Moves with Simple Harmonic Motion



Analysis of cam and follower motion

Let

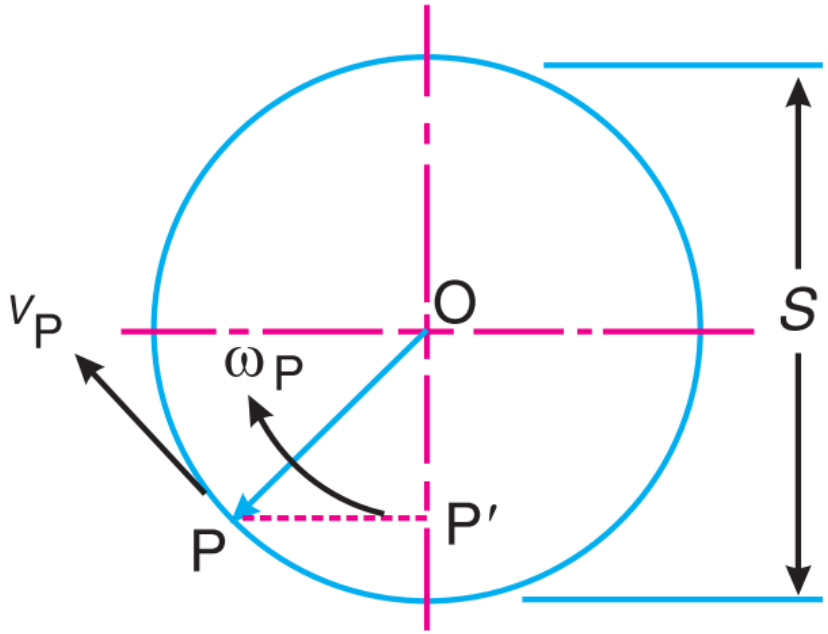
S = Stroke of the follower,

θ_O and R_θ = Angular displacement of the cam during out stroke and return stroke of the follower respectively, in radians, and

ω = Angular velocity of the cam in rad/s.

Time required for the out stroke of the follower in seconds,

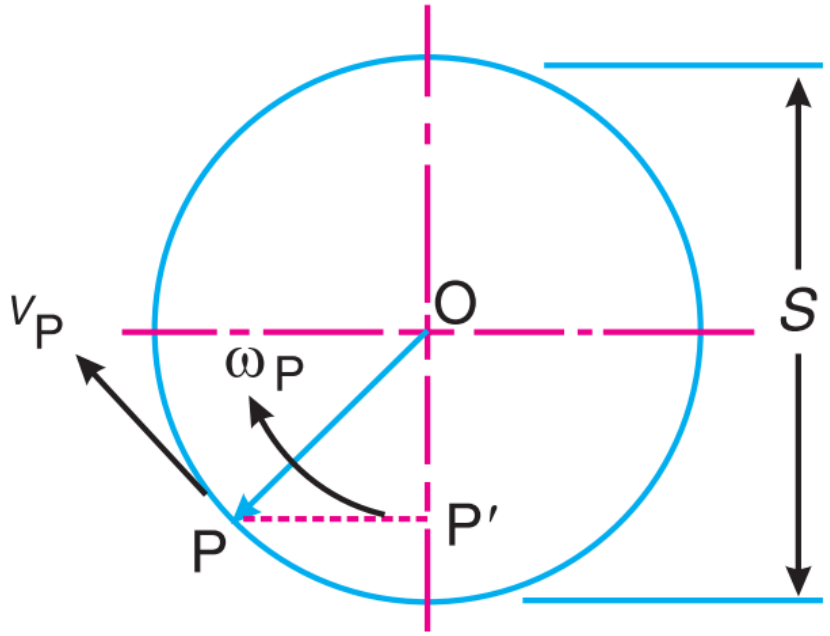
$$t_O = \theta_O / \omega$$



Peripheral Speed

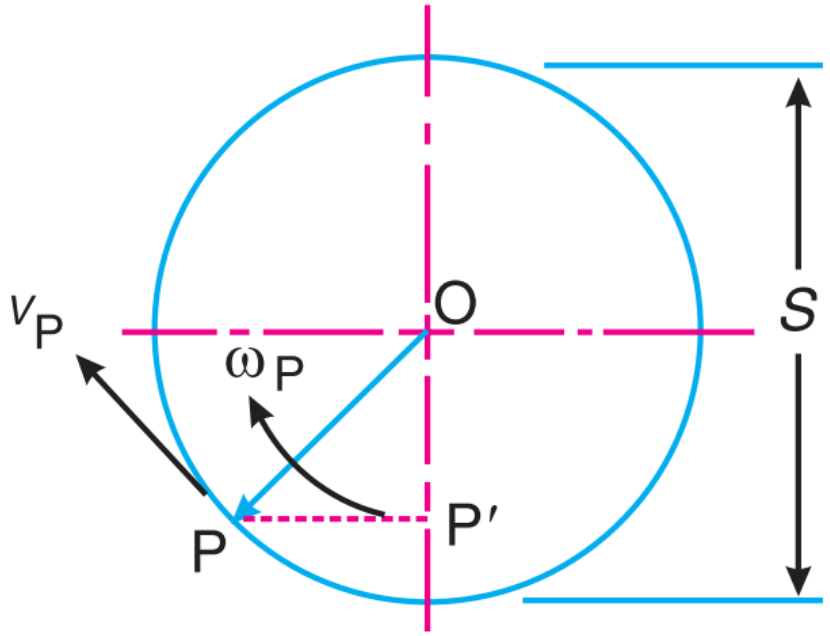
Peripheral speed of the point P',

$$v_P = \frac{\pi S}{2} \times \frac{1}{t_O} = \frac{\pi S}{2} \times \frac{\omega}{\theta_O}$$



Centripetal acceleration of the point P

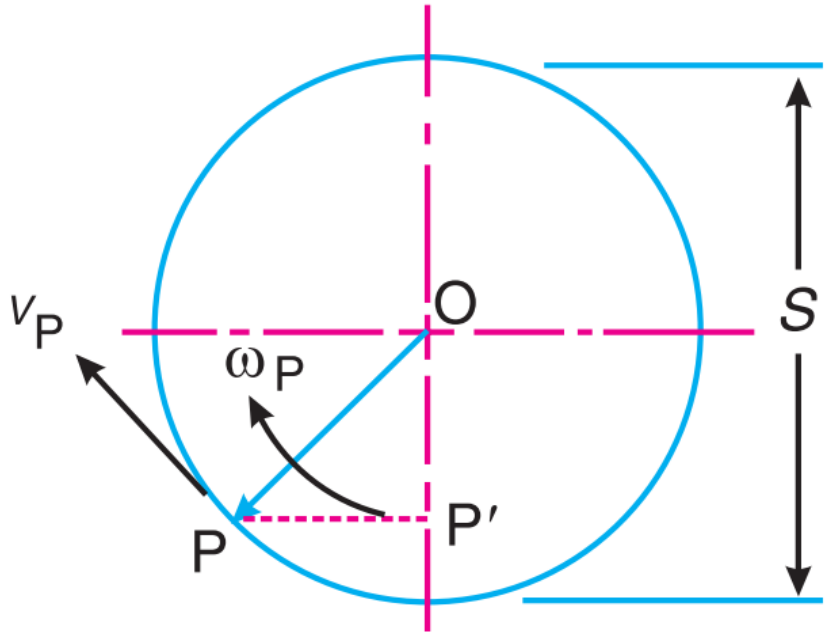
$$a_P = \frac{(v_P)^2}{OP} = \left(\frac{\pi \omega . S}{2\theta_O}\right)^2 \times \frac{2}{S} = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$



Maximum velocity and acceleration of the follower on the outstroke

$$v_O = v_P = \frac{\pi S}{2} \times \frac{\omega}{\theta_O} = \frac{\pi \omega S}{2\theta_O}$$

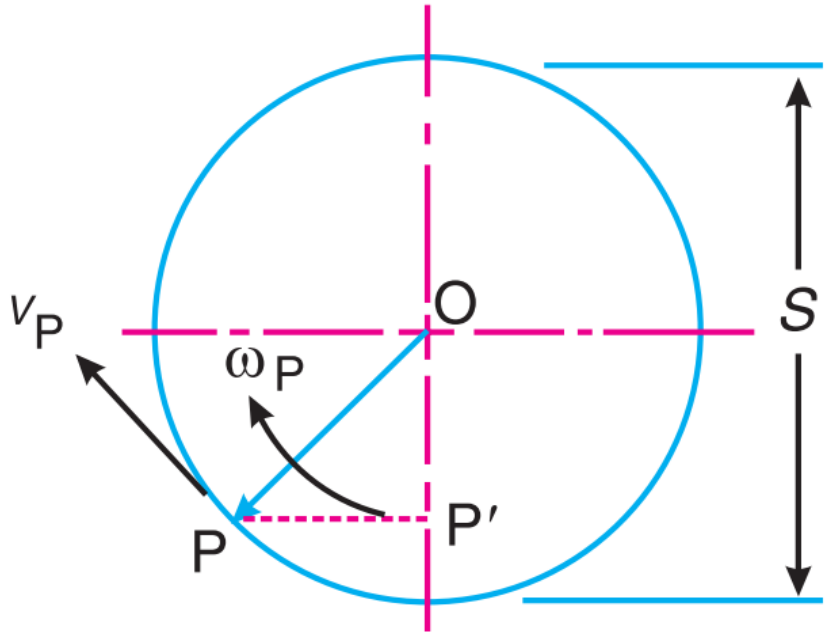
$$a_O = a_P = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2}$$



Maximum velocity and acceleration of the follower on the return stroke

$$v_R = \frac{\pi \omega . S}{2\theta_R}$$

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2}$$



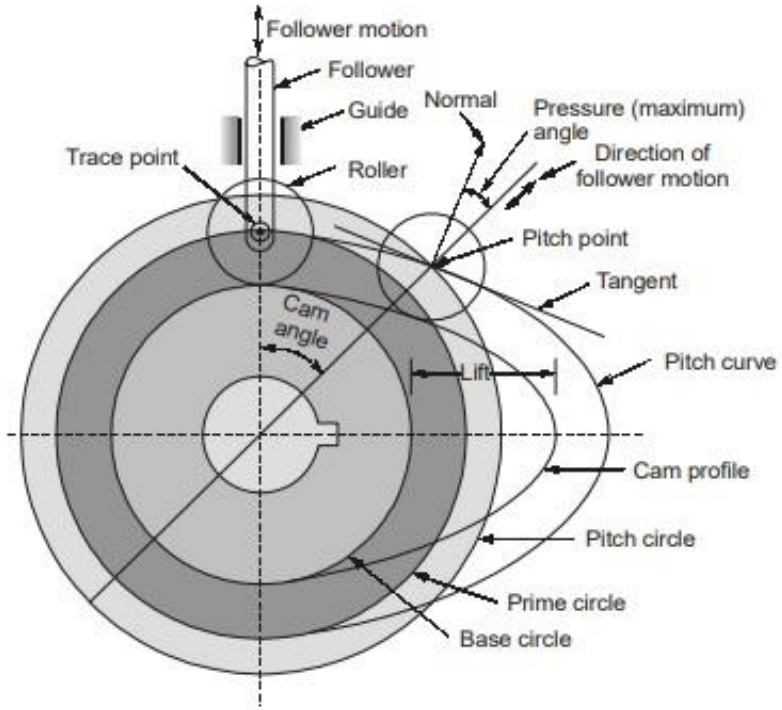


Fig. Cam Nomenclature

Development of cam profile

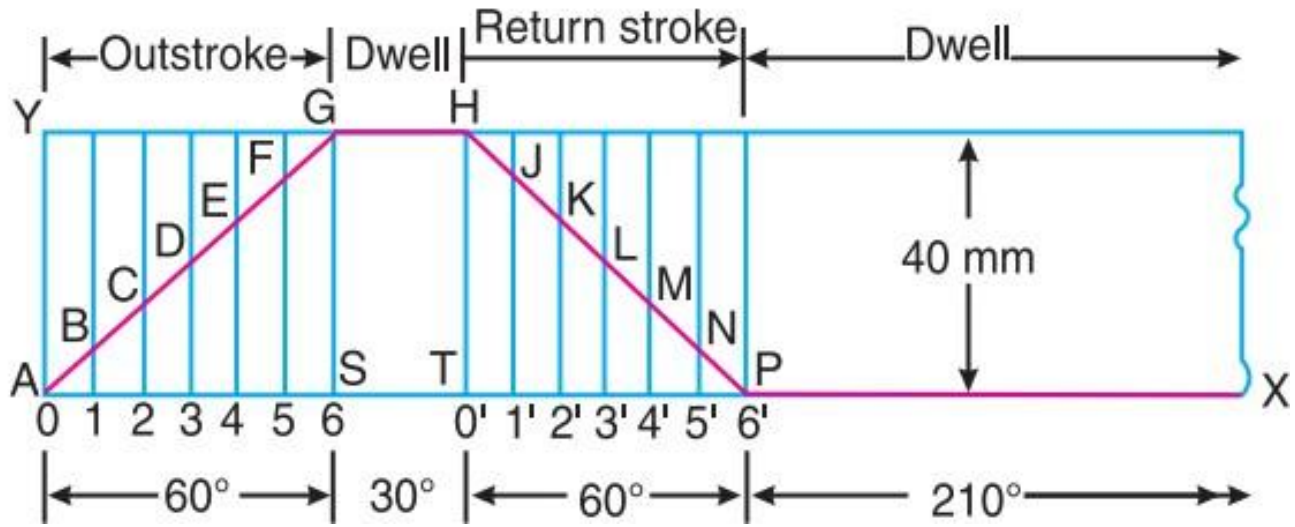
Section 2

A cam is to give the following motion to a knife-edged follower : 1. Outstroke during 60° of cam rotation ; 2. Dwell for the next 30° of cam rotation ; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

First of all, the displacement diagram, as shown in Fig. 20.10, is drawn as discussed in the following steps :

1. Draw a horizontal line $AX = 360°$ to some suitable scale. On this line, mark $AS = 60°$ to represent outstroke of the follower, $ST = 30°$ to represent dwell, $TP = 60°$ to represent return stroke and $PX = 210°$ to represent dwell.
2. Draw vertical line AY equal to the stroke of the follower (*i.e.* 40 mm) and complete the rectangle as shown in Fig. 20.10.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join AG and HP.
5. The complete displacement diagram is shown by $AGHPX$ in Fig. 20.10.

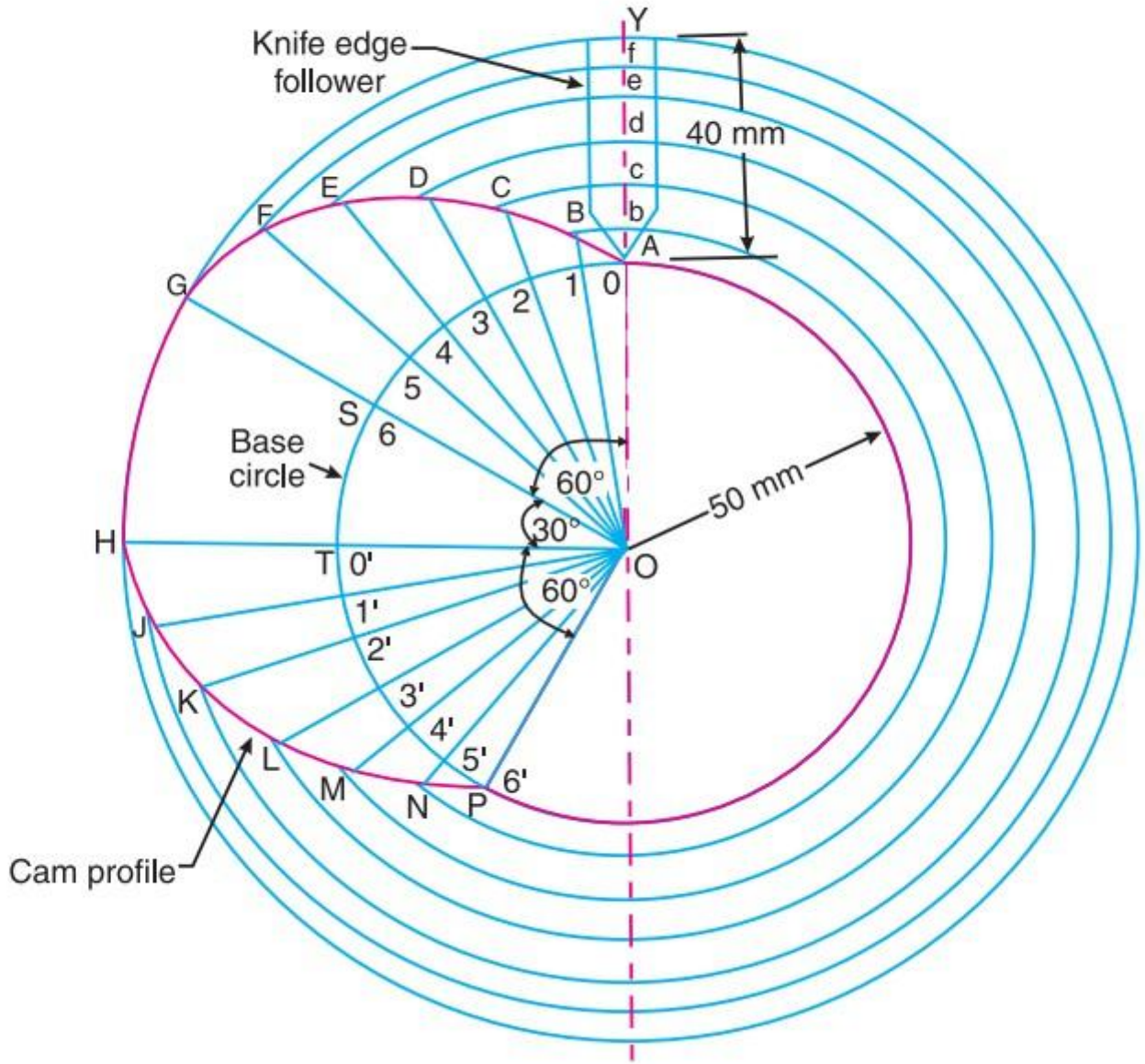
Construction



1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with O as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point A, as shown in Fig. 20.11.
3. From OA, mark angle $AOS = 60°$ to represent outstroke, angle $SOT = 30°$ to represent dwell and angle $TOP = 60°$ to represent return stroke.
4. Divide the angular displacements during outstroke and return stroke (*i.e.* angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.
5. Join the points 1, 2, 3 ...etc. and $0', 1', 2', 3'$, ... etc. with centre O and produce beyond the base circle as shown in Fig. 20.11.
6. Now set off $1B$, $2C$, $3D$ ... etc. and $0'H, 1'J$ ... etc. from the displacement diagram.
7. Join the points A, B, C,... M, N, P with a smooth curve. The curve $AGHPA$ is the complete profile of the cam.

(a) *Profile of the cam when the axis of follower passes through the axis of cam shaft*

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, as shown in Fig. 20.11, is drawn as discussed in the following steps :



1. Draw a base circle with radius equal to the minimum radius of the cam (*i.e.* 50 mm) with O as centre.
2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at A.
3. Join AO and draw an offset circle of radius 20 mm with centre O.
4. From OA, mark angle $AOS = 60°$ to represent outstroke, angle $SOT = 30°$ to represent dwell and angle $TOP = 60°$ to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (*i.e.* angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.
6. Now from the points 1, 2, 3 ... etc. and $0', 1', 2', 3'$ ... etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle as shown in Fig. 20.12.
7. Now set off $1B$, $2C$, $3D$ ... etc. and $0'H, 1'J$ ... etc. from the displacement diagram.
8. Join the points A, B, C ...M, N, P with a smooth curve. The curve $AGHPA$ is the complete profile of the cam.

(b) *Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft*

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, as shown in Fig. 20.12, is drawn as discussed in the following steps :

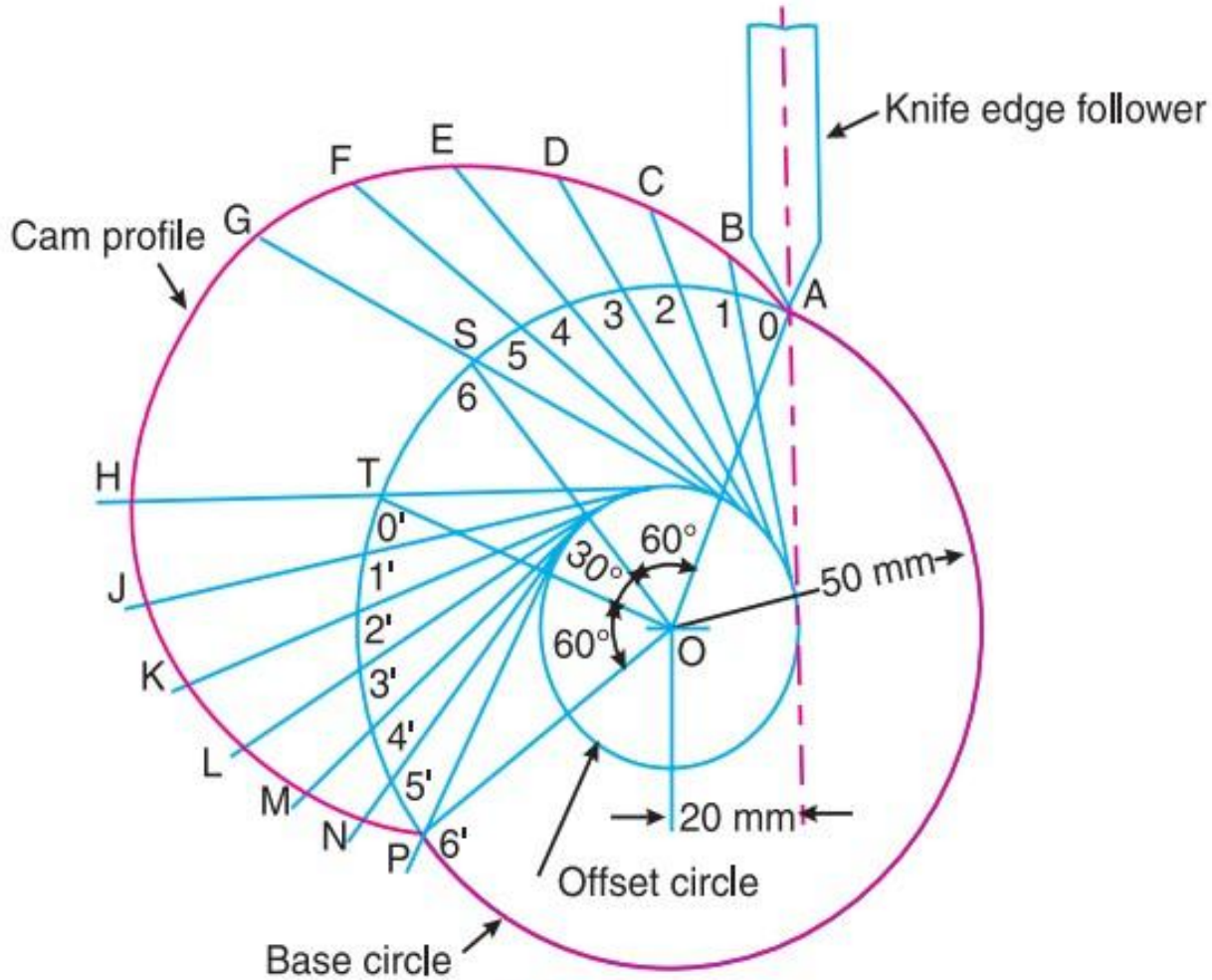


Fig. 20.12

A cam is to be designed for a knife edge follower with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion.

2. Dwell for the next 30°.

3. During the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion.

4. Dwell during the remaining 180°.

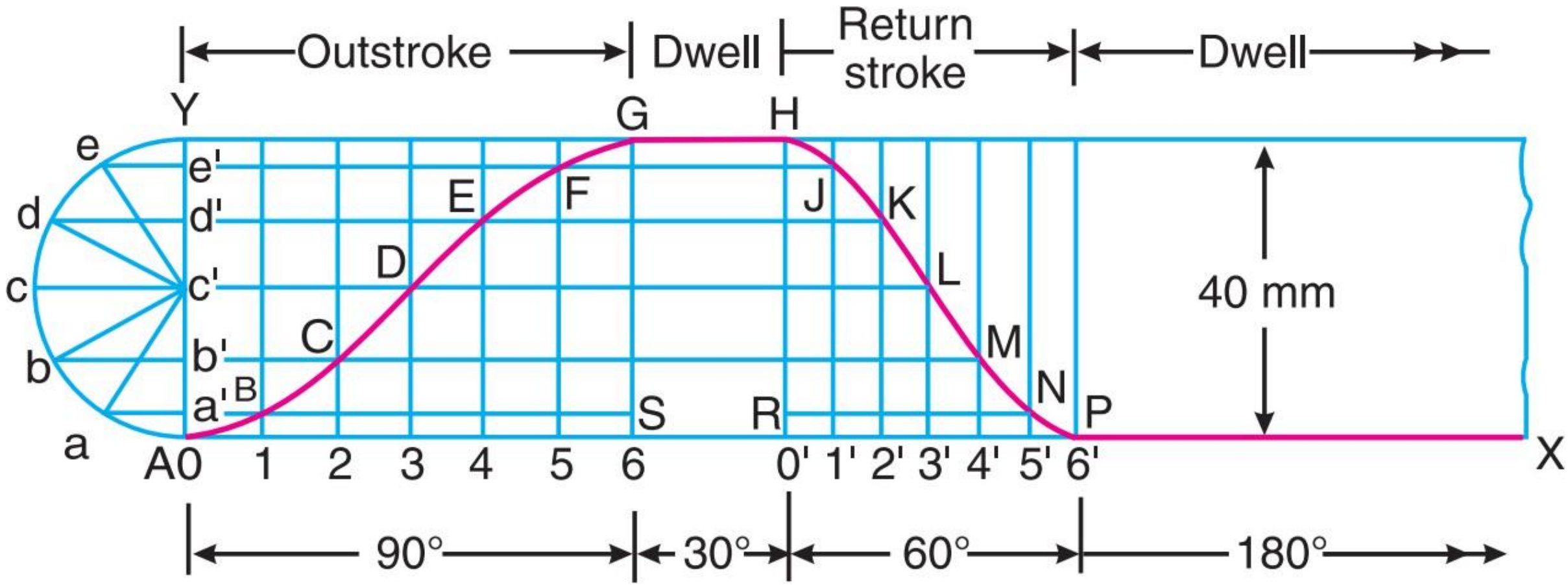
Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shaft, and

(b) the line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm.

Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

1. Draw horizontal line $AX = 360°$ to some suitable scale. On this line, mark $AS = 90°$ to represent out stroke ; $SR = 30°$ to represent dwell ; $RP = 60°$ to represent return stroke and $PX = 180°$ to represent dwell.
2. Draw vertical line $AY = 40$ mm to represent the cam lift or stroke of the follower and complete the rectangle as shown in Fig. 20.13.
3. Divide the angular displacement during out stroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with simple harmonic motion, therefore draw a semicircle with AY as diameter and divide into six equal parts.
5. From points a, b, c ... etc. draw horizontal lines intersecting the vertical lines drawn through 1, 2, 3 ... etc. and $0', 1', 2'$ ...etc. at B, C, D ... M, N, P.
6. Join the points A, B, C ... etc. with a smooth curve as shown in Fig. 20.13. This is the required displacement diagram.


Outstroke
Dwell
Return stroke
Dwell
Y
G
H
e
d
c
b
a
e'
d'
c'
b'
a'
B
C
D
E
F
J
K
L
M
N
P
S
R
40 mm
A0
1
2
3
4
5
6
0'
1'
2'
3'
4'
5'
6'
X
90°
30°
60°
180°

(a) *Profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft*

The profile of the cam when the line of stroke of the follower passes through the axis of the cam shaft, as shown in Fig. 20.14, is drawn in the similar way as is discussed in Example 20.1.

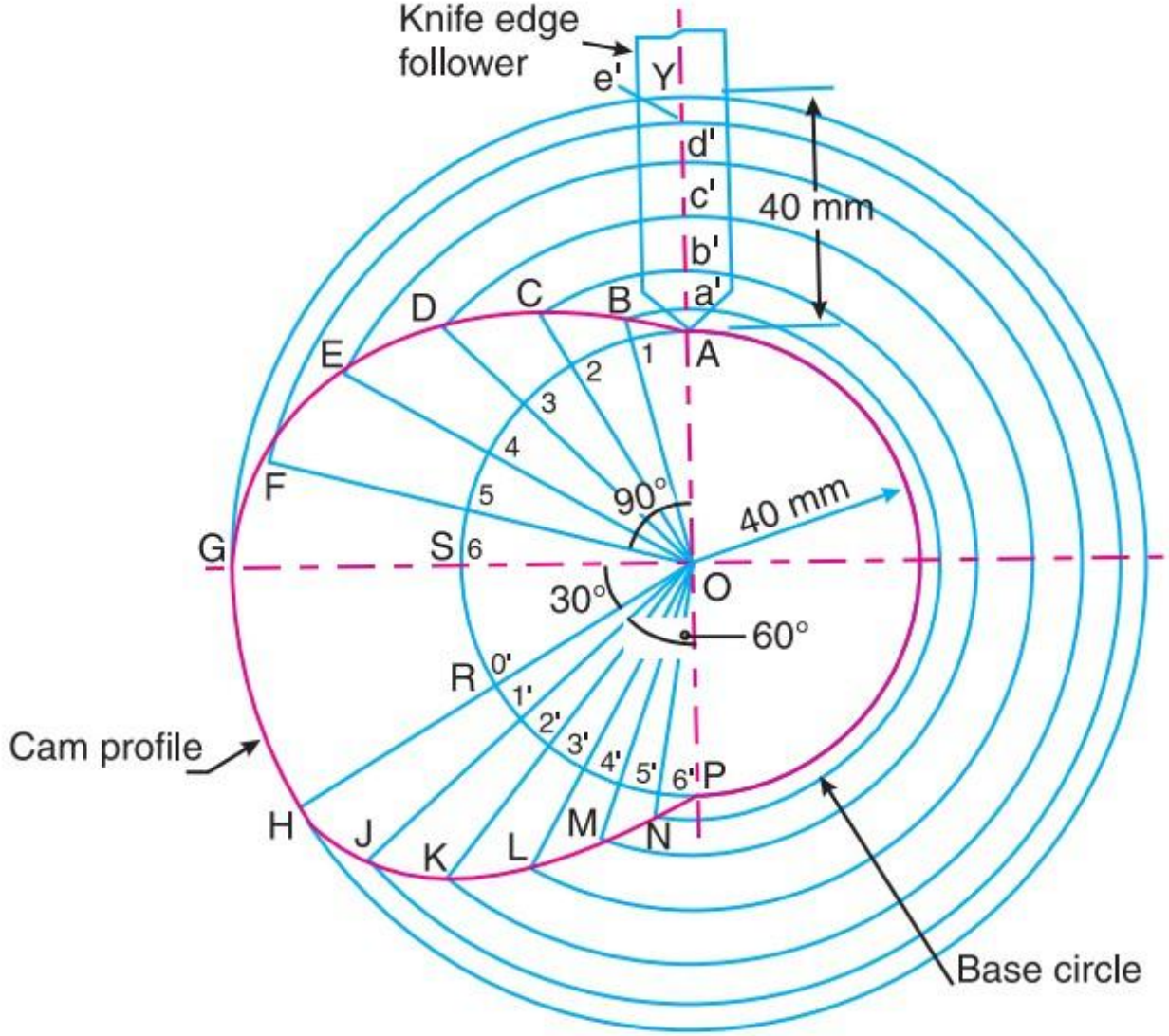


Fig. 20.14

(*b*) *Profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft*

The profile of the cam when the line of stroke of the follower is offset 20 mm from the axis of the cam shaft, as shown in Fig. 20.15, is drawn in the similar way as discussed in Example 20.1.

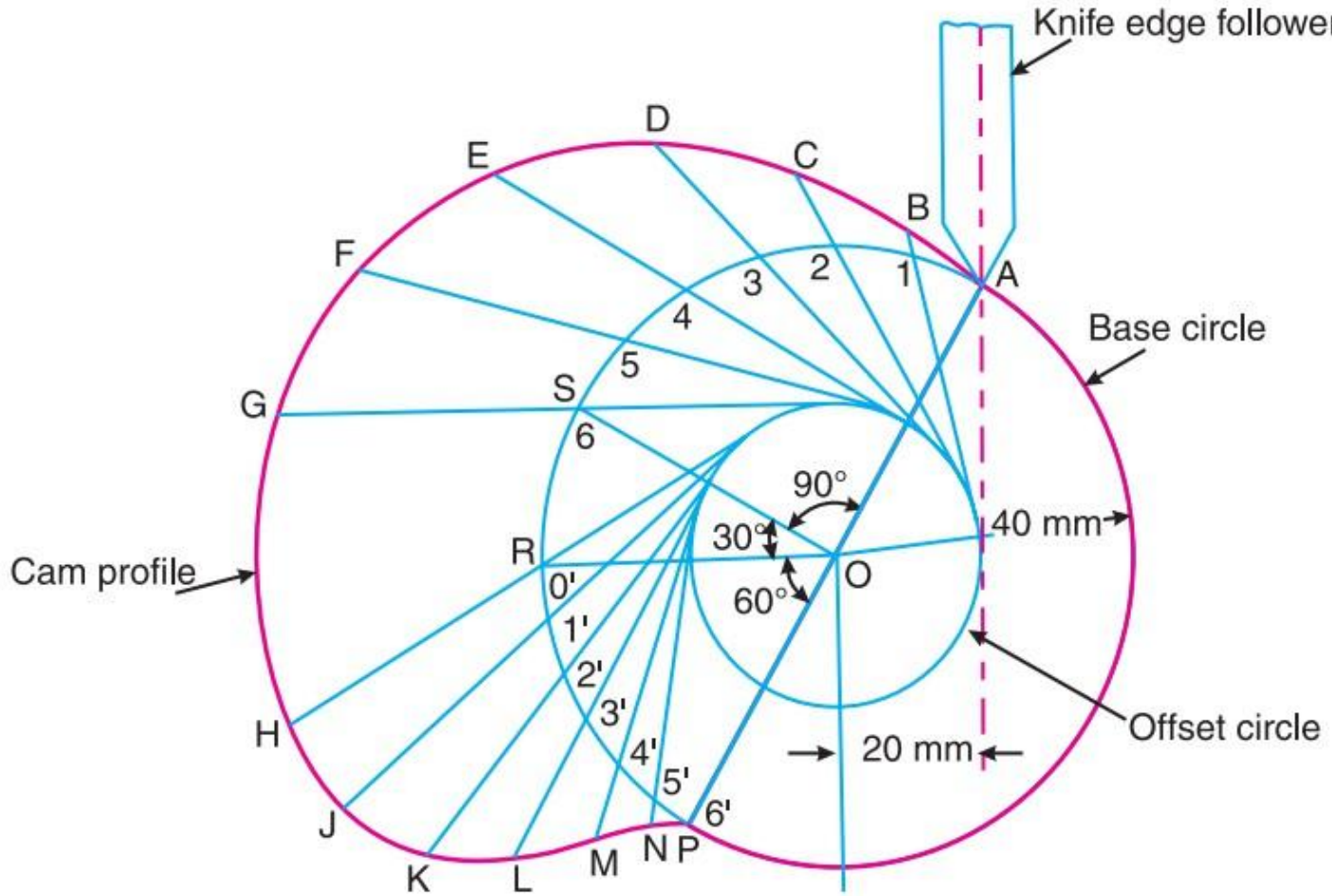


Fig. 20.15

Maximum velocity of the follower during its ascent and descent

We know that angular velocity of the cam,

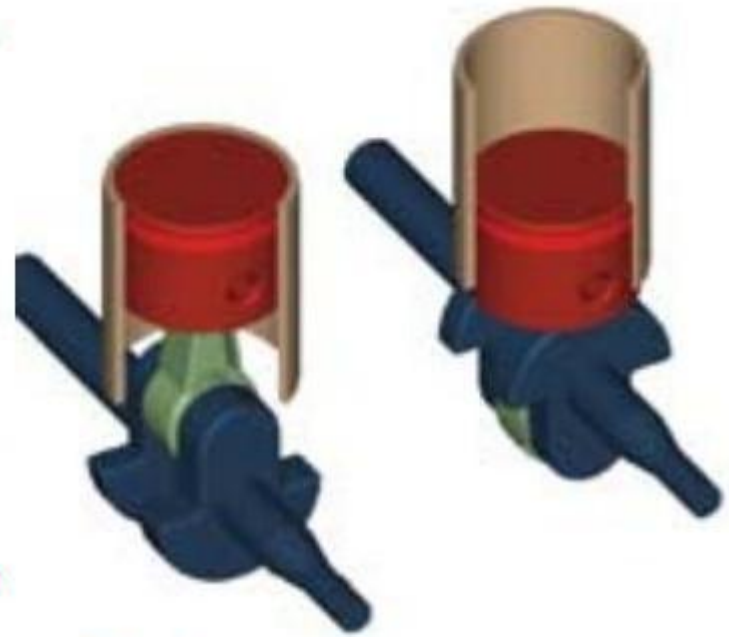
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

We also know that the maximum velocity of the follower during its ascent,

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s}$$ **Ans.**

and maximum velocity of the follower during its descent,

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s}$$ **Ans.**



Role of cams in piston movement.

Maximum acceleration of the follower during its ascent and descent

We know that the maximum acceleration of the follower during its ascent,

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2$$ **Ans.**

and maximum acceleration of the follower during its descent,

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2$$ **Ans.**

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;

2. To keep the valve fully raised through next 30°;

3. To lower the valve during next 60°; and

4. To keep the valve closed during rest of the revolution i.e. 150° ; The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

Draw the profile of the cam when

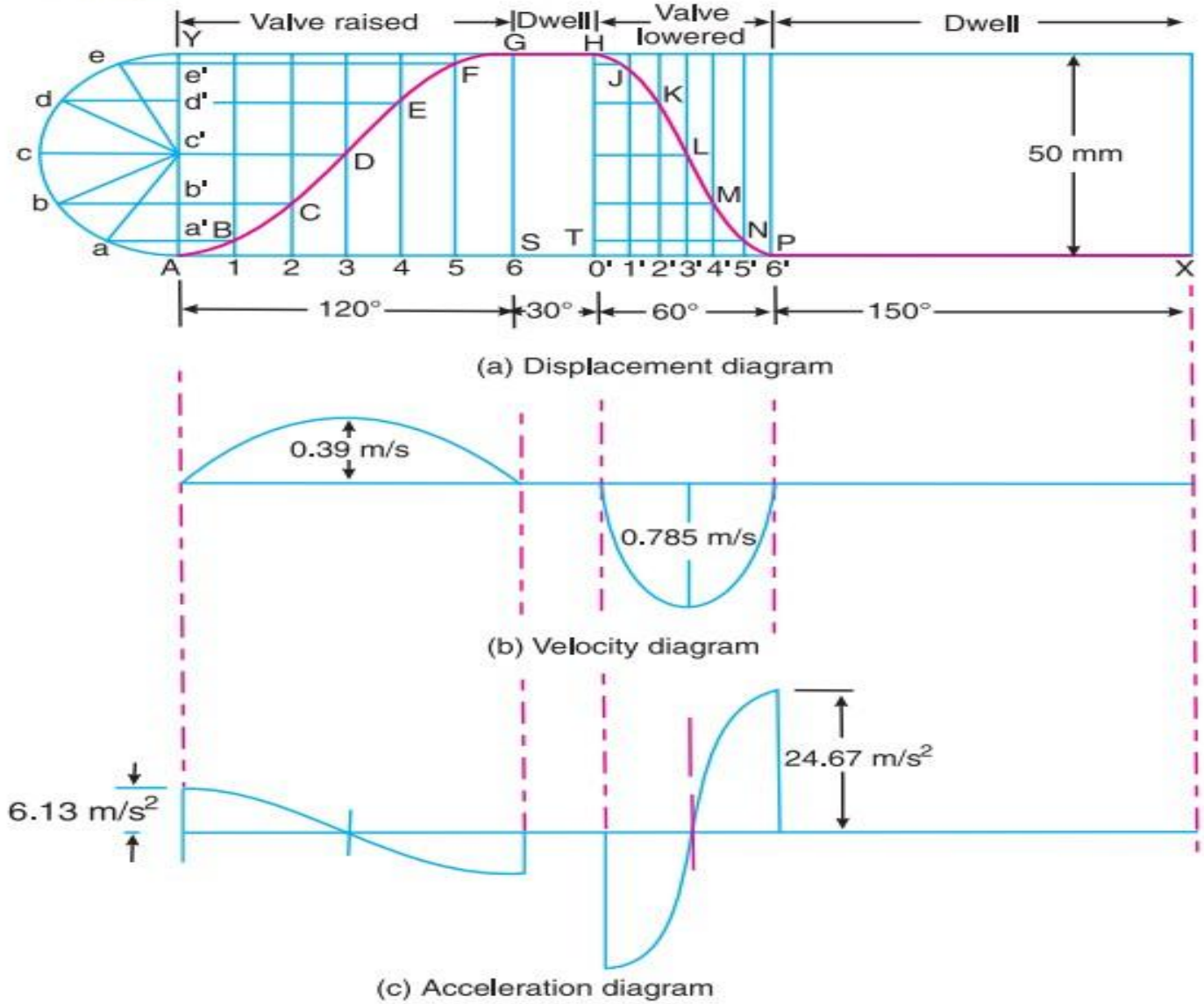
(a) the line of stroke of the valve rod passes through the axis of the cam shaft, and

(b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m. Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam.

(*a*) *Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft*

The profile of the cam, as shown in Fig. 20.17, is drawn as discussed in the following steps :

1. Draw a base circle with centre *O* and radius equal to the minimum radius of the cam (*i.e.* 25 mm).

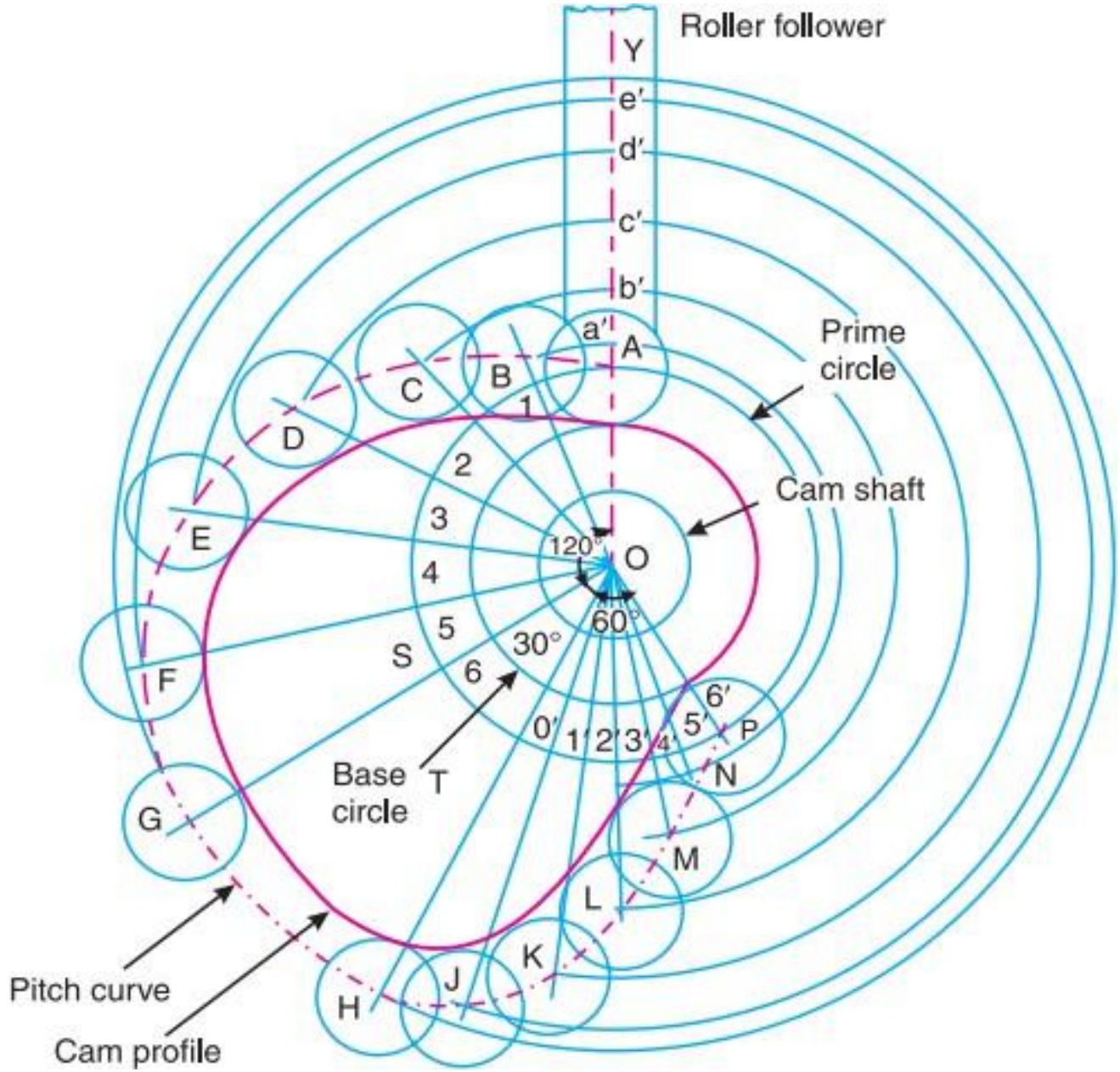


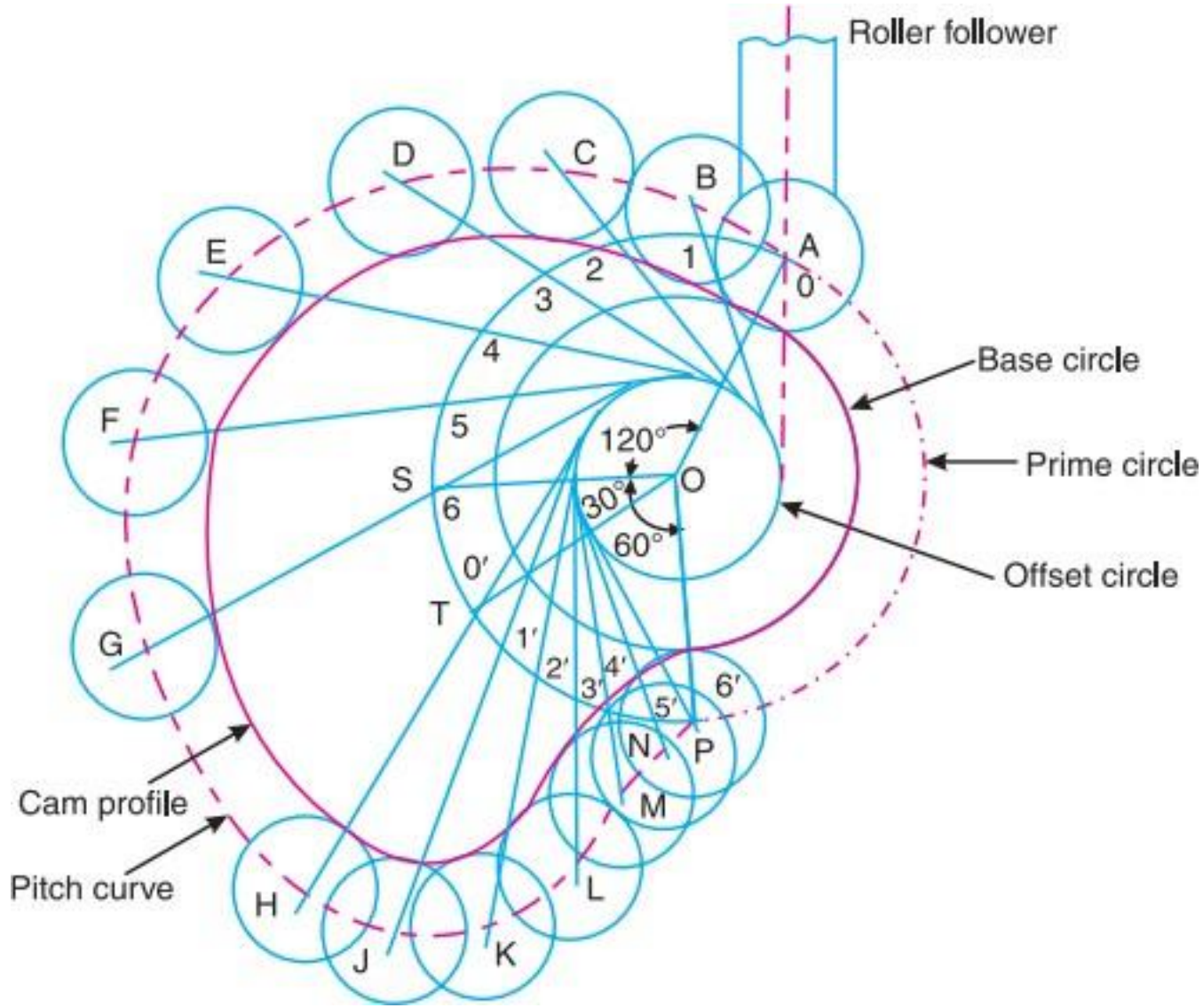
(a) Displacement diagram

(b) Velocity diagram

(c) Acceleration diagram

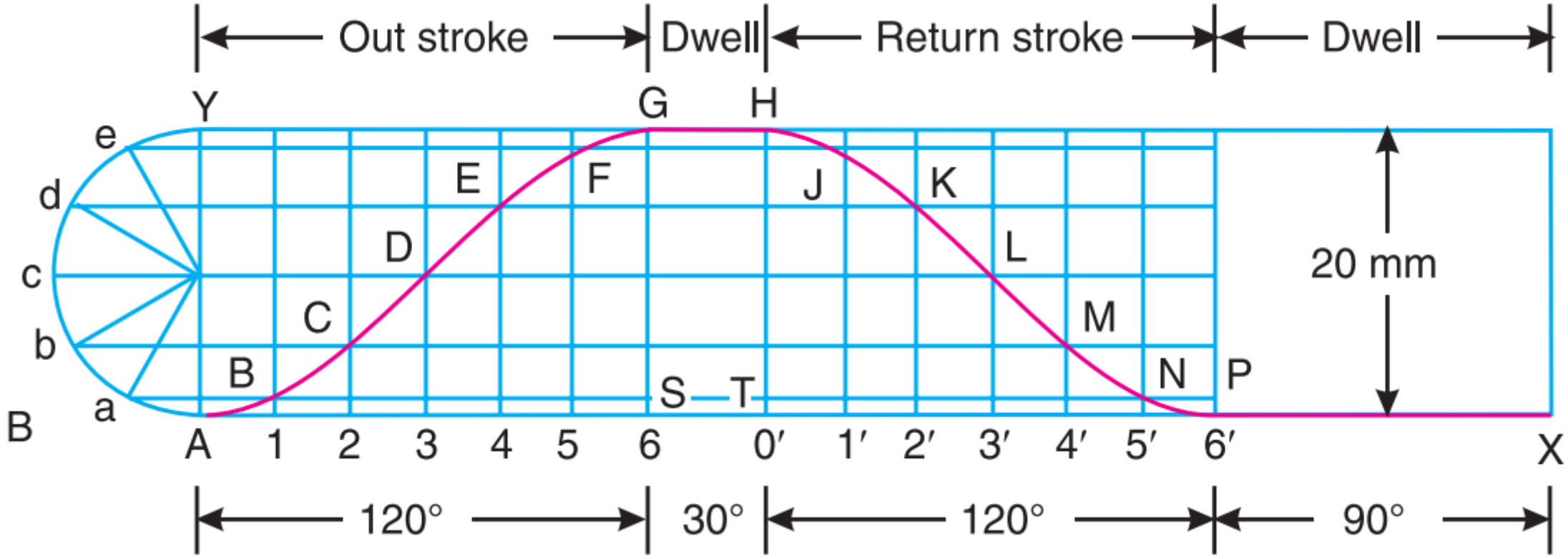
Fig. 20.16


Roller follower
Y
e′
d′
c′
b′
a′
A
B
C
D
E
F
G
H
J
K
L
M
N
P
1
2
3
4
5
6
S
T
O
120°
60°
30°
0′
1′
2′
3′
4′
5′
6′
Prime circle
Cam shaft
Base circle
Pitch curve
Cam profile


Roller follower
Base circle
Prime circle
Offset circle
Cam profile
Pitch curve
A
B
C
D
E
F
G
H
J
K
L
M
N
P
O
S
T
0
1
2
3
4
5
6
0′
1′
2′
3′
4′
5′
6′
120°
30°
60°

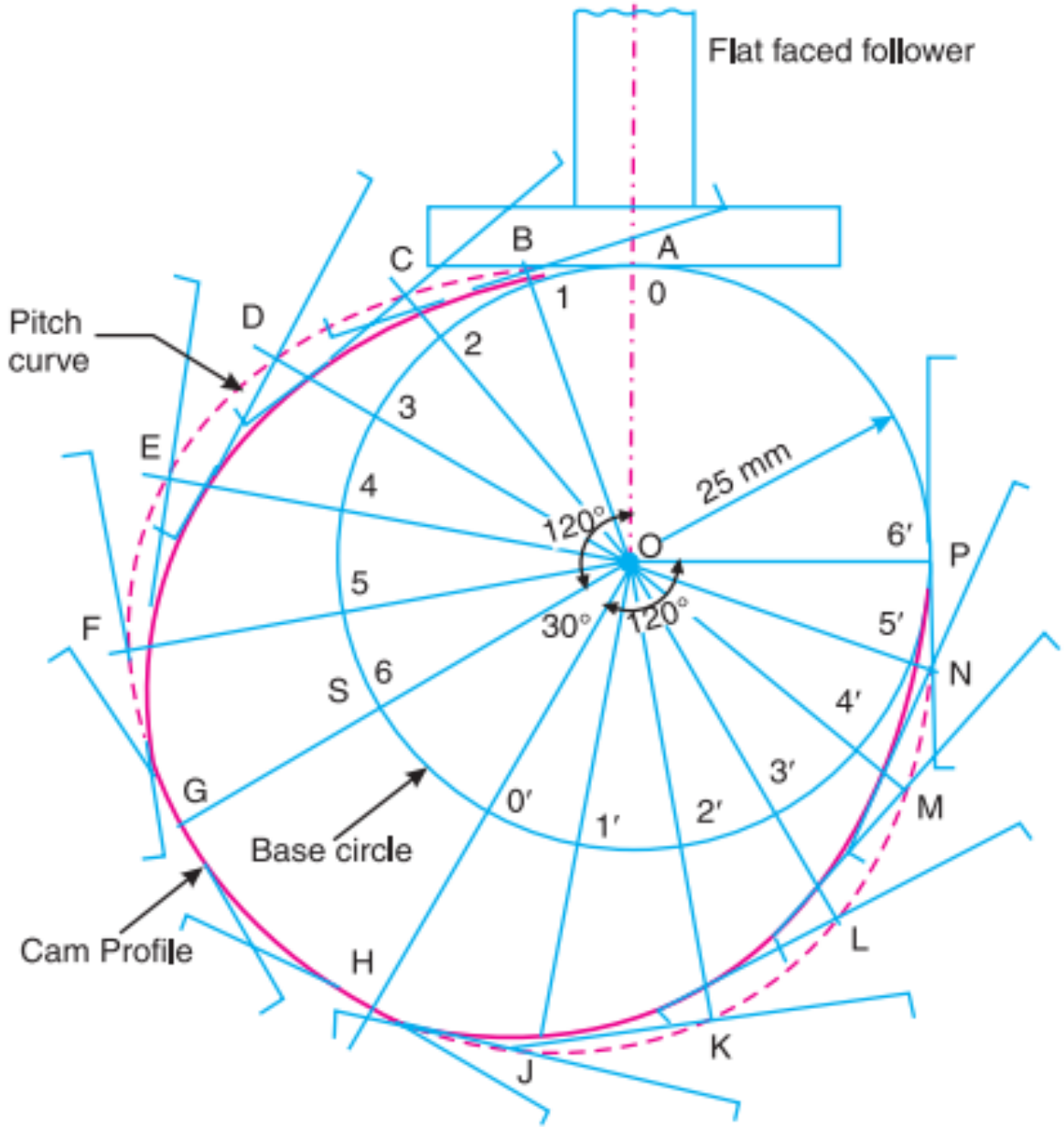
A cam drives a flat reciprocating follower in the following manner : During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.

Displacement Diagram:

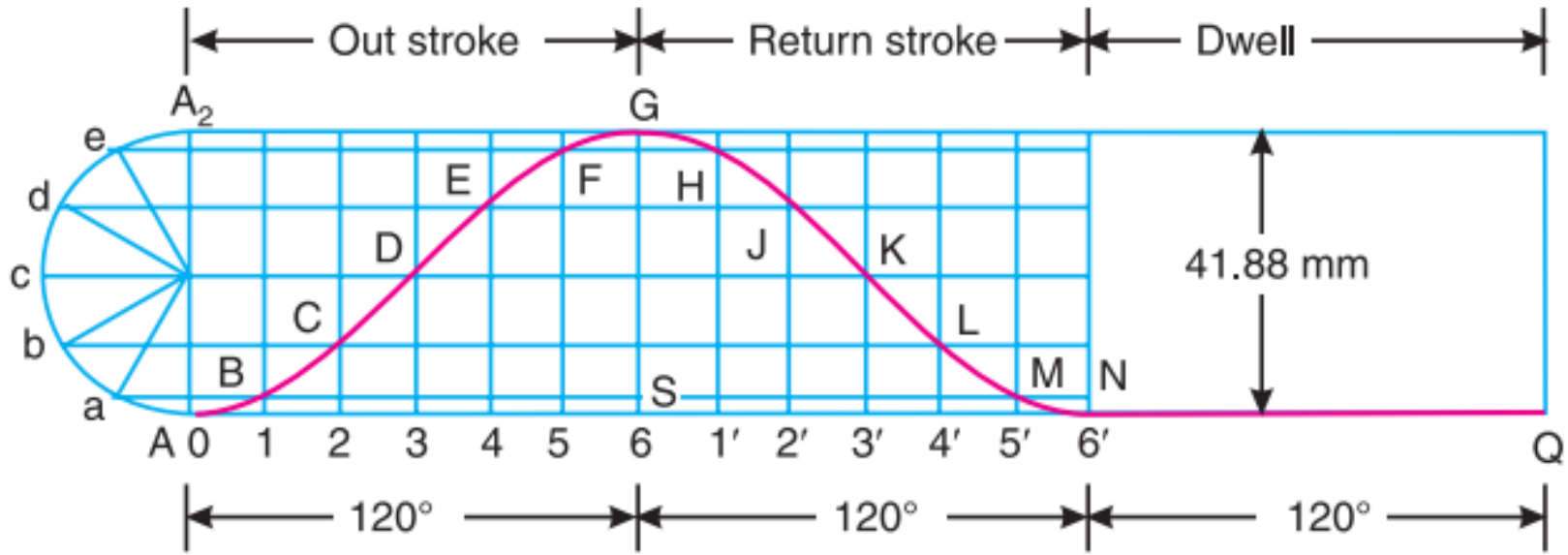


Now the profile of the cam driving a flat reciprocating follower, as shown in Fig. 20.20, is drawn as discussed in the following steps :

1. Draw a base circle with centre O and radius OA equal to the minimum radius of the cam (*i.e.* 25 mm).
2. Draw angle $AOS = 120°$ to represent the outward stroke, angle $SOT = 30°$ to represent dwell and angle $TOP = 120°$ to represent inward stroke.
3. Divide the angular displacement during outward stroke and inward stroke (*i.e.* angles AOS and TOP) into the same number of equal even parts as in the displacement diagram.
4. Join the points 1, 2, 3 . . . etc. with centre O and produce beyond the base circle.
5. From points 1, 2, 3 . . . etc., set off $1B$, $2C$, $3D$ . . . etc. equal to the distances measured from the displacement diagram.
6. Now at points B, C, D . . . M, N, P, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.
7. The curve drawn tangentially to the flat side of the follower is the required profile of the cam, as shown in Fig. 20.20.


Flat faced follower
Pitch curve
Cam Profile
Base circle
25 mm
120°
120°
30°
O
A
B
C
D
E
F
G
H
J
K
L
M
N
P
S
0
1
2
3
4
5
6
0′
1′
2′
3′
4′
5′
6′

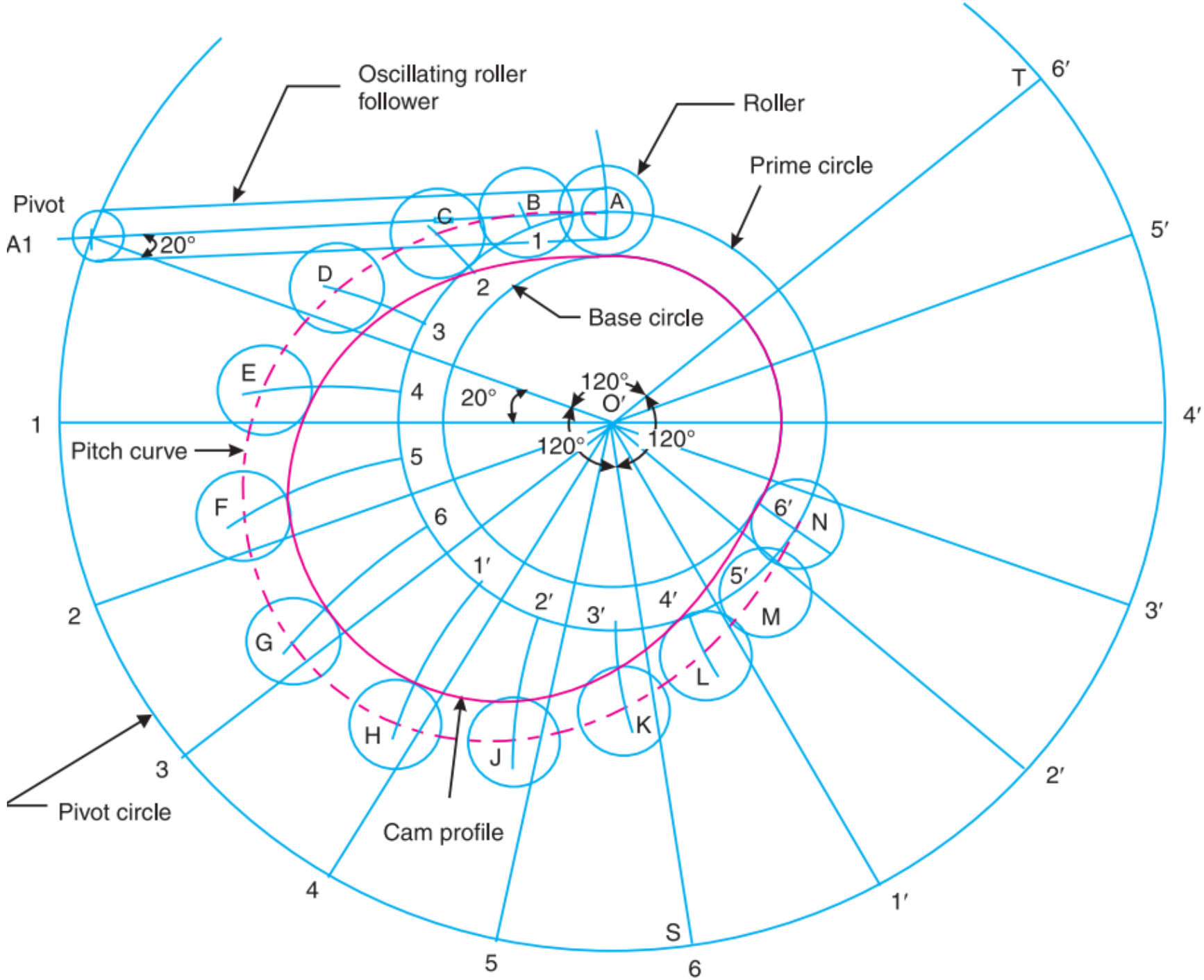
Draw a cam profile to drive an oscillating roller follower to the specifications given below :
(a) Follower to move outwards through an angular displacement of 20° during the first 120° rotation of the cam ;
(b) Follower to return to its initial position during next 120° rotation of the cam ;
(c) Follower to dwell during the next 120° of cam rotation.
The distance between pivot centre and roller centre = 120 mm ; distance between pivot centre and cam axis = 130 mm ; minimum radius of cam = 40 mm ; radius of roller = 10 mm ; inward and outward strokes take place with simple harmonic motion.


Out stroke
Return stroke
Dwell
A2
G
e
d
c
b
a
B
C
D
E
F
H
J
K
L
M
N
S
41.88 mm
A 0 1 2 3 4 5 6 1′ 2′ 3′ 4′ 5′ 6′ Q
120°
120°
120°

The profile of the cam to drive an oscillating roller follower, as shown in Fig. 20.23, is drawn as discussed in the following steps :

1. First of all, draw a base circle with centre O and radius equal to the minimum radius of the cam (*i.e.* 40 mm)
2. Draw a prime circle with centre O and radius OA

   = Min. radius of cam + radius of roller = 40 + 10 = 50 mm
3. Now locate the pivot centre A_1 such that OA_1 = 130 mm and AA_1 = 120 mm. Draw a pivot circle with centre O and radius OA_1 = 130 mm.
4. Join OA_1. Draw angle A_1OS = 120° to represent the outward stroke of the follower, angle SOT = 120° to represent the inward stroke of the follower and angle TOA_1 = 120° to represent the dwell.
5. Divide angles A_1OS and SOT into the same number of equal even parts as in the displacement diagram and mark points 1, 2, 3 . . . $4', 5', 6'$ on the pivot circle.
6. Now with points 1, 2, 3 . . . $4', 5', 6'$ (on the pivot circle) as centre and radius equal to A_1A (*i.e.* 120 mm) draw circular arcs to intersect the prime circle at points 1, 2, 3 . . . $4', 5', 6'$.
7. Set off the distances $1B$, $2C$, $3D$... $4'L$, $5'M$ along the arcs drawn equal to the distances as measured from the displacement diagram.
8. The curve passing through the points A, B, C....L, M, N is known as pitch curve.
9. Now draw circles with A, B, C, D....L, M, N as centre and radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.23. This is the required profile of the cam.


Oscillating roller follower
Roller
Prime circle
Pivot
A1
20°
Base circle
Pitch curve
120°
O′
Pivot circle
Cam profile
A
B
C
D
E
F
G
H
J
K
L
M
N
T
S
1
2
3
4
5
6
1′
2′
3′
4′
5′
6′

A cam, with a minimum radius of 50 mm, rotating clockwise at a uniform speed, is required to give a knife edge follower the motion as described below :

1. *To move outwards through 40 mm during 100° rotation of the cam ;*
2. *To dwell for next 80° ;*
3. *To return to its starting position during next 90°, and*
4. *To dwell for the rest period of a revolution i.e. 90°.*

Draw the profile of the cam

(i) when the line of stroke of the follower passes through the centre of the cam shaft, and

(ii) when the line of stroke of the follower is off-set by 15 mm.

The displacement of the follower is to take place with uniform acceleration and uniform retardation.

Determine the maximum velocity and acceleration of the follower when the cam shaft rotates at 900 r.p.m. Draw the displacement, velocity and acceleration diagrams for one complete revolution of the cam

Design a cam for operating the exhaust valve of an oil engine. It is required to give equal uniform acceleration and retardation during opening and closing of the valve each of which corresponds to 60° of cam rotation. The valve must remain in the fully open position for 20° of cam rotation. The lift of the valve is 37.5 mm and the least radius of the cam is 40 mm. The follower is provided with a roller of radius 20 mm and its line of stroke passes through the axis of the cam.

A cam rotating clockwise at a uniform speed of 1000 r.p.m. is required to give a roller follower the motion defined below :

1. *Follower to move outwards through 50 mm during 120° of cam rotation,*
2. *Follower to dwell for next 60° of cam rotation,*
3. *Follower to return to its starting position during next 90° of cam rotation,*
4. *Follower to dwell for the rest of the cam rotation.*

The minimum radius of the cam is 50 mm and the diameter of roller is 10 mm. The line of stroke of the follower is off-set by 20 mm from the axis of the cam shaft. If the displacement of the follower takes place with uniform and equal acceleration and retardation on both the outward and return strokes, draw profile of the cam and find the maximum velocity and acceleration during out stroke and return stroke.

Construct the profile of a cam to suit the following specifications : Cam shaft diameter = 40 mm ; Least radius of cam = 25 mm ; Diameter of roller = 25 mm; Angle of lift = 120° ; Angle of fall = 150° ; Lift of the follower = 40 mm ; Number of pauses are two of equal interval between motions.

During the lift, the motion is S.H.M. During the fall the motion is uniform acceleration and deceleration. The speed of the cam shaft is uniform. The line of stroke of the follower is off-set 12.5 mm from the centre of the cam.

A cam operating a knife-edged follower has the following data :
(a) Follower moves outwards through 40 mm during 60° of cam rotation.
(b) Follower dwells for the next 45°.
(c) Follower returns to its original position during next 90°.
(d) Follower dwells for the rest of the rotation.
The displacement of the follower is to take place with simple harmonic motion during both the outward and return strokes. The least radius of the cam is 50 mm. Draw the profile of the cam when
1. the axis of the follower passes through the cam axis, and
2. the axis of the follower is offset 20mm towards right from the cam axis. If the cam rotates at 300 r.p.m., determine maximum velocity and acceleration of the follower during the outward stroke and the return stroke.
[Ans. 1.88 m/s, 1.26 m/s ; 177.7 m/s2, 79 m/s2]

References: Theory of Machines by Khurmi