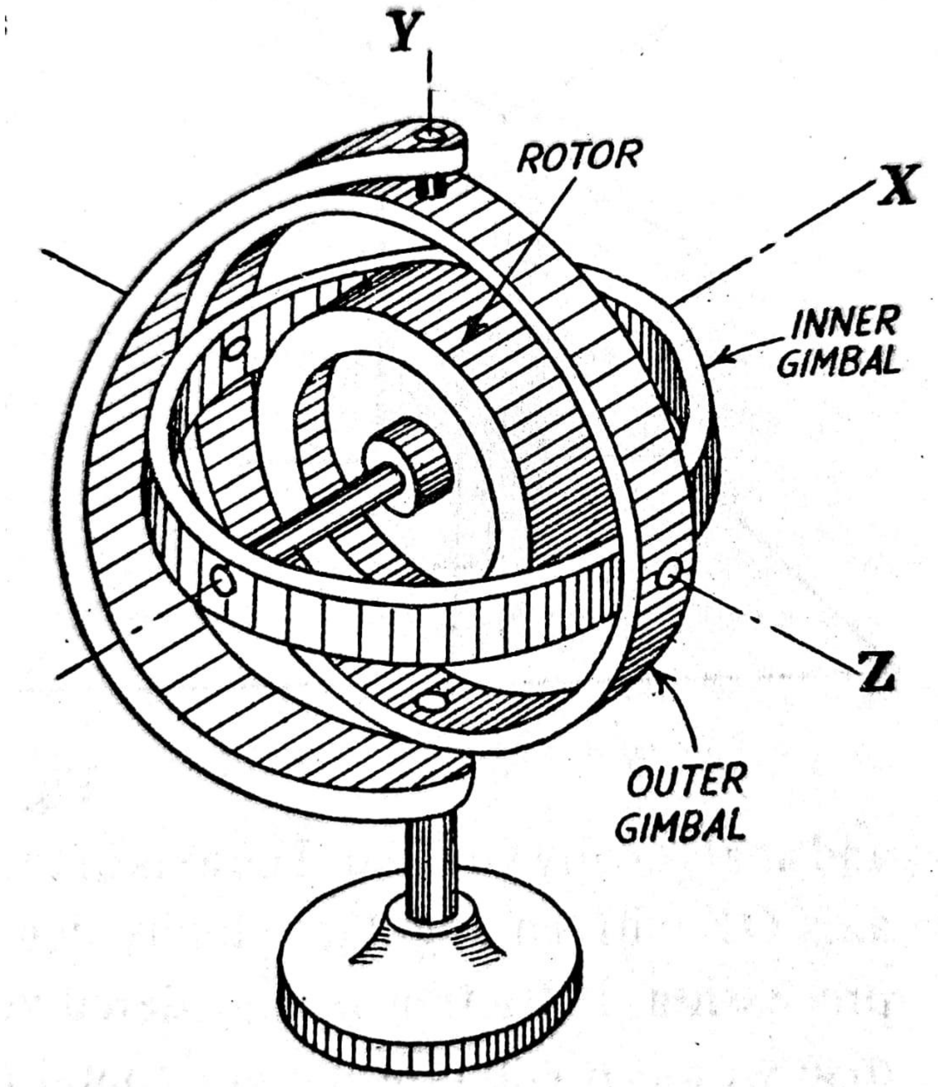
Gyroscope

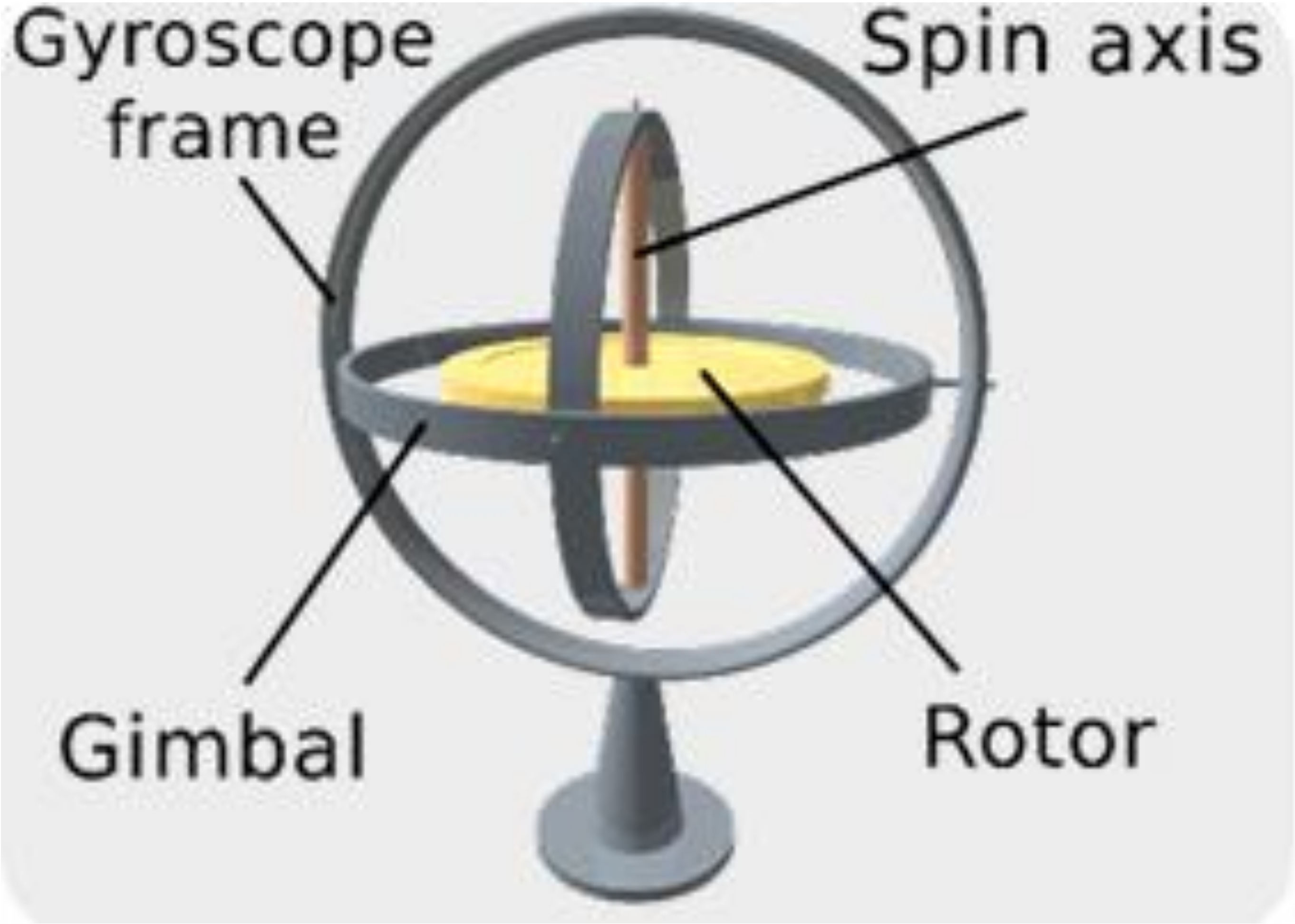


A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum.

The spinning mass will resist change in its angular momentum .

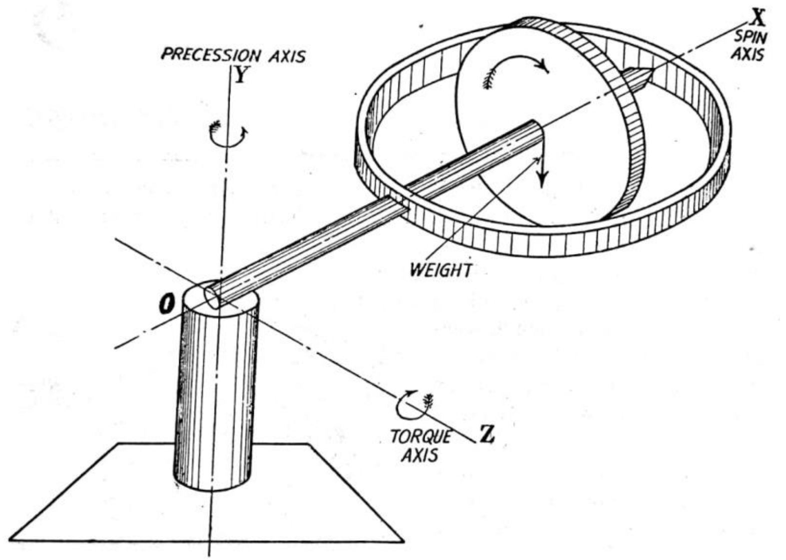
Angular momentum $H = I\omega$

By placing the gyro in a pair of frictionless gimbals
it is free to maintain its inertial spin axis.


Gyroscope
frame
Spin axis
Gimbal
Rotor

- Gyroscopes have two basic properties:

    Rigidity and Precession



The properties are defined as follows:

Rigidity: The axis of rotation (spin axis) of the gyro wheel tends to remain in a fixed direction in space if no force is applied to it.

Precession: The axis of rotation has a tendency to turn at a right angle to the direction of an applied force.

Types of gyroscopes.

1. Mechanical Gyroscopes
2. Gas-Bearing Gyroscopes
3. Optical Gyroscopes
4. Rate Gyro
5. Hemispherical Resonator Gyroscope (Wine-Glass Gyroscope Or Mushroom Gyro)
6. MEMS Gyroscope
7. FOG (Fiber Optical Gyro)
8. RLG (Ring Laser Gyro)
9. VSG (Vibrating Structure Gyroscope)
10. CVG (Coriolis Vibratory Gyroscope)
11. Piezoelectric Gyroscopes

Applications

- Mechanical Gyroscopes
  1. Navigation of large aircraft
  2. Missile guidance and control
- Gas-Bearing Gyroscopes
  1. By NASA in the development of the Hubble Telescope
- Optical Gyroscopes
  1. Modern rocketry and spacecraft
- Rate gyro
  1. Used in rate integrating gyroscopes
  2. In attitude control systems for vehicles
  3. In combination with other sensors to make inertial navigation systems

- Hemispherical resonator gyroscope (wine-glass gyroscope or mushroom gyro)
    1. In space applications (satellites and spacecraft)
    2. Marine maintenance-free gyrocompass and Altitude Heading Reference Systems
    3. In Commercial Air Transport navigation system
- FOG
    1. Navigation aid in remotely operated vehicles and autonomous Under-water vehicles
    2. High performance space applications
- RLG
    1. Air transport vehicles
    2. Missile guidance system
    3. Fighter plane remote control
    4. Fighter plane guidance system
    5. Ship's Internal Navigation System

- MEMS gyroscope
  1. In automotive roll-over prevention
  2. Airbag systems
  3. Image stabilization
  4. Spacecraft positioning
  5. Automotive
  6. Industrial robotics
  7. Entertainment
  8. Photography

- Vibrating structure gyroscopes

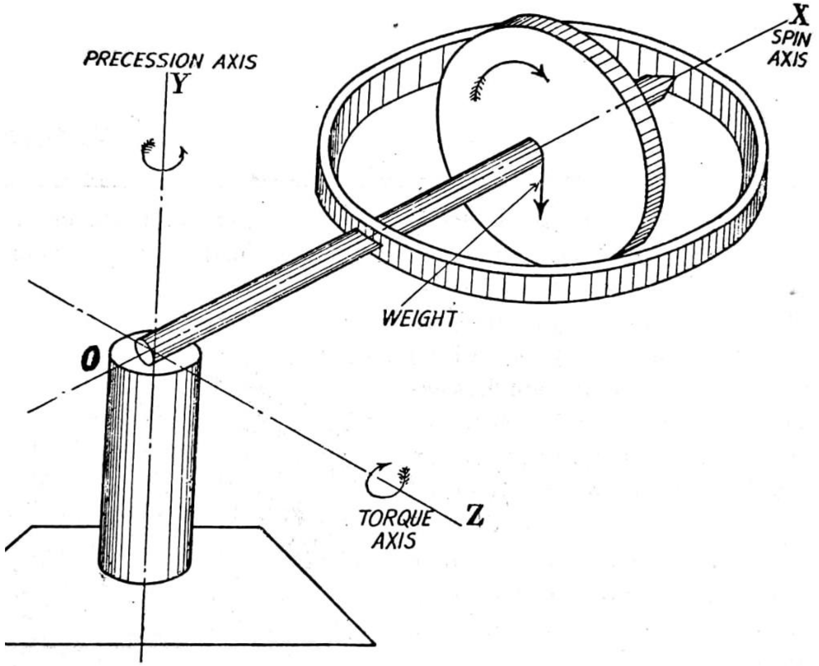
  Used in many Image stabilization systems on video and still cameras

Principles of gyroscope

A rotating disc supported on gimbal ring, rotates with an angular velocity called the velocity of spin denoted by ω, Gimbal ring has a projected rod which is supported on frictionless surface.

The axis of the rotating shaft rotates about the vertical axis OY with an angular velocity denoted by ω_p. It is called the velocity of precession.

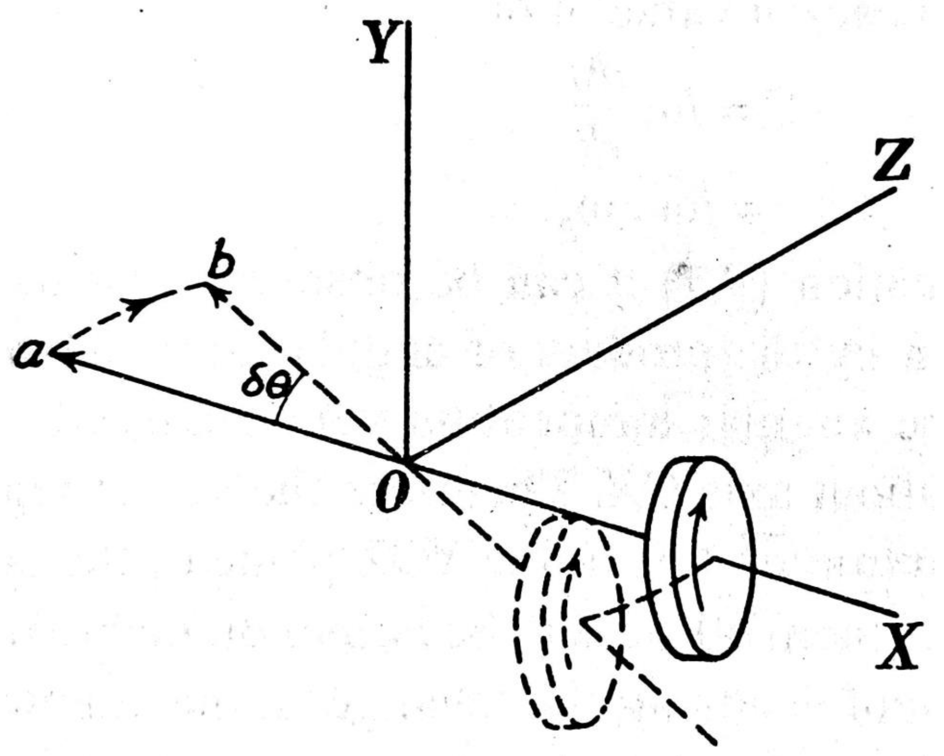
If friction is considered zero, the axis of rotation of the rotating disc would revolve in the horizontal plane XOZ, but normally the rotating disc will gradually drop.



Gyroscopic couple

The plane in which the disc is rotating is parallel to plane YOZ, it is called plane of spin and the axis of the rotor is along axis OX, it is called axis of spin.

XOZ is a horizontal plane, and the axis of spin is rotating in a plane parallel to the horizontal plane about an axis OY, thus XOZ is the plane of precession and OY is called the axis of precession.



Angular velocity of rotor - ω

Precessing angular velocity - ω_p

The rotor is rotating in clockwise direction, and the direction of action of the angular momentum will be inwards.

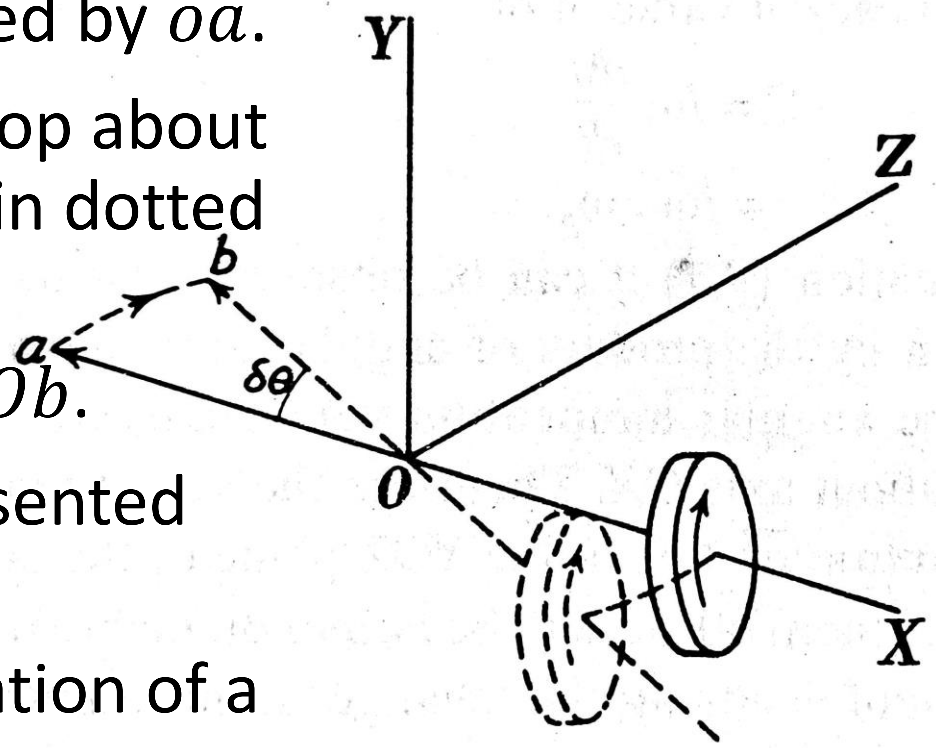
This vector of angular momentum is represented by $\overrightarrow{oa}$.

The axis of spin is rotating clockwise from the top about axis OY, the next position occupied by rotor is in dotted line.

The angular momentum is now by dotted line Ob.

The change in the angular momentum is represented by vector ab.

The change of momentum is due to the application of a couple to the rotor.



Couple applied = Rate of change of momentum

$$C = \frac{\delta(I\omega)}{\delta t}$$

$\delta(I\omega)$ - the change in angular momentum

δt – the time interval in which this change takes place

$$\delta(I\omega) = ab \approx Oa \times \delta\theta$$

$\delta\theta$ – angle in radians turned through by the axis of spin in time δt

$$C \approx Oa\frac{\delta\theta}{\delta t} \approx I\omega\frac{\delta\theta}{\delta t}$$

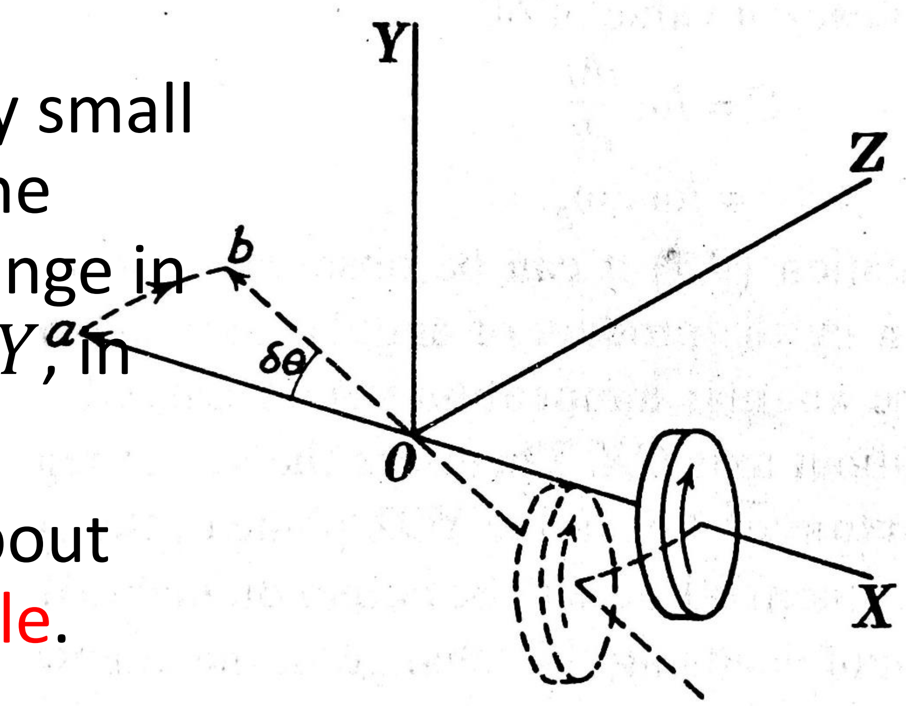
For infinitesimal value of $\delta\theta$

$$C = I\omega\frac{d\theta}{dt} = I\omega \cdot \omega_p$$

$$C = I\omega \cdot \omega_p$$

The applied gyroscopic couple C is given by the product of angular momentum $I\omega$ and the angular velocity ω_p of the angular momentum vector Oa.

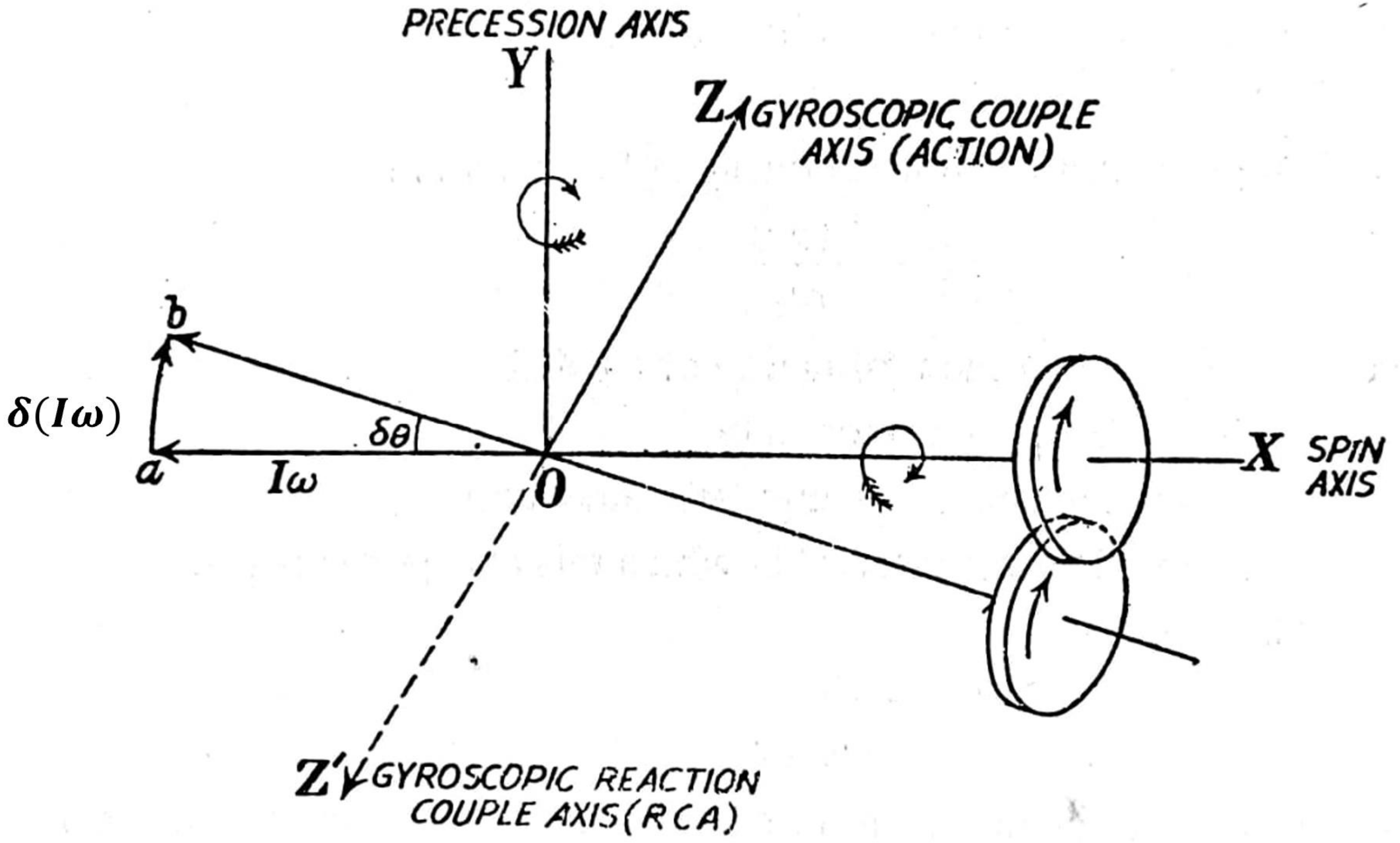
The vector representing the change in angular momentum $\overrightarrow{ab}$ lies in the XOZ plane. For a very small displacement $\delta\theta$, ab will be perpendicular to the vertical plane XOY. The couple causing this change in the angular momentum will lie in the plane XOY, in clockwise direction.



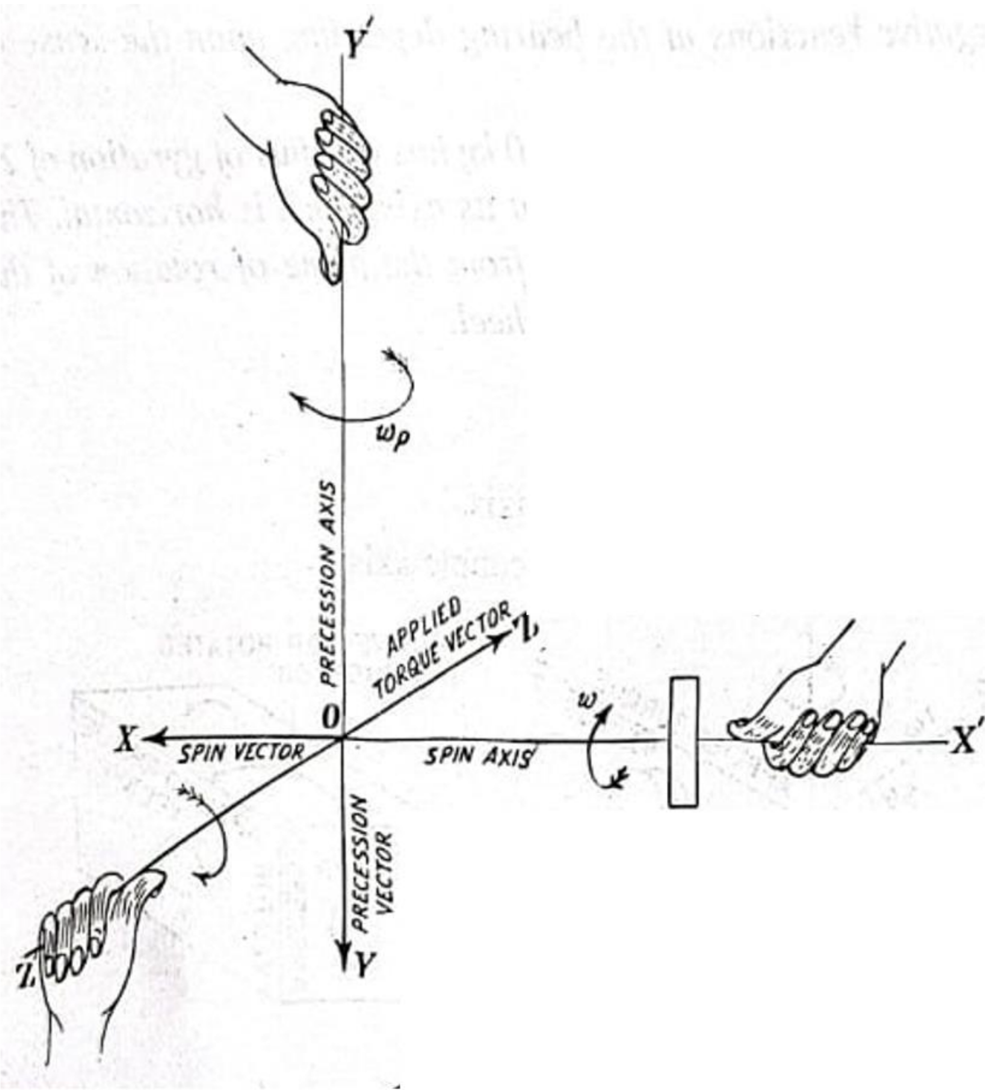
OZ, an axis perpendicular to the plane XOY, about which the couple acts, is called the axis of couple.

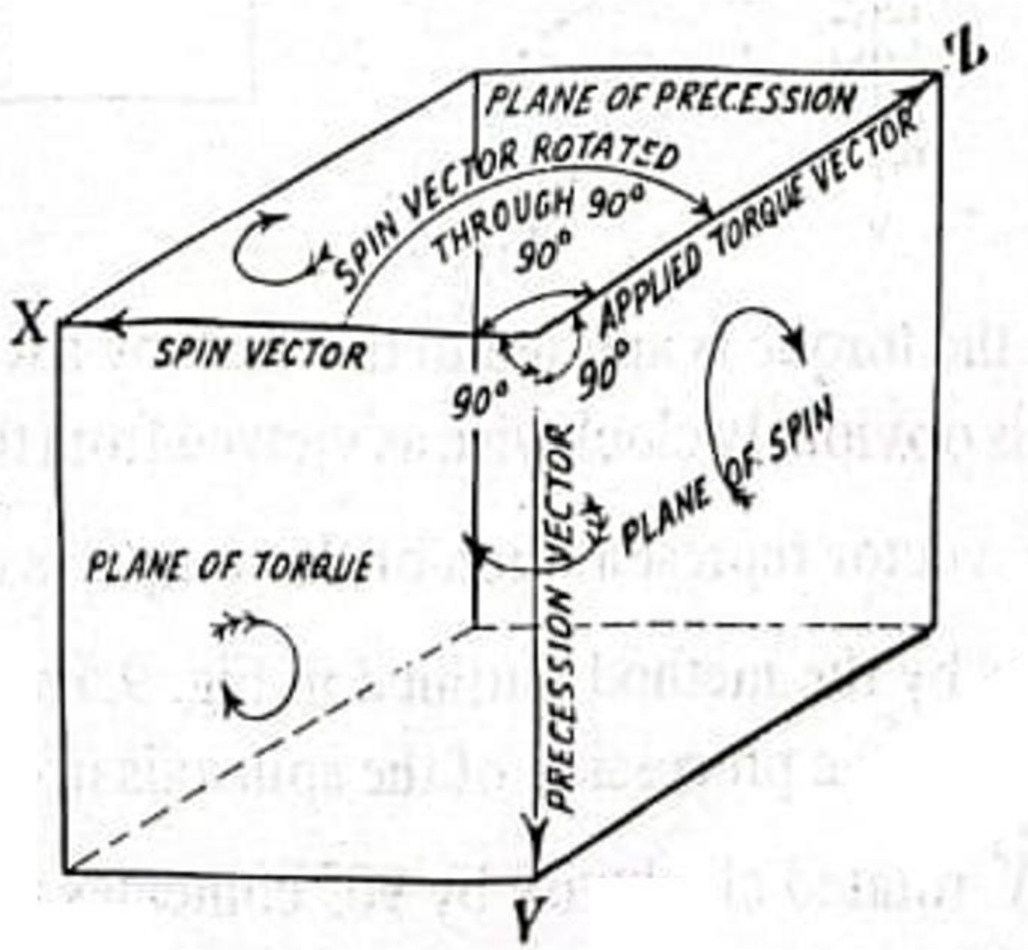
Gyroscopic couple causes gyroscopic acceleration, which causes change in direction of the angular velocity keeping its magnitude constant.

The change of the direction of the axis of rotation is caused by a gyroscopic couple applied to it, Gyroscopic couple is applied through the bearings which support the shaft.

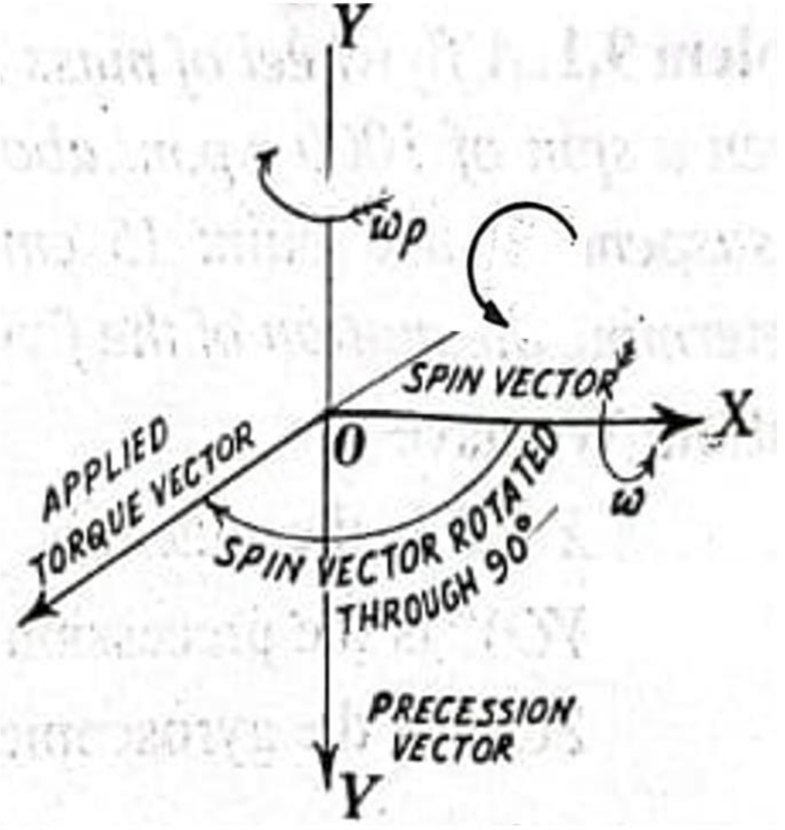


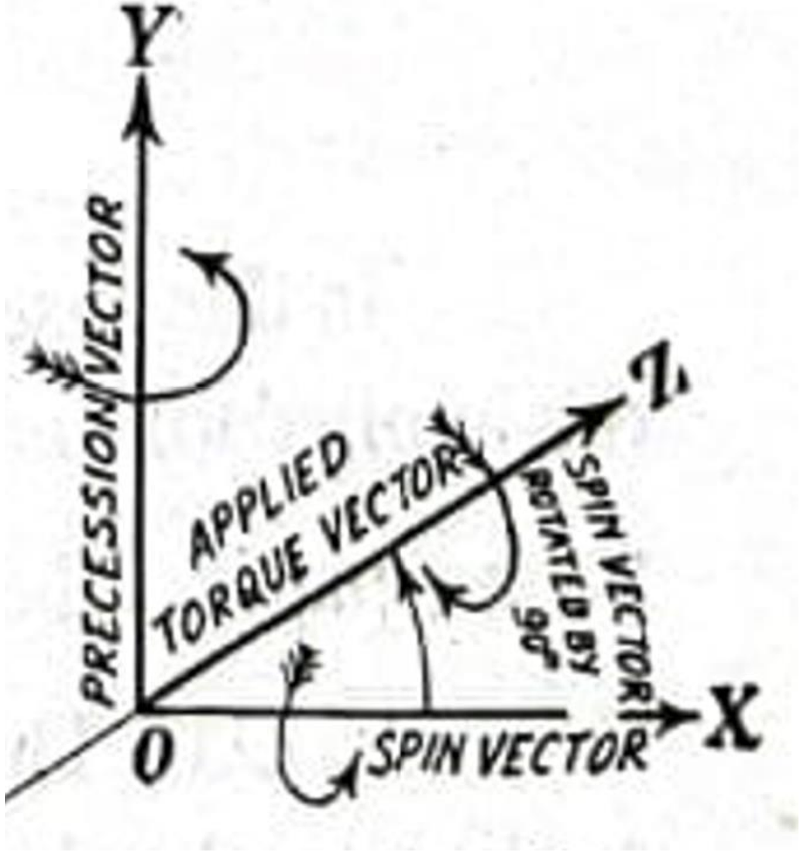
Directions of Spin Vector, Precession Vector and Torque Vector with forced precession.

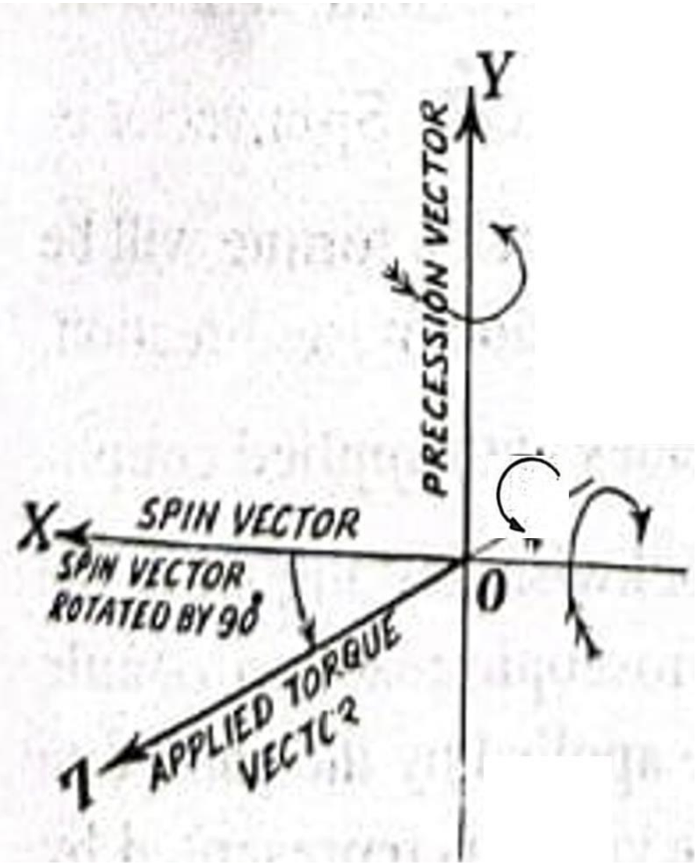




The method of determining the direction of torque is as follows: Rotate the spin vector through 90° about the precession axis in the direction of precession, then the spin vector will correspond to the direction of the applied torque vector.

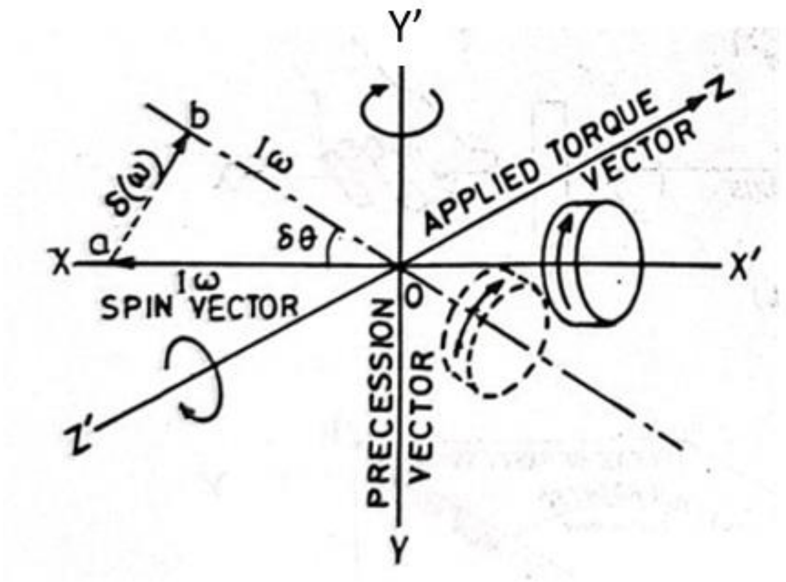






Example 1

A flywheel of mass 10 kg has a radius of gyration of 20 cm. It is given a spin of 1000 rev/min in clockwise direction about its axis which is horizontal. The flywheel is suspended at a point 15 cm from the plane of rotation of the flywheel. Determine the motion of the flywheel.



XOX' – Spin axis
YOY' – Precession axis
ZOZ' – gyroscopic couple axis

$$C = I\omega \cdot \omega_p$$

$$C = mgl = 10 \times 9.81 \times 15 \times 10^{-2}$$

$$I\omega \cdot \omega_p = 10 \times \left(20 \times 10^{-2}\right)^2 \times \frac{2\pi}{60} \times 1000 \times \omega_p$$

$$\omega_p = \frac{10 \times 9.81 \times 15 \times 10^{-2}}{10 \times (20 \times 10^{-2})^2 \times \frac{2\pi}{60} \times 1000} = 0.351\, rad/s$$

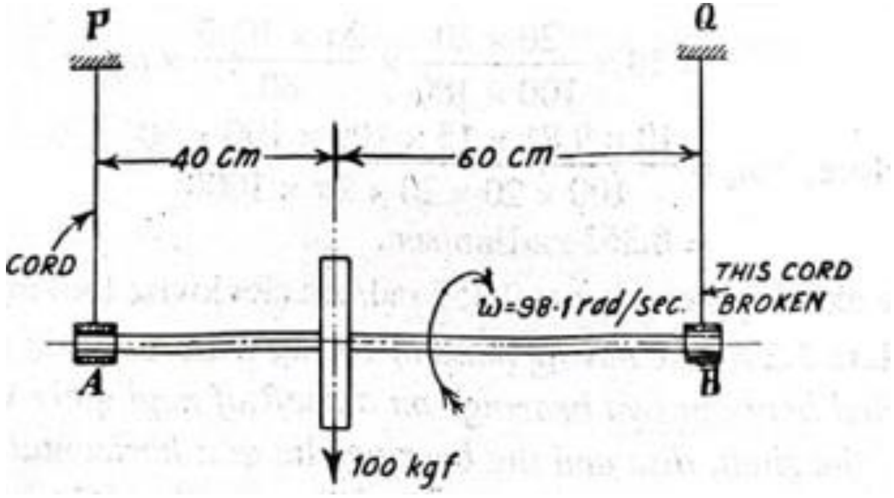
Example 2

A disc having a mass of 100 kg with radius of gyration 40 cm is supported between two bearings on a shaft of negligible weight. The centre line of the shaft, disc and bearing lie in a horizontal plane. These bearings are supported by vertical thin cords. When the disc is rotating at 98.1 rad/s in the clockwise direction, the cord supporting the right-hand side bearing gets broken.

i. Discuss the motion of the disc at the instant the cord is cut.

ii. Determine the magnitude of the motion of precession.

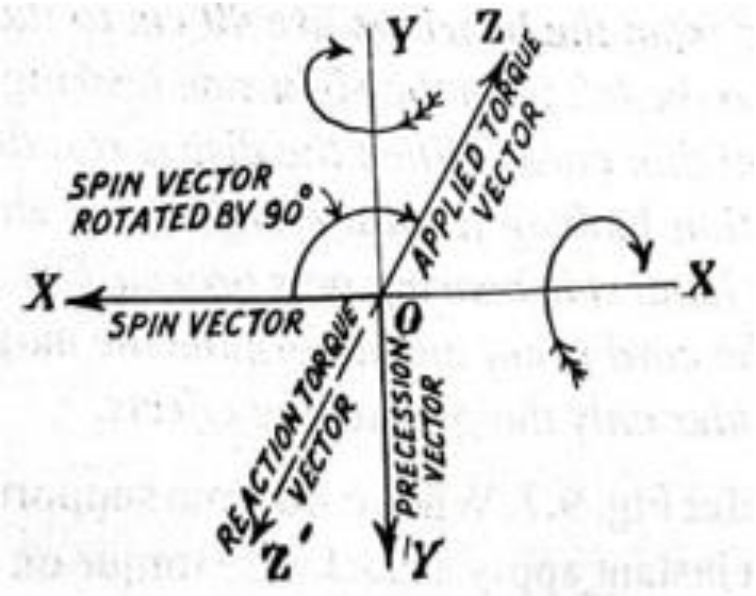
Consider only the gyroscopic effects.



When cord from support Q is broken, the mass 100 kg will apply a clockwise torque on the shaft, represented by Applied couple vector $\overrightarrow{OZ}$.

The spinning disc is represented by Spin vector $\overrightarrow{OX}$.

i. The spinning disc under only gyroscopic effects, precesses about vertical string PA in a clockwise sense as viewed from the top.



ii.

$$C = I\omega \cdot \omega_p$$

$$C = mgl = 100 \times 9.81 \times 40 \times 10^{-2}$$

$$I\omega \cdot \omega_p = 100 \times \left(40 \times 10^{-2}\right)^2 \times 98.1 \times \omega_p$$

$$\omega_p = \frac{100 \times 9.81 \times 40 \times 10^{-2}}{100 \times (40 \times 10^{-2})^2 \times 98.1} = 0.25\, rad/s$$

The spin axis starts precessing about cord PA in clockwise direction as viewed from the top at an angular velocity of 0.25 rad/s.

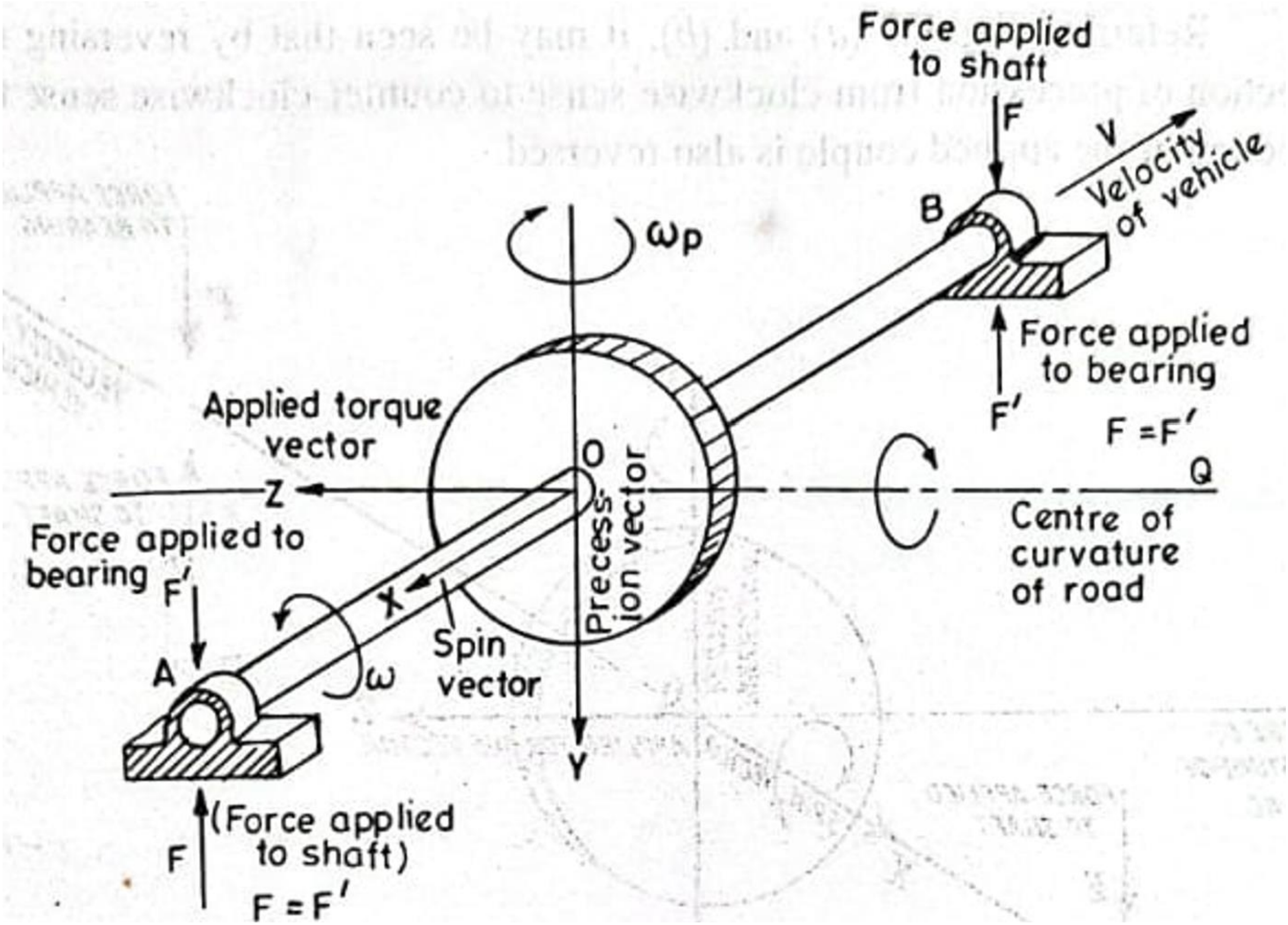
Gyroscopic effect

Gyroscopic effect is ability (tendency) of the rotating body to maintain a steady direction of its axis of rotation.

The gyroscopes are rotating with respect to the axis of symmetry at high speed. This gives them big kinetic energy $E = \frac{1}{2} I\omega^2$, where ω is angular speed and I moment of inertia of the form $I = \int_Q \; r^2 dm$, where Q is entire mass.

The gyroscopic effect is dependent on the weight of the rotating body, the weight distribution around the rotation axis, and the angular speed.

Analysis of the forces on bearing due to the forced precessing of rotating disc mounted on shafts.



The shaft AB carry a disc spinning with angular velocity ω in counter-clockwise direction as viewed from bearing A.

The shaft with bearings move with velocity V around a curve with the radius of curvature R.

The vehicle is rounding a curve in clockwise direction as viewed from the top.

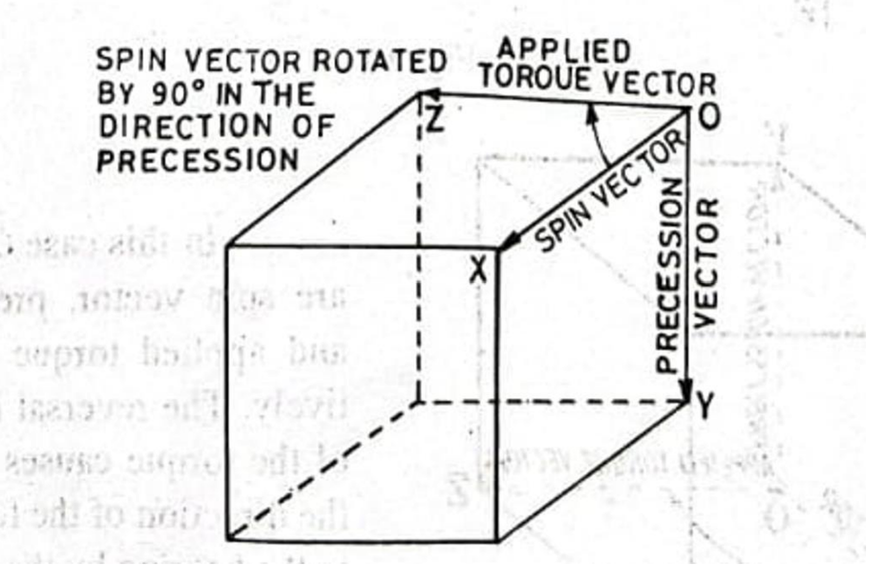
The spinning disc in bearings is precessing with angular velocity $\omega_P = \frac{V}{R}$ in the clockwise direction as viewed from the top.

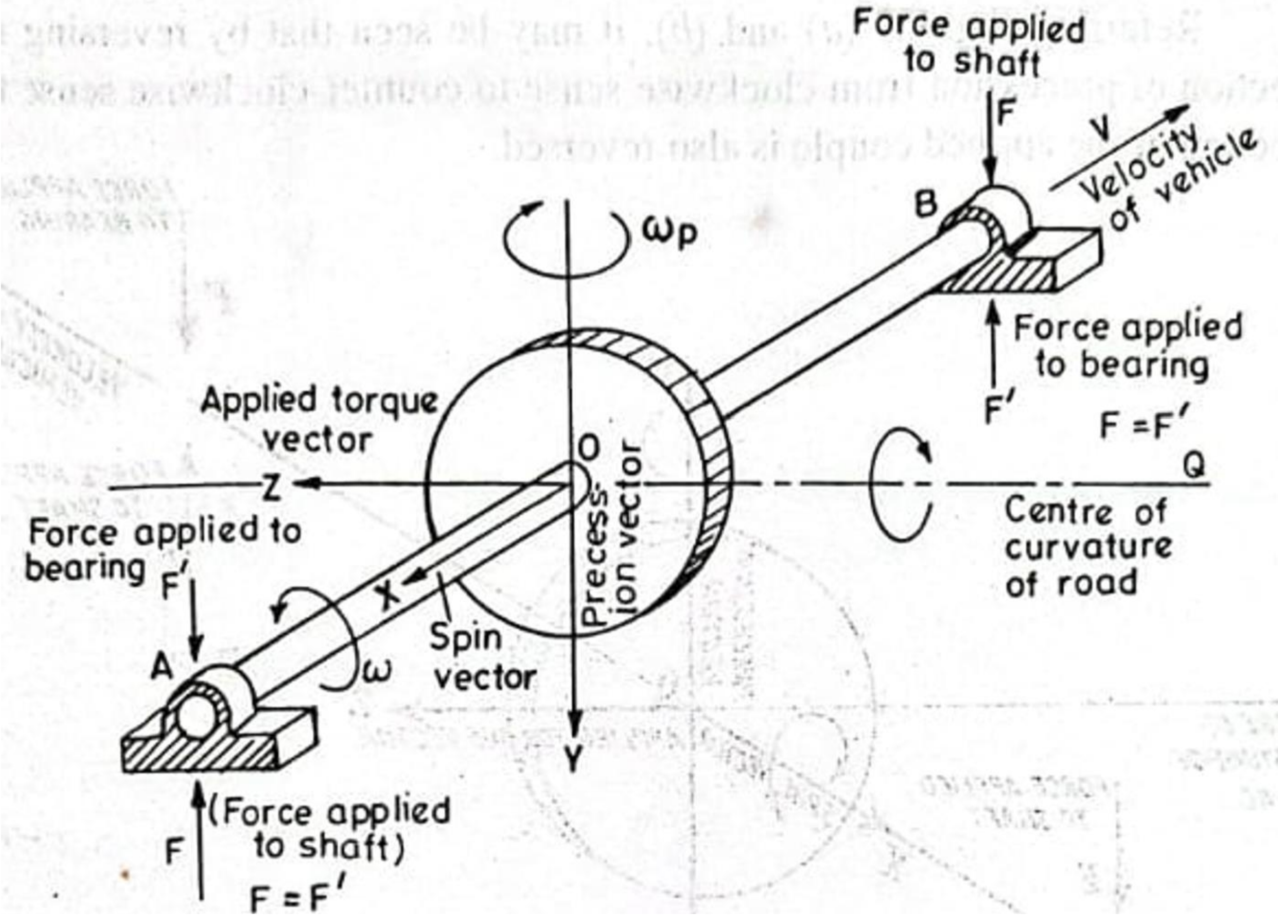
The mass moment of inertia of the disc is I and its radius of gyration is k.

Spin vector is $\overrightarrow{OX}$.

Precession vector is $\overrightarrow{OY}$.

The applied torque vector is $\overrightarrow{OZ}$.



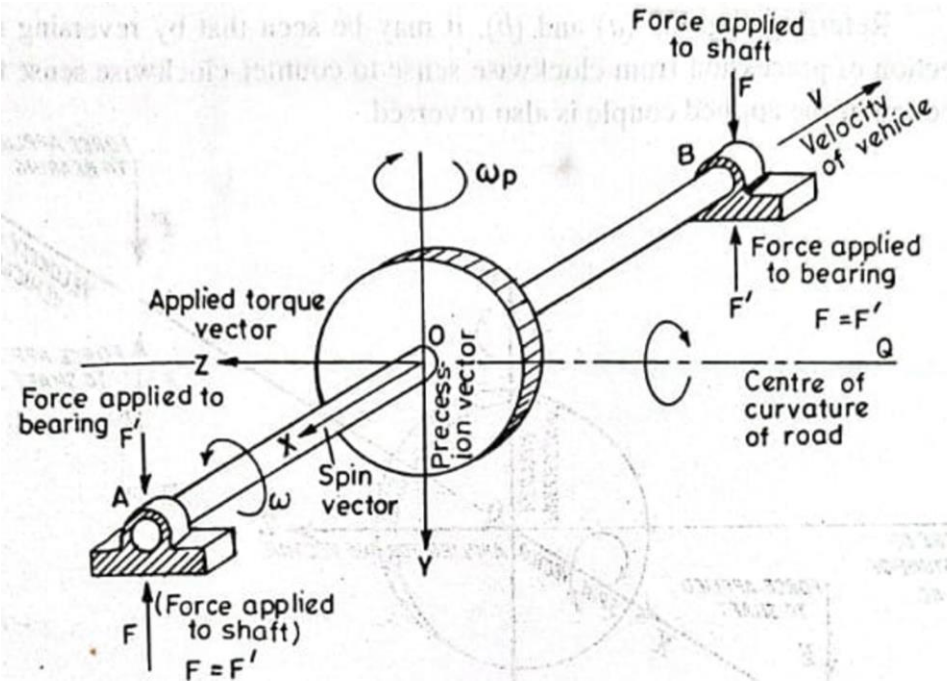


Viewed from the front applied torque vector acts in the clockwise sense.

This torque is applied to the shaft by two forces of magnitude F acting on the shaft through the bearings.

This will set up reaction forces of magnitude $F' = F$, which shaft will apply on the bearings.

The magnitude of the forces are given by,



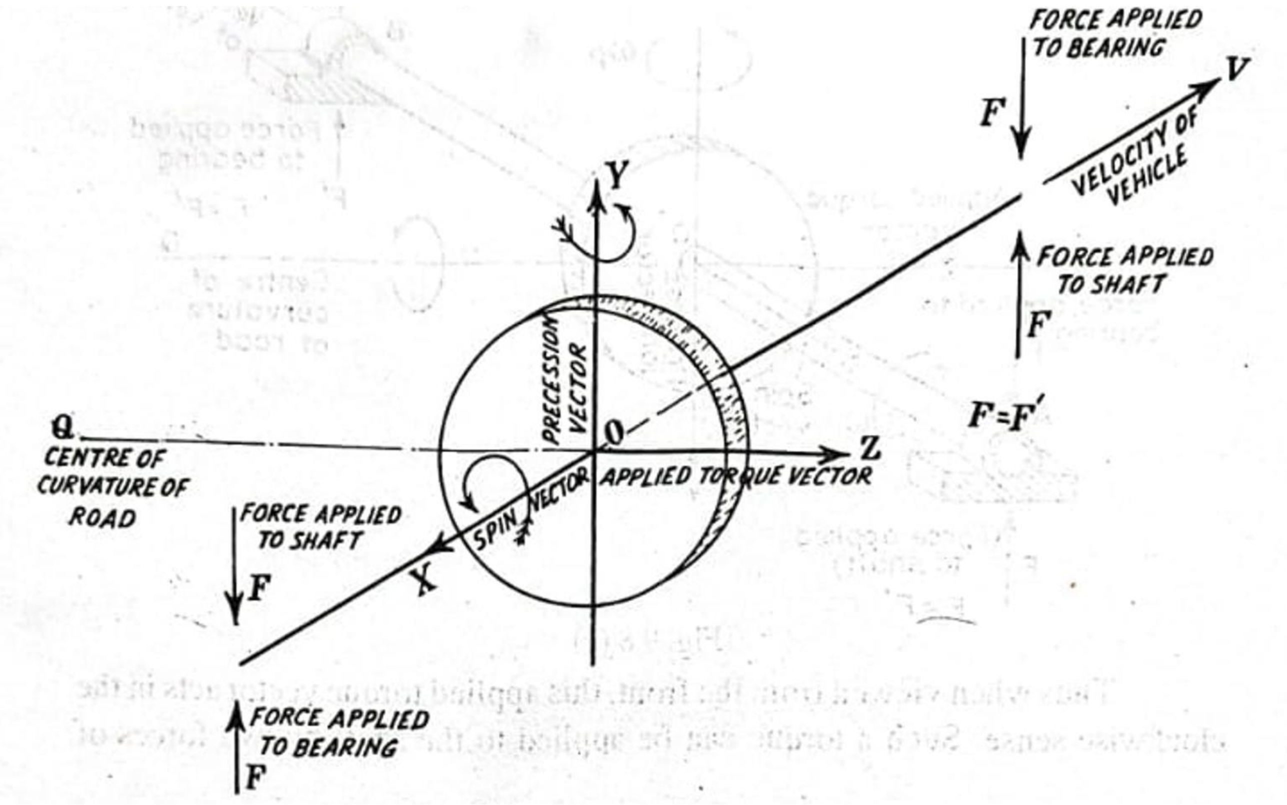
$$Force\ applied\ to\ shaft = \frac{Applied\ torque}{AB}$$

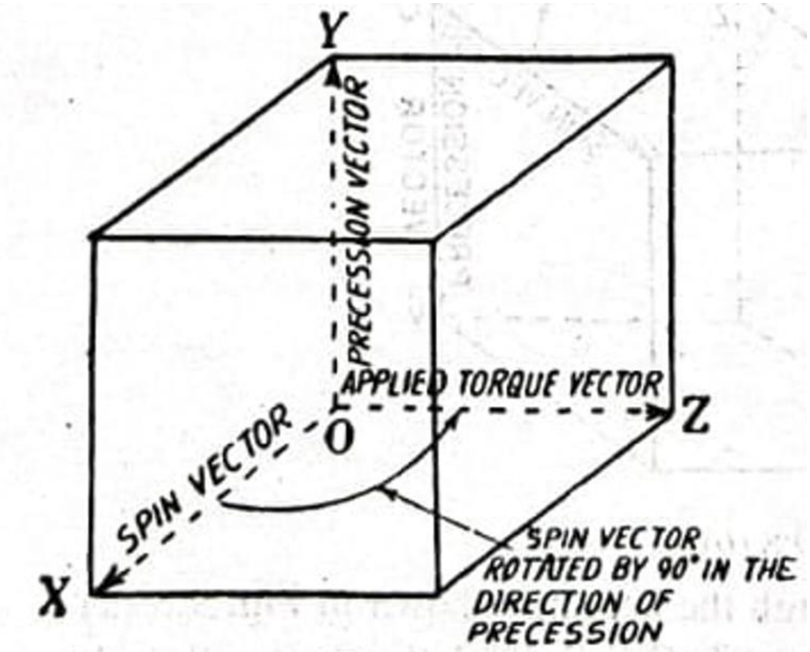
$$F = \frac{I \cdot \omega \cdot \omega_P}{AB} = F'$$

$$\Rightarrow\ Gyroscopic\ applied\ couple\ =\ Gyroscopic\ reaction\ couple$$

By reversing the direction of precession from clockwise sense to counter-clockwise sense the direction of the applied couple is also reversed.

OX, OY and OZ are spin vector, precession vector and applied torque vector respectively. The reversal in the direction of torque causes the reversal in the direction of the forces F applied to the shaft.

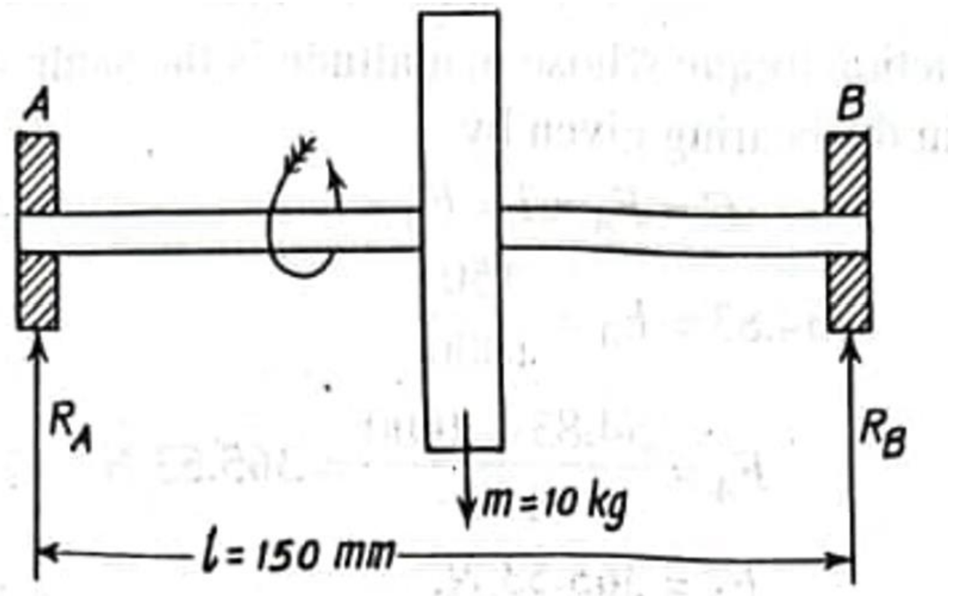




Example 3

A uniform disc of 200 mm diameter has a mass of 10 kg. It is mounted centrally on the horizontal shaft which runs in bearings which are 150 mm apart. The disc spins with a uniform speed of 2000 rev/min in vertical plane in counter-clockwise direction looking from right hand side bearing. The shaft precesses with a uniform velocity of 50 rev/min in the horizontal plane in counter-clockwise direction looking from top.

Determine the reactions at each bearing due to the mass and gyroscopic effects.



Gyroscopic couple acting on the disc

$$C = I\omega\omega_P$$

$$= \frac{10 \times (0.1)^2 \times 2000 \times 50 \times (2\pi)^2}{2 \times 60^2} = 54.83\ Nm$$

Reaction torque causes reaction in the bearing

$$C = F_A \times l = F_B \times l$$

$$F_A = \frac{C}{l} = \frac{54.83}{0.15} = 365.53\ N$$

The resistive torque from bearings acts in counter-clockwise sense as looking from front.

Reaction on bearings due to mass,

$$R_A = R_B = \frac{mg}{2} = \frac{10 \times 9.81}{2} = 49.05\ N$$

Resultant reaction at bearings

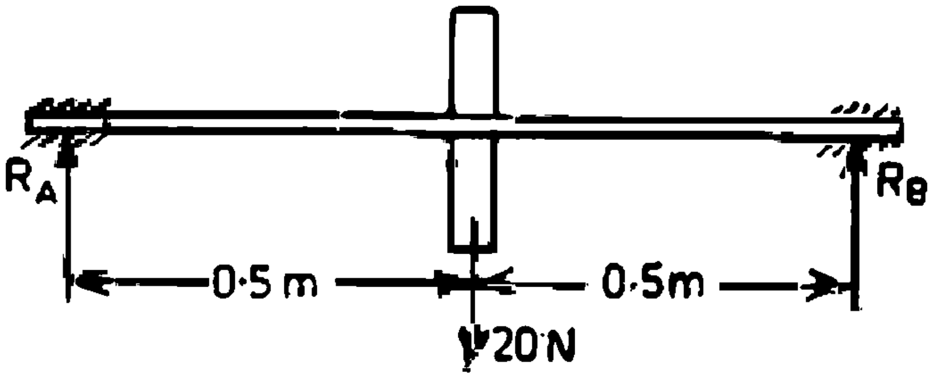
$$R_{A(resultant)} = F' - R_A = 365.53 - 49.05 = 316.48\ N\ (\downarrow)$$

$$R_{B(resultant)} = F' + R_B = 365.53 + 49.05 = 414.58\ N\ (\uparrow)$$

Example 4

As shown in Figure bellow, a disc is mounted on a shaft between two bearings. Mass moment of inertia of disc is $0.05\ Nms^2$, spin velocity is $200\ rad/s$, counter-clockwise looking from right; precessional velocity is $1\ rad/s$, counter-clockwise looking from top. Weight of disc is $20\ N$.

Determine the reactions at bearing R_A and R_B.



$$R_A = 0$$
$$R_B = 20\ N(\uparrow)$$

Example 5

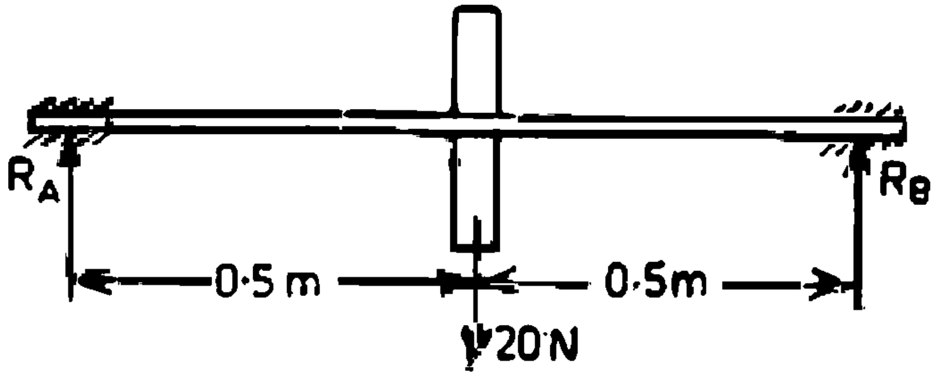
Same data as Example 4, except that the spin velocity direction is clockwise, Determine the reactions at bearing R_A and R_B.

$$R_A = 20\ N\ (\uparrow)$$
$$R_B = 0$$

Example 6

Same data as Example 4, except that the precessional direction is clockwise, Determine the reactions at bearing R_A and R_B.

$$R_A = 20\ N\ (\uparrow)$$
$$R_B = 0$$



Applications

A. In Aeronautics and Aviation:-

1. Remote control flying devices, helicopters, some hovercraft, some planes, etc. rely on gyroscopes to prevent them from flipping over or going into a spin.

2. Spacecraft rely on gyroscopes for orientation while in space.

3. Aircraft and aircraft autopilots rely on gyroscopes to account for changes in direction & altitude

B. In Naval field

1. Gyroscope are used in ships to maintain stability as the effect of gyroscopic couple is on:-

(a.) Steering

(b.) Pitching

(c.) Rolling

C. In automobiles

1. Gyroscopic behaviour is used in the racing car industry. This is because car engines act just like big gyroscopes. Because of the of the gyroscopic forces from the engine depending on whether the engine is spinning clockwise or anti-clockwise the cars nose will be forced up or down.




LIT

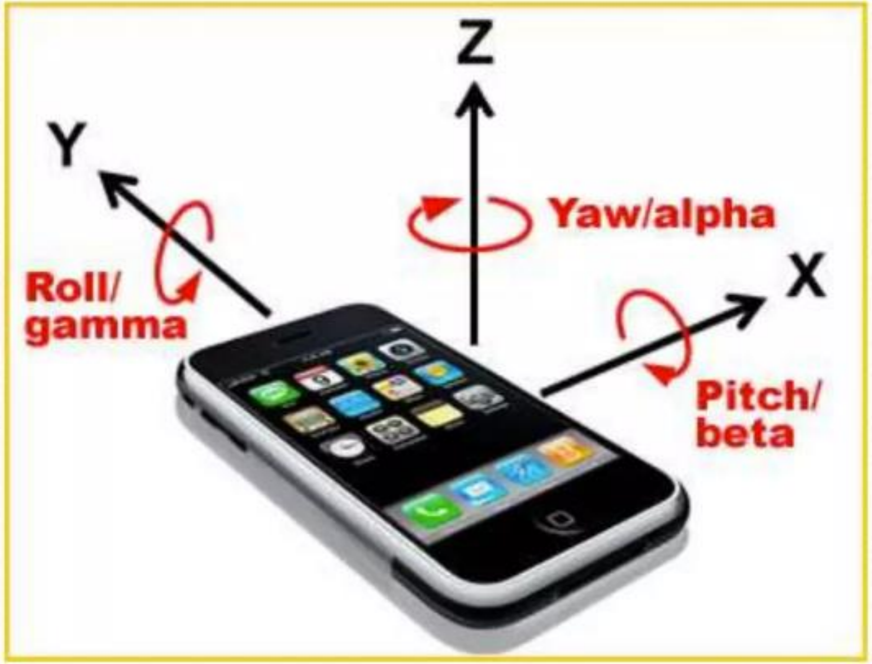

2. Wheels on motorbikes act as gyroscopes and make the bike easier to balance (stay up right) when moving.

D. In toys and everyday uses

1.Gyroscopes are used in various toys such as yo-yos and Frisbees. Some other toy products are Gravitron and ChikyuGoma.

E. In electronics and gadgets

1. Gyroscopes are used in various fields nowadays such as in smartphones, video game controllers, computer mouses and presentation mouses.




GYROSCOPE CONTROL


L3G3200D