Cairo University, I.S.S.R.
Mathematical Department
Statistics / OR Diploma

First Term Exam
2014-2015

[illegible]
[illegible]
[illegible]

Answer the following questions:

Question (1):-

a) Let $u = (-3, 1, 2)$, $v = (4, 0, -8)$ and $w = (6, -1, -4)$, find the component of:

i. $6u + 2v$.
ii. $-3(v - 8w)$.
iii. The distances $d(u, v)$ and $d(v, w)$.
iv. Normalize the vector $w = (6, -1, -4)$.

b) For a diagonal matrix A:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

i. Find A^{-1}.
ii. Show that $(A^5)^{-1} = (A^{-1})^5$.

c) Verify that $\det(AB) = \det(A)\det(B)$ for

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}.$$

Question (2):-

a) Find the value of each of x and y to satisfy that $A = B^T$, where:

$$A = \begin{pmatrix} 1 & -3 & 6 \end{pmatrix}_{1\times 3} \text{ and } B = \begin{pmatrix} 1 \\ -y \\ 3x \end{pmatrix}_{3\times 1}.$$

أنظر بقية الأسئلة بالخلف

Cairo University	Cairo University – Institute of Statistical Studies & Research Department of Mathematical Statistics Academic Year: 2019-2020 Mid Exam. Diploma Level: Summer Term			
Course Title: Linear Algebra	Course Code: MS 506	Time: 1.30 Hours	Exam. Marks: 25	# Exam. Sheets: 1

Answer the following questions

Question (1):

Consider the following four matrices

$$W=\begin{bmatrix}1 & -3\\ 2 & -2\\ 3 & 1\end{bmatrix},\quad X=\begin{bmatrix}3 & 2 & 1\\ -1 & 2 & 3\\ 2 & -1 & 1\end{bmatrix},\quad Y=\begin{bmatrix}1 & 2 & 0\\ 0 & -1 & 2\end{bmatrix}\ \text{and}\ Z=\begin{bmatrix}0 & 1\\ 2 & 3\end{bmatrix}.$$

Compute the following (if exists)

a) W^T b) W^T+Y c) $2W$

d) YW e) $|YW|$ f) $(YW)^{-1}$

g) $|Z|$ h) Z^{-1} i) $|X|$

j) X^{-1} using Gauss elimination technique.

Question (2): Solve the following questions

1] Find the determinant of the following matrix:

$$B=\begin{pmatrix}2 & 3 & -1 & 5\\ 0 & -3 & 1 & 6\\ 1 & 3 & -1 & 2\\ 3 & 6 & -2 & 0\end{pmatrix}.$$

2] If $A^2=0$, show that $(I-A)(I+A)=I$.

3] Find x, y, z, t where

$$3\begin{bmatrix}x & y\\ z & t\end{bmatrix}=\begin{bmatrix}x & 6\\ -1 & 2t\end{bmatrix}+\begin{bmatrix}4 & x+y\\ z+t & 3\end{bmatrix}.$$

4] Suppose that $B=\begin{bmatrix}4 & x+2\\ 2x-3 & x+1\end{bmatrix}$ is a symmetric matrix. Find x and B.

"Best Wishes" Dr: Heba Fathy Mohamed

Cairo University, I.S.S.R.
Mathematical Department

Midterm Exam
2014-2015

Linear Algebra
Math [illegible]
Time: 1.5 hours

Answer the following questions:

Question (1):-

Let $A=\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

Compute the following (if they exist).

i. AB and BA.

ii. $A+C$.

iii. B^{-1}.

iv. The rank of matrix C.

Question (2):-

a) Determine the inverse of the following matrix if it exists, using the method of Adjoint.

$$A=\begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}.$$

b) If the matrix $A=\begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B^T + C^T = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$. Find the matrix X such that $X = (AB + AC)^T$.

"With my best wishes"

Dr. Salwa M. Assar

Cairo University

Cairo University – Institute of Statistical Studies & Research

Department of Mathematical Statistics

Academic Year: 2016-2017		First Semester		
Date: 11/1/2017		Diploma (Stat)		
Course Title: Linear Algebra	Course code: MS 506	Time: 3 Hours	Exam marks: 75	# Exam. Sheets: 2

Exam. Instructions :

Question One: (21 Marks)

a) Using the matrices

$$B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}, \text{ verify that:}$$

1. $(B^T)^{-1} = (B^{-1})^T$
2. $(B+C)^T = B^T + C^T$
3. $(BC)^{-1} = C^{-1}B^{-1}$

b) Find the adjoint and inverse matrices for a matrix A:

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}.$$

Question Two: (16 Marks)

a) Consider the following matrices:

$$A = \begin{bmatrix} -1 & 2 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -8 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

1. Find the product AB.
2. Compute the inverse of A (if possible).

b) For which values of a will the following system of linear equations have exactly one solution?

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

Question Three: (20 Marks)

a) Find the values of λ for which the matrix A is not invertible:

$$A = \begin{pmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{pmatrix}.$$

b) Solve the following system of linear equations by using Cramer's Rule.

$$\begin{aligned} x_1 + \quad\quad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8. \end{aligned}$$

Question Four: (18 Marks)

a) Find the rank of the following matrix [i.e. $r(A)$]

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

b) Solve the following homogeneous system of linear equations by any method

$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 0 \\ x_1 + 2x_2 \quad\quad &= 0 \\ x_2 + x_3 &= 0. \end{aligned}$$

End of Exam

Question Three: (20 Marks)

a) Find the values of λ for which the matrix A is not invertible:

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$$\begin{aligned} 2x_1 + x_2 + 3x_3 &= 0 \\ x_1 + 2x_2 \quad\quad &= 0 \\ x_2 + x_3 &= 0. \end{aligned}$$

End of Exam

b) Solve the system of linear equations by inverting the coefficient matrix (A^{-1}):-

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 5 \\ 2x_1 + 5x_2 + 3x_3 &= 3 \\ x_1 \qquad\quad + 8x_3 &= 17 \end{aligned}$$

Question (3):-

a) Solve the following system of linear equations by using Gauss-Jordan elimination.

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0. \end{aligned}$$

b) Consider the vectors $u = (2,3)$ and $v = (5,-7)$. Find

i. The dot product $u.v$

ii. The cosine of the angle θ between u and v.

With My Best Wishes

Dr. Salwa Mahmoud.

(5)

Question Three: (18 Marks)

(a) If $(I + 2A) = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$, use the given information to find the matrix A where I is the 2×2 identity matrix.

(b) For the following system of linear equations

$$\begin{aligned} x_1 \quad\quad\quad &= -1 \\ -x_1 + 2x_2 + 3x_3 &= 4 \\ x_2 + 2x_3 &= 1 \end{aligned}$$

$x_1 = -1$

$x_2 = 1 - 2x_3$

$1 + 2x_2 - 3 = 4$

$2x_2 - 2 = 4$

$2x_2 = 2 = 1$

$1 + 2(1 - 2x_3) + 3x_3 = 4$

$1 + 2 - 4x_3 + 3x_3 = 4$

$3 - 4x_3 + 3x_3 = 4$

$3 - x_3 = 4$

$3 - 4 = x_3$

$x_3 = -1$

(i) Write the system of linear equations in the form $AX = b$.

(ii) Use Guass-Jordan method to find A^{-1}.

(iii)Use A^{-1} to solve the system of linear equations.

Question Four: (18 Marks)

(a) Find the rank of the following matrix A (i.e. $r(A)$).

$$A = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(b) Consider the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Find the eigenvalues and the corresponding eigenvectors of matrix A.

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$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \lambda-2 & 1 \\ 1 & \lambda-2 \end{bmatrix}$

$\det(A) = \begin{bmatrix} \lambda-2 & 1 \\ 1 & \lambda-2 \end{bmatrix}$

$(\lambda-2)(\lambda-2)-1$

$= \lambda^2 - 2\lambda - 2\lambda + 4 - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda-1)(\lambda-3)$

$\lambda = 1 \quad \lambda = 3$

$\begin{bmatrix} 1-2 & 1 \\ 1 & 1-2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \lambda_1$

$\begin{bmatrix} 3-2 & 1 \\ 1 & 3-2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \lambda_2$

$\lambda_1 = 1$

$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}$

$x_1 - x_2 = 0$

$x_1 = x_2$

$x_2 = t$

$v \begin{bmatrix} t \\ t \end{bmatrix} t = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 3$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{-R_1+R_2} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$x_1 + x_2 = 0$

$x_1 = -x_2$

$v \begin{bmatrix} -t \\ t \end{bmatrix} t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

	Cairo University–Faculty of Graduate Studies for Statistical Research		
Cairo University	Department of Mathematical Statistics		
	Academic Year: 2019-2020	Summer Semester	
	Date: 1-10-2019	Diploma Level	

Course Title: Linear Algebra	Course Code: MS 506	Time: 3 Hours	Exam. Marks: 75	# Exam. Sheets: 2

Answer the Following Questions:

Question One: (21 Marks)

(a) Let

$$A = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 4 & -5 \end{bmatrix}$$

(i) Verify that

1. $(AB)^{-1} = B^{-1}A^{-1}$. 2. $(A+B)^T = A^T + B^T$.

(ii) Find $tr(A)$.

(b) For the following matrix

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

(i) Find the adjoint matrix of A.

(ii) Use the adjoint defined in (i) to find A^{-1}.

Question Two: (18 Marks)

(a) Find the value of x for which the matrix A is not invertible (singular)

$$A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$$

(b) Use Cramer's rule to solve the following system of linear equations

$$2x + 3y - z = 1$$
$$4x + y - 3z = 11$$
$$3x - 2y + 5z = 21$$

x = 4, y = 0.9 z = 4.6