MS506

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lecture 1

- Midterm 25%
- Final 75%

Book :-
- Elementary linear Algebra

Authors :
- Howard Anton &
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DR. Marwa :
Chapter 1, 2, 3 11th Edition or 9th

1. Matrices (Matrix)
- operation on Matrices
- properties of Matrices operations
- the transpose, Inverse of Matrix

2. System of linear equations
- Gaussian elemintaion Method
- Gauss - Jordan

3. Determinats

4. Vector space

$$\begin{array}{c} \\ 1st \\ 2nd \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 2 & 0.04 & 5 \\ 3 & 0 & 1 & 2 \end{bmatrix} \end{array}$$

named element → and must be small letter.

- Matrix must be named by Capital letters A, B

- Matrix must have size "rows & Coulmns"

$A = \underset{m \times n}{2 \times 4} \rightarrow$ size

- General Form:-

element

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ a_{31} & a_{32} & a_{33} & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ a_{mn} & a_{m2} & a_{m3} & a_{mn} \end{bmatrix}$$

ex.

let $\underset{2\times2}{A} = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

- square Matrix →

$a_{11} = 2$ $a_{21} = 1$

$a_{12} = 3$ $a_{22} = 1$

- Square Matrix:

is amatrix the rows numbers are equal Coulmns numbers.

- Rectangle Matrix:

is amatrix the rows numbers are unequal Coulmns numbers.

- Two Matrices A & B

- are equal if they have the same size and $a_{ij} = B_{ij}$.

ex.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

A & B are equal? No $\quad A \neq B$

ex.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1}$$

- A → Row Matrix = Row vector
- B → Coulmns Matrix = Coulmn vector

- operations on Matrices :-

1. Addition (+) → if $A = [a_{ij}]$ & $B = [b_{ij}]$

- are matrices of the same size $m \times n$, then their sum is matrix given by

$$[A+B] = [a_{ij} + b_{ij}]$$

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ex.

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{Zero Matrix or } [0]$$

Find $\Rightarrow$ 1. $A+B \rightarrow 2\times2 + 2\times2$
2. $A+D \rightarrow$ is undefined Matrix
3. $C+D \rightarrow 2\times3 + 2\times3$

$$A+B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$$

$$C+D = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} \rightarrow C$$

2. subtraction → like addition

$$A-B=\begin{bmatrix}-1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}=\begin{bmatrix}-2 & -1\\ 1 & -1\end{bmatrix}$$

3. scalar multiplication

$$C\cdot A\begin{bmatrix}Ca_{11} & Ca_{12} & \cdots & Ca_{1n}\\ Ca_{21} & Ca_{22} & \cdots & Ca_{2n}\\ \vdots & \vdots & & \vdots\\ Ca_{m1} & Ca_{m2} & \cdots & Ca_{mn}\end{bmatrix}$$

ex. let → $C=5$, $A\begin{bmatrix}1 & 2 & 3\\ 4 & 1 & -1\end{bmatrix}$

Find $CA = 5\times\begin{bmatrix}1 & 2 & 3\\ 4 & 1 & -1\end{bmatrix}$

$$=\begin{bmatrix}5 & 10 & 15\\ 20 & 5 & -5\end{bmatrix}$$

4. Matrix Multiplication

if → $A = [a_{ij}]$ is an $m \times n$ Matrix and

$B = [b_{ij}]$ is an $n \times p$ Matrix then the

prouduct of $A \cdot B$ is an $m \times p$ Matrix.

$$AB = [C_{ij}]_{m \times p}$$

$$C_{ij} = \sum_{K=1}^{n} a_{iK} \, b_{Kj}$$

$$\underset{m \times n}{A} \times \underset{n \times P}{B}$$

$\boxed{\text{note:-}}$ if n of a equal n of B then the result will be $m \cdot p$

- to make matrix multiplication must be the Coulmns of matrix one must be equal the rows of matrix two.

ex.

let $A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}$

Find → $\underset{2\times 2}{AB}$ & $\underset{2\times 2}{BA}$ → undefined

$$AB = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \times \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} =$$

- First row X First Coulmns

$$\begin{bmatrix} -1\times -3 + 3\times -4 & -1\times 2 + 3\times 1 \\ 4\times -3 + -2\times -4 & 4\times 2 + -2\times 1 \\ 5\times -3 + 0\times -4 & 5\times 2 + 0\times 1 \end{bmatrix} =$$

$$\begin{bmatrix} -9 & -1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$$

- properties of matrix addition and scalar multiplication

- let A, B, C are m x n matrices &
s, d are scalar

1. $A+B = B+A$

2. $(A+B)+s = A+(B+s)$

3. $(sd)A = s(dA) = d(sA)$

4. $s(A+B) = sA+sB$

5. $(s+d)A = sA+dA$

6. $A+0 = 0+A = A$

7. $A+(-A) = 0$

8. if $sA = 0$ then
$s=0$ or $A=0$

9. $I(A) = A$

- Identy Matrix $\Rightarrow$ square matrix the Major diameter = 1

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad , I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Major Diameter : a_{11}, a_{22}, a_{33}

$$\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 3 \\ 4 & 5 & 1 \end{bmatrix}$$

- properties of matrix multiplication

let A, B, C are $m \times n$ matrices & s, d are scalars

1. $A(BC) = (AB)C$

2. $A(B+C) = AB + AC$

3. $(A+B)C = AC + BC$

4. $s(AB) = (sA)B = A(sB)$

5. $AB \neq BA$, 6. $AI = IA = A$

ex. let $A = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 8 \end{bmatrix}$, $B \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}$

Find

same size of A

1. $A+O$, 2. BO, 3. BI

$$1.\ A+O = \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & -3 \\ 4 & 5 & 8 \end{bmatrix} \rightarrow A$$

$$2.\ B\,O = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2x2 / 2x1

any size but must rows = Coulmns

$$3.\ BI = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \rightarrow B$$

must be size of B

let $A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -7 \\ 2 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$

Find $\rightarrow$ $2A + 3B - 5C$

$$= 2\begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix} + 3\begin{bmatrix} 3 & -7 \\ 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 \\ -8 & 10 \end{bmatrix} + \begin{bmatrix} 9 & -21 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 10 \\ 15 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & -25 \\ -17 & 18 \end{bmatrix}$$

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ex. $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Find $\underset{2\times3\ 3\times2}{AB}$, $\underset{3\times2\ 2\times3}{BA}$, ABC

Solve → $AB = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 11 \\ 2 & 2 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & -1 \\ 0 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 8 & 4 & 0 \\ 4 & 7 & 0 \end{bmatrix}$$

$$ABC = AB \cdot C = \underset{2\times2}{\begin{bmatrix} 1 & 11 \\ 2 & 2 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 1 \\ -2 \end{bmatrix}} = \begin{bmatrix} -21 \\ -2 \end{bmatrix} \text{ or } -1 \begin{bmatrix} 21 \\ 2 \end{bmatrix}$$

Find a, b, c, d such that

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

$a - b = 8 \rightarrow a = 8 + b$

$b + c = 1 \rightarrow c = 1 - b$

$3d + c = 7 \rightarrow 3d + 1 - b$

$2a - 4d = 6 \rightarrow 2(8+b) - 4d$

$3d - b + 1 = 7 \quad = \quad 3d - b = 6 \qquad \times 2$

$16 + 2b - 4d = 6 \quad = \quad 2b - 4d = -10 \qquad +$

$$\begin{array}{r} 6d - \cancel{2b} = 12 \\ -4d - \cancel{2b} = -10 \\ \hline \end{array}$$

$\frac{2d}{2} = \frac{2}{2} = d = 1$

$3d - b = 6$

$3(1) - b = 6$

$3 - b = 6$

$b = -3$

$a - b = 8$

$a - (-3) = 8$

$a + 3 = 8$

$a = 8 - 3$

$a = 5$

$c = 1 - b$

$c = 1 - (-3)$

$c = 4$

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- Linear → unknown power 1

- power of matrices

$A^K = A \times A \times A \ldots\ldots A$ → K times

- properties of matrices powers:-

if r & s are non negative integers then

1. $A^r \cdot A^s = A^{r+s}$

2. $(A^r)^s = A^{r \cdot s}$

3. $A^0 = I$

ex. simplify the following matrix expression

$A(A+2B)+3B(2A-B)-A^2+7B^2-5AB$

$A^2+2AB+6BA-3B^2-A^2+7B^2-5AB$

$6BA-3AB+4B^2$