$\Rightarrow$ Consensus term (for minimization).

$\Rightarrow A t_1 \,\&\, A' t_2 \Rightarrow t_1 t_2$

$\Rightarrow A'BC + ACd \Rightarrow BCd$ is a Consensus term.

$\Rightarrow A'BC + AB'Cd \Rightarrow$ there isn't any Consensus term because more than (variable & its Complement) are found.

EX:

Simplify:

$F = a'b'c' + a'bc' + a'bc + ab'c'$

$= b'c'(a' + a) + a'b(c' + c)$

$= b'c' + a'b$ (Consensus Term لايوجد)

EX:

Simplify: $F = a'bc' + abc' + abd + acd$

$= bc'(a' + a) + abd + acd$

$= bc' + \cancel{abd} + acd$

(abd) is a Consensus Term

حد مشترك

$= bc' + acd$

EX:

Simplify: $F = \underline{A'BCD} + \underline{A'BC'D} + \underline{B'EF} + CDE'G + A'DEF + \underline{A'B'EF}$

$$\underbrace{A'BCD + A'BC'D}_{A'BD} \qquad \underbrace{B'EF + A'B'EF}_{B'EF}$$

$= A'\underline{B}D + \underline{B}'EF + CDE'G + \cancel{A'DEF}$

$$\underbrace{A'BD \; ¢ \; B'EF}_{A'DEF}$$

$A'DEF$ is a consensus term

نحذفه لأنه Consensus Term ناتج من $A'BD \, ¢ \, B'EF$

$F = A'BD + B'EF + CDE'G$

EX:

Minimize:

$¢ \; XYZ$

$F = W'Y' + \underline{W}'XZ + \underline{W}XY + WYZ'$

علينا إيجاد Consensus term لتسهيل ال Simplification

$F = W'\underline{Y}' + W'XZ + WXY + WYZ' + X\underline{Y}Z$ ← تم إضافته

نحذفه

$¢ \; W'XZ$

$F = W'Y' + WXY + WYZ' + XYZ$

نحذفه

$¢ \; WXY$

$F = W'Y' + WYZ' + XYZ$

Simplify:

No. 9) Assignment 1

$$(A+B)(A'+C)$$

$$= \cancel{A.A'}_{0} + AC + A'B + BC$$

$$= AC + A'B + \cancel{BC}$$

$\Phi\ BC$

$$= AC + A'B$$

No. 10) Assignment 1

$$F = a + a'b + a'b'c + (a'b'c'd + a'b'c'd'e)$$

$a'b'c'(\quad)$

$$= a + a'b + a'b'c + a'b'c'(d + d'e)$$

$$\underbrace{(d+d')}_{1}(d+e)$$

$$= a + a'b + [a'b'c + a'b'c'(d+e)]$$

$a'b'(\quad)$

$$= a + a'b + a'b'(c + c'(d+e))$$

$$\underbrace{(c+c')}_{1}(c+d+e)$$

$$= a + [a'b + a'b'(c+d+e)]$$

$$= a + a'(b + b'(c+d+e))$$

$$\underbrace{(b+b')}_{1}(b+c+d+e)$$

$$= a + a'(b+c+d+e)$$

$$\underbrace{(a+a')}_{1}(a+b+c+d+e)$$

$$= (a+b+c+d+e)$$

Demorgan's Theorems:

1) $(A \cdot B)' = A' + B'$

2) $(A + B)' = A' * B'$

Ex:

Find the Complement of $F = a \cdot bd' + b'c' + a'cd + a'bc'd$

Solution

$$F' = (abd' + b'c' + a'cd + a'bc'd)'$$

$$= [(abd')' \cdot (b'c')' \cdot (a'cd)' \cdot (a'bc'd)']$$

$$= (a' + b' + d)(b + c) \cdot (a + c' + d')(a + b' + c + d')$$

Ex:

if $f = (a+b)(b'+c) + d'(a'b+c)$ find f'

Solution

$$f' = [(a+b)(b'+c) + d'(a'b+c)]'$$

$$= [(a+b)(b'+c)]' \cdot [d'(a'b+c)]'$$

$$= [(a+b)' + (b'+c)'][d + (a'b+c)']$$

$$= (a'b' + bc')(d + (a+b') \cdot c')$$

$$= (a'b' + bc')(d + ac' + b'c')$$

Date:..............

Simplify..

No. 13) Assignment 1 $F = AB + (AC)' + AB'C(AB + C)$

$= AB + (A' + C') + \cancel{AB'C \cdot AB}_{0} + AB'C \cdot C$

$= (AB + A') + (C' + AB'C)$

$= \underbrace{(A + A')}_{1}(A' + B) + \underbrace{(C' + C)}_{1}(C' + AB')$

$= A' + B + C' + AB'$

$= (A' + AB') + B + C'$

$= \underbrace{(A' + A)}_{1}(A' + B') + B + C'$

$= A' + \underbrace{(B' + B)}_{1} + C' = A' + 1 + C' = \boxed{1}$

lecture 2 slide 82 $F = \bar{c}((a+b)' + bcd' + ab\bar{c})(b\bar{c}d)$

$= b\bar{c}d(a'b' + bcd' + ab\bar{c})$

$= \underbrace{\cancel{b\bar{c}d \cdot a'b'}}_{0} + \underbrace{\cancel{b\bar{c}d \cdot bcd'}}_{0} + b\bar{c}d \cdot ab\bar{c}$

$= \boxed{ab\bar{c}d}$

From SOP to POS & VS

⇒ To convert from SOP (Sum of Product) to POS (Product of Sum) and VS. , We use These Rules :-

1) $a(b+c) = ab + ac$

2) $a + bc = (a+b)(a+c)$

3) $ab' + a'c = (a+c)(a'+b)$

EX:

Convert from SOPs to POSs :-

$F = wxy' + xyz + w'x'z'$

Solution:

$F = x\cdot(wy' + yz) + w'x'z'$

$= x\cdot(wy' + yz) + x'\cdot w'z'$

$= (x + w'z')(x' + (wy' + yz))$

$= (x + w')(x + z')(x' + (w+y)(y'+z))$

$= (x + w')(x + z')(w + x' + y)(x' + y' + z)$

EX:

Convert from SOPs To POSs:

$$x' + yz' + y'z$$

$$x' + (y+z')(y'+z)$$

$$(x'+y+z')(x'+y'+z)$$

EX:

Convert from SOPs To POS:

$$a'c'd + a'cd' + bc$$

$$a'c'd + c(a'd' + b)$$

$$(c + a'd) \cdot (c' + (a'd' + b))$$

$$(a'+c)(c+d)(c' + (a'+b)(b+d'))$$

$$(a'+c)(c+d)(a'+b+c')(b+c'+d')$$

EX: $wxy' + xy'z + wx'z'$

$$x \cdot (wy' + y'z) + x' wz'$$

$$(x + wz')(x' + (wy' + y'z))$$

$$(x+w)(x+z')(x' + (y'(w+z)))$$

$$(x+w)(x+z')(x'+y')(x'+w+z)$$

EX:

Given the following expression:

$F(a,b,c,d) = bc'd' + cd + bc'd + abc + bd + abc'd$

a) find the minimum Sum of Products.

b) find the minimum Product of Sums.

Solution:

a) Minimum SoPs:

$F = bc'd' + cd + bc'd + abc + bd + abc'd$

bc' $\qquad\qquad$ ab

$= bc'(\cancel{d'+d}) + cd + ab(c + c'd) + bd$

$= bc' + cd + ab(\cancel{c+c'})(c+d) + bd$

$= bc' + cd + abc + \underbrace{abd + bd}_{bd}$

$= bc' + cd + abc + bd(\cancel{a+1})$

$\cancel{c}$

bd يحذف

$= bc' + cd + abc$

$= b(c' + ac) + cd$

$= b(c'+a)(\cancel{c'+c}) + cd$

$= bc' + ab + cd$

b) Minimum PoS:

$$F = b\grave{c} + cd + ab$$

$$= (\grave{c} + d)(b + c) + ab$$

$$= (ab + (\grave{c} + d))(ab + (b + c))$$

$$= (a + \grave{c} + d)(b + \grave{c} + d)(a + b + c)(b + b + c)$$

$$= (a + \grave{c} + d)(b + \grave{c} + d)\underline{(a + b + c)}\,\underline{(b + c)}$$

$$= (a + \grave{c} + d)(b + \grave{c} + d)[(b + c) + (\underbrace{a \cdot 1}_{a})]$$

$$= (a + \grave{c} + d)(b + \grave{c} + d)(a + b + c)$$

Ex:

Given the following expression :

$$F = a'b'c' + a'bc' + a'bc + ab'c + abc' + abc$$

a) find the Minimum SoP.
b) find the Minimum PoS.

Solution :

a)

$$F = \underbrace{a'b'c' + a'bc'}_{a'c'} + a'bc + ab'c + \underbrace{abc' + abc}_{ab}$$

$$= \underbrace{a'c' + a'bc}_{a'} + \underbrace{ab'c + ab}_{a}$$

$$= a'(c' + bc) + (a(b'c + b))$$

$$= a'(c' + b)(\cancel{c' + c}) + (a(\cancel{b' + b})(c + b))$$

$$= a'(c' + b) + (a(c + b))$$

$$= a'c' + a'b + ac + ab$$

$$= a'c' + b(\cancel{a' + a}) + ac$$

$$= a'c' + ac + b$$

$\Rightarrow$

b) The Minimum PoS:-

$$F = a\grave{}\,.\,c + a\,c\grave{} + b$$

$$= (a\grave{} + c)(a + c\grave{}) + b$$

$$= (a\grave{} + b + c)(a + b + c\grave{})$$

⇒ Manipulate the following to Sum of Product expression:-

$$F = (b\grave{} + c)(a\grave{} + c + d\grave{})(c\grave{} + d)\cdot(a + c\grave{} + d\grave{})$$

Solution.

$$F = [c + (b\grave{}\,.\,(a\grave{} + d\grave{}))][c\grave{} + (d\cdot(a + d\grave{}))]$$

$$= [c + (a\grave{}b\grave{} + b\grave{}d\grave{})][c\grave{} + (ad + \cancel{dd\grave{}})]$$

$$= [c + (a\grave{}b\grave{} + b\grave{}d\grave{})][c\grave{} + ad]$$

$$= (c\cdot ad) + (c\grave{}\cdot(a\grave{}b\grave{} + b\grave{}d\grave{}))$$

$$= acd + a\grave{}b\grave{}c\grave{} + b\grave{}c\grave{}d\grave{}$$

From POS to SOP

Rules

1) $(a+b)(a+c) = a+bc$

2) $a(b+c) = ab+ac$

3) $(a+c)(\bar{a}+b) = ab+\bar{a}c$

Ex:

Manipulate the following function to SOP :

$$F = (x+y)(x+\bar{z})(\bar{x}+\bar{y}+z)$$

$$= [x + (y\bar{z})](\bar{x}+\bar{y}+z)$$

$$= x\cdot(\bar{y}+z) + \bar{x}\cdot(y\bar{z})$$

$$= x\bar{y} + xz + \bar{x}y\bar{z}$$

Truth Table (TT)

Ex:

For the function:-

$F = x'y + z$.

a) Show the truth table

b) Show the equation F in:-

Sum of minterms

or Sum of Product (SoP)

or Disjunctive form

c) Show F in:

Product of Maxterms.

or Product of sum (Pos)

or Conjunctive form.

d) Draw the block diagram of "F"

ANS:

(a)

3 Variable (x, y, z)

$2^3 = 8$ Prob.

		x	y	z	x'	$x'y$	$F = x'y + z$
M_0	m_0	0	0	0	1	0	0
M_1	m_1	0	0	1	1	0	$\boxed{1}$
M_2	m_2	0	1	0	1	1	$\boxed{1}$
M_3	m_3	0	1	1	1	1	$\boxed{1}$
M_4	m_4	1	0	0	0	0	0
M_5	m_5	1	0	1	0	0	$\boxed{1}$
M_6	m_6	1	1	0	0	0	0
M_7	m_7	1	1	1	0	0	$\boxed{1}$

b)

The sum of minterms. $\Rightarrow$ from one's of "F"

$\rightarrow$ in numeric form: $m_1 + m_2 + m_3 + m_5 + m_7$

$F(x,y,z) = \Sigma m(1,2,3,5,7)$

$\rightarrow$ in Algebraic form:

$F(x,y,z) = x'y'z + x'yz' + x'yz +$
$xy'z + xyz$

c)

The Product of Maxterms $\Rightarrow$ from Zero's of "F"

$\rightarrow$ in numeric form: $M_0 + M_4 + M_6$

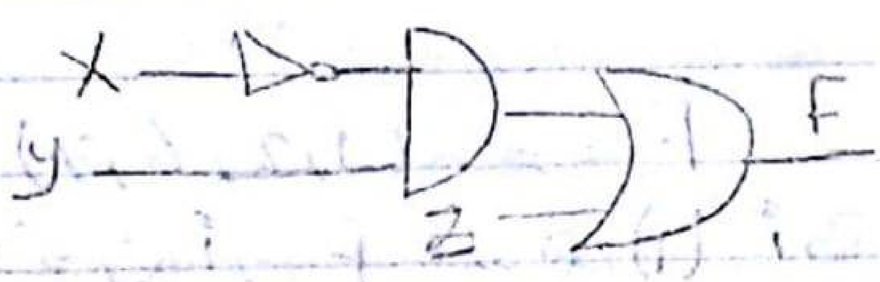
$F(x,y,z) = \Pi(0,4,6)$

$\rightarrow$ in Algebraic form:

$F(x,y,z) = (x+y+z)(x'+y+z)(x'+y'+z)$

d)

$F = x'y + z$



Note

The Sum of minterms
Called:
(Canonical form)

ونعني أن كل ترم بـ كل الـVariable الموجودة بالـ "F"
X, y, Z

Ex:-

For the function:

$F = \sum m(1, 2, 6, 7)$.

a) Show the truth table.

b) Show an algebraic expression in sum of product form.

c) Show the minimum sum of product expression

d) Show the minterms of F' (Complement of F) in numeric form & in Algebraic form.

ANS

a)

from $0 \rightarrow 7$ is 3 Var.

let that 3 variable are a, b, c

m	a	b	c	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

$F = \sum m(1, 2, 6, 7)$

تعني أن F عندها (1) أي:

تتحقق عند ~~ط~~

m_1

or m_2

or m_6

or m_7

b) Algebraic form in SOP:

$F(a, b, c) = a'b'c + a'bc' + abc' + abc$

c) The minimum of SOP:

$$F(a,b,c) = a'b'c + a'bc' + \underbrace{abc' + abc}_{ab}$$

$$= a'b'c + \underbrace{a'bc' + ab}_{b}$$

$$= a'b'c + b(a'c' + a)$$

$$= a'b'c + b(\underbrace{\cancel{a'+a}}_{1})(c' + a)$$

$$= a'b'c + b(c' + a)$$

$$= a'b'c + ab + bc'$$

d) The minterms of f' in:

numeric form ⇒ From Truth table "from 0's"

$$f'(a,b,c) = \sum m(0,3,4,5)$$

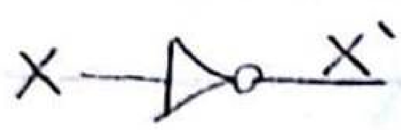
Algebraic form ⇒ Every 0's term as SOP.

$$f'(a,b,c) = a'b'c' + a'bc + ab'c' + ab'c$$

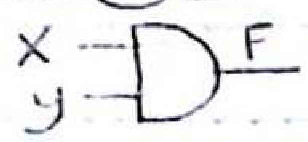
* Gates

⇒ Schematic of AND, OR, Not, NAND, NOR, XOR and XNOR

NOT gate



AND gate



$F = x \cdot y$

AND $\frac{0}{0}{0}$ $\frac{0}{1}{0}$ $\frac{1}{0}{0}$ $\frac{1}{1}{1}$

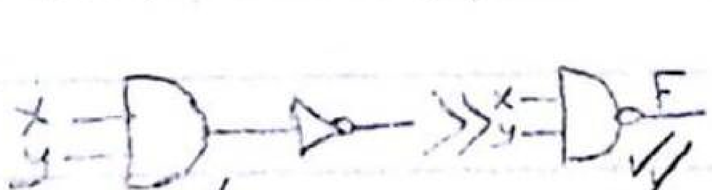
AND	0,0	0,1	1,0	1,1
	0	0	0	1

NAND gate

⇒ Not AND

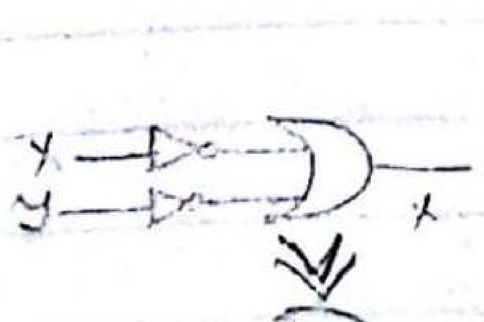
⇒ AND Then AND

⇒ Complement of AND

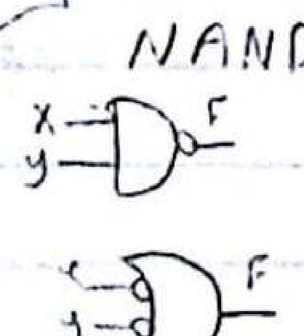


$F = (x \cdot y)'$

$= x' + y'$



2 shape for NAND



OR gate

$F = x + y$

OR	0,0	0,1	1,0	1,1
	0	1	1	1

NOR gate

XOR

$F = x \oplus y$

$= x'y + xy'$

XOR	0,0	1,1	0,1	1,0
	0	0	1	1

الاختلاف 1

التشابه 0

XNOR

$F = (x \oplus y)'$

$= x'y' + xy$

⇒ show the block diagram of :

or ⇒ show the schematic for the following function :

a) $F = ab'd' + bde' + bc'd + a'ce$

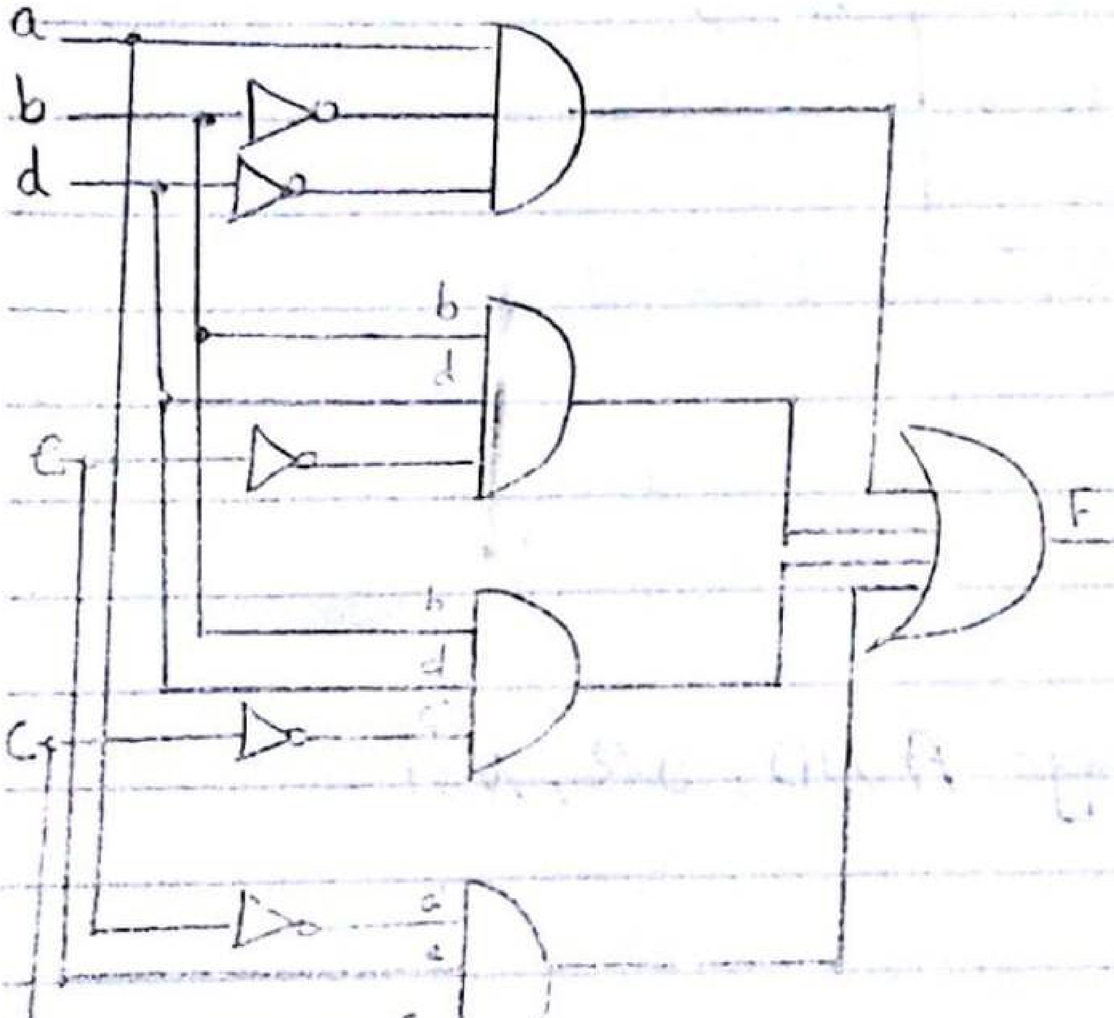
b) $g = b(c'd + c'e') + (a + ce)(a' + b'd')$

① using AND, OR, NOT

② using only NAND

ANS

a)



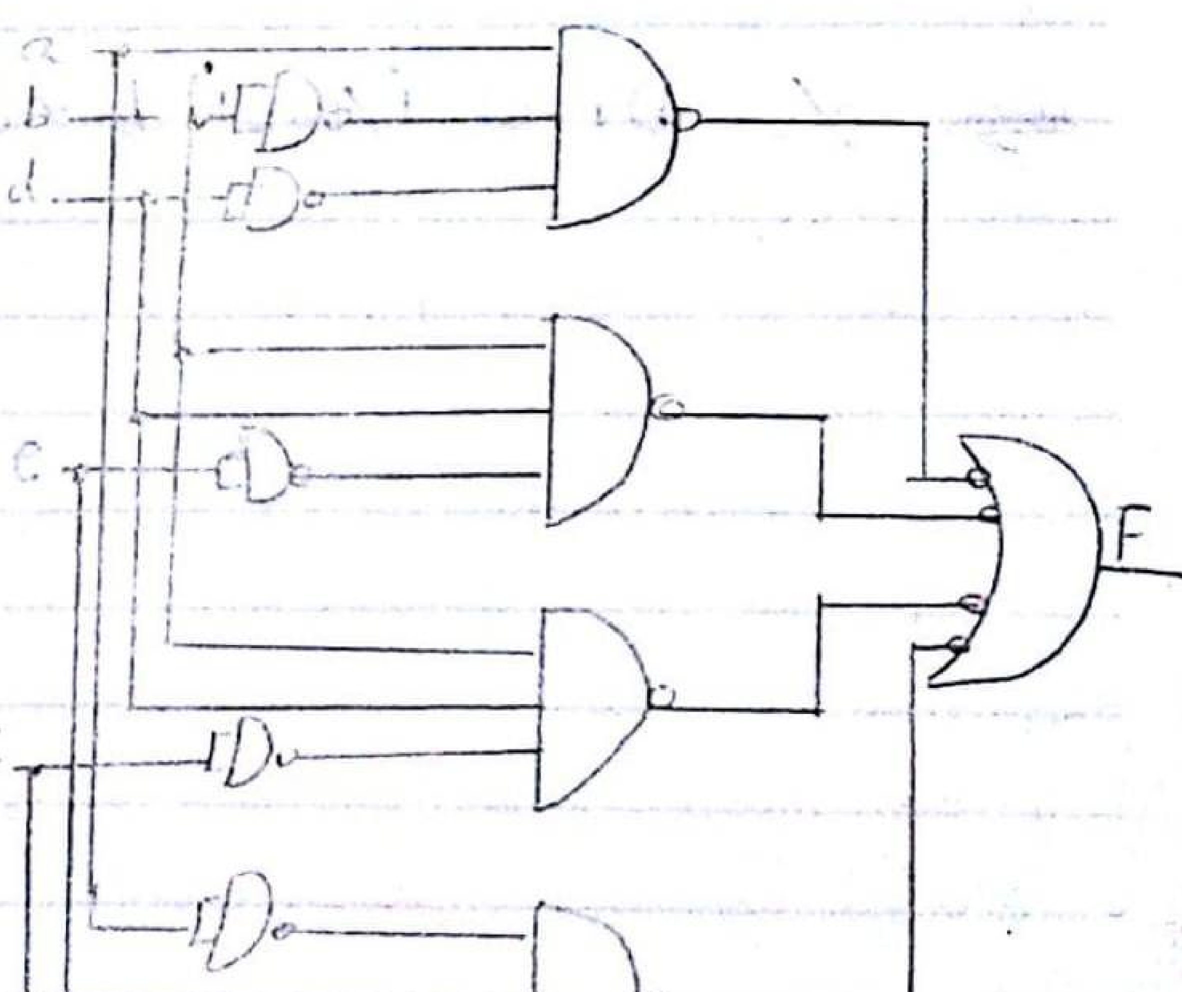
①by using NOT, AND, OR

To use NAND

Draw by AND, OR, NOT Then convert each AND to NAND

& each OR to NAND

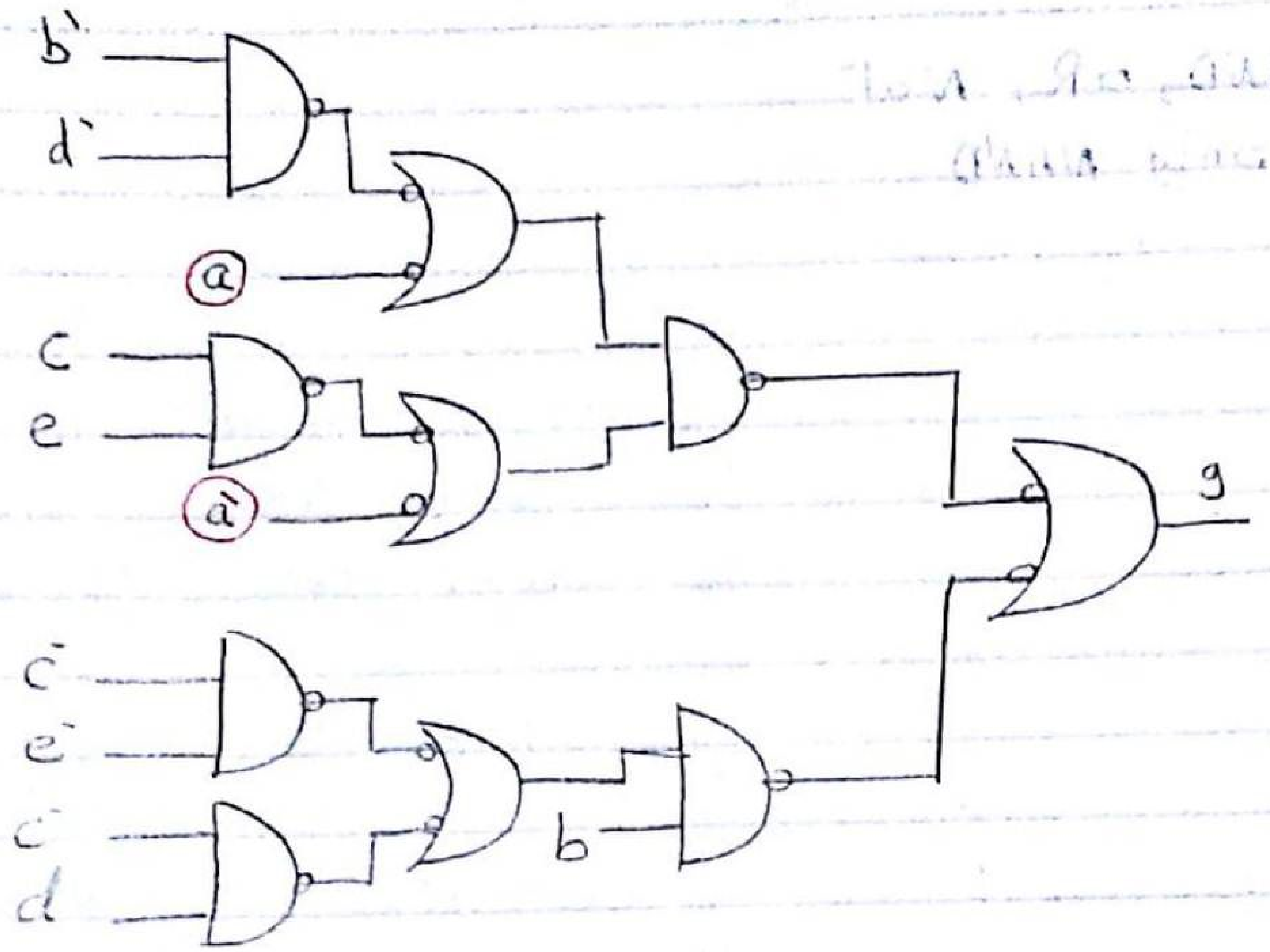
& each NOT to NAND



b)

For the function "g" $\Rightarrow$ if All Variable are Available.

$$g = b(\bar{c}d + \bar{c}\bar{e}) + (\bar{a} + ce)(\bar{a} + \bar{b}\bar{d})$$



② by using NAND.

$\Rightarrow$ You Draw (No. 1) by using AND, OR, NOT.

⇒ Using "ONly" OR and NOT gates: draw aschematic for this function:

$$F = xy + x'y' + y'z$$

ANS:

You must manipulate the function to can draw it by using OR, NOT only.

By Complement The function twice

$$(F')' = [[xy + x'y' + y'z]']'$$

$$= [(x'+y')(x+y)(y+z')]'$$

$$= (x'+y')' + (x+y)' + (y+z')'$$