

Q 5.6

A sequential circuit with two D flip-flops A and B, two inputs X and Y, and one output Z is specified by the following next state and output equations:

$$A(t+1) = \bar{X}Y + XB$$

$$B(t+1) = \bar{X}A + XB$$

$$Z = A$$

- Draw the logic diagram of the circuit.
- List the state table for the sequential circuit.
- Draw the corresponding state diagram.

ANS

→ In DFF → The $(\overset{\text{next}}{\underset{\text{state}}{\alpha}})$ equation α^* or $\alpha(t+1) = D$

So:

$$D_A = \bar{X}Y + XB$$

$$D_B = \bar{X}A + XB$$

$$Z = A$$

a)

* The logic Diagram of the Circuit

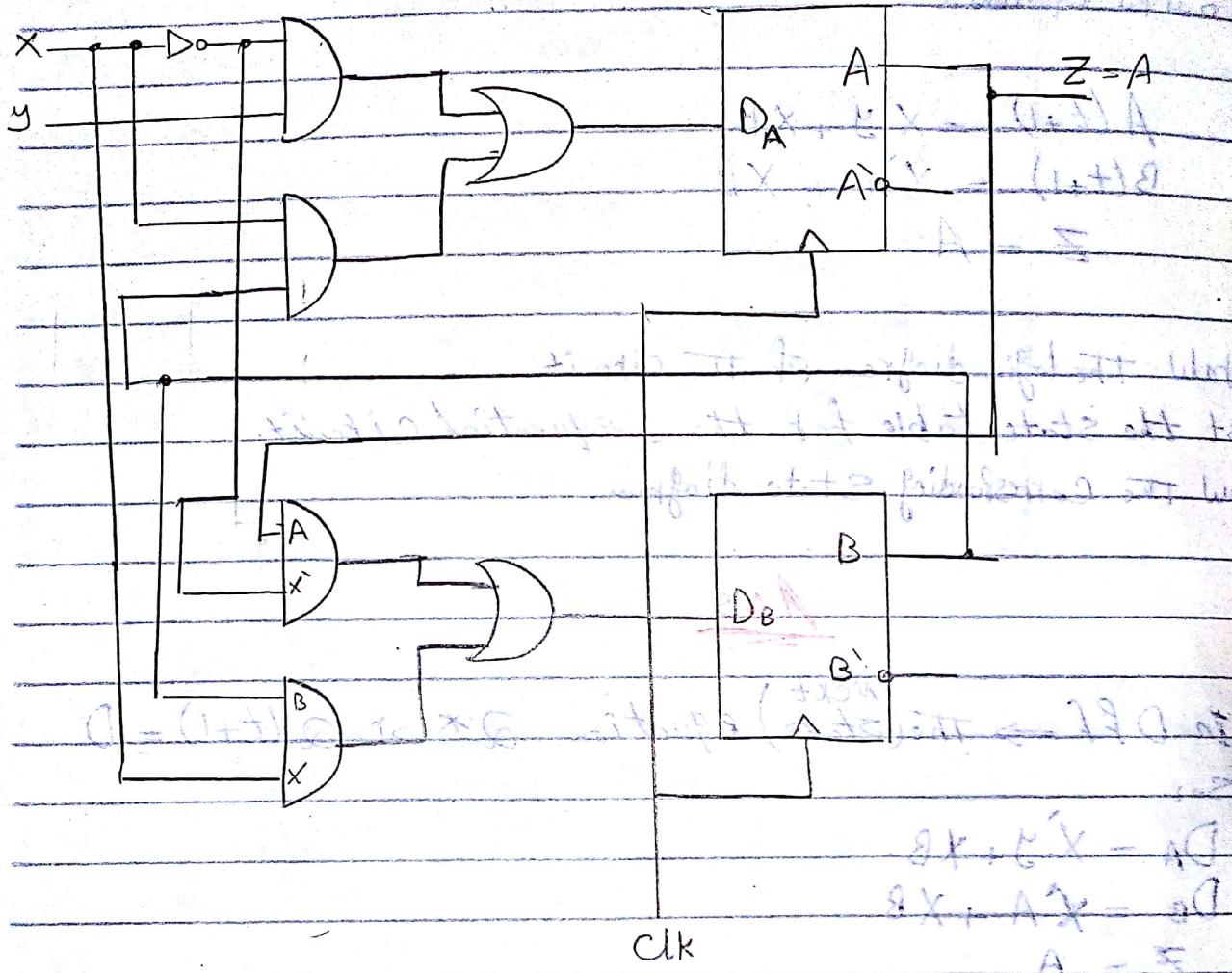


* the logic diagram of the circuit

$$D_A = \bar{X}Y + XB$$

$$D_B = \bar{X}A + XB$$

$$Z = A$$



For State table :-

$$A^* = \bar{X}Y + XB$$

$$B^* = \bar{X}A + XB$$

When $X=0, Y=0 [00]$

$$A^* = 0$$

$$B^* = A$$

When $X=0, Y=1 [01]$

$$A^* = 1$$

$$B^* = A$$

When $X=1, Y=0 [10]$

$$A^* = B$$

$$B^* = B$$

When $X=1, Y=1 [11]$

$$A^* = B$$

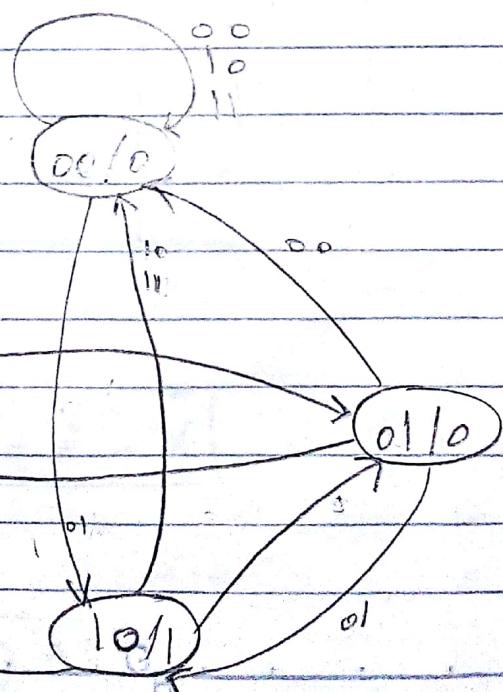
$$B^* = B$$

② The state table:

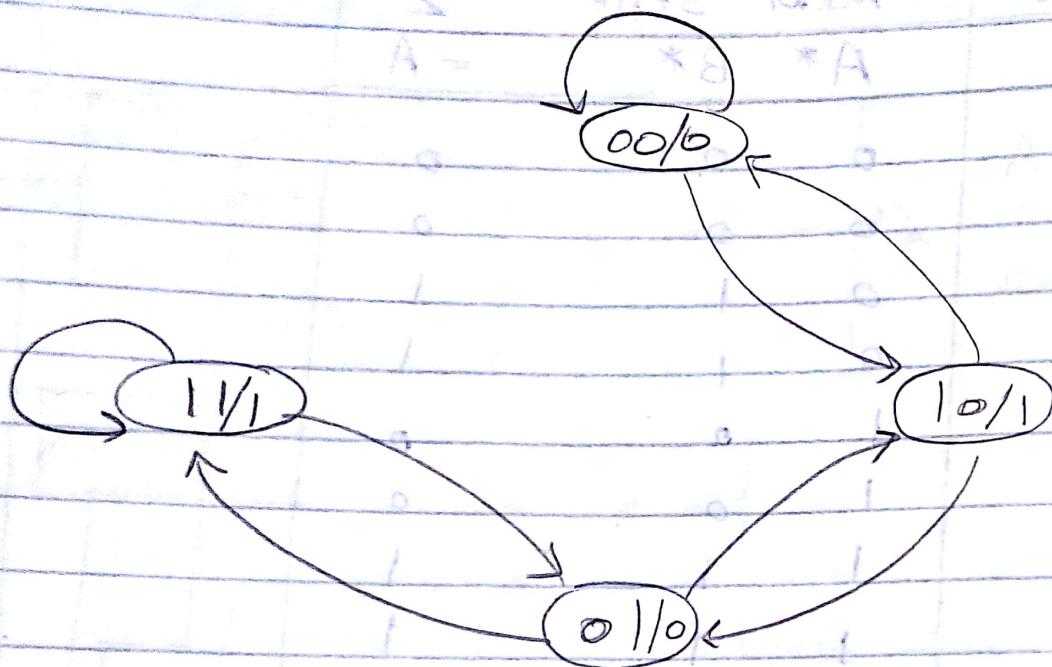
$$A^* = \bar{x}y + xB$$

$$B^* = \bar{x}A + XB$$

inputs x y	Present State		Next State		Z = A
	A	B	A^*	B^*	
0 0	0	0	0	0	0
0 0	0	1	0	0	0
0 0	1	0	0	1	1
0 0	1	1	0	1	1
0 1	0	0	1	0	0
0 1	0	1	1	0	0
0 1	1	0	1	1	1
0 1	1	1	1	1	1
1 0	0	0	0	0	0
1 0	0	1	1	1	0
1 0	1	0	0	0	1
1 0	1	1	1	1	1
1 1	0	0	0	0	0
1 1	0	1	1	1	0
1 1	1	0	0	0	1
1 1	1	1	1	1	1



States لخطى بحسب المدخلات وخطى المخرجات



أمثلة على خطى مدخلات

(Q 5.2 in Summer Exam) : ✓

A synchronous sequential circuit has two T-FlipFlops A and B, one input X and one output Y. The flip flop input equations and circuit output equations are respectively:

$$\bar{T}_A = AX + BX$$

$$\bar{T}_B = AX + \bar{B}X$$

$$Y = AB$$

a) Derive the state equations for $A(t+1)$ and $B(t+1)$

b) Draw the logic diagram of the circuit.

c) Tabulate the state table.

d) Draw the state diagram.



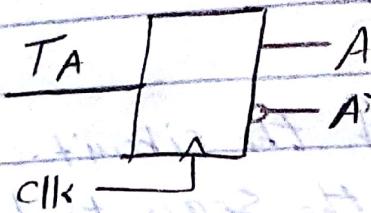
Answer

a) The State Equations for $A(t+1)$ and $B(t+1)$

In Toggle Flip Flop the next state equation

$$Q^* = T \oplus Q$$

Then (for A Flip Flop)



$$A^* \text{ or } A(t+1) = \bar{T}A \oplus A$$

$$\Rightarrow \boxed{\bar{T}A = AX + BX}$$

$$A(t+1) = (AX + BX) \oplus A$$

Similarly (For B Flip Flop)



$$B(t+1) = \bar{T}B \oplus B$$

$$\Rightarrow \boxed{\bar{T}B = AX + B\bar{X}}$$

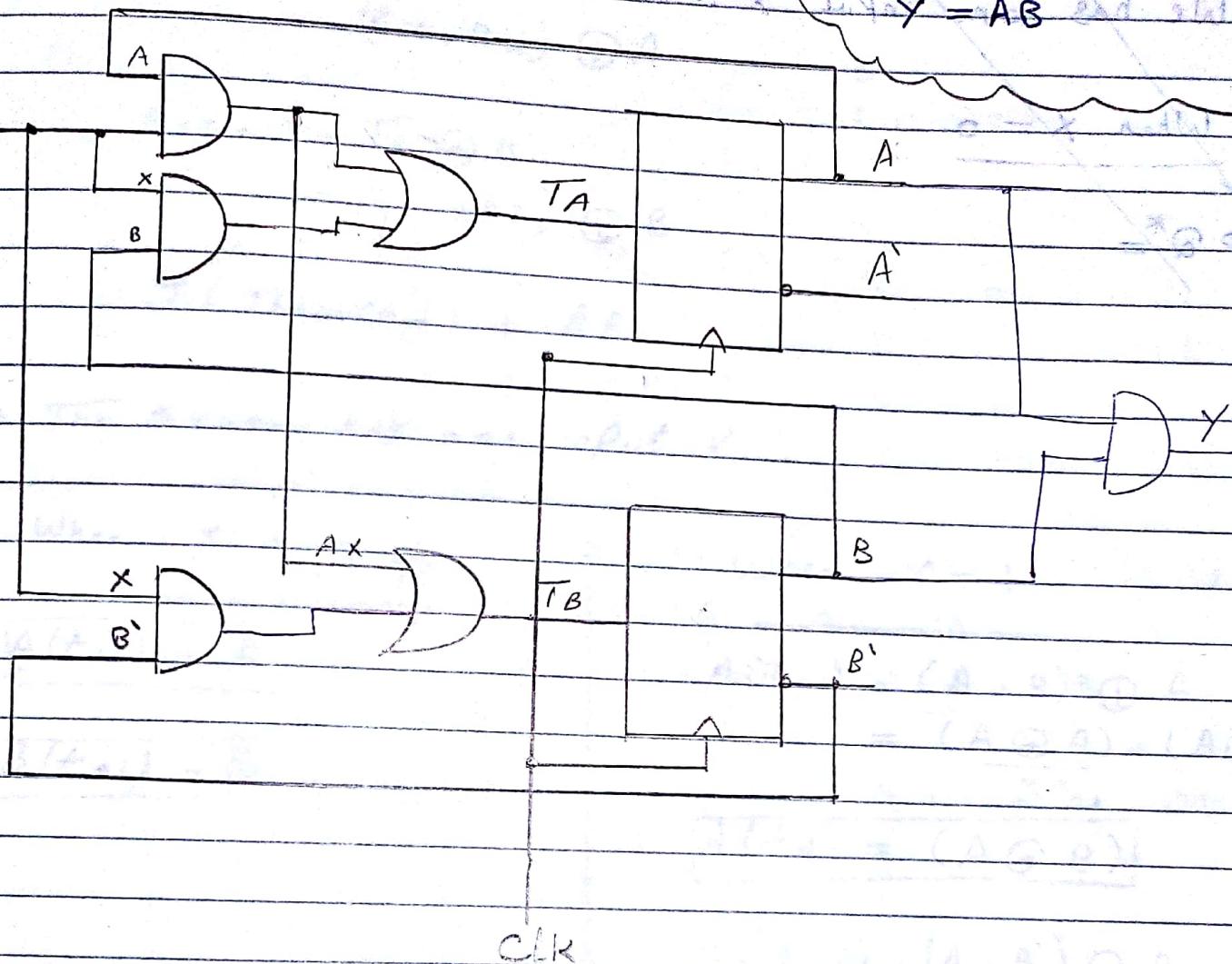
$$B(t+1) = (AX + B\bar{X}) \oplus B$$

b) The logic Diagram of the circuit.

$$TA = A \bar{X} + B \bar{X}$$

$$TB = A \bar{X} + B' \bar{X}$$

$$Y = AB$$



c) The State table



c) The state table:

$$A(t+1) = (AX + BX) \oplus A$$

$$B(t+1) = (AX + \bar{B}X) \oplus B$$

$$Y(\text{Output}) = AB$$

* The system has one input X :

When $X=0$

$$\begin{aligned} A^* &= 0 \oplus A \\ &= A \end{aligned}$$

$$\begin{aligned} B^* &= 0 \oplus B \\ &= B \end{aligned}$$

$$y = AB$$

When $X=1$

$$A^* = (A+B) \oplus A$$

$$B^* = (A+\bar{B}) \oplus B$$

$$y = A B$$



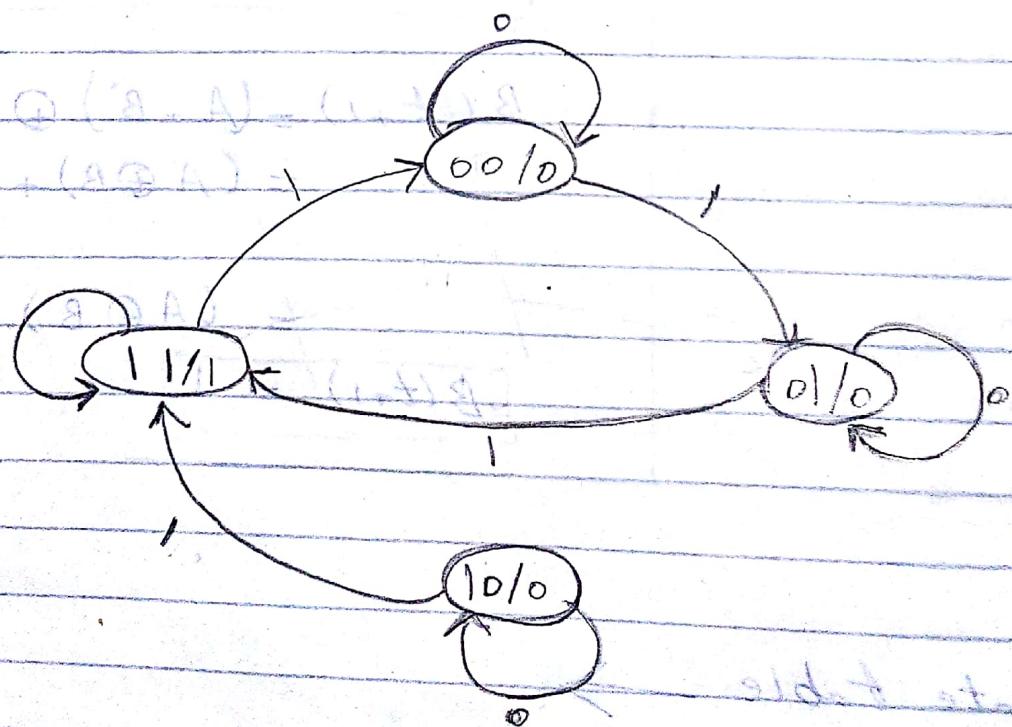
inputs	present state		next state		output
X	A	B	A^*	B^*	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	1
1	0	0	0	1	0
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	0	0	

When $(X=0) \rightarrow A^* = A$, $B^* = B$.

$\therefore (X=1) \rightarrow A^* = (A+B) \oplus A$, $B^* = (A+B) \oplus B$

$\therefore A \oplus B$ is the output, $Y = AB$.

d) The State Diagram.



Q5.9)

A Sequential circuit has two JK flip-flops A and B and one input X. The circuit is described by the following flip flop input equations:

$$J_A = X \quad K_A = \bar{B}$$

$$J_B = X \quad K_B = A$$

- Derive the state equations $A(t+1)$ and $B(t+1)$ by substituting the input equations for the J and K variables.
- Draw the state diagram of the circuit.

Answer

a) Q^* for JK flipflop is $= J\bar{Q} + \bar{K}Q$

For A Flip Flop:

$$\begin{aligned}A(t+1) &= J_A \bar{A} + \bar{K}_A A \\&= X \bar{A} + (\bar{B})^* A\end{aligned}$$

$$\boxed{A(t+1) = X \bar{A} + BA}$$

For B Flip Flop:

$$B(t+1) = J_B \bar{B} + \bar{K}_B B$$

$$\boxed{B(t+1) = X B + A^* B}$$

b) To draw the state diagram, first, we must list the state table.

→ J-K register (State diagram) (done)
→ State table

Inputs	Present State		Next State	
X	A	B	A*	B*
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	1	1
1	0	1	1	1
1	1	0	0	1
1	1	1	1	0

پسیه $A(t+1)$, $B(t+1)$ کیا کہلے تھے؟

$$X=0 \quad , \quad X=1 \quad [A(t+1) = XA + BA \quad \& \quad B(t+1) = XB + A^*B]$$

When $X=0$

$$A(t+1) = AB$$

$$B(t+1) = A^*B$$

When $X=1$

$$A(t+1) = A^* + AB$$

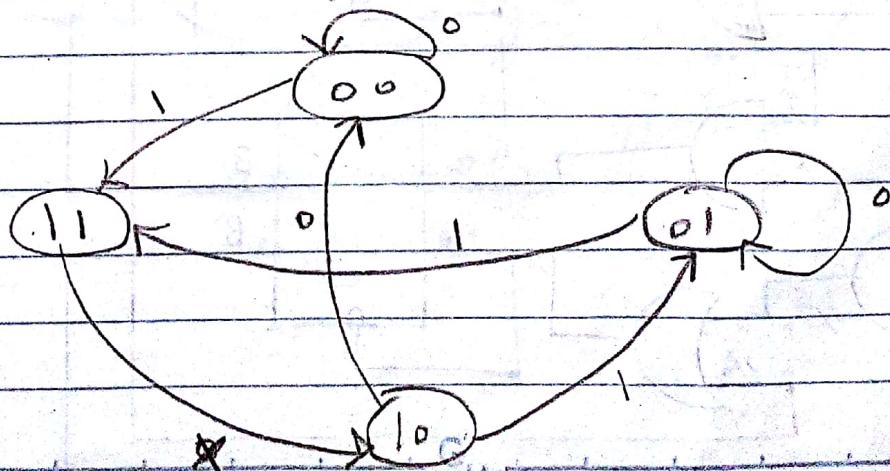
$$= (A^* + A)(A + B)$$

$$A(t+1) = (A^* + B)$$

$$B(t+1) = B^* + A^*B$$

$$= (A^* + B^*)(B + B^*)$$

$$B(t+1) = (A^* + B^*)$$



Q5.10)

A sequential circuit has two JK flip flops A and B, two inputs x and y , and one output Z . The flip flop input equations and circuit output equation are:

$$J_A = Bx + \bar{B}y \quad K_A = \bar{B}xy$$

$$J_B = A\bar{x} \quad K_B = A + xy$$

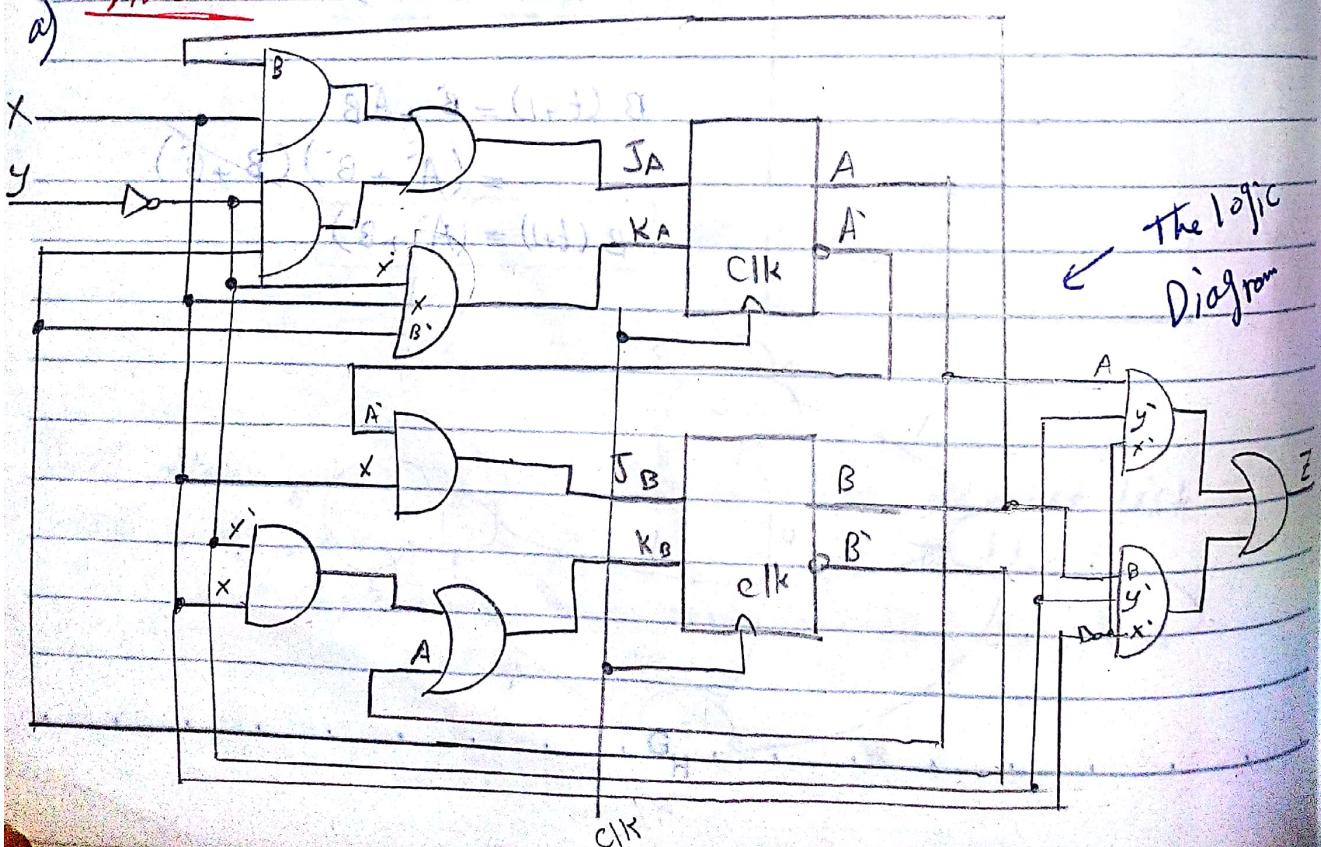
$$Z = Ax\bar{y} + Bx\bar{y}$$

a) Draw the logic diagram of the circuit.

b) Tabulate the state table.

c) Define the state equations for A & B .

ANSWER: $(Q, A) = (1, 1)$



b) To tabulate the state table: $\bar{Q}^*(JK) = J\bar{Q} + K\bar{Q}$

$$\begin{aligned}
 A(t+1) &= J_A \bar{A} + K_A A \\
 &= (\bar{B}x + \bar{B}\bar{y}) \bar{A} + (\bar{B}x\bar{y}) A \\
 &= \bar{A}\bar{B}x + \bar{A}\bar{B}\bar{y} + AB + Ax + Ay
 \end{aligned}$$

	$x\bar{y}$	$x\bar{y}$	$x\bar{y}$	xy	xy'
$\bar{A}\bar{B}$	1				1
$\bar{A}B$			1	1	
$A\bar{B}$	1	1	1		1
AB	1	1			

$$(A(t+1)) = A\bar{x} + A\bar{y} + Bx + \bar{B}\bar{y}$$

$$B(t+1) = J_B \bar{B} + K_B B$$

$$\begin{aligned}
 &= \bar{A}\bar{x}\bar{B} + (A + x\bar{y}) B \\
 &= \bar{A}\bar{x}\bar{B} + \bar{A}(x\bar{y}) B \\
 &= \bar{A}\bar{x}\bar{B} + \bar{A}\bar{B}(x\bar{y}) \\
 &= \bar{A}\bar{x}\bar{B} + \bar{A}\bar{B}x + \bar{A}\bar{B}y
 \end{aligned}$$

	$x\bar{y}$	$x\bar{y}$	$x\bar{y}$	xy	xy'
$\bar{A}\bar{B}$				1	1
$\bar{A}B$	1	1		1	
$A\bar{B}$					
AB					

$$(B(t+1)) = \bar{A}\bar{B}\bar{x} + \bar{A}\bar{B}x + \bar{A}\bar{x}y$$



$$A(t+1) = A\bar{X} + AY + BX + AB\bar{Y} \quad / \quad Z = A\bar{X}\bar{Y} + B\bar{X}Y$$

$$B(t+1) = A' B\bar{X} + A' B' X + A' X Y$$

This system has two inputs X, Y

When $X=0 \& Y=0$

$$A(t+1) = A + A'\bar{B}$$

$$\boxed{A^* = A + B'}$$

$$\boxed{B(t+1) = A'B}$$

$$\boxed{Z = A + B}$$

When $X=0 \& Y=1$

$$A^* = A + A$$

$$\boxed{A^* = A}$$

$$\boxed{B^* = A'B}$$

$$\boxed{Z = 0}$$

When $X=1 \& Y=0$

$$\boxed{A^* = B + A'\bar{B}}$$

$$\boxed{A^* = B + A'}$$

$$\boxed{B^* = A'\bar{B}}$$

$$\boxed{Z = 0}$$

$$\boxed{A^* = A + B}$$

$$\boxed{B^* = A'\bar{B} + A'}$$

$$\boxed{B^* = A'}$$

$$\boxed{Z = 0}$$

b) The State table \Rightarrow

Inputs	Present State		Next State	Output
*	A	B	A*	Z
Y				
0 0	0 0		1 0	0
0 0	0 1		0 1	1
0 0	1 0		1 0	1
0 0	1 1		1 0	1
0 1	0 0		0 0	0
0 1	0 1		0 1	0
0 1	1 0		1 0	0
0 1	1 1		1 0	0
1 0	0 0		1 1	0
1 0	0 1		1 0	0
1 0	1 0		0 0	0
1 0	1 1		1 0	0
1 1	0 0		0 1	0
1 1	0 1		1 1	0
1 1	1 0		1 0	0
1 1	1 1		1 0	0

c) The state equations:

$$A(t+1) = Ax + Ay + Bx + A'B'y$$

$$B(t+1) = A'bx + A'b'x + Ax'y$$

Counters :-

A Synchronous Counter

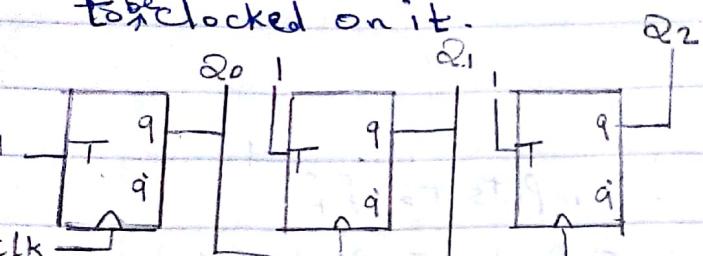
→ Asynchronous

→ Called Ripple Counter.

→ The flipflops aren't clocked using same clock.

→ each ff wait the output signal of the previous ff.

to be clocked on it.

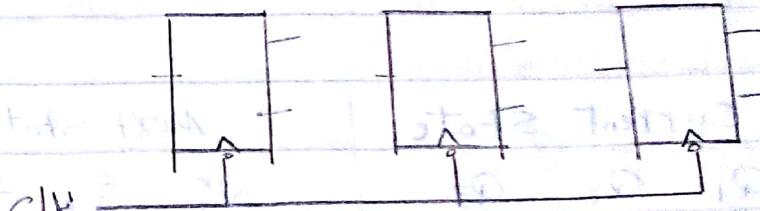


A Synchronous Counter.

→ clk is same

→ All ffs clock inputs are directly connected to the clock signal.

→ And all ffs outputs change at the same time.



→ The same clock for all ffs.

Examples :-

Ex1:

* Design a 3 bit up Counter by using 5 RFFs. That counts.

0 1 2 3 4 5 6 7 and Repeat.

Ans.

↳ 3 bit mean from 0 : 7 \Rightarrow 8 states.

↳ We will use 3 FFs.

↳ 1 bit (each FF) for a bit.

↳ (up) mean \rightarrow Counter from 0 to 7 \Rightarrow (0 1 2 3 4 5 6 7 0 ...)

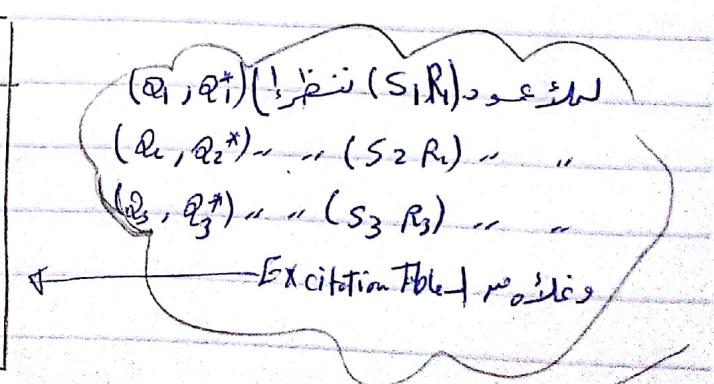
(Modified State Transition Table)

\Rightarrow Modified State Transition Table:

Current State			Next State			Inputs to FF.		
Q_1	Q_2	Q_3	Q_1^*	Q_2^*	Q_3^*	S_1, R_1	S_2, R_2	S_3, R_3
0 0 0	0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	0 X	0 X	1 0
0 0 1	0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	0 X	1 0 0	1
0 1 0	0 1 1	1 0 0	1 0 1	1 1 0	1 1 1	0 X	X 0	1 0
0 1 1	1 0 0	1 0 1	1 1 0	0 1 0	0 1 1	1 0	0 1	0 1
1 0 0	1 0 1	1 1 0	0 1 0	0 1 1	0 1 1	X 0	0 X	1 0
1 0 1	1 1 0	0 1 0	0 1 1	1 0 0	1 0 1	X 0	1 0	0 1
1 1 0	0 1 0	0 1 1	1 0 0	1 0 1	1 0 1	X 0	X 0	1 0
1 1 1	0 1 1	0 1 1	0 0 0	0 0 0	0 0 0	0 1	0 1	0 1

\rightarrow SRFF \rightarrow Excitation Table \rightarrow get logic

S	R	Q	Q^*	S	R
0 0	No Change	0 0	0 0	0 X	
0 1	Reset	0 0	0 0	0 X	
1 0	Set	0 1	0 1	1 0	
0 1	Reset	1 0	1 0	0 1	
0 0	No Change	1 1	1 1	X 0	
1 0	Set	1 1	1 1	X 0	



$$S_1 = \Sigma m(3) + \Sigma d(4, 5, 6)$$

Q ₃	0	1	X	X
Q ₂	1	0	1	X
Q ₁	0	1	1	0
	1	0	0	1

$S_1 = Q_1 Q_2 Q_3$

$$S_2 = \Sigma m(1, 5) + \Sigma d(2, 6)$$

0	2	X	X	4
1	3	1	1	1

$S_2 = Q_2 Q_3$

$$R_1 = \Sigma m(7) + \Sigma d(0, 1, 2)$$

Q ₃	0	X	2	1	0	1
Q ₂	1	X	X	1	0	1
Q ₁	0	1	0	1	1	0
R ₁	1	X	0	1	1	0

$R_1 = Q_1 Q_2 Q_3$

$$R_2 = \Sigma m(3, 7) + \Sigma d(0, 4)$$

0	2	6	4
1	X	1	1

$R_2 = Q_2 Q_3$

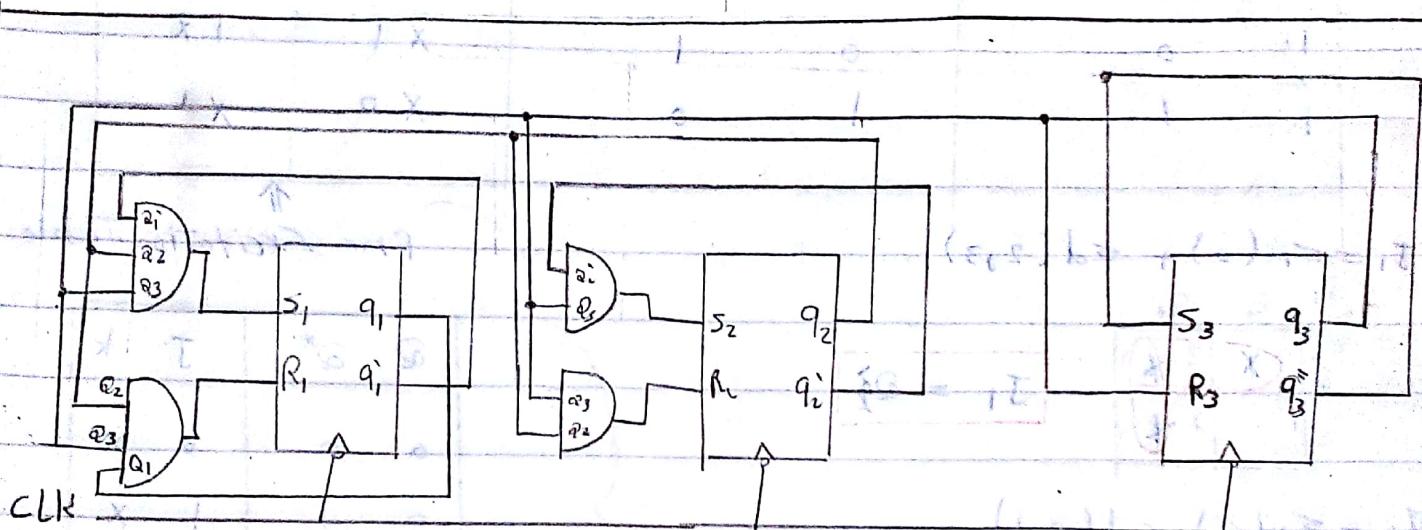
$$S_3 = \Sigma m(0, 2, 4, 6)$$

1	1	X	1
X	1	X	1

$S_3 = Q_3$

$$R_3 = \Sigma m(1, 3, 5, 7)$$

0	2	6	4
1	1	1	1



(Sequential Circuit by using SR ff.)

(Implementation)

Note :-

This System is Synchronous Counter. \Rightarrow "The same clock"

Ex 2:

* Design a 2 bit down counter by using JK ff.

Ans:

* 2 bits from 0 : 3

* We need 2 ffs.

⇒ (down) means Counter from 3 to 0,

3 2 1 0 3 2 1 0

* Modified State Transition Table: ~~(State Model)~~

Current State		next State		Inputs to ff.	
Q_1	Q_2	Q_1^*	Q_2^*	J_1, K_1	J_2, K_2
0	0	1	1	1 X	1 X
0	1	0	0	0 X	X 1
1	0	0	1	X 1	1 X
1	1	1	0	X 0	X 1

$$J_1 = \sum_m(0) + \sum_d(2, 3)$$

Q_1	Q_2	Q_1^*	Q_2^*
0	0	1	1
0	1	0	0

$$J_1 = \bar{Q}_2$$

From Excitation Table

Q	Q^*	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$$K_1 = \sum_d(0, 1)$$

Q_1	Q_2	Q_1^*	Q_2^*
0	0	1	1
1	0	0	1

$$K_1 = \bar{Q}_2$$

$$J_2 = \sum_m(0, 2) + \sum_d(1, 3)$$

Q_1	Q_2	Q_1^*	Q_2^*
0	1	1	1
1	X	1	X

$$J_2 = 1$$

$$K_2 = \sum_m(1, 3) + \sum_d(0, 2)$$

Q	Q^*	J	K
0	1	1	X
1	0	X	1

$$J_2 = 1$$

* sequential circuit by using JKff ::

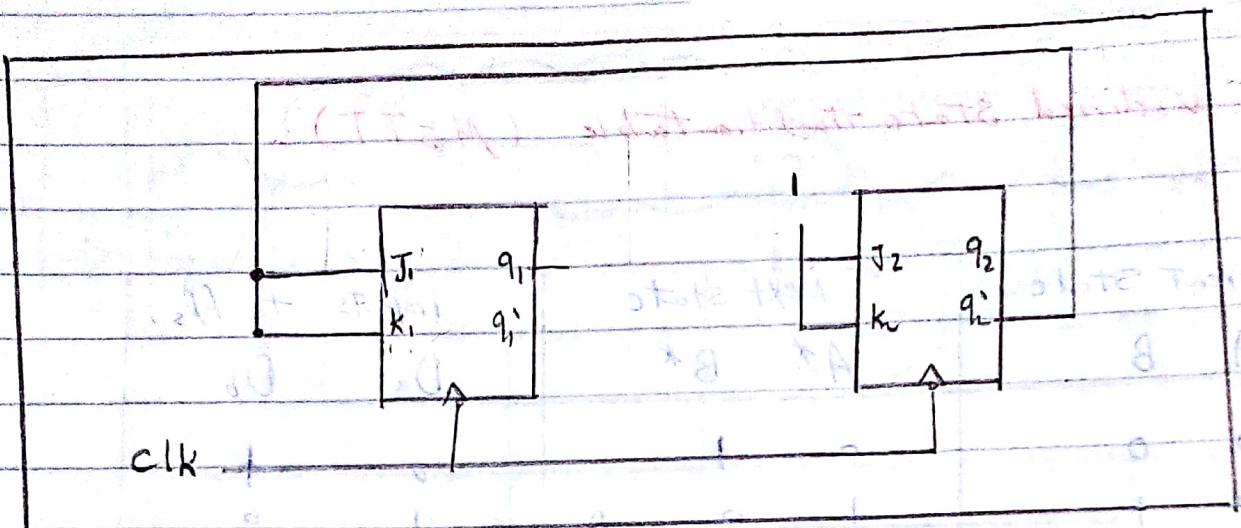
(Implementation)

$$J_1 = Q_2$$

$$K_1 = \bar{Q}_2$$

$$J_2 = 1$$

$$K_2 = 1$$



$$S_A + (Q, A)X = S_A + (Q, A)X$$

Ex3:

Design and implement a counter with 2 DFFs A and B, with a one bit control line X. if $X=0$, it counts $0, 1, 2, 3, 3, \dots$.
 if $X=1$, it counts $3, 2, 1, 0, 0, \dots$.

Ans:

(~~State Table~~)

* The Modified State Transition Table (MSTT).

Current State			Next state		Inputs to FFs.	
X	A	B	A^*	B^*	D_a	D_b
G_0	0	0	0	1	0	1
G_1	0	0	1	0	$D = Q^*$	1
G_2	0	1	0	1	1	1
G_3	1	1	1	1	1	1
G_0	1	0	0	0	0	0
G_1	1	0	0	0	0	0
G_2	1	1	0	1	0	1
G_3	1	1	1	0	1	0

$$D_a = \Sigma_m(1, 2, 3, 7)$$

$\bar{X}A$	$X\bar{A}$	$\bar{X}A$	$X\bar{A}$	$\bar{X}A$
0	1	1	0	0
1	0	0	1	1

$$D_a = \bar{X}A + AB + \bar{X}B$$

$$= \bar{X}(A+B) + AB$$

$$D_b = \Sigma_m(0, 2, 3, 6)$$

$\bar{X}A$	\bar{B}	$\bar{X}A$	\bar{B}	$\bar{X}A$	\bar{B}
1	1	1	0	0	0
0	0	0	1	1	1

$$D_b = \bar{X}A + \bar{X}B + AB$$

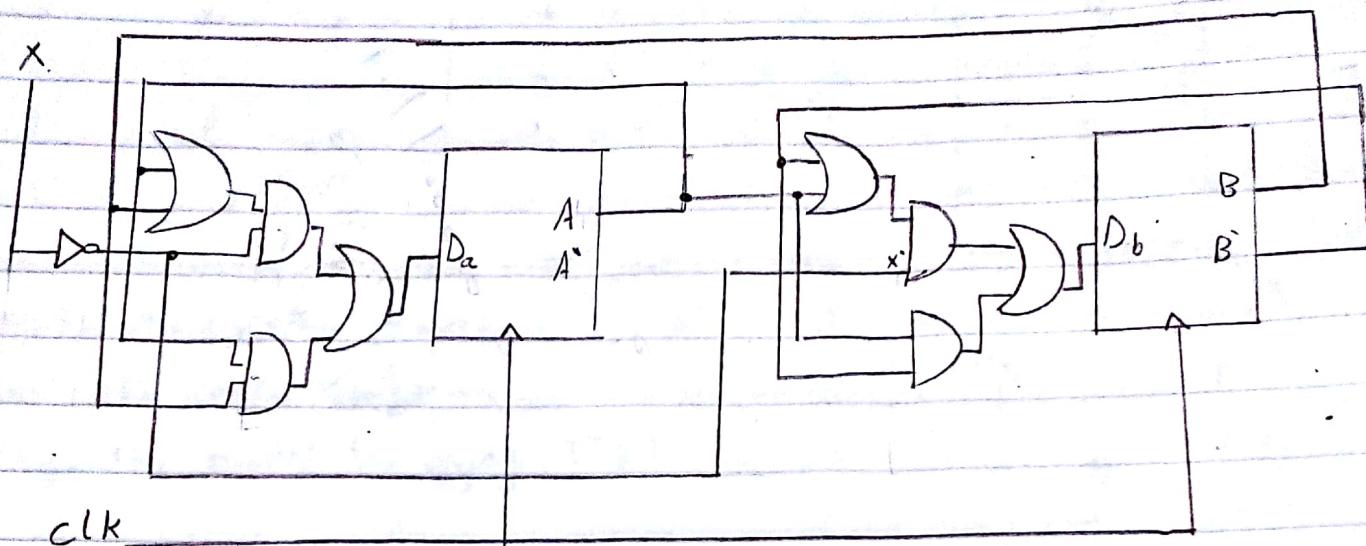
$$= \bar{X}(A+B) + AB$$

* A sequential circuit by using DFF. (Implementation)

⇒ A solution implementation:

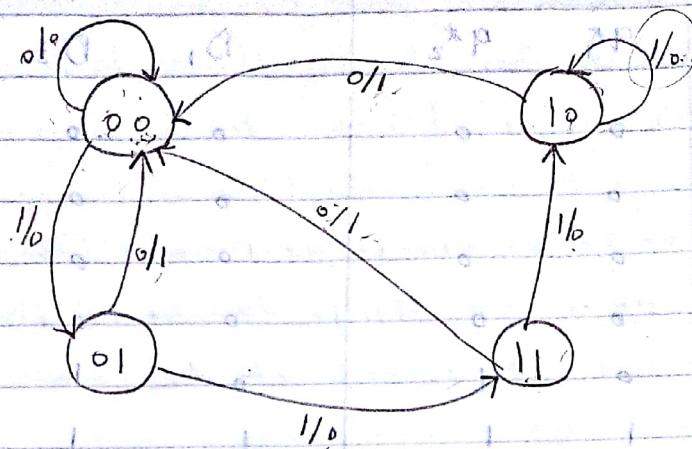
$$D_a = \bar{x}(A+B) + AB$$

$$D_b = \bar{x}(A+\bar{B}) + A\bar{B}$$



EX6:

- For the following state diagram. Starting from state 00
 → Determine the state transitions and Design this system by using Dff.



Ans:

The State table.

Present state (q)	Next state (q')	Output (Z)
$q_1 \quad q_2 \quad q$	$X = 0 \quad X = 1$ $q'_1 \quad q'_2$	$X = 0 \quad X = 1$
0 0	0 0 0 1	0 0
0 1	0 0 1 1	1 0
1 0	0 0 1 0	1 0
1 1	0 0 1 0	1 0

MSTT \Rightarrow

* Modified State transition table:

Present state	Next state	Inputs to ff's.	Output
$X \ q_1 \ q_2$	$q'_1 \ q'_2$	$D_1 \ D_2$	Z
0 0 0	0 0	0 0	0
0 0 1	0 0	0 0	1
0 1 0	0 0	0 0	1
0 1 1	0 0	0 0	1
1 0 0	0 1	0 1	0
1 0 1	1 1	1 1	0
1 1 0	1 0	1 0	0
1 1 1	1 0	1 0	0

$$D \neq q'$$

* The equations:

$$D_1 = \Sigma_m(5, 6, 7)$$

$X \ q_1 \ q_2$	$X' \ q_1 \ q_2$	$\bar{X} \ q_1 \ q_2$	$X \bar{q}_1 \ q_2$	$X \bar{q}_1 \bar{q}_2$
0 0 0	1 0 0	0 1 0	0 0 1	0 0 0
0 0 1	1 0 1	0 1 1	0 0 0	0 0 1

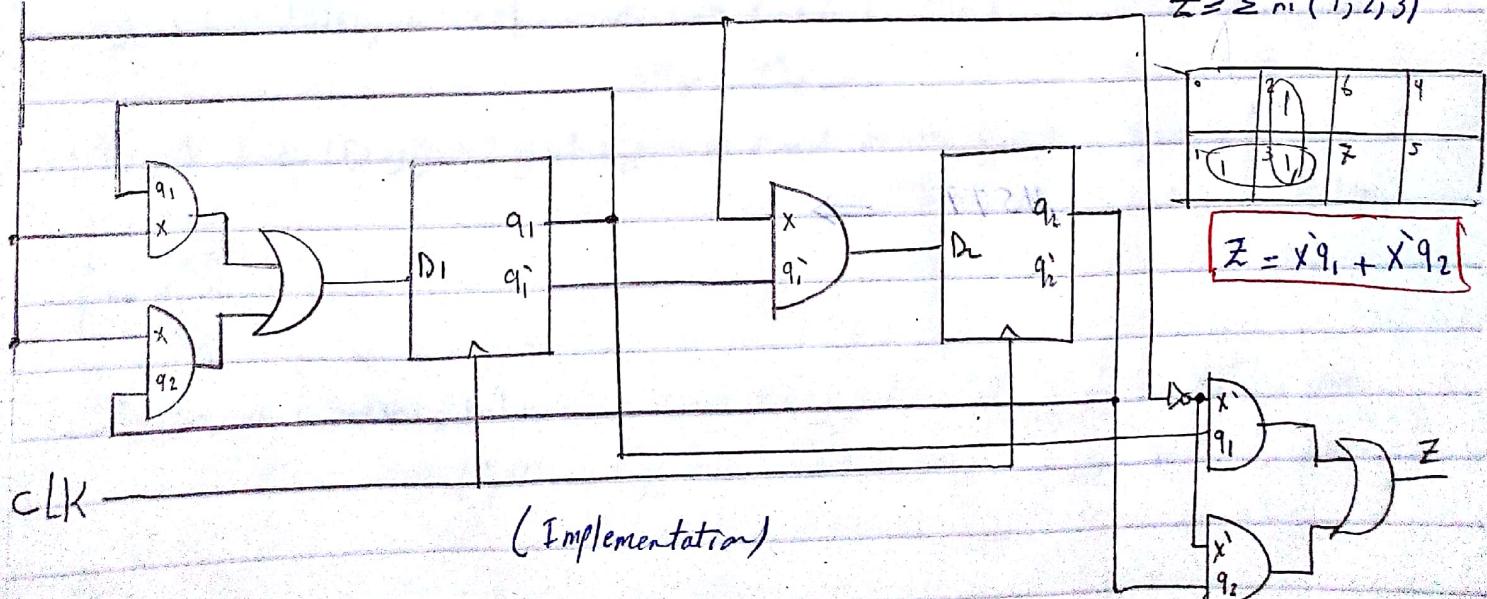
$$D_2 = \Sigma_m(4, 5)$$

0	2	6	4
1	3	7	5

$$D_1 = X q_1 + X q_2$$

$$D_2 = X' q_1$$

$$Z = \Sigma_n(1, 2, 3)$$



EX 7:

* Design a sequential circuit with two DFFs A, B and one ~~X~~ input X

a) When $X=0$, the state of the circuit remains the same, ~~When $X=1$~~

When $X=1$, the circuit goes through the state transitions from: 00, to 01, to 11, to 10, back to 00 and repeats.

b) When $X=0$, the state of the circuit remains the same.

When $X=1$, the circuit goes through the state transitions from: 00 to 11, to 01, to 10 back to 00 and repeats.

Ans:

a) MTT:

current state			next state		inputs to DFF	
X	q_1	q_2	q_1^*	q_2^*	D _a	D _b
A	q_1	B	A*	B*		
0	0	0	0	0	0	0
0	0	1	0	1	0	1
0	1	0	1	0	1	0
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	1	1	1	1
1	1	0	0	0	0	1
1	1	1	1	0	1	0

You complete it \Rightarrow * extract the equations of D_1 , D_2

* minimize them by k-map.

* Build the design by DFFs.

b) MTT.

Current state	Next state	Inputs to DFFs
$X \ q_1 \ q_2$	$q_1^* \ q_2^*$	$D_a \ D_b$
A B	A* B*	
0 0 0	0 0	0 0
0 0 1	1 1	1 1
0 1 0	0 0	0 0
0 1 1	1 1	1 1
1 0 0	1 0	1 0
1 0 1	0 0	0 0
1 1 0	0 1	0 1
1 1 1	X X	X X

→ You complete it.

EX8:

- * Design a sequential circuit with two JK flip flops A & B and two inputs E and F.
- * If $E=0$, the circuit remains in the same state regardless of the value of F.
- * When $E=1$ and $F=1$, the circuit goes through the state transitions from 00 to 01 to 10 to 11 back to 00 and repeats.
- * When $E=1$ and $F=0$, the circuit goes through the state transitions from 00 to 11 to 10 to 01 back to 00 and repeats.

Ans:

\Rightarrow MTT

Q	Q^*	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Current state				Next state		Inputs to JKffs			
E	F	A	B	A^*	B^*	J_1	K_1	J_2	K_2
0	0	0	0	0	0	0	X	0	X
0	0	0	1	0	1	0	X	X	0
0	0	1	0	1	0	X	0	0	X
0	0	1	1	1	1	X	0	X	0
0	1	0	0	0	0	0	X	0	X
0	1	0	1	0	1	0	X	X	0
0	1	1	0	1	0	X	0	0	X
0	1	1	1	1	1	X	0	X	0
\rightarrow				\rightarrow Q (The same)		\rightarrow			
1	0	0	0	0	0	0	X	1	X
1	0	0	1	0	1	0	X	X	1
1	0	1	0	1	0	X	0	0	X
1	0	1	1	1	1	X	0	X	0
\rightarrow				\rightarrow E=0		\rightarrow			
1	0	0	0	1	0	1	X	1	X
1	0	0	1	0	1	0	X	X	1
1	0	1	0	0	1	X	1	1	X
\rightarrow				\rightarrow E=1, F=0		\rightarrow			
1	1	0	0	0	1	0	0	X	1
1	1	0	1	1	0	1	X	X	1
1	1	1	0	1	1	X	0	1	X
\rightarrow				\rightarrow E=1, F=1		\rightarrow			
1	1	1	1	0	0	0	X	1	X