Number Systems:

Famous Numbering System:

Number system.	base or radix	Symbols.
$\boxed{\text{Decimal}}$ ✓	Base 10	0, 1, 2, ..., 9
$\boxed{\text{Binary}}$ ✓	Base 2	0, 1
Ternary	Base 3	0, 1, 2
Quaternary	Base 4	.
Quinary	Base 5	.
senary	Base 6	.
septenary	Base 7	.
$\boxed{\text{Octal}}$ ✓	Base 8	0, 1, 2, 3, 4, 5, 6, 7
—	Base 9	.
	⋮	⋮
$\boxed{\text{Hexadecimal}}$ ✓	Base 16	0, 1, 2, ..., 9, A, B, C, D, E, F 10 → A 11 → B 14 → E 12 → C 15 → F 13 → D

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⇒ يمكن يطلب التحويل من أي نظام للآخر:

① Any Number system to decimal [to $(\ )_{10}$]:

⇒ Binary system $(-)_2$ to Decimal system $(?)_{10}$

Ex.1

integer → $(10110)_2 = (?)_{10}$

$$(\overset{4}{1}\ \overset{3}{0}\ \overset{2}{1}\ \overset{1}{1}\ \overset{0}{0})_2$$

$$= (1 * 2^4) + (0 * 2^3) + (1 * 2^2) + (1 * 2^1) + (0 * \underset{1}{2^0})$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= (22)_{10}$$

EX.

$(10110.11)_2 = (?)_{10}$

integer & fraction ?

$$(\overset{4}{1}\ \overset{3}{0}\ \overset{2}{1}\ \overset{1}{1}\ \overset{0}{0}\ .\ \overset{-1}{1}\ \overset{-2}{1})_2$$

$$= (1 * 2^4) + (0 * 2^3) + (1 * 2^2) + (1 * 2^1) + (0 * 2^0) + (1 * 2^{-1}) + (1 * 2^{-2})$$

$$= 16 + 0 + 4 + 2 + 0 + \frac{1}{2} + \frac{1}{4}$$

$$= (22.75)_{10}$$

X

Ternary system $()_3$ to Decimal system $()_{10}$

Ex.

$$(\overset{3}{2}\overset{2}{0}\overset{1}{2}\overset{0}{1}.\overset{-1}{2}\overset{-2}{0}\overset{-3}{0}\overset{-4}{1})_3 = (?)_{10}$$

$$= (2*3^3) + (\cancel{0*3^2}) + (2*3^1) + (1*3^0) + (2*3^{-1}) + (\cancel{0*3^{-2}}) + (\cancel{0*3^{-3}}) + (1*3^{-4})$$

$$= 54 + 0 + 6 + 1 + \frac{2}{3} + \frac{1}{81}$$

$$= (61.67901235)_{10} \simeq (61.679)_{10}$$

⇒ with aprecition of 3. digits.
↳ that is for fraction part.

Base 4 : (0, 1, 2, 3)

Ex:

$$(\overset{3}{3}\overset{2}{0}\overset{1}{1}\overset{0}{2}.\overset{-1}{0}\overset{-2}{1})_4 = (?)_{10}$$

$$(3*4^3) + (\cancel{0*4^2}) + (1*4^1) + (2*\underset{1}{4^0}) + (\cancel{0*4^{-1}}) + (1*4^{-2})$$

$$= 192 + 0 + 4 + 2 + 0 + \underset{(0.0625)}{\frac{1}{16}}$$

$$= (198.0625)_{10}$$

---

octal system $( )_8$ to Decimal system $( )_{10}$:-

Ex.

$(\overset{2}{1}\overset{1}{2}\overset{0}{7}.\overset{-1}{6}\overset{-2}{4})_8$

$= (1*8^3) + (2*8^1) + (7*8^0) + (6*8^{-1}) + (4*8^{-2})$

$= 64 + 16 + 7 + 0.75 + 0.0625$

$= (87.8125)_{10}$

Hexadecimal system $( )_{16}$ to decimal system $( )_{10}$:-

Ex.

$(\overset{3}{1}\underset{10}{\overset{2}{A}}\underset{12}{\overset{1}{C}}\underset{15}{\overset{0}{F}}.\underset{14}{\overset{-1}{E}}\overset{-2}{3})_{16}$

$(1*16^3) + (10*16^2) + (12*16^1) + (15*16^0) + (14*16^{-1}) + (3*16^{-2})$

$= (6863.886719)_{10}$

② From Decimal to any number system :

Decimal system To Binary system.

$(174.390625)_{10} = (?)_2$

Solu.

	Integer part	remainder.
2 ÷	174	
	87	0
	43	1
	21	1
	10	1
	5	0
	2	1
	1	0
	0	1
	(10101110)	

	Fraction Part.	integer.
2 x	0.390625	
	0.78125	0
	0.5625	1
	0.125	1
	0.25	0
	0.5	0
	0.000	1
	(.011001)	

$(10101110.011001)_2$

EX:

$(53.763)_{10} = (?)_2$ with a precision of 7 binary digits.

	integer part	remainder
2 ÷	53	
	26	1
	13	0
	6	1
	3	0
	1	1
	0	1

	fraction part	integer
2 x	0.763	
	0.526	1
	0.052	1
	0.104	0
	0.208	0
	0.416	0
	0.832	0
	0.664	1
	0.328	1

$(110101.1100001)_2$

Decimal system To Ternary system $()_3$.

EX:

$(87\frac{1}{81})_{10} = (?)_3$

	integer part	remainder
3 %	87	
	29	0
	9	2
	3	0
	1	0
	0	1

	Fraction part	Integer
3 *	$\frac{1}{81}$	
	$\frac{1}{27}$	0
	$\frac{1}{9}$	0
	$1/3$	0
	0	1

$(10020.0001)_3$

Decimal system To octal

$(167.390625)_{10} = (?)_8$

	integer	remainder
8x	167	
	20	7
	2	4
	0	2

	fraction	integer
8x	0.390625	
	0.25	3
	0.00	1

$(247.31)_8$

$(138.236)_{10} = (?)_8$ with aprecision up to 4 octal digits.
for fraction part

	integer	remainder
8x	138	
	17	2
	2	1
	0	2

	fraction	integer
8x	0.236	
	0.888	1
	0.104	7
	0.832	0
	0.656	6 → 4 digits
	0.248	5
	0.984	1

$(212.1706)_8$

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Decimal system to Hexadecimal system :-

$(247.390625)_{10} = (?)_{16}$

	integer	remainder
16 ÷	247	
	15	7
	0	15 F

	Fraction.	integer.
16×	0.390625	
	0.25	6
	0.000	4

$(F7.64)_{16}$

$(335.171801)_{10} = (?)_{16}$ with a precision up to 4 Hexa. digits.

	integer	remainder
16 ÷	335	
	20	15 → F
	1	4
	0	1

	Fraction.	integer.
16×	0.171801	
	0.748816	2
	0.981056	11 B
	0.696896	15 F
	0.150336	11 B
	0.405376	2
		:

$(14F.2BFB)_{16}$

③ From Binary To octal system & vs.

Binary To octal.

$$(1001101.1011)_2 = (?)_8$$

$$\underbrace{\overset{4\,2\,1}{001}}\ \underbrace{\overset{4\,2\,1}{001}}\ \underbrace{\overset{4\,2\,1}{101}}\ .\ \underbrace{\overset{4\,2\,1}{101}}\ \underbrace{\overset{4\,2\,1}{100}}$$

$$(1\quad 1\quad 5\ .\ 5\quad 4)_8$$

octal To Binary.

$$(7431.640)_8 = (?)_2$$

$$\begin{array}{ccccccc} 7 & 4 & 3 & 1. & 6 & 4 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ (111) & (100) & (011) & (001). & (110) & (100) & (000) \end{array}$$

$$= (111100011001.110100000)_2$$

④ From Binary to Hexadecimal & V.S.

Hexadecimal to Binary

$(3BC.2E)_{16} = (?)_2$

3 → (0011), B (11) → (1011), C (12) → (1100) . 2 → (0010), E (14) → (1110)

$(0011)(1011)(1100).(0010)(1110)$

$(001110111100.00101110)_2$

$(4FA9.17B)_{16} = (?)_2$

4 → (0100), F (15) → (1111), A (10) → (1010), 9 → (1001) . 1 → (0001), 7 → (0111), B → (1011)

$(0100)(1111)(1010)(1001).(0001)(0111)(1011)$

$(100111110101001.000101111011)_2$

Binary To Hexadecimal.

$(111010.11011)_2 = (?)_{16}$

$\underline{0011}\ \underline{1010}\ .\ \underline{1101}\ \underline{1000}$

0011 → 3, 1010 → $\boxed{10}$ A, 1101 → $\boxed{13}$ D, 1000 → 8

$(3A.D8)_{16}$

From octal to Hexadecimal & Vs :-

$(375.231)_8 = (?)_{16}$

⇓ 3 bits

Binary

⇓ 4 bits.

Hexadecimal

3	7	5	.	2	3	1	← octal.
↓	↓	↓		↓	↓	↓	
011	111	101	.	010	011	001	

0000	1111	1101	.	0100	1100	1000	← Binary.
↓	↓	↓		↓	↓	↓	
0	$\boxed{15}$ F	$\boxed{13}$ D	.	4	$\boxed{12}$ C	8	

$(0FD.4C8)_{16}$ ← Hexa.

⇒

Ex:

Convert The following decimal numbers to binary. Assume All numbers are unsigned (Positive) and represented by 12 bits:

a) 98 b) 3163

Solu.

a) 98

	integer	remainder
2%	98	
	49	0
	24	1
	12	0
	6	0
	3	0
	1	0
	0	1

$(1100010)_2$

↳ in 12 bits, will be:

(000001100010)

b) 3163

	integer	remainder
2%	3163	
	1581	1
	790	1
	395	0
	197	1
	98	1
	49	0
	24	1
	12	0
	6	0
	3	0
	1	1
	0	1

$=(110001011011)_2$

↳ it is in 12 bits ✓

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Addition & Subtraction in Binary System

⇐ العمليات الحسابية لـ (Binary) هي الجمع والطرح :-

1) Addition :

$$\begin{array}{r} 0 \\ +\ 0 \\ \hline 0 \end{array} \qquad \begin{array}{r} 0 \\ +\ 1 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ 0 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1 \\ +\ 1 \\ \hline 0 \end{array}$$

Carry 1 ← 0
for the next column.

Ex1:
Find the sum of the binary numbers 1111 & 1010 and verify the result using decimal numbers.

ANS :

$$\begin{array}{r} \overset{\text{carry}}{1}\ \ \overset{\text{carry}}{1} \\ 1\ 1\ 1\ 1 \\ +\ \ 1\ 0\ 1\ 0 \\ \hline 1\ 1\ 0\ 0\ 1 \end{array}$$

The verification :

$$\begin{array}{r} 15 \\ +\ 10 \\ \hline 25 \end{array}$$

مهم ٢٠٠٪ لا تنساه .

EX.2:-

Perform the following binary Addition operation then verify the Result using decimal numbers.

$$101010.111 + 10111.011$$

ANS:

$$\begin{array}{rl} & 1\,1\,1\,1\,1\,1\;\;\;1\;1\\ & 1\,0\,1\,0\,1\,0\,.\,1\,1\,1\\ + & 0\,1\,0\,1\,1\,1\,.\,0\,1\,1\\ \hline & 1\,0\,0\,0\,0\,1\,0\,.\,0\,1\,0 \end{array}$$

The verification :-

$$\begin{array}{rl} & 42.875\\ + & 23.375\\ \hline & 66.25 \end{array}$$

Binary Subtraction

0	1	1	0
− 0	− 0	− 1	− 1
0	1	0	1 with 1 borrowed from The next Column

أنواع :-

→ Direct subtraction.

→ 1's Complement.

→ 2's Complement.

EX1:-

Use the Direct binary subtraction to get The result of:-

111101011 _ 11110100

ANS:

$$\begin{array}{rcccccccccc} & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ - & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ \hline & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \end{array} \qquad \begin{array}{r} 491 \\ -\ 244 \\ \hline 247 \end{array}$$

EX: 2

Use the direct binary subtraction to get the result of :

1100101 _ 100111 . Verify the Result in decimal system.

ANS:

$$\begin{array}{rcccccccc} & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ - & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ \hline & 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{array} \qquad \begin{array}{r} 101 \\ -\ 39 \\ \hline 62 \end{array}$$

EX. 3 :

Use the direct subtraction to get the result of :

11101101 − 111100011

<u>ANS:</u>

$$
\begin{array}{r}
11101101 \\
-\,111100011 \\
\hline
\end{array}
$$

← نلاحظ أن الرقم الثاني هو الرقم الأكبر

- لذلك يجب ترتيب الرقم بحيث نطرح الصغير من الكبير.

- ونراعي أن الإشارة سوف تكون $\boxed{\text{سالب}}$.

$$
\begin{array}{r}
11101101 \\
-\;111100011 \\
-\;011101101 \\
\hline
\boxed{-}\;011110110
\end{array}
$$

$$
\begin{array}{r}
237 \\
-\;483 \\
\hline
-\,246
\end{array}
$$

Types of Subtraction :

1) Direct Subtraction :

2) 1's Complement.

3) 2's Complement ⟹

* 1's Complement in "Binary Numbers" :

⇒ Replacing every "1" by "0" and every "0" by "1"

* ⇒ EX: 1's complement of "0111010100 1"

⇓

(1000101011 0)

---

* 2's Complement : "in Binary Numbers" :

⇒ 2 Ways :

1) Add $\boxed{1}$ to the 1's Complement.

OR

2) Start the (binary number) from right & Replacing each 1 → 0 and each 0 → 1, That After first 1

لما نبدأ من اليمين وبعد اول "1" يظهر نحول كل "1" لـ "0" وكل "0" لـ "1"

⇒ EX : 2's Complement of 1011100 is.

أول واحد ظاهر من اليمين نتركه ونحول ما بعده.

(0100100)

---

* Find 1's & 2's Complements for :

1) 110111101000 ⇒ 1's Complement is :

(001000010111)

⇒ 2's Complement is :

(001000011000)

EX:

Calculate the following binary subtraction in direct, 1's Comp. and 2's Comp. & verify the Result:

$$1101001.111 - 1100110.110$$

ANS:

(Direct method:)

$$\begin{array}{r} 1101001.111 \\ -\ 1100110.110 \\ \hline 0000011.001 \end{array} \Rightarrow$$

Direct subtraction:
105.875
−102.750
3.125

(1's Complement:)

① * ننزل أول رقم كما هو:

② * نحول الإشارة من السالب إلى موجب (من الطرح للجمع)

③ * نحول الرقم الثاني لـ (1's)

④ * لو ظهر ① زيادة عن عدد الـ bits نجمعه على الناتج.

$$\begin{array}{r} 1101001.111 \\ +\ 0011001.001 \\ \hline \boxed{1}\,0000011.000 \\ +\ 1 \\ \hline 0000011.001 \end{array} \Rightarrow$$

نجمع الـ bit الزائد

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1's Complement:

```
  1 1 1   1 1   1 1
  1 1 0 1 0 0 1 . 1 1 1
+ 0 0 1 1 0 0 1 . 0 0 1
-------------------------
[1] 0 0 0 0 0 1 1 . 0 0 0
                      + [1]
-------------------------
  0 0 0 0 0 1 1 . 0 0 1
```

2's Complement:

* نترك الأول كما هو.
* نحول الرقم الثاني إلى (2's).
* نحول الإشارة من الطرح إلى الجمع.
* أي رقم زيادة نحذفه.

```
  1 1 1     1 1   1
  1 1 0 1 0 0 1 . 1 1 1
+ 0 0 1 1 0 0 1 . 0 1 0
-------------------------
~~1~~ 0 0 0 0 0 1 1 . 0 0 1
```

Direct Subtraction:

```
  105.875
- 102.750
---------
    3.125
```

⇐ في حالة إننا نطرح الرقم الكبير من الصغير (يعني الرقم التاني هو الكبير) :-

* Calculate the following binary subtraction :-

01 1010 . 10 − 111011 . 11

ANS :-

Direct subtraction :-

$$\begin{array}{r} 011010.10 \\ -\ 111011.11 \\ \hline \end{array}$$

$$\begin{array}{r} -\ 011010.10 \\ \hline -100001.01 \end{array}$$

Direct verification :-

$$\begin{array}{r} 26.5 \\ -\ 59.75 \\ \hline -33.25 \end{array}$$

1's Complement :-

$$\begin{array}{r} 011010.10 \\ +\ 000100.00 \\ \hline \boxed{\ }011110.10 \end{array}$$

نلاحظ عدم وجود (bit) زيادة لذلك الناتج سالب ويساوي :-

The Result = − (1's Complement of (011110.10)

= − (100001.01)

⇐ في حالة إننا نطرح الرقم الكبير من الصغير (يعني الرقم الثاني هو الكبير :-)

* Calculate the following binary subtraction :-

$011010.10 - 111011.11$

ANS :

Direct subtraction :-

$$
\begin{array}{r}
011010.10 \\
-\ 111011.11 \\
\hline
-\ 011010.10 \\
\hline
-100001.01
\end{array}
$$

Direct verification :-

$$
\begin{array}{r}
26.5 \\
-\ 59.75 \\
\hline
-33.25
\end{array}
$$

1's Complement :-

$$
\begin{array}{r}
011010.10 \\
+\ 000100.00 \\
\hline
\boxed{}\ 011110.10
\end{array}
$$

نلاحظ عدم وجود (bit) زيادة لذلك الناتج سالب ويساوي :-

The Result $= -($ 1's Complement of $(011110.10))$

$= -(100001.01)$

2's Complement:

$$
\begin{array}{r}
0\,1\,1\,0\,1\,0\,.\,1\,0 \\
+\;0\,0\,0\,1\,0\,0\,.\,0\,1 \\
\hline
\boxed{-}\;0\,1\,1\,1\,1\,0\,.\,1\,1
\end{array}
$$

↳ The Result = − (2's Complement of (011110.11))

= −(100001.01)

No:......

Date:...............

* Signed 2's Complement Addition & subtraction:

By using the signed 2's complement notation calculate the following arithmetic operations :

(4 bits)

a) +2 +5

b) -3 +5

c) +3 -5

d) -3 -5

e) -4.75 -2.5

f) +3.25 -6.5

طبق ال 2's Comp.
إلا على السالب

The solution :

a) +2 +5

$$
\begin{array}{rrr}
 & 0010 & +2 \\
+ & 0101 & +5 \\
\hline
 & 0111 & +7
\end{array}
$$

$\underbrace{0}_{+}\underbrace{111}_{7}$

$\underline{5+9} \rightarrow 14$

No:......

Date:..............

b) $-3 + 5$

يحولها 2's لانه سالب

$$\begin{array}{r} -3 \\ +5 \\ \hline +2 \end{array} \qquad \begin{array}{r} \text{2's of}(0011) \\ +(0101) \end{array} \Rightarrow \begin{array}{r} 1101 \\ +\,0101 \\ \hline \cancel{1}0010 \\ (+2) \end{array}$$

c) $+3 - 5$

سيحولان لانها سالب

$$\begin{array}{r} +3 \\ -5 \\ \hline -2 \end{array} \qquad \begin{array}{l} \rightarrow (0011) \rightarrow \\ \rightarrow \text{2's of}(0101) \rightarrow \end{array} \qquad \begin{array}{r} 0011 \\ +\,1011 \\ \hline 1110 \end{array}$$

$= -\text{2's of } (1110)$

$= -(0010)$

$= (-2)$

d) $-3 - 5$

سيحولون لانهم سالب

$$\begin{array}{r} -3 \\ -5 \\ \hline -8 \end{array} \qquad \begin{array}{l} \rightarrow \text{2's comp. of } (0011) \rightarrow \\ \rightarrow \text{2's comp. of } (0101) \rightarrow \end{array} \qquad \begin{array}{r} 1101 \\ +\,1011 \\ \hline \cancel{1}1000 \end{array}$$

$= -\text{2's of } (1000)$

$= -1000$

$= (-8)$

No:......

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e) $\boxed{-4.75 - 2.5}$

هيحولوا الـ 2's Comp. لأنهم بالسالب

$\ominus 4.75 \rightarrow$ 2's Comp $(0100.11) \rightarrow 1011.01$

$\ominus 2.5 \rightarrow$ 2's Comp $(0010.10) \rightarrow +1101.10$

$\cancel{1}1000.11$

$\ominus 7.25$

$= -$ 2's Comp of

$= -0111.01$

$= (-7.25)$

f) $+3.25 - 6.5$

سيحولان لأنها بالسالب

$+3.25 \rightarrow (0011.01) \rightarrow (0011.01)$

$\ominus 6.5 \rightarrow$ 2's Comp $(0110.10) \rightarrow +(1001.10)$

1100.11

$\ominus 3.25$

$= -$ 2's Comp of

$= -(0011.01)$

$= (-3.25)$

(MR) GREEN

No:......

Date:..............

→ By using signed 2's Complement notation Calculate The following arithmetic operations :-

a) +1101.1 + 110.11

b) -1101.1 + 110.11

c) -1101.1 - 110.11

The Solution :-

a) +1101.1 + 110.11

الـ 2's comp لـ الموجب (+)

+13.5 → 01101.10

+6.75 → + 00110.11

+20.25 → 10100.01

↓

+(20.25) ⚠ over flow

$[-(2^{n-1}), +(2^{n-1}-1)] = [-(2^{5-1}), +(2^{5-1}-1)] = [-16, +15]$

out of Range.

b) -1101.1 + 110.11

⊖13.5 → 2's Comp. (01101.10) → 10010.10

+6.75 → (00110.11) → + 00110.11

⊖6.75 ← إشارة الـ 2's الأكبر

11001.01

= - 2's Comp. of it

= -(00110.11)

= -(6.75)

No:......

Date:..............

c) $-1101.1 - 110.11$

$\ominus 13.5 \rightarrow$ 2's Comp. $(01101.10) \rightarrow 10010.10$

$\ominus 6.75 \rightarrow$ 2's Comp. $(00110.11) \rightarrow {}^{+}11001.01$

$\ominus 20.25$

$\cancel{1}01011.11$

سيحولون للـ 2's لأنهم سالب

out of Range (over flow)

$\hookrightarrow = -$ 2's Comp. of $\square$

$= -10100.01$

$= -(20.25)$

$\hookrightarrow$ overflow.

⇒ Signed 2's Complement subtraction:

Calculate the following operation:

$(-10) - (-5)$

The solution

$(-10) - (-5) \Rightarrow (-10) + 5$

$\ominus 10 \rightarrow$ 2's Comp. of $(01010) \rightarrow 10110$

$+5 \rightarrow (00101) \rightarrow 00101$

$\ominus 5$

11011

يحولون للـ 2's لأنها سالب

$\hookrightarrow = -$ 2's of (11011)

$= -00101$

$= (-5)$

(MR) GREEN

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Date:..............

overflow Problem

← عندما ناتج الجمع أو الطرح يكون (out of Range) هنا تحدث مشكلة الـ (overflow).

EX:

$+5 \rightarrow 0101$

$+4 \rightarrow 0100$

$+9 \qquad 1001$

$(+9)$ is out of the Range.

The Range of 4 bits in 2's

$= [-(2^{n-1})_{10}, +(2^{n-1}-1)_{10}]$

$= [-(2^3)_{10}, +(2^3-1)_{10}]$

$= [-8, +7]$

-14

$+3$

-11

← قبل حل المسألة يتبين أن (-11)
(↳ is out of the Range)

⇒ To Avoid The Problem of overflow, The number of binary bits must be increased.

↳ instead of using 4 bits. We Will use 5 bit, & Repeat The Solution on 5 bits.
