بسم الله الرحمن الرحيم

* Number system:

$(\ )_2$

$(\ )_3$

$(\ )_4 \longrightarrow (\ )_{10}$ index.

$(\ )_8$

$(\ )_{16}$

$(\ )_{10} \longrightarrow (\ )_2$, $(\ )_7$, $(\ )_8$, $(\ )_{16}$ $\div$, $*$

$(\ )_2 \xleftrightarrow[\text{2bits}]{} (\ )_4$

$(\ )_2 \xleftrightarrow[\text{3bits}]{} (\ )_8$

$(\ )_2 \xleftrightarrow[\text{4bits.}]{} (\ )_{16}$

Q: let $X = (\underline{247}.390625)_{10}$
find the value of X in Hexadecimal.

$(\;)_{16}$ 0 → 9
A → F

Ans:

integer	remainder		frac.	integer
16 ÷ 247			16 * 0.390625	
16 ÷ 15	7		0.25	6
0	15 $\boxed{F}$		0	4

$(F\,7.6\,4)_{16}$

integer fraction

$\boxed{15.\_\_\_}$

$\frac{15}{16}$

$\boxed{6.25}$

EX.

$\frac{27}{100}$ →

②

Q let $X = (1001101.1011)_2$

find The value of X in octal & Hexadecimal.

$(\ )_2 \xrightarrow[\text{3 bits.}]{} (\ )_8$

$$\begin{array}{ccc|ccc|ccc|c|ccc|ccc}
 & & & & & & & & & & & & & 2^2 & 2^1 & 2^0 \\
4 & 2 & 1 & 4 & 2 & 1 & 4 & 2 & 1 & & 4 & 2 & 1 & 4 & 2 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & . & 1 & 0 & 1 & 1 & 0 & 0
\end{array}$$

$$(1 \quad 1 \quad 5 \ . \ 5 \quad 4)_8$$

$$\begin{array}{cccc|cccc|c|cccc}
8 & 4 & 2 & 1 & 8 & 4 & 2 & 1 & & 8 & 4 & 2 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & . & 1 & 0 & 1 & 1
\end{array} \rightarrow (\ )_{16}$$

$$(4 \quad D \ . \ B)_{16}$$

A 10

B 11

C 12

D 13

E 14

F 15

* Convert the binary number $(10110.10101)_2$ to its equivalent numbers in:

a) octal $\quad ( )_2 \xrightarrow[3\text{ bits}]{} ( )_8$

b) Hexa. $\quad ( )_2 \xleftrightarrow[4\text{ bits}]{} ( )_{16}$

c) decimal $\quad ( )_2 \xRightarrow[\text{index}]{} ( )_{10}$

d) Base 4

c)

$$\begin{array}{ccccccccccc} 4 & 3 & 2 & 1 & 0 & & -1 & -2 & -3 & -4 & -5 \\ 1 & 0 & 1 & 1 & 0 & . & 1 & 0 & 1 & 0 & 1 \end{array}_2 \longrightarrow (?)_{10}$$

$$(1*2^4)+(1*2^2)+(1*2^1)+$$
$$(1*2^{-1})+(1*2^{-3})+(1*2^{-5})$$

$$16+4+2+\frac{1}{2}+\frac{1}{2^3}+2^{-5}$$

$$(\boxed{\qquad\quad}.\boxed{\qquad})_{10}$$

$$( )_2 \xrightarrow[2\text{ bits}]{} ( )_4$$

$$(112.222)_4$$

$$\underbrace{\overset{2\,1}{01}}\;\underbrace{\overset{2\,1}{01}}\;\underbrace{\overset{2\,1}{10}}.\underbrace{\overset{2\,1}{10}}\;\underbrace{\overset{2\,1}{10}}\;\underbrace{\overset{2\,1}{10}}$$

Q: let $X = (\text{—})_{10}$

$y = (\text{—})_{16} \rightarrow (\ )_{10}$ index

$Z = (\text{—})_{8} \rightarrow (\ )_{10}$ index

find: $(X - y) * (X + y)$

Q * Find the sum of binary numbers:

11010.110 ~~&~~ + 01110.101

$$\begin{array}{r} 11010.110 \\ +\ 01110.101 \\ \hline 101001.011 \end{array}$$

* Calculate the following operations:

a) 1100101 - 0100111

(Direct , 1's Comp. , 2's Comp.)

بتأثر على السالب
فقط
(أو إشارة الطرح)

→ **Direct**

$$
\begin{array}{r}
1100101 \\
-\;0100111 \\
\hline
0111110
\end{array}
$$

→ **1's Comp.**

$$
\begin{array}{rcr}
1100101 & \longrightarrow & 1100101 \\
-\,0100111 & \xrightarrow{\text{1's comp.}} & +\;1011000 \\
\hline
 & & \cancel{1}\,0111101 \\
 & \text{carry} \leftarrow & 1+ \\
\hline
 & & (0111110)
\end{array}
$$

2's Comp.

$$
\begin{array}{r}
1100101 \\
-\ 0100111 \\
\hline
\end{array}
\xrightarrow{2\text{'s comp}}
\begin{array}{r}
1100101 \\
+\ 1011001 \\
\hline
\cancel{1}\,0111110
\end{array}
$$

b) $0100111 - 1100101 = -$

(using Direct, 1's comp. , 2's comp.)

* **Direct**

$$
\begin{array}{r}
1100101 \\
-\ 0100111 \\
\hline
-\ 0111110
\end{array}
$$

* 1's Comp:

$$\begin{array}{rr} & 0100111 \\ - & 1100101 \\ \hline \end{array} \xrightarrow{} \quad \begin{array}{rr} & 0100111 \\ + & 0011010 \\ \hline & 1000001 \end{array}$$

$1'\text{s Comp.}$

the result = − 1's Comp. of (100000)

− 0111110

* 2's Comp.

$$\begin{array}{rr} & 0100111 \\ - & 1100101 \\ \hline \end{array} \xrightarrow{} \quad \begin{array}{rr} & 0100111 \\ + & 0011011 \\ \hline & 1000010 \end{array}$$

$2'\text{s Comp.}$

the result = − 2's Comp. of (100010)

− 0111110

c) $(-6) + (+3)$

(using signed 2's Comp. in 4 bits)

$\square$

$$\begin{array}{lcl} \ominus 6 \rightarrow -0110 & \xrightarrow{2\text{'s Comp.}} & 1010 \\ +3 \rightarrow +0011 & \longrightarrow & +0011 \\ \hline \ominus 3 & & \end{array}$$

Weight of 4 bits

8	4	2	1
0	1	1	0
0	0	1	1

$$\begin{array}{r} \overset{1}{1}\,0\,1\,0 \leftarrow 2\text{'s Comp.} \\ +\,0\,0\,1\,1 \\ \hline \boxed{1\,1\,0\,1} \end{array}$$

The result $= -$ 2's Comp. of (1101)

$$-0011$$

Range of 2's Comp. in 4 bits:

$$[-2^{n-1}, +2^{n-1}-1]$$

$$[-8, +7]$$

d) $(-6) + (-3)$

(using signed 2's comp. in 4 bits)

$\boxed{1}$ Range of 2's Comp. in 4 bits.

$[-2^{n-1}, +2^{n-1}-1]$

$[-8, +7]$

$-6 \longrightarrow -0110 \xrightarrow{\text{2's Comp}} 1010$

$-3 \longrightarrow -0011 \xrightarrow{\text{2's Comp}} +1101$

$\boxed{\cancel{1}0111}$

$\boxed{-9}$

↳ in 4 bits ⟶ (over flow)

EX: $-10 - 3$ ⟶ using 2's comp in 4 bits. $[-8, +7]$

-10 ⟶ out of Range

-3

-13

in 5 bits

$[-16, +15]$

2 cases

Case ① صغير - كبير

Direct

1's Comp.

+ 1's Comp.

①

+1

2's comp.

+ 2's Comp.

~~①~~

Case ② كبير - صغير

Direct

صغير
- كبير

1's Comp.

+ 1's Comp.

the result = - 1's Comp.

2's Comp

+ 2's Comp.

the result = - 2's comp

9

$(-6) + (-3)$ using 2's Comp. in 4 bits

↳ Calculate the Operation

and if any Problem is founded show ~~see~~ it and How to solve it?

→ $\boxed{\text{overflow}}$

→ increase the number of bits to be 5 bits:

* Calculate the following operation:

$13.625 - 27.750$

(using 9's Comp. & 10's Comp.)

1's Comp. 2's Comp.

1) 9's Comp.

$$\begin{array}{r} 13.625 \\ -\ 27.750 \\ \hline \ominus 14.125 \end{array}$$

Direct

$13.625 \longrightarrow 13.625$

$27.750 \xrightarrow{9's} +\ 72.249$

$$\begin{array}{r} 13.625 \\ +\ 72.249 \\ \hline 85.874 \end{array}$$

the result = $-$9's Comp. of (27.750)

the result = $-$9's Comp. of (85.874)

(−14.125)

2) 10's Comp.

$$\begin{array}{r} 13.625 \\ -27.750 \end{array} \xrightarrow{\text{10's Comp.}} \begin{array}{r} 13.625 \\ +72.250 \\ \hline 85.875 \end{array}$$

$\boxed{72.250}$

10's Comp

the result =

$-$ 10's Comp. of (85.875)

(-14.125)

* Show the upper & lower bounds of decimal value that are allowed for binary bit Patterns of length 4, 5, 6, 7 & 8 in cases of:

1) ~~un~~signed
2) signed magnitude
3) 1's Comp.
4) 2's Comp.
5) Excess notation.

$n = 4$

1) unsign: $[0, 2^{n}-1]$

$[0, 15]$

lower bound   upper bound

2) signed magnitude:

$[\mp(2^{n-1}-1)] = [\mp 7]$

3) 1's Comp.: $[\mp 7]$

4) 2's Comp.

if $n = 4$

$[-(2^{n-1})_{10}, +(2^{n-1}-1)_{10}]$

$[-8, +7]$

5) Excess notation

EX)

$\circled{4}\ 2\ 1$

Excess 4 $\longrightarrow$ 3 bits $\longrightarrow$ $\boxed{n=3}$

$\hookrightarrow [-4, +3]$

$[-2^{3-1}, +2^{3-1}-1]$

$8\ 4\ 2\ 1$

Excess 8 $\longrightarrow$ 4 bits

$\hookrightarrow [-8, +7]$

$16\ 8\ 4\ 2\ 1$

Excess 16 $\longrightarrow$ (5 bits)

$\hookrightarrow [-16, +15]$

Q: Find the integer decimal Value of binary number $(\underline{10101})_2$ in the following systems:

$$\begin{array}{ccccc} 16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{array}$$

→ unsign: $(16+4+1) = (21)_{10}$

→ signed.

$$\boxed{1}\,\underbrace{0101}$$

$(-5)_{10}$

→ 1's comp. $= $ 1 0 1 0 1 $= -$ 1's Comp. of (0101)

$-\ 1010$

$(-10)_{10}$

→ 2's Comp.

→ Excess-16

2's Comp:

$\boxed{1}\,0101$

$= -$ 2's comp. of (0101)

$$\begin{array}{ccccc} & 8 & 4 & 2 & 1 \\ - & 1 & 0 & 1 & 1 \end{array}$$

$(-11)_{10}$

(B

$(10101)_2 \rightarrow (?)_{10}$

↳ Excess 16

$\overline{\text{5 bits}}$

16 8 4 2 1

$$\begin{array}{ccccc} 16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{array} - 16$$

$= 21 - 16$

$= (+5)_{10}$

زيادة

$$\begin{tikzcd}
\text{Excess}(\ )_2 \arrow[d, bend left, "-\text{not}"] \\
(\ )_{10} \arrow[u, bend left, "+\text{not.}"]
\end{tikzcd}$$

* find the binary value of integer number $(-13)_{10}$ in the following systems: in 5 bits

1) unsign. $[0, 2^n - 1] = [0, 31]$

$(-13) \to (\ )_2$

$(0\,1\,1\,0\,1)$

$2 \div 13$

2) signed-magnitude

16	8	4	2	1
0	1	1	0	1

$(-13)_{10}$

$\boxed{2}$ (13) in 5bits.

$\boxed{0}$ 1 1 0 1

سالب

1	1	1	0	1

$\boxed{1}$ Range $[\mp(2^{n-1}-1)] \to [\mp 15]$

3) 1's Comp. of $(-13)_{10}$ in 5 bits

$[\mp(2^{n-1}-1)] \Rightarrow [\mp 15]$

① binary of 13 in 5 bits

0 1 1 0 1

② if the number − or +

(−) → Convert it to 1's.

1's Comp. of (0 1 1 0 1)

$(1\ 0\ 0\ 1\ 0)_2$ in 1's Comp.

4) 2's Comp. [−16, +15]

1 (0 1 1 0 1) $\boxed{-13}$

2 2's Comp. of (0 1 1 0 1)

$(1\ 0\ 0\ 1\ 1)_2$ in 2's Comp.

$(-18)_{10} \to (\ )_2$ using exc 16.

16 8 4 2 1

5 bits

$[-16, +15]$

(−18) out of Range

$(-13)_{10} \to (\ )_2$ in exc 16

$-13 + 16 \to \boxed{3}$

$(0\,0\,0\,\overset{2}{1}\,\overset{1}{1})$

$$\begin{tikzcd}
\text{exc } (\ )_2 \arrow[d, bend left, "\text{-not}"] \\
\text{decimal} \arrow[u, bend left, "\text{not}"]
\end{tikzcd}$$

* Assuming floating-point binary pattern in IEEE notation.

1) Code the decimal value $(+43)_{10}$ as a binary floating point number.

2) Decode the Hexadecimal pattern $(C0A40000)_{16}$ to its equivalent decimal value.

3) Decode the Hexadecimal pattern $(7FB76E5B)_{16}$ to its equivalent decimal value.

*(IEEE)

① Representation. 32 bits

1 S	e (8 bits)	f (23 bits)

② $1.f \times 2^{e}$

③ exponent → excess 127

1) Code $(+43)_{10} \rightarrow (?)_2$ in floating Point. ~~fet~~

① $(+43)_{10}$

$\Downarrow$

$+1.01011$

remain

$2 \div \overline{|\,43}$

64	32	16	8	4	2	1
	1	0	1	0	1	1

② $1.f \times 2^e$

$+1.\underbrace{01011}_{f} \times 2^{+5}$ e

e → exponent

③ $f = +0.01011$

$e = +5$

④ $e = +5 \rightarrow$ to $(\ )_2$ ~~in~~

using exam 127

$+5 + 127$

$\boxed{132}$

256	128	64	32	16	8	4	2	1
	1	0	0	0	0	1	0	0

(10000100)

5

1sp	e(8bits)	f(23bits)
0	1 0 0 0 0 1 0 0	0 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...

$f = + \, 0.\underline{01011}$

$+ \leftrightarrow 0$
$- \leftrightarrow 1$

$e = (10000100)$

$(\underbrace{0100}_{4}\,\underbrace{0010}_{2}\,\underbrace{0010}_{2}\,\underbrace{1100}_{C}\,\ldots\ldots)_2$ 18 zeros.

$(4\;2\;2\;C\;0000)_{16}$

2) Decode

$(C\,0\,A\,4\,0\,0\,0\,0)_{16} \rightarrow (\ )_{10}$

ANS.

① C 0 A 4 0 0 0 0

| 1 | 0 0 0 0 0 0 | 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

C → 12

	8	4	2	1
C → 12 →	1	1	0	0
A → 10 →	1	0	1	0
4 →	0	1	0	0

②

S	e (8 bits)	f (23 bits)
1	1 0 0 0 0 0 0 1	0 1 0 0 1 0 0 . . . . .

③

$f = -0.0100100$

$$e = \begin{array}{cccccccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}_{\,2} \rightarrow (\ )_{10} \text{ using excess } 127$$

$129 - 127 \rightarrow (+2)_{10}$

$f = \underline{-0.01001}$

$e = (+2)_{10}$

(4) $1.f \times 2^{e}$

$(-1.01001 \times 2^{+2})$

$-101.001 \longrightarrow (-5.125)_{10}$

3) Decode $(7FB76E5B)_{16}$

$\hookrightarrow (\qquad)_{10}$

7 F B 7 6 E 5 B

1011

0111 1111 1011 0111 0110 1110 0101

1st	e (8 bits)	f (23 bits)
0	11111111	011 . . . . .

Special Case

$e = 1 \qquad f \neq 0$

Not a Number

NaN.

* 4 Special Case in IEEE:

$e = 1$

$f = 0 \rightarrow \pm\infty$

$f \neq 0 \rightarrow NaN$

$e = 0$

$f = 0 \rightarrow \pm 0$

$f \neq 0$

↳ the number is less than the allowed minimum value