trapezodial Proof:-

$$m=\frac{y-y_0}{x-x_0} \Rightarrow y-y_0=m(x-x_0) \Rightarrow y=y_0+m(x-x_0)\ ,\ \because m=\frac{y_n-y_0}{x_n-x_0}$$

$$\therefore y=y_0+\frac{y_n-y_0}{x_n-x_0}(x-x_0)\ ,\ I=\int_{x_0}^{x_n}\left(y_0+\frac{y_n-y_0}{x_n-x_0}(x-x_0)\right)dx \Rightarrow I=\int_{x_0}^{x_n} y_0+\frac{y_n-y_0}{x_n-x_0}\cdot x-\frac{y_n-y_0}{x_n-x_0}\cdot x_0$$

$$I=\int_{x_0}^{x_n}\left(\frac{y_n-y_0}{x_n-x_0}\cdot x+y_0-\frac{x_0y_n-x_0y_0}{x_n-x_0}\right)dx \Rightarrow I=\int_{x_0}^{x_n}\left[\frac{y_n-y_0}{x_n-x_0}\cdot x+\frac{y_0(x_n-x_0)-(x_0y_n-x_0y_0)}{x_n-x_0}\right]dx$$

ثابت متغير ثابت

$$I=\int_{x_0}^{x_n}\left[\frac{y_n-y_0}{x_n-x_0}\cdot x+\frac{x_ny_0\cancel{-x_0y_0}-x_0y_n\cancel{+x_0y_0}}{x_n-x_0}\right]dx \Rightarrow I=\int_{x_0}^{x_n}\left(\frac{y_n-y_0}{x_n-x_0}\cdot x+\frac{x_ny_0-x_0y_n}{x_n-x_0}\right)dx$$

$$I=\frac{y_n-y_0}{x_n-x_0}\cdot\frac{x^2}{2}\Big|_{x_0}^{x_n}+\frac{x_ny_0-x_0y_n}{x_n-x_0}\cdot x\Big|_{x_0}^{x_n}$$ by substitute $x \Rightarrow x_n$, $x \Rightarrow x_0$

$$I=\left(\frac{y_n-y_0}{x_n-x_0}\cdot\frac{x_n^2}{2}-\frac{y_n-y_0}{x_n-x_0}\cdot\frac{x_0^2}{2}\right)+\left(\frac{x_ny_0-x_0y_n}{x_n-x_0}\cdot x_n-\frac{x_ny_0-x_0y_n}{x_n-x_0}\cdot x_0\right)$$

$$I=\left(\frac{1}{2}\cdot\frac{y_n-y_0}{x_n-x_0}(x_n^2-x_0^2)\right)+\left(\frac{x_ny_0-x_0y_n}{(x_n-x_0)}\cdot(x_n-x_0)\right)$$

$-x_ny_0$
$+x_0y_n$

$$I=\frac{1}{2}\cdot\frac{y_n-y_0}{\cancel{(x_n-x_0)}}\cdot\cancel{(x_n-x_0)}(x_n+x_0)+(x_ny_0-x_0y_n) \Rightarrow I=\frac{1}{2}(y_n-y_0)(x_n+x_0)+(x_ny_0-x_0y_n)$$

$$I=\frac{x_ny_n-x_ny_0+x_0y_n-x_0y_0+2x_ny_0-2x_0y_n}{2} \Rightarrow I=\frac{x_ny_n-x_0y_0+x_ny_0-x_0y_n}{2}$$ take x_n , x_0

$$I=\frac{x_n(y_n+y_0)-x_0(y_0+y_n)}{2} \Rightarrow I=\frac{(y_n+y_0)(x_n-x_0)}{2} \Rightarrow I=(x_n-x_0)\cdot\frac{(y_n+y_0)}{2}$$

$x_n-x_0=h$

$$\boxed{I=h\cdot\frac{y_n+y_0}{2}}$$

general form $I=\int_{x_{n-1}}^{x_n} f(x)\,dx=h\left(\frac{y_n+y_{n-1}}{2}\right)$

where $h=(x-x_{n-1})$

[1] Bisection method

iteration	a	f(a)	b	f(b)	ε
initial ⇐ 0					—
↓ 1					0

$$\frac{a|b \text{ current} - a|b \text{ previous}}{a|b \text{ current}} \times 100 = \text{Percentage error}$$

[2] Newton – Raphson method

[1] $f'(x)$ & $f''(x)$

[2] find two Points that achieve ⇒ $f(a) = +ve$ & $f(b) = -ve$

[3] using each one of previous two Points a, b to find initial value x_0 using

$$\boxed{f(a|b) \cdot f''(a|b) = +ve}$$

[4] get $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$

$$\boxed{\varepsilon = |x_n - x_{n-1}|}$$

X_s	Value	ε
X_0	⊖	—
↓ X_1	⊖	

$$\frac{\text{new } x - \text{old } x}{\text{new } x} \times 100$$

→ Percentage error

---

[1] relative error (ε) $= \dfrac{\text{true} - \text{approximate}}{\text{true}} = \dfrac{\text{Current} - \text{Previous}}{\text{Current}}$

[2] taylor series

$$f(x) = f(0) + f'(0)(x-c) + f''(0)\frac{(x-c)^2}{2!} + f'''(0)\frac{(x-c)^3}{3!} \ldots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots$$

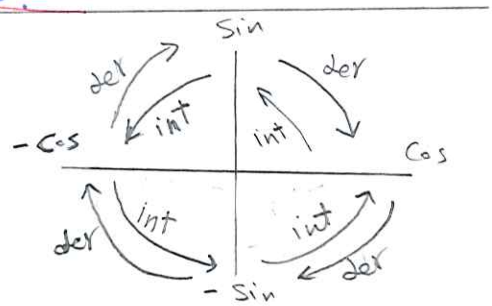
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$$

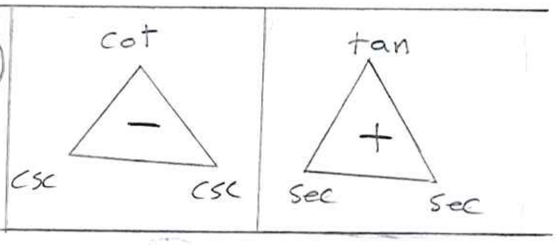
$$\tan^{-1}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

تفاضل
[redacted] مع عقارب الساعه
[redacted] عكس عقارب الساعه
تكامل



. اخفاء واحدة يبقى تفاضل
. اخفاء اثنين يبقى تكامل



- $\dfrac{d}{dx}\ln(x) = \dfrac{1}{x}$ & $\dfrac{d}{dx}\log_a(x) = \dfrac{1}{a \cdot \ln(x)}$
- $\dfrac{d}{dx}\dfrac{a}{b} = \dfrac{a'b - ab'}{b^2}$
- $\dfrac{d}{dx}(1-x)^{-1} = -1(1-x)^{-1-1} \Rightarrow -1(1-x)^{-2}$

1) trapezodial rule $\Rightarrow \boxed{n = 1}$

$$\boxed{I = \frac{h}{2}\left[f(a) + f(b)\right]}$$

2) Composite trapezodial rule $\Rightarrow \boxed{n > 1}$

$$\boxed{I = \frac{h}{2}\left[f(a) + f(b) + 2(f(x))\right]}$$

3) Simpson $\frac{1}{3}$ rule $\Rightarrow \boxed{n = 2}$

$$\boxed{I = \frac{h}{3}\left[f(a) + f(b) + 4(f(x))\right]}$$

4) Composite simpson $\frac{1}{3}$ rule $\Rightarrow \boxed{n = \quad}$

$$\boxed{I = \frac{h}{3}\left[f(a) + f(b) + 4(f\, x_{\underline{odd}}) + 2(f\, x_{\underline{even}})\right]}$$

5) Simpson $\frac{3}{8}$ rule $\Rightarrow \boxed{n = 3}$

$$\boxed{I = \frac{3h}{8}\left[f(a) + f(b) + 3(f(x))\right]}$$

6) Composite simpson $\frac{3}{8}$ rule $\Rightarrow \boxed{n = \quad}$

$$\boxed{I = \frac{3h}{8}\left[f(a) + f(b) + 3(f_{\text{\underline{not} divided by 3}}) + 2(f_{\text{\underline{divided} by 3}})\right]}$$

7) midpoint rule

$$\boxed{I = f\left(\frac{a+b}{2}\right)\cdot(b-a)}$$

---

1) Forward difference approximation $\underline{\text{using 2 Points}}$ $\underline{\text{تناقص } i}$

$$\left.\frac{df}{dx}\right|_{x=x_i} = \frac{f(x_{i+1}) - f(x_i)}{x_{i+1} - x_i}$$

2) backword difference approximation $\underline{\text{using 2 Points}}$ $\underline{\text{تناقص } i}$

$$\left.\frac{df}{dx}\right|_{x=x_i} = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

3) Central difference approximation $\underline{\text{using 2 Points}}$ $\underline{\text{تناقص } i}$

$$\left.\frac{df}{dx}\right|_{x=x_i} = \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}}$$

$i \quad i+1 \quad i+2$

$0 \quad 1 \quad 2 \quad \longrightarrow$ $\underline{\text{تزايد } i}$

$-3 \quad +4 \quad -1$

4) Forward difference approximation using 3 Points

$$f'(x_i) = \frac{-3f(x_i) + 4f(x_{i+1}) - f(x_{i+2})}{2h}$$

$h = x_{i+1} - x_i$

$-2 \quad -1 \quad 0 \quad \longrightarrow$ $\underline{\text{تزايد } i}$

$1 \quad -4 \quad +3$

5) backword difference approximation using 3 Points

$$f'(x_i) = \frac{f(x_{i-2}) - 4f(x_{i-1}) + 3f(x_i)}{2h}$$

$h = x_i - x_{i-1}$

$$\ln(1+x) = f(0) + f'(0)(x-c) + f''(0)\frac{(x-c)^2}{2!} + f'''(0)\frac{(x-c)^3}{3!}$$

$$= \ln(1+0) + \left[(1+x)^{-1}\cdot(x-0)\right] + \left[-1(1+x)^{-2}\cdot\frac{(x-0)^2}{2!}\right] + \left[2(1+x)^{-3}\cdot\frac{(x-0)^3}{3!}\right]$$

$$= 0 + x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$= \boxed{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \ldots}$$

$$\frac{1}{1-x} = f(0) + f'(0)(x-c) + f''(0)\frac{(x-c)^2}{2!} + f'''(0)\frac{(x-c)^3}{3!}$$

$$= \frac{1}{1-0} + \left[-1(1-x)^{-2}\cdot -1\cdot(x-0)\right] + \left[-2(1-x)^{-3}\cdot -1\cdot\frac{(x-0)^2}{2!}\right] + \left[-6(1-x)^{-4}\cdot -1\cdot\frac{(x-0)^3}{3!}\right]$$

$$= 1 + x + x^2 + x^3 + \ldots$$

$$e^x = f(0) + f'(0)(x-c) + f''(0)\frac{(x-c)^2}{2!} + f'''(0)\frac{(x-c)^3}{3!}$$

$$= e^0 + \left[e^0\cdot(x-0)\right] + \left[e^0\cdot\frac{(x-0)^2}{2!}\right] + \left[e^0\cdot\frac{(x-0)^3}{3!}\right]$$

$$\boxed{= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots}$$

$$\sin x = f(0) + f'(0)(x-c) + f''(0)\frac{(x-c)^2}{2!} + f'''(0)\frac{(x-c)^3}{3!}$$

$$= \sin(0) + \left[\cos(0)\cdot(x-0)\right] + \left[-\sin(0)\cdot\frac{(x-0)^2}{2!}\right] + \left[-\cos(0)\cdot\frac{(x-0)^3}{3!}\right]$$

$$= 0 + 1\cdot x + 0\cdot\frac{x^2}{2!} + -1\cdot\frac{x^3}{3!}$$

$$= \boxed{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}}$$
