20/10/2019 MS621

Lecture ①

Discrete Mathematics

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Discrete Mathematics with applications
by:- Susanna S. Epp

1- the logic
2- set theory
3- Mathematical Induction
4- Recursion
5- difference equations
6- Functions
7- graph theory

the logic

1) year is 266 days ✓ statement
2) he is a student ✗
3) 4+2=7 ✓

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statement → truth value < T / F

p, q, r, — p = Cairo is the capital of egypt

compond statement ~p = cairo isn't ~ ~

connective	meaning	symbol	called
① Negation	not	~ or ¬	Tilled
② conjunction	and	$\wedge$	hat
③ disjunction	or	$\vee$	vel
④ conditional	if...then	$\rightarrow$	arrow
⑤ Biconditional	if and only if	$\leftrightarrow$	double arrow

q = today is sunny

* $p \wedge q$ * $p \vee q$

* $p \rightarrow q$ * $p \leftrightarrow q$

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ex. ①

let p = it is hot , q = it is sunny

represent the following sentence in a symbolic form

1- it is not hot $\sim p$

2- it is hot and sunny $p \wedge q$

3- it is not hot but sunny $\sim p \wedge q$

but $\downarrow$ and

4- it is neither hot nor sunny $\sim p \wedge \sim q$

ex ②

m = Ali is good in Maths

c = Ali is a computer science Student

* translate the following symbols statement into plan english

1- $\sim c$ Ali isn't a computer ~ ~

2- $c \vee m$ Ali is a computer or Ali is good in Maths

3- $m \wedge \sim c$ Ali is good in Math and isn't a computer science student

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Homwork.:

h = Zia is healthy
w = Zia is wealthy
s = zia is wise
1 - $h \wedge w \wedge \sim s$
2 - Ziad is not wealthy but he is healthy and wise
3 - Ziad is neither heathy, wealthy nor wise.

① Zia is healthy and wealthy and wise

② $\sim w \wedge h \wedge s$

③ $\sim h \wedge \sim w \wedge \sim s$

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Truth Table

① p

p	~p
T	F
F	T

② p∧q

p	q	p∧q	p∨q
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

③ p → q

p = I am egyption
q = I am African

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

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④ $p \leftrightarrow q$

p = x^2 is negative

q = x is complex number

p	q	$p \leftrightarrow q$
T	T	T
T	f	f
f	T	f
f	f	T

Hierarchy of operations

1- bracket

2- Negation

3- conj & disj $\wedge, \vee$

4- $\rightarrow$, $\leftrightarrow$

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عبد الله سعيد الجبول 2 number of simple عبد الله سعيد

$p \vee \sim q \rightarrow \sim p$

p	q	~q	~p	p∨~q	p∨~q → ~p
T	T	F	F	T	F
T	F	T	F	T	F
F	T	F	T	F	T
F	F	T	T	T	T

$(p \rightarrow q) \wedge (\sim p \rightarrow r)$

p	q	r	p → q	~p	~p → r	
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

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tautology & contradiction
t & c

p	~p	p∨~p
T	F	T
F	T	T

p∨~p

↳ tautology statement

p	~p	p∧~p
T	F	F
F	T	F

p∧~p

↳ contradiction statement

ex. Find using Truth table ~p∧(q∨~r)

p	q	r	~r	q∨~r	~p	~p∧(q∨~r)
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F		T	F
F	F	F	T		T	T

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$p \wedge q \equiv 0$

$\sim(\sim p) \equiv p$

p	$\sim p$	$\sim(\sim p)$
T	F	T ✓
F	T	F ✓

$\sim(p \wedge q) \equiv \sim p \wedge \sim q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	F
F	T	F	T	T	F	F
F	F	F	T	T	T	T

these two statments aren't equvelent since they have different truthe values in rows 2&3

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① if "you get ≥ 50" (p) then "you will succeed" (q)
② if "you faild" then "you get < 50"
are these two statements equivalent?

$p \to q \equiv \sim q \to \sim p$

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

these two statments are equivelent since they have the same truth values in all rows

$p \vee q \to r$ & $(p \to r) \wedge (q \to r)$

p	q	r	$p \vee q$	$p \vee q \to r$	$p \to r$	$q \to r$	$p \to r \wedge q \to r$	
T	T	T	T	T	T	T	T	✓
T	T	F	T	F	T	F	F	✓
T	F	T	T	T	F	T	F	X
T	F	F	T	F	F	T	F	✓
F	T	T	T	T	T	T	T	✓
F	T	F	T	F	T	F	F	✓
F	F	T	F	T	T	T	T	✓
F	F	F	F	T	T	T	T	✓

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① $\sim(p \wedge q) \equiv \sim p \vee \sim q$
② $\sim(p \vee q) \equiv \sim p \wedge \sim q$
DeMorgan's laws

①

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

these two statement are equivlant since they have the same truth value in all rows

②

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

these two statement are equavlent since they have the same truth values in all rows

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Dr. Marwa

lecture 2

Negation for if then statment

* $\sim(p \to q) \equiv p \wedge \sim q$

ex. write a negation of
if Ali lives in egypt then he lives in cairo

① Ali lives in egypt and he doesn't live in cairo

② If my car is in the repair shop then I can't get to class

my car is in the repair shop and I can get to class

Inverse, converse, contrapositive
(for $p \to q$)

* Inverse $p \to q$, $\sim p \to \sim q$
* converse for $p \to q$ $q \to p$
* contrapositive for $p \to q$ $\sim q \to \sim p$

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→ if today is easter, then tomorrow is Monday
write the Invese, converse, conterapositive

* Inverse
if today isn't easter, then tomorrow isn't Monday
* converse
if ~~today is~~, tomorrow is Monday then today is easter
* contrapositive
if tomorrow isn't Monday then today isn't easter

---

if john can swim across the lake then john can swim to the island
* Inverse

* converse

* conterpositive

Valid and invalid argument

Statement 1 }
Statement 2 } premesis

Statement ← conclusion

∴ therefore

→ Show that the following argument is valid

$p \vee (q \vee r)$
$\sim r$
$\therefore p \vee q$

premesis ↓ ↓ conclusion ↓

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$\sim r$	$p \vee q$	
T	T	T	T	T	F	T	
T	T	F	T	T	T	T	✓
T	F	T	T	T	F	T	
T	F	F	F	T	T	T	✓
F	T	T	T	T	F	T	
F	T	F	T	T	T	T	✓
F	F	T	T	T	F	F	
F	F	F	F	F	T	F	

This is a valid argument since all true premesis have true conclusion

$p \to q$
p
$\therefore q$

premssis — conclusion

p	q	$p \to q$	p	q	
T	T	$\boxed{T}$	$\boxed{T}$	T	✓
T	F	F	T	F	
F	T	T	F	T	
F	F	T	F	F	

ex3
State whether the following argument is valid or not

$p \to q \vee \sim r$
$q \to p \wedge r$
$\therefore p \to r$

conclusion — premssis

$p \to r$	p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$
T ✓	T	T	T	F	T	T	$\boxed{T}$	$\boxed{T}$
F	T	T	F	T	T	F	T	F
T	T	F	T	F	F	T	F	T
F ×	T	F	F	T	T	F	$\boxed{T}$	$\boxed{T}$
T	F	T	T	F	T	F	T	F
T	F	T	F	T	T	F	T	F
T ✓	F	F	T	F	F	F	$\boxed{T}$	$\boxed{T}$
T ✓	F	F	F	T	T	F	$\boxed{T}$	$\boxed{T}$

this is invalid argument since the fourth row has true premesis and false conclusion.

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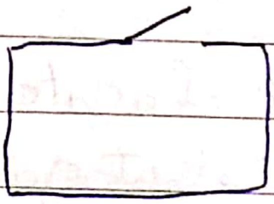
logical circuts



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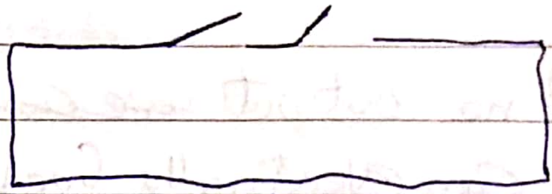
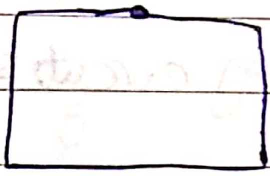


open

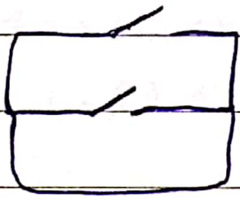
1	1	→ 1
1	0	→ 0
0	1	→ 0
0	0	→ 0



one

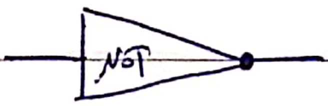
close

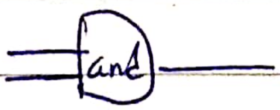
like and

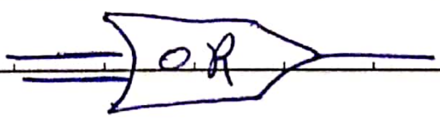


1	0	→ 1
0	1	→ 1
1	1	→ 1
0	0	→ 0

like or







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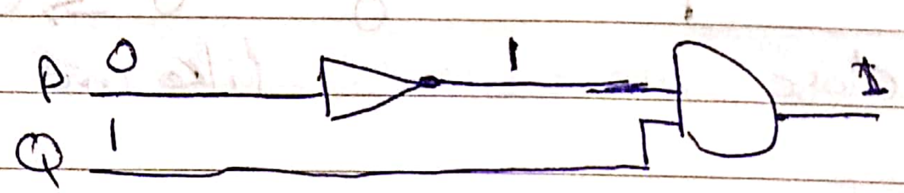
① never combine 2 input wires without a gate
② A single input wire can be split partway and used as input for two separate gates

③ an output wire can be used as input

④ no output ~~wire can be used~~ of a gate can eventually feed back into that gate.

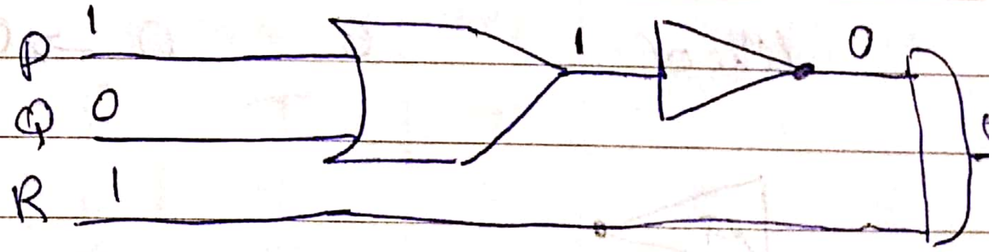
ex.
indicate the output for the following circuits

1-



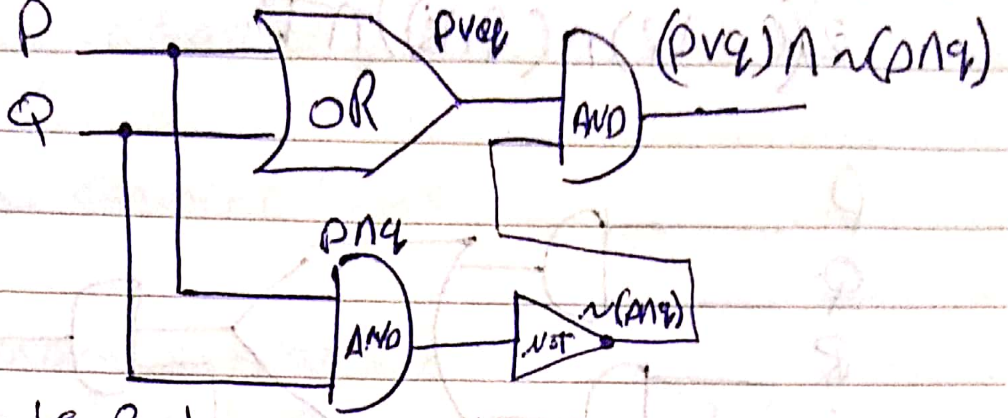
P = 0 S = 1
q = 1

2-



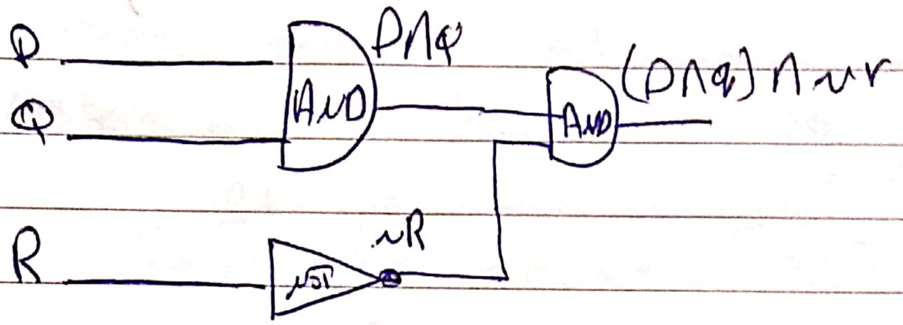
P = 1
Q = 0
R = 1

S = 0



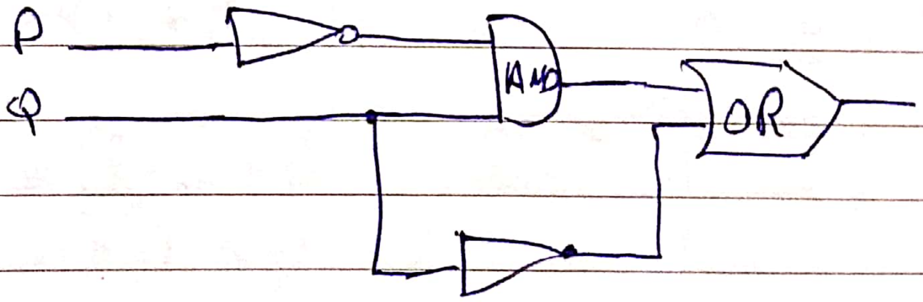
Write the Boolean expression

~~write~~



construct circuit for the following Boolean expressions

1-(~p∧Q)∨~Q



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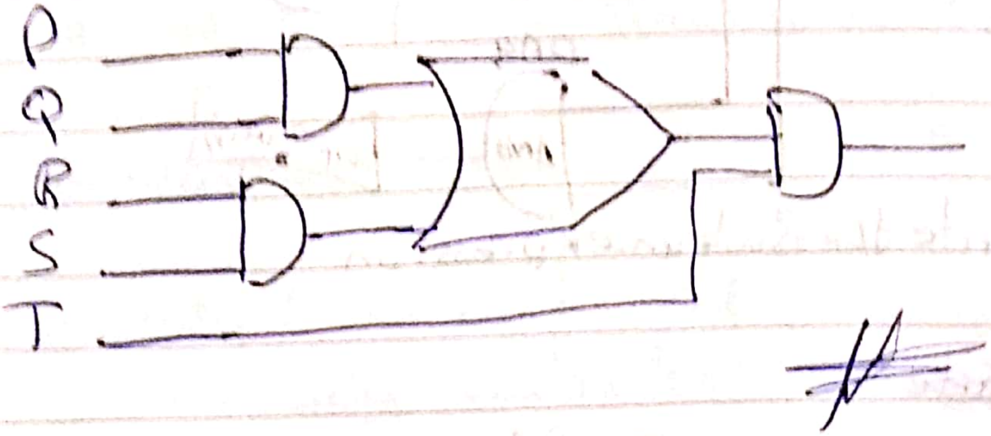
2) $(P \wedge Q) \wedge (R \wedge S)) \wedge T$



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Lecture 3

Dr. Marwa

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

Demorgan's Laws

1. commutative law $p \wedge q \equiv q \wedge p$
   $p \vee q \equiv q \vee p$
2. Associative law $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. distributive law $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
   $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity law $p \wedge t \equiv p$
   $p \vee c \equiv p$
5. negation law $p \vee \sim p \equiv t$
   $p \wedge \sim p \equiv c$
6. double negative law $\sim(\sim p) \equiv p$
7. Idempotent law $p \wedge p \equiv p$ $\quad p \vee p \equiv p$
8. universal law $p \vee t \equiv t$ $\quad p \wedge c \equiv c$

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(9) Demorgan law $\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$

(10) Absorption law $p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$

(11) Negation of t & c. $\sim t \equiv c$
$\sim c \equiv t$

ex1.
* verify the following logical Equivelente
simplify the following steatment

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim q$

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$
Demorgan's
$\equiv \sim p \wedge \sim(\sim p) \vee \sim q$ Demorgan's
$\equiv \sim p \wedge (p \vee \sim q)$ double negation law

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ distributive law
$\equiv c \vee (\sim p \wedge \sim q)$ Negation law
$\equiv \sim p \wedge \sim q$ Identity law

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ex.2

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv (\sim(\sim p) \vee \sim q)$
$\wedge (p \vee q)$ Demorgan
$\equiv (p \vee \sim q) \wedge (p \vee q)$ double negative
$\equiv p \vee (\sim q \wedge q)$ distributive law

$\equiv p \vee c$ Negation law

$\equiv p$ Identity law

ex3.
$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$
$\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv$
$\equiv (\sim p \wedge (\sim \sim q)) \vee (\sim p \wedge \sim q)$ Demorgan
$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ double negative
$\equiv \sim p \wedge (q \vee \sim q)$ Distributive law
$\equiv \sim p \wedge t$ Negation law
$\equiv \sim p$ Identity law

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Recursion

sequence

1- $2 = 2^1$
2 $4 = 2^2$
3 $8 = 2^3$
4 $16 = 2^4$

1, 3, 5, 7, 9, 11
, 7, 11

recurrence relation

$b_k = b_{k-1} + b_{k-2}$
$b_0 = 1 \quad b_1 = 3$
$b_0, b_1, b_2, b_3, b_4, \ldots b_6 \qquad b_{10}$
1, 3, 4, 7, 11, 18, 29

$b_k = 3b_{k-1}$ متغير ١
$S_k = S_{k-1} + S_{k-2} + 3\cancel{S}_{k-3}$ ٣ متغيرات

Solving Recurrence relation using iteration
① start from initial
② calculate successive terms of the sequence until you see a pattern (recommanded # 4)
③ guess an expellat formala

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ex,

Let $a_0, a_1, a_2, \ldots$ be a sequence defined recursively as follow:-

① $a_k = a_{k-1} + 2$ → Arithmetic sequence $1, 3, 5, 7, \ldots$

② $a_0 = 1$

Sol

$a_0 = 1$

$a_1 = a_0 + 2$

$a_1 = 1 + 2$

$a_2 = a_1 + 2$

$= 1 + 2 + 2 = 1 + 2 \times 2$

$a_3 = a_2 + 2$

$= 1 + 2 + 2 + 2 = 1 + 3 \times 2$

$a_4 = a_3 + 2$

$= 1 + 2 + 2 + 2 + 2 = 1 + 4 \times 2$

$a_5 = 1 + 5 \times 2$

$\boxed{a_n = 1 + 2n}$

N-B $2 + 2 + 2 + 2$

$= k.2$

$2 \times 2 \times 2 \ldots$

$= 2^k$

any arithmetic sequence ← $a_n = a_0 + d.n$

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ex $a_k = a_{k-1} + 5$ $\quad a_0 = 4$

$a_n = 4 + 5.n$

ex $a_k = a_{k-1} + 10$ $\quad a_0 = 0$

$= 0 + 10n$

---

ex, $a_k = 5 a_{k-1}$, $a_0 = 1$ $\quad k \geq 1$

a_0, a_1, a_2

$1, 5, 25, 125,$

$a_k = r a_{k-1}$

geometric Sequance

$a_n = a_0 r^n$

$a_0 = 1$

$a_1 = 5 a_0$

$= 5 \times 1$

$a_2 = 5 a_1$

$= 5 \times 5 \times 1$ $\quad 5^2$

$a_3 = 5 a_2$

$= 5 \times 5 \times 5 \times 1$ $\quad 5^3$

$a_4 = 5 a_4$

$= 5 \times 5 \times 5 \times 5 \times 1$ $\quad 5^4$

$a_5 = 5^5$

$a_n = 5^n$

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ex.

$a_k = 10\ a_{k-1} \qquad a_0 = 5$

$a_n = 5(10)^n$

$a_k = 3a_{k-1} \qquad a_1 = 1$

$a_n = 1(3)^n = 3^n$

4%

100000

① How much the account will be at end of 21 years

$a_k = 0.04\ a_{k-1}$

$a_k = 1.04\ a_{k-1}$

$a_n = a_0 \cdot r^n$

$= 100000\ (1.04)^n$

$= 100000\ (1.04)^{21} = 227,876.81$

② in How many years the account will be one million?

$a_n = a_0 \cdot r^n$

$1000000 = 100000 \cdot (1.04)^n$

$10 = (1.04)^n$

Taking ln to both sides

$\ln 10 = n \ln 1.04$

$n = \frac{\ln 10}{\ln 1.04} = 58.7$

ex.

$m_k = 2m_{k-1} + 1$

$m_1 = 1 \qquad k \geq 2$

Sol

$m_1 = 1$

$m_2 = 2 \times m_1 + 1$

$= 2 \times 1 + 1 = 2 + 1$

$m_3 = 2m_2 + 1$

$= 2(2+1) + 1 = 2 \times 2 + 2 + 1$

$m_4 = 2m_3 + 1 \qquad 2^2 + 2 + 1$

$= 2(2 \times 2 + 2 + 1) + 1$

$= 2 \times 2 \times 2 + 2 \times 2 + 2 + 1$

$2^3 + 2^2 + 2 + 1$

$m_n = 2^{n-1} + 2^{n-2} + \dots + 2 + 1$

$m_n = 1 + 2 + 2^2 + 2^3 + \dots + 2^{n-2} + 2^{n-1}$

geometric Series

Sum of geometric series

$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1} \longrightarrow$ تحفظ [illegible]

$r^0 + r^1 + r^2 + r^3 + \dots r^n$

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$$m_n = \frac{2^{n-1+1}+1}{2-1} = \frac{2^n-1}{1} = 2^n-1$$

$m_n = 2^n - 1$

Q.

$e_k = 4e_{k-1} + 5$ 2, 13,

$e_0 = 2$ $k \geq 1$

Sol

$e_0 = 2$

$e_1 = 4e_0 + 5$

$= 4 \times 2 + 5$

$e_2 = 4e_1 + 5$

$= 4 \times (4 \times 2) + 5 = 4 \times 4 \times 2 + 5 \times 4 + 5$

$e_3 = 4e_2 + 5$

$= 4(4 \times 4 \times 2 + 5 \times 4 + 5) + 5$

$= 4 \times 4 \times 4 \times 2 + 5 \times 4 \times 4 + 4 \times 5 + 5$

$e_4 = 4e_3 + 5$

$= 4(4 \times 4 \times 4 \times 2 + 5 \times 4 \times 4 + 4 \times 5 + 5) + 5$

$= 4 \times 4 \times 4 \times 4 \times 2 + 5 \times 4 \times 4 \times 4 + 4 \times 4 \times 5 + 4 \times 5$
$+ 5$

$= 2 \times 4^4 + 4^3 \times 5 + 4^2 \times 5 + 4 \times 5 + 5$

$e_5 = 2 \times 4^5 + 4^4 \times 5 + 4^3 \times 5 + 4^2 \times 5 + 4 \times 5 + 5$

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$$e_5 = 2 \times 4^5 + 5(4^4 + 4^3 + 4^2 + 4^3 + 4^4)$$

$$e_n = 2 \times 4^n + 5(1 + 4 + 4^2 + \ldots 4^{n-1})$$

$$= 2 \times 4^n + 5\left(\frac{4^{n-1+1} - 1}{4-1}\right)$$

$$= 2 \times 4^n + 5\left(\frac{4^n - 1}{3}\right)$$

$$= 2 \times 4^n + \frac{5}{3}(4^n - 1)$$

$$e_n = 2 \cdot 4^n + 5\left(\frac{4^n - 1}{3}\right)$$

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MS 521

Lecture 4 Dr Maria

$e_k = 4e_{k-1} + 5$ $e_0 = 2$

$\frac{r^{n+1} - 1}{r - 1}$

$e_0 = 2$

$e_1 = 4 \cdot e_0 + 5$

$e_1 = 4 \times 2 + 5$ من المحاضرة السابقة مراجعة

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* Second order linear Homogenous recurrence relation with constant coefficient

$y_{n+2} = A y_{n+1} + B y_n$

$y_{n+2} = 5 y_{n+1} + 3 y_n$ $y_0 = 1$ $y_1 = 3$

$y_{n+2} = 3 y_n$

y_0, y_1, y_2, y_3 → ch. equation

1, 3, 18

$a_k = A a_{k-1} + B a_{k-2}$

$r^2 - Ar - B = 0$

2 similar roots

$a_n = r^n (c_1 + n c_2)$

2 Different roots

$a_n = c_1 (r_1)^n + c_2 (r_2)^n$

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ex. solve the recurrence relation

$$a_k = a_{k-1} + 2a_{k-2}$$

$a_0 = 1 \quad a_1 = 8 \qquad k \geq 2$

sol

Ch. eq $r^2 - Ar - B = 0$

$$r^2 - 1 \times r - 2 = 0$$
$$r^2 - r - 2 = 0$$
$$(r + 1)(r - 2) = 0$$

$r + 1 = 0$ or $r - 2 = 0$

$r_1 = -1 \qquad r_2 = 2$

$$a_n = c_1 (r_1)^n + c_2 (r_2)^n$$

when $n = 0$

$a_0 = c_1 (r_1)^0 + c_2 (r_2)^0$

$a_0 = c_1 + c_2$

$1 = c_1 + c_2 \rightarrow$ eq 1

when $n = 1$

$a_1 = c_1 r_1^1 + c_2 r_1^1$

$8 = (-1)c_1 + 2c_2 \rightarrow$ eq 2

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solveing eq1 & eq2

$\cancel{c_1} + c_2 = 1$

$-\cancel{c_1} + 2c_2 = 8$ adding

$3c_2 = 9$

$c_2 = \frac{9}{3} = 3$

$c_2 = 3$

$c_1 + c_2 = 1$

Since $c_1 + 3 = 1$

$c_1 = 1 - 3$

$c_1 = -2$

$a_n = -2(-1)^n + 3(2)^n$

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ex.

find and explicit formula for the sequence

$$x_{n+2} = x_{n+1} + x_n$$

$x_0 = 0 \quad x_1 = 1 \quad n \geq 2$

sol

Ch. eq $\quad r^2 - Ar - B = 0$

$r^2 - (1)r - 1 = 0$

$r^2 - r - 1 = 0$

$N.B = ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = 1,\ b = -1,\ c = -1$$

$$r_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$r_{1,2} = \frac{1 \pm \sqrt{1+4}}{2}$$

$$r_{1,2} = \frac{1 \pm \sqrt{5}}{2} \qquad r_1 = \frac{1+\sqrt{5}}{2}$$

$$r_2 = \frac{1-\sqrt{5}}{2}$$

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$$X_n = c_1(r_1)^n + c_2(r_2)^n$$

when $n = 0$

$$X_0 = c_1(r_1)^0 + c_2(r_2)^0$$

$$X_0 = 0$$

$$0 = c_1 + c_2 \rightarrow eq\ 1$$

when $n = 1$

$$X_1 = c_1 r_1^1 + c_2 r_2^1$$

$$1 = c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right) \quad eq\ 2$$

$$\cancel{\left(\frac{1+\sqrt{5}}{2}\right)c_1} + \left(\frac{1-\sqrt{5}}{2}\right)c_2 = 1 \rightarrow *$$

$$c_1 + c_2 = 0 \rightarrow -\left(\frac{1+\sqrt{5}}{2}\right)$$

$$\cancel{-\left(\frac{1+\sqrt{5}}{2}\right)c_1} - \left(\frac{1+\sqrt{5}}{2}\right)c_2 = 0 \rightarrow **$$

$$\frac{\cancel{1} - \sqrt{5} - \cancel{1} - \sqrt{5}}{2}c_2 = 1$$

$$\frac{-2\sqrt{5}}{\cancel{2}}c_2 = 1$$

$$-\sqrt{5}\,c_2 = 1$$

$$\boxed{c_2 = \frac{-1}{\sqrt{5}}}$$

Since

$$c_1 + c_2 = 0$$

$$c_1 + \frac{-1}{\sqrt{5}} = 0$$

$$\boxed{c_1 = \frac{1}{\sqrt{5}}}$$

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$$X_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$g_{n+2} - 4g_{n+1} + 4g_n = 0$$

$$g_{n+2} = 4g_{n+1} - 4g_n$$

$g_0 = 1$ $g_1 = 1$

Sol

Chqe $r^2 - Ar - B = 0$

$$r^2 - 4r + 4 = 0$$

$$(r-2)(r-2) = 0$$

$r - 2 = 0$ or $r - 2 = 0$

$r_1 = 2$ $r_2 = 2$

$$g_n = r^n(c_1 + nc_2)$$

when $n = 0$

$$g_0 = r^0(c_1 + 0c_2)$$

$1 = c_1 \rightarrow$ eq 1

when $n = 1$

$$g_1 = r^1(c_1 + 1c_2)$$

$$1 = r(c_1 + c_2)$$

from equation 1

$eq_2 = 1 = r(1 + c_2 = 2(1 + c_2)$

$= 2 + 2c_2$

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$$2c_2 = 1-2$$

$$2c_2 = -1$$

$$c_2 = \frac{-1}{2}$$

$$y_n = 2^n\left(1+n-\frac{1}{2}\right)$$

$$y_n = 2^n\left(1+\frac{n}{2}\right)$$

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ex.

$$y_{n+2} = 5y_{n+1} - 6y_n$$

$y_0 = 1 \qquad y_1 = 1$

sol

ch.qe $\quad r^2 - Ar - B = 0$

$r^2 - 5r + 6 = 0$

$(r-3)(r-2) = 0$

$r_1 = 3 \quad$ or $\quad r_2 = 2$

$a_n = c_1 r_1^n + c_2 r_2^n$

when $n = 0$

$y_0 = c_1 r_1^0 + c_2 r_2^0$

$1 = c_1 + c_2 \longrightarrow$ eq 1

when $n = 1$

$y_1 = c_1 r_1^1 + c_2 r_2^1$

$1 = r_1(3) + c_2(2) \longrightarrow$ eq 2

$c_1 + c_2 = 1 \longrightarrow \times -2$

$3c + 2c_2 = 1$

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$$-2c_1 - 2\cancel{c_2} = -2$$
$$3c_1 + \cancel{2}c_2 = 1 \quad \text{adding}$$

$$\boxed{c_1 = -1}$$

since $c_1 + c_2 = 1$
$$-1 + c_2 = 1$$
$$c_2 = 1+1$$
$$\boxed{c_2 = 2}$$

$$y_n = -1(3)^n + 2(2)^n$$

$$y_n = 2(2)^n - 3^n$$

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MS521

Lecture 5 Dr. Marwa

Mathimetical Induction

step1 → initial step
I proof that the rule is true at the initial

step2 → hypothesis step
we suppose that the rule is true at any number k
step3 → ~~we~~ Induction step
we proof that the rule is true at k+1

---

ex.1 use Mathimetical Induction to prove that $1+2+3+\ldots+n = \frac{n(n+1)}{2}$ $\forall\ n \geq 1$

$1+2+3+4 = 10$

$\frac{4(5)}{2} = 10$

sol

step 1
prove that rule is true at the initial
L.H.S at $n=1$ $= 1$
R.H.S at $n=1$ $= \frac{1(1+1)}{2} = \frac{2}{2} = 1$
the rule is true at the initial

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step 2 Suppose that the rule is true at $n = k$

$$1+2+3 \cdots +k = \frac{k(k+1)}{2}$$

Step 3 induction step
We need to true that the rule is true at $n = k+1$

$$1+2+3+ \cdots + k + k+1 = \frac{k+1(k+1+1)}{2}$$

$$1+2+3+ \cdots + k + k+1 = \frac{(k+1)(k+2)}{2}$$

L.H.S

$$\underbrace{1+2+3+ \cdots + k} + (k+1)$$

From that the hypothesis step

$$\frac{k(k+1)}{2} + \frac{k+1}{} = \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$\frac{k^2+k+2k+2}{2} = \frac{(k+1)(k+2)}{2}$$

Sinse L.H.S = R.H.S then the rule is true at any K ex 2 prove that $\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1} \quad \forall n \geq 0$

Sol

$r^0 + r^1 + r^2 + \cdots + r^n$

① intial step

prove that the rule is true at $n=0$

$L.H.S = r^0 = 1$

$R.H.S = \frac{r^{0+1}-1}{r-1} = \frac{r-1}{r-1} = 1$

Since $L.H.S = R.H.S$ then the rule is true at the initial

② hypothesis step

suppose that the rule is true at $n=k$

$\sum_{i=0}^{k} r^i = \frac{r^{k+1}-1}{r-1}$

③ Induction step

we need to prove that the rule is true at $n=k+1$

$\sum_{i=0}^{k+1} r^i = \frac{r^{k+1+1}-1}{r-1}$

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From the hypothesis step

$$L.H.S = \sum_{r=0}^{k+1} r^i = r^0 + r^1 + r^2 + r^3 + \dots + r^k + r^{k+1}$$

$$= \sum_{i=0}^{k} r^i + r^{k+1}$$

$$= \frac{r^{k+1} - 1}{r-1} + r^{k+1}$$

$$= \frac{r^{k+1} - 1}{r-1} + \frac{(r-1)(r^{k+1})}{r-1}$$

$$= \frac{r^{k+1} - 1}{r-1} + \frac{r \cdot r^{k+1} - r^{k+1}}{r-1}$$

$$= \frac{\cancel{r^{k+1}} - 1 + r^{k+2} - \cancel{r^{k+1}}}{r-1}$$

$$= \frac{r^{k+2} - 1}{r-1}$$

R.H.S = L.H.S
sinse then the rule is true at any k

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ex3

$$1+6+11+16+\dots+(5n-4) = \frac{n(5n-3)}{2}$$

$n \geq 1$

Sol

① intial step

prove that the rule is true at $n=1$

$L.H.S = 1$

$$R.H.S = \frac{1(5\times1-3)}{2} = \frac{2}{2} = 1$$

since $L.H.S = R.H.S$

then the rule is true at the initial

② hyposis step

suppose that the rule is true at k

$$1+6+11+16+\dots+5k-4 = \frac{k(5k-3)}{2}$$

③ Induction step

we need to prove that the rule is true at $k+1$

$$1+6+11+16+\dots+(5k-4)+(5(k+1)-4)$$

$$= \frac{(k+1)(5(k+1)-3)}{2}$$

$$= \frac{(k+1)(5k+5-3)}{2}$$

$$= \frac{(k+1)(5k+2)}{2}$$

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$$\underbrace{1+6+11+16+\dots+(5k-4)}+(5(k+1)-4)$$

L.H.S

$$1+6+11+16+\dots+(5k-4)+(5(k+1)-4)$$

From the hypothesis step

$$= \frac{k(5k-3)}{2} + 5(k+1)-4$$

L.H.S

$$= \frac{k(5k-3)}{2} + 5k+5-4$$

$$= \frac{k(5k-3)}{2} + 5k+1$$

$$= \frac{k(5k-3)}{2} + \frac{2(5k+1)}{2}$$

$$= \frac{5k^2-3k+10k+2}{2} = \frac{5k^2+7k+2}{2}$$

$$R.H.S = \frac{(k+1)(5k+2)}{2} = \frac{5k^2+2k+5k+2}{2}$$

$$= \frac{5k^2+7k+2}{2}$$

Since L.H.S = R.H.S

then the rule is true

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ex4

Proving an Inequality

ex. use Mathimatical induction to prove that

$2n+1 < 2^n \qquad n \geq 3$

sol.

① initial step

we need to prove that the rule is true at $n = 3$

L.H.S $= 2 \times 3 + 1 = 7$

R.H.S $= 2^3 = 8$

Since L.H.S $<$ R.H.S then the rule is true at the initial

② hypothesis step

suppose That the rule is true at k

$2k+1 < 2^k$

③ Induction step

we need to prove that the rule is true at $k+1$

$2(k+1)+1 < 2^{k+1}$

$2k+2+1 < 2^{(k+1)}$

$2k+3 < 2^{k+1}$

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From the hypothesis step

$2k+1 < 2^k$ by adding 2 to both sides

$2k+3 < 2+2^k$

Since $2.2^k > 2+2^k$

then $2k+3 < 2.2^k$

$2k+3 < 2^{k+1}$

since the rule is true at k+1 then the rule is true

ex. $n^2 < 2^n \quad n \geq 5$

① Initial step

$n=5$

$L.H.S = 5^2 = 25$

$R.H.S = 2^5 = 32$

Since $L.H.S < R.H.S$

then the rule is true at the initial

② hypothsis step

suppose that $k^2 < 2^k$

③ Induction step

$(k+1)^2 < 2^{k+1}$

$k^2+2k+1 < 2^{k+1}$

from hypothesis step

$k^2 < 2^k$ adding $2k+1$

$k^2+2k+1 < 2^k+2k+1$ to both sides

$k^2+2k+1 < 2^k+2^k$

$k^2+2k+1 < 2 \cdot 2^k$

$k^2+2k+1 < 2^{k+1}$

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Ex.

$n^3 > 2n+1 \quad \forall\, n \geq 2$

① Initial step $n=2$

$L.H.S = 2^3 = 8$

$R.H.S = 2\times 2+1 = 5$

then $L.H.S > R.H.S$ then the rule is true
at the initial

② hypothesis step
Suppose the rule is true at k $\boxed{k^3 > 2k+1}$

③ induction step
$n = k+1$

$(k+1)^3 > 2(k+1)+1$

$k^3 + 3k^2 + 3k + 1 > 2k+3$

$(k+1)(k+1)$

$k^2 + 2k + 1$

from hypothesis

$(k+1)(k^2+2k+1)$

$k^3 > 2k+1$ by adding $3k^2+3k+1$

$k^3 + 2k^2 + k + k^2 + 2k + 1$

$k^3 + 3k^2 + 3k + 1 >$

$k^3 + 3k^2 + 3k + 1$

$2k+1+\underbrace{3k^2+3k+1}$

since $2 < 3k^2+3k+1$

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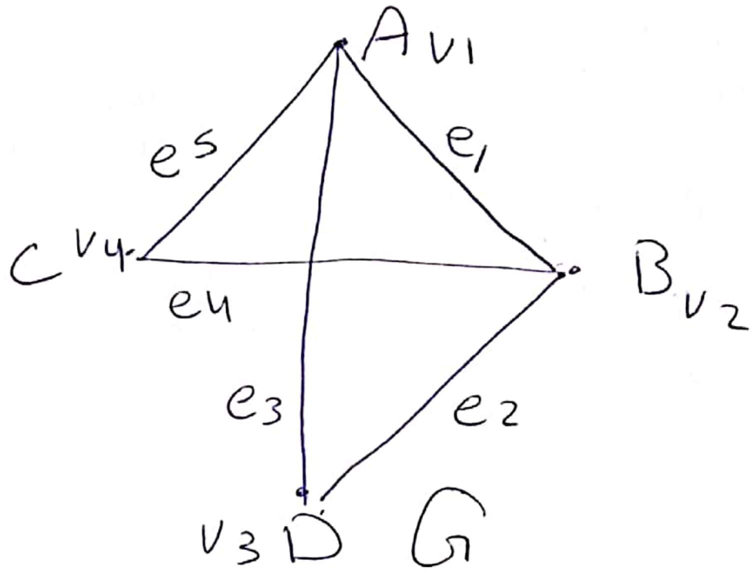
$k^3 + 3k^2 + 3k + 1 \geq 2k + 1 + 2$

$k^3 + 3k^2 + 3k + 1 > 2k + 3$

the rule is true

graph theory

A	B, C
B	A, D
C	B, A
D	A, B



G

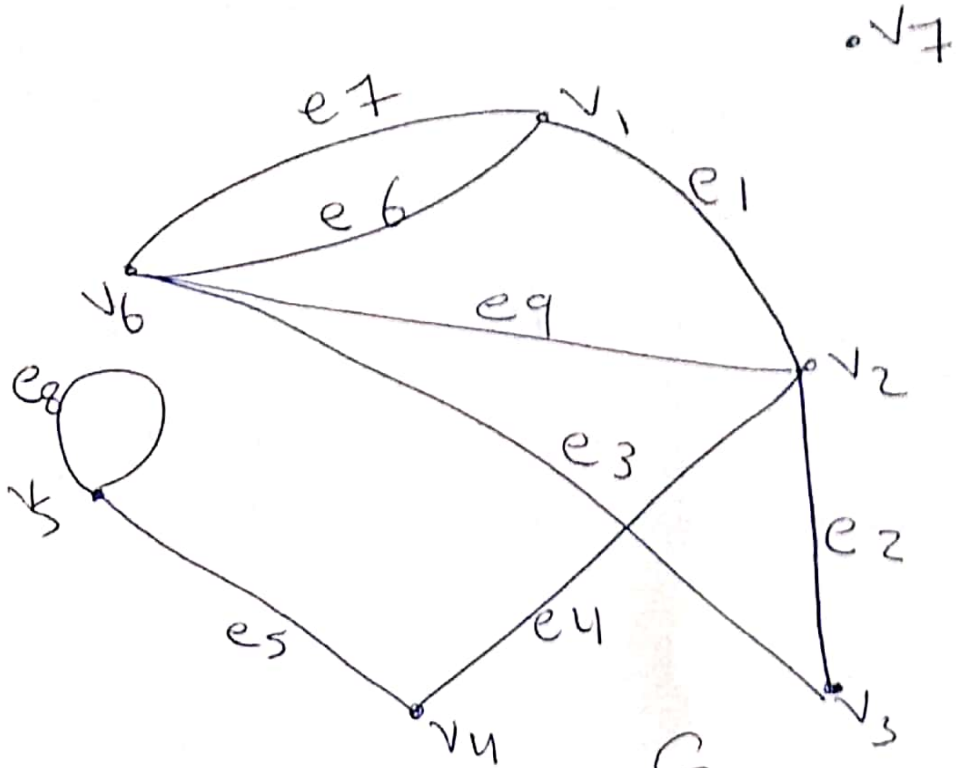
set of Vertices (vertex)

set of edges

G

set of vertices $= \{v_1, v_2, v_3, v_4\}$

set of edges $= \{e_1, e_2, e_3, e_4, e_5\}$



set of vertices
V(G) {V1, V2, V3, V4, V5, V6, V7}

set of edges
E(G) = {e1, e2, e3, e4, e5, e6, e7, e8}

* isolated vertex = {V7}

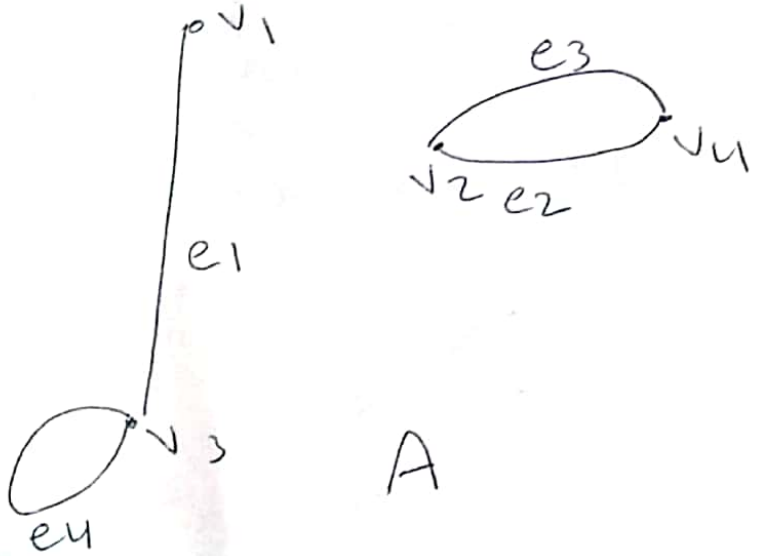
edge	end points	edge	endpoints
e1	{V1, V2}	e8	V5 *
e2	{V2, V3}	e9	{V2, V6}
e3	{V3, V6}		
e4	{V2, V4}		
e5	{V4, V5}		
e6	{V1, V6} *		
e7	{V1, V6} *		

* parallel edges
  {e6, e7}

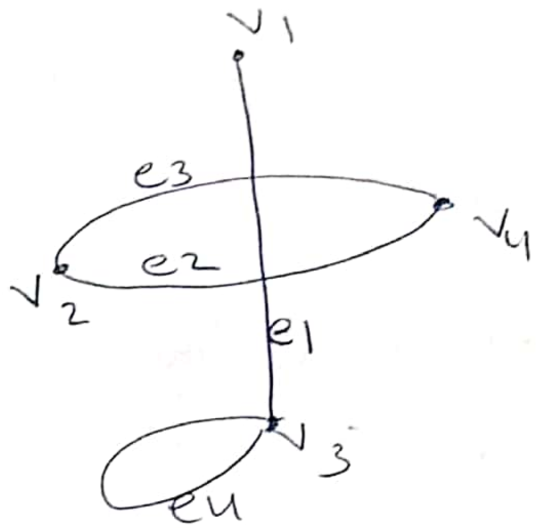
* loop = {e8}

* incident edges on v_2 {e1, e2, e4, e9}

* adjacent vertices to v_2 {v_1, v_6, v_4, v_3}



A



B

E(A) = {e1, e2, e3, e4}

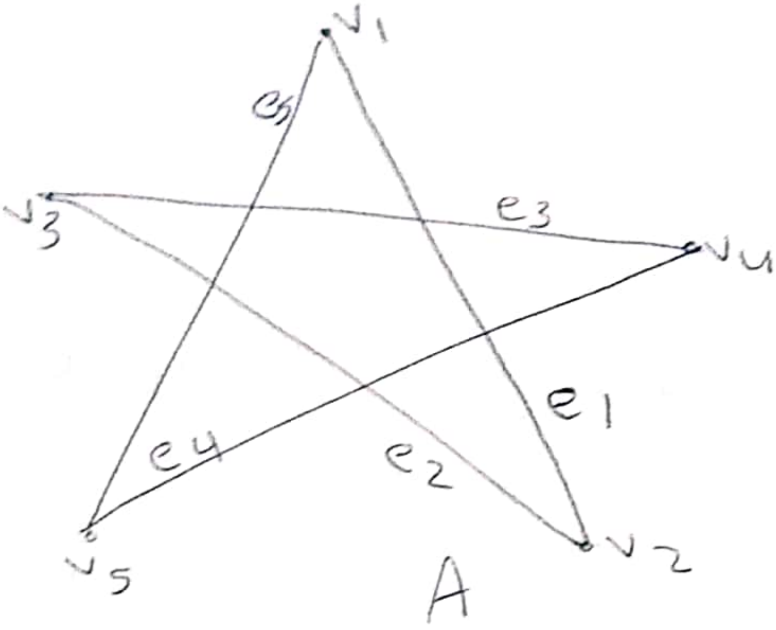
V(A) = {v_1, v_2, v_3, v_4}

edge	endpoints
e1	{v_1, v_3}
e2	{v_2, v_4}
e3	{v_2, v_4}
e4	{v_3}

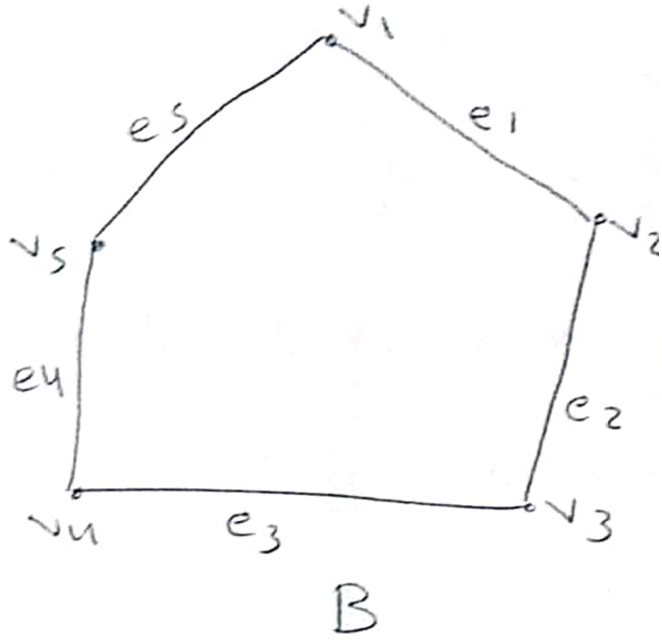
E(B) = {e1, e2, e3, e4}

V(B) = {v_1, v_2, v_3, v_4}

edge	endpoints
e1	{v_1, v_3}
e2	{v_2, v_4}
e3	{v_2, v_4}
e4	{v_3}



A



B

$V(A) = \{v_1, v_2, v_3, v_4, v_5\}$

$E(A) = \{e_1, e_2, e_3, e_4, e_5\}$

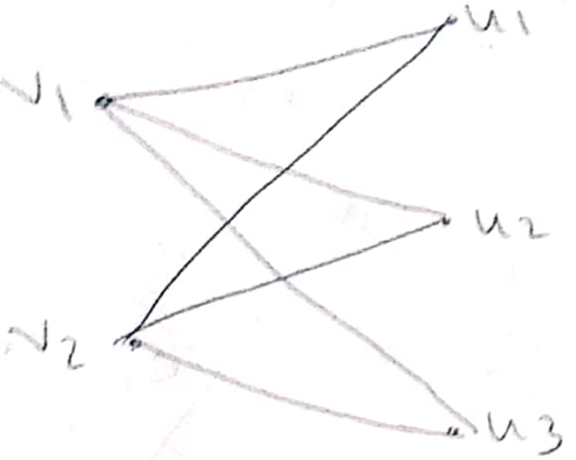
edge	endpoints
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_3, v_4\}$
e_4	$\{v_4, v_5\}$
e_5	$\{v_1, v_5\}$

$V(B) = \{v_1, v_2, v_3, v_4, v_5\}$

$E(A\} = \{e_1, e_2, e_3, e_4, e_5\}$

edge	end point
e_1	$\{v_1, v_2\}$
e_2	$\{v_2, v_3\}$
e_3	$\{v_3, v_4\}$
e_4	$\{v_4, v_5\}$
e_5	$\{v_1, v_5\}$

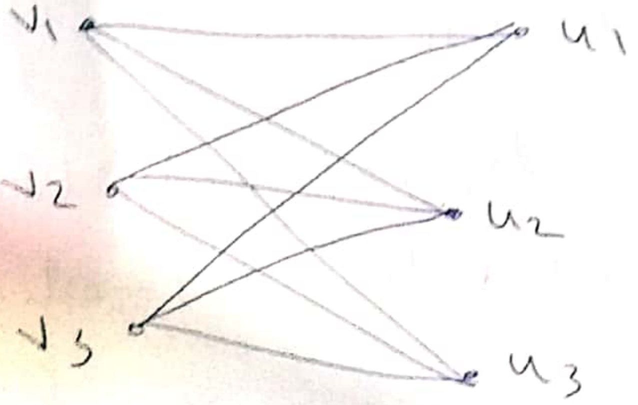
* Complete biparted graph $K_{2,3}$



$\{v_1, v_2, u_1, u_2, u_3\}$

$V\{v_1, v_2\}$ $U = \{u_1, u_2, u_3\}$

$K_{3,3}$



degree of vertex and total degree of graph

*degree of vertex

$deg(v_1) = 6$

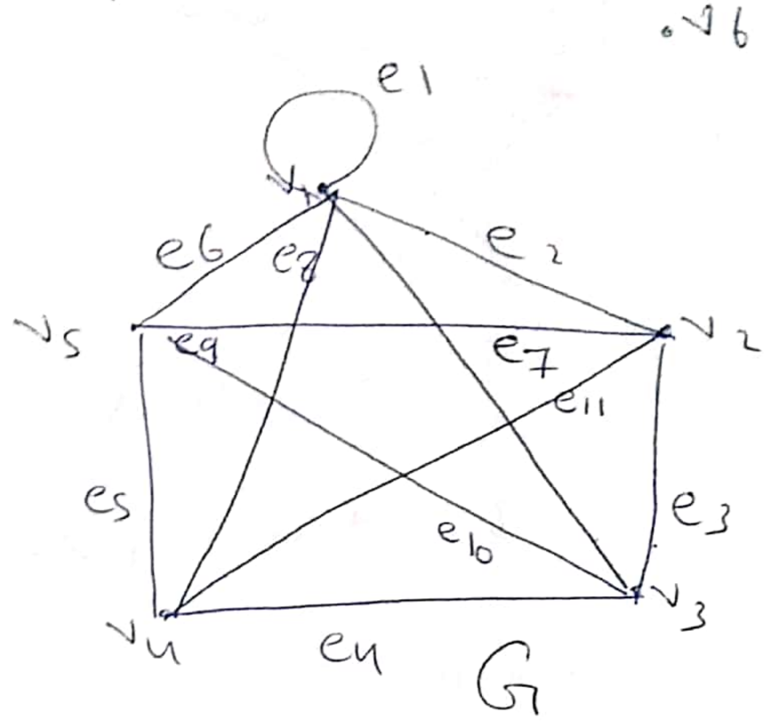
$deg(v_2) = 4$

$deg(v_3) = 4$

$deg(v_4) = 4$

$deg(v_5) = 4$

$deg(v_6) = 0$



total degree of graph

$deg(G) = deg(v_1) + deg(v_2) + deg(v_3)$
$+ deg(v_4) + deg(v_5) + deg(v_6)$

$= 6 + 4 + 4 + 4 + 4 + 0$

$= \underline{22}$ even number

طريقة أخرى لإيجاد ← degree of graph

7

$deg(G) = [2 * edges]$

$= 2 * 11 = 22$

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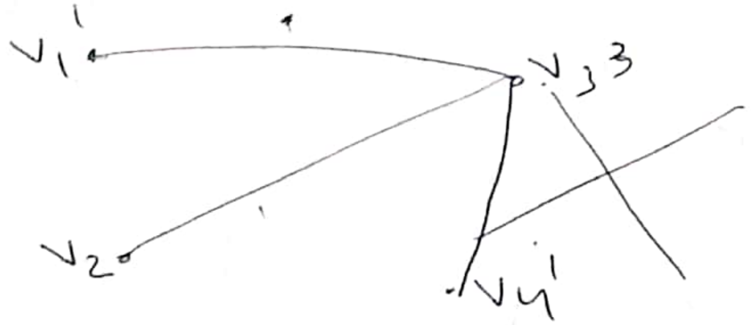
ex

draw a graph with

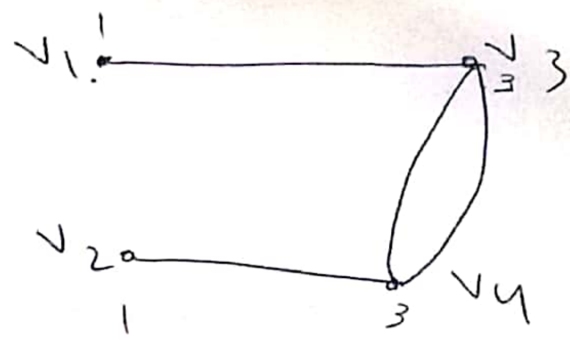
1- Four vertices of degree 1, 1, 2 and 3

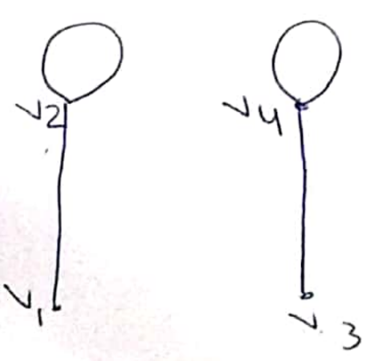


$1 + 1 + 2 + 3 = 7$ odd

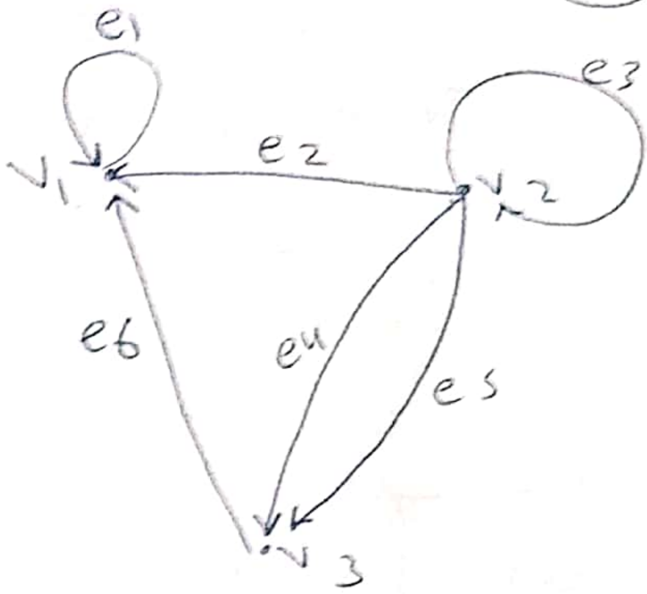
2- Four vertices of degree 1, 1, 3 & 3

$1 + 1 + 3 + 3 = 8$ even





directed graph



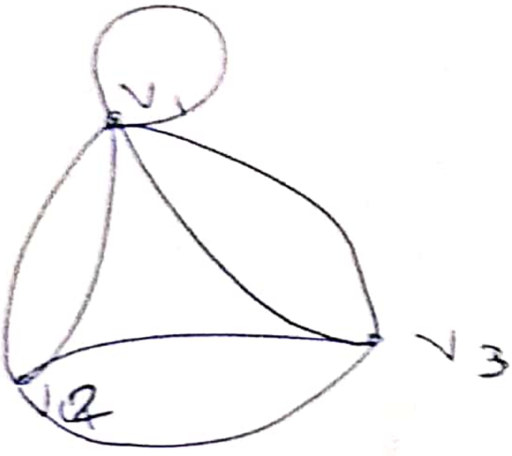
$$\begin{array}{c|ccc} & v_1 & v_2 & v_3 \\ \hline v_1 & & & \\ v_2 & & & \\ v_3 & & & \end{array} \quad \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}_{3 \times 3}$$

the Matrix of directed graph Maybe a symmetric matrix or symmetric
غير متماثلة

$$\text{Transpose} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix} \neq$$

the Matrix of undirect graph is always Symmetric Matrix adjiancy matrix

$$\begin{array}{c|c} & \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \hline \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} & \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \end{array}$$



transpose $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$

Symmetric Matrix

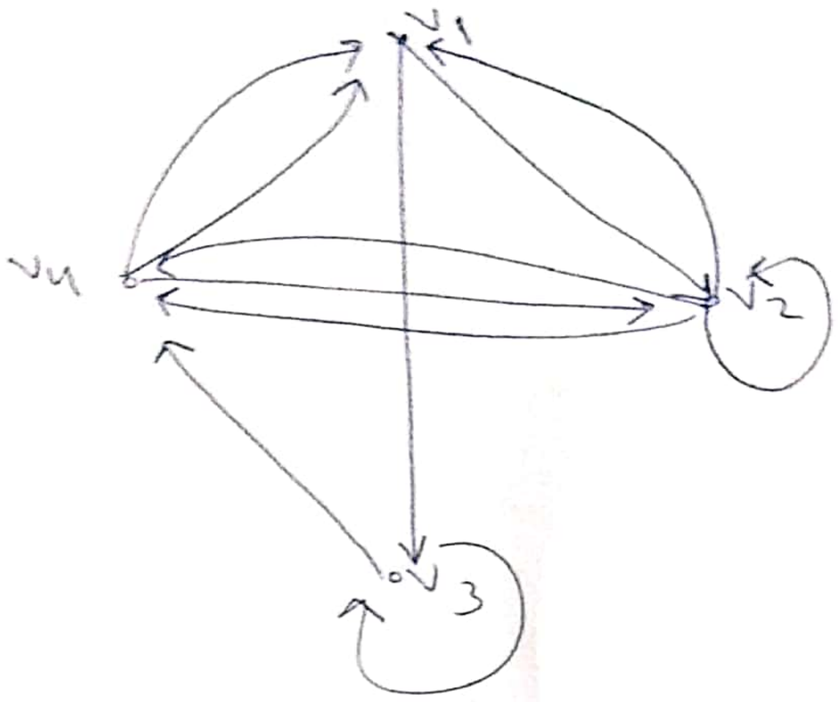
---

ex

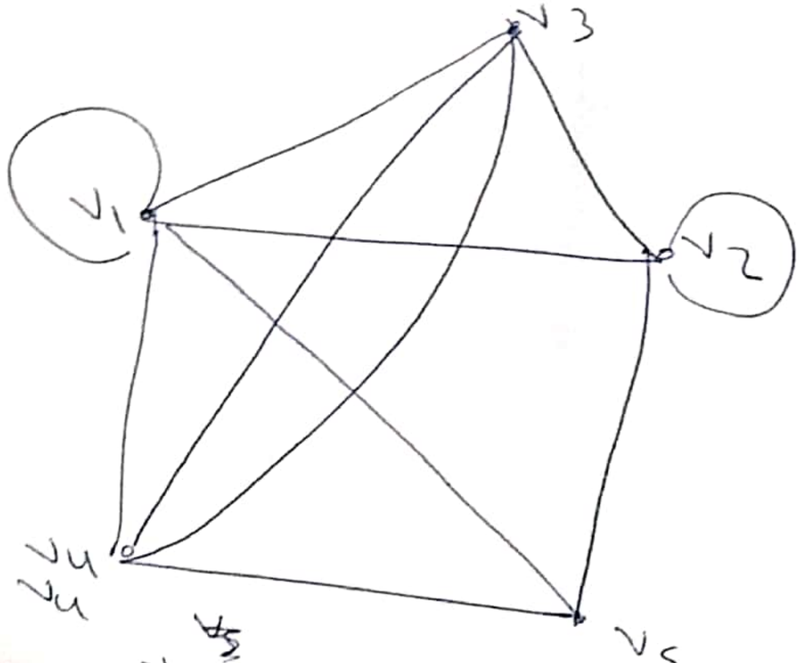
draw a graph for the following Matrix

$$\begin{array}{c|c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \hline \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

directed



Find the Matrix for the following graph



$$
\begin{array}{c|ccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 \\
\hline
v_1 & 1 & 1 & 1 & 1 & 1 \\
v_2 & 1 & 1 & 1 & 0 & 1 \\
v_3 & 1 & 1 & 0 & 2 & 0 \\
v_4 & 1 & 0 & 2 & 0 & 1 \\
v_5 & 1 & 1 & 0 & 1 & 0
\end{array}
$$