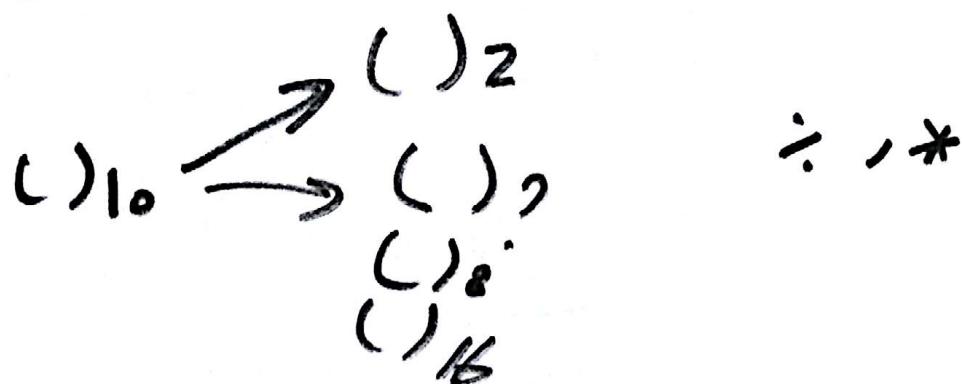
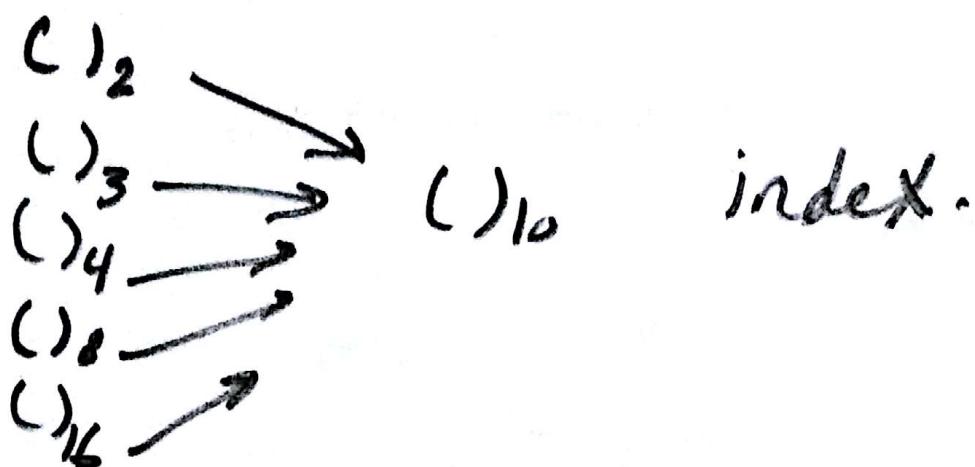


النظام العددي

## \* Number System:



$$( )_2 \leftrightarrow ( )_4$$

$$( )_2 \leftrightarrow ( )_8$$

3 bits

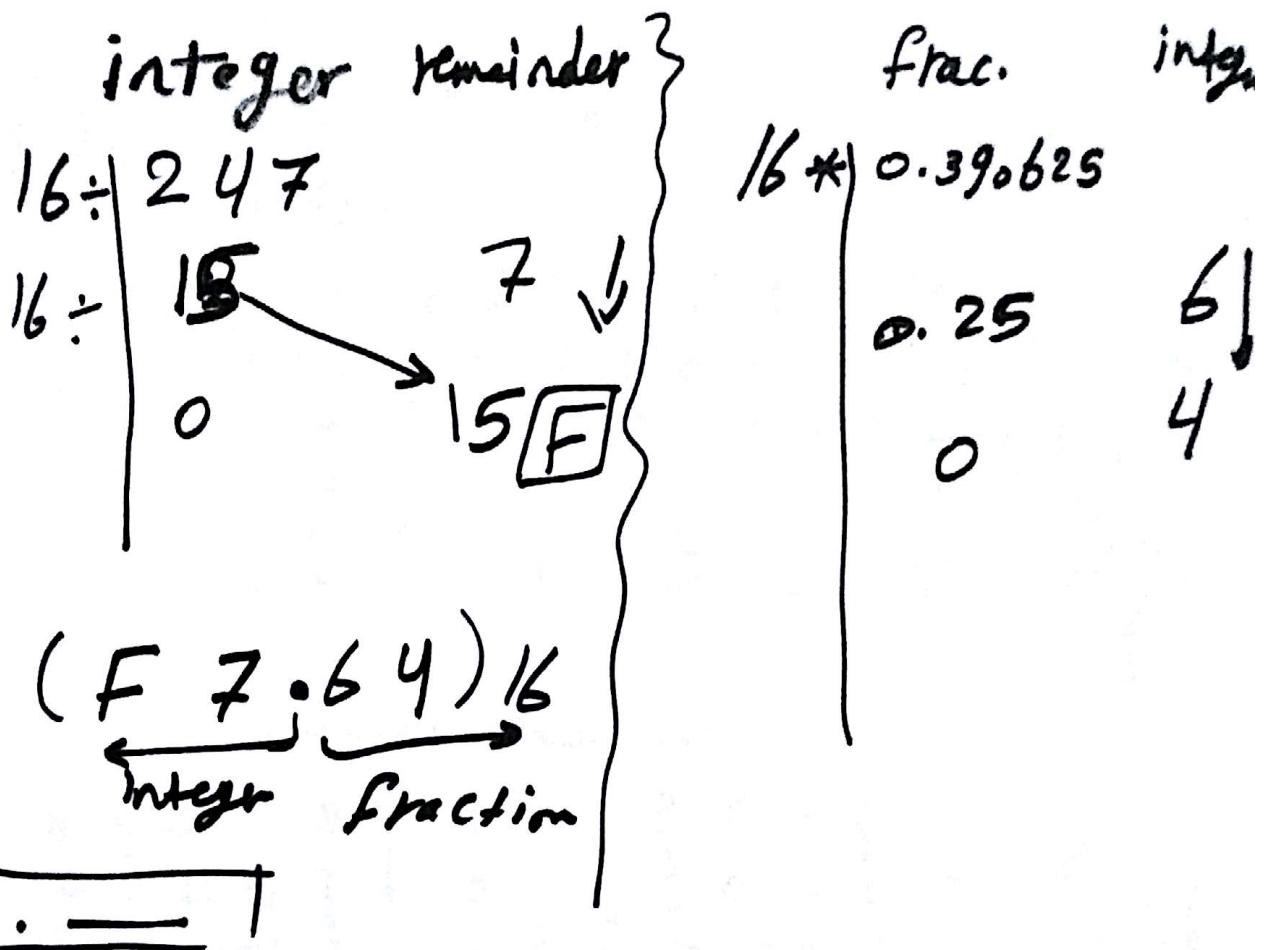
$$( )_2 \leftrightarrow ( )_{16}$$

4 bits.

Q: let  $X = (\underline{247} \cdot 390625)_{10}$   
 find the value of  $X$  in Hexadecimal.

$$(\ )_{16} = \begin{matrix} o \rightarrow g \\ A \rightarrow F \end{matrix}$$

Ans:



$\boxed{6.25}$

Ex.

$$\frac{27}{100} \rightarrow$$

②

$$Q \text{ & let } X = (1001101 \cdot 1011)_2$$

Find the value of  $X$  in octal  
& Hexadecimal.

$$(.)_2 \xrightarrow{3 \text{ bits.}} (.)_3$$

$$\begin{array}{cccccc} 4 & 2 & 1 & 4 & 2 & 1 \\ \boxed{00} & \boxed{00} & \boxed{10} & \vdots & \boxed{10} & \boxed{100} \\ & & & & \downarrow & \downarrow \\ (1 & 1 & 5 & . & 5 & 4) \\ & & \hline & & & \end{array}$$

$$\begin{array}{cccccc} 3 & 4 & 2 & 1 & 3 & 4 & 2 & 1 \\ \boxed{0} & \boxed{00} & \boxed{10} & \vdots & \boxed{101} & \rightarrow (.)_{16} \\ & & & & \downarrow & \\ (4 & D & . & B)_{16} & & & & A & 10 \\ & & & & & B & 11 \\ & & & & & C & 12 \\ & & & & & D & 13 \\ & & & & & E & 14 \\ & & & & & F & 15 \end{array}$$

(3)

\* Convert the binary number  
 $(\underline{1} \underline{0} \underline{1} \underline{1} \underline{0} \cdot \underline{1} \underline{0} \underline{1} \underline{0})_2$  to its  
equivalent numbers in :

- a) octal      b) Hexa.      c) decimal  
 $(\ )_2 \xrightarrow[3 \text{ bits}]{} (\ )_8$        $(\ )_2 \xleftrightarrow[4 \text{ bits}]{} (\ )_{16}$        $(\ )_2 \xrightarrow[\text{index}]{} (\ )_{10}$
- d) Base 4

c)

$$(\underline{1} \underline{0} \underline{1} \underline{1} \underline{0} \cdot \underline{1} \underline{0} \underline{1} \underline{0})_2 \rightarrow (\ ? )_{10}$$

$$(1 * 2^4) + (1 * 2^2) + (1 * 2^1) + \\ (1 * 2^{-1}) + (1 * 2^{-3}) + (1 * 2^{-5})$$

$$16 + 4 + 2 + \frac{1}{2} + \frac{1}{2^3} + 2^{-5}$$

$$(\underline{\quad} \cdot \underline{\quad})_{10}$$

$$(\ )_2 \xrightarrow[2 \text{ bits}]{} (\ )_4$$

$(112.222)_4$

$\underline{\underline{1}} \underline{\underline{0}} \underline{\underline{1}} \underline{\underline{0}} \cdot \underline{\underline{1}} \underline{\underline{0}} \underline{\underline{1}} \underline{\underline{0}}$

$$Q: \text{let } X = (-)_{10} \rightarrow ^\leftarrow$$

$$y = (-)_{16} \rightarrow ()_{10} \text{ index}$$

$$z = (-)_8 \rightarrow ()_{10} \text{ index}$$

Find:  $\underbrace{(x-y)}_{\downarrow} * \underbrace{(x+y)}_{\downarrow}$

Q \* Find the sum of binary numbers:

$$\begin{array}{r} 11010.110 \\ + 01110.101 \\ \hline \end{array}$$

$$\begin{array}{r} 11010.110 \\ + 01110.101 \\ \hline 101001.011 \end{array}$$

\* Calculate the Following operations:

a)  $1100101 - 0100111$

(Direct, 1's Comp., 2's Comp.)

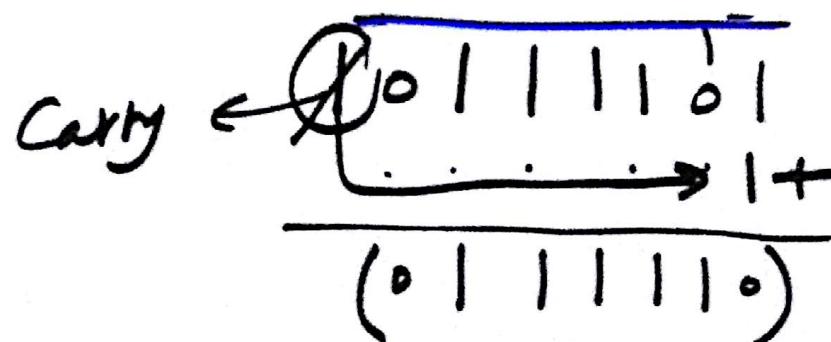
→ Direct

$$\begin{array}{r} \overset{2}{\cancel{0}} \overset{1}{\cancel{1}} \overset{1}{\cancel{1}} \overset{2}{\cancel{0}} \\ + + \cancel{0} \cancel{0} + \cancel{0} \\ - 0100111 \\ \hline 0111110 \end{array}$$

الجواب  
الخطوة  
(الخطوة الأولى)

→ 1's Comp.

$$\begin{array}{r} 1100101 \\ - 0100111 \\ \hline \end{array} \xrightarrow{\text{1's Comp.}} \begin{array}{r} 1100101 \\ + 1011000 \\ \hline \end{array}$$

Carry ← 

$$\begin{array}{r} 1100101 \\ + 1011000 \\ \hline 0111110 \end{array}$$

## 2's Comp.

$$\begin{array}{r}
 1100101 \\
 - 0100111 \\
 \hline
 \end{array}
 \xrightarrow{\text{2's Comp}}
 \begin{array}{r}
 1100101 \\
 + 1011001 \\
 \hline
 \cancel{0} \quad 111110
 \end{array}$$

← →

b)  $0100111 - 1100101 =$

(using Direct, 1's Comp., 2's Comp.)

### \* Direct

$$\begin{array}{r}
 0100111 \\
 - 1100101 \\
 \hline
 0100111 \\
 \hline
 - 011110
 \end{array}$$

\* 1's Comp:

$$\begin{array}{r} 0 \mid 00111 \\ - 1100101 \\ \hline \end{array} \rightarrow \begin{array}{r} 0 \mid 00111 \\ 1's Comp. + 0011010 \\ \hline 1000001 \end{array}$$

→ the result =

$$\begin{aligned} & - 1's Comp. of (10.000) \\ & - 0111110 \end{aligned}$$

\* 2's Comp.

$$\begin{array}{r} 0 \mid 00111 \\ - 1100101 \\ \hline \end{array} \rightarrow \begin{array}{r} 0 \mid 00111 \\ 2's Comp. + 0011011 \\ \hline 1000010 \end{array}$$

→ the result =

$$\begin{aligned} & - 2's Comp. of (10.000) \\ & - 0111110 \end{aligned}$$

$$c) (-6) + (+3)$$

(using signed 2's Comp. in 4 bits)

$$\begin{array}{r}
 06 \rightarrow -0110 \xrightarrow{\text{2's Comp.}} 1010 \\
 +3 \rightarrow +0011 \xrightarrow{\text{2's Comp.}} 0011 \\
 \hline
 03
 \end{array}$$

Weight of 4 bits

8	4	2	1
---	---	---	---

$$\begin{array}{r}
 1010 \leftarrow \text{2's Comp.} \\
 +0011 \\
 \hline
 \boxed{1101}
 \end{array}$$

$\rightarrow$  The result = - 2's Comp. of  
 $(\underbrace{1101}_{-0011})$

Range of 2's Comp in 4 bits:

$$[-2^{n-1}, +2^{n-1} - 1]$$

$$[-8, +7]$$

⑨

$$d) (-6) + (-3)$$

(using signed 2's comp. in 4 bits)

III Range of 2's Comp. in 4 bits.

$$[-2^{n-1}, +2^{n-1} - 1]$$

$$[-8, +7]$$

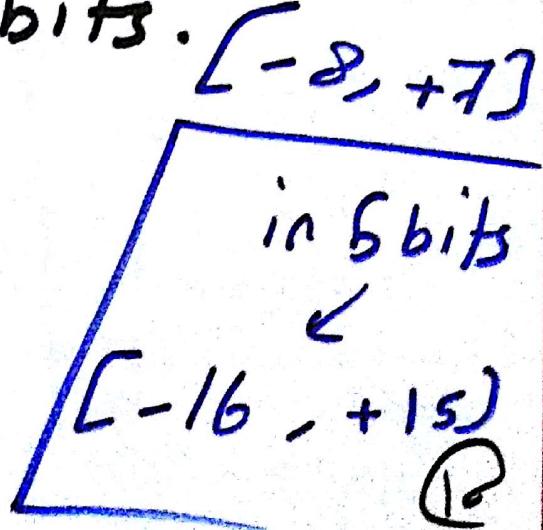
$$\begin{array}{r}
 -6 \rightarrow -0110 \xrightarrow{\text{2's Comp}} 1010 \\
 -3 \rightarrow -0011 \xrightarrow{\text{2's Comp}} +1101 \\
 \hline
 \boxed{-9}
 \end{array}$$

in 4 bits  $\rightarrow$  (Overflow)

Eg:  $-10 - 3 \rightarrow$  using 2's comp  
in 4 bits.

$-10 \rightarrow$  out of Range

$$\begin{array}{r}
 -3 \\
 \hline
 -13
 \end{array}$$



10

## 2 Cases

Case ①  $\text{out} - \text{inp}$

Direct  $1\sigma S \text{ Comp.}$

$+ 1\sigma S \text{ Comp.}$



? 'S Comp.

+ 2'S Comp.



Case

②  $\text{inp} - \text{out}$

③

Direct

$- \text{inp}$   
 $- \text{out}$

$1\sigma S \text{ Comp.}$

$+ 1\sigma S \text{ Comp.}$

$2\sigma S \text{ Comp.}$

$+ 2\sigma S \text{ Comp.}$

$\rightarrow \text{the result} = - 1\sigma S \text{ Comp.}$

$\rightarrow \text{the result} = - 1\sigma S \text{ Comp.}$

$(-6) + (-3)$  using 2's comp.  
in 4 bits  
↳ Calculate the Operation

and if any problem is founded  
~~Show~~ it and how to solve it?

→ overflow

→ increase the number of bits to  
be 5 bits:

.....

\* Calculate the following operation:

$$13.625 - 27.750$$

(using 9's Comp. & 10's Comp.)

1) 9's Comp.    10's Comp.    2.5 Comp.

$$\begin{array}{r} \cancel{13.625} \\ - 27.750 \\ \hline \cancel{13.625} \end{array} \xrightarrow{\text{9's Comp.}} \begin{array}{r} 13.625 \\ + 72.249 \\ \hline 85.874 \end{array}$$

$$\Theta 14.125$$

Direct

→ The result =  
- 9's Comp. of

$$\begin{array}{r} .9 .9 .9 .9 .9 \\ 27.750 \end{array}$$

→ The result =

$$\begin{array}{r} -.9 -.9 .9 .9 .9 \\ - 9's \text{ Comp. of } (85.874) \\ (-14.125) \end{array}$$

2) 10's Comp.

$$\begin{array}{r}
 13.625 \xrightarrow{\text{10° S Comp.}} 13.625 \\
 -27.750 \xrightarrow{\text{10° S Comp.}} 72.250 \\
 \hline
 72.250
 \end{array}$$

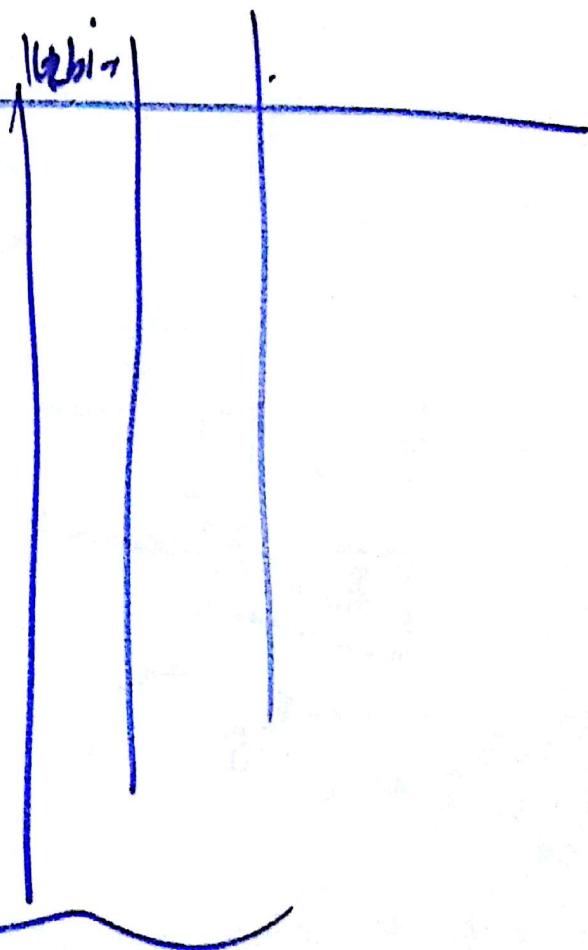
10's Comp → the result =

- 10<sup>9</sup>s Comp. of (85.875)  
(- 14.125)

\* Show the upper & lower bounds of decimal value that are allowed for binary bit patterns of length 4, 5, 6, 7 & 8 in cases.

of:

- 1) unsigned
- 2) signed magnitude
- 3) 1's comp.
- 4) 2's comp.
- 5) Excess notation.



$n = 4$

1) unsigned:  $[0, 2^{n-1}]$

$\xrightarrow{\text{lower bound}} [0, 15]$        $\downarrow \text{upper bound}$

- 2) Signed magnitude:

$$[-(2^{n-1}-1)] = [-7]$$

(14)

3) 1's Comp.:  $\{ \pm 7 \}$

4) 2's Comp.

$$[-(2^{n-1})_{10} + (2^{n-1} - 1)_B]$$

if  $n = 4$

$$[-8, +7] \quad \text{↗}$$

5) Excess notation

Excess 4  $\rightarrow$  3 bits  $\rightarrow$   $n=3$

$\hookrightarrow [-4, +3]$   $\quad [-2^{3-1}, +2^{3-1}]$

Excess 8  $\rightarrow$  4 bits  $\quad \textcircled{8} \quad 4 \ 2 \ 1$

$\hookrightarrow [-8, +7]$

Excess 16  $\rightarrow$  (5 bits)  $\quad \textcircled{16} \ 3 \cdot 4 \ 2 \ 1$

$\hookrightarrow [-16, +15]$

Q: Find the integer decimal values of binary number  $(\underline{1} \underline{0} \underline{1} \underline{0} \underline{1})_2$

in the following systems:

$\begin{array}{r} 16 \\ 8 \\ 4 \\ 2 \\ 1 \end{array}$

→ unsigned:  $(16 + 4 + 1) = (21)_{10}$

→ signed.

$\begin{array}{r} 16 \\ 8 \\ 4 \\ 2 \\ 1 \\ \hline 0 \ 1 \ 0 \end{array}$   
 $(-5)_{10}$

→ 1's comp. =  $\neg$  1's comp. of  $(\underline{1} \underline{0} \underline{1} \underline{0} \underline{1})_2$   
 $= (\underline{0} \underline{1} \underline{0} \underline{1} \underline{0})_2$   
 $= (-10)_{10}$

→ 2's Comp.

→ excess-16

2's Comp.:

$\begin{array}{r} 16 \\ 8 \\ 4 \\ 2 \\ 1 \\ \hline 0 \ 1 \ 0 \ 1 \end{array}$   
 $= -2^{\text{'}}\text{s Comp. of } (0101)_2$   
 $= -1011$   
 $= (-11)_{10}$

⑧

$$(10101)_2 \rightarrow (?)_{10}$$

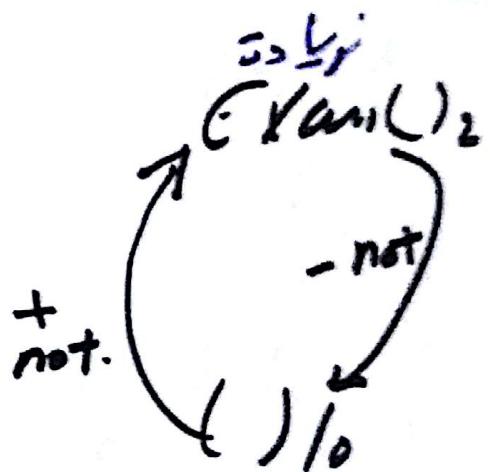
↳ Excess 16

$$\begin{array}{r} \text{5 bits} \\ \textcircled{16} \quad \begin{array}{r} 3 & 4 & 2 & 1 \end{array} \end{array}$$

$$\begin{array}{r} 16 \ 8 \ 4 \ 2 \ | \\ 10101 - 16 \end{array}$$

$$= 21 - 16$$

$$= (+5)_{10}$$



17

\* find the binary value of integer number  $(-13)_{10}$  in the following systems: in 5 bits

1) unsigned.  $[0, 2^n - 1] = [0, 31]$

$$(-13) \rightarrow (?)_2$$

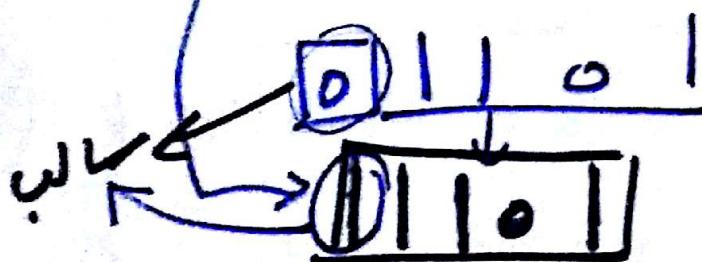
$$\underbrace{(01101)}_{2 \div 113^{10}}$$

2) Signed-magnitude

$$(-13)_{10}$$

$$\begin{array}{r} 16 & 8 & 4 & 2 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array}$$

2 (-13) in 5 bits.



1 Range  $[-(2^{n-1}-1)] \rightarrow [-15]$

3) 1's comp. of  $(-13)_{10}$  in 5 bits

$$[= (2^{n-1} - 1)] \Rightarrow [= 15]$$

① binary of -13 in 5 bits

0 | 1 0 1

② if the number - or +

$\rightarrow$  Convert it to 2's.

1's comp. of (01101)

(10010)

$2 \text{ in } 1's \text{ comp.}$

4) 2's Comp. [-16, +15]

① (01101)  $\boxed{-13}$

② 2's Comp. of (01101)

(10011) $_2$  in 2's comp.

19

$(-18)_{10} \rightarrow (\ )_2$   
using ex. 16.  
 $16 \ 8 \ 4 \ 2 \ 1$   
5 bits  
 $[-16, +15]$

$(-18)$  out of Range

$(-13)_{10}$   $\rightarrow (\ )_2$  in ex. 16

$$-13 + 16 \rightarrow \boxed{3}$$

$\downarrow$

$$(00\overset{2}{\underset{1}{\phi}}1)$$

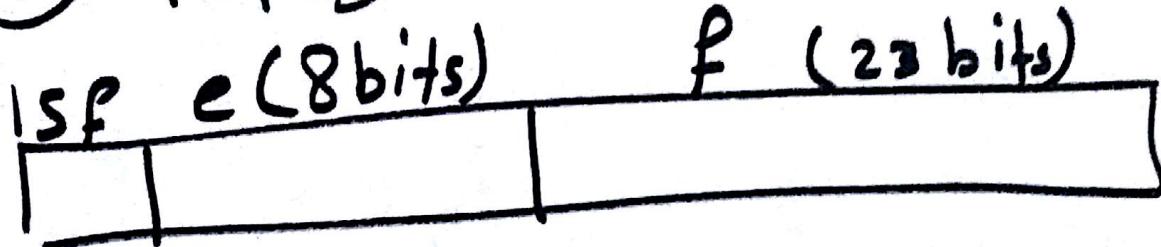
ex.  $(\ )_2$   
not decimal

\* Assuming Floating-Point binary pattern in IEEE notation.

- 1) Code the decimal value  $(+43)_{10}$  as binary floating point number.
- 2) Decode the Hexadecimal pattern  $(\text{C}0\text{A}4\ 0\ 0\ 0)_{16}$  to its equivalent decimal value.
- 3) Decode the Hexadecimal pattern  $(\text{7}\text{F}\text{B}\ 7\ \text{6}\text{E}\ 5\text{B})_{16}$  to its equivalent decimal value.

\* (IEEE)

① Representation. 32 bits



②  $1 \cdot f \times 2^e$

③ Exponent  $\rightarrow e + 127$

1) Code  $(+43)_{10} \rightarrow (?)_2$   
in floating Point ~~format~~

①  $(+43)_{10}$  }  $2 \div | 43$  remain  
 $\downarrow$   
 $+1.01011 \dots$  }  
-----  
64 32 16 8 4 2 1  
| . 0 1 0 1 1

②  $1.F \times 2^e$   
 $+1.\underline{01011}_f \times 2^{+5}_e$

e → Exponent

③ f = +0.01011

e = +5

④ e = +5  $\rightarrow$  to  $(?)_2$

+5 + 127 using exor 127

256	128	64	32	16	8	4	2	1
1	0	0	0	0	1	0	0	0

(10000100)

23

$$f = \oplus 0.\underline{0}1011$$

$\oplus \leftrightarrow 0$   
 $\ominus \leftrightarrow 1$

$$e = (100\ 001\ 00)$$

$$\rightarrow \left( \begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \quad \text{18 zeros.}$$

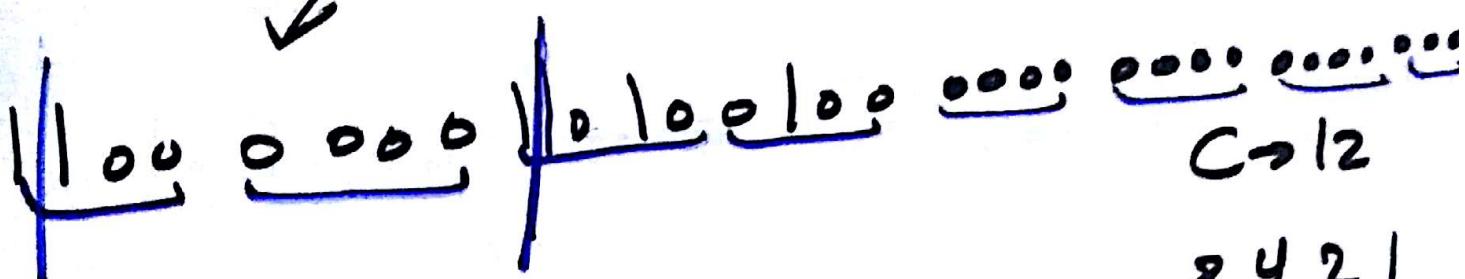
$$(4 \quad 2 \quad 2 \quad \underset{\text{c}}{0} \quad 0 \quad 0 \quad 0)_{16}$$

## 2) Decode

(CoA 40000)<sub>16</sub> → ()<sub>10</sub>

ANS.

① Co A 40000



$$\begin{array}{r}
 & 8 & 4 & 2 & 1 \\
 C \rightarrow 12 \rightarrow & | & 1 & 0 & 0 \\
 A \rightarrow 1^0 \rightarrow & | & 0 & | & 0 \\
 4 \rightarrow 0 & | & 0 & 0
 \end{array}$$

2

(2)	$c(8\text{ bits})$	$f(23\text{ bits})$
+SF	10000001	0100100 ....

$$③ f = -0.0100100$$

$$e = \left( \begin{smallmatrix} 128 & 64 & 32 & 16 & 8 & 4 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{smallmatrix} \right) \frac{1}{2} \rightarrow ( )_{10} \text{ using excess 127}$$

$$129 - 127 \Rightarrow (+2)_{10}$$

$$f = \underline{-0.01001}$$

$$e = (+2)_{10}$$

④  $1.\textcircled{f} \times 2^{\textcircled{e}}$

$$(-1.\underset{\rightarrow}{0}1001 \times 2^{+2})$$

$$-101.001 \rightarrow (5.125)_{10}$$

~~~~~

3) Decode  $(7FB76E5B)_{16}$   
to  $( )_{10}$

7 F B 7.6E5 B  
↓ ↓ .  
0 1 1 1 1 1 1 1 0 1 1 1 0 1 1 1 0 1 1 0 1 1 1 0 0 1 0 1

| ISF | $e$ (8 bits) | $F$ (23 bits) |
|-----|--------------|---------------|
| 0   | 1111111      | 011.....      |

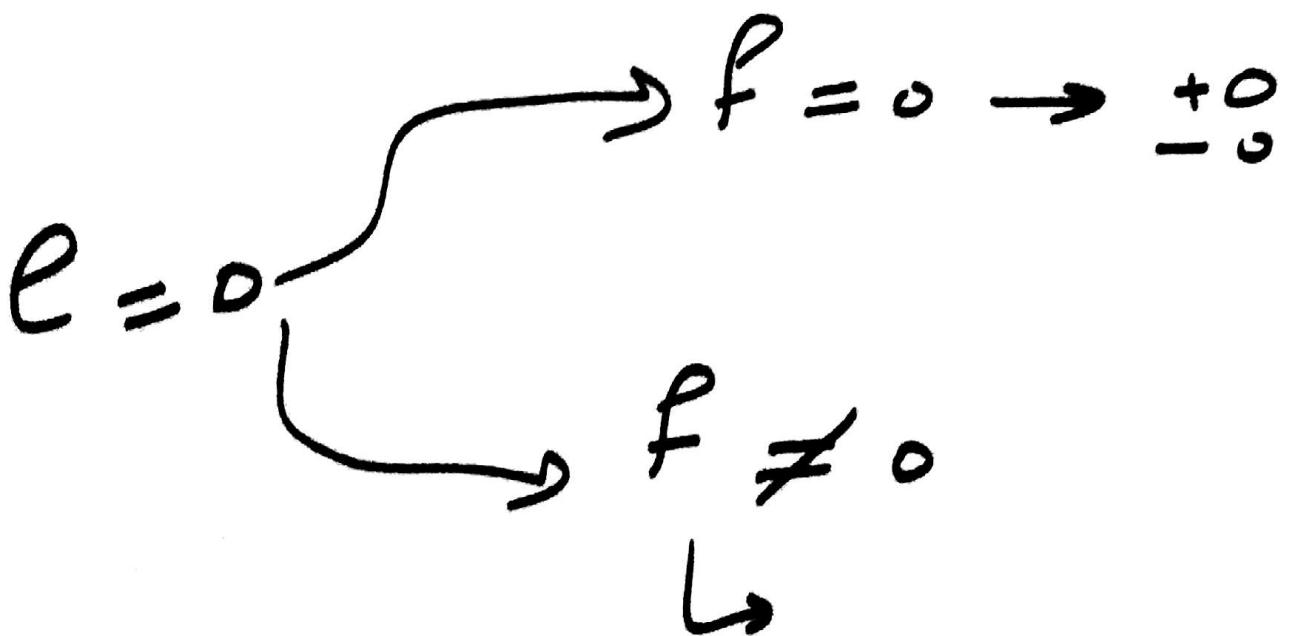
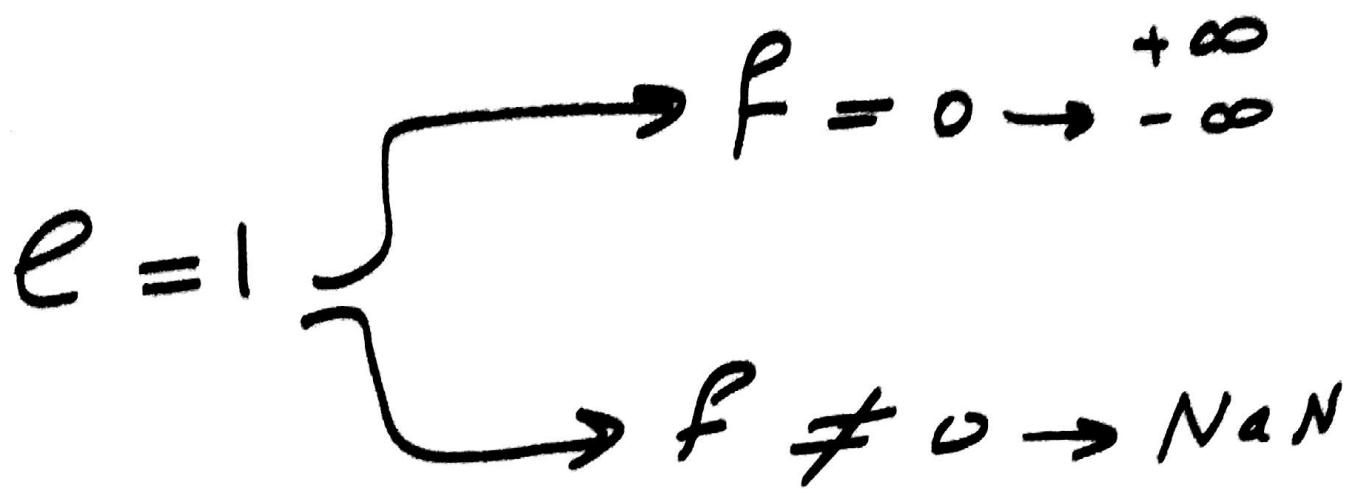
## Special Case

$$e = 1 \quad f \neq 0$$

Not a number

Nan.

\* 4 Special Case in IEEE:



the number is less than  
the allowed minimum value