Date Subject

MS 506 Linear algebra د/ مروة عبدالله

Course outlines

1)- Matrices المصفوفات
* operations on matrices
* properties of matrix operation
* inverse of a matrix

2)- System of Linear Equation نظم المعادلة الخطية
* Gaussian elimination Method
* Gauss jordan

3)- Determinants المحددات
* Determinant of matrix
* evaluating determinant
* properties of determinant
* eigen values & eigen vectors

4)- Vector Space
* Vector space
* Vector subspace
* spanning basis and Dimension

(1)_ Matrix المصفوفة

هي عبارة عن ارقام جنب بعضها ، بتتكون من صفوف واعمدة

$$A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}_{2\times2}$$

$a_{11}=1$ $a_{12}=2$ ← صف اول عمود ثاني

$a_{21}=3$ $a_{22}=5$

$A=[a_{ij}]_{m\times n}$ → $\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & & \vdots\\ a_{m1} & a_{m2} & & a_{mn}\end{bmatrix}$

صفوف ، اعمدة ، عدد الصفوف ، عدد الاعمدة

$$B=\begin{bmatrix}1 & 2 & 0\\ -1 & 3 & 4\end{bmatrix}_{2\times3}$$

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* Square Matrix

* اذا كان عدد الصفوف = عدد الاعمدة ، تسمى مصفوفة مربعة

$n = m$

ex1 $\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}_{2\times2}$ ex2 $\begin{bmatrix}-1 & 3 & 2\\ -3 & 5 & 2\\ 0 & 3 & 0\end{bmatrix}_{3\times3}$

4x4
5x5
6x6

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* Vector

* Column Matrix $A=\begin{bmatrix}1\\3\\2\end{bmatrix}$ عمود واحد

* Row Matrix $B=[1\ 3\ 2]$ صف واحد

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ex) $A=\begin{bmatrix}1 & 2\\ 4 & 0\end{bmatrix}_{2\times2}$, $B=[1, 2]_{1\times2}$, $C=\begin{bmatrix}2\\0\end{bmatrix}_{2\times1}$

$D=\begin{bmatrix}1 & 2\\ 4 & 0\end{bmatrix}_{2\times2}$, $E=\begin{bmatrix}1\\2\end{bmatrix}_{2\times1}$ → $A=D$

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* operations on Matrices

1) Addition

$A=[a_{ij}]$ and $B=[b_{ij}]$

are Matrices of the same size $m\times n$, then their sum in the $m\times n$ Matrix given

$$A+B=[a_{ij}+b_{ij}]$$

ex) let $A=\begin{bmatrix}-1 & 2\\ 0 & 1\end{bmatrix}_{2\times2}$ $B=\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}_{2\times2}$

$C=\begin{bmatrix}0 & 1 & -2\\ 1 & 2 & 3\end{bmatrix}_{2\times3}$ $D=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}_{2\times3}$

find $A+B$, $C+D$, $A+C$

Solve

$$A+B=\begin{bmatrix}-1 & 2\\ 0 & 1\end{bmatrix}_{2\times2}+\begin{bmatrix}1 & 3\\ -1 & 2\end{bmatrix}_{2\times2}=\begin{bmatrix}0 & 5\\ -1 & 3\end{bmatrix}_{2\times2}$$

$$C+D=\begin{bmatrix}0 & 1 & -2\\ 1 & 2 & 3\end{bmatrix}_{2\times3}+\underbrace{\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}_{\text{محايد جمعي}}=\begin{bmatrix}0 & 1 & -2\\ 1 & 2 & 3\end{bmatrix}$$

zero Matrix

$\underset{2\times2}{A}+\underset{2\times3}{C}\longrightarrow$ is undefined

2)- Scalar Multiplication ضرب المصفوفات من رقم ثابت

if $A = [a_{ij}]_{m \times n}$ Matrix & c is scalar

then Scalar Multiple A by C $m \times n$ Matrix given by

$CA = [Ca_{ij}]$

$A = \begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix}$, $3A = 3\begin{bmatrix} 1 & 5 \\ -2 & 0 \end{bmatrix}$, $3A = \begin{bmatrix} 3 & 15 \\ -6 & 0 \end{bmatrix}$

$3 - 2 = 3 + (-2)$

3)- Subtraction

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix}$

$\underset{2\times2}{A} - \underset{2\times2}{B} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}_{2\times2}$

$B - A = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 2 & 2 \end{bmatrix}_{2\times2}$

ex) let $A = \begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$

find ⓐ $3A$
ⓑ $-B$
ⓒ $3A - B$

Solve

$$3A = 3\begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix}$$

$$-B = -\begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix}$$

$$3A - B = 3A + (-B) = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 0 & 0 \\ -1 & +4 & -3 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 6 & 12 \\ -10 & 4 & -6 \\ 7 & 0 & 4 \end{bmatrix}$$

$3A - B$ -

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* Properties of Matrix Addition & Scalar Multiplication

let A, B & C matrices & c, d scalars

1)- $A + B = B + A$

$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$A + B = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$ & $B + A = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix}$

2)- $A + (B + C) = (A + B) + C$

$C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$B + C = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

$A + (B + C) = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$

$(A + B) + C = \begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 2 & 2 \end{bmatrix}$

3)- $(cd)A = c(dA)$

Scalar $\longrightarrow$ $c=2$, $d=3$

$$(2*3)A = 6A = \begin{bmatrix} 6 & 12 \\ 0 & -6 \end{bmatrix}$$

$$2(3A) = 2\begin{bmatrix} 3 & 6 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 0 & -6 \end{bmatrix}$$

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4)- $IA = A$

(I) محايد ضربي للمصفوفات

حيث (I) هي مصفوفة مربعة (square matrix)

(عدد الصفوف = عدد الأعمدة)

$a_i = j$ $\quad a_{11} \quad a_{22} \quad a_{33} \quad \ldots\ldots = 1$

$$I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ مصفوفة وحدة من الدرجة الثانية

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ مصفوفة وحدة من الدرجة الثالثة

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5)- $c(A+B) = cA + cB$ (Home Work) بأي مثال

ك (مصفوفات)

6)- $(c+d)\,A = cA + dA$ (Home Work)

Scalar مصفوفة

7)- $A + 0 = 0 + A = A$ $\quad$ 0 zero Matrix

8)- $A + (-A) = 0$

9)- if $cA = 0$ then $c = 0$ OR $A = 0$

* Matrix Multiplication

If $A=[a_{ij}]$ is $m\times n$ Matrix

and $B=[b_{ij}]$ is an $n\times p$ Matrix.

then the product of AB is $m\times p$ Matrix.

$$AB=[C_{ij}]_{m\times p} \qquad A_{m\times n}\ B_{n\times p}=AB_{m\times p}$$

where $C_{ij}=\sum_{k=1}^{n} a_{ik}b_{kj}=a_{i1}b_{1j}+a_{i2}b_{2j}+\dots+a_{in}b_{nj}$

$$A=\begin{bmatrix}-1 & 3\\ 4 & -2\\ 5 & 0\end{bmatrix}_{3\times 2} \qquad B=\begin{bmatrix}-3 & 2\\ -4 & 1\end{bmatrix}_{2\times 2}$$

$$\underset{3\times2}{A}\ \underset{2\times2}{B}=\underset{3\times2}{AB} \qquad AB=\begin{bmatrix}-9 & 1\\ -4 & 6\\ -15 & 10\end{bmatrix}_{3\times 2}$$

$$\begin{aligned}
(-1\times-3)+(3\times-4) &= 3+(-12)=-9\\
(-1\times2)+(3\times1) &= (-2)+3=1\\
(4\times-3)+(-2\times-4) &= (-12)+8=-4\\
(4\times2)+(-2\times1) &= 8+-2=6\\
(5\times-3)+(0\times-4) &= -15+0=-15\\
(5\times2)+0\times1 &= 10+0=10
\end{aligned}$$

$AB\neq BA$ * عملية الضرب غير تبادلية

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let $C = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix}$ $\underset{3\times2}{A}\ \underset{3\times2}{C}$ لا تحقق شروط الضرب

let $C = \begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix}$ $\underset{2\times2}{B}\ \underset{2\times2}{C} = \begin{bmatrix} 11 & -13 \\ 8 & -14 \end{bmatrix}$

$(-3\times-1)+(2\times4) = 3+8 = 11$

$(-3\times3)+(2\times-2) = -9+(-4) = -13$

$(-4\times-1)+(1\times4) = 4+4 = 8$

$(-4\times3)+(1\times-2) = -12+(-2) = -14$

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* properties of Matrix Multiplication ضرب المصفوفات

A, B, C are Matrices, d is scalar

1)- $A(BC) = (AB)C$

2)- $A(B+C) = AB+AC$

3)- $(A+B)C = AC+BC$

4)- $d(AB) = dA(B) = A(dB)$

5)- $AB \neq BA$

6)- $AI = IA = A$ الحالة التبادلية الوحيدة

7)- if $AC = BC$ then $A = B$

ex) $A = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -7 \\ 2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$

find AB & BA & ABC

$$\underset{2\times2}{A}\,\underset{2\times2}{B} = \begin{bmatrix} 9 & -4 \\ -2 & 33 \end{bmatrix}$$

$$\underset{2\times2}{B}\,\underset{2\times2}{A} = \begin{bmatrix} 31 & -26 \\ -2 & 8 \end{bmatrix}$$

$$ABC = (AB)C = \begin{bmatrix} 9 & -4 \\ -2 & 33 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -12 & 22 \\ 99 & -37 \end{bmatrix}$$

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* find a, b, c, d such that

$$\begin{bmatrix} a-b & b+c \\ 3d+c & 2a-4d \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 7 & 6 \end{bmatrix}$$

$a - b = 8$
$b + c = 1$ $\rightarrow$ $a - b + b + c = 9 \Rightarrow a + c = 9$
$3d + c = 7$
$2a - 4d = 6$

$a + c = 9$
$-3d - c = -7$ $\Rightarrow$ $a - 3d = 2$ $(\times -2)$

$-2a + 6d = -4$
$2a - 4d = 6$ $\rightarrow$ $2d = 2$ $\Rightarrow$ $d = 1$

$3d + c = 7$
$3 + c = 7 \rightarrow c = 4$

$2a - 4d = 6 \rightarrow a = 5$
$2a - 4 = 6$

$a - b = 8$
$5 - b = 8 \Rightarrow b = -3$

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* Power of Matrices

if A is a Square Matrix

then $A^K = A.A.A.\dots A$ K times

* Properties

if r & s are non negative integers then

1)- $A^r A^s = A^{r+s}$

2)- $(A^r)^s = A^{r.s}$

3)- $A^0 = I$

let $A = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}$ Compute $A^4 \Rightarrow A^4 = (A^2)^2$

$$A^2 = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$$

$$A^4 = A^2 A^2 = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 11 & -10 \\ -5 & 6 \end{bmatrix}$$

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* simplify the following Matrix expression

$A(A+2B)+3B(2A-B)-A^2+7B^2-5AB$

مطلوب تبسيط المعادلات ابسط صورة ، باستخدام الخصائص

$=A^2+2AB+6BA-3B^2-A^2+7B^2-5AB$

$=-3AB+6BA+4B^2$

انواع المصفوفات :-

1)- Zero Matrix

2)- Identity "

3)- Column "

4)- Row "

5)- Square "

6)- diagonal " (مصفوفة مربعة دايما)

* عناصر القطر الرئيسي ارقام وليس (واحد)

$$\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{nn} \end{bmatrix}_{n\times n}$$

* عند الحصول على (Power)
ارفع (الاس) على عناصر القطر الرئيسي

ex) $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $A^4=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7)- upper triangular Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 4 \end{bmatrix}$$

* القطر الرئيسي وما فوقه ارقام
والباقي اصفار

8)- lower triangular Matrix

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 5 & 4 \end{bmatrix}$$

* القطر الرئيسي وما اسفله ارقام
والباقي اصفار

$A^T = B$

مدور $B = A$

* Transpose of a Matrix

the transpose of a matrix denoted by A^T is the matrix whose Columns are rows of the given Matrix A

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & & a_{mn} \end{bmatrix}$$

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ex) find the transpose of the following

$$A = \begin{bmatrix} 2 \\ 8 \end{bmatrix}_{2\times 1} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times 3} \quad D = \begin{bmatrix} 0 & 1 \\ 2 & 4 \\ 1 & -1 \end{bmatrix}_{3\times 2}$$

$$A^T = \begin{bmatrix} 2 & 8 \end{bmatrix}_{1\times 2} \quad B^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}_{3\times 3} \quad D^T = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & -1 \end{bmatrix}_{2\times 3}$$

* Symmetric Matrix المصفوفة المتماثلة

* يجب أن تكون مربعة

if A is a square Matrix & if $A^T = A$

then A is Called a symmetric Matrix

ex)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 3 & 0 & 1 \end{bmatrix} \quad , \quad A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Since $(A = A^T)$ then $\longrightarrow$ A is (Symmetric Matrix)

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* Properties of transpose

if A, B are Matrices, C is a scalar, then

1) $(A+B)^T = A^T + B^T$

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 & -1 \\ 3 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 1 \\ 3 & 5 \end{bmatrix} \quad , \quad (A+B)^T = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} \quad , \quad B^T = \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix} \quad , \quad A^T + B^T = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$$

2)- $(CA)^T = C A^T$ $\quad C = 3$

$(CA) = 3\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 12 \end{bmatrix}$ $\quad (CA)^T = \begin{bmatrix} 3 & 0 \\ 6 & 12 \end{bmatrix}$

$3A^T = 3\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 12 \end{bmatrix}$

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3)- $(AB)^T = B^T A^T$

$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ ، $B = \begin{bmatrix} 0 & -1 \\ 3 & 1 \end{bmatrix}$

$AB = \begin{bmatrix} 6 & 1 \\ 12 & 4 \end{bmatrix}$ ، $(AB)^T = \begin{bmatrix} 6 & 12 \\ 1 & 4 \end{bmatrix}$

$B^T = \begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix}$ ، $A^T = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ ، $B^T A^T = \begin{bmatrix} 6 & 12 \\ 1 & 4 \end{bmatrix}$

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4)- $(A+B+G)^T = A^T + B^T + G^T$

5)- $(A^T)^T = A$ $\quad (A^T)^T = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

6)- $(ABG)^T = G^T . B^T . A^T$

* هذه الخصائص لا تنطبق على (square) فقط ،
بل تنطبق على كل المصفوفات ، بشرط تحقيق شروط الضرب .

ex) let $A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}_{3\times3}$, $B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}_{3\times2}$

Find $(AB)^T$, $B^T A^T$

$AB = \begin{bmatrix} 2 & 1 \\ 6 & -1 \\ -1 & 2 \end{bmatrix}_{3\times2}$ $\quad (AB)^T = \begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 2 \end{bmatrix}_{2\times3}$

$B^T = \begin{bmatrix} 3 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}_{2\times3}$ $\quad A^T = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \\ -2 & 3 & 1 \end{bmatrix}_{3\times3}$ $\quad A^T B^T = \begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 2 \end{bmatrix}_{2\times3}$

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* Trace of the Matrix

let A be a square Matrix, the Trace of A denoted by $tr(A)$ is the sum of the Main diagonel elements of A

$$tr(A) = a_{11} + a_{22} + a_{33} + \cdots + a_{nn}$$

ex/ Determine the trace of A

$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & -5 & 6 \\ 7 & 3 & 0 \end{bmatrix}_{3\times3} \longrightarrow tr(A) = 4 - 5 + 0 = -1$

شرط ان المصفوفة مربعة

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* properties of the trace

1) $tr(A+B) = tr(A) + tr(B)$

$A = \begin{bmatrix} 4 & 1 & -2 \\ 2 & -5 & 6 \\ 7 & 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ $A+B = \begin{bmatrix} 6 & 2 & -2 \\ 5 & -6 & 10 \\ 8 & 5 & 1 \end{bmatrix}$

$tr(A) = 4 - 5 + 0 = -1$ $tr(B) = 2 - 1 + 1 = 2$ $tr(A+B) = 6 - 6 + 1 = 1$

$tr(A) + tr(B) = 2 - 1 = 1 = tr(A+B)$

2) $tr(AB) = tr(BA)$ H.W

3) $tr(cA) = c\, tr(A)$

$$3A = \begin{bmatrix} 12 & 3 & -6 \\ 6 & -15 & 18 \\ 21 & 9 & 0 \end{bmatrix} \qquad tr(3A) = 12 - 15 + 0 = -3$$

$$3\, tr(A) = 3(-1) = -3$$

4) $tr(A^T) = tr(A)$

تتحقق في اي مصفوفة ، حتى ان لم تكن (Symmetric)

$$A^T = \begin{bmatrix} 4 & 2 & 7 \\ 1 & -5 & 3 \\ 2 & 6 & 0 \end{bmatrix} \qquad tr(A^T) = 4 - 5 + 0 = -1$$

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* Inverse of the Matrix

Definition: if A is a square matrix, and if B is a matrix of the same degree of A, can be found such that $AB = BA = I$

* then B is called an Invarse of A
* if no such Matrix B can be found

then A is said to be (Singular Matrix)
(مصفوفة ليس لها معكوس)

ex) $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}_{2\times2}$, $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}_{2\times2}$

is B is inverse of A ? اريد التأكد

$$AB = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \overset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{2\times2}$$

$$BA = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$$

$AB = BA = I$

then B is inverse of A

* لإيجاد معكوس المصفوفة من الدرجة الثانية

* finding inverse of Matrix 2x2

let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

* استبدال عناصر القطر الرئيسي.
* تغيير الإشارة من القطر الآخر.

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ex)

Find inverse Matrix for A where

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{1\times3-2\times1} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

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* لإيجاد معكوس المصفوفة من أى درجة

* Finding Inverse of any Square Matrix

* Using Row operation

1)- Interchange Two Rows تبديل صفين، وليس للأعمدة.

2)- multiply a row by a non zero Constant
اضرب الصف في رقم، ولا يكون صفر

3)- add a multiply of a row to another
حاصل ضرب صف، وأجمعه على صف آخر.

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* Steps of finding A^{-1}

1). adjoint the Identify Matrix to the original Matrix $[A \vdots I]$

2). Apply Row operations to this Matrix until left side is Reduced to $I \to [I : A^{-1}]$

ex)

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ find A^{-1}

* طالما هو (واحد) يفضل زي ما هو ويسمى (leading)

$$\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 1_{-1} & 3_{-2} & 0_{-1} & 1_{0} \end{array}\right]$$

* يظل السطر الأول كما هو
* نقوم بعملية $(-R_1 + R_2)$ على السطر الثاني

$$\left[\begin{array}{cc:cc} 1 & 2^{-2} & 1^{2} & 0^{-2} \\ 0 & 1 & -1 & 1 \end{array}\right]$$

* السطر الثاني كما هو
* نقوم بعملية $(-2R_2 + R_1)$ على السطر الأول

$$\left[\begin{array}{cc:cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \end{array}\right] \longrightarrow A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

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ex2)

Find A^{-1} for $A = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 3 \\ 1 & 0 & 8 \end{bmatrix}_{3\times 3}$

$$\left[\begin{array}{ccc:ccc} 2 & 5 & 3 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 1^{-1} & 2^{0} & 3^{-8} & 0^{0} & 1^{0} & 0^{-1} \\ 2^{-2} & 5^{0} & 3^{-16} & 1^{0} & 0^{0} & 0^{-2} \end{array}\right] \quad \begin{array}{l} -R_1 + R_2 \\ -2R_1 + R_3 \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 2 & -5 & 0 & 1 & -1 \\ 0 & 5 & -13 & 1 & 0 & -2 \end{array}\right] \quad \tfrac{1}{2} R_2$$

* لكي اصل وسط العمود الثاني الى (1)

baraka

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -5/2 & 0 & 1/2 & -1/2 \\ 0 & 5 & -13 & 1 & 0 & -2 \\ 0 & -5 & +25/2 & 0 & -5/2 & 5/2 \end{array}\right] \quad -5R_2+R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -5/2 & 0 & 1/2 & -1/2 \\ 0 & 0 & -1/2 & 1 & -5/2 & 1/2 \end{array}\right] \quad -2R_3$$

$$\left[\begin{array}{ccc|ccc} 1^{0} & 0^{0} & 8^{-8} & 0^{16} & 0^{-40} & 1^{8} \\ 0^{0} & 1^{0} & -5/2^{\,5/2} & 0^{-5} & 1/2^{\,25/2} & -1/2^{\,-5/2} \\ 0 & 0 & 1 & -2 & 5 & -1 \end{array}\right] \quad \begin{array}{l} \frac{5}{2}R_3+R_2 \\ -8R_3+R_1 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & -40 & 9 \\ 0 & 1 & 0 & -5 & 13 & -3 \\ 0 & 0 & 1 & -2 & 5 & -1 \end{array}\right] \Longrightarrow A^{-1} = \begin{bmatrix} 16 & -40 & 9 \\ -5 & 13 & -3 \\ -2 & 5 & -1 \end{bmatrix}$$

$$\underbrace{\qquad\qquad}_{I} \qquad \underbrace{\qquad\qquad}_{A^{-1}}$$

EX3)

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 2^{-2} & 4^{-12} & -1^{-8} & 0^{-2} & 1^{0} & 0^{0} \\ -1^{1} & 2^{6} & 5^{4} & 0^{1} & 0^{0} & 1^{0} \end{array}\right] \quad \begin{array}{l} -2R_1+R_2 \\ R_1+R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right] \quad R_2/-8$$

$$\begin{array}{cccccc} 0 & -6 & -54/8 & -12/8 & 6/8 & 0 \\ \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & 9/8 & 2/8 & -1/8 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right] \quad \begin{array}{l} -6R_2 + R_1 \\ -8R_2 + R_3 \end{array}$$

$$\begin{array}{cccccc} 0 & -8 & -9 & -2 & 1 & 0 \\ \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & -22/8 & -1/2 & 6/8 & 0 \\ 0 & 1 & 9/8 & 2/8 & -1/8 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

(لوجود (3) اصفار، لن استطيع الحصول على المعكوس)

then A is a singular matrix.

* Properties of inverse matrix

1)- $(A^{-1})^{-1} = A$

2)- $(A^{k})^{-1} = (A^{-1})^{k} = A^{-k} \rightarrow A^{-3} = (A^{3})^{-1} = (A^{-1})^{3}$

3)- $(cA)^{-1} = \frac{1}{c}(A^{-1})$

4)- $(A^{T})^{-1} = (A^{-1})^{T}$

5)- $(AB)^{-1} = B^{-1}A^{-1}$

$$D = \begin{bmatrix} d_1 & 0 & 0 & \cdots & 0 \\ 0 & d_2 & 0 & \cdots & \\ \vdots & \vdots & & & \\ 0 & 0 & d & & \end{bmatrix} \qquad D^{-1} = \begin{bmatrix} 1/d_1 & & \\ & 1/d_2 & \\ & & 1/d_3 \end{bmatrix}$$

Date Subject

Problems

H.W

1)- Show that the matrix A is a singuler

where $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 0 \end{bmatrix}$

2)- let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ find A^{-3}

3)- let $A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$ find A

4)- Find the matrix A given that

$$(5A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$$

5)- Simplify $(AB)^{-1}(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1}$

Kamal $\longrightarrow K$

Doaa $\longrightarrow D$

$K = 3D + 5$

$K = D + 29$

$K = ?$ $D = ?$

baraka

Date L3 Subject 19/8/2017

* System of Linear Equation

المعادلة الخطية تكون بها كل المجاهيل مرفوعة لأس (واحد)

$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$

$a_1, a_2, \ldots, a_n \longrightarrow$ Coefficients معاملات

$b \longrightarrow$ Constant ثابت

ex/

$3x + 2y = 7$ linear

$\frac{1}{2}x + y - \pi z = \sqrt{2}$ linear

خطية طالما المتغيرات بأس (واحد)

$\left(\sin\left(\frac{\pi}{2}\right)\right) x_1 - 4x_2 = e^2$ linear

$e^x - 2y = 4$ nonlinear غير خطية

$1/x + 1/y = 4$ 〃 〃

baraka

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Date Subject

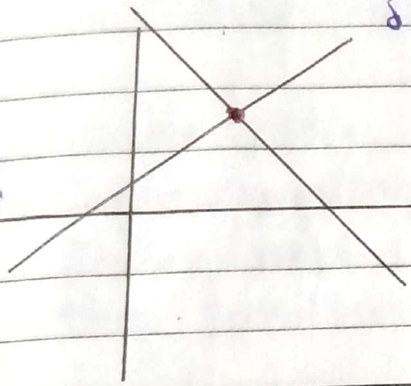
* Solving system of linear Equations

$$x + y = 3$$
$$x - y = -1$$

الحلول المتوقعة ؛ حالة من ثلاث حالات

1)- Exactly one solution

الناتج يكون حل وحيد
وذلك عند تقاطع الخطين في نقطة واحدة

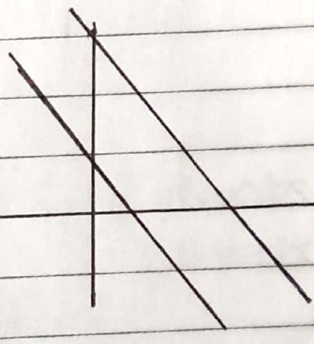


مثل المثال السابق

$x = 1$ ، $y = 2$

2)- no solution

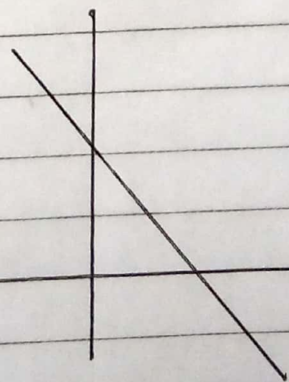
وذلك عند تواجد خطين متوازيين



مثال

$$x + y = 3$$
$$x + y = 1$$

3)- Many solutions
or infinite numbers of solution



وذلك عند تطابق خطين على بعض

مثال

$$x + y = 3$$
$$2x + 2y = 6$$

1) Gaussian elimination method

steps

1) write the augmented Matrix of the system of linear equation

2) use elementary row operations to rewrite the augmented Matrix in row echelon form

3) write the system of linear equation, corresponding to the Matrix in row echelon form and use back substitution to find the solution

---

$$
\begin{aligned}
x - 4y + 3z &= 5 \\
-x + 3y - z &= -3 \\
2x \qquad - 4z &= 6
\end{aligned}
$$

$AX = b$

A → Coefficients Matrix

X → variables Matrix

b → Costants Matrix

* augmented Matrix

هي مصفوفة نكتب فيها المعاملات

وأيضا الثوابت

$$
\underbrace{\begin{bmatrix} 1 & -4 & 3 \\ -1 & 3 & -1 \\ 2 & 0 & -4 \end{bmatrix}}_{\text{Coefficients}}
\begin{matrix} : \\ : \\ : \end{matrix}
\underbrace{\begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix}}_{\text{Constants}}
$$

* Row echelon Form Matrix

* لا يشترط ان تكون مربعة

* all rows Consisting entirely of zero's occur at the bottom of the matrix.

لو عندي صف كله اصفار، لابد ان يكون اسفل المصفوفة، وليس في الوسط او اعلى

* For each row that doesn't Consist entirely of zero's, the first non zero entry is 1, Called leading 1

الصفوف التي تحتوي على ارقام، لابد ان تبدأ ب (1) على اليسار

* For two successive (non zero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

الصفين المتتاليين اولهما (واحد) يكون (واحد) الصف الاعلى على يسار (واحد) الصف الاسفل.

* Reduced Row echelon form Matrix

- له شرط رابع، بالاضافة للثلاث شروط السابقة.

* every Column that has a leading 1 has zero's in every position

العمود الذي يحتوي على (1) يحتوي على اصفار في باقي المواضع

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ex)

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

هذا المثال يحقق ثلاث شروط فقط
row - not reduced
* لأن الأعمدة التي تحتوي على واحد ، تحتوي على أرقام غير الصفر ، سواء بأعلى (الواحد) او اسفله .

$$\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

* هذا المثال حقق الاربع شروط
reduced row echelon form

$$\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

* تحقيق 3 شروط
row echelon form
* اذا تم تغيير (الواحد) الى (3) فقط شروط ، واصبحت مصفوفة عادية

$$\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

* حققت (4) شروط
reduced

$$\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

* لا تحقق الشروط .
لان صف الاصفار ليس الاسفل .

$$\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

row - not reduced *

Ex1)

Solve the system using Gaussian elimination

$x_1 - 2x_2 = -3$

$2x_1 + 3x_2 = 8$

Sol

step 1:

$$\begin{bmatrix} 1 & -2 & -3 \\ 2 & 3 & 8 \end{bmatrix}$$

step 2:

$$\begin{bmatrix} 1 & -2 & -3 \\ 2 & 3 & 8 \\ -2 & 4 & 6 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & -2 & -3 \\ 0 & 7 & 14 \end{bmatrix} \xrightarrow{R_2/7}$$

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 2 \end{bmatrix}$$

step 3:

$x_1 - 2x_2 = -3$

$x_2 = 2 \longrightarrow x_2 = 2$

$x_1 - 2(2) = -3 \longrightarrow x_1 = 1$

ثم التأكد في المعادلات الاساسية

$x_1 - 2x_2 = -3$ $\quad 1 - 2(2) = -3$ ✓

$2x_1 + 3x_2 = 8$ $\quad 2(1) + 3(2) = 8$ ✓

ex2) Solve the system using Gaussian elimination

$$
\begin{aligned}
x_2 + x_3 - 2x_4 &= -3 \\
x_1 + 2x_2 - x_3 &= 2 \\
2x_1 + 4x_2 + x_3 - 3x_4 &= -2 \\
x_1 - 4x_2 - 7x_3 - x_4 &= -19
\end{aligned}
$$

sol

$$
\begin{bmatrix}
0 & 1 & 1 & -2 & -3 \\
1 & 2 & -1 & 0 & 2 \\
2 & 4 & 1 & -3 & -2 \\
1 & -4 & -7 & -1 & -19
\end{bmatrix}
\qquad R_1 \leftrightarrow R_2
$$

$$
\begin{bmatrix}
1 & 2 & -1 & 0 & 2 \\
0 & 1 & 1 & -2 & -3 \\
2^{-2} & 4^{-4} & 1^{2} & -3^{0} & -2^{-4} \\
1^{-1} & -4^{-2} & -7^{1} & -1^{0} & -19^{-2}
\end{bmatrix}
\qquad \begin{array}{l} -2R_1 + R_3 \\ -R_1 + R_4 \end{array}
$$

$$
\begin{bmatrix}
1 & 2 & -1 & 0 & 2 \\
0 & 1 & 1 & -2 & -3 \\
0 & 0 & 3 & -3 & -6 \\
0 & \underset{6}{-6} & \underset{6}{-6} & \underset{-12}{-1} & \underset{-18}{-21}
\end{bmatrix}
\qquad \begin{array}{l} 6R_2 + R_4 \\ \frac{1}{3}R_3 \end{array}
$$

$$
\begin{bmatrix}
1 & 2 & -1 & 0 & 2 \\
0 & 1 & 1 & -2 & -3 \\
0 & 0 & 1 & -1 & -2 \\
0 & 0 & 0 & -13 & -39
\end{bmatrix}
\qquad R_4/_{13}
$$

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$$\left[\begin{array}{ccccc} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

ومنه

$x_4 = 3$

$x_3 - x_4 = -2 \qquad x_3 - 3 = -2 \rightarrow x_3 = 1$

$x_2 + x_3 - 2x_4 = -3$

$x_2 + 1 - 2(3) = -3 \longrightarrow x_2 = 2$

$x_1 + 2x_2 - x_3 = 2$

$x_1 + 2(2) - 1 = 2 \longrightarrow x_1 = -1$

ثم بالتعويض في المعادلات الاساسية

$x_2 + x_3 - 2x_4 = -3 \longrightarrow 2 + 1 - 2(3) = -3$ ✓

$x_1 + 2x_2 - x_3 = 2 \longrightarrow -1 + 2(2) - 1 = 2$ ✓

$2x_1 + 4x_2 + x_3 - 3x_4 = -2 \longrightarrow 2(-1) + 4(2) + (1) - 3(3) = -2$ ✓

$x_1 - 4x_2 - 7x_3 - x_4 = -19 \longrightarrow (-1) - 4(2) - 7(1) - (3) = -19$ ✓

ex/ Solve the system using Gaussian elimination

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 4 \\ x_1 \quad\quad + x_3 &= 6 \\ 2x_1 - 3x_2 + 5x_3 &= 4 \\ 3x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

sol

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 4 \\ 1^{1} & 0^{1} & 1^{-2} & 6^{-4} \\ 2^{2} & -3^{2} & 5^{-4} & 4^{-8} \\ 3^{3} & 2^{3} & -1^{-6} & 1^{-12} \end{array}\right] \qquad \begin{array}{l} -R_1 + R_2 \\ -2R_1 + R_3 \\ -3R_1 + R_4 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0^{0} & -1^{1} & 1^{-1} & -4^{2} \\ 0^{0} & 5^{-5} & -7^{5} & -11^{10} \end{array}\right] \qquad \begin{array}{l} R_2 + R_3 \\ -5R_2 + R_4 \end{array}$$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & -21 \end{array}\right] \qquad R_3 \longleftrightarrow R_4$$

$$\left[\begin{array}{cccc} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -2 & -21 \\ 0 & 0 & 0 & -2 \end{array}\right]$$

* مفيش حاجة في الدنيا اسمها (أصفار = -2)

يتعارض

* this means that system is inconsistent the system has no solution

Date Subject

$$3x_1 - 3x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + 4x_3 = 7$$
$$3x_1 - 5x_2 - x_3 = 7$$

sol

$$\begin{bmatrix} 3 & -3 & 3 & 9 \\ 2 & -1 & 4 & 7 \\ 3 & -5 & -1 & 7 \end{bmatrix} \quad \frac{R_1}{3}$$

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ 2^{-2} & -1^{2} & 4^{-2} & 7^{-6} \\ 3^{-3} & -5^{3} & -1^{-3} & 7^{-9} \end{bmatrix} \quad \begin{matrix} -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -2^{2} & -4^{4} & -2^{2} \end{bmatrix} \quad 2R_2 + R_3$$

$$\begin{bmatrix} 1 & -1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow$$

* the system has many solutions

* اذا كان الصف الاخير اصفار، معناه عدد لا نهائي من الحلول
* اذا كان الصف الاخير اصفار مع رقم في الاخر ← معناه لا يوجد حل

* هنا عدد المجاهيل (ثلاث مجاهيل) بينما المعادلات = (2)
فيتم حل المجاهيل بدلالة مجهول

$x_2 + 2x_3 = 1 \quad (x_2 = 1 - 2x_3)$

$x_1 - (1 - 2x_3) + x_3 = 3$

$x_1 - 1 + 2x_3 + x_3 = 3$

$x_1 + 3x_3 = 4 \quad (x_1 = 4 - 3x_3)$

let $x_3 = r$

$x_2 = 1 - 2r$

$x_1 = 4 - 3r$

* Gauss jordan

فيها يتم التعامل مع (Reduced Row echelon form Matrix)
بدلاً من (Row echelon form Matrix)

ex/ use Gauss-jordan Method to Solve

$$x - 2y + 3z = 9$$
$$-x + 3y \quad = -4$$
$$2x - 5y + 5z = 17$$

Sol

$$\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1^{1} & 3^{-2} & 0^{3} & -4^{9} \\ 2^{-2} & -5^{4} & 5^{-6} & 17^{-18} \end{bmatrix} \qquad \begin{matrix} -2R_1 + R_3 \\ R_1 + R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & -2^{2} & 3^{6} & 9^{10} \\ 0 & 1 & 3 & 5 \\ 0 & -1 & 1 & -1 \end{bmatrix} \qquad \begin{matrix} 2R_2 + R_1 \\ R_2 + R_3 \end{matrix}$$

$$\begin{matrix} \quad & 1 & 3 & 5 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 9 & 19 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{bmatrix} \qquad R_3/2$$

$$\begin{bmatrix} 1 & 0 & 9^{-9} & 19^{-18} \\ 0 & 1^{0} & 3^{-3} & 5^{-6} \\ 0 & 0 & 1 & 2 \end{bmatrix} \qquad \begin{matrix} -3R_3 + R_2 \\ -9R_3 + R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} \longrightarrow x = 1 \\ \longrightarrow y = -1 \\ \longrightarrow z = 2 \end{matrix}$$

ex2) use Gauss-jordan method to solve

$2x_1 + 4x_2 - 2x_3 = 0$
$3x_1 + 5x_2 \qquad = 1$

solve

$$\begin{bmatrix} 2 & 4 & -2 & 0 \\ 3 & 5 & 0 & 1 \end{bmatrix} \quad R_1/2$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 3_{-3} & 5_{-6} & 0_{3} & 1_{0} \end{bmatrix} \quad -3R_1 + R_2$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 3 & 1 \end{bmatrix} \quad \cancel{1/2 R_2 + R_1} \; -R_2$$

$$\begin{bmatrix} 1 & 2^{-2} & -1^{6} & 0^{2} \\ 0 & 1 & -3 & -1 \end{bmatrix} \quad -2R_2 + R_1$$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$x_1 + 5x_3 = 2 \Longrightarrow x_1 = 2 - 5x_3$
$x_2 - 3x_3 = -1 \Longrightarrow x_2 = -1 + 3x_3$

let $x_3 = t$ $\qquad x_1 = 2 - 5t$
$\qquad\qquad\quad x_2 = -1 + 3t$

* So this system has infinite number of solution.

Date Subject

* Homogeneous Linear system *

Definition: A system of linear equations is said to be homogeneous if the constants terms are all zeroed

$$a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n = 0$$
$$a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n = 0$$
$$\vdots$$
$$a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n = 0$$

---

every homogeneous linear system have the trivial solution as a solution

$$(X_1 = 0,\ X_2 = 0, \dots, X_n = 0)$$

---

ex) Solve the system

$$X_1 + 2X_2 = 0$$
$$2X_1 + X_2 = 0$$

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix} \\ \scriptstyle -2 \quad -4 \quad 0 \end{array} \xrightarrow{-2R_1+R_2} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 0 \end{bmatrix} \xrightarrow{R_2/-3} \begin{array}{c} \scriptstyle -2 \\ \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \xrightarrow{-2R_2+R_1}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{array}{l} \longrightarrow X_1 = 0 \\ \longrightarrow X_2 = 0 \end{array}$$

ex) Solve the system

$$
\begin{aligned}
2x_1 + 2x_2 - x_3 \quad\quad\quad + x_5 &= 0 \\
-x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0 \\
x_1 + x_2 - 2x_3 \quad\quad\quad - x_5 &= 0 \\
x_3 + x_4 + x_5 &= 0
\end{aligned}
$$

Solution

$$
\left[\begin{array}{cccccc}
2 & 2 & -1 & 0 & 1 & 0 \\
-1 & -1 & 2 & -3 & 1 & 0 \\
1 & 1 & -2 & 0 & -1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0
\end{array}\right]
\quad R_1 \leftrightarrow R_3
$$

$$
\left[\begin{array}{cccccc}
1 & 1 & -2 & 0 & -1 & 0 \\
-1 & -1 & 2 & -3 & 1 & 0 \\
2^{-2} & 2^{-2} & -1^{4} & 0 & 1^{2} & 0 \\
0 & 0 & 1 & 1 & 1 & 0
\end{array}\right]
\quad \begin{array}{l} R_1 + R_2 \\ -2R_1 + R_3 \end{array}
$$

$$
\left[\begin{array}{cccccc}
1 & 1 & -2 & 0 & -1 & 0 \\
0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 3 & 0 & 3 & 0 \\
0 & 0 & 1 & 1 & 1 & 0
\end{array}\right]
\quad R_2 \leftrightarrow R_4
$$

$$
\left[\begin{array}{cccccc}
1 & 1 & -2^{2} & 0^{2} & -1^{2} & 0 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 3\;{-3} & 0\;{-3} & 3\;{-3} & 0 \\
0 & 0 & 0 & -3 & 0 & 0
\end{array}\right]
\quad \begin{array}{l} 2R_2 + R_1 \\ -3R_2 + R_3 \end{array}
$$

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$$\begin{bmatrix} 1 & 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} R_3/-3 \\ R_4/-3 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 2^{-2} & 1 & 0 \\ 0 & 0 & 1 & 1^{-1} & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1^{-1} & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} -2R_3 + R_1 \\ -R_3 + R_2 \\ -R_3 + R_4 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + x_2 + x_5 = 0$$
$$x_3 + x_5 = 0$$
$$x_4 = 0 \longrightarrow x_4 = 0$$

* ويمكن الحصول على المجاهيل بدلالة مجهول اخر

* let $x_5 = r$ $x_2 = t$

$x_1 = -t - r$

$x_2 = t$

$x_3 = -r$

$x_4 = 0$

$x_5 = r$

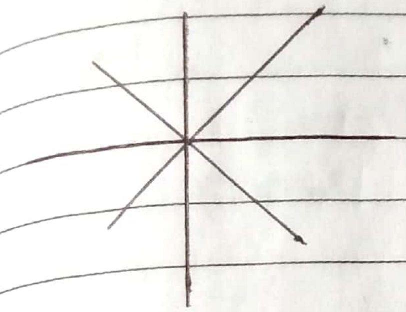
* عدد لا نهائي من الحلول

(this system has many solutions) والافضل

baraka

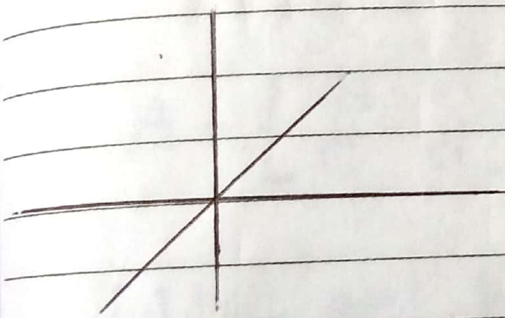
* theorem

A homogeneous system of linear equation is that has (more variables, than equations) has many solutions,
one of these solutions if the trivial ذات solution.



$$a_1 x + b_1 y = 0$$
$$a_2 x + b_2 y = 0$$

trivial solution التقاطع عند الصفر



many solution

among them the trivial solution
من بينها الحل التافهة

---

* Solve the system

$$x_1 + x_2 - x_4 = 0$$
$$x_1 + 2x_2 - x_3 + 2x_4 = 0$$
$$x_1 - 2x_2 + 2x_3 - 3x_4 = 0$$

* اذا كان عدد المجاهيل اكثر من عدد المعادلات = 0 نظام (homogeneous)
اذن الناتج (Many solutions)

Date L4 Subject 26/8/2017

Determinants المحددات

فقط (square Metrices) المحددات لا يتم حسابها الا لل ★
2X2 , 3X3 , 4X4, →

★ عندما نحسب (A^{-1})

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^{-1} = \frac{1}{(ad-cb)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(محدد)

(1) 2X2 Matrix

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A| = ad - bc$
or $\det(A)$

ex) Find $|A|$ where $A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$

$|A| = 2 \times 2 - (-3)(1) = 4 + 3 = 7$ → موجب

Date Subject

$B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$|B| = 2 \times 2 - (4)(1) = 4 - 4 = 0 \longrightarrow$ صفر

تعني (singuler Matrix)
(has no inverse)

$C = \begin{bmatrix} 0 & 3 \\ 2 & 4 \end{bmatrix}$

$|C| = 0 \times 4 - (2)(3) = 0 - 6 = -6 \longrightarrow$ سالب

---

(2) For 3x3 Matrix (only)

$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

* يتم اعادة اول عمودين من الاحمر

$$A = \begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}$$

الخطوط السوداء.

$$|A| = (a_{11} \times a_{22} \times a_{33}) + (a_{12} \times a_{23} \times a_{31}) + (a_{13} \times a_{21} \times a_{32}) - (a_{13} \times a_{22} \times a_{31}) - (a_{11} \times a_{23} \times a_{32}) - (a_{12} \times a_{21} \times a_{33})$$

الخطوط الزرقاء.

ex1) Find det for $A=\begin{bmatrix}0&2&1\\3&-1&2\\4&-4&1\end{bmatrix}_{3\times3}$

$$|A|=\begin{matrix}0&2&1&0&2\\3&-1&2&3&-1\\4&-4&1&4&-4\end{matrix}$$

$$|A|=(0\times-1\times1)+(2\times2\times4)+(1\times3\times-4)-(2\times3\times1)-(0\times2\times-4)-(1\times-1\times4)$$

$$|A|=0+16-12-6-0+4=2$$

---

ex2) $A=\begin{bmatrix}7&-8&7\\-4&5&0\\-6&7&-5\end{bmatrix}_{3\times3}$

$$|A|=\begin{matrix}7&-8&7&7&-8\\-4&5&0&-4&5\\-6&7&-5&-6&7\end{matrix}$$

$$|A|=(7\times5\times-5)+(-8\times0\times-6)+(7\times-4\times7)-(-8\times-4\times-5)-(7\times0\times7)-(7\times5\times-6)$$

$$|A|=-175+0-196+160-0+210=-1$$

Date Subject

* General Method

* بطلع اصب محدد (4X4) اصب (3X3) بعد النزول درجة

* General Method from 2X2 and more
- Method of minors and Cofactors

definition: if A is a square Matrix, then the minor M_{ij} of the element a_{ij} is the Determinant of the matrix, obtained by deleting the i^{th} row & j^{th} Column of A the Cofactor

C_{ij} is given by $C_{ij} = (-1)^{i+j} M_{ij}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \cancel{a_{21}} & \cancel{a_{22}} & \cancel{a_{23}} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

* بعد حذف صفها وعمودها

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} \qquad M_{21} = (a_{12} \times a_{33}) - (a_{13} \times a_{32})$$

$$C_{21} = (-1)^{2+1} M_{21} = (-1)^3 M_{21} = -M_{21}$$

* وتفس هذه الخطوة يمكن الوصول اليها بالتعامل مع الاشارات
البداية تكون بالموجب ثم سالب وهكذا .
وبداية الصف الثاني تكون بالسالب ثم موجب وهكذا .

$$\begin{matrix} \overset{+}{M_{11}} & \overset{-}{M_{12}} & \overset{+}{M_{13}} \\ - & + & - \\ + & - & + \end{matrix}$$

baraka

Ex) Find all the minors & Cofactors of

$$A = \begin{bmatrix} 0^{+} & 2^{-} & 1^{+} \\ 3^{-} & -1^{+} & 2^{-} \\ 4^{+} & 0^{-} & 1^{+} \end{bmatrix}$$

هو محدد المصفوفة المتبقية بعد حذف الصف الأول، والعمود الأول (M_{11})

$M_{11} = \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = (-1)(1) - (2)(0) = -1 - 0 = -1$

$C_{11} = +(-1) = -1$

$\left((-1)^{1+1} \; M_{11} = (+1)(-1) = -1 \right) \leftarrow$ توضيح

$M_{12} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = (3)(1) - (2)(4) = 3 - 8 = -5$

$C_{12} = -(-5) = 5$

$M_{13} = \begin{vmatrix} 3 & -1 \\ 4 & 0 \end{vmatrix} = (3)(0) - (-1)(4) = 0 + 4 = 4$

$C_{13} = +(4) = 4$

---

* Find the det(A) using first row

$$A = \begin{bmatrix} 0^{+} & 2^{-} & 1^{+} \\ 3^{-} & -1^{+} & 2^{-} \\ 4^{+} & 0^{-} & 1^{+} \end{bmatrix}$$

* اذا لم يحدد، فإننا نختار الصف او العمود الذي يحتوي على اكبر عدد من الأص

$$\det(A) = 0\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 4 & 0 \end{vmatrix}$$

$$= 0(-1) - 2(-5) + 1(4) = 14$$

ex2) Find the det(A)
$$A = \begin{bmatrix} 4 & 2 & 1 \\ -2 & -6 & 3 \\ -7 & 5 & 0 \end{bmatrix}$$

using 3rd row

$$\det(A) = -7\begin{vmatrix} 2 & 1 \\ -6 & 3 \end{vmatrix} - 5\begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} + 0\begin{vmatrix} 4 & 2 \\ -2 & -6 \end{vmatrix}$$

$$= -7(12) - 5(14) + 0$$
$$= -84 - 70 = -154$$

Date Subject

Ex/ Find the det(A) using Cofactors Method

$$A = \begin{bmatrix} 5^{+} & -2^{-} & 2^{+} & 7^{-} \\ 1^{-} & 0^{+} & 0^{-} & 3^{+} \\ -3^{+} & 1^{-} & 5^{+} & 0^{-} \\ 3^{-} & -1^{+} & -9^{-} & 4^{+} \end{bmatrix}$$

using 2nd row

$|A| = -|B| + 0 - 0 + 3|C|$

$$= -1 \begin{vmatrix} -2 & 2 & 7 \\ 1 & 5 & 0 \\ -1 & -9 & 4 \end{vmatrix} + 3 \begin{vmatrix} 5 & -2 & 2 \\ -3 & 1 & 5 \\ 3 & -1 & -9 \end{vmatrix}$$

$$|B| = -1 \begin{vmatrix} 2 & 7 \\ -9 & 4 \end{vmatrix} + 5 \begin{vmatrix} -2 & 7 \\ -1 & 4 \end{vmatrix}$$

$= -1(8+63) + 5(-8+7) = -71 - 5 = -76$

$$|C| = \begin{matrix} 5 & -2 & 2 & 5 & -2 \\ -3 & 1 & 5 & -3 & 1 \\ 3 & -1 & -9 & 3 & -1 \end{matrix}$$

$= (5\times1\times-9) + (-2\times5\times3) + (2\times-3\times-1)$
$-(-2\times-3\times-9) - (5\times5\times-1) - (2\times1\times3)$
$= -45 - 30 + 6 + 54 + 25 - 6 = 4$

$|A| = -|B| + 3|C| = -(-76) + 3(4) = 76 + 12 = 88$

baraka

theorem: if A is a triangular matrix of order then its determinant is the product of the entries on the main diagonal that is

$|A| = a_{11}\, a_{22}\, a_{33} \dots a_{nn}$

(Lower ، upper) إذا كانت المصفوفة

ex1)

$$A = \begin{bmatrix} 4 & 1 & 3 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

since A is upper triangular Matrix ويجب كتابة هذه الجملة

then $|A| = a_{11}\, a_{22}\, a_{33}\, a_{44}$

$|A| = 4 \times 1 \times 1 \times 3 = 12$

ex2)

$$A = \begin{bmatrix} -3 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 5 & 0 & 0 \\ 0 & 1 & 2 & 3 & 0 \\ -2 & 1 & 4 & 3 & 1 \end{bmatrix}$$

since A is lower triangular Matrix

then $|A| = -3 \times 1 \times 5 \times 3 \times 1 = -45$

ex3)

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

since C is a diagonal Matrix - then

$|C| = 1 \times 2 \times 1 \times 3 = 6$

Date Subject

* properties of det

*خواص المحددات -

1) $\det(AB) = \det(A) \cdot \det(B)$

* if A & B are matrices C is Constant

2) $|CA| = C^n |A|$

3) $|A^T| = |A|$

4) if $|A| \neq 0$, then A is invertible

A^{-1} exist موجودة

(non singular)

5) $|A^{-1}| = \dfrac{1}{|A|}$

---

ex/

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ $AB = \begin{bmatrix} 4 & 7 \\ 10 & 15 \end{bmatrix}$

Find $|A|, |B|, |AB|, |A^{-1}|, |A^T|$

$|A| = 4 - 6 = -2$ $|B| = 6 - 1 = 5$

$|AB| = -2 \times 5 = -10$ $|AB| = (4 \times 15) - (7 \times 10) = -10$

$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $|A^T| = 4 - 6 = -2$ $|A| = |A^T|$

$|A^{-1}| = \dfrac{1}{-2}$

الإثبات في الصفحة التالية

Date Subject

$$A^{-1} = \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \quad \underline{-3R_1+R_2}$$

$$= \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array}\right] \quad \underline{-R_2/2}$$

$$= \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right] \quad \underline{-2R_2+R_1}$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$|A^{-1}| = (-2)(-1/2) - (1)(3/2) = 1 - \frac{3}{2} = -\frac{1}{2}$$

$$|A^{-1}| = \frac{1}{|A|} = -1/2$$

---

ex) Find $|A|$

$$A = \begin{bmatrix} 20 & 30 & 50 \\ -20 & -10 & 40 \\ -10 & 50 & 20 \end{bmatrix}$$

$$A = 10 \begin{bmatrix} 2 & 3 & 5 \\ -2 & -1 & 4 \\ -1 & 5 & 2 \end{bmatrix}_{3\times3} = (10)^3 \begin{vmatrix} 2 & 3 & 5 \\ -2 & -1 & 4 \\ -1 & 5 & 2 \end{vmatrix}$$

$$= (10)^3(-99) = -99000$$

baraka

Date Subject

* the effect of elementary row operations on det

* if A, B are square Matrices. *

1) if A, B are square matrices, & if (B) is obtained from (A) by interchanging by 2 (Rows or Columns) of (A) then det (B) = _ det (A)

ex)

$$|A| = \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} \qquad |A| = 2\times 4 - (-3)(1) = 11$$

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & -3 \end{vmatrix} \qquad |B| = (1)(-3) - (2)(4) = -11$$

عند تبديل صفين او عمودين للمحدد ، تختلف الإشارة .

---

2) if (B) is obtained from (A) by adding a multipled row to another then det (B) = det (A)

ex)

$$|A| = \begin{vmatrix} 1 & -3 \\ 2 & -4 \end{vmatrix} \qquad |A| = (1)(-4) - (-3)(2) = 2$$

$$\xrightarrow{-2R_1+R_2} \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} \qquad |A| = (1)(2) - (-3)(0) = 2$$

3) if (B) is obtained from (A) by Multiplaying a row of A by a non zero Constant (C)

then det (B) = C det (A)

ex/

$$|A| = \begin{vmatrix} 2 & -8 \\ -2 & 9 \end{vmatrix} \qquad |A| = (2)(9) - (-8)(-2) = 2$$

$$\xrightarrow{\frac{1}{2}R_1} \begin{vmatrix} 1 & -4 \\ -2 & 9 \end{vmatrix} \qquad |B| = (1)(9) - (-4)(-2) = 1$$

$$|B| = \tfrac{1}{2}|A|$$

---

theorem: if (A) is a square Matrix, and any one of the following Conditions is (true) then (det(A) = 0)

1) an entries row or Column Consists of Zero

$$|A| = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 0 & 4 \\ -3 & 0 & 2 \end{vmatrix} = 0$$

العمود الثاني كله اصفار

2) Two rows or Columns are equal

$$|A| = \begin{vmatrix} 2 & 1 & 3 \\ 2 & 1 & 3 \\ 1 & 5 & 2 \end{vmatrix} = 0$$

(الصف الاول مثل الصف الثاني (بالضبط

3) one row or Column is a multipled of another

$$|A| = \begin{vmatrix} 2 & 0 & 4 \\ 1 & 2 & 2 \\ 3 & 0 & 6 \end{vmatrix} = 0$$

Date Subject

* Cramer's Rule

$$A x = b$$

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = b_1$$
$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = b_2$$
$$a_{31} X_1 + a_{32} X_2 + a_{33} X_3 = b_3$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$X_1 = \dfrac{\det A_1}{\det A}$ $\quad X_2 = \dfrac{\det A_2}{\det A}$ $\quad X_3 = \dfrac{\det A_3}{\det A}$

A_i = is the matrix obtained by replacing entries of the j^{th} column of (A) by entries of the Constants matrix

---

ex/ Use Cramer's rule to solve this system of linear equations

$$X_1 + 2X_3 = 6$$
$$-3X_1 + 4X_2 + 6X_3 = 30$$
$$-X_1 - 2X_2 + 3X_3 = 8$$

Date Subject

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix} \quad |A| = 44$$

$$A_1 = \begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix} \quad |A_1| = -40$$

$$A_2 = \begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix} \quad |A_2| = 72$$

$$A_3 = \begin{bmatrix} 1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix} \quad |A_3| = 152$$

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-40}{44}$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{72}{44}$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{152}{44}$$

Date 15

Theorem, if (A) is an invertible Matrix, then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$ للحصول على المعكوس بطريقة المحددات

ex/ find A^{-1} $A = \begin{bmatrix} a^{+} & b^{-} \\ c^{-} & d^{+} \end{bmatrix}$

$$\operatorname{Cof}(A) = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Date Subject

ex/ Find A^{-1} using adj M for

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

sol/

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} -2 & 1 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} & +\begin{vmatrix} 0 & -2 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 3 & 2 \\ 0 & -2 \end{vmatrix} & +\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} -1 & 3 \\ 1 & 0 \end{vmatrix} \\ +\begin{vmatrix} 3 & 2 \\ -2 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} \end{bmatrix}^T$$

$$\operatorname{adj}(A) = \begin{bmatrix} 4 & 1 & 2 \\ 6 & 0 & 3 \\ 7 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & 6 & 7 \\ 1 & 0 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

Date Subject

$|A|$ Using 2nd row

$$|A| = -2\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} -1 & 3 \\ 1 & 0 \end{vmatrix}$$

$$|A| = -2(0) - 1(-3) = 0 + 3 = 3$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 4 & 6 & 7 \\ 1 & 0 & 1 \\ 2 & 3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4/3 & 2 & 7/3 \\ 1/3 & 0 & 1/3 \\ 2/3 & 1 & 2/3 \end{bmatrix}$$

Date Subject

ex) Find A^{-1} using adj M for

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} +\begin{vmatrix} 6 & 3 \\ -4 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 2 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 6 \\ 2 & -4 \end{vmatrix} \\ -\begin{vmatrix} 2 & -1 \\ -4 & 0 \end{vmatrix} & +\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 2 & -4 \end{vmatrix} \\ +\begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} & -\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 1 & 6 \end{vmatrix} \end{bmatrix}^T$$

$$\text{adj}(A) = \begin{bmatrix} 12 & 6 & -16 \\ 4 & 2 & 16 \\ 12 & -10 & 16 \end{bmatrix}^T = \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$$

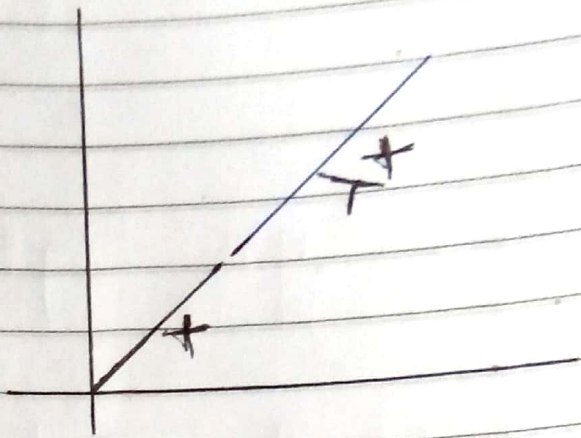
$|A|$ using 3rd row

$$|A| = 2\begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} + 4\begin{vmatrix} 3 & -1 \\ 1 & 3 \end{vmatrix} = 24 + 40 = 64$$

$$A^{-1} = \frac{1}{64}\begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix} = \begin{bmatrix} 12/64 & 1/16 & 12/64 \\ 6/64 & 1/32 & -10/64 \\ -1/4 & 1/4 & 1/4 \end{bmatrix}$$

baraka

* eigen Values & eigen vectors



$A \quad X$

$AX = \lambda X$

لها

eigen value
and the Corresponding
eigen vectors

$AX = \lambda X$

$AX = \lambda I X$ مصفوفة الوحدة (I)

$\lambda I X - AX = 0$

$(\lambda I - A) X = 0$

$\det(\lambda I - A) = 0 \longrightarrow$ characteristic equation

---

steps

1) _ Ch. equation
2) _ Find roots (λ)
3) _ Find eigen vectors

Date Subject

ex) Find All eigen Values & eigen vectors for $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$

sol

step 1 Ch. equation

$$|\lambda I - A| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - 1 & 0 - 1 \\ 0 + 2 & \lambda - 4 \end{vmatrix} = 0$$

$$(\lambda - 1)(\lambda - 4) - ((-1)(2)) = 0$$

$$\lambda^2 - 4\lambda - \lambda + 4 + 2 = 0$$

$$\lambda^2 - 5\lambda + 6 = 0$$

step 2 Find roots (λ)

$$(\lambda - 2)(\lambda - 3) = 0$$

$$\lambda - 2 = 0 \longrightarrow \lambda_1 = 2$$

$$\lambda - 3 = 0 \longrightarrow \lambda_2 = 3$$

step 3 , eigen Vector

for $\lambda_1 = 2$

$$(\lambda I - A) X = 0$$

$$\left[\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ \underset{-2}{2} & \underset{2}{-2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2R_1 + R_2$. ثم يتم تحويلها لابسط صورة

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 = 0 \longrightarrow x_1 = x_2$$

let $x_1 = t$ then $x_2 = t$

$$V_1 = \begin{bmatrix} t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

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for $\lambda_2 = 3$

$$(\lambda_2 I - A)X = 0$$

$$\left[\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-R_1 + R_2$

$$\begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 - x_2 = 0 \qquad 2x_1 = x_2$

let $x_1 = t$ then $x_2 = 2t$

$$V_2 = \begin{bmatrix} t \\ 2t \end{bmatrix} = t\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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ex2) Find All eigen values & eigen vectors

For $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

sol

step 1 ch. equation $|\lambda I - A| = 0$

$$\left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda - 2 & -1 & 0 \\ 0 & \lambda - 2 & 0 \\ 0 & 0 & \lambda - 2 \end{vmatrix} = 0$$

step 2

كونها (upper triangle) فسيكون الناتج حاصل ضرب عناصر القطر الرئيسي

$$(\lambda_1 - 2)(\lambda_2 - 2)(\lambda_3 - 2) = 0$$

$$\lambda - 2 = 0$$

$$\lambda_{1,2,3} = 2$$

step 3 : eigen vector $(\lambda I - A) X = 0$

$$\left[\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_2 = 0$

baraka

let $x_1 = t$, $x_3 = s$

$$V = \begin{bmatrix} t \\ 0 \\ s \end{bmatrix} = \begin{bmatrix} 1t + 0s \\ 0t + 0s \\ 0t + 1s \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ s \end{bmatrix}$$

$$= t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

---

theorem: if (A) is an $n \times n$ Matrix (upper triangle, lower triangle, diagonal) then the given values of (A) are the entries on the main diagonal

ex)
$$A = \begin{bmatrix} 1/2 & 0 & 0 \\ -1 & 2/3 & 0 \\ 5 & -8 & -1/4 \end{bmatrix}$$

$\lambda_1 = 1/2$ $\lambda_2 = 2/3$ $\lambda_3 = -1/4$

---

ex3) Find All eigen values & eigen vectors for $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

sol

step 1. Ch. equation

$$|\lambda I - A| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda-1 & 1 & 0 \\ 1 & \lambda-2 & 1 \\ 0 & 1 & \lambda-1 \end{vmatrix} = 0$$

Using 1st Row

$$(\lambda-1)\begin{vmatrix} \lambda-2 & 1 \\ 1 & \lambda-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 0 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-1)\big((\lambda-2)(\lambda-1)-1\big) - 1(\lambda-1-0) = 0$$

$$(\lambda-1)\big((\lambda-2)(\lambda-1)-1-1\big) = 0$$

$$(\lambda-1)\big((\lambda-2)(\lambda-1)-2\big) = 0$$

$$(\lambda-1)(\lambda^2-\lambda-2\lambda+2-2) = 0$$

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$(\lambda - 1)(\lambda^2 - 3\lambda) = 0$

$\lambda(\lambda - 1)(\lambda - 3) = 0$

$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 3$

step (3) find eigen vectors

for $\lambda_1 = 0$ $\quad (\lambda_1 I - A)X = 0$

$$\left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

use Row echelon form

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{2R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - x_3 = 0 \longrightarrow x_1 = x_3$

$x_2 - x_3 = 0 \longrightarrow x_2 = x_3$

let $x_3 = t \longrightarrow x_1 = t \;,\; x_2 = t$

$$V_1 = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

---

for $\lambda_2 = 1$ $(\lambda I - A)X = 0$ $(1 I - A)X = 0$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \xrightarrow[-R_2 + R_3]{R_2 + R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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$x_1 + x_3 = 0 \longrightarrow x_1 = -x_3$

$x_2 = 0$

let $x_3 = t \rightarrow x_1 = -t \quad , \quad x_2 = 0$

$$V_2 = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

---

for $\lambda_3 = 3 \qquad (\lambda I - A)X = 0 \qquad (3I - A)X = 0$

$$\left[\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

use Row echelon form

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 1 \\ 2_{-2} & 1_{-2} & 0_{-2} \\ 0 & 1 & 2 \end{bmatrix} \xrightarrow{-2R_1 + R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow[R_2 + R_3]{-R_2 + R_1} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - x_3 = 0 \longrightarrow x_1 = x_3$

$x_2 + 2x_3 = 0 \longrightarrow x_2 = -2x_3$

let $x_3 = t$, $x_1 = t$, $x_2 = -2t$

$$V_3 = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

---

ex4)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

sol

step 1 Ch-equation $|\lambda I - A| = 0$

$$\left| \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} \lambda-1 & 0 & 0 & 0 \\ 0 & \lambda-1 & -5 & 10 \\ -1 & 0 & \lambda-2 & 0 \\ -1 & 0 & 0 & \lambda-3 \end{vmatrix} = 0$$

Using 1st Row

$$(\lambda-1)\begin{vmatrix} \lambda-1 & -5 & 10 \\ 0 & \lambda-2 & 0 \\ 0 & 0 & \lambda-3 \end{vmatrix} = 0$$

$$(\lambda-1)\left[(\lambda-3)\begin{vmatrix} \lambda-1 & -5 \\ 0 & \lambda-2 \end{vmatrix}\right] = 0$$

$$(\lambda-1)\Big((\lambda-3)(\lambda-1)(\lambda-2)\Big) = 0$$

$\lambda-1=0 \longrightarrow \lambda_1 = 1$

$\lambda-3=0 \longrightarrow \lambda_2 = 3$

$\lambda-1=0 \longrightarrow \lambda_3 = 1$

$\lambda-2=0 \longrightarrow \lambda_4 = 2$

$\lambda_1, \lambda_3 = 1$ $\lambda_2 = 3$ $\lambda_4 = 2$

step 3. For $\lambda = 1$ $(1I - A)X = 0$

$$\left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}\right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -5 & 10 \\ -1 & 0 & -1 & 0 \\ -1 & 0 & 0 & -2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_4} \begin{bmatrix} -1 & 0 & 0 & -2 \\ 0 & 0 & -5 & 10 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-R_1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -5 & 10 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 + R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & -5 & 10 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{-\frac{R_2}{5}}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_4 = 0 \longrightarrow x_1 = -2x_4$

$x_3 - 2x_4 = 0 \longrightarrow x_3 = 2x_4$

let $x_4 = t \Rightarrow x_1 = -2t$, $x_3 = 2t$, let $x_2 = s$

$$V = \begin{bmatrix} -2t \\ s \\ 2t \\ t \end{bmatrix} = \begin{bmatrix} -2t + 0s \\ 0t + 1s \\ 2t + 0s \\ t + 0s \end{bmatrix} = t \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

تم استكمال الحل بنفس الخطوات

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ex5) $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}$

sol

Step 1: ch-equation $|\lambda I - A| = 0$

(λ - عناصر القطر الرئيسي
تغيير الاشارة للباقي)

$$\begin{vmatrix} \lambda-1 & -2 & 1 \\ -1 & \lambda & -1 \\ -4 & 4 & \lambda-5 \end{vmatrix} = 0$$

Using 1st Row

$$(\lambda-1)\begin{vmatrix} \lambda & -1 \\ 4 & \lambda-5 \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ -4 & \lambda-5 \end{vmatrix} + 1\begin{vmatrix} -1 & \lambda \\ -4 & 4 \end{vmatrix} = 0$$

$$(\lambda-1)(\lambda(\lambda-5)+4) + 2(-1(\lambda-5)-4) + (-4+4\lambda) = 0$$

$$(\lambda-1)(\lambda^2-5\lambda+4) + 2(-\lambda+1) + 4(\lambda-1) = 0$$

$$\lambda^3-5\lambda^2+4\lambda-\lambda^2+5\lambda-4+2-2\lambda-4+4\lambda = 0$$

$$\lambda^3-6\lambda^2+11\lambda-6 = 0$$

الحلول الممكنة حسب

$\pm 1, \pm 6, \pm 2, \pm 3$

بتجربة $(+1)$

$(1)^3 - 6(1)^2 + 11(1) - 6 = 0$ ✓

اذن يمكن اخذ عامل مشترك $(\lambda-1)$

لان ($\lambda - 1 = 0 \xrightarrow{\text{تعطي}} \lambda = 1$)

وقد تم التأكد من صحة الحل عند التعويض ب (1)

baraka

باستخدام القسمة المطولة

$$
\begin{array}{r|l}
& \lambda^2 - 5\lambda + 6 \\
\hline
\lambda - 1 & \lambda^3 - 6\lambda^2 + 11\lambda - 6 \\
& \lambda^3 - \lambda^2 \\
\hline
& 0 - 5\lambda^2 + 11\lambda - 6 \\
& -5\lambda^2 + 5\lambda \\
\hline
& 0 + 6\lambda - 6 \\
& 6\lambda - 6 \\
\hline
& 0 \quad 0
\end{array}
$$

$$(\lambda - 1)(\lambda^2 - 5\lambda + 6) = 0$$

$$(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

$\lambda_1 = 1$ $\lambda_2 = 2$ $\lambda_3 = 3$

ويمكن استخدام القانون

$$ax^2 + bx + c = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for $\lambda = 1$ $(1I - A)X = 0$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 0 & -2 & 1 \\ -1 & 1 & -1 \\ -4 & 4 & -4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -1 & 1 & -1 \\ 0 & -2 & 1 \\ -4 & 4 & -4 \end{bmatrix} \xrightarrow{-R_1}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -4 & 4 & -4 \end{bmatrix} \xrightarrow{4R_1 + R_3} \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2/-2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 + R_2} \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + 1/2\, x_3 = 0 \longrightarrow x_1 = -1/2\, x_3$$

$$x_2 - 1/2\, x_3 = 0 \longrightarrow x_2 = 1/2\, x_3$$

let $x_3 = t \longrightarrow x_1 = -1/2\, t \,,\ x_2 = 1/2\, t \,,\ x_3 = t$

$$V_1 = \begin{bmatrix} -1/2\, t \\ 1/2\, t \\ t \end{bmatrix} = t \begin{bmatrix} -1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

(استكمال الحل)

for $\lambda_2 = 2$

$$V_2 = t \begin{bmatrix} -1/2 \\ 1/4 \\ 1 \end{bmatrix}$$

for $\lambda_3 = 3$

$$V_3 = t \begin{bmatrix} -1/4 \\ 1/4 \\ 1 \end{bmatrix}$$

---

ex6)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$$

(استكمال الحل)

$\lambda_1 = 4$

$\lambda_2 = 2 + \sqrt{3}$

$\lambda_3 = 2 - \sqrt{3}$

* لحل مسائل على المنهج - من كتاب

elementary linear algebra 6th

Larson, Edwards, Falvo

1) section 1.1, P11 1-16, 37-56, 65-68, 77-86
2) section 1.2, P26 1-36, 43-48, 49-50
3) section 2.1, P56 1-40, 43-46, 49-52
4) section 2.2, P70 1-14, 19-28, 31-34, 39-40
5) section 2.3, P84 1-28, 33-42
6) section 3.1, P130 1-34, 41-46, 49-54, 55, 58

P149 1-18, 27-44, 45, 48

تم بحمد الله وفضله

م/ إسلام سليم

اللهم وفقنا لما تحب وترضى