MS506 1/3/2020

# Transpose of Matrix & symmetric Matrix

Lecture 2

$A = A^T$

ex. $A = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$, $A^t = \begin{bmatrix} 2 & 8 \end{bmatrix}$

ex. $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 0 & 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 0 \\ 3 & 6 & 2 \end{bmatrix}$

if A is square Matrix & $A = A^T$ then A is Called "symmetric Matrix"

ex. if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 5 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 5 \end{bmatrix}$

symmetric Matrix

- properties of Transpose:-

1. $(A+B)^T = A^T + B^T$

2. $(AB)^T = B^T \cdot A^T$

3. $(cA)^T = cA^T$

4. $(A^T)^T = A$

5. $(A+B+C)^T = A^T + B^T + C^T$

6. $(A \cdot B \cdot C)^T = C^T + B^T + A^T$

ex. show that $(AB)^T = B^T A^T$ are equal if

$$A = \begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}$$

$$AB = \underset{3\times3}{\begin{bmatrix} 2 & 1 & -2 \\ -1 & 0 & 3 \\ 0 & -2 & 1 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 3 & 1 \\ 2 & -1 \\ 3 & 0 \end{bmatrix}}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 6 & -1 \\ -1 & 2 \end{bmatrix}, \quad (AB)^T = \begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 3 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \quad A^T = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -2 \\ -2 & 3 & 1 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 2 & 6 & -1 \\ 1 & -1 & 2 \end{bmatrix} \qquad (AB)^T = B^T A^T$$

## Trace of Matrix

Let $A$ be square Matrix. the [trace] of $A$ denoted by $tr(A)$ is the sum of the diagonal elements of $A$

$$tr(A) = a_{11} + a_{22} + a_{33} + \cdots + a_{nn}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 1 & 0 & 2 \end{bmatrix} \qquad tr(A) = 1 + 0 + 2 = 3$$

## - properties of trace :-

1. $tr\,(A+B) = tr\,(A) + tr\,(B)$

2. $tr\,(AB) = tr\,(BA)$

3. $tr\,(\overset{k}{c}A) = c\,tr(A)$

4. $tr\,(A^T)^{\text{or}\,T} = tr\,(A)$

- stochastic Matrix :- a Matrix that all its amatrix are nonnegative and the sum of each row is 1

$$\begin{bmatrix} 0.3 & 0.3 & 0.4 & 0 \\ 0.4 & 0.1 & 0.3 & 0.2 \\ 0.25 & 0.25 & 0 & 0.5 \end{bmatrix}$$

- Diagonal Matrix :- is a square Matrix which all non negative entries are zeros ex.

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D^K = \begin{bmatrix} d_1^K & 0 & 0 & 0 \\ 0 & d_2^K & 0 & 0 \\ 0 & 0 & d_3^K & 0 \\ 0 & 0 & 0 & d_4^K \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad A^5 = \begin{bmatrix} (1)^5 & 0 & 0 \\ 0 & (2)^5 & 0 \\ 0 & 0 & (3)^5 \end{bmatrix}$$

- upper triangular Martix :- is square Matrix that has only zeros below the diagonal

ex.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

* Lower triangluar matrix is a square matrix has only zeros above the diagonal

ex:

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 3 & 1 & 3 \end{bmatrix}$$

- Inverse of a Matrix:-

if A is a square matrix, and if B is a matrix of the same size can be found such that $AB = BA = I$, then A is said to be invertible and B is called its inverse. if no such matrix B can be found then A is said to be singular. (has no Inverse).

ex:

$$B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$$

is B the Inverse of A?!

$$AB = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Finding the Inverse of a matrix

1. 2x2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $A^{-1} = \frac{1}{ad - cb} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ex.

$A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$ find $A^{-1}$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$$

2. For any Matrix

steps

1. adjoint the Identity matrix to original Matrix $[A \vdots I]$ From the same size

2. apply row operators to this matrix until the left side reduced to $I \rightarrow [I \vdots A^{-1}]$

- row operations :-

1. multiply row, by non zero Costant

2. interchanging two rows

3. add amultiple of arow to another

ex. Find $A^{-1}$ for $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

leading entry $\rightarrow$ $\left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] \xrightarrow{-R_1+R_2} \left[\begin{array}{cc:cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$

$-R_2+R_1$

$\left[\begin{array}{cc:cc} 1 & 0 & 3 & -2 \\ 0 & 1 & -1 & 1 \end{array}\right]$

$A^{-1}$

ex.//  Find $A^{-1}$

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$$

$$A=\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 2 & 5 & 3 & 0 & 1 & 0\\ 1 & 0 & 8 & 0 & 0 & 1\end{array}\right]\quad \begin{array}{l}-2R_1+R_2\\ -R_1+R_3\end{array}$$

$$\left[\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & -3 & -2 & 1 & 0\\ 0 & -2 & 5 & -1 & 0 & 1\end{array}\right]\quad \begin{array}{l}-2R_2+R_1\\ 2R_2+R_3\end{array}$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 9 & 5 & -2 & 0\\ 0 & 1 & -3 & -2 & 1 & 0\\ 0 & 0 & -1 & -5 & 2 & 1\end{array}\right]\quad -R_3$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & 9 & 5 & -2 & 0\\ 0 & 1 & -3 & -2 & 1 & 0\\ 0 & 0 & 1 & 5 & -2 & -1\end{array}\right]\quad \begin{array}{l}-9R_3+R_1\\ 3R_3+R_2\end{array}$$

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$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$A^{-1}$

ex.

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix}$$

$$A = \left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 2^{-2} & 4^{-12} & -1^{-8} & 0^{-2} & 1^{0} & 0^{0} \\ -1^{1} & 2^{6} & 5^{4} & 0^{1} & 0^{0} & 1^{0} \end{array}\right] \begin{array}{l} -2R_1+R_2 \\ R_1+R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & -8 & -9 & -2 & 1 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right] \quad \frac{R_2}{-8}$$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{9}{8} & \frac{2}{8} & \frac{-1}{8} & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right] \quad -8R_2+R_3$$

$$\begin{array}{cccccc} 0 & -8 & -9 & -2 & 1 & 0 \end{array}$$

- there is no inverse for A

$$\left[\begin{array}{ccc|ccc} 0 & 1 & \frac{9}{8} & \frac{2}{8} & \frac{-1}{8} & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{array}\right]$$

which means that A is singular



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- properties of Inverse :-

1. $(A^{-1})^{-1} = A$

2. $(A^{K})^{-1} = (A^{-1})^{K}$

3. $(CA)^{-1} = \frac{1}{c} A^{-1}$

4. $(A^{T})^{-1} = (A^{-1})^{T}$

5. $(AB)^{-1} = B^{-1} A^{-1}$

- the diagonal matrix is :- invertable iff all of its diagonal entries are non zero

$$D = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & & d_n \\ 0 & 0 & & \end{bmatrix} \quad D^{-1} = \begin{bmatrix} \frac{1}{d_1} & 0 & 0 \\ 0 & \frac{1}{d_2} & 0 \\ 0 & 0 & \frac{1}{d_n} \end{bmatrix}$$