MS506

8/3/2020

lecture 3

- system of linear equations :-

- $x + y + xy = 0$ → non linear equation due to any 2 unknown it's mean like unknown power 2

- $3x = 1$ → this linear equation

1. $3x + 2y = 7$ → linear
2. $\frac{1}{2}x + y - \pi 2 = \sqrt{2}$ → linear
3. $xy + 2 = 2$ → non
4. $-(\sin\frac{\pi}{2})x_1 - 4x_2 = e^2$ → linear
5. $e^x - 2y = 4$ → non
6. $1/x + 1/y = 4$ → linear

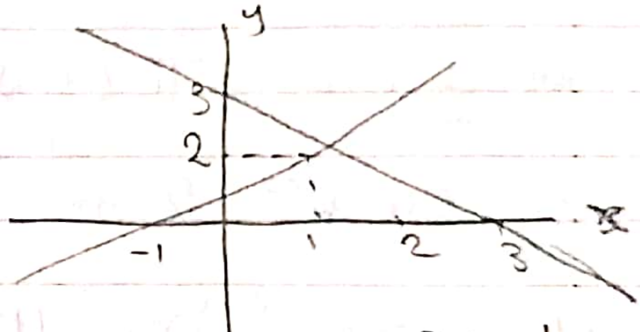
- General Form of Linear equations:-

$$
\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n = b_2 \\
&\vdots \\
&a_{m1}x_1 + a_{m2}x_2 + \cdots\cdots + a_{mn}x_n = b_m
\end{aligned}
$$

ex.

$x + y = 3 \rightarrow y = 3 - x$

$x - y = -1 \rightarrow y = x + 1$



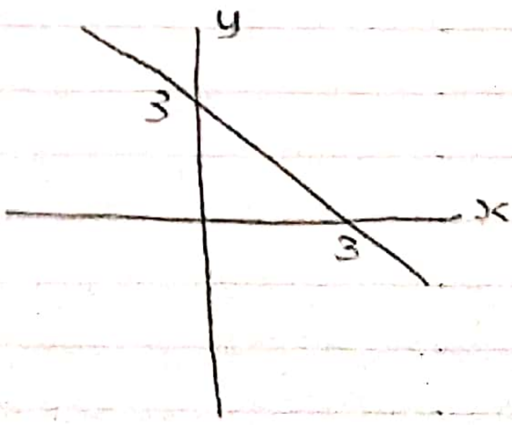
$x = 1$

$y = 2$

ex.

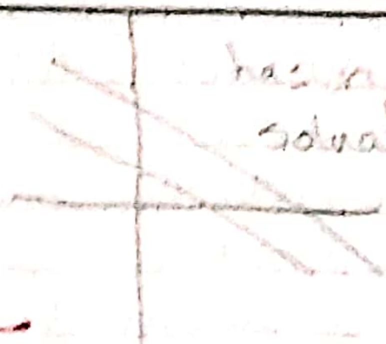
$x + y = 3$

$2x + 2y = 6$



ex.

$$x+y=3$$
$$x+y=1$$



$$A\,X = b$$

Coefficient matrix (A) — unknown matrix (X) — Constant matrix (b)

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & a \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}_{m \times n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \\ \\ x_n \end{bmatrix}}_{n \times 1} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \\ \\ b_m \end{bmatrix}}_{m \times 1}$$

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- Gaussion - elemination method :-

steps

1. write the augumented Matrix $[A : b]$
2. use row operations to rewrite the augumented. matrix in [Row Echolen Form]
3. write the system of equations Corresponding to the matrix in Row echelon Form and use. back substitution to find the solution.

- row echelon Form :-

1. if there exist rows Consists of only Zeros are at the botton of the Matrix
2. For each non zero rows, the first non zero antry is one " leading one "entery" "
3. For two successive non zero rows, the leading 1 in higher row is farther to the left than the Leading 1 in the Lower row

- Reduced Row echelon Form :-

if it is row echelon Form and every Coulmn Contain leading 1 Consists of zeros

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1. $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ row echelon form

2. $\begin{bmatrix} 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Reduced row echelon form

3. $\begin{bmatrix} 1 & -5 & 2 & -1 & 3 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ row echelon form

4. $\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 2 & 1 & -1 \\ 0 & 0 & 1 & -3 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ -2 & 3 & 1 & 0 \end{bmatrix}$

non echelon form

ex. $x_1 - 2x_2 = -3$
$2x_1 + 3x_2 = 8$

solve the system of equations using Gaussian elemination method

step 1

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 2^{-2} & 3^{4} & 8^{6} \end{array}\right] \xrightarrow{-2R+R_2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0/1 & 7/1 & 14/1 \end{array}\right] \xrightarrow{\frac{R_2}{7}}$$

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 1 & 2 \end{array}\right] \Longrightarrow \begin{aligned} x_1 - 2x_2 &= -3 \\ x_2 &= 2 \end{aligned}$$

$x_1 - 2(2) = 3$
$x_1 - 4 = -3$
$x_1 = 1$

ex: solve the following system of equation using Gaussian elemination

$$x_2 + x_3 - 2x_4 = -3$$
$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 4x_2 + x_3 - 3x_4 = -2$$
$$x_1 - 4x_2 + 7x_3 - x_4 = -19$$

step 1

$$\left[\begin{array}{cccc|c} 0 & 1 & 1 & -2 & -3 \\ 1 & 2 & -1 & 0 & 2 \\ 2 & 4 & 1 & -3 & -2 \\ 1 & -4 & -7 & -1 & -19 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 2^{-2} & 4^{-4} & 1^{2} & -3^{0} & -2 \\ 1^{-1} & -4^{-2} & -7^{1} & -1^{0} & -19 \end{array}\right] \begin{array}{c} -2R_1+R_3 \\ -R_1+R_4 \end{array} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & 3 \\ 0 & 0 & 3 & -3 & -6^{18} \\ 0^{0} & -6^{6} & -6^{6} & -1^{-12} & -21 \end{array}\right]$$

$6R_2+R_4$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 3 & -3 & -6 \\ 0 & 0 & 0 & -13 & -39 \end{array}\right] \frac{R_3}{3} \left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & -13 & -39 \end{array}\right] \frac{R_4}{-13}$$

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$x_1 + 2x_2 - x_3 = 2$$
$$x_2 + \cancel{3}x_3 - 2x_4 = -3$$
$$x_3 - x_4 = -2$$
$$\boxed{x_4 = 3}$$

$$x_3 - 3 = -2 \longrightarrow x_3 = 1$$
$$x_2 + 1 - 2(3) = -3 \longrightarrow x_2 + 1 - 6 = -3 \rightarrow x_2 = 2$$
$$x_1 + 2(2) - 1 = 2$$
$$x_1 + 4 - 1 = 2$$
$$x_1 = -1$$

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ex.

$$x_1 - x_2 + 2x_3 = 4$$
$$x_1 + x_3 = 6$$
$$2x_1 - 3x_2 + 5x_3 = 4$$
$$3x_1 + 2x_2 - x_3 = 1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 1^{-1} & 0^{1} & 1^{-2} & 6^{-4} \\ 2^{-2} & -3^{2} & 5^{-4} & 4^{-8} \\ 3^{-3} & 2^{3} & -1^{-6} & 1^{-12} \end{array}\right] \begin{array}{l} \\ -R_1 + R_2 \\ -2R_1 + R_3 \\ -3R_1 + R_4 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & -4 \\ 0^{0} & 5^{-5} & -7^{5} & -11^{-10} \end{array}\right] \xrightarrow{\substack{R_2+R_3 \\ -5R_2+R_4}} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & - & -1 \end{array}\right] \xrightarrow{R_3 \leftrightarrow R}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -2 & -21 \\ 0 & 0 & 0 & -2 \end{array}\right] \xrightarrow{\frac{R_3}{-2}} \left[\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & \frac{21}{2} \\ 0 & 0 & 0 & -2 \end{array}\right]$$

$$x_1 - x_2 + 2x_3 = 4$$
$$x_2 - x_3 = 2$$
$$x_3 = \frac{21}{2}$$
$$\boxed{0 = -2}$$

These system of equations has
No solution

Homework

$$3x_1 - 3x_2 + 3x_3 = 9$$
$$2x_1 - x_2 + 4x_3 = 7$$
$$3x_1 - 5x_2 - x_3 = 7$$