lec (1) 10-11-2020

MS 517

Course Content

① Error analysis.

② Finding roots

③

single → Multiple

1- Bisection.

2. ~~Net~~ Newont Method.

3- Secant Method.

4- Fixed point Method.

③ Solving System of linear equations.

① Gauss elemination.

② Decomposition Method.

Crout's — Doolittle — Cholsky

④ Interpolation.

① Lagrage interpolation.

② Divided difference interpolation.

⑤ Neumerical differentrations.

⑥ Neumerical integration.

⑦ Statistical applications "May be"

Text book * Numerical techniques.

* Computer Based Numerical and Statistical techniques.

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17.11.2020

Lec (2) MS 517

Error Analysis

- Numerical Solutions gives an approximate solution to problems.

Mathematics

Exact

$5x = 10$

$\boxed{x = 2}$

Rational $\frac{1}{3} = 0.3333---$

approximate

$\pi \simeq 3.14$

$\pi = 3.14159265---$

$\frac{1}{\pi}$

Rational: 1.1315 1315 1315 --- ⇒ has a pattern ⇒ exact

irrational: 1.1315 1275 1970 --- irrational bec. it has no pattern ⇒ approximated

$\frac{10}{6} = 1.6666$

if we take four digits → total digits ~~but significant digits~~

$\frac{10}{6} = 1.667$

Siginificant digits

total

# of all digits represent numbers

decimals

# of digits after decimal point represent number.

* 1.667 have 4 total digits but has 3 significant digits.

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* floating Point number } task
  maintessa } Search about them

task — How to represent number system on Computer.

Accuracy vs Precesian

① Accuracy : defined as the Closness of calculated Value to exact Solution.

Ex True Value was 4.2138

Approximate is 4.1182 — less accurat
closed — $\boxed{4.2146}$ — more accurat.
3.9392 — least accurat.

② Precision : means repeativness of the value.

Ex True value 4.2138

Approximate	
4.1182	1st trial
4.1183	2nd " ... so on
4.1181	all are more
4.1182	Precise

Ex True — 4.2138

Approximate	
4.2137	more accurt
4.2139	& more
4.2137	precision
4.2136	

- Accurt means that percision exist but the inverse is not true.

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Types of errors : we have 2 Commons types of error.

① Truncation error: "Cut off error" "Ch

Truncation errors are generated when only significant digits are considered and ~~rep~~ remaing ~~and~~ are discarded. ← استبعد

$\frac{1}{6} = 0.16666---$

truncate after 4 digits(decimal).

$\frac{1}{6} = 0.1666$ Error $0.0000666---$ more accurate

Truncate after 1 point

$\frac{1}{6} = 0.1$ Error $0.06666---$

* more Smaller error the more accurate

e^x, $\sin x$, $\cos x$, $(1+x)^n$ $(1.01)^{10}$

* Tailor & Macrone.

* $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ---$ $\sum \frac{x^r}{r!}$

$e^0 = 1$

$e = 2.718$

$e^2 = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}$

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[2] Rounding error: Rounding errors take place because of rounding the last significant to the nearest value.

Ex $\frac{1}{6} = 0.1666\boxed{6}\ldots$ ($\rightarrow > 5$)

use only 4 significant (decimal)

$\frac{1}{6} = 0.1667$

≥ 5 +1

< 5 not change

$= 5$ ^_^ read about it

دور عليها

Ex 0.2314012

Round off to 4 significant decimal.

0.2314

* Analytical and Estimation errors.

Error = Actual (True) value − Approximate

ϵ (espilon)

[1] absolut error $|\epsilon| = |\text{actual value} - \text{Approximate}|$

[2] Relative error: measure the distance (True) bet. actual and Approximate

$$= \left| \frac{\text{actual} - \text{Approximate}}{\text{actual}} \right|$$

$$= \frac{\text{Absolut error}}{|\text{True value}|}$$

Actual

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* Calculate absolute & Relative error for the following Cases:

① true value $= 1 \times 10^{-6}$ Approximate value 0.5×10^{-6}

$|\epsilon| = |1 \times 10^{-6} - 0.5 \times 10^{-6}| = 0.5 \times 10^{-6}$

Realtive $= \dfrac{0.5 \times \cancel{10^{-6}}}{1 \times \cancel{10^{-6}}} = 0.5$

② true value $= 1 \times 10^{6}$ Approximate Value 0.99×10^{6}

$|\epsilon| = |1 \times 10^{6} - 0.99 \times 10^{6}| = 0.01 \times 10^{6} = 10000$

Realtive error $= \dfrac{0.01 \times \cancel{10^{6}}}{1 \times \cancel{10^{6}}} = 0.01$

- it is not allowed to compare bet. 2↑ absloute errors. Computations with

- absloute may be large but RE may be to be Small.

How to ~~Compute~~ Calculate error:

- to Calculate error when the true value is not known.

absloute error $= |$ Current Approximate $-$ Perivous Approximate $|$

Relative error $= \left| \dfrac{\text{Current} - \text{Perivous}}{\text{Current}} \right|$

- Taylor's series for approximate of functions.

~~f(x)~~ $F(x_{i+1}) \simeq f(x_i)$

→ Zero order approximation.

we ~~call~~ Can also use

$$f(x_{i+1}) \simeq f(x_i) + f'(x_i)(x_{i+1} - x_i)$$

(straight line approximate)
(first order approximation.

$$* \quad f(x_{i+1}) \simeq f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2$$

" Second order approximation"

- In general, it becomes Complete taylor series

~~Ex~~ $$f(x_{i+1}) \simeq f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \frac{f'''(x_i)}{3!}(x_{i+1} - x_i)^3 + \ldots + \frac{f^{(n)}(x_i)}{n!}(x_{i+1} - x_i) + R_n$$

Ex Example: obtain MacLaurin expansion for e^x using taylor series.

Set $x_i = 0$

$$f(x_i) = e^{x_i} = e^0 = 1$$

$$f'(x_i) = e^{x_i} = e^0 = 1$$

let $h = x_{i+1} - x_i \Rightarrow h = x_{i+1} - 0$

$$h = x_{i+1} = x$$

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 \cdots + \frac{f^{(n)}(0)}{n!} x^n$$

$$e^x = 1 + 1x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \cdots \frac{1}{n!} x^n + R$$

بالتالي $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \frac{x^n}{n!}$

if $x = 2$ $\quad e^2 = 7.38205609 9$

find zero order $\quad e^2 \simeq 1$

First order $\quad e^x \simeq = 1 + x$

$$e^2 = 1 + 2 = \boxed{3}$$

second order $e^x = 1 + x + \frac{x^2}{2!}$

$$e^2 = 1 + 2 + \frac{2^2}{2!} = 5$$

fourth order $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

$$e^2 = 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} =$$

$$1 + 2 + 2 + \frac{8}{6} + \frac{16}{24} = 7$$

Ex find the value of e^x using the expansion

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

for $x = 0.5$ with abslote value error

less than 0.005

Sol.

let n^{th} term $\frac{x^n}{n!}$

اكبر رقم فيه error

$$\cancel{n+} \boxed{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^n}{n!}} + \boxed{\frac{x^{n+1}}{n+1!} \cancel{x}^{n+1}}$$

$(n+1)^{th}$ from

$$\frac{x^{n+1}}{(n+1)!} < 0.005$$

take log for both sides

$\log \frac{u}{v} = \log u - \log v$

$$(n+1) \log x - \log (n+1)! < \log (0.005)$$

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$(n+1)\log x - \log(n+1)! < -2.30103$

$\log (n+1)! - (n+1)\log x > 2.30103$

$\log (n+1)! - (n+1) \underbrace{\log 0.5}_{(-0.30103)} > 2.30103$

$n=2 \quad \log(3!) - 3\log 0.5 = 1.68 < 2.30103$

$n=3 \quad \log((3+1)!) - 3\log 0.5 =$

$\log(4!) - 4\log 0.5 = 2.58 > 2.30103$ ✓

$\boxed{n=3}$ take 3 terms

$e^x = 1 + x + \frac{x^2}{2!}$

$e^{0.5} = 1 + 0.5 + \frac{(0.5)^2}{2!}$

$= 1.6458333$ (Approximated)

$e^{0.5} = 1.6487213$ (true value)

absolute error $= 1.6487213 - 1.6458333$

$= 0.0028879$

P 78, 79, 80

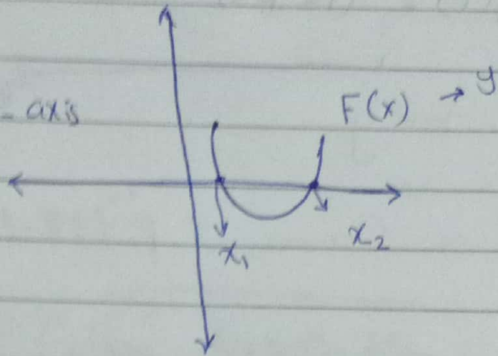
24/11/2020

lec (3)

Roots of equation

The root is the value of x that satisfy $f(x)=0$

$F(x)$ intersect x-axis



we know that
algebric ~~bion~~ Polynomial of degree one, two, three, four can be solved algebrically analytically

$F(x) = x+2$ Find roots

put $f(x)=0$

$x+2=0$

$x=-2$ → root

$f(x) = x^2+5x+6$

$f(x)=0$

$x^2+5x+6=0$

$(x+2)(x+3)=0$

$x=-2$ or $x=-3$

Find the roots of $x^7+x^5+3x^4+2x+C=0$ (Constant)

In this cases we can not find algebric solution.

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we will use numerical Methods to find an approximated Solution

- find solution of $f(x)=0$

$$f(x) = x^3 - \cos x + 3$$

$$f(x) = \ln x - \sin 3x + x$$

→ different numerical methods can be used to find roots for higher Polynomial and transcandental function.

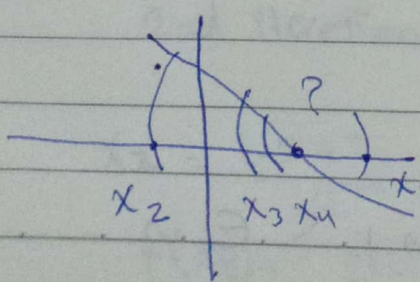
↙

trig. function

log. & ln function

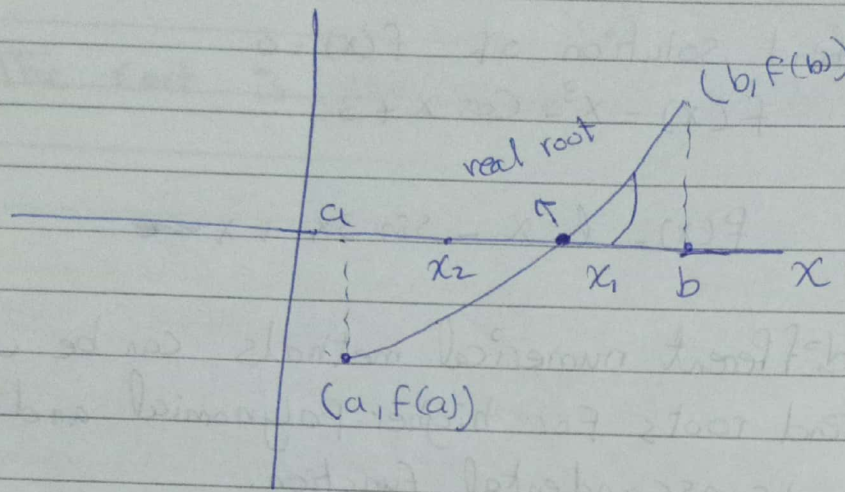
$\boxed{3}$ Types of methods to obtain root.

Bracketing Method	Open Methods
- two intial values to start.	- one or more initial values.
- always convergent (متقاربة)	- some time can be divergent.
- Need more numbers of iterations (عدوة)	- less number of iterations.
Ex - Bisection Method.	Ex → Newton Method
- ~~Secant Method~~.	→ Secant Method.
- ~~secant M~~	
- Fixed point.	
- regular False	



Continous

* Bisection Method: this Method is called interval halving Method



Suppose we want to find the solution of $f(x) = 0$

$f(x)$ is Continous in (a, b)

let $f(a)$ $-ve$ and $f(b)$ $+ve$

$$f(a) \cdot f(b) < 0$$

$$x_1 = \frac{a+b}{2} \qquad f(x_1) \; +ve$$

$$x_2 = \frac{x_1 + a}{2}$$

$f(x) = 0$? Yes $\rightarrow$ root
No $\rightarrow$ Continous.

$F(x)$ $-ve$

$$X_3 = \frac{X_1 + X_2}{2}$$

⋮

and stop when $X_n - X_{n-1} < \epsilon$

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$\epsilon = 0.0001$ or $f(x_n) \simeq 0$

Find a root for the following Polynomial using bisection Method and error ~~0.0001~~ 0.01

$$F(x) = x^3 - x - 2$$

using interval $[1,2]$

Solution let $a = 1$ $b = 2$

$f(a) = f(1) = 1^3 - 1 - 2 = -2$ $-ve$

$f(b) = f(2) = 2^3 - 2 - 2 = 4$ $+ve$

$f(a) \cdot f(b) = f(1) \cdot f(2) = -2 \times 4 = -8 < 0$

So the ~~functation~~ function has aroot

Note if $f(a) \cdot f(b) < 0$ So the function has a root.

First iteration (iteretion No.1)

$$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$f(x_1) = (1.5)^3 - (1.5) - 2 = -0.125$

$-ve$ value

نحطها مكان ال a لأن x_1 عشان نفس الإشا

علامة ال

2nd iteretion $a = 1.5$ $b = 2$

$$x_2 = \frac{1.5+2}{2} = 1.75$$

$f(x_2) = (1.75)^3 - 1.75 - 2 = 1.609375$ $+ve$

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$a = 1.5$ $-ve$ $b=2$ $+ve$ → sign of function

$\epsilon = |x_2 - x_1| = |1.75 - 1.5| = 0.25$

x_3 $a = 1.5$ $b = 1.75$

$x_3 = \frac{1.5 + 1.75}{2} = 1.625$

$f(x_3) = f(1.625) = (1.625)^3 - (1.625) - 2$

$= 0.666056$

$\epsilon = x_3 - x_2 = \cancel{0.666056} = |1.625 - 1.75|$

$= 0.125$

$a = 1.5$ $-ve$ $b^2 = +ve$ x 1.625

$x_4 = \frac{1.5 + 1.625}{2} = 1.5625$

$f(1.5625) = 0.2521973$

$\epsilon = |x_4 - x_3| = |1.5625 - 1.625| = 0.0625$

fifth iteration

$x_5 = \frac{1.5 + 1.5625}{2} = 1.53125$

$f(1.53125) = 0.054113$ $+ve$

$\epsilon = |x_5 - x_4| = |1.53125 - 1.5625| = 0.03125$

$x_6 = 1.515625$

$\epsilon =$

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