Métodos Numéricos

→ Atividade 6

Prof. Dr. Rafael Gabler

Aluno Eng. Lucas Wanick

3.1 - Para Casa

→ $f(x) = a_0 + a_1 x + a_2 x^2$

$$S_r = \sum_{i=1}^{N} e^2 = \sum_{i=1}^{N} (y_i - a_0 - a_1 x_i - a_2 x_i^2)^2$$

Ex1

1-

$$\rightarrow \frac{\partial S_r}{\partial a_0} = -\cancel{2}\sum_{i=1}^{N} (y_i - a_0 - a_1 x_i - a_2 x_i^2) = 0 \rightarrow \sum y_i = a_0 \sum 1 + a_1 \sum x_i + a_2 \sum x_i^2$$

$$\rightarrow \frac{\partial S_r}{\partial a_1} = -\cancel{2}\sum_{i=1}^{N} (y_i - a_0 - a_1 x_i - a_2 x_i^2) x_i = 0 \rightarrow \sum x_i y_i = a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3$$

$$\rightarrow \frac{\partial S_r}{\partial a_2} = -\cancel{2}\sum_{i=1}^{N} (y_i - a_0 - a_1 x_i - a_2 x_i^2) x_i^2 = 0 \rightarrow \sum x_i^2 y_i = a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4$$

2-

- $b_1 = \sum y_i$
- $b_2 = \sum x_i y_i$
- $b_3 = \sum x_i^2 y_i$

$$\begin{cases} c_{11} a_0 + c_{12} a_1 + c_{13} a_2 = b_1 \\ c_{21} a_0 + c_{22} a_1 + c_{23} a_2 = b_2 \\ c_{31} a_0 + c_{32} a_1 + c_{33} a_2 = b_3 \end{cases}$$

- $c_{11} = N$
- $c_{12} = c_{21} = \sum x_i$
- $c_{13} = c_{22} = c_{31} = \sum x_i^2$
- $c_{23} = c_{32} = \sum x_i^3$
- $c_{33} = \sum x_i^4$

Ex2.) Tabela 2

	x_i	y_i	$x_i y_i$	x^2	x^3	x^4	$x_i^2 y_i$
	0	2.1	0	0	0	0	0
	1	7.7	7.7	1	1	1	7.7
	2	13.6	27.2	4	8	16	54,4
	3	27.2	81.6	9	27	81	244,8
	4	40.9	163,6	16	64	256	654,4
	5	61.1	305,5	25	125	625	1527,5
$\sum =$	15	152,6	585,6	55	225	979	2488,8

$$\begin{cases} 6a_0 + 15a_1 + 55a_2 = 152,6 \\ 15a_0 + 55a_1 + 225a_2 = 585,6 \\ 55a_0 + 225a_1 + 979a_2 = 2488,8 \end{cases}$$

$$a_0 = \frac{1}{6}(152,6 - 15a_1 - 55a_2)$$

$$a_1 = \frac{1}{55}(585,6 - 15a_0 - 225a_2)$$

$$a_1 = \frac{1}{55}\left[585,6 - \frac{15}{6}(152,6 - 15a_1 - 55a_2) - 225a_2\right]$$

$$a_1 = \frac{1}{55}\left[\underbrace{585,6 - 381,5}_{} + \frac{75}{2}a_1 + \frac{275}{2}a_2 - 225a_2\right]$$

$$a_1 = \frac{15}{22}a_1 + \frac{1}{55}\left(204,1 - \frac{175}{2}a_2\right)$$

$$\frac{7}{\cancel{22}}a_1 = \frac{1}{\cancel{55}}\left(204,1 - \frac{175}{2}a_2\right) \times 11$$

$$a_1 = \frac{2}{35}\left(204,1 - \frac{175}{2}a_2\right)$$

$$a_0 = \frac{1}{6}\left[152,6 - 15 \cdot \frac{2}{35}\left(204,1 - \frac{175}{2}a_2\right) - 55a_2\right]$$

$$a_0 = \frac{1}{6}\left[152,6 - 174,94 + 75a_2 - 55a_2\right]$$

$$a_0 = \frac{1}{6}(-22,34 + 20a_2)$$

$$\boxed{a_0 = 2,48}$$

$$\boxed{a_1 = 2,36}$$

$\rightarrow 55a_0 + 225a_1 + 979a_2 = 2488,8$

$$55\left[\frac{1}{6}(20a_2 - 22,34)\right] + \cancel{225}^{45}\left[\frac{2}{\cancel{35}_7}\left(204,1 - \frac{175}{2}a_2\right)\right] + 979a_2 = 2488,8$$

$$183,3a_2 - 204,78 + 2624,14 - 1125a_2 + 979a_2 = 2488,8$$

$$37,33a_2 = 69,44$$

$\boxed{a_2 = 1,86}$ $\boxed{R: f(x) = 2,48 + 2,36x + 1,86x^2}$

Ex 3 $\rightarrow f(x_1, x_2) = a_0 + a_1x_1 + a_2x_2$

$$S_r = \sum_{i=1}^{n} e^2 = \sum_{i=1}^{n}(y_i - a_0 - a_1x_{1i} - a_2x_{2i})^2$$

$\rightarrow \frac{\partial S_r}{\partial a_0} = -2\sum(y_i - a_0 - a_1x_{1i} - a_2x_{2i}) = 0 \quad \rightarrow \sum y_i = a_0\sum 1 + a_1\sum x_{1i} + a_2\sum x_{2i}$

$\rightarrow \frac{\partial S_r}{\partial a_1} = -2\sum(y_i - a_0 - a_1x_{1i} - a_2x_{2i})x_{1i} = 0 \quad \sum x_{1i}y_i = a_0\sum x_{1i} + a_1\sum x_{1i}^2 + a_2\sum x_{2i}x_{1i}$

$\rightarrow \frac{\partial S_r}{\partial a_2} = -2\sum(y_i - a_0 - a_1x_{1i} - a_2x_{2i})x_{2i} = 0 \quad \sum x_{2i}y_i = a_0\sum x_{2i} + a_1\sum x_{1i}x_{2i} + a_2\sum x_{2i}^2$

$$\begin{cases} c_{11}a_0 + c_{12}a_1 + c_{13}a_2 = b_1 \\ c_{21}a_0 + c_{22}a_1 + c_{23}a_2 = b_2 \\ c_{31}a_0 + c_{32}a_1 + c_{33}a_2 = b_3 \end{cases}$$

$b_1 = \sum y_i$ $\quad c_{13} = c_{31} = \sum x_{2i}$

$b_2 = \sum x_{1i}y_i$ $\quad c_{12} = c_{21} = \sum x_{1i}$

$b_3 = \sum x_{2i}y_i$ $\quad c_{23} = c_{32} = \sum x_{1i}x_{2i}$

$c_{11} = N$ $\quad c_{22} = \sum x_{1i}^2$

$c_{33} = \sum x_{2i}^2$

Ex 4

x_{1i}	x_{2i}	y_i	x_{1i}^2	x_{2i}^2	$x_{1i}x_{2i}$	$x_{1i}y_i$	$x_{2i}y_i$
0	0	5	0	0	0	0	0
2	1	10	4	1	2	20	10
2,5	2	9	6,25	4	5	22,5	18
1	3	0	1	9	3	0	0
4	6	3	16	36	24	12	18
7	2	27	49	4	14	189	54
Σ = 16,5	14	54	76,25	54	48	243,5	100

$$\begin{cases} 6a_0 + 16,5a_1 + 14a_2 = 54 \\ 16,5a_0 + 76,25a_1 + 48a_2 = 243,5 \\ 14a_0 + 48a_1 + 54a_2 = 100 \end{cases}$$

$\rightarrow a_0 = \frac{1}{6}\left[54 - 16,5a_1 - 14a_2\right]$

$a_0 = \frac{1}{6}\left[54 - \frac{\cancel{165}^{33}}{\cancel{10}_{2}}\frac{\cancel{4}^{2}}{13}(10 - a_2)\right) - 14a_2\right]$

$a_0 = 9 - \frac{11}{13}(10 - a_2) - \frac{7}{3}a_2$

$\rightarrow \frac{165}{10}\left[\frac{1}{6}\left(54 - \frac{165}{10}a_1 - 14a_2\right)\right] + 76,25a_1 + 48a_2 = 243,5$

$\frac{1485}{10} - \frac{165^2}{600}a_1 - \frac{385}{10}a_2 + 76,25a_1 + 48a_2 = 243,5$

$\frac{18525}{60\cancel{0}}a_1 + \frac{95}{\cancel{10}}a_2 = \frac{950}{\cancel{10}}$

$\frac{18525}{60}a_1 = 950 - 95a_2$

$a_1 = \frac{60}{18525}(950 - 95a_2)$

$a_1 = \frac{\cancel{60}^{\cancel{20}\,4}}{\cancel{195}_{\cancel{65}\,13}}(10 - a_2) = \frac{4}{13}(10 - a_2)$

$\rightarrow \boxed{a_1 = 4}$

$\rightarrow \boxed{a_0 = 5}$

$\rightarrow 14\left[9 - \frac{11}{13}(10 - a_2) - \frac{7}{3}a_2\right] + 48\left[\frac{4}{13}(10 - a_2)\right] + 54a_2 = 100$

$+126 - \frac{154}{13}(10 - a_2) - \frac{98}{3}a_2 + \frac{192}{13}(10 - a_2) + 54a_2 = 100$

$\overset{718}{}$

$\frac{+462 - 1274 - 576 + 2106}{39}a_2 = -26 - \frac{380}{13}$

$\frac{718}{39}a_2 = -\frac{718}{13}$

$\boxed{a_2 = -3}$

$\boxed{R: f(x_1, x_2) = 5 + 4x_1 - 3x_2}$

4.1 - $y = \alpha_2 x^{\beta_2} \rightarrow \ln(y) = \ln(\alpha_2) + \ln(x^{\beta_2})$

$\ln(y) = \ln(\alpha_2) + \beta_2 \ln(x)$

$\bar{y} = \bar{\alpha}_2 + \beta_2 \bar{x}$

x_i	y_i	$\bar{x}_i$	$\bar{y}_i$	$\bar{x}_i^2$	$\bar{x}_i\bar{y}_i$
1	0,5	0	-0.69	0	0
2	1,7	0.69	0,53	0,48	0,37
3	3,4	1.10	1.22	1.21	1.34
4	5,7	1.39	1.74	1.93	2.42
5	8,4	1.61	2.13	2.59	3.43
$\Sigma =$ 15	19.7	4.79	4.93	6.21	7.56

$C_{11} = \Sigma N$

$C_{12} = C_{21} = \Sigma x_i$

$C_{22} = \Sigma x_i^2$

$b_1 = \Sigma y_i$

$b_2 = \Sigma x_i y_i$

$S_r = \sum_{i=1}^{N} e^2 = \sum_{i=1}^{N} (\bar{y}_i - \bar{\alpha}_n - \beta_n \bar{x}_i)^2$

$\frac{\partial S_r}{\partial \bar{\alpha}} = -2\Sigma(\bar{y}_i - \bar{\alpha}_n - \beta_n \bar{x}_i) = 0$

$\Sigma \bar{y}_i = \bar{\alpha}\Sigma 1 + \beta \Sigma \bar{x}_i$

$\frac{\partial S_r}{\partial \beta} = -2\Sigma(\bar{y}_i - \bar{\alpha}_n - \beta_n \bar{x}_i)\bar{x}_i = 0$

$\Sigma \bar{x}_i \bar{y}_i = \bar{\alpha}\Sigma \bar{x}_i + \beta \Sigma \bar{x}_i^2$

$$\begin{cases} C_{11}\bar{\alpha} + C_{12}\beta = b_1 \\ C_{21}\bar{\alpha} + C_{22}\beta = b_2 \end{cases}$$

$$\begin{bmatrix} 5 & 4.79 \\ 4.79 & 6.21 \end{bmatrix} \cdot \begin{Bmatrix} \bar{\alpha} \\ \beta \end{Bmatrix} = \begin{Bmatrix} 4.93 \\ 7.56 \end{Bmatrix}$$

$\bar{\alpha} = \frac{4,93 - \beta\, 4,79}{5}$

$4,79\left[\frac{4,93 - \beta 4,79}{5}\right] + 6,21\beta = 7,56$

$\frac{23,61 - 22,94\beta}{5} + 6,05\beta = 7,56$

$1,66\beta = 2,84$

$\boxed{\beta = 1,71}$

$\boxed{\bar{\alpha} = -0,65}$

$\ln(\alpha) = \bar{\alpha}$

$e^{\ln(\alpha)} = e^{\bar{\alpha}}$

$\alpha = e^{\bar{\alpha}}$

$\boxed{\alpha = 0,52}$

R: $y = 0,52\, x^{1,71}$

$\approx 0,5 \cdot x^{1,75}$

5.2 -

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ // $f(x) = \ln(x)$

Ponto	x	$\ln(x)$
x_0	1	0
x_1	4	1,386294
x_2	6	1.79179
x_3	5	1.609438

$f(x) = \ln(x)$

1) Diferenças Divididas

$\rightarrow f[x_0] = 0$

$f[x_0, x_1] = \frac{1,386294 - 0}{4 - 1} = 0,462098$

$f[x_1, x_2] = \frac{1,79179 - 1,386294}{6 - 4} = 0,202748$

$f[x_2, x_3] = \frac{1,609438 - 1,79179}{5 - 6} = 0,182352$

$\rightarrow f[x_0, x_1, x_2] = \frac{0,202748 - 0,462098}{6 - 1} = -0,05187$

$f[x_1, x_2, x_3] = \frac{0,182352 - 0,202748}{5 - 4} = -0,020396$

$\rightarrow f[x_0, x_1, x_2, x_3] = \frac{-0,020396 - (-0,05187)}{5 - 1} = 0,007869$

2) Polinômio Interpolador de Newton

$\rightarrow P_3(x) = f[x_0] + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1) + f[x_0,x_1,x_2,x_3](x-x_0)(x-x_1)(x-x_2)$

$P_3(x) = 0 + 0,462098(x-1) + (-0,05187)(x-1)(x-4) + 0,007869(x-1)(x-4)(x-6)$

$P_3(2) = 0,462098(2-1) - (0,05187)(2-1)(2-4) + 0,007869(2-1)(2-4)(2-6)$

$P_3(2) = 0,462098 + 0,10374 + 0,062952 = 0,62879$

$\rightarrow E_{\%} = \frac{\ln(2) - P_3(2)}{} = 9,28\ \%$

Ponto	x	$\ln(x)$ $\ln 2$
x_0	1	0
x_1	2	0,62879
x_2	4	1,386294
x_3	5	1.609438
x_4	6	1.79179

$\rightarrow f[x_0] = 0$

$\rightarrow f[x_0,x_1] = 0,62879$

$f[x_1,x_2] = 0,378752$

$f[x_3,x_2] = 0,223144$

$f[x_4,x_3] = 0,182352$

$\rightarrow f[x_0,x_1,x_2] = -0,083346$

$f[x_1,x_2,x_3] = -0,051869333$

$f[x_2,x_3,x_4] = -0,040792$

$\rightarrow f[x_0,x_1,x_2,x_3] = 7,86916675 \times 10^{-3}$

$f[x_1,x_2,x_3,x_4] = 2,76933325 \times 10^{-3}$

$\rightarrow f[x_0,x_1,x_2,x_3,x_4] = -1,0199667 \times 10^{-3}$

$\rightarrow P_4(x) = 0 + 0,62879(x-1) - 0,083346(x-1)(x-2) + 7,86916675 \times 10^{-3}(x-1)(x-2)(x-4) - 1,0199667 \times 10^{-3}(x-1)(x-2)(x-4)(x-5)$

$P_4(2) = 0,62879$

6.2 -

$$\begin{bmatrix} 4,5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 20,25 & 4,5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 49 & 7 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 49 & 7 & 1 \\ 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -9 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 14 & 1 & 0 & -14 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \\ a_3 \\ b_3 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2,5 \\ 2,5 \\ 2,5 \\ 0,5 \\ 0 \\ 0 \end{bmatrix}$$

x	$f(x)$
3	2,5
4,5	1
7	2,5
9	0,5

$f(5) = 0,66$

→ ① $c_1 = 1 - 4{,}5 b_1$

$c_1 = 1 + 4{,}5$

$c_1 = 5{,}5$

⑤ $3b_1 + c_1 = 2{,}5$

$3b_1 + 1 - 4{,}5 b_1 = 2{,}5$

$-1{,}5 b_1 = 1{,}5$

$b_1 = -1$

⑦ $-1 - 9a_2 - b_2 = 0$

$b_2 = -9a_2 - 1$

$b_2 = -6{,}76$

② $20{,}25 a_2 + 4{,}5 b_2 + c_2 = 1$

$-40{,}5 a_2 - 4{,}5 + 20{,}25 a_2 + c_2 = 1$

$c_2 = 5{,}5 + 20{,}25 a_2$

$c_2 = 18{,}46$

③ $49a_2 + 7b_2 + c_2 = 2{,}5$

$49a_2 + 7(-9a_2 - 1) + 5{,}5 + 20{,}25 a_2 = 2{,}5$

$49a_2 - 63a_2 - 7 + 5{,}5 + 20{,}25 a_2 = 2{,}5$

$6{,}25 a_2 = 4$

$a_2 = 0{,}64$

$f_2(x) = 0{,}64x^2 - 6{,}76x + 18{,}46$

$\boxed{f_2(5) = 0{,}66}$

6.3-

x_i	$f_i(x_i)$
3	2,5
4,5	1
7	2,5
9	0,5

x_1

$h_i = (x_i - x_{i-1})$

$f_i''(x_i) = M_i$

$f_i(x_{i-1}) = f_{i-1}$

$f_i(x_i) = f_i$

$f_i(x_i) = a_i(x - x_{i-1})^3 + b_i(x - x_{i-1})^2 + c_i(x - x_{i-1}) + d_i$

$f_i'(x_i) = 3a(x - x_{i-1})^2 + 2b(x - x_{i-1}) + c$

$f_i''(x_i) = 6a(x - x_{i-1}) + 2b$

→ $f_i(x_{i-1}) = a \cdot 0 + b \cdot 0 + c \cdot 0 + d$

$\boxed{d = f_{i-1}}$

→ $f_i = ah_i^3 + bh_i^2 + ch_i + f_{i-1}$

$f_i = \left(\frac{M_i - M_{i-1}}{6h_i}\right)h_i^3 + \frac{M_{i-1}}{2}h_i^2 + ch_i + f_{i-1} = f_i$

[...]

→ $f_i''(x_{i-1}) = 6a \cdot 0 + 2b = M_{i-1}$

$\boxed{b = \frac{M_{i-1}}{2}}$

→ $f_i''(x_i) = 6ah_i + 2b = M_i$

$\boxed{a = \frac{M_i - M_{i-1}}{6h_i}}$

[...] $\frac{M_i + 2M_{i-1}}{6} h_i^2 + c\,h_i = f_i - f_{i-1}$

$\boxed{c = \frac{f_i - f_{i-1}}{h_i} - \frac{M_i - 2M_{i-1}}{6} h_i}$

$$\rightarrow f_i(x_i) = \frac{M_{i-1}}{6h_i}(x_i - x)^3 + \frac{M_i}{6h_i}(x - x_{i-1})^3 + \left(\frac{f(x_i)}{h_i} - \frac{M_i h_i}{6}\right)(x - x_{i-1}) + \left(\frac{f(x_{i-1})}{h_i} - \frac{M_{i-1} h_i}{6}\right)(x_i - x)$$

$$f_i'(x) = -\frac{M_{i-1}}{2h_i}(x_i - x)^2 + \frac{M_i}{2h_i}(x - x_{i-1})^2 + \left(\frac{f_i}{h_i} - \frac{M_i h_i}{6}\right) - \left(\frac{f_{i-1}}{h_i} - \frac{M_{i-1} h_i}{6}\right)$$

$$f_i'(x) = -\frac{M_{i-1}}{2h_i}(x_i - x)^2 + \frac{M_i}{2h_i}(x - x_{i-1})^2 + \left[\frac{f_i - f_{i-1}}{h_i} - \frac{(M_i - M_{i-1})h_i}{6}\right]$$

$\rightarrow$ Nós internos: $f_i'(x) = f_{i+1}'(x)$

- p/ $x = x_i$

$$f_i'(x_i) = -\frac{M_{i-1}}{2h_i}(\cancel{x_i - x_i})^2 + \frac{M_i}{2\cancel{h_i}}\overbrace{(x_i - x_{i-1})}^{h_i^{\cancel{2}}}{}^2 + \left[\frac{f_i - f_{i-1}}{h_i} - (M_i - M_{i-1})\frac{h_i}{6}\right]$$

$$f_i'(x_i) = \frac{M_i h_i}{2} + \left[\frac{f_i - f_{i-1}}{h_i} - (M_i - M_{i-1})\frac{h_i}{6}\right]$$

$$f_i'(x_i) = \frac{f_i - f_{i-1}}{h_i} + (2M_i + M_{i-1})\frac{h_i}{6}$$

- p/ $x = x_{i+1}$

$$f_i'(x_{i+1}) = -\frac{M_i}{2\cancel{h_{i+1}}}\overbrace{(x_{i+1} - x_i)}^{h_{i+1}^{\cancel{2}}}{}^2 + \frac{M_{i+1}}{2h_{i+1}}(\cancel{x_i - x_i})^2 + \left[\frac{f_{i+1} - f_i}{h_{i+1}} - \frac{(M_{i+1} - M_i)h_{i+1}}{6}\right]$$

$$f_i'(x_{i+1}) = -\frac{M_i h_{i+1}}{2} + \frac{f_{i+1} - f_i}{h_{i+1}} - (M_{i+1} - M_i)\frac{h_{i+1}}{6}$$

$$f_i'(x_{i+1}) = \frac{f_{i+1} - f_i}{h_{i+1}} - (2M_i + M_{i+1})\frac{h_{i+1}}{6}$$

Igualando

$$\rightarrow \frac{f_{i+1} - f_i}{h_{i+1}} - (2M_i + M_{i+1})\frac{h_{i+1}}{6} = \frac{f_i - f_{i-1}}{h_i} + (2M_i + M_{i-1})\frac{h_i}{6}$$

$$(2M_i + M_{i+1})\frac{h_{i+1}}{6} + (2M_i + M_{i-1})\frac{h_i}{6} = \frac{f_{i+1} - f_i}{h_i} - \frac{f_i - f_{i-1}}{h_{i+1}}$$

$$h_i M_{i-1} + 2(h_i + h_{i+1})M_i + h_{i+1}M_{i+1} = 6\left(\frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}\right)$$

→ p/ o 1º intervalo $[x_0, x_2]$

$x_1 = 4{,}5$ $f(x_1) = 1$ $h_i = 2{,}5$ $h_{i+1} = 2$ $f_{i+1} = 2{,}5$ $f_{i-1} = 2{,}5$

$x_2 = 7$ $f(x_2) = 2{,}5$ $h_i = 2$ $h_{i+1} = 2$ $f_{i+1} = 0{,}5$ $f_{i-1} = 1$

→ P/ $M_0 = 0$

$$2{,}5(0) + 2(2{,}5+2)M_1 + 2M_2 = 6\left(\frac{2{,}5-1}{2{,}5} - \frac{1-2{,}5}{2{,}0}\right) \quad 3 \cdot \frac{27}{\cancel{20}\ 10} \to \frac{71}{10}$$

$$9M_1 + 2M_2 = 7{,}1$$

→ P/ $M_3 = 0$

$$2{,}5M_1 + 2(2{,}5+2)M_2 + 2\cdot(0) = 6\left(\frac{0{,}5-2{,}5}{2} - \frac{2-1}{2{,}5}\right) \quad -2\ (-1) \quad -\frac{35}{25} = -\frac{7}{5}$$

$$2{,}5M_1 + 9M_2 = -\frac{42}{5} = -8{,}4$$

$$\begin{cases} 4{,}5M_1 + M_2 = 3{,}55 \\ 2{,}5M_1 + 9M_2 = -8{,}4 \end{cases} \qquad M_2 = 3{,}55 - 4{,}5M_1$$

$$2{,}5M_1 + 9(3{,}55 - 4{,}5M_1) = -8{,}4$$

$$-38M_1 = -40{,}35$$

$$\boxed{M_1 = 1{,}062}$$

$$\boxed{M_2 = -1{,}229}$$

15 +

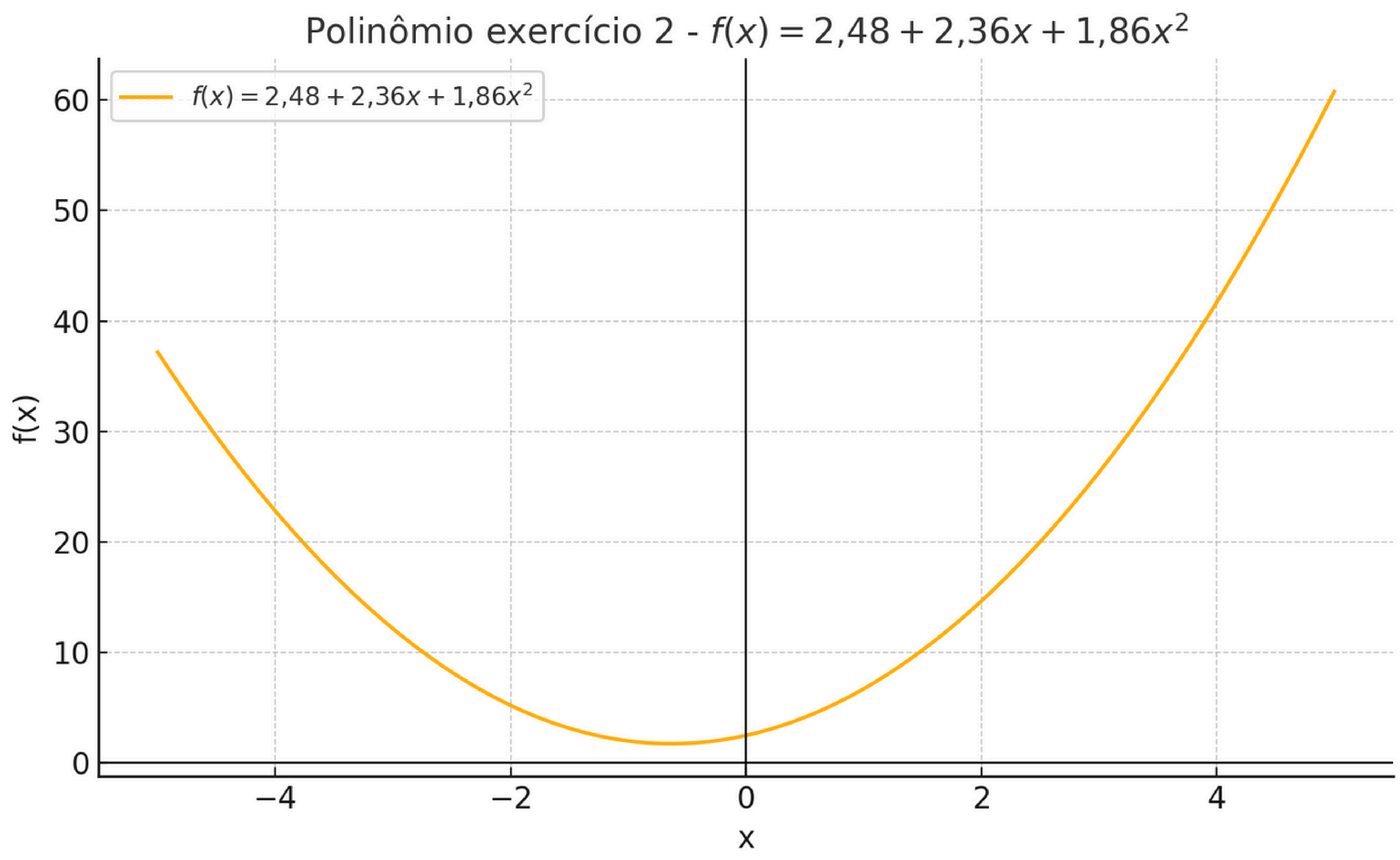
$$\to f_2(x) = +\frac{1{,}062}{6\cdot 2}(7-x)^3 + \frac{(-1{,}229)}{6\cdot 2}(x-4{,}5)^3 + \left(\frac{2{,}5}{2} - \frac{(-1{,}229)\cancel{2}}{\cancel{6}3}\right)(x-4{,}5) + \left(\frac{1}{2} - \frac{1{,}062\cdot\cancel{2}}{\cancel{6}3}\right)(7-x)$$

$$\boxed{f_2(x) = 0{,}0885(7-x)^3 - 0{,}102(x-4{,}5)^3 + 1{,}66(x-4{,}5) + 0{,}876(7-x)}$$

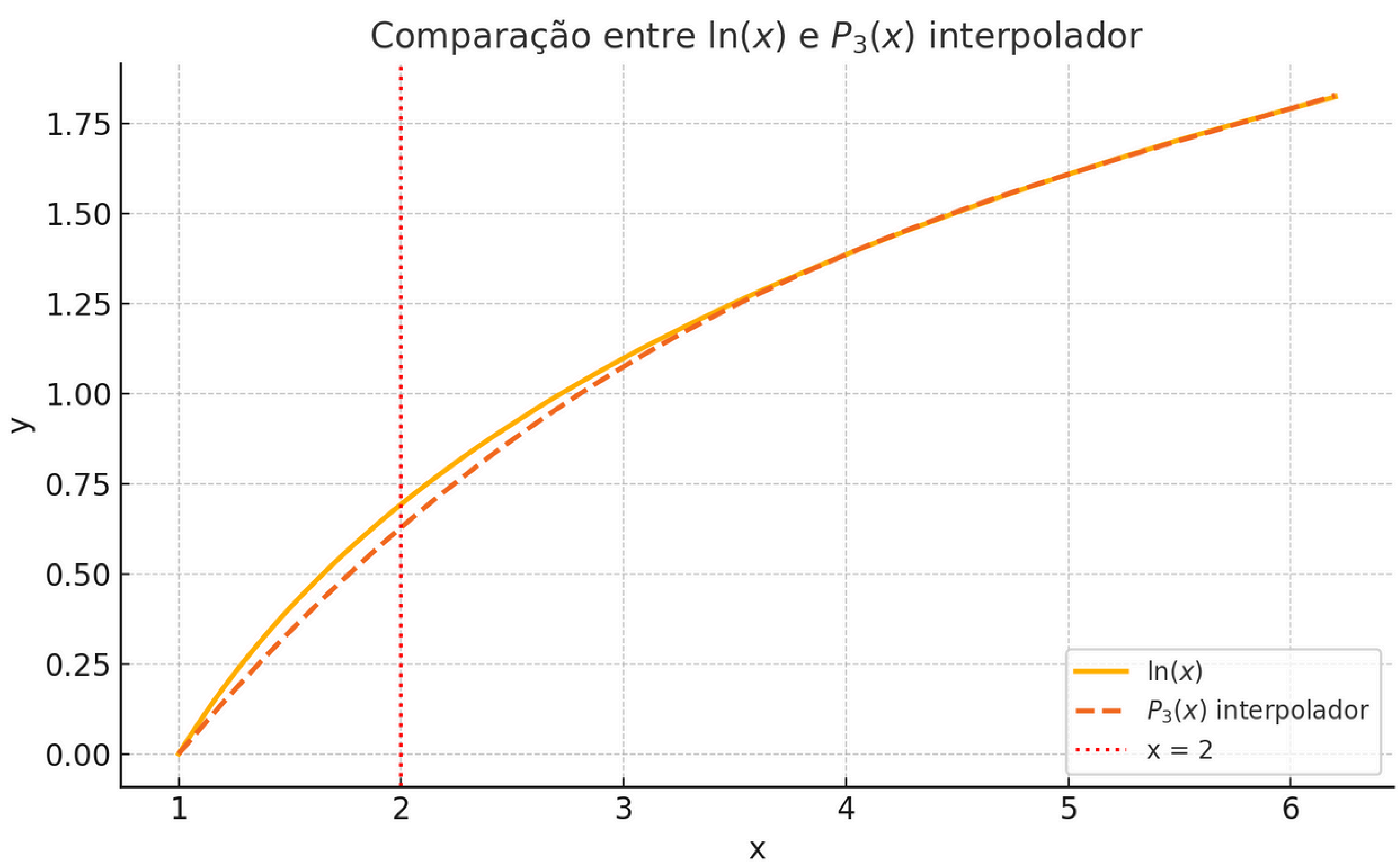
$$\to f_3(x) = \frac{(-1{,}229)}{6\cdot 2}(9-x)^3 + \cancel{\frac{0}{6\cdot 2}(x-7)^3}^{\,0} + \left(\frac{0{,}5}{2} - \cancel{\frac{0\cdot 2}{6}}\right)(x-7) + \left(\frac{2{,}5}{\cancel{2}} - \frac{(-1{,}229)\cancel{2}}{\cancel{6}3}\right)(9-x)$$

$$\boxed{f_3(x) = -0{,}102(9-x)^3 + \frac{1}{4}(x-7) + 1{,}66(9-x)}$$

Tópico 3.1 - Exercício 2



Tópico 5.2 - Exercício 3



Tópico 5.2 - Exercício 5

O erro permaneceu o mesmo pois eu defini meu quinto ponto como sendo x=2, e tentei qualcular o valor de ln(2) com o novo polinômio, tendo como saída o mesmo valor encontrado para o valor do polinômio com 4 pontos.