"UNIT-2"

"BOOLEAN ALGEBRA AND LOGIC GATES"

Boolean ALGEBRA →

Boolean algebra is used to analyze and simplify the digital circuits. Since it used only in binary Numbers ie 0 and 1. It is also called bool 'binary Algebra' or 'Logical Algebra'

→ It was invented by George Boole in year 1854

Imp **Rules**.

1. Variable used can have only two values. Binary 1 for High and Binary 0 for Low.
2. Symbols used in Boolen algenbra do not represent numerical values.
3. ORing of the variable is represented by a plus (+) sign b/w them.
4. Logical ANDing of two or more vairables is represent by writing a dot b/w them.
5. Arithematic operations are not performed in boolean Algebra. also there are no fractions, negative numbers square, square root etc.

Boolean function - Maps inputs from the set {0,1} to set {0,1}.

Boolean Expression - An algebraic statement of Boolean variables and operators.

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Basic Theorems of Boolen Algebra

Imp

Duality →

Acc. to this theorem-

1. change each AND operation to an OR operation.
2. change each OR operation to an AND operation.
3. Complement any 1 or 0 appearing in the expression.

Ex-① if the given expression is $A + 1 = 1$ then replace the OR (+) operation by (AND (·) operation and take complement of 1 write the dual of given relation as.

$$A \cdot 0 = 0$$

Ex- ② $A(B+C) = AB + AC$

Ans $A + (B \cdot C) = (A+B) \cdot (A+C)$

BASIC LAWS

(i) AND LAWS → These laws use the AND operation therefore they are called as 'AND' Laws

Summary -

1. $A \cdot 0 = 0$
2. $A \cdot 1 = A$
3. $A \cdot A = A$
4. $A \cdot \bar{A} = 0$

(2) OR LAWS :→ These laws use the OR operation. Therefore they are called as OR Laws.

Summary -

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A + A = A$
4. $A + \bar{A} = 1$

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(3) Inversion Law →

This law uses the "NOT" operation. The inversion law states that double inversion of a variable result in the original varible itself i.e.

$$\overline{\overline{A}} = A$$

(4) Associative Law →

A binary operator on a set S can be said to be associative if following equation is satisfied.

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$
$$(A + B) + C = A + (B + C)$$

(5) Commutative law → A binary operator on a set S is said to be commutative if following equation satisfied.

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

(6) Distributive law →

$$A \cdot (B + C) = AB + AC$$

Imp (7) Other important Laws →

$$A + BC = (A+B)(A+C)$$
$$\overline{A} + AB = \overline{A} + B$$
$$\overline{A} + A\overline{B} = \overline{A} + \overline{B}$$
$$A + AB = A$$
$$A + \overline{A}B = A + B$$

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V.V. Imp

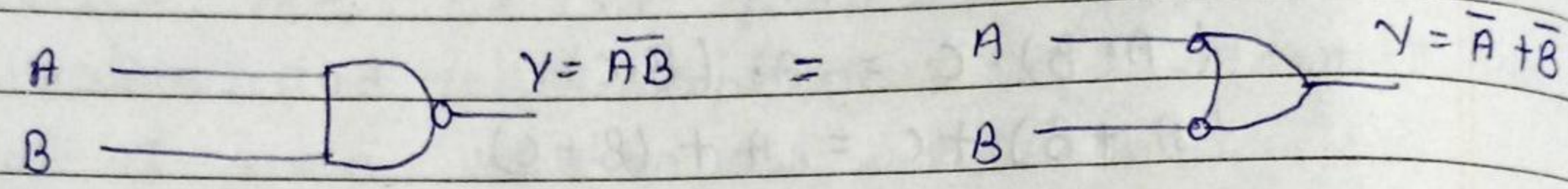
De-Morgan's Theorem

There are two theorem which are suggested by De-Morgan

1. $\overline{AB} = \bar{A} + \bar{B}$: NAND = Bubbled OR

This states that, the complement of a product of a varibles is equal to the sum of the complemented varibles.

applying first theorem to gates

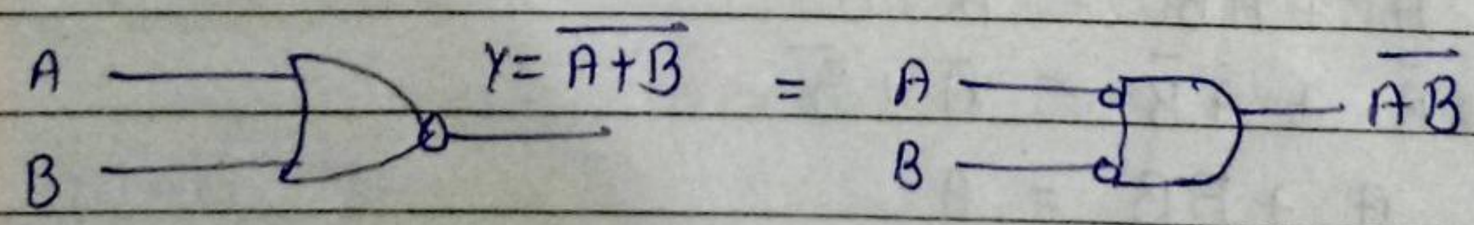


Input		Output	
A	B	$\overline{AB}$	$\bar{A}+\bar{B}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

2. $\overline{A+B} = \bar{A} \cdot \bar{B}$: NOR = Bubbled AND

This states that, The complement of a sum of varibles is equal to the product of the complemented varibles.

applying Second theorem to gates



Input		Output	
A	B	$\overline{A+B}$	$\overline{AB}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

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Examples

① Prove $A + AB = A$

L.H.S $A(1+B)$ But we know $1+B=1$

$= A \cdot 1$ & $A \cdot 1 = A$

$= A$ R.H.S

② $A + \bar{A}B = A + B$ Prove

L.HS $A + \bar{A}B$

$= A + AB + \bar{A}B$ $\therefore A = A + AB$

$= A + B(A + \bar{A})$ & $A + \bar{A} = 1$

$= A + B$ R.H.S

③ $(A+B)(A+C) = A + BC$ Prove

L.HS $(A+B)(A+C)$

$= (AA + BA + AC + BC)$

$= (A + AC + BA + BC)$ & $AA = A$

$= (A(1+C) + BA + BC)$ & $\because 1 + C = 1$

$= (A + AB + BC)$

$= A(1+B) + BC$ & $1 + B = 1$

$= A + BC$ R.H.S

④ $(A + \bar{B} + AB)(A + \bar{B})(\bar{A}B)$ Solve this

L.HS $\therefore A + AB = A$

$= (A+\bar{B})(A+\bar{B})(\bar{A}B)$

$= (A+\bar{B})(\bar{A}B)$ $\therefore$ & $\bar{A} \cdot A = A$

$= A\bar{A}B + \bar{B}\bar{A}B$ & $A\bar{A} = 0$ or $B\bar{B} = 0$

$= 0$ Ans

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(5) Simplify, $A\bar{B}C + \bar{A}BC + ABC$

L.H.S $AC(\bar{B}+B) + \bar{A}BC$ $\quad \{ \bar{B}+B = 1$

$= AC + \bar{A}BC$ $\quad \cancel{\{ 1+B = 1}$

$= C(A + \bar{A}B)$ Ans

(6) Simplify

$Y = (AB+C)(AB+D)$

Sol $Y = ABAB + ABD + ABC + CD$ $\quad \therefore \{ AA = A \quad B \cdot B = B$

$Y = AB + ABD + ABC + CD$

$Y = AB(1 + D + C) + CD$ $\quad \therefore \{ (1+D+C) = 1$

$Y = AB + CD$ Ans

(7) For the given function $F = x\bar{y} + x\bar{y}$, find the complement of F.

Sol- $F = x\bar{y} + x\bar{y}$ $\quad \{ A + A = A$

$F = x\bar{y}$

Take complement, $\bar{F} = \overline{x\bar{y}}$

$= \bar{x} + \bar{\bar{y}}$

$= \bar{x} + y$ Ans $\quad \{ \bar{\bar{y}} = y$

(8) Find the complement-

(1) $F = \bar{A}B\bar{C} + \bar{A}\bar{B}C$

Sol $\bar{F} = \overline{(\bar{A}B\bar{C} + \bar{A}\bar{B}C)}$

$= \overline{(\bar{A}B\bar{C})} \cdot \overline{(\bar{A}\bar{B}C)}$

$= (A + \bar{B} + C) \cdot (A + B + \bar{C})$ Ans $\quad \{ \bar{\bar{A}} = A$

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(ii) $F = A(\bar{B}\bar{C} + BC)$

Sol $\bar{F} = \overline{A(\bar{B}\bar{C} + BC)}$

$= [\bar{A} + \overline{(\bar{B}\bar{C} + BC)}]$

$= [\bar{A} + (\overline{\bar{B}\bar{C}}) \cdot (\overline{BC})]$

$= [\bar{A} + (B+C)(\bar{B}+\bar{C})]$ Ans

(9) $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$ Prove that

L.H.S $(A\cdot\bar{A} + AC + \bar{A}B + BC)(B+C)$

$= (AC) + \bar{A}B) + BC)(B+C)$ $\{A\cdot\bar{A} = 0$

$= ABC + \bar{A}BB + BBC + ACC + \bar{A}BC + BCC$

$= ABC + \bar{A}B + BC + AC + B\bar{A}C + BC$

$= BC(A+\bar{A}) + \bar{A}B + BC + AC$

$= BC + \bar{A}B + BC + AC$ $\{A+A = A$

$= \bar{A}B + BC + AC$

R.H.S $(A+B)(\bar{A}+C)$

$= (A\bar{A}) + \bar{A}B + AC + BC$ $\{A\cdot\bar{A} = 0$

$= \bar{A}B + AC + BC$

L.H.S = R.H.S

(10) To Prove -

$A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C = B+C$

L.H.S = $A\bar{B}C + B(1 + \bar{D} + A\bar{D}) + \bar{A}C$

$= A\bar{B}C + B(1 + \bar{D}(A+1)) + \bar{A}C$ $\{1+A = 1$

$= A\bar{B}C + B(1+\bar{D}) + \bar{A}C$ $\{1+\bar{D} = 1$

$= A\bar{B}C + B + \bar{A}C$

$= B + C(\bar{A} + A\bar{B})$

$= B + C(\bar{A} + \bar{B})$ Acc. to Imp law,

$= B + \bar{A}C + \bar{B}C \Rightarrow (B + \bar{B}C) + \bar{A}C$

$= B + C + \bar{A}C$ $\{B + \bar{B}C = B+C$

$= B + C(\bar{A}+1) \Rightarrow B + C$ R.H.S

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(11) Prove that,

$$AB + BC + \bar{A}C = AB + \bar{A}C$$

L.H.S $AB + BC + \bar{A}C$

$= AB + BC(A+\bar{A}) + \bar{A}C$ $\{\because A+\bar{A}=1$

$= AB + ABC + \bar{A}BC + \bar{A}C$

$= AB(1+C) + \bar{A}C(B+1)$

$= AB + \bar{A}C$ R.H.S $\{1+C=1$, $B+1=1$

(12) Prove that a dual of exclusive or is also its complement.

Sol- The Boolean Expression Ex-OR gate

$$Y = A \oplus B$$

i.e $Y = A\bar{B} + \bar{A}B$

Dual of EX-OR gate $Y = (A+\bar{B}) \cdot (\bar{A}+B)$

$= A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}$

$= 0 + AB + \bar{A}\bar{B} + 0$ $\{A\bar{A}=0$

$\therefore Y = AB + \bar{A}\bar{B} = \overline{A \oplus B}$

Imp "Concepts of Minterm and Maxterm"

Minterm →

A product term in which all the variables appear exactly once, either complemented or uncomplement.

→ Represents exactly one combination in truth table.

→ Denoted by m_j, where j is the decimal equivalent of minterm's corresponding binary combination (b_j).

→ A variable in m_j is complemented if its value in b_j is 0 otherwise is uncomplemented.

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Ex→ Assume 3 variables (A, B, C) and $j = 3$ then, $b_j = 011$ and its corresponding minterm is denoted by $m_j = A'BC$

Maxterm →

A sum term in which in which all the variables appear excatly once, either complemented or uncomplemented.

→ Represents excatly one combination in truth table.

→ Denoted by M_j, where j is decimal equivalent of the maxterm's corresponding binary combination (b_j).

→ A variable in M_j is complemented if its value in b_j is 1 otherwise is uncomplemented.

Ex→ Assume 3 variables (A, B, C), and $j = 3$, then $b_j = 011$ and its corresponding maxterm is denoted by $M_j = A + B' + C'$

"TRUTH TABLE for minterm Maxterms"

Variables			Minterms	Maxterms
A	B	C	m_i	M_i
0	0	0	$\bar{A}\bar{B}\bar{C} = m_0$	$A + B + C = M_0$
0	0	1	$\bar{A}\bar{B}C = m_1$	$A + B + \bar{C} = M_1$
0	1	0	$\bar{A}B\bar{C} = m_2$	$\bar{A} + \bar{B} + C = M_2$
0	1	1	$\bar{A}BC = m_3$	$A + \bar{B} + \bar{C} = M_3$
1	0	0	$A\bar{B}\bar{C} = m_4$	$\bar{A} + B + C = M_4$
1	0	1	$A\bar{B}C = m_5$	$\bar{A} + B + \bar{C} = M_5$
1	1	0	$AB\bar{C} = m_6$	$\bar{A} + \bar{B} + C = M_6$
1	1	1	$ABC = m_7$	$\bar{A} + \bar{B} + \bar{C} = M_7$

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Imp

$$\boxed{\begin{array}{l} \text{Minterm} = \Sigma m \\ \text{Maxterm} = \pi M \end{array}}$$

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Canonical forms (Unique) :

→ Any boolean Function F () can be expressed as a unique sum of minterms and a unique Product of maxterms.

→ every function F () has two canonical forms -

(1) Canonical Sum-of-Products (sum of minterms)
(2) Canonical Product-of-Sum (Product of maxterms)

★ Canonical Sum-of-Products → The minterms included are those m_j such that F() = 1 in row j of the truth table for F().

★ Canonical Product of Sum → The maxterms included are those M_j such that F() = 0 in row j of the truth table for F().

① For truth table of two variables write minterm and maxterm.

Solution

Variable		Minterm	Maxterm
A	B	m_i	M_i
0	0	$m_0 = \bar{A}\,\bar{B}$	$M_0 = A + B$
0	1	$m_1 = \bar{A}\,B$	$M_1 = A + \bar{B}$
1	0	$m_2 = A\bar{B}$	$M_2 = \bar{A} + B$
1	1	$m_3 = AB$	$M_3 = \bar{A} + \bar{B}$

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Representation of Minterms and Maxterms :-

① Given logic expression

$$Y = \underbrace{ABC}_{m_7} + \underbrace{\bar{A}BC}_{m_3} + \underbrace{A\bar{B}\bar{C}}_{m_4}$$ → corresponding minterms

$\therefore Y = m_7 + m_3 + m_4$

$\therefore Y = \Sigma m(3, 4, 7)$ → other way of representation

where Σ denotes sum of products.

② Given logic expression

$$Y = \underbrace{(A+\bar{B}+C)}_{M_2}\underbrace{(A+B+C)}_{M_0}\underbrace{(\bar{A}+\bar{B}+C)}_{M_6}$$ → corresponding maxterms

$Y = M_2 \quad M_0 \quad M_6$

$Y = \pi M(0, 2, 6)$ → other way of representation.

where π denots Product of Sum.

Note → Table is compulsory to learn for solving numerical problems.

Standard forms (NOT UNIQUE) :-

Standard forms are "like" canonical forms, except that not all variables need appears in the individual product (SOP) or sum (POS) terms.

Ex-

$f_1(a, b, c) = a'b'c + bc' + ac'$

is a standard sum of product form.

$f_1(a, b, c) = (a+b+c) * (b'+c') * (a'+c')$

is a standard Product of sum form.

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① $Y(A,B,C) = \Sigma m(0,2,4,7)$ convert this to SOP.

Solution - $Y(A,B,C) = m_0 + m_2 + m_4 + m_7$

$m_0 \to 000$, $m_2 \to 010$, $m_4 \to 100$, $m_7 \to 111$

$Y(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

② $Y(A,B,C) = A\bar{B}C + \bar{A}B + \bar{A}\bar{B}$

convert standard SOP to canonical SOP.

Solution $= A\bar{B}C + \bar{A}B(C+\bar{C}) + \bar{A}\bar{B}(C+\bar{C})$

$= A\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$

$A\bar{B}C \to 101$, $\bar{A}BC \to 011$, $\bar{A}B\bar{C} \to 010$, $\bar{A}\bar{B}C \to 001$, $\bar{A}\bar{B}\bar{C} \to 000$

$= \Sigma m(0,1,2,3,5)$

$Y(A,B,C) = \Sigma m(0,1,2,3,5)$

③ $Y(A,B,C) = (A+\bar{B}+C)(B+\bar{C})(A+B)$

convert Standard POS to canonical POS

Solution → $= (A+\bar{B}+C)(A\cdot\bar{A}+B+\bar{C})(A+B+C\cdot\bar{C})$

$= (A+\bar{B}+C)(A+B+\bar{C})(\bar{A}+B+\bar{C})(A+B+C)$
$(A+B+\bar{C})$

$= (A+\bar{B}+C)(A+B+\bar{C})(\bar{A}+B+\bar{C})(A+B+C)$

$(A+\bar{B}+C) \to 010$, $(A+B+\bar{C}) \to 001$, $(\bar{A}+B+\bar{C}) \to 101$, $(A+B+C) \to 000$

$Y(A,B,C) = M_0 M_1 M_2 M_5$

$Y(A,B,C) = \pi M(0,1,2,5)$

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(4) $F(A, B, C) = \Sigma m(1, 4, 5, 6, 7)$

convert into canonical POS form.

Solution → first convert into Maxterm

$F(A, B, C) = \pi M(0, 2, 3)$ (maxterm form)

$= M_0, M_2, M_3$

$= 000 \quad 010 \quad 011$

$F(A, B, C) = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C})$

This is canonical POS form.

To write canonical SOP Expression for a given Truth Table-

step-1 consider only those combinations of inputs which correspond to $Y = 1$

Step-2 Write down a product term for each such combination.

Step-3 OR all these products term to get the canonical SOP.

To write a Canonical POS Expression for a given Table-

step-1 consider only those combinations of inputs which produce a low output $(Y=0)$

Step-2 Write the maxterms only for such combination.

Step-3. AND these maxterms to obtain the expression in canonic POS form.

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① find minterm, maxterm, SOP and POS (canonical) by the truth table -

Input			Output
A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Solution-

Minterm,

$f(A,B,C) = \Sigma m(0,2,4,6)$

$= m_0 + m_2 + m_4 + m_6$

000 010 100 110

SOP

$f(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

Maxterm,

$f(A,B,C) = \Pi M(1,3,5,7)$

$= M_1 \cdot M_3 \cdot M_5 \cdot M_7$

001 011 101 111

POS

$f(A,B,C) = (A+B+\bar{C})(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$

Note for minterm always take those output whose value is 1.
for maxterm always take those output whose value is 0.

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(2) $Y = A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$, convert SOP (canonical) to standard SOP.

$$Y = A\bar{B}C + A\bar{B}\bar{C} + AB\bar{C}$$
$$= A\bar{B}(C+\bar{C}) + AB\bar{C}$$
$$= A\bar{B} + AB\bar{C}$$

This is Standard SOP.

Imp (3) Prove that -

$(A+B)(A+\overline{AB})C + \bar{A}(B+\bar{C}) + \bar{A}B + ABC = C(A+B) + \bar{A}(B+\bar{C})$

L.H.S $(A+B)(A+\overline{AB})C + \bar{A}(B+\bar{C}) + \bar{A}B + ABC$

$= (A+B)(A+\bar{A}+\bar{B})C + \bar{A}(B+\bar{C}) + \bar{A}B + ABC$ $\quad \{\overline{AB} = \bar{A}+\bar{B}$

$= (A+B)(1+\bar{B})C + \bar{A}(B+\bar{C}) + \bar{A}B + ABC$ $\quad \{A+\bar{A}=1$

$= (A+B)C + \bar{A}(B+\bar{C}) + \bar{A}B + ABC$ $\quad \{1+\bar{B}=1$

$= (A+B)C + \bar{A}(B+\bar{C}) + B(\bar{A}+AC)$

$= (A+B)C + \bar{A}(B+\bar{C}) + B(\bar{A}+C)$ $\quad \{\bar{A}+AC = \bar{A}+C$

$= AC + BC + \bar{A}B + \bar{A}\bar{C} + \bar{A}B + BC$

$= AC + BC + \bar{A}B + \bar{A}\bar{C}$ $\quad \{A+A=A$

$= C(A+B) + \bar{A}(B+\bar{C}) =$ R.H.S

(4) Find SOP form of $f = AB + BC + \bar{A}$

Solution-

$$f = AB + BC + \bar{A}$$
$$= AB(C+\bar{C}) + BC(A+\bar{A}) + \bar{A}(B+\bar{B})(C+\bar{C})$$
$$= ABC + AB\bar{C} + ABC + \bar{A}BC + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$f = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

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(5) Prove that if $A+B = A+C$ and $A'+B = A'+C$ then $B=C$

Solution

$$A+B = A+C \quad -(i)$$
$$\bar{A}+B = \bar{A}+C \quad -(2)$$

adding equation (1) and (2)

$(\because A+\bar{A}=1)$

$$1+2B = 1+2C$$
$$2B = 2C$$
$$\underline{\underline{B = C}}$$

"KARNAUGH MAP"

+ This is another simplification technique to reduce the Boolean equation.
→ K-Maps are graphical representations of boolean functions
→ K-Maps can be written for 2, 3, 4 --- upto 6 variables.
→ one map cell corresponds to a row in truth table.
→ Also, one map cell corresponds to a minterm or a maxterm in the boolean expression.
→ Multiple-cell areas of the map corresponed to Standard terms.

K-Map

→ 2 Variable → AB $(2^2=4)$
→ 3 Variable → ABC $(2^3=8)$
→ 4 Variable → ABCD $(2^4=16)$

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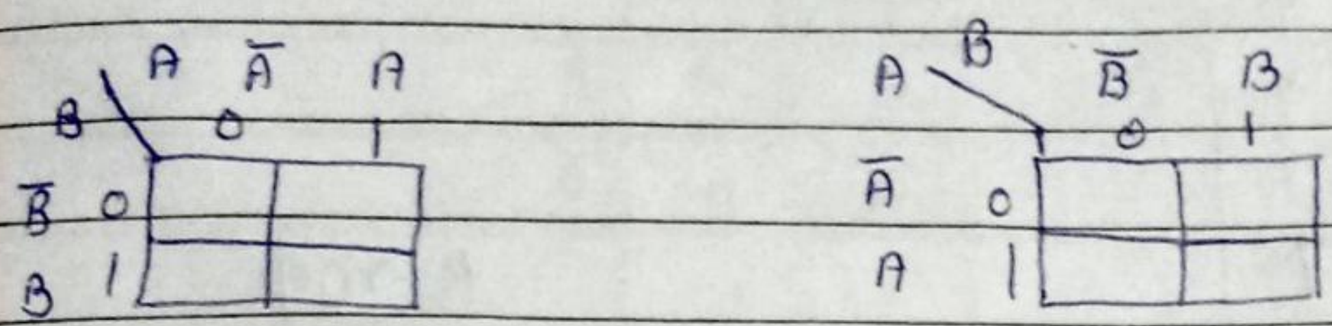
HARSH JINDAL

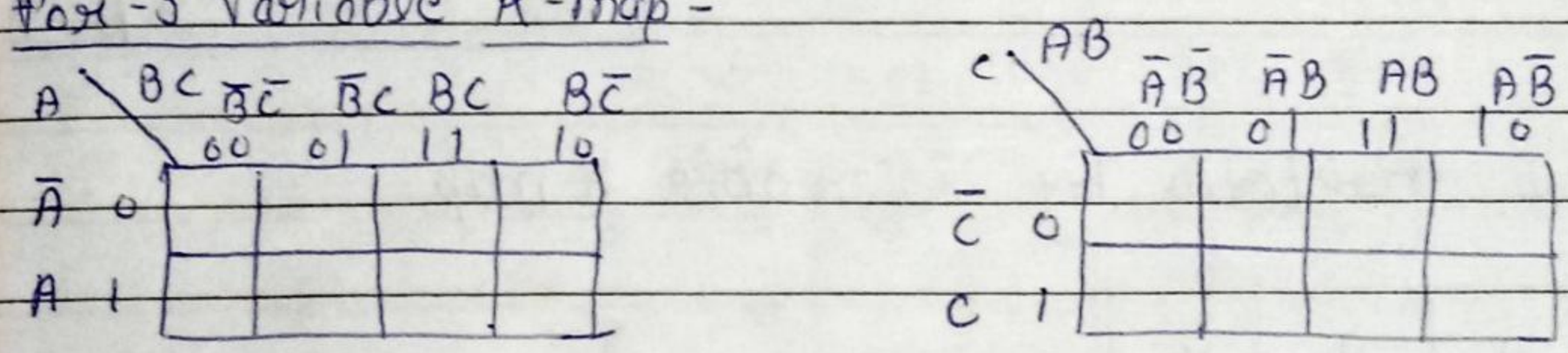
"Structures"

for - 2 Variable K-map -



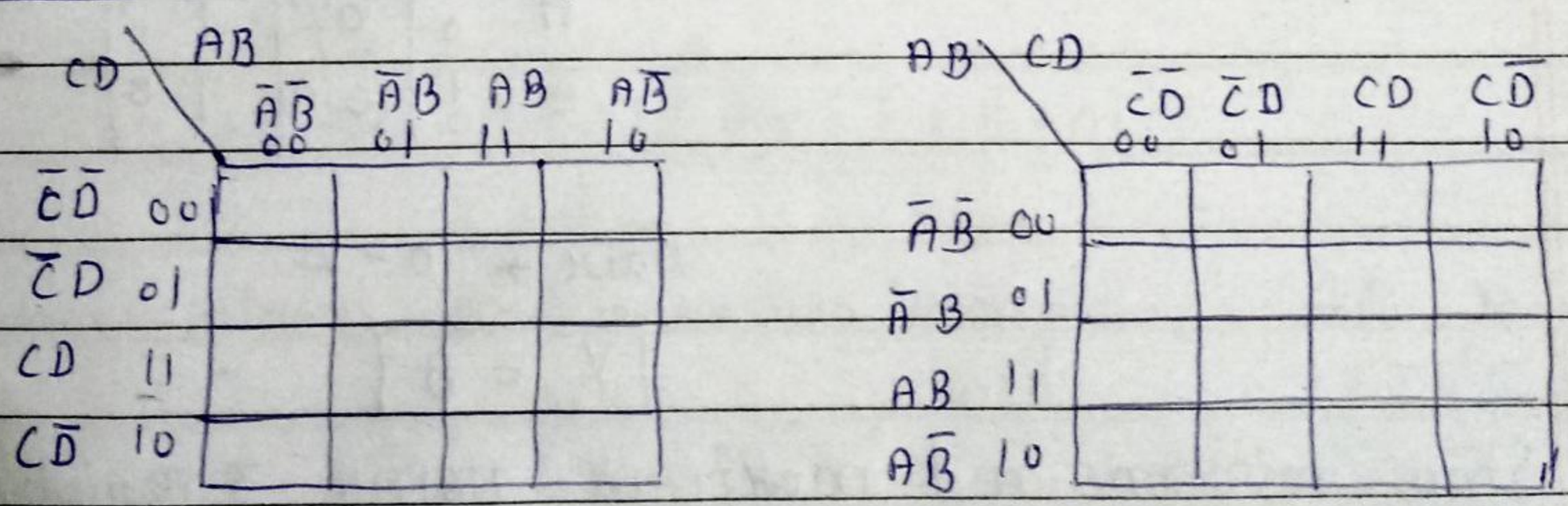
- A and B are the inputs or Variables.
- 0 and 1 are value of A or B.
- inside 4 boxes we have to enter the value of Y i.e output.

For - 3 Variable K-map -



- The positions of variables, A, B, C are interchangeable.

For - 4 Variable K-map -



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① Show minterm by 2 Variable K-map.

Solution -

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	0

K-map

A \ B	$\bar{B}$ (0)	B (1)
$\bar{A}$ (0)	0 (0)	1 (1)
A (1)	1 (2)	0 (3)

pair → 1-2

$$\boxed{Y = \bar{A}B + A\bar{B}}$$

② Show maxterm by 2variable K-map.

Solution -

	A	B	Y
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	1

K-map

A \ B	B_0 (0)	$\bar{B}$ (1)
A (0)	0 (0)	1 (1)
$\bar{A}$ (1)	0 (2)	1 (3)

pair → 0-2

$$\boxed{Y = B}$$

③ Show minterm and maxterm using 3 variable K-map.

	A	B	C	Y
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

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K-map for mintcrm,

A \ BC	$\bar{B}\bar{C}$ (00)	$\bar{B}C$ (01)	BC (11)	$B\bar{C}$ (10)
$\bar{A}$ 0	0 (0)	1 (1)	1 (3)	0 (2)
A 1	1 (4)	0 (5)	0 (7)	0 (6)

mintcrm show only by output value - 1 i.e 1, 3, 4
so pairs = 1-3 , 4

$$\boxed{Y = \bar{A}C + A\bar{B}\bar{C}}$$

K-map for Maxterm,

A \ BC	$B+C$ (00)	$B+\bar{C}$ (01)	$\bar{B}+\bar{C}$ (11)	$\bar{B}+C$ (10)
A 0	0 (0)	1 (1)	1 (3)	0 (2)
$\bar{A}$ 1	1 (4)	0 (5)	0 (7)	0 (6)

maxterm show only by output value - 0 i.e. 0-2-5-6-7
so pairs, = 0-2 , 5-7 , 2-6

$$\boxed{Y = (A+C)(\bar{A}+\bar{C})(\bar{B}+C)}$$

(4) Show mintcrm and maxterm using 4-variable K-map-

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1

	A	B	C	D	Y
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

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for minterm

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	1 (1)	1 (3)	0 (2)
$\bar{A}B$ 01	0 (4)	1 (5)	1 (7)	1 (6)
AB 11	1 (12)	1 (13)	0 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	0 (11)	0 (10)

Pairs = 1-3-5-7, 12-13, 6-7

1-3-5-7 = $\bar{A}D$

12-13 = $AB\bar{C}$

6-7 = $\bar{A}BC$

so $\boxed{Y = \bar{A}D + AB\bar{C} + \bar{A}BC}$

for Maxterm

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0 (0)	1 (1)	1 (3)	0 (2)
$A+\bar{B}$ 01	0 (4)	1 (5)	1 (7)	1 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	1 (13)	0 (15)	0 (14)
$\bar{A}+B$ 10	0 (8)	0 (9)	0 (11)	0 (10)

Pairs = 8-9-10-11, 10-11-14-15, 0-2, 8-10, 0-4

8-9-10-11 = $\bar{A}+B$

10-11-14-15 = $\bar{A}+\bar{C}$

0-2-8-10 = $B+D$

0-4 = $A+C+D$

so $Y = (\bar{A}+B)(\bar{A}+\bar{C})(B+D)(A+C+D)$

Note → in all above Numerical Problems, in all table output will be taken randomely, so don't be confuse

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(5) $Y = B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + ABCD$

Reduce following using K-map.

Solution -

$(A+\bar{A})B\bar{C}\bar{D} = \underbrace{AB\bar{C}\bar{D}}_{1100 \;\; 12} + \underbrace{\bar{A}B\bar{C}\bar{D}}_{0100 \;\; 4}$

$\bar{A}B\bar{C}D = 0101 \rightarrow 5$

$AB\bar{C}D = 1101 \rightarrow 13$

$\bar{A}BCD = 0101 \rightarrow 7$

$ABCD = 1111 \rightarrow 15$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	0 (1)	0 (3)	0 (2)
$\bar{A}B$ 01	1 (4)	1 (5)	1 (7)	0 (6)
AB 11	1 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	0 (11)	0 (10)

pairs - 1-5-12-13 , 5-7-13-15

$$\boxed{Y = B\bar{C} + BD}$$

(6) $Y(A,B,C,D) = \Sigma m(3,5,9,11,13,15)$

Simplify using K-Map.

Solution -

Pairs

13-15-9-11 $\rightarrow AD$

5-13 $\rightarrow B\bar{C}D$

3-11 $\rightarrow \bar{B}CD$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	0 (1)	1 (3)	0 (2)
$\bar{A}B$ 01	0 (4)	1 (5)	0 (7)	0 (6)
AB 11	0 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	1 (9)	1 (11)	0 (10)

$$\boxed{Y = AD + B\bar{C}D + \bar{B}CD}$$

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(7) $Y = (A+B+C+D)(\bar{A}+B+\bar{C}+D)(A+\bar{B}+\bar{C}+\bar{D})$
$(A+B+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C+\bar{D})(A+\bar{B}+C+\bar{D})$

Simplify using K-Map.

Solution -

$A+B+C+D = 0000 \rightarrow 0$
$\bar{A}+B+\bar{C}+D = 1010 \rightarrow 10$
$\bar{A}+\bar{B}+C+\bar{D} = 0101 \rightarrow 5$
$A+\bar{B}+\bar{C}+\bar{D} = 0111 \rightarrow 7$
$A+B+\bar{C}+D = 0011 \rightarrow 3$
$\bar{A}+\bar{B}+C+\bar{D} = 1101 \rightarrow 13$

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0 (0)	1 (1)	0 (3)	1 (2)
$A+\bar{B}$ 01	1 (4)	0 (5)	0 (7)	1 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	0 (13)	1 (15)	1 (14)
$\bar{A}+B$ 10	1 (8)	1 (9)	1 (11)	0 (10)

Pairs - $5-13 = \bar{B}+C+\bar{D}$
$3-7 = A+\bar{C}+\bar{D}$
$0-10 = B+D$

$Y = (\bar{B}+C+\bar{D})(A+\bar{C}+\bar{D})(B+D)$

(8) $Y(A,B,C,D) = \Sigma m(0,3,5,7,11,15)$

Simplify using K-Map.

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	0 (1)	1 (3)	0 (2)
$\bar{A}B$ 01	0 (4)	1 (5)	1 (7)	0 (6)
AB 11	0 (12)	0 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	1 (11)	0 (10)

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Pair → $0 = \bar{A}\bar{B}\bar{C}\bar{D}$

$3-7-5-11 = CD$

$5-7 = \bar{A}BD$

$$\boxed{Y = \bar{A}BD + CD + \bar{A}\bar{B}\bar{C}\bar{D}}$$

(9) $Y = (A+B+C+D)(\bar{A}+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})(A+\bar{B}+C+\bar{D})$
$(\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})$

Simplify using K-Map.

Solution- $A+B+C+D = 0000\ (0)$

$\bar{A}+B+\bar{C}+D = 1010\ (10)$

$A+B+\bar{C}+\bar{D} = 0011\ (3)$

$\bar{A}+\bar{B}+C+\bar{D} = 0101\ (5)$

$\bar{A}+\bar{B}+\bar{C}+D = 1111\ (15)$

$A+\bar{B}+\bar{C}+\bar{D} = 0111\ (7)$

AB \ CD		C+D	C+$\bar{D}$	$\bar{C}+\bar{D}$	$\bar{C}+D$
		00	01	11	10
A+B	00	0 (0)	1 (1)	0 (3)	1 (2)
A+$\bar{B}$	01	1 (4)	0 (5)	0 (7)	1 (6)
$\bar{A}+\bar{B}$	11	1 (12)	1 (13)	0 (15)	1 (14)
$\bar{A}$+B	10	1 (8)	1 (9)	1 (11)	0 (10)

Pair → $0-10 = B+D$

$3-7 = A+\bar{C}+\bar{D}$

$5-7 = A+\bar{B}+\bar{D}$

$7-15 = \bar{B}+\bar{C}+\bar{D}$

$$\boxed{Y = (B+D)(A+\bar{C}+\bar{D})(A+\bar{B}+\bar{D})(\bar{B}+\bar{C}+\bar{D})}$$

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(10) $Y = \pi M(0, 2, 5, 7, 9, 11, 13)$
simplify using k-Map.

Solution -

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0 (0)	1 (1)	1 (3)	0 (2)
$A+\bar{B}$ 01	1 (4)	0 (5)	0 (7)	1 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	0 (13)	1 (15)	1 (14)
$\bar{A}+B$ 10	1 (8)	0 (9)	0 (11)	1 (10)

Pairs - 0 - 2 = $(A+B+D)$
5 - 13 = $(\bar{B}+C+\bar{D})$
4 - 11 = $(\bar{A}+B+\bar{D})$
5 - 7 = $(A+\bar{B}+\bar{D})$

$$\boxed{Y = (A+B+D)(\bar{A}+B+\bar{D})(A+\bar{B}+\bar{D})(\bar{B}+C+\bar{D})}$$

5-Marks Imp

(11)

	A	B	C	D	Y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	0
15	1	1	1	1	0

find
Draw 4-K map
for (Minterm / SOP)
and
for (Maxterm / POS)

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for minterm → taking which show output-1

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0 (0)	1 (1)	1 (3)	1 (2)
$\bar{A}B$ 01	1 (4)	0 (5)	1 (7)	0 (6)
AB 11	1 (12)	1 (13)	0 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	1 (11)	0 (10)

Pairs

1-3 = $\bar{A}\bar{B}D$

2-3 = $\bar{A}\bar{B}C$

3-7 = $\bar{A}CD$

3-11 = $\bar{B}CD$

4-12 = $B\bar{C}\bar{D}$

12-13 = $AB\bar{C}$

$Y = \Sigma m(1,2,3,4,7,11,12,13)$

$Y = \bar{A}\bar{B}D + \bar{A}\bar{B}C + \bar{A}CD + \bar{B}CD + B\bar{C}\bar{D} + AB\bar{C}$

for Maxterm → taking which show output-0

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0 (0)	1 (1)	1 (3)	1 (2)
$A+\bar{B}$ 01	1 (4)	0 (5)	1 (7)	0 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	1 (13)	0 (15)	0 (14)
$\bar{A}+B$ 10	0 (8)	0 (9)	1 (11)	0 (10)

Pairs

$5 = A+\bar{B}+C+\bar{D}$

$0-10 = B+D$

$6-14 = \bar{B}+\bar{C}+D$

$14-15 = \bar{A}+\bar{B}+\bar{C}$

$8-9 = \bar{A}+B+C$

$Y = \Pi M(0,5,6,8,9,10,14,15)$

$$\boxed{Y = (A+\bar{B}+C+\bar{D})(B+D)(\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+\bar{C})(\bar{A}+B+C)}$$

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Don't Care Conditions :-

In such situation, we have a freedom to assume a 0 or 1 as output for each of these combinations. These conditions are known as the "Don't care conditions" and in the K-map it is represented as x (cross) marks in corresponding cell.

- for SOP form, we enter 1's corresponding to the combinations of input variables which produce a high output, and we shell enter 0's in the remaining cells of the K-map.
- for the POS form we enter 0's corresponding to the combinations of inputs which produce a low output and enter 1's in the remaining cells of the K-map.
- But it is not always true that the cells not containing 1's (in SOP) will contain 0's. because some combinations of input varible do not occur.
- Also for some functions the outputs corresponding to certain combination of input variables do not matter.

Imp **Note** The don't care condition (x) may be assumed to be 0 or 1 as per the need for simplifications.

① Simplify -

$$Y(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$$

Sol-

Pairs

0-1-2-3 = $\bar{A}\bar{B}$

3-7-15-11 = CD

$$\boxed{Y = \bar{A}\bar{B} + CD}$$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	X 0	1 1	1 3	X 2
$\bar{A}B$ 01	0 4	X 5	1 7	0 6
AB 11	0 12	0 13	1 15	0 14
$A\bar{B}$ 10	0 8	0 9	1 11	0 10

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② Simplify using K-Map-

$F(A, B, C, D) = \Sigma m(0, 4, 5, 6, 7, 11, 12, 13, 14, 15) + \Sigma d(2, 3)$

solution-

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1 (0)	0 (1)	X (3)	X (2)
$\bar{A}B$ 01	1 (4)	1 (5)	1 (7)	1 (6)
AB 11	1 (12)	1 (13)	1 (15)	1 (14)
$A\bar{B}$ 10	0 (8)	0 (9)	1 (11)	0 (10)

Pairs -

$0-4 = \bar{A}\bar{C}\bar{D}$

$4-5-6-7-12-13-14-15$
$= B$

$3-7-11-15 = CD$

$$\boxed{F = CD + B + \bar{A}\bar{C}\bar{D}}$$

③ Obtain POS Expression for-

$F(A, B, C, D) = \Sigma m(2, 4, 6, 7, 8, 9, 12, 13) + d(4, 10, 14$

Solution First write Expression for maxterm

$F(A, B, C, D) = \pi M(0, 1, 5, 11, 15) \cdot d(4, 10, 14)$

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	0 (0)	0 (1)	1 (3)	1 (2)
$A+\bar{B}$ 01	X (4)	0 (5)	1 (7)	1 (6)
$\bar{A}+\bar{B}$ 11	1 (12)	1 (13)	0 (15)	X (14)
$\bar{A}+B$ 10	1 (8)	1 (9)	0 (11)	X (10)

Pairs -

$0-1-4-5 = A+C$

$10-11-14-15 = \bar{A}+\bar{C}$

$$\boxed{Y = (A+C)(\bar{A}+\bar{C})}$$

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(4) $Y = \Sigma m(0, 2, 9, 11, 13, 15) + \Sigma d(1, 3, 4, 5)$

Simplify using K-Map in SOP form.

Solution-

AB \ CD	$\bar{C}\bar{D}$ (00)	$\bar{C}D$ (01)	CD (11)	$C\bar{D}$ (10)
$\bar{A}\bar{B}$ 00	1 (0)	X (1)	X (3)	1 (2)
$\bar{A}B$ 01	X (4)	X (5)	0 (7)	0 (6)
AB 11	0 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	1 (9)	1 (11)	0 (10)

Pairs-

0-1-2-3 = $\bar{A}\bar{B}$

9-11-13-15 = AD

$$\boxed{Y = AD + \bar{A}\bar{B}}$$

(5) $Y = \Sigma m(0, 2, 9, 11, 13, 15) + \Sigma d(1, 3, 4, 5)$

Simplify using K-Map in POS form.

Solution- $Y = \Pi M(6, 7, 8, 10, 12, 14) \cdot \Pi d(1, 3, 4, 5)$

AB \ CD	$C+D$ (00)	$C+\bar{D}$ (01)	$\bar{C}+\bar{D}$ (11)	$\bar{C}+D$ (10)
$A+B$ 00	1 (0)	X (1)	X (3)	1 (2)
$A+\bar{B}$ 01	X (4)	X (5)	0 (7)	0 (6)
$\bar{A}+\bar{B}$ 11	0 (12)	1 (13)	1 (15)	0 (14)
$\bar{A}+B$ 10	0 (8)	1 (9)	1 (11)	0 (10)

Pairs-

8-10-12-14 = $\bar{A}+D$

4-5-6-7 = $A+\bar{B}$

$Y = (A+\bar{B})(\bar{A}+D)$

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(6) $Y = \pi M(3,4,7,8,9,12,13) \cdot \pi d(1,6,10)$

simplify using K-map in POS form

Solution -

AB \ CD	$C+D$ 00	$C+\bar{D}$ 01	$\bar{C}+\bar{D}$ 11	$\bar{C}+D$ 10
$A+B$ 00	1 (0)	X (1)	0 (3)	1 (2)
$A+\bar{B}$ 01	0 (4)	1 (5)	0 (7)	X (6)
$\bar{A}+\bar{B}$ 11	0 (12)	0 (13)	1 (15)	1 (14)
$\bar{A}+B$ 10	0 (8)	0 (9)	1 (11)	X (10)

Pairs

$3-7 \rightarrow A+\bar{C}+\bar{D}$

$4-6 \rightarrow A+\bar{B}+D$

$8-9-12-13 \rightarrow \bar{A}+C$

$$\boxed{Y = (\bar{A}+C)(A+\bar{C}+\bar{D})(A+\bar{B}+D)}$$

(7) $Y = \Sigma m(2,4,6,7,9,11,13,15) + \Sigma d(0,1,5)$

simplify using K-map in SOP form

Solution -

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	X (0)	X (1)	0 (3)	1 (2)
$\bar{A}B$ 01	1 (4)	X (5)	1 (7)	1 (6)
AB 11	0 (12)	1 (13)	1 (15)	0 (14)
$A\bar{B}$ 10	0 (8)	1 (9)	1 (11)	0 (10)

Pairs-

$9-11-13-15 = AD$

$4-5-6-7 = \bar{A}B$

$0-2-4-6 = \bar{A}\bar{D}$

$$\boxed{Y = \bar{A}\bar{D} + \bar{A}B + AD}$$

"THANK-YOU"

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