ELECTRICAL AND COMPUTER ENGINEERING

Room D36, Head Hall

LABORATORY / ASSIGNMENT / REPORT
COVER PAGE

Course # : _____ECE2701_____________ Experiment # or Assignment # ___Lab 2___

Date: ____Friday, October 25________ Course Instructor: ____Howard Li____________

Title: _______EE2701 Experiment 2 – Voltage Source Equivalence__________________
(if applicable)

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Name of Author: ___Justen Di Ruscio & Stephen Cole_______
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Signature of Author: ____
Student Name (signature)

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Comments:

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Grade

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Initials of Marker

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Contents

Abstract

Any network of resistors can be simplified into an equivalent single resistance. This equivalent resistor exhibits the same properties between the terminals as the network, and allows for a single representation of the entire network and its effect in the circuit [1] [2]. In addition, any single linear voltage source may contain a network of discrete or intrinsic resistances. Tying this together with the fact that any resistor network can be modelled as a single equivalent resistance, any linear voltage source can be modelled as an ideal voltage source in series with a single series resistance; this is *Thevenin's Theorem* [2] [3]. As a consequence, the terminal voltage of the voltage source varies based on the current drawn [3]. Understanding this, linear voltage sources are often characterized by their v-i characteristics - the drop in terminal voltage for a given increase in current draw [3]. This lab examines the equivalence between voltage sources using Thevenin's Theorem.

1 Introduction

In EE2701 Experiment 2 - Voltage Source Equivalence, we examined voltage source equivalence using Thevenin's Theorem. The background needed for this lab includes basic circuit analysis techniques, such as the operation of a digital multimeter(DMM), the ability to measure current and voltage in a circuit, and simplification techniques of resistor networks. The theory required to complete this experiment is Thevenin's Theorem, alongside basic concepts such as Ohm's Law and Kirchoff's Laws. These concepts are used to convert more complex circuits into a single supply, single resistor circuit which can easily be solved. In this lab, two source networks will be compared. Each consists of a DC supply, which is assumed to have no internal resistance, and a network of discrete resistors. The $v - i$ characteristics of the two sources are measured by measuring the source characteristics under no load, a 100Ω load, and a 200Ω load. Using the above theories and techniques, the following values were extrapolated: a Thevenin voltage of 5.054 V for Source 1 and 5.017 V for Source 2. Along with Thevenin Resistance of 149.58Ω for Source 1 and 148.121Ω for Source 2, these values resulted in a $V_L = 3.246v$ for Source 1 and $v_L = 3.238V$ for Source 2. After comparing the values with the percent difference formula, the two sources were determined to be ideal and Thevenin's Theorem held.

2 Experiment

2.1 Apparatus

2.1.1 Instruments

The instruments used in this lab are listed below [3]:

Elenco XP-720 DC Power-Supply A dual output DC power-supply used to deliver power to the circuit under test. This device included a fixed $5V$ rail, and a variable output rail, which was set to $7.35V$ in this experiment.

Banana Connectors 11 Banana Connectors were required. Nine of these were used to make electrical connections between the components of the Experimental Sources, and the remaining two were used as probes on the multi-meter.

Digital Multimeter To calibrate the power supply, and to measure the load voltages, a digital multimeter was used.

Resistor Blocks For convenience over bare resistor components, resistor blocks were used in the Test Network. These consisted of a resistor connected to two female banana plugs, all embedded within a small block of wood. Five different resistor blocks were needed to build the resistor networks, these consisted of the following values:

- 100Ω - R_x
- 150Ω - R_1
- 220Ω - R_2
- 270Ω - R_x
- 470Ω - R_3

These pieces of equipment slightly vary from that listed in the lab manual. Instead of resistor blocks, single resistors are listed in the lab manual, which do not contain female banana plugs. In addition, banana connectors were completely omitted from the lab manual. This discrepancy introduced resistances which the lab procedure did not account for, introducing errors in the results [3].

2.1.2 Test Networks

External resistors are connected to a DC power-supply, assuming it has zero series resistance, to create the following two experimental voltage sources.

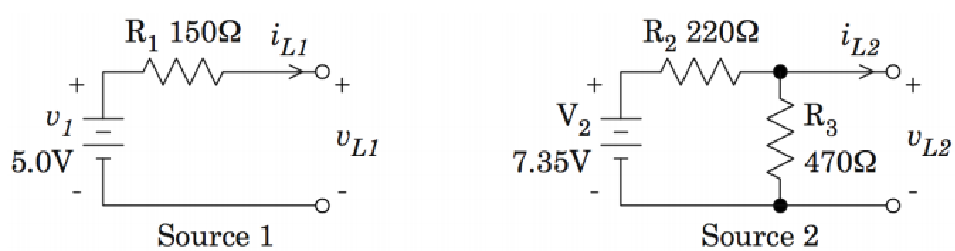


Figure 1: Schematic Diagram of the Experimental Sources [3]

2.2 Descriptions

2.2.1 Theory

This lab depends upon the understanding that resistor networks can be simplified into a single equivalent resistor. This equivalent resistor exhibits the same properties between its terminals as that of the network, thereby allowing for a single representation of the entire network and its effects on the circuit [1] [2]. Kirchoff's voltage and current laws are the fundamental principles behind the techniques in circuit simplification, which was examined and proved in the previous lab [2].

As a consequence of the ability to simplify resistor networks, the network of discrete or intrinsic resistances within a voltage source can be modelled as a single equivalent resistance [2]. This leads to *Thevenin's Theorem*, which states that any linear voltage source can be modelled as an ideal voltage source, in series with the internal equivalent resistance [3]. An illustration of Thevenin's Theorem is shown in figure 2.

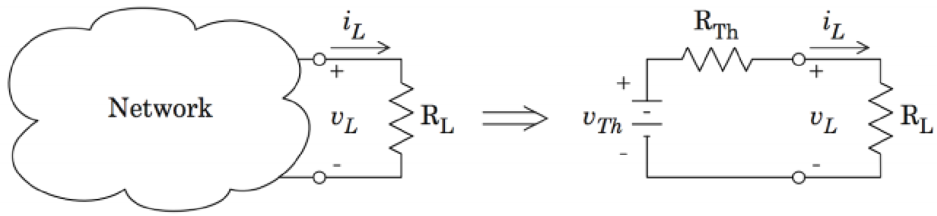


Figure 2: Illustration of Thevenin's Theorem [3]

Real voltage sources are characterized by their v-i characteristics, which is a plot relating the terminal voltage to the current drawn. This relation can be expressed mathematically as follows:

$$v_L = v_s - R_{int} i_L$$

Where the load voltage is related to the ideal source voltage, minus the voltage drop across the equivalent internal resistance. As such, voltage sources which exhibit the same v-i characteristics are considered equivalent. This lab utilizes this definition of equivalence to compare multiple voltage sources.

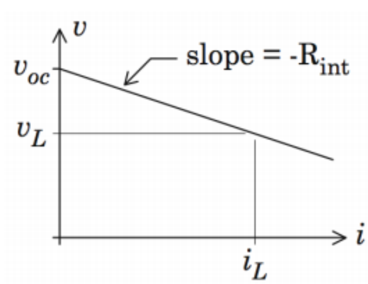


Figure 3: Example $v - i$ Characteristics [3]

Comparisons between quantities of the two sources in this lab are quantified using the formula for percent difference, shown below:

$$\%Difference = \frac{source2_{value} - source1_{value}}{source1_{value}} \times 100\%$$

Measuring Source Characteristics To measure the source characteristics of a linear voltage source, the terminal voltage is first measured without a load attached, producing v_{o1}. Since the multi-meter provides a significantly large resistance, $10M\Omega$, this measurement effectively the Thevenin voltage, $V_{Th} \approx v_{o1}$. Next, a known load can be applied and the load voltage, v_{o2}, can be measured to examine how it drops while undergoing a current draw. The following equation provides the equivalent series Thevenin resistance, based on the two measurements of load voltage:

$$R_{Th} = \left(\frac{v_{o1}}{v_{o2}} - 1 \right) R_x$$

This is derived from the equation describing the voltage divider formed between R_x and R_{Th}.

$$v_{o2} = \frac{R_x}{R_{Th} + R_x} v_{o1}$$

Figure 4 depicts the described method from the above paragraph:

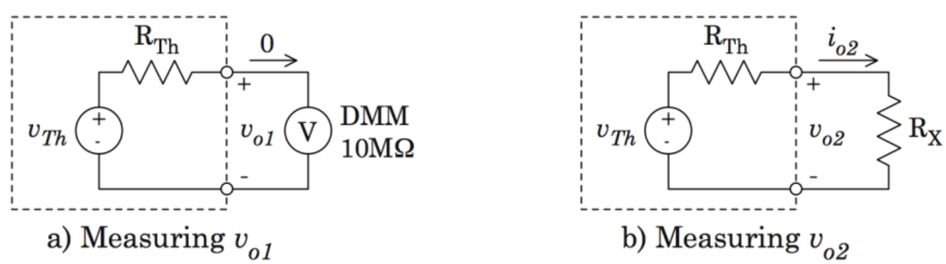


Figure 4: Method for Measuring Source Resistance [3]

2.2.2 Procedure

1. After acquiring the listed materials, the two sources depicted in figure 1 were constructed. Before connecting them to the DC power-supply, the variable output of the supply was set to 7.35 V.

2. Using the method described to measure the source characteristics in the Theory section, the source characteristics of each source was measured. A 100Ω resistor was used for R_x, and the respective measured values of R_x, v_{o1}, and v_{o2} were recorded in table 2.

3. Using the assumption that $v_{o1} = v_{Th}$, since $i_L \approx 0$, the 'measured' values of V_{Th} were recorded in the appropriate cells of tables 3 4. The percent difference formula was used to compare the 'measured' values between the two sources. Results of this comparison was listed in table 3, including a comment of the results.

4. Next, the 'measured' values of the Thevenin equivalent resistances for each source, R_{Th} were calculated and recorded in table 3. The values of R_{Th} between the two sources were then compared and commented on, which were then recorded in table 3.

5. After this, each source was loaded with a 270Ω resistor, and the load voltage that developed from each source was measured and recorded in table 3. Comparisons and comments on the two 'measured' values were carried out and recorded in table 3.

6. Using the results from the above measurements, the two sources were compared for equivalence. It was realized that the results above described the $v - i$ characteristics of each source. Comparing these characteristics therefore determined whether the voltage sources were equivalent.

7. Finally, table 4 was filled with the calculated values and the measured results from the lab. Comparisons between the calculated and the measured values were carried out - the results of which can be found in table 4. With these results, the equivalence between the Thevenin Equivalent circuit and the networks could be carried out, allowing the validation of Thevenin's Theorem.

8. Once the lab was complete, the aparatus was cleaned and powered down.

The procedure follows that outlined in the lab manual [3].

2.3 Calculations and Results

Before any lab activities were carried out, the sources depicted in figure 1 were analyzed for their Thevenin voltages, Thevenin resistances, and load voltage when a 270Ω load was applied. The results of these calculations are shown in table 1, but the pre-labs can be referenced for further information, which are included in the *Appendices* section.

Table 1: Pre-lab Calculation Results

Quantity	**Source 1**	**Source 2**	**% Difference**	**Comment**
$v_{Th}(V)$	5.000	5.007	0.1304	$v_{Th1} \approx v_{Th2}$ & $\%Diff. \approx 0$
$R_{Th}(\Omega)$	150.0	149.9	0.0967	$R_{Th1} \approx R_{Th2}$ & $\%Diff. \approx 0$
$v_L(V)$	3.214	3.220	0.1650	$v_{L1} \approx v_{L2}$ & $\%Diff. \approx 0$

The results from the preparation indicate that the two sources are equivalent, within a negligible percent difference, as they have the same Thevenin voltage and resistance, and provide the same load voltage to a given load. This indicates that their $v - i$ characteristics are equivalent, indicating that the voltage sources are equivalent, as governed by the definition of equivalent voltage sources.

Once the two sources were constructed, the values of R_x, v_{o1}, and v_{o2} were measured, as per the method to measure source characteristics with a load resistance of 100Ω. Both sources were measured in order to compare their properties. The results of these measurements are shown in table 2

Table 2: Source Measurement Data

Quantity	**Source 1**	**Source 2**
$v_{o1}(V)$	5.054	5.017
$v_{o2}(V)$	2.025	2.022
$R_x(\Omega)$	100	

From these measurements, the values of v_{Th} for Source 1 and 2 can be extrapolated. In these measurements, v_{o1} is essentially the open circuit voltage, and as such, it is equivalent to v_{Th}, since the current drawn by the multi-meter was negligible.

In addition to this, the Thevenin voltage values, v_{Th}, between the two Sources were compared using the calculations shown below.

$$\%Difference = \frac{source2_{value} - source1_{value}}{source1_{value}} \times 100\%$$
$$\%Difference = \frac{5.054V - 5.017V}{5.017V} \times 100\%$$
$$\%Difference \approx 0.7375\%$$

As a result of the very low percent difference between the measured Thevenin voltage of the two sources, it can be said that they are equivalent. Not only this, but the value of Thevenin resistance can also be calculated using these measurements.

Source 1

$$R_{Th} = \left(\frac{v_{o1}}{v_{o2}} - 1\right) R_x$$
$$R_{Th} = \left(\frac{5.054V}{2.025V} - 1\right) 100\Omega$$
$$R_{Th} \approx 149.580\Omega \approx 149.6\Omega$$

Source 2

$$R_{Th} = \left(\frac{v_{o1}}{v_{o2}} - 1\right) R_x$$
$$R_{Th} = \left(\frac{5.017V}{2.022V} - 1\right) 100\Omega$$
$$R_{Th} \approx 148.121\Omega \approx 148.1\Omega$$

Similarly, comparing the calculated results of R_{Th} for each of the sources can be seen below.

$$\%Difference = \frac{source2_{value} - source1_{value}}{source1_{value}} \times 100\%$$
$$\%Difference = \frac{148.121\Omega - 149.580\Omega}{149.580\Omega} \times 100\%$$
$$\%Difference \approx 0.9758\%$$

Again, the percent difference between these results is negligible, illustrating that the Thevenin resistances between the two circuits is essentially equivalent.

After this, the 100Ω load resistors, R_x, were replaced with 270Ω ones, and the load voltages presented by each source, v_L was measured. These measurements, along with the results of the above calculations are contained within tables 3 and 4. Comparisons between the measured load voltages presented by each source are calculated as follows:

$$\%Difference = \frac{source2_{value} - source1_{value}}{source1_{value}} \times 100\%$$
$$\%Difference = \frac{3.239V - 3.246V}{3.246V} \times 100\%$$
$$\%Difference \approx 0.2157\%$$

Such a low percent difference indicates that the measured load voltages are equivalent.

Table 3: Measured Source Parameter Comparisons

Quantity	**Source 1**	**Source 2**	**% Difference**	**Comment**
$v_{Th}(V)$	5.054	5.017	0.7375	$v_{Th1} \approx v_{Th2}$ & $\%Diff. \approx 0$
$R_{Th}(\Omega)$	149.6	148.1	0.9758	$R_{Th1} \approx R_{Th2}$ & $\%Diff. \approx 0$
$v_L(V)$	3.246	3.239	0.2157	$v_{L1} \approx v_{L2}$ & $\%Diff. \approx 0$

The calculated models for Source 1 and 2 are compared in table 4 using the following calculations:

Source 1 - v_{Th}

$$\%Difference = \frac{calculated_{value} - measured_{value}}{calculated_{value}} \times 100\%$$
$$\%Difference = \frac{5.000V - 5.054V}{5.000V} \times 100\%$$
$$\%Difference = 1.080\%$$

Source 1 - R_{Th}

$$\%Difference = \frac{calculated_{value} - measured_{value}}{calculated_{value}} \times 100\%$$
$$\%Difference = \frac{150\Omega - 149.58\Omega}{150\Omega} \times 100\%$$
$$\%Difference = 0.2800\%$$

Source 2 - v_{Th}

$$\%Difference = \frac{calculated_{value} - measured_{value}}{calculated_{value}} \times 100\%$$
$$\%Difference = \frac{5.00652V - 5.017V}{5.00652V} \times 100\%$$
$$\%Difference \approx 0.2093\%$$

Source 2 - R_{Th}

$$\%Difference = \frac{calculated_{value} - measured_{value}}{calculated_{value}} \times 100\%$$
$$\%Difference = \frac{149.855\Omega - 148.121\Omega}{149.855\Omega} \times 100\%$$
$$\%Difference \approx 1.157\%$$

Table 4: Calculated and Measured Model Comparisons

	vth		**% Diff.**	**Rth**		**% Diff.**
	Calculated	**Measured**		**Calculated**	**Measured**	
Source 1	5.000	5.054	1.080	150.0	149.6	0.2800
Source 2	5.007	5.017	0.2093	149.9	148.1	1.157

2.3.1 Analysis

Based on the experimental results in table 3, the two sources are equivalent. Both present equivalent Thevenin voltages, and Thevenin resistances. These are the features that determine the $v - i$ characteristics of the circuit, and since all the features are the same, so are the $v - i$ characteristics. To further exemplify the equal $v - i$ characteristics between the two sources, when a given load of 270Ω was placed on the terminals, the same load voltage was reached, indicating that the variation in terminal voltage is the same. Applying our definition of equivalent voltage sources, the result from this experiment indicate that the voltage sources are equivalent [3].

Thevenin's Theorem absolutely appears to be valid for these sources. The calculated Thevenin values of Source 2 are those used in Source 1, which is proven above. As such, Source 1 represents the Thevenin equivalent circuit of Source 2. Since it can be seen from the above calculations that both circuits are equivalent, as proven by their equivalent $v - i$ characteristics. This leads to the conclusion that since both sources are equivalent, and one is the Thevenin equivalent of the other, Thevenin's Theorem must be valid for these sources [2].

3 Discussion

Overall, the results of the experiment followed those calculated in the preparation. The encountered errors were small enough that the compared values could always be considered equal, allowing the two sources to exhibit the same $v - i$ characteristics, and therefore be declared equivalent. Nevertheless, errors still existed in the results, which can mostly be attributed to systematic errors in the procedure. Some of the errors include:

- The tolerances of the resistors. All resistors used had 5% manufacturing tolerances and the procedure did not declare to measure them [3]. Performing resistance measurements prior to constructing the sources would minimize this error.

- The assumption of a perfect voltage source. A fictitious voltage source was constructed in this lab, consisting of a DC voltage source and a resistor network. The procedure and equations assume that the DC voltage source has a series resistance low enough to be considered zero [3]. No voltage source is perfect, and this introduced a resistance not accounted for by the lab. Simply not making this assumption, and performing a measurement on the supply would mitigate this error.

- The assumption of infinite multi-meter resistance. When measuring the open-circuit voltages, the $10M\Omega$ internal resistance of the multimeter was assumed to be infinite, allowing the simplification of setting the drawn current to zero [3]. This is not the case, and using a multimeter with a larger internal resistance would reduce this error slightly.

- Not accounting for the resistance of the banana connectors. Banana connectors weren't even mentioned in the lab, and although it's small, they do provide resistance to the circuit which was not accounted for [3]. Expanding the procedure to account for these resistances would reduce error in the measurements.

4 Conclusion

In this lab, voltage source equivalence was examined using Thevenin's Theorem [3]. This theorem states that regardless of the internal network of intrinsic or discrete resistors in a linear voltage source, it can be modelled as an ideal voltage source in series with a resistor [2]. Two circuits were compared and contrasted through comparisons of measurements of their $v - i$ characteristics. To do this, discrete resistors were connected in a network on the output of DC power supply, in a fashion depicted in figure 1. These mimicked two voltage sources with different resistance networks to compare. Through a series of measurements, it was found that the Thevenin equivalent voltages, the Thevenin equivalent resistances, and the load voltages were the same for each source for given loads of 100Ω and 200Ω. Comparisons were made for each measured value using the percent-difference equation, and the results of these were small enough to characterize the measured values between the two sources, and the measured values against the calculated values as being the same. The slight errors in these results are mostly attributed to systematic errors in the experiment. The most significant of these consists of the procedure outlining to use the listed resistor values instead of measuring their actual resistances, as manufacturing tolerances were $+/-5\%$ [3]. Simply measuring these values before using the resistors in circuit could provide more accurate results [2]. Nevertheless, the objectives of this lab, to prove the equivalence of the two sources and to prove Thevenin's Theorem, were reached.

5 Summay of Roles

Justen G. Di Ruscio

1. Formatting and Outline
2. Abstract
3. Experiment
4. Conclusion
5. Bibliography

Stephen W.W. Cole

1. Introduction
2. Procedure
3. Discussion

References

[1] W. McAllister, "Simplifying resistor networks," 2019.

[2] H. Li, "Ece2701 lectures," October 2019.

[3] I. Veach, *EE2701 Experiment 2. Voltage Source Equivalence*, March 2011.

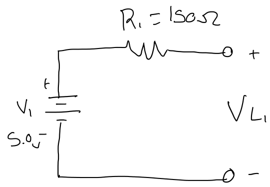
[4] H. Li, "Lab marking scheme," Winter 2008. EE3312.

6 Appendicies

6.1 Prelabs - Stephen, Justen

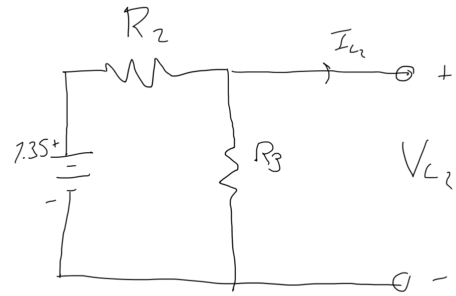
Prelab 2

Thursday, October 17, 2019 7:22 PM



$V_{th} = 5.0v$
$R_{th} = 150\Omega$

$$V_L = \frac{5.0v \cdot 270\,\Omega}{(270\Omega + 150\Omega)} = 3.214v$$

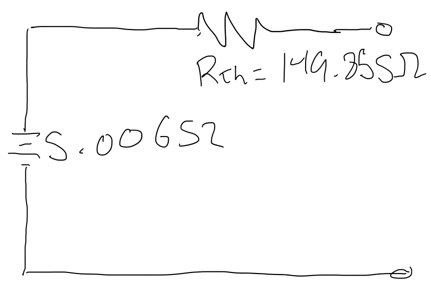


$R_2 = 220\Omega$
$R_3 = 470\Omega$

$R_{eq} = 149.855\Omega = R_{th}$

$$V_{th} = \frac{7.35v\,(470\,\Omega)}{(220+470)\,\Omega} = 5.00652\,v$$

Theivenin



$$V_L = \frac{5.00652 \cdot 270\Omega}{(270\Omega + 149.855\Omega)} = 3.220v$$

Comment on the relation of the Values

- They have the Same theivenin equivalent Circuit therefore they have the Same drop across VL when the Same load is placed on it.

Percent difference

$$V_L = \frac{3.220v - 3.214v}{3.214v} \cdot 100 = 0.1867\%$$

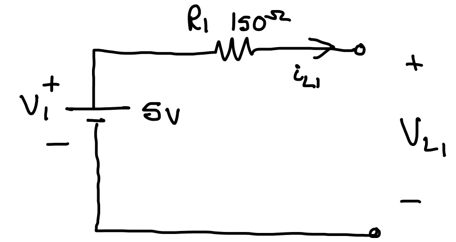
$$R_{Th} = \frac{149.855 - 150}{150} \cdot 100 = 0.0967\%$$

$$V_{th} = \frac{5.00652v - 5v}{5v} \cdot 100 = 0.1304\%$$

Pre-Lab 2

Tuesday, October 15, 2019 12:26 PM

Source 1:

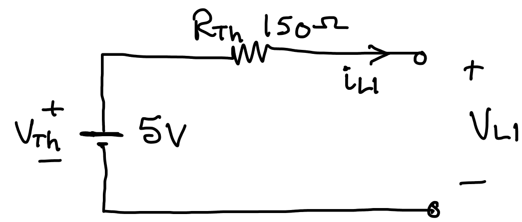


$i_{L1,open} = 0A \Rightarrow V_{R_1} = 0V$

$\therefore V_{open} = V_1 = V_{Th}$

$R_{Th} = 150\Omega$

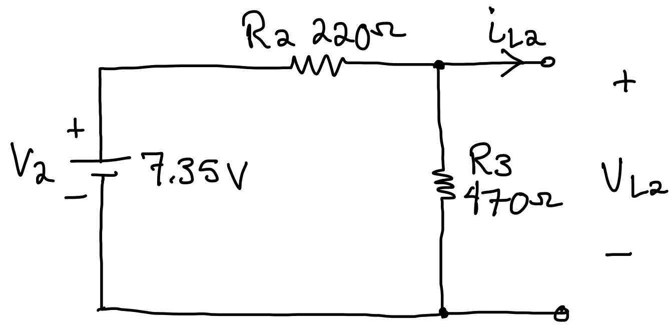
Thevenin equivalent Circuit:



adding a 270Ω resistor

$$V_{L1} = V_{th} \frac{R_{Th}}{R_{th} + 270\Omega} = 5V \frac{270\Omega}{420\Omega} \doteq 3.2143V$$

Source 2:

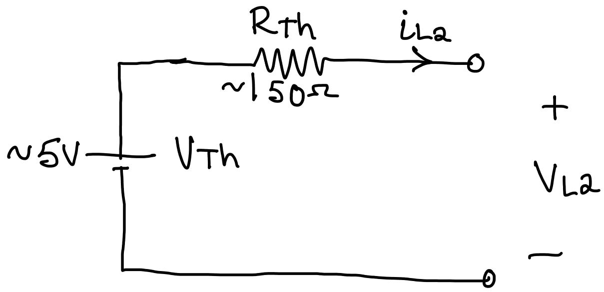


$i_{L2,open} = 0A$

$$V_{open} = V_2 R_3 / (R_2 + R_3) = \frac{(7.35V)(470\Omega)}{(220\Omega + 470\Omega)} \doteq 5.00652V \sim 5V$$

$$R_{Th} = \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{220\Omega} + \frac{1}{470\Omega}\right)^{-1} \doteq 149.85507\Omega \sim 150\Omega$$

Thevenin Equivalent Circuit



adding a 270Ω Resistor:

$$V_{L2} = V_{Th} \frac{R_{Th}}{R_{Th} + 270\Omega} \doteq 5V \frac{270\Omega}{420\Omega}$$

$$V_{L2} \doteq 3.21959V$$

Error Calculations

$$\varepsilon_{V_{Th}} = \frac{|V_{Th_1} - V_{Th_2}|}{V_{Th_1}} \times 100\% \doteq \frac{|5V - 5.00652V|}{5V} \times 100\% = 0.1304\%$$

$$\varepsilon_{V_L} = \frac{|V_{L_1} - V_{L_2}|}{V_{L_1}} \times 100\% \doteq \frac{|3.2143V - 3.2196V|}{3.2143V} \times 100\% \doteq 0.1650\%$$

$$\varepsilon_{R_{Th}} = \frac{|R_{Th_1} - R_{Th_2}|}{R_{Th_1}} \times 100\% \doteq \frac{|150\Omega - 149.8551V|}{150\Omega} \times 100\% \doteq 0.0966\%$$

Comment On Similarity:

In both circuits, the Thevenin voltages and the Thevenin resistances are essentially the same. This indicates that the two sources are the same, as seen by the load. To further emphasize this, the load voltages are equivalent.