1) $T_0 = 2\pi$, $k = 1\,mV$, $f_s = 20\,Hz$, $T = 10\,secs.$

$f_0 = \frac{1}{T_0} = \frac{1}{2\pi}$

$g(t) = \begin{cases} -k & \text{if } -\frac{T_0}{2} \le t < 0 \quad -\pi \le t < 0 \\ +k & \text{if } 0 \le t < \frac{T_0}{2} \quad 0 \le t < \pi \end{cases}$

* − $a_0 = ?$

$$a_0 = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g_{T_0}(t)\,dt$$

$$= \frac{2}{2\pi} \int_{-\pi}^{\pi} g(t)\,dt$$

$$= \frac{2}{2\pi} \left[\int_{-\pi}^{0} -k\,dt + \int_{0}^{\pi} k\,dt \right] = \frac{2}{2\pi} \left[0 + \cancel{-\pi k} + \cancel{\pi k} - 0 \right]$$

$$= 0 //$$

* − $a_n = ?$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g_{T_0}(t) \cdot \cos(2\pi f_0 n t)\,dt = \frac{2}{2\pi} \int_{-\pi}^{\pi} g(t) \cos(2\pi f_0 n t)\,dt$$

$$= \frac{2}{2\pi} \left[\int_{-\pi}^{0} -k \cos(2\pi f_0 n t)\,dt + \int_{0}^{\pi} k \cos(2\pi f_0 n t)\,dt \right]$$

$$= \frac{2k}{2\pi} \left[-1 \int_{-\pi}^{0} \cos(2\pi f_0 n t)\,dt + \int_{0}^{\pi} \cos(2\pi f_0 n t)\,dt \right]$$

Since

$$\int \cos(2\pi f_0 n t)\,dt$$

$u = 2\pi f_0 n t$

$\frac{du}{dt} = 2\pi f_0 n$, $dt = \frac{du}{2\pi f_0 n}$; $\int \cos u \frac{du}{2\pi f_0 n}$

$\cancel{\frac{1}{\sin(2\pi f_0 n}}$

$$= \frac{1}{2\pi f_0 n} \left(\sin(2\pi f_0 n t) \right)$$

$$\frac{2K}{2\pi}\cdot\frac{1}{2\pi f_0 n}\left[-1\left[\sin(2\pi f_0 n t)\right]_{-\pi}^{0} + \left[\sin(2\pi f_0 n t)\right]_{0}^{\pi}\right]$$

$$\frac{2k}{4\pi^2 f_0 n}\left[-\cancel{\sin(0)} + \cancel{\sin(-2\pi^2 f_0 n)} + \cancel{\sin(2\pi^2 f_0 n)} - \cancel{\sin(0)}\right]$$

$\sin n\pi = 0$

$= 0$ //

$$b_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{T_0}(t)\cdot\sin(2\pi f_0 n t)\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(t)\cdot\sin(2\pi f_0 n t)\,dt$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} -k\sin(2\pi f_0 n t)\,dt + \int_{0}^{\pi} k\sin(2\pi f_0 n t)\,dt\right]$$

$$= \frac{k}{2\pi}\left[-\int_{0}^{-\pi}\sin(2\pi f_0 n t)\,dt + \int_{0}^{\pi}\sin(2\pi f_0 n t)\,dt\right]$$

Since

$$\int \sin(2\pi f_0 n t)\,dt$$

$u = 2\pi f_0 n t$, $\frac{du}{dt} = 2\pi f_0 n$, $dt = \frac{du}{2\pi f_0 n}$, $\int \sin u \frac{du}{2\pi f_0 n}$

$$= \frac{1}{2\pi f_0 n}\left[-\cos(2\pi f_0 n t)\right]$$

$$= \frac{k}{4\pi^2 f_0 n}\left[-\cos(-2\pi^2 f_0 n) + \cos 0 - \cos(2\pi^2 f_0 n) + \cos 0\right]$$

$$- \quad b_n = \frac{2}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g_{T_0}(t)\cdot \sin(2\pi f_0 n t)\,dt$$

$$\cancel{T_0 = \frac{2\pi}{w_0}}$$

$$w_0 = 2\pi f_0$$

$$\frac{w_0}{2\pi} = f_0 \qquad T_0 = \frac{2\pi}{w_0}$$

$$\frac{1}{T_0} = \frac{2 w_0}{2\pi} \qquad \frac{T_0}{2} = \frac{\pi}{w_0}$$

$$b_n = \frac{\cancel{2} w_0}{\cancel{2}\pi}\int_{-\frac{\pi}{w_0}}^{\frac{\pi}{w_0}} g(t)\sin(w_0 n t)\,dt$$

$$= \frac{w_0}{\cancel{2}\pi}\left[\int_{-\frac{\pi}{w_0}}^{0} -k\sin(w_0 n t)\,dt + \int_{0}^{\frac{\pi}{w_0}} k\sin(w_0 n t)\,dt\right]$$

$$= \frac{k w_0}{\cancel{2}\pi}\left[\int_{0}^{-\frac{\pi}{w_0}} \sin(w_0 n t)\,dt + \int_{0}^{\frac{\pi}{w_0}} \sin(w_0 n t)\,dt\right]$$

$$\int \sin(w_0 n t)\,dt$$

$$u = w_0 n t \qquad \int \sin u \,\frac{du}{w_0 n}$$

$$\frac{du}{dt} = w_0 n$$

$$dt = \frac{du}{w_0 n} \qquad \frac{-\cos(w_0 n t)}{w_0 n}$$

$$\to = \frac{k\cancel{w_0}}{\pi \cancel{w_0} n}\left[\underbrace{-\cos\left(\cancel{w_0}\cdot n\cdot \frac{-\pi}{\cancel{w_0}}\right)}_{\cos(-x) = \cos(x)} + \underbrace{\cos 0}_{1} - \cos\left(\cancel{w_0}\cdot n\cdot\frac{\pi}{\cancel{w_0}}\right) + \underbrace{\cos 0}_{1}\right]$$

$$\cos(n\pi) = (-1)^n$$

$$= \frac{k}{\pi n}\left[2 - 2\cos(n\pi)\right]$$

$$= \frac{2k}{\pi n}\left[1 - (-1)^n\right]$$

~~→ if n is even~~

$= 0$ if n is even.

$= \frac{4k}{\pi n}$ if n is odd.

(3)

- When a_0 is 0, it tells that the AC signal has equal positive and negative half cycle areas. The average or mean value of signal is described by a_0

- When a_n is 0, it tells us that sine components is in the Fourier series expansion of the signal. It makes sense, the sine wave looks close to the square wave.

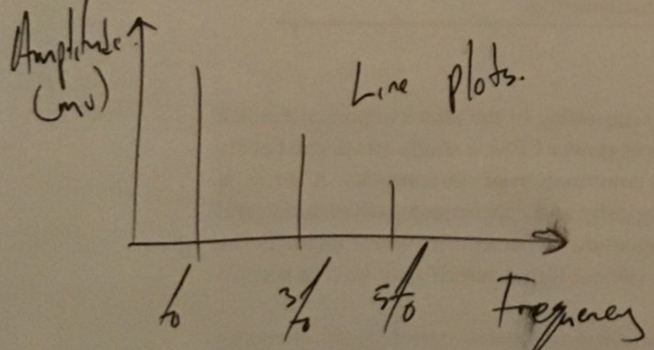
- With the increase in N, the square waves approaches the ideal shape. However, increase in the number of terms (N) produces undesirable ripple in the square wave. The amplitude of the ripples decreases as N increases.

2) - A quick glance at the plots of Fourier transform and line spectrum of the square wave reveals that they do not correspond to each other

For $N = 3$.

$$y(t) = b_1 \sin(2\pi f_0 t) + b_3(\sin 2\pi 3 f_0 t) + b_5 \sin(2\pi 5 f_0 t)$$

$$= \frac{4k}{\pi}\sin(2\pi f_0 t) + \frac{4k}{3\pi}\sin(6\pi f_0 t) + \frac{4k}{5\pi}\sin(10\pi f_0 t)$$



$$A = \left[\frac{4k}{\pi}, \frac{4k}{3\pi}, \frac{4k}{5\pi}\right]$$

$$f = [f_0, 3f_0, 5f_0]$$

stem (f, a, 'r')