UNB

UNIVERSITY OF NEW BRUNSWICK

ELECTRICAL AND COMPUTER ENGINEERING

Room D36, Head Hall

LABORATORY / ASSIGNMENT / REPORT COVER PAGE

Course # : ECE 2701 Experiment # or Assignment # 1

Date: October 11th 2019 Course Instructor: Dr. Howard Li

Title: ECE 2701 Experiment 1: An Examination of Kirchoff's Laws
(if applicable)

Group # N/A
(if applicable)

Dominic Geneau
Name (print)

Syed Abdul Wasey Naqvi
Name (print)

Name (print)

Name (print)

Name of Author: Dominic Geneau
Name (print)

Signature of Author: [signature]
Student Name (signature)

*****PLEASE ENSURE YOU FILL OUT THE BACK OF THIS PAGE*****

Comments:

___/___	___
Grade	Initials of Marker

Group # N/A
(if applicable)

3675144 / Dominic Geneau
Student # *Name (print)*

3609233 / Syed Abdul Wasey Naqvi
Student # *Name (print)*

___ / ___
Student # *Name (print)*

___ / ___
Student # *Name (print)*

Name of Author: 3675144 / Dominic Geneau
Student # **Name (print)**

ECE 2701 Experiment 1:

An Examination of Kirchoff's Laws

Dominic Geneau

Syed Abdul Wasey Naqvi

Prepared for

Dr. Howard Li

Due: Friday, October 11th 2019

Table of Contents

Abstract

Experiment 1: An Examination of Kirchoff's Laws investigates the physical relationship of current and voltage across resistors in a constructed circuit. More specifically, each respective team of ECE 2701 was tasked with proving the theoretical reasoning behind Kirchoff's Laws (Voltage and Current Laws): each law contributing theoretically and in practice inside the circuit.

This analysis was accomplished through the measurement of voltages across three resistors, followed by the systematic calculation of current using Ohm's Law (V = iR). Then, using current and voltage analysis, each team was meant to prove Kirchoff's laws. Furthermore, each group was tasked with writing a formal report reflecting their findings: ultimately supporting or refuting the given objective mentioned above.

In the end, it was found that both laws held true across the entire circuit, even if there were slight deviations to the theoretical values calculated beforehand.

Seeing as this experiment was relatively straightforward, the team did not encounter any major issues. Nevertheless, experimental error was still present throughout the procedure, especially human mistakes due to faulty data recording and/or possible calculation errors.

Introduction

Ohm's law is of paramount importance in all aspects of electricity and circuitry. The law relates voltage and current through an ideal resistor. Through Eq.1, Allan R. Hambley (2018, p.26) explains how the voltage is directly proportional to the current in an ideal resistor:

$$V = iR \tag{1}$$

Where:

V = Voltage/Potential difference (Volts, V)
i = Current (Amperes, A)
R = Resistance (Ohms, Ω)

Through this fundamental statement, further analysis can be performed using Kirchoff's Current and Voltage Laws, which will further be elaborated herein. As mentioned multiple times, the results obtained throughout the experiment have proven Kirchoff's Laws to hold true. Also, for the purpose of simplicity, both laws will be referred to as KCL and KVL, respectively.

Regarding the experiment, the team was tasked with investigating the validity of Kirchoff's Laws using a pre-designated circuit, Ohm's law and measuring instruments (e.g., digital multimeter). The authenticity of Kirchoff's laws proves to be incredibly meaningful in society. In fact, if not for these laws, charge flow and potential difference could not function in a circuit.

For example, KCL supposes that charges cannot linger in a circuit node. If this were not the case, an unequal distribution of charge would spread throughout each node, creating forces of attraction across a circuit that would disrupt the flow of electrons (Hambley, 2018, p.18). A similar conclusion can be drawn from KVL as well.

Experiment

This section will further elaborate on the theory as well as the systematic procedure of the experiment. Additionally, the content will be accompanied by an apparatus of materials used for the lab and pertinent tables and figures.

3.1 Apparatus

List of equipment (courtesy of I. L. Veach (2011, p.1)):

- Single 150Ω, 270Ω and 470Ω resistors
- Two digital multimeters (DMM)
- An Elenco XP-720 Power Supply
- Colour coated conducting wire

3.2 Theory and Procedure

Before entering the laboratory, students were tasked with finding the values of the current (i_1, i_2 and i_3) and voltage (v_{ab}, v_{bc}, and v_b) across each resistor in Figure 1 below. These results could be found in a variety of ways, including the use of KCL, KVL and Ohm's law to systematically evaluate the voltage around each mesh (i_x and i_y) and the currents in node b.

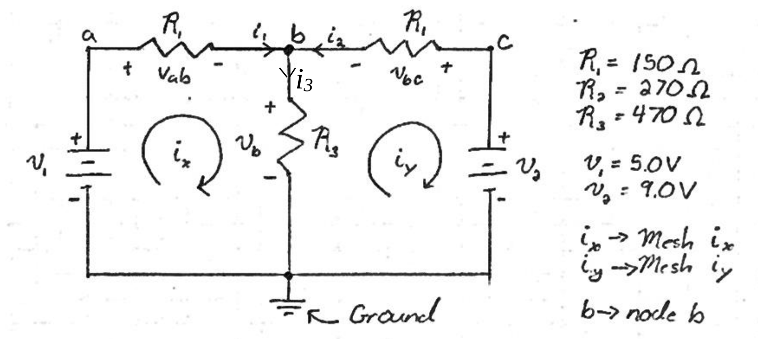


Figure 1. **Electrical circuit constructed and analyzed during the experiment.**

KCL states that: "The sum of the currents entering a node equals the sum of the currents leaving a node" (Hambley, 2018, p.17). Simply put, there should never be residual charges that idle in a node (points in a circuit where two or more currents join). An example of this would be node b where Eq.2 shows that:

$$i_1 + i_2 = i_3 \tag{2}$$

where both currents i_1 and i_2 enter the node and must therefore equal i_3.

KVL involves isolating each loop into their own equations. According to Hambley (2018): The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit" (p.20). Plus, the polarity of each voltage in the loop inverts if the current flows in the opposite direction of the potential difference (i.e., voltage drop). Eq.3 uses mesh i_x as an example of KVL:

$$-v_1 + R_1 i_1 + R_2 i_2 = 0 \tag{3}$$

where the power source v_1 has its polarity inverted because of a voltage drop and, using Ohm`s law, we substitute v_{ab} and v_b by their respective currents and resistances.

Once the students arrived at the lab and presented their pre-lab calculations, they could begin the experimental procedure explained below (Veach, 2011, p.2):

1. Students were to use a digital multimeter (DMM) to measure and record each resistor that would be used in their circuit. It was very important to measure them separately before implementing them in the circuit, since the DMM injects a current through the resistor and any additional current source will botch the measurement. This can be seen

in Figure 2 below, using R_1 as an example, I_T as the current source and V_m as the voltage.

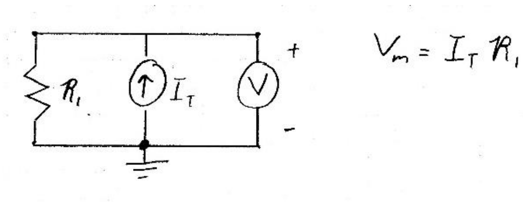


***Figure 2.* Internal workings of a DMM using the ohmmeter function to evaluate R_1.**

2. Next, the students needed to construct a circuit mirroring the electrical system in Figure 1. They then used the power supply to create a 5.0V source at v_1 and, using the variable positive supply, created a positive 9.0V source at v_2.
3. Using the voltmeter function of the DMM, students measured the unknown voltage across each resistance. The results were then saved and recorded in another table. In this case, the measuring instrument was connected to each polarity of the resistance and the direct current provided the necessary charge flow to give a positive reading. This is represented in Figure 3, using v_{ab} as an example.

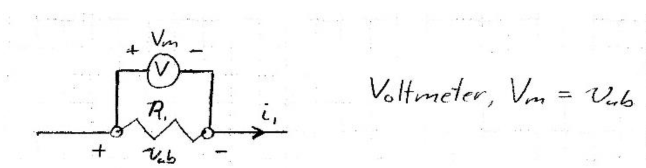


***Figure 3.* Internal workings of a DMM using the voltmeter function across R_1.**

4. This part of the experiment was meant to prove the validity of KVL. Using the results calculated in part 3, the students were tasked to sum all voltages around each mesh of the circuit to make sure Kirchoff's Voltage Law held true. In the case of this team, both meshes were valid and equaled approximately zero. The exact values calculated and the reason behind the small variation will be explained in the following sections.
5. Finally, using Ohm's law, the measured resistances from part 1 and the voltages in part 2, the team isolated the current in Ohm's equation and calculated each individual current. Eq. 4 is a derived version of Ohm's Law to calculate the current across each resistor:

$$i = \frac{V}{R} \tag{4}$$

Using KCL with the measured currents, the students evaluated the currents flowing through node b (refer to Figure 1) and successfully proved that the currents entering the node equaled the current exiting the node.

3.3 Calculations, Graphs, Results, and Analysis

The body of this section contains a series of tables representing the data collected throughout the experiment. Each individual table will be explained and analyzed to the best degree possible.

Table 1 represents the measured resistances of the resistors. The values were determined by connecting the multimeter individually to the resistors. It is relatively pertinent to mention the slight difference in resistances compared to the factory value. This is mainly due to the performance deterioration of the resistor over time as well as temperature variance.

***Table 1*. Network Component Values (Resistors)**

Resistors	Factory Value	Measured
R_1 (Ω)	150.0	150.0
R_2 (Ω)	270.0	268.7
R_3 (Ω)	470.0	466.5

Table 2 represents the measured and calculated values of the voltage at different points in the circuit. The voltage values in the Calculated column were determined through the calculations done in the 'Pre-Lab' section of this experiment. A thorough breakdown of these calculations can be found in Appendix A. The voltage values determined under the Measured column were also found with the DMM.

***Table 2*. Test Circuit Voltage (Measured & Calculated Results)**

Quantity	Calculated	Measured
v_1 (V)	5.000	5.122
v_{ab} (V)	-0.335	-0.279
v_b (V)	5.335	5.404
v_{cb} (V)	3.667	3.606
v_2 (V)	9.000	9.000

Table 3 represents the theoretical check performed using KVL for each individual loop: i_x and i_y. That is: the sum of the voltages in each circuit should equal zero. As seen in Eq.5, the sum of loop i_x was found by calculating the values of v_1, v_{ab} and v_b:

$$\sum v_{i_x} = v_1 + v_{ab} + v_b = (-5.122\text{V}) + (-0.279\text{V}) + (5.404\text{V}) = 0.0028\text{V} \qquad (5)$$

where, as mentioned in the previous section, the polarity of v_1 is inverted since a voltage drop occurs. The detailed calculations for loops i_x and i_y can be found in Appendix B.

Table 3. **KVL Checks and Comments**

Quantity	$\sum v_i$ (V)	Comment on KVL
i_x Mesh	0.0028	KVL applies since $\sum v_i$ is almost zero
i_y Mesh	-0.0060	KVL applies since $\sum v_i$ is almost zero

Table 4 represents the calculated and the measured values of the currents in the various sections in the circuit. Like the KVL check, the detailed calculated values are found in Appendix B. However, a brief view of the formulae used can be seen in Eq.6, indicating the currents i_1 and i_2 entering the node equal the current i_3 (variation of -0.16mA, as seen in Table 5):

$$\boldsymbol{RHS} = i_1 + i_2 = (-1.86\text{mA}) + (13.40\text{mA}) = 11.54\text{mA} \cong (11.70\text{mA}) = i_3 = \boldsymbol{LHS} \quad (6)$$

The measured values were found by a set of calculations since we were informed that the electric current readings from the multimeter are not correct. This is due to the DMM's inability to process small currents correctly through a preset voltage.

Table 4. **KCL Calculated & Measured Current (Using Ohm's Law)**

Quantity	Calculated	Measured
i_1 (mA)	-2.23	-1.86
i_2 (mA)	13.58	13.40
i_3 (mA)	11.35	11.70

Table 5. **KCL Check and Comments**

Quantity	$\sum i_i$ (mA)	Comment on KCL
Node b	-0.16	KCL applies since $\sum i_i$ is almost zero

Discussion

An examination of Kirchoff's Laws allowed us to compare the calculated and measured values of the various components in the circuit: resistance, voltage, and current.

Comparisons were conducted by measuring the resistances, currents, and voltages using multimeters. These provided experimental values and calculated values through given parameters (i.e. the source voltages and the resistance values of the resistors). After conducting the experiment, various values were found experimentally and theoretically; some of which were given, like resistance values and voltage values.

Percent error between theoretical and experimental values can also be calculated using Eq.7 (shown below). While it is not particularly useful to show all %errors, the team has found it worthwhile to elaborate on the %error of the measured voltages (Table 2) using the formula for % error:

$$\%error = \left|\frac{experimental - theoretical}{theoretical}\right| \times 100 \tag{7}$$

Using Eq.7 and v_1 as an example, it was found that:

$$\%error_{v_1} = \left|\frac{(5.122\text{V}) - (5.000\text{V})}{(5.000\text{V})}\right| \times 100 = 2.44\% \tag{8}$$

While 2.44% is still within reasonable parameters (%error ≤ 5%), it still raises the question of why this value is higher than the rest. This could be due to many variables, but, for the purpose of this report, the following paragraphs elaborate on the most common errors that the team could think of.

During the lab, the team encountered various errors that were caused experimentally or by themselves. Potential human errors encountered in the lab which could have altered our readings range from misreading the value on the multimeter or not waiting for the value displaying on the multimeter to stabilize. Mathematically, not using appropriate significant digits during the calculations could have equally caused a varying percent difference between the theoretical values and the experimental values

Experimental errors consisted of some being unavoidable. Environmental factors such as a change in temperature would evidently alter the readings on the multimeter (e.g. an increase in resistance of the circuit that results from an increase in temperature). Faulty calibration of the devices used in the lab such as multimeter and voltage source may have resulted in a variation of the readings as well. As the name states, unavoidable errors cannot be altered directly, but being aware of these variables could equally reduce the margin of error while also creating a better understanding of the phenomena.

Conclusion

The experimental analysis of Kirchoff's Laws has provided the team with plausible evidence supporting both KVL and KCL rules. Meticulously following the procedure of the lab, the team measured voltages and resistances across resistors in a circuit. Then, using Ohm's law, each respective current was experimentally calculated. As demonstrated in the discussion and section 3.3 of this report, the voltage sum of both meshes in the constructed circuit were very close to zero ($\sum v_{i_x} = 0.0028\text{V} \cong 0\text{V}$ & $\sum v_{i_y} = -0.0060\text{V} \cong 0\text{V}$). This, in turn, draws on the team's conclusion which states that the sum of voltages in a closed loop must indeed equal zero.

Similarly, Kirchoff's Current Law was proven by calculating the difference between the sum of currents entering a node and the sum of currents exiting the node. Yielding a value of -0.16 mA, the percent error was relatively high compared to the values obtained in KVL analysis. While this value is not high enough to raise concerns regarding the validity of KCL, it does give the team an example of experimental mistakes caused by human and random error.

In order to improve the accuracy of this experiment (given more time), the team could have performed the experiment repeatedly using the same instruments. Because of possible inaccurate visual recording of the DMM or any other human mistakes, this could provide a basis of uncertainty which would fall in the expected results of the experiment (Li, n.d.). Conversely, the team could have used multiple DMM's: increasing the spectrum of results that could be obtained through the measuring instrument.

Summary of Each Member's Role

Dominic Geneau was present and involved in the entire experimental procedure of the lab as well as the completion of this report. Accompanied by his teammate, he recorded measurements and calculated the results during the experiment. He wrote a draft of the abstract, introduction, apparatus, theory and procedure, and the conclusion. Additionally, Dominic also played a substantial part in the formatting and proof-reading of the document itself.

Syed Abdul Wasey Naqvi was present and involved in the throughout the entire experiment in this lab and during the completion of this report. Accompanied by his teammate, he conducted the experiment and calculated the mathematical calculation for questions 5, 6, and 8. He wrote section 3.3: Calculations, graphs, results, and analysis and Discussion section of the lab. Additionally, Abdul Wasey reviewed over the entire lab section by section to ensure no misunderstandings and clarity.

References

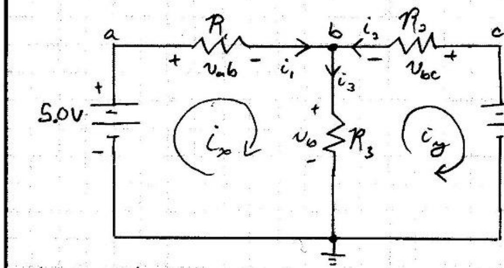
Hambley, A. R. (2018) *Electrical engineering: Principles and Applications* (7th ed.). Hoboken, NJ: Pearson Education, Inc.

Li, H. (n.d.). *Lab common mistakes* [PDF document]. Retrieved from D2L UNB website: https://lms.unb.ca/d2l/le/content/160443/viewContent/1616315/View

Veach, I. L. (2011) *ECE2701 experiment 1: An examination of Kirchoff's laws* [PDF document]. Retrieved from D2L UNB website: https://lms.unb.ca/d2l/le/content/160443/viewContent/1616299/View

Appendix A – Pre-Lab Calculations

UNIVERSITY OF NEW BRUNSWICK	FACULTY OF ENGINEERING	
Course No. ECE 2701	Assignment No. LAB#1 Date Oct-3-2019	Page
Problem No. Calculations	By Dominic Geneau	of



$R_1 = 150\,\Omega$
$R_2 = 270\,\Omega$
$R_3 = 470\,\Omega$

$v_1 = 5\text{ V}$
$v_2 = 9\text{ V}$

KVL loop i_x

$-5V + R_1 i_1 + R_3 i_3 = 0 \quad (1)$

KVL loop i_y

$9V - R_3 i_3 - R_2 i_2 = 0 \quad (2)$

KCL Node b

$i_3 = i_1 + i_2 \quad (3)$

Substituting eqn (3) in eqns (1) & (2)

$R_1 i_1 + R_3(i_1 + i_2) = 5$

$R_3(i_1 + i_2) + R_2 i_2 = 9$

$R_1 i_1 + R_3 i_1 + R_3 i_2 = 5$

$i_1 = \frac{5 - R_3 i_2}{(R_1 + R_3)}$

$i_1 = \frac{5 - (470)(13.58\times10^{-3}A)}{(150\Omega + (470\Omega))}$

$\boxed{i_1 = -2.23\,mA}$

$i_2(R_3 + R_2) + R_3\left[\frac{5 - R_3 i_2}{(R_1 + R_3)}\right] = 9$

$i_2(R_3 + R_2) + \frac{5R_3}{(R_1 + R_3)} - \frac{(R_3)^2 i_2}{(R_1 + R_3)} = 9$

$i_2\left[(R_3 + R_2) - \frac{(R_3)^2}{(R_1 + R_3)}\right] = 9 - \frac{5R_3}{(R_1 + R_3)}$

$i_2 = \frac{9 - \frac{5(470\Omega)}{(150+470)\Omega}}{\left[(470+270)\Omega - \frac{(470)^2\Omega}{(470+150)\Omega}\right]}$

$i_2 = \frac{(5.209677)}{(383.7097)}$

$i_2 = 0.01358\,A$

$\boxed{i_2 = 13.58\,mA}$

$i_3 = i_1 + i_2 = (-2.23\,mA) + (13.58\,mA)$

$\boxed{i_3 = 11.35\,mA}$

$V_{bc} = I_2 R_2 = (13.58\,mA)(270\,\Omega)$

$\boxed{V_{bc} = 3.67\text{ V}}$

$V_{ab} = I_1 R_1 = (-2.23\,mA)(150\,\Omega)$

$\boxed{V_{ab} = -0.335\text{ V}}$

$V_b = I_3 R_3 = (11.35\,mA)(470\,\Omega)$

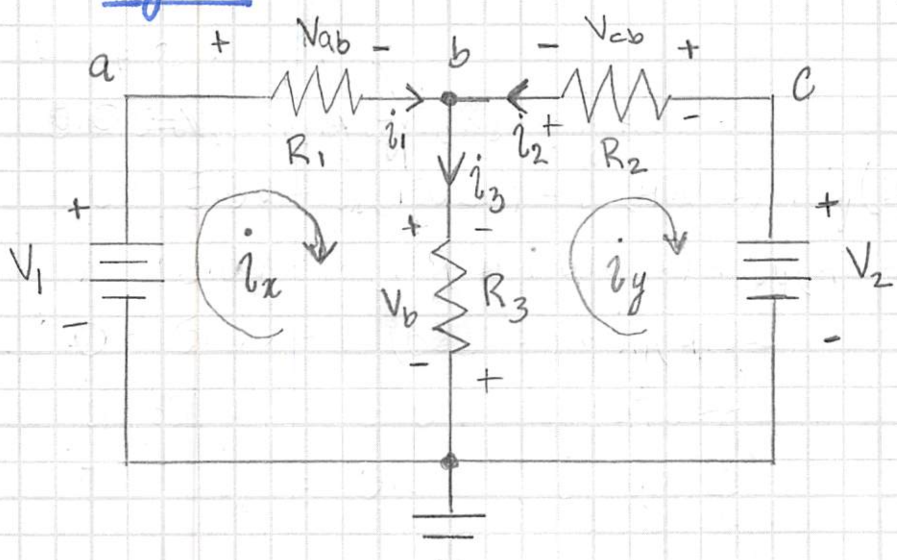
$\boxed{V_b = 5.335\text{ V}}$

Appendix B – Mathematical Analysis

Analyze the Circuit diagram to find the values of V_{ab}, V_b, V_{cb}, i_1, i_2, and i_3.

Solution:

Diagram:



$V_1 = 5.0V$

$V_2 = 9.0V$

$R_1 = 150\Omega$

$R_2 = 270\Omega$

$R_3 = 470\Omega$

Calculations:

Using Mesh Analysis for loops i_x and i_y:

Loop 1: (i_x)

$$V_1 = R_1 i_x + R_3 i_x - R_3 i_y$$

$$5.0V = i_x(150\Omega) + i_x(470\Omega) - i_y(470\Omega)$$

$$5.0V = i_x(150\Omega + 470\Omega) - i_y(470\Omega)$$

$$5.0V + i_y(470\Omega) = i_x(620\Omega)$$

$$i_x = \frac{5.0V}{620\Omega} + \frac{i_y(470\Omega)}{620\Omega}$$

$$i_x = 0.00806 + i_y(0.758\Omega)$$

UNIVERSITY OF NEW BRUNSWICK | FACULTY OF ENGINEERING

Course No. ECE 2701 | Assignment No. 1 | Date Sept. 20. 2019 | Page 2 of

Problem No. Pre-lab Cont'd | By Syed Abdul Wasey Naqvi

Loop 2: (i_y)

$$-V_2 = R_3 i_x - R_3 i_y - R_2 i_y$$

$$-9.0V = i_x(470\Omega) - i_y(470\Omega) - i_y(270\Omega)$$

$$-9.0V = i_x(470\Omega) - i_y(470\Omega + 270\Omega)$$

$$-9.0V = i_x(470\Omega) - i_y(740\Omega)$$

$$i_y(740\Omega) = i_x(470\Omega) + 9.0V$$

$$i_y = \frac{i_x(470\Omega)}{740\Omega} + \frac{9.0V}{740\Omega}$$

$$i_y = i_x(0.635\Omega) + 0.0121$$

Now sub in the value of i_x into i_y:

$$i_y = [0.00806 + i_y(0.758\Omega)](0.635\Omega) + 0.0121$$

$$i_y = (0.00806)(0.635\Omega) + [(i_y)(0.758\Omega)](0.635\Omega) + 0.0121$$

$$i_y = 0.00518 + i_y(0.481) + 0.0121$$

$$i_y = i_y(0.481) + 0.1728$$

$$i_y - i_y(0.481) = 0.1728$$

$$i_y(0.519) = 0.1728$$

$$i_y = \frac{0.1728}{0.519}$$

$$i_y = 0.3329$$

Subing in value of i_y in i_x

Therefore, i_x is equal to:

$$i_x = 0.00806 + (0.3329)(0.758\Omega)$$

$$i_x = 0.2603$$

Finding the Currents:

$$i_1 = i_x = 0.2603 \text{ A}$$

$$i_2 = -i_y = -0.3329 \text{ A}$$

$$i_3 = i_x - i_y$$
$$= 0.2603 - (-0.3329)$$
$$= 0.5932 \text{ A}$$

Finding the Voltages:

$$V_{ab} = i_1 R_1 = (0.2603\text{A})(150\,\Omega)$$
$$= \quad \text{V}$$

$$V_b = i_3 R_3 = (0.5932\text{A})(470\,\Omega)$$
$$= \quad \text{V}$$

$$V_{cb} = i_2 R_2 = (-0.3329\text{A})(270\,\Omega)$$
$$= \quad \text{V}$$

UNIVERSITY OF NEW BRUNSWICK	FACULTY OF ENGINEERING		Page
Course No.	Assignment No.	Date	of
Problem No. 8	By		

Apply KCL at Node b :

$\cancel{i_1 + i_2 - i_3} = 0$

$\cancel{-1.86mA + 13.4mA - 11.\overset{70}{54}mA} = 0$

$-0.16mA = 0$

UNIVERSITY OF NEW BRUNSWICK | FACULTY OF ENGINEERING | Page

Course No. | Assignment No. | Date | of

Problem No. 6 | By

Measured Currents:

$$i_1 = \frac{V_{ab}}{R_1} = \frac{-0.2792\,V}{150.0\,\Omega} = -0.00186\,A$$
$$= -1.86\,mA$$

$$i_2 = \frac{V_{cb}}{R_2} = \frac{3.606\,V}{268.7\,\Omega} = 0.0134\,A$$
$$= 13.4\,mA$$

$$i_3 = \frac{V_b}{R_3} = \frac{5.404\,V}{466.5\,\Omega} = 0.0117\,A$$
$$= 11.7\,mA$$

UNIVERSITY OF NEW BRUNSWICK	FACULTY OF ENGINEERING	Page
Course No. ECE 2701	Assignment No. 1 Date Oct. 4.2019	of
Problem No. 5	By	

Total Voltage for each loop: (Using KVL)

Loop i_x:

$$0 = -V_1 + V_{ab} + V_b$$

$$0 = -(5.122V) + (-0.2792V) + 5.404V$$

$$0 = 0.0028V$$

Loop i_y:

$$0 = V_2 - V_b - V_{cb}$$

$$= 9.00V - (5.404V) - (3.606V)$$

$$= -0.006V$$