



**ELECTRICAL AND COMPUTER ENGINEERING**  
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**LABORATORY / ASSIGNMENT / REPORT**  
**COVER PAGE**

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(if applicable)

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**ECE 2701 Experiment 3:**

**Simple Filters**

**Dominic Geneau**

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**Prepared for**

**Dr. Howard Li**

**Date Due: November 8<sup>th</sup>, 2019**

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## **Abstract**

Experiment 3: Simple Filters investigates the behavior of first order capacitive filter networks. More specifically, each respective team of ECE 2701 was initially tasked with observing the behaviour of two first order filter circuits in order to determine their relative output voltage and phase angle behaviour when these circuits were put through various frequencies.

Using prior knowledge in AC filter response networks, the team initially observed the waveform changes relative to the change in frequencies that the input voltage was subjected to. This was accomplished using two specific measuring instruments: an oscilloscope and a function generator.

In short, the team found results that respected the theory behind low-pass and high-pass filter networks and managed to conclude the following statement: as the frequency increases in a first order low-pass filter, the output signal will decrease until it reaches approximately zero. The opposite can be said for high-pass filters as well: as the frequency decreases, the output voltage will eventually reach zero.

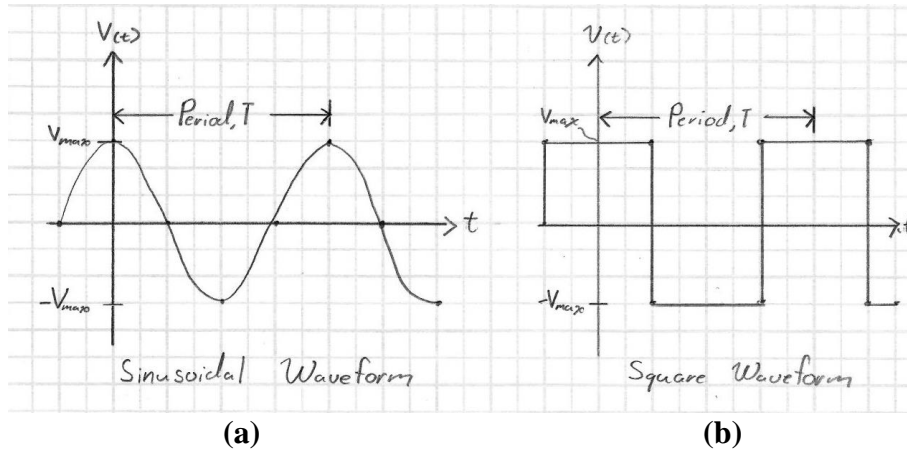
Once these results were explored, the team was tasked with finding the time constants of each respective circuit. Using the function generator square wave input signal, an exponential curve was generated from both circuits: one being an exponentially increasing function (low-pass) and the other being an exponentially decreasing function (high-pass).

It was then found, after calculating the theoretical values of each time constant, that the measured values from the oscilloscope worked very well with the calculated values. This could equally be taken as supporting the behaviour of the simple filter networks as well.

Seeing as this experiment was relatively straightforward, the team did not encounter any major issues. Nevertheless, experimental and random errors were still present throughout the procedure, especially human mistakes due to faulty data recordings and calculation errors.

## Introduction

In AC circuitry, waveform signals can be emitted using a voltage source. These waveforms can adopt many shapes: the most common ones being sinusoidal and square wave functions (illustrated by Figure 1 below).



**Figure 1.** AC common waveform functions where (a) is a sinusoidal waveform and (b) is a square waveform.

These waves are generally formed by the superposition of many sinusoidal functions that form a single wave response function. In other words, these functions are mathematically analysed through *Fourier Series*. This finite sum of derived sinusoids is well demonstrated by I. L. Veatch in the experimental lab manual. Using the form shown in an example equation for a square wave function like Figure 1b (Eq. 1) (2010, p.1):

$$v(t) = \frac{4V_c}{\pi} \sin(\omega_o t) + \frac{4V_c}{3\pi} \sin(3\omega_o t) + \frac{4V_c}{5\pi} \sin(5\omega_o t) + \dots \quad (1)$$

However, there are situations where these waveforms need to be bounded above or below by specified frequencies. In such cases, circuits are designed with *frequency response filters*. The nature and analysis of these transfer functions will be elaborated in the following sections, but, for now, it is simply important to note that two simple filters exist: *first order low-pass and high-pass filters*.

In the former case, Allan R. Hambley states that an input signal is guided through a simple two-port network which retains low frequency responses and discards frequencies higher than the permitted threshold. Conversely, the latter only accepts high frequency responses and rejects

frequencies below a pre-designated boundary (2018, p.290). These, in turn, will form new waveforms that will flow through the rest of the circuit.

In terms of this group's experiment, *Experiment 3: Simple filters* was meant to observe the behaviour of the first order filters mentioned above, while providing a baseline understanding of the frequency trend that these filters made in relation to the RC (resistor-capacitor) circuits produced in-lab.

In real world applications, filters are extremely important in audio extracting and sound isolation. A simple example would be radio frequencies. FM radio uses first order filters to target specific transmitted waveforms to obtain a requested frequency while also drowning out frequencies that would otherwise interfere with the audio emanating from the waveform.

## **Experiment**

In this section, the group will provide a thorough analysis of the procedure executed throughout the experiment, as well as an arrangement of tables and figures relating the results of the experiment with theoretical facts. Additionally, a short list of equipment used has been provided herein as well.

### **3.1. Apparatus**

In this experiment, the team used:

- 10k  $\Omega$  resistor;
- One of each of the following capacitors: 3.3 nF and 100 nF;
- Oscilloscope;
- Function generator;
- Black and red conducting wire.

### **3.2. Description of Theory and Procedure**

When constructing AC input functions, as demonstrated by Eq.1, input voltage generally involves one or more input functions at various frequencies. To properly evaluate these functions, it is useful to turn them into *Transfer Functions*. In short, these functions receive input signals and, using voltage divider analysis, can produce a new signal of the same shape. The general equation

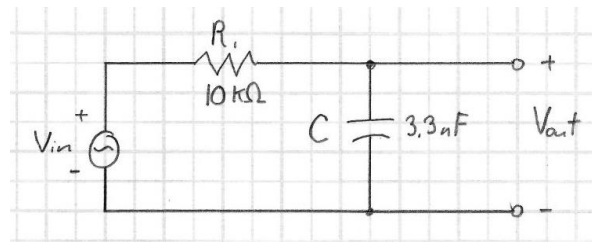
of a transfer function is described as the “gain” ratio between the output function,  $V_{out}$  with respect to the input function,  $V_{in}$ . The general equation can be seen through Eq.2 below (Hambley, 2018, p.297):

$$H(f) = \frac{\vec{V}_{out}}{\vec{V}_{in}} \quad (1)$$

Where:

- $H(f)$  = Gain ratio of the transfer function

This phenomenon is best explained by using the lowpass filter constructed in lab (Figure 2).



**Figure 2.** Low-pass filter circuit constructed and analysed in experiment 3.

Using voltage divider analysis and knowing capacitor,  $C$ , and the resistor,  $R$ , have impedances represented by Eq.2 & Eq.3, respectively:

$$Z_C = \frac{1}{j2\pi fC} \quad (2)$$

$$Z_R = R \quad (3)$$

Where:

- $j$  = imaginary axis unit vector (unitless)
- $f$  = frequency (Hz)
- $C$  = Capacitance (Farads)
- $R$  = Resistance ( $\Omega$ )

Now, referring to Figure 2 and using voltage divider analysis, we find that the output function can be found using Eq.4 (Veatch, 2010, p.2)

$$\vec{V}_{out} = \vec{V}_{in} \frac{Z_C}{Z_R + Z_C} = \vec{V}_{in} \frac{\frac{1}{j2\pi fC}}{R + \frac{1}{j2\pi fC}} = \vec{V}_{in} \frac{1}{1 + j2\pi fRC} \quad (4)$$

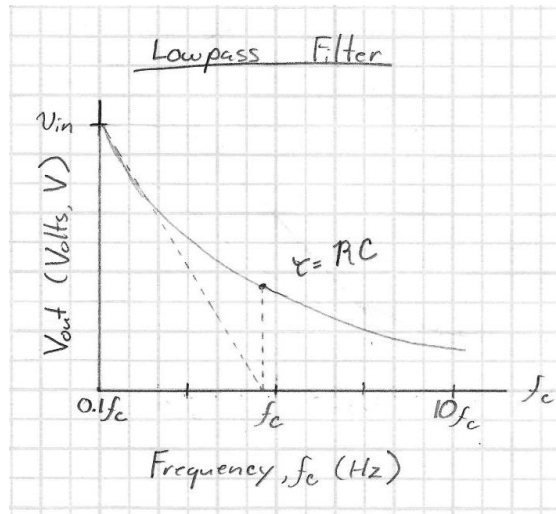


Furthermore, the magnitude and phase angle,  $\alpha$ , of the transfer function can be found by simply evaluating Eq.4 as a polar form function, where:

$$|H(f)| = \left| \frac{\vec{V}_{out}}{\vec{V}_{in}} \right| = \frac{1}{\sqrt{1^2 + (2\pi fRC)^2}} \quad (5)$$

$$\alpha = -\arctan\left(\frac{1}{2\pi fRC}\right) \quad (6)$$

It is important to note that both the phase angle and magnitude of the transfer function are directly influenced by the frequency of the circuit. As seen in both Eq.5 & Eq.6, a high frequency will cause the denominators to approach infinity, which, in turn, will cause the phase angle to plateau at 90 degrees and the gain to reach zero. The exponential decrease of the output function can be seen in Figure 3, where the time constant,  $\tau$ , represents the time it takes for its initial slope to reach the x-axis of the curve (typically  $\approx 0.632$  of initial y-axis value). This is observed through Figure 3, showing a decaying exponential curve as the frequency,  $f_c$ , approaches infinity.

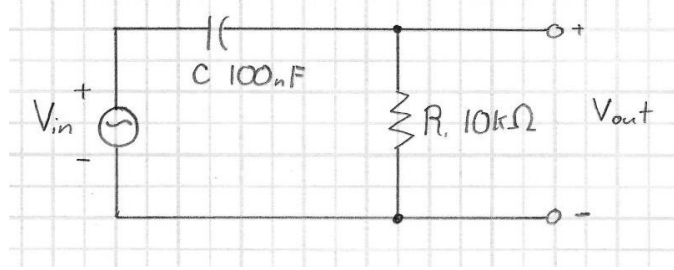


**Figure 3.** Low-pass filter decaying output voltage exponential with respect to increasing frequency (Hz).

The notation  $f_c$  is used in this report to act in tandem with the experimental data obtained through multiplying/dividing the cut-off frequency by a factor of ten to observe the change in output voltage. As stated by Veach, the cut-off frequency for both high-pass and low-pass filters is the frequency at which the power to a resistor is reduced by half when the output voltage is reduced by  $\frac{1}{\sqrt{2}}$  (see Eq.5 for proof) (2010, p.2). This value can be found by rearranging the time constant with the transfer function's factored imaginary impedance (Eq.7):

$$\omega_c = 2\pi f_c RC = 1 \rightarrow f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi\tau} \quad (7)$$

In terms of high-pass filters, the exact same process is true, save for a few details. For the experiment, Figure 4 was used as a first order high-pass filter circuit and can be analysed the same way as the low-pass filter.



**Figure 4.** High-pass filter circuit constructed and analysed in Experiment 3.

Using the same voltage divider analysis, it is found that the transfer function is “reversed” in the sense that it now only accepts high frequencies and rejects lower frequencies. This is seen through the general form (Eq.8), magnitude (Eq.9) and phase angle (Eq.10) functions similar to the low-pass functions (Veach, 2010, p.3):

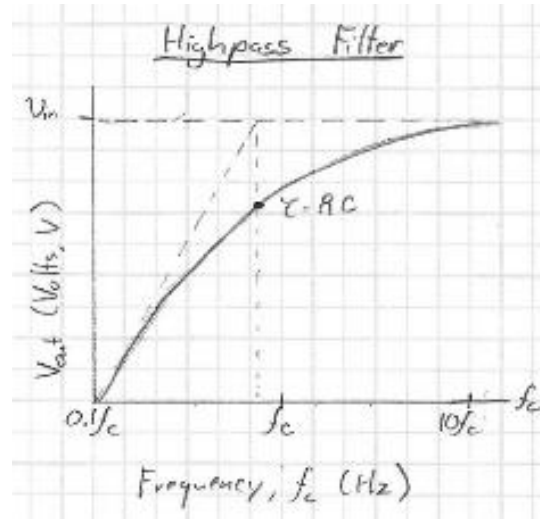
$$\vec{V}_{out} = \vec{V}_{in} \frac{Z_R}{Z_R + Z_C} = \vec{V}_{in} \frac{R}{R + \frac{1}{j2\pi fC}} = \vec{V}_{in} \frac{j2\pi fRC}{1 + j2\pi fRC} \quad (8)$$

$$|H(f)| = \left| \frac{\vec{V}_{out}}{\vec{V}_{in}} \right| = \frac{2\pi fRC}{\sqrt{1^2 + (2\pi fRC)^2}} \quad (9)$$

$$\alpha = 90^\circ - \arctan\left(\frac{1}{2\pi fRC}\right) \quad (10)$$

Now, again, both magnitude and phase angles are directly influenced by the applied frequency except, in this case, a high frequency will induce a gain of 1 whereas a lower frequency will reduce the output voltage until a theoretical gain of zero is attained and the phase angle will shift by  $90^\circ$ .

As one would expect, the exponential function of the output voltage over the applied frequency creates an increasing exponential curve as the frequency increases, and its maximum value will be the input voltage applied to the circuit. Figure 5 approximates this behaviour by depicting a rough diagram of voltage with respect to increasing frequencies.



**Figure 5.** General high-pass filter exponential behaviour of voltage with respect to applied frequency.

Now, in relation with the actual experiment, the group had to prepare by finding the theoretical time constant of the low-pass filter (Figure 2) and the high-pass filter (Figure 4). Then using Eq.7, they were to find the cut-off frequencies of the low-pass filter network in order to apply these frequencies throughout the experiment. These results were then collected and written down in a table of values, where they could be used to analyze the behaviour of the filters. Mathematical pre-lab calculations can be found in Appendix A of the lab report.

Then, after constructing the low-pass filter capacitive circuit mentioned above, the input voltage was connected to a function generator, where it applied an 8 V<sub>pp</sub> sinusoid wave with no DC offset to the circuit. The input and output voltages were also connected to distinct channels on an oscilloscope where their amplitudes and period could be measured and compared with respect to the change in frequency that could be applied by the frequency generator.

Amplitude, time delay and period values were measured at frequencies of  $0.1f_c$ ,  $f_c$  and  $10f_c$ . Just like in the lab preparation, the values were considered and inputted into a table of values (found in the following section). Then, using the measured values of the input voltage and the output voltage, the gain was calculated using Eq.5. And, now knowing the time delay between the two signals,  $T_d$ , and the period of the input waveform,  $T$ , a new equation (Eq.11) was used to calculate the phase angle of each output signal (Veach, 2010, p.6):

$$\theta = \frac{T_d}{T} \times 360^\circ \quad (11)$$

Following this part of the experiment, the function generator was changed to a 1 kHz square waveform and the resulting output waveform was printed for further analysis. More specifically, this was meant to manually find the time constant of the waveform so that it could be directly compared with the theoretical findings of the pre-lab.

A similar analysis was conducted using a constructed high-pass filter capacitive circuit (Figure 4). In this case, a square wave output of 100 Hz was emitted by the function generator and the resulting output waveform was printed to measure the time constant as well.

As elaborated by the following sections, the team's findings directly correlated with the theoretical evidence of the first order filters. In brief, it was found that phase delayed the output waveform proportionally to the increase in frequency of a low-pass filter circuit and delayed the output waveform with respect to the decrease in frequency of a high-pass filter.

### 3.3. Calculations, Graphs, Results and Analysis

Herein lies a list of figures, tables and graphs to further analyze the obtained results of the experiment. To avoid confusion, the section will follow the lab results recorded in chronological order, starting from procedure from the lab manual.

Table 1 is a direct tabulation of all the results (measured and calculated) taken from the low-pass filter oscilloscope and pre-lab calculations. As explained in the prior section, the measured outgoing and incoming voltage signals were used to calculate the gain at each respective frequency. And, using Eq.11, the phase at each frequency was calculated and a trend was established.

**Table 1.** Experiment 3 tabulated waveform parameters measured at various frequencies.

Frequency, $f$ (Hz)		$ \vec{V}_{in} $ (V <sub>pp</sub> )	$ \vec{V}_{out} $ (V <sub>pp</sub> )	Period, T (sec)	Delay, T <sub>d</sub> (sec)	Gain (V/V)	Phase angle, $\theta$ (deg)
$0.1 f_c$	482.3	8.00 V	7.84 V	$2.080 \times 10^{-3}$	$\sim 100.0 \times 10^{-6}$	0.98	-0.17
$f_c$	4 822.9	8.00 V	5.52 V	$207.2 \times 10^{-6}$	$25.6 \times 10^{-6}$	0.69	-44.45
$10 f_c$	48 229.0	8.00 V	0.80 V	$20.80 \times 10^{-6}$	$5.00 \times 10^{-6}$	0.10	-86.81

Using Eq.5, a sample calculation is performed that found the gain created through a frequency of 482.3 Hz (Eq.12):

$$|H(f)| = \left| \frac{\vec{V}_{out}}{\vec{V}_{in}} \right| = \left| \frac{(7.84 \text{ V})}{(8.00 \text{ V})} \right| = 0.98 \quad (12)$$

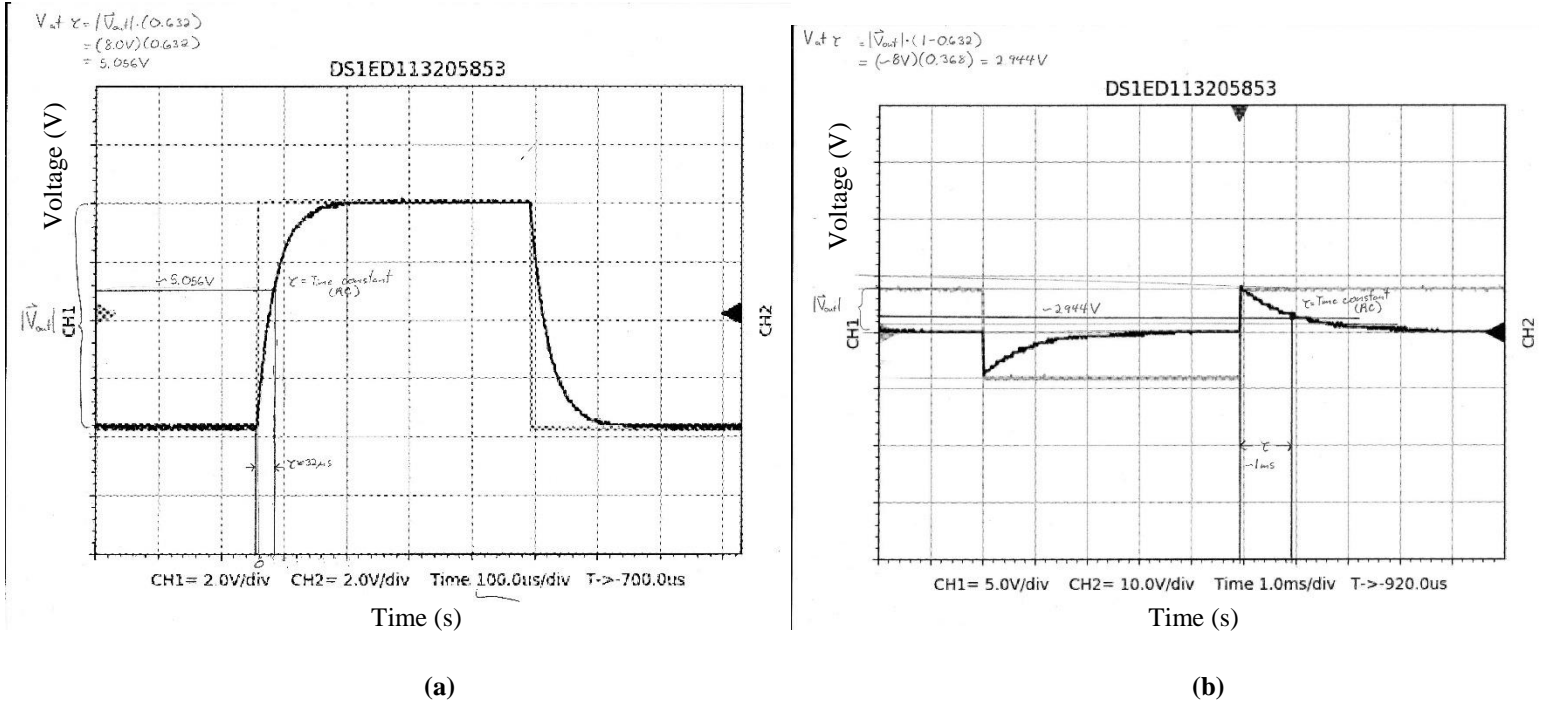
This bodes well with the expected results, since a low frequency provides a practical gain peak of approximately one, which means there is almost no interference toward the input voltage. And, to further prove this fact, the overall gain decreases as the frequency is increased ( $0.1f_c = 0.98 > f_c = 0.69 > 10f_c = 0.1$ ).

Furthermore, if the phase angle is investigated, a similar trend can be found. By using Eq.11, one can find the phase angle indicating a delay in the output voltage. Eq.13 uses the largest frequency,  $10f_c$ , as a sample calculation to find the phase angle of a given frequency:

$$\theta = \frac{T_d}{T} \times 360^\circ = \frac{(-5.00 \times 10^{-6} \text{ s})}{(20.80 \times 10^{-6} \text{ s})} \times 360^\circ = -86.81^\circ \quad (13)$$

Such a large phase angle corresponds with the initial theoretical objective, since it indicates a large delay in the output waveform that goes with a large frequency. It is equally worthwhile to note that a negative phase angle states that the waveform *lags* by that angle. If it were positive, the output voltage would lead the input voltage (which does not make sense). Additionally, Table 1 supports this theory as the phase angle increases in the clockwise direction as the frequency decreases ( $0.1f_c = -0.17^\circ > f_c = -44.45^\circ > 10f_c = -86.81^\circ$ ).

Regarding the last part of the experiment, the purpose was to use a square wave function emitted by the function generator to approximate a “step response” (Veatch, 2010, p.7). This would, in turn, allow the team to approximate the time it takes for the waveform to reach 0.632 of its final voltage (i.e., the first order time constant). Figures 6a and 6b represent the low-pass filter outgoing waveform function and the high-pass filter outgoing waveform, respectively.



**Figure 6.** Printed response waveform from (a) a low-pass filter circuit and (b) a high-pass filter circuit

A simple method discussed in-lab that was used to manually measure the time constant was to look at the vertical axis of the response. The measured amplitude of the output waveform was multiplied by the time constant ratio (i.e., 0.632) to find the voltage corresponding to the time constant. Then, a horizontal line was traced, intersecting the exponential increase/decrease which coincided with the time at which the time constant occurs. Then, by calculating the difference between the initial “start” time of the period, the time constant was found. In the case of the low-pass filter, the time constant voltage,  $V_t$  was found to be  $\sim 5.056$  V through Eq.14:

$$V_t = |\vec{V}_{out}| \times (0.632) = (8.00 \text{ V}) \times (0.632) = 5.056 \text{ V} \quad (14)$$

Then, by tracing a line across this value on the vertical axis, this voltage is reached at approximately  $32 \times 10^{-6}$  s, or 32  $\mu$ s. Now, if we calculate the time constant using the regular equation relating capacitance, C, with resistance, R, from the low-pass filter circuit (Figure 2), we find that:

$$\tau = RC = (10 \text{ k}\Omega)(3.3 \text{ nF}) = 33 \mu\text{s} \quad (15)$$

At a glance, these values are extremely close: further supporting the theoretical validity of time constants in waveform signal responses. Furthermore, if the percent difference is calculated using Eq. 16:

$$\%Difference = \left| \frac{measured - calculated}{calculated} \right| \times 100 \quad (16)$$

the percentage falls between 0% and 5% (demonstrated by Eq. 17). This is widely considered to be a typical boundary for accurate readings and, therefore, can be considered valid within uncertainties.

$$\%Difference_{\tau} = \left| \frac{(32\mu s) - (33\mu s)}{(33\mu s)} \right| \times 100 = 3.03\% \quad (17)$$

These calculations can be performed with Figure 6b as well, but is rather expressed in a table alongside the low-pass filter time constant measured and calculated results (Table 2):

**Table 2.** Measured and calculated time constants for the low-pass and high-pass filter circuits

	Calculated (sec)	Measured (sec)	% Difference
Low-Pass Filter	$33 \times 10^{-6}$	$32 \times 10^{-6}$	3.03%
High-Pass Filter	0.001	0.001	0.00%

As expressed above, the time calculated and measured values of the time constants of each filter circuit coincides very well with one another. In fact, the high pass filter has an approximate 0% difference, which further supports experimental theory of the results.

Lastly, it is important to note the relative shapes of Figures 6a and 6b, and to explain the reasoning behind their natural curves. Simply put, frequency ( $s^{-1}$ ) is the inverse of the period (s) of a wave function. This would cause the opposite behaviour to occur relative to one another. This can, in fact, be seen when observing the exponential behaviour of Figure 3 (a frequency dependent function) and Figure 6a (a time dependant function). Thus, their exponential curves are flipped: one being increasing (Figure 6a) and the other being a decaying exponential function (Figure 3).

## Discussion

An examination of first order capacitive filter networks allowed the team to compare the calculated and measured values of the low-pass and high-pass filters of first order RC circuits. More specifically, the input/output signal voltages, the period of the initial input function and the time delay between the two waveforms.

Comparisons were conducted by measuring the period of the input signal voltage response using the oscilloscope. Prior to this part of the experiment, the team examined the behavior of period as frequency changes. This relationship gave rise to the phase angle caused by the delay in between both waveforms. After conducting this experiment, various values were found experimentally and theoretically; some of which were given, like resistance values and capacitance values.

During the experiment, the behavior of the period with the change in frequency was examined. To better understand, one needs to know the inner workings of waves and frequencies. While conducting the experiment, the team examined that as the frequency increased, the phase angle increased for the low-pass filter. As a result, we encountered time delay,  $T_d$  which increased as the frequency increased. Therefore, as the frequency of a function decreases, the period would increase, and the delay would be eliminated (as seen in Table 1).

Additionally, percent difference between theoretical and experimental values were calculated using Eq.16 as well. While it is not particularly useful to discuss all %differences, the team has found it worthwhile to elaborate on the %difference of the measured time constant,  $\tau$ , of the low-pass filter network (Table 2). As an example, it was found that (through Eq.17) the calculated and measured values differed by a margin of approximately 3.03%.

While values remained within reasonable parameters (%difference  $\leq 5\%$ ), it still raises the question of why this value is higher than the other. This could be due to many variables, but, for the purpose of this report, the following paragraphs elaborate on the most common errors that the team could think of.

Potential human errors encountered in the lab which could have altered our readings range from misreading the value on the oscilloscope, misinterpreting readings given by the function



generator, or simply having a faulty wire in the network. Mathematically, not using appropriate significant digits during the calculations could have equally caused a varying percent difference between the theoretical values and the experimental values.

Experimental errors consisted of some being unavoidable. Environmental factors such as a change in temperature would evidently alter the readings on the oscilloscope (e.g., varying resistance, which was measured to be  $9.87\text{ k}\Omega$  instead of the theoretical  $10\text{ k}\Omega$ ). Faulty calibration of the devices used in the lab such as oscilloscope and function generator may have resulted in a variation of the readings as well. As the name states, unavoidable errors cannot be altered directly, but being aware of these variables could equally reduce the margin of error while also creating a better understanding of the phenomena.

## **Conclusion**

Experiment 3 provided a wide range of results that all revolved around the same thing: examination of behavior of first order RC capacitive filter networks: low-pass and high-pass filter. After a thorough analysis of the obtained results throughout the experiment, it was found that the experiment supported the effective use of low-pass and high-pass filter circuits, by allowing or removing certain frequencies from continuing further along in the system. Thus, improving the input signal clarity. Although the period ( $T$ ) of the calculated versus measured did not have satisfactory results within the perfect range, the values were in the range suited for the frequencies being tested. Noticeable differences in the results and the accuracy of the hand measured graphs used to obtain the time constant could have introduced significant errors experimentally as well.

Therefore, it was found very important to reiterate the human and random errors that were encountered during the experiment that may have altered the actual results. Firstly, when measuring the period, the team used the measured value of the resistor ( $9.87\text{ k}\Omega$  instead of  $10\text{ k}\Omega$ ) which would have obviously changed the true value, even if only slightly. Additionally, rounding errors during calculations could have ensued slight deviations as well.

It is also important to understand that, during the experiment, due to an instrument malfunction, the team may not have been able to achieve correct values. This meant that the measured time delay from the oscilloscope between the two waveforms could have shifted; thereby

producing an incorrect signal voltage ( $V_{in}$ ,  $V_{out}$ ), period ( $T$ ) and time delay ( $T_d$ ) in the process by the machine and manually adjusted by hand, which could have. It is also worthy to note that not all resistances were measured. Because of this, measured values might not have been the same if the resistances did not hold their indicated values.

In conclusion, as with any experiment, the results could have been more accurate if it was performed multiple times. Given more time, the team could have meticulously gone through every circuit to assure a strong and reliable connection through each conductor as well. Through it all, however, the team successfully completed what they were set out to do: proving the effective use of first order RC low-pass and high-pass filter circuits.

### **Summary of Roles**

Dominic Geneau played a marginal role in the making of the report as well as the experimental procedure of the lab. Working alongside Abdul Wasey, he analyzed the waveform behaviour that was projected by the oscilloscope and constructed each first order simple filter network accordingly. In terms of the report, Dominic wrote most of the abstract, introduction and experiment (except for the apparatus). He also proof read the entire document to ensure adequate and professional formatting was implemented throughout the report.

Syed Abdul Wasey Naqvi contributed an equitable amount during the lab, and during the writing stage of this report. Working along side Dominic Geneau, he analyzed the behavior of period with the changes in frequency and phases changes. In terms of the report, Syed Abdul Wasey Naqvi contributed to the abstract, discussion, apparatus and conclusion. He also proof read the entire document to ensure adequate and professional report was formed for submission.

## References

Hambley, A. R. (2018) *Electrical engineering: Principles and applications* (7<sup>th</sup> ed.). Hoboken, NJ: Pearson Education Inc.

Veatch, I. L. (2010) *EE2701 experiment 3: Simple filters* [PDF Document]. Retrieved from UNB D2L online website:

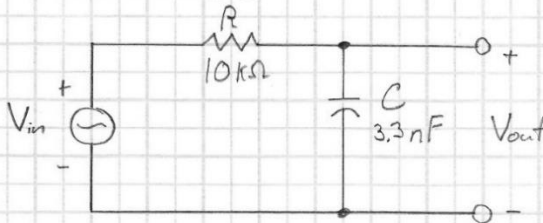
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## Appendix A: Pre-Lab Calculations

UNIVERSITY OF NEW BRUNSWICK	FACULTY OF ENGINEERING	
Course No. <u>ECE 2701</u>	Assignment No. <u>LAB 3</u> Date <u>Oct 30<sup>th</sup> 2019</u>	Page
Problem No. <u>Pre-Lab Calculations</u>	By <u>Dominic Gagneau</u>	of

RC Circuit (Low Pass Filter)



$\left| \frac{\vec{V}_{out}}{\vec{V}_{in}} \right| = G \angle \theta$ 

Time constant

 $\tau = RC = (10k\Omega)(3.3nF)$ 
 $\tau = 33 \times 10^{-6} s$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{Z_c}{Z_R + Z_c} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

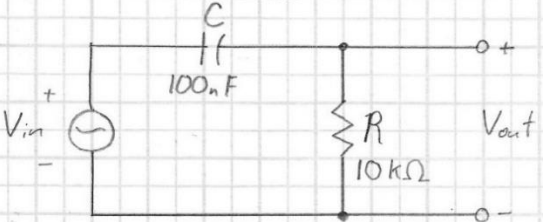
Half power frequency

$$\omega_c RC = 1$$

$$\omega_c = \frac{1}{\tau} = \frac{1}{(33 \times 10^{-6} s)} = 30.303 \times 10^3 \frac{rad}{s} = \omega_c$$

$$\omega_c = 2\pi f_c = 30.303 \times 10^3 Hz \Rightarrow f_c = 4822.9 Hz$$

RC Circuit (High Pass Filter)



$\frac{\vec{V}_{out}}{\vec{V}_{in}} = G \angle \theta$ 

Time constant

 $\tau = RC = (10k\Omega)(100nF)$ 
 $\tau = 0.001 s$

$$\left| \frac{V_{in}}{V_{out}} \right| = \frac{Z_R}{Z_R + Z_c} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega CR}{j\omega CR + 1}$$

must equal 1

Half Power Frequency

$$\omega_c RC = 1 \Rightarrow \omega_c = \frac{1}{RC} = \frac{1}{\tau}$$

$$\omega_c = \frac{1}{(0.001s)} \Rightarrow \omega_c = 1000 rad/s$$

$$2\pi f_c = \omega_c \Rightarrow f_c = \frac{\omega_c}{2\pi} = \frac{(1000 rad/s)}{2\pi} \Rightarrow f_c = 159.2 Hz$$

Extra Information if Required

- 1)  $\tau = RC \rightarrow$  Determine - the time constant.  
 (Referring to figure 3.2)   
 - Associated half power frequencies  
 -  $\omega_c$   
 -  $f_c$   
 of simple RC low-pass filter

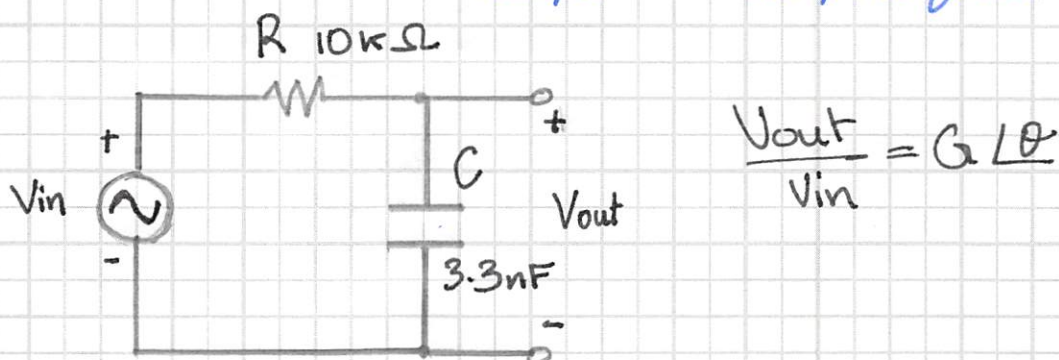


Figure 1: Circuit used for question 1 of pre-lab from Lab Manual Figure 3.2

$$\tau = RC = \frac{1}{2\pi f_c}$$

$$\begin{aligned}\tau &= (10\text{ k}\Omega)(3.3\text{ nF}) \\ &= (10 \times 10^3 \Omega)(3.3 \times 10^{-9}\text{ F}) \\ &= 3.3 \times 10^{-5}\text{ s} \quad \text{or} = 0.000033\text{ s}\end{aligned}$$

$$f_c = \frac{1}{2\pi\tau}$$

$$f_c = \frac{1}{2\pi(3.3 \times 10^{-5}\text{ s})}$$

$$= 4.8 \times 10^3\text{ Hz} \quad \text{or} = 4822.87\text{ Hz}$$

$$0.1f_c = 0.1(4.8 \times 10^3\text{ Hz}) = 482.3\text{ Hz}$$

$$10f_c = 10(4.8 \times 10^3\text{ Hz}) = 48229\text{ Hz}$$

$$\omega_c = \frac{1}{\tau}$$

$$= \frac{1}{3.3 \times 10^{-5}\text{ s}}$$

$$= 3.03 \times 10^4\text{ rad/s} \quad \text{or} = 30300\text{ rad/s}$$

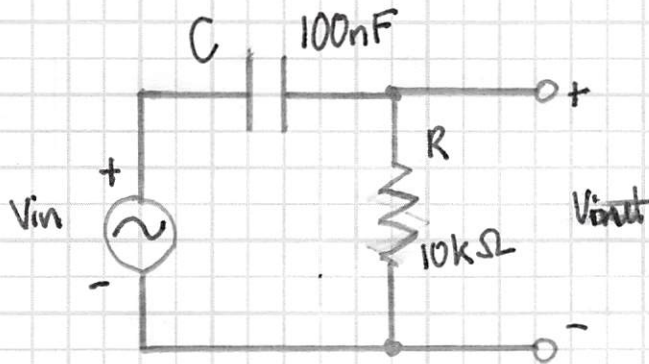
Handwritten notes at the top of the page, including the word "SPECIAL" and other illegible text.

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Handwritten notes at the bottom of the page, including the word "TOTAL" and other illegible text.



2)  $\tau = RC \rightarrow$  Determine - the time constant  
(Referring to Figure 3.3)



$$\frac{V_{out}}{V_{in}} = G \angle \theta$$

Figure 2 : Circuit used for question 2 of pre-lab  
from Lab Manual Figure 3.3.

$$\tau = RC$$

$$= (10\text{ k}\Omega)(100\text{ nF})$$

$$= (10 \times 10^3 \Omega)(100 \times 10^{-9} \text{ F})$$

$$= 1 \times 10^{-3} \text{ s} \text{ or } 0.001 \text{ s}$$

$$\omega_c = \frac{1}{\tau}$$

$$= \frac{1}{0.001 \text{ s}}$$

$$= 1000 \text{ rad} \text{ or } 1.0 \times 10^3 \text{ rad/s}$$

$$f_c = \frac{1}{2\pi\tau}$$

$$= \frac{1}{2\pi(0.001 \text{ s})}$$

$$= 159.15 \text{ Hz}$$



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Handwritten text in the middle section, possibly a list or series of notes.

Handwritten text in the lower middle section, appearing as a separate paragraph.

Handwritten text in the lower section, continuing the notes or list.

Handwritten text at the bottom of the page, possibly a conclusion or signature area.

## **Appendix B – Mathematical Calculations**

Name: Dominic GeneauID number: 3675144Date: Nov-1<sup>st</sup> 2019

## EE2701 Experiment 3

### Summary of Results

1. Use the value obtained in the preparation for the "cut-off" frequency,  $f_c$ , of the low-pass filter of Figure 3-2 to fill in the frequency column of Table 3-1.

Record the calculated values of the time constants obtained in the preparation for the filters of Figure 3-2 and Figure 3-3 in the appropriate "Calculated" cells of Table 3-2.

2. Record the values measured in part 3 of the procedure in the appropriate cells of table 3-1.

Table 3-1. Frequency Response Data for the RC Low-Pass Filter of Figure 3-2.

f (Hz)		$ v_{in} $ (V <sub>pp</sub> )	$ v_{out} $ (V <sub>pp</sub> )	T (sec)	T <sub>d</sub> (sec)	G (V/V)	θ (deg)
0.1 f <sub>c</sub>	482.3	8.00V	7.84 V	2.080ms	0.100ms	0.98	-0.17°
f <sub>c</sub>	4822.9	8.00V	5.52 V	207.2μs	25.6μs	0.69	-44.45°
10 f <sub>c</sub>	48229	8.00V	800 mV	20.8μs	5.0μs	0.1	-86.81°

From the data in Table 3-1, calculate the gains at the frequencies of 0.1  $f_c$ ,  $f_c$  and 10  $f_c$ .

From the data in Table 3-1, calculate the phase at the frequencies of 0.1  $f_c$ ,  $f_c$  and 10  $f_c$ .

Do the results obtained for  $G$  and  $\theta$  consistent with this network being a low-pass filter? Explain.

3. Attach a printout of the response of the filter of Figure 3-2 to a 1.0kHz square wave input. Use this printout to estimate the time constant of the filter and record the result in the "Measured" column of Table 3-2. Annotate the printout to indicate how you obtained your value for the time constant.

Table 3-2. The Time Constant for Figure 3-2.

	Calculated	Measured	% difference
Figure 3-2	$33 \times 10^{-6} \text{ s}$	<del>3.3 x 10<sup>-6</sup> s</del>	3.03 %
Figure 3-3	0.001 s	0.001 s	0 %

Is the observed time constant consistent with the calculated value obtained in the preparation?

4. Attach a printout of the response of the network of Figure 3-2 to a 1.0 kHz square wave. Comment on the shape of the response.

## EE2701 Experiment 3

### Summary of Results

1. Use the value obtained in the preparation for the "cut-off" frequency,  $f_c$ , of the low-pass filter of Figure 3-2 to fill in the frequency column of Table 3-1.  
Record the calculated values of the time constants obtained in the preparation for the filters of Figure 3-2 and Figure 3-3 in the appropriate "Calculated" cells of Table 3-2.
2. Record the values measured in part 3 of the procedure in the appropriate cells of table 3-1.

Table 3-1. Frequency Response Data for the RC Low-Pass Filter of Figure 3-2.

f (Hz)	$ v_{in} $ (V <sub>pp</sub> )	$ v_{out} $ (V <sub>pp</sub> )	T (sec)	T <sub>d</sub> (sec)	G (V/V)	$\theta$ (deg)
0.1 $f_c$ 482.3	8V	7.84V	2.080ms	100.0 $\mu$ s	0.98	-0.17°
$f_c$ 4822.8	8V	5.22V	207.2 $\mu$ s	25.6 $\mu$ s	0.69	-44.45°
10 $f_c$ 48229	8V	800mV	20.8 $\mu$ s	5.0 $\mu$ s	0.1	-86.81°

From the data in Table 3-1, calculate the gains at the frequencies of 0.1  $f_c$ ,  $f_c$  and 10  $f_c$ .

From the data in Table 3-1, calculate the phase at the frequencies of 0.1  $f_c$ ,  $f_c$  and 10  $f_c$ .

Do the results obtained for  $G$  and  $\theta$  consistent with this network being a low-pass filter? Explain.

3. Attach a printout of the response of the filter of Figure 3-2 to a 1.0kHz square wave input. Use this printout to estimate the time constant of the filter and record the result in the "Measured" column of Table 3-2. Annotate the printout to indicate how you obtained your value for the time constant.

Table 3-2. The Time Constant for Figure 3-2.

	Calculated	Measured	% difference
Figure 3-2	$3.3 \times 10^{-5} s$	$3.2 \times 10^{-5} s$	3.03%
Figure 3-3	$1 \times 10^{-3} s$	0.001s	0%

Is the observed time constant consistent with the calculated value obtained in the preparation?

4. Attach a printout of the response of the network of Figure 3-2 to a 1.0 kHz square wave. Comment on the shape of the response.

Gains at the Frequencies  $0.1 f_c$ ,  $f_c$ , and  $10 f_c$  :

$$\begin{aligned} \underline{0.1 f_c} : \frac{V_{out}}{V_{in}} &= \frac{7.84 V_{pp}}{8.0 V_{pp}} \\ &= 0.98 \end{aligned}$$

$$\begin{aligned} \underline{f_c} : \frac{V_{out}}{V_{in}} &= 6 \angle \theta \\ &= \frac{5.52 V_{pp}}{8.0 V_{pp}} \\ &= 0.69 \end{aligned}$$

$$\begin{aligned} \underline{10 f_c} : \frac{V_{out}}{V_{in}} &= \frac{800 mV_{pp}}{8.0 V_{pp}} \\ &= \frac{0.8 V_{pp}}{8.0 V_{pp}} \\ &= 0.1 \end{aligned}$$

$\theta$ , Angle at the Frequencies of  $0.1 f_c$ ,  $f_c$ , and  $10 f_c$  :

$$\begin{aligned} \underline{0.1 f_c} : \quad & 0.1 f_c \times T_d \times 360^\circ \\ &= 0.1 (482.3 \text{ Hz}) (100 \mu s) \times 360^\circ \\ &= -0.17^\circ \end{aligned} \quad \begin{aligned} \underline{10 f_c} : \quad & 10 f_c \times T_d \times 360^\circ \\ &= 10 (48229 \text{ Hz}) (5.0 \mu s) \times 360^\circ \\ &= -86.81^\circ \end{aligned}$$

$$\begin{aligned} \underline{f_c} : \quad & f_c \times T_d \times 360^\circ \\ &= (482.287 \text{ Hz}) (25.6 \mu s) (360^\circ) \\ &= -44.45^\circ \end{aligned}$$

1. The first part of the document is a letter from the President of the United States to the Congress.

2. The second part is a report on the state of the Union.

3. The third part is a report on the state of the Treasury.

4. The fourth part is a report on the state of the Navy.

5. The fifth part is a report on the state of the Army.

6. The sixth part is a report on the state of the Marine Corps.

7. The seventh part is a report on the state of the Coast Guard.

8. The eighth part is a report on the state of the Air Force.

9. The ninth part is a report on the state of the Department of the Interior.

10. The tenth part is a report on the state of the Department of Justice.

11. The eleventh part is a report on the state of the Department of Education.

12. The twelfth part is a report on the state of the Department of Agriculture.

13. The thirteenth part is a report on the state of the Department of Commerce.

14. The fourteenth part is a report on the state of the Department of Labor.

15. The fifteenth part is a report on the state of the Department of Health and Human Services.



Table 3.2 % Difference:

$$\% \text{ Difference} = \left| \frac{\text{measured} - \text{calculated}}{\text{calculated}} \right| \times 100\%$$

$$\begin{aligned} \text{(Figure 3.2)} \quad &= \left| \frac{3.2 \times 10^{-5} \text{ s} - 3.3 \times 10^{-5} \text{ s}}{3.3 \times 10^{-5} \text{ s}} \right| \times 100\% \\ &= 3.03\% \end{aligned}$$

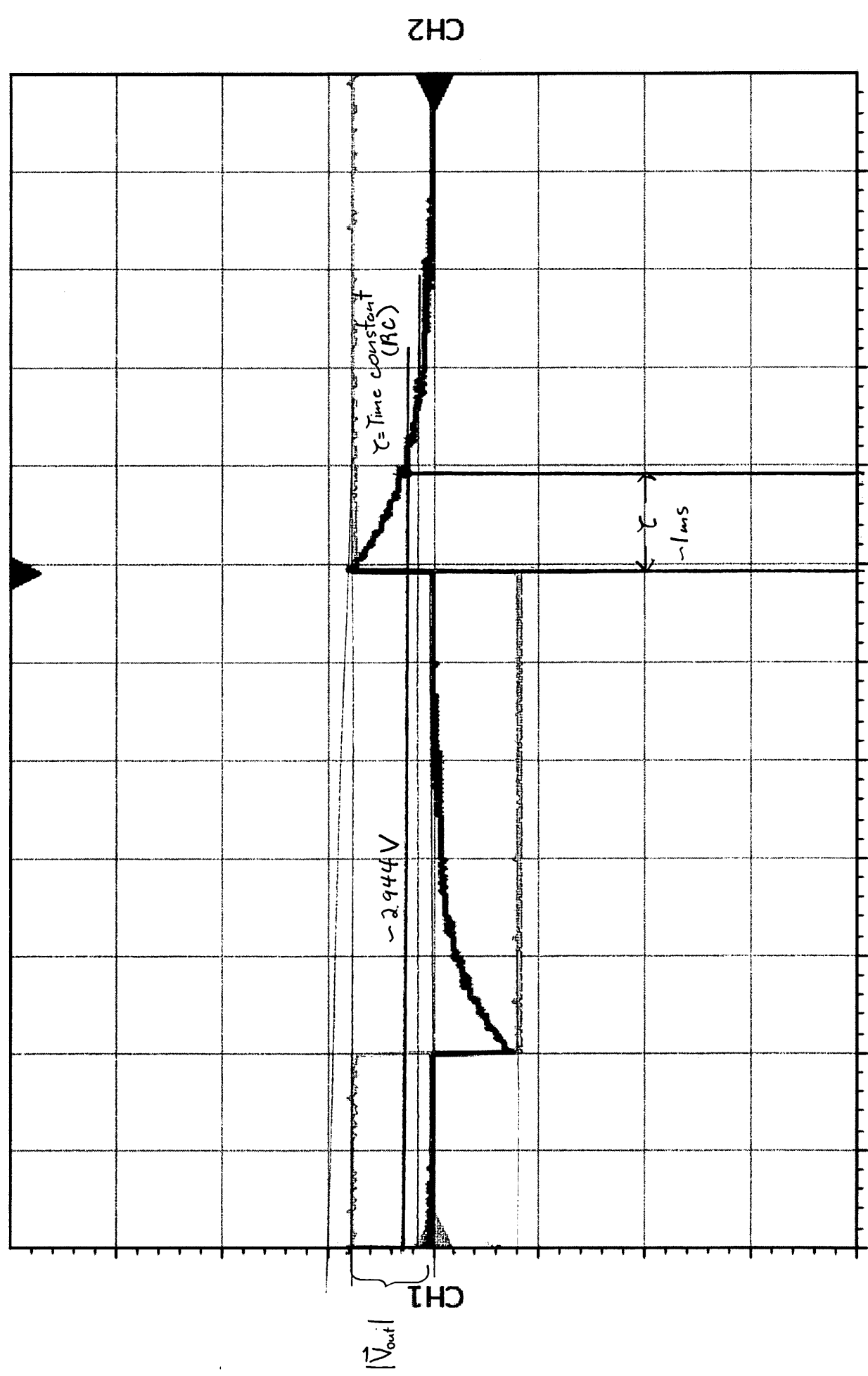
$$\begin{aligned} \text{(figure 3.3)} \quad &\% \text{ Difference} = \left| \frac{0.001 \text{ s} - 1.0 \times 10^{-3} \text{ s}}{1.0 \times 10^{-3} \text{ s}} \right| \times 100\% \\ &= 0\% \end{aligned}$$

## **Appendix C – Simple Filter Graphs**



$$V_{at} \tau = |\vec{V}_{out}| \cdot (1 - 0.632) = (\sim 8V)(0.368) = 2.944V$$

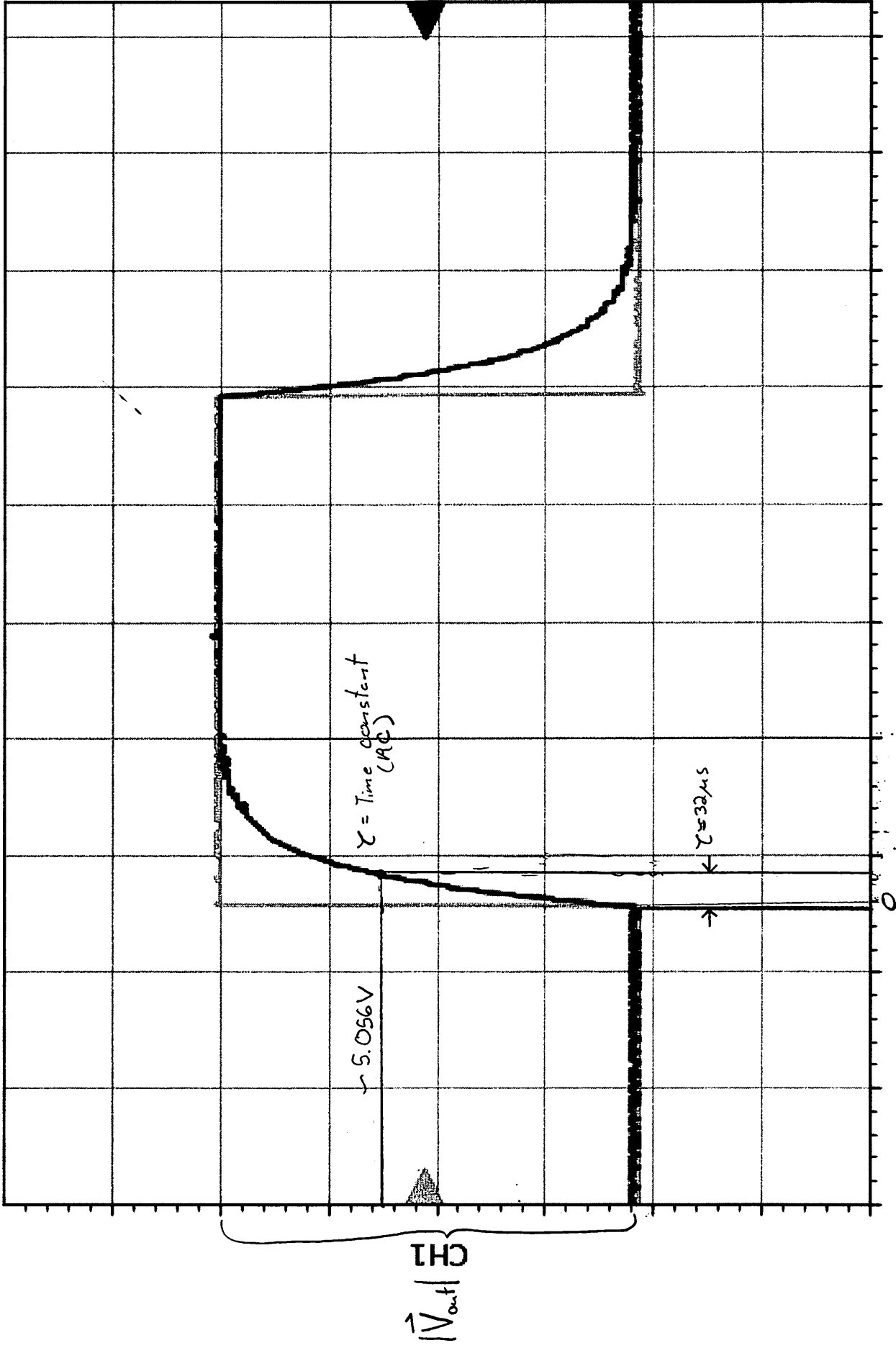
DS1ED113205853



CH1= 5.0V/div CH2= 10.0V/div Time 1.0ms/div T->-920.0us

$$\begin{aligned}
 V_{at} \tau &= |\vec{V}_{out}| \cdot (0.632) \\
 &= (8.0V)(0.632) \\
 &= 5.056V
 \end{aligned}$$

DS1ED113205853



CH1 = 2.0V/div CH2 = 2.0V/div Time 100.0us/div T -> 700.0us