

ELECTRICAL AND COMPUTER ENGINEERING

Room D36, Head Hall

LABORATORY / ASSIGNMENT / REPORT COVER PAGE

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ECE 2701 Experiment 2:

Voltage Source Equivalence

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Prepared for

Dr. Howard Li

Date Due: October 25th, 2019

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Abstract

Experiment 2: Voltage Source Equivalence investigates the concepts of circuit equivalence using Thevenin's Theorem. More specifically, each respective team of ECE 2701 was tasked with observing two seemingly different circuits (at first glance) in order to determine if they held voltage and resistance equivalencies. Both circuits were analyzed: each having a different voltage source and varying resistors.

This analysis was accomplished by initially finding the Thevenin equivalent circuit of both sources experimentally using Thevenin's Theorem (i.e., open circuit voltage, short circuit current and equivalent resistance strategies). Then, using current/voltage divider analysis as well as the digital multimeter recorded measurements, each team was meant to prove Thevenin's Theorem. Furthermore, each group was tasked with writing a formal report reflecting their findings: ultimately supporting or refuting the given objective mentioned above.

In the end, it was found that Thevenin's Theorem held true, even if there were slight deviations to the theoretical values calculated beforehand.

Seeing as this experiment was relatively straightforward, the team did not encounter any major issues. Nevertheless, experimental and random errors were still present throughout the procedure, especially human mistakes due to faulty data recordings and calculation errors.

Introduction

In electricity, circuits are generally created to complete a given function in the most efficient way possible. However, it comes a time where these circuits become an amalgamation of multiple smaller circuits, nodes or loops: creating a maze of conductors, voltage sources and resistors. In order to correctly analyze these two-terminal circuits, it was determined that there were ways to "simplify" the layout of a complex circuit into one single equivalent voltage source and resistor. This is what is known as the *Thevenin Equivalent Circuit* (demonstrated in Figure 1).

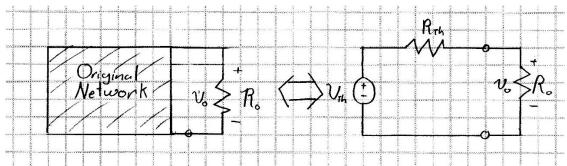


Figure 1. Representation of Thevenin equivalence of a complex circuit network.

As seen in Figure 1, and as Allan R. Hambley summarizes: "... The Thevenin source voltage is equal to the open-source voltage of the original network" (2018, p.91). There are strategies to properly analyze these networks into a Thevenin equivalent circuit which will be discussed in more detail in the following sections.

For this experiment, the group investigated the theoretical properties of the Thevenin equivalent circuit. More specifically, the team was tasked with constructing and analyzing two circuits to determine if they were equivalent (i.e., if they had the same open-source voltage, V_{oc} , and resistance, R_{Th} . This, in turn, would aid in supporting this fundamental theorem as well as giving the team an idea of Thevenin equivalency.

As mentioned before, Thevenin equivalence is mainly used in the simplification of two terminal circuitry. It is debatably not as paramount as Ohm's law or Kirchoff's laws, but it still provides a considerable amount of importance in successfully simplifying and analysing complex circuits. Thus, proving this theorem is important in the sense that it eases the analysis of a complex circuit (similar to phasors in relation to alternating currents (AC) circuits).

Experiment

In this section, the group will provide a thorough analysis of the procedure executed throughout the experiment, as well as an arrangement of tables and figures relating the results of the experiment with theoretical facts. Additionally, a short list of equipment used has been provided herein as well.

3.1. Apparatus

In this experiment, the team used:

- One of each of the following resistors: 100Ω , 150Ω , 220Ω , 270Ω , and 470Ω ;
- A Digital Multimeter (DMM);
- An Elenco XP-720 power supply;
- An assortment of red and black conducting wire.

3.2. Description of Theory and Procedure

Before expanding on the procedure implemented during the experiment, it was deemed important to elaborate further on the process of finding the Thevenin equivalent circuit. This will help readers understand the reasoning behind the pre-lab calculations while also providing a baseline for the theory that follows.

As seen in figure 1, the Thevenin equivalent of a circuit is simply the open circuit voltage (represented as a voltage source) in series with the equivalent resistance of the original network. This holds true for any complex two-source terminal circuit, and there is a systematic approach that facilitates the analysis of these circuits. All three steps are represented in Figure 2 below.

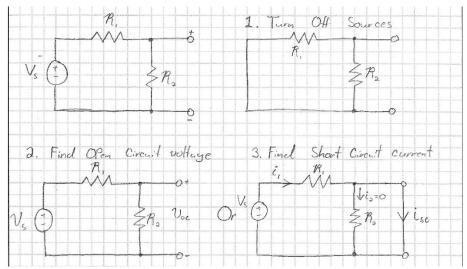


Figure 2. Systematic steps used to theoretically determine the Thevenin equivalent of a circuit.

First and foremost, it is ideal to turn off all independent sources in the circuit in order to find the Thevenin equivalent resistance, R_{Th} . In the case above (Figure 2), we would solve R_{Th} using the parallel resistance equation, as shown in Eq.1:

$$\frac{1}{R_{Th}} = \sum_{i=1}^{n} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} = R_{Th}$$
 (1)

Then, depending on the situation, one could choose to find the open circuit voltage across the terminals or to compute the short circuit current. To find the open circuit voltage, we use the voltage divider formula to compute the voltage in parallel with R₂: since voltages remain the same if they are in parallel. This can be observed in Eq.2 below:

$$V_{oc} = V_s \frac{R_2}{R_1 + R_2} = V_{Th}$$
 (2)

Third, the short circuit current, i_{sc} can be found by creating a short circuit through the terminals. Then, the current through them can be found using KCL or any other pertinent circuit analysis strategy. Referring to Figure 2, knowing that the current through R_2 is zero, we can find i_{sc} using Eq. 3:

$$i_{sc} = \frac{V_s}{R_1} \tag{3}$$

Although, after finding two of the three characteristics of the Thevenin equivalent circuit, it is easiest to simply compute Ohm's law using Eq. 4 (Hambley, 2018, p.92):

$$V_{oc} = i_{sc} R_{Th} \tag{4}$$

where V_{oc} , i_{sc} and R_{Th} are found using the the steps mentioned above.

Finally, once all three values are found, all that is left is to create an equivalent circuit consisting of the voltage source, now V_{Th} , in series with the Thevenin resistance, R_{Th} . While this process may take a long time, it has proven to be very versatile during many circuit analysis problems. In fact, this approach was used in the computation of the pre-lab results, as discussed in the following paragraphs.

Initially, just like Experiment 1, both team members were to complete a pre-lab sheet before entering the laboratory. In this case, the group was tasked with theoretically computing the open-circuit voltage, V_{oc} , the short circuit current, i_{sc} and the Thevenin equivalent resistance, R_{Th} , of the two circuits (represented in Figure 3 below). Then, after adding a load resistance of 270 Ω , each member was tasked with calculating the voltage, V_L , across this resistor. Once this was done, they were to compare all the calculated values between source 1 and source 2 to determine if they were theoretically equivalent. A procedural breakdown of how each circuit's Thevenin equivalent was found has been provided in Appendix A.

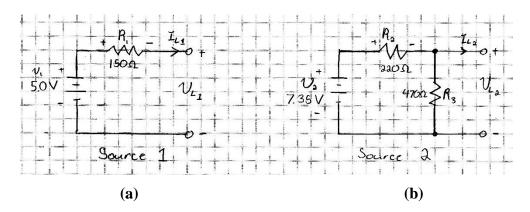


Figure 3. Diagram of each circuit constructed in lab where (a) is Source 1 and (b) is Source 2

In terms of procedure, the group was to set up each source with its respective power source: 5V for source 1 and 7.35V using a variable power supply for source 2. Then, following the *Measuring Source Characteristics* procedure in lab, we were to measure the voltage across the two terminals of each source using the DMM. As mentioned in the lab manual, it is important to note that since there is no current flowing through the multimeter, it is safe to assume that the recorded voltage, V_{o1} is, in fact, the Thevenin equivalent for the respective source (as seen in figure 4) (Veach, 2011, p.2).

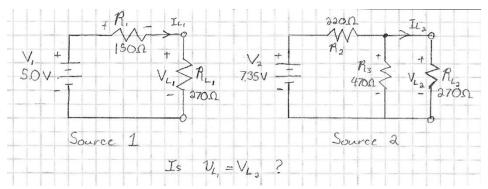


Figure 4. Updated source 1 and source 2 circuits containing a 270 Ω loaded resistor.

Next, the team was asked to link a 100 Ω resistor, R_x , across the two terminals while measuring the new voltage, V_{o2} . Then, using voltage divider analysis and relating V_{o1} to V_{o2} , we can find the Thevenin equivalent resistance, as seen in Eq. 5 (Veach, 2011, p.2):

$$V_{o2} = \frac{R_x}{R_x + R_{Th}} V_{01} \rightarrow R_{Th} = \frac{V_{o1} - V_{o2}}{V_{o2}} R_x = \left(\frac{V_{o1}}{V_{o2}} - 1\right) R_x = R_{Th}$$
 (5)

It is important to note, however, that since both V_{o1} and V_{o2} are experimentally measured results, this creates a very big chance for R_{Th} to become inaccurate due to experimental error (Veach, 2011, p.2).

Once R_{Th} and V_{Th} were recorded in the tables listed in the experiment sheet, the remaining task involved loading a new resistor of 270 Ω across the two terminals in order to calculate a new voltage: V_{L1} and V_{L2} (represented in Figure 4). The reason behind this new measurement is simple: a circuit is deemed "equivalent" if both circuits obtain the same values across various loaded resistors.

Once all of this was done, all measured values were to be compared with their calculated counterparts (from the pre-lab). The percent difference was also calculated in hopes to analyse possible sources of error as well as proving the true objective of the experiment. The variant equations are represented by Eq. 6 & 7 (Veach, 2011, p.4), and the results obtained from them can be found in the following section of the report.

$$\%Difference = \left| \frac{measured-calculated}{calculated} \right| \times 100$$
 (6)

$$\%Difference = \left| \frac{Source \, 1_{value} - Source \, 2_{value}}{Source \, 1_{value}} \right| \times 100 \tag{7}$$

In the end, the group found that both sources were, in fact, experimentally and theoretically "equivalent". Both had very similar measured V_{Th} of approximately 5.00 V (within uncertainties) as well as their Thevenin equivalent resistances which varied around 150 Ω (percent difference of 1.49%), This supports Thevenin's Theorem, and the reasoning behind this conclusion will be further discussed in the following section.

3.3. Calculations, Graphs, Results and Analysis

Herein lies a list of figures, tables and graphs to further analyse the obtained results of the experiment. To avoid confusion, the section will follow the lab results recorded in chronological order, starting from procedure 1 from the lab manual.

As referenced in section 3.2, the team observed the theoretical equivalence between sources 1 and 2. Table 1 (seen below) expresses the calculated values of the Thevenin voltage, the Thevenin resistance and the voltage across the 270 Ω resistor, V_L . Even if the percent difference is put into consideration, it is safe to assume that the sources observed in-lab are, in fact, equivalent. This holds true, especially since the percent difference is marginally small for all component pairs ($\geq 1\%$).

More importantly, however, are the values of V_L . According to Veach: "(...) for any given load, V_L and I_L will be the same for both sources. Thus, these circuits should be experimentally equivalent.

Table 1. Pre-lab calculated values and percent differences between sources

Component	Source 1	Source 2	% Difference (%)
V _{Th} (V)	5.00	5.01	0.20
R _{Th} (Ω)	150.00	149.86	0.09
V _L (V)	3.21	3.22	0.31

After using the DMM to measure the actual resistance of the $\sim \! 100~\Omega$ resistor, R_x , and the voltages across the two open terminals (with and without the small resistor), the team recorded the values in Table 1.2. These values were then used for the experimental calculation of R_{Th} , as explained above.

Table 2. Measured voltages of sources 1 & 2 under condition $V_{\rm o1}$ and $V_{\rm o2}$

Component	Source 1	Source 2
V ₀₁ (V)	5.017	4.992
V ₀₂ (V)	2.012	2.020
$\mathbf{R}_{\mathbf{x}}\left(\Omega\right)$	101.0	
Calculated R _{Th} (Ω)	149.35	147.13

In proof of this, using Eq. 5, R_{Th} was calculated by substituting the variables with the measured values obtained from Table 2. Eq. 8 shows the mathematical calculation used to obtain R_{Th} of source 1:

$$R_{Th} = \left(\frac{V_{01}}{V_{02}} - 1\right) R_{x} = \left(\frac{(5.017 \text{ V})}{(2.012 \text{ V})} - 1\right) (100.0 \Omega) = 149.35^{*}$$
(8)

*NOTE: The incorrect value of R_x was used throughout the report in its entirety. While 101.0Ω is the actual measured value, 100.0Ω was used instead.

In hopes to prove Thevenin's Theorem of equivalence, sources 1 and 2's equivalent voltage, resistance and load voltage were directly compared using Table 3 (expressed below). This was another way to determine whether both sources were equivalent or not. Using the percent difference equation (Eq. 7), it was found that all values fell under acceptable parameters of less than 2%: further supporting the sources' equivalence. A sample calculation of percent difference is represented in Eq.9 below as well.

Table 3. Measured parameters evaluating the validity of Thevenin equivalence

Component	Source 1	Source 2	% Difference (%)
V _{Th} (V)	5.017	4.992	0.50
R _{Th} (Ω)	149.35	147.13	1.49
V _L (V)	3.16	3.21	1.65

$$\%Difference_{V_{Th}} = \left| \frac{V_{Th_1} - V_{Th_2}}{V_{Th_1}} \right| \times 100 = \left| \frac{(4.992V) - (5.017V)}{(5.017V)} \right| \times 100 = 0.50\%$$
 (9)

Furthermore, another method which was deemed quite relevant to the overall experiment was referenced in the introduction of the lab manual: v-i characteristics. The linear equation for any two terminal voltage equivalent source is represented by Eq.10, where R_{int} corresponds to the equivalent resistance and v_{oc} is the open circuit voltage of a given source. Using this equation, we can graph functions which can be directly compared in order to confirm their equivalence (Veach, 2011, p.1).

$$v_{L} = v_{oc} - R_{int}i_{L} \tag{10}$$

Using Figure 5's graphs as reference, we can see that both sources have slopes that fall within uncertainties ($-149.35 \approx -147.49$): with an open circuit voltage that correlates well with the expected behaviour of the circuits. We can therefore prove, again, that sources 1 and 2 are equivalent.

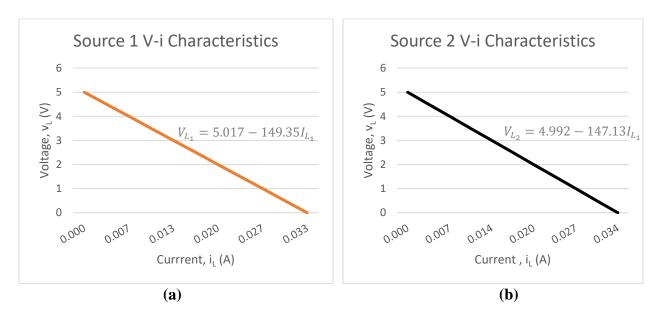


Figure 5. Direct Thevenin equivalent comparison between sources using terminal characteristic linear equation where **(a)** is source 1 and **(b)** is source 2.

Lastly, Table 4 investigates the experimental authentication of the results obtained. In other words, it was created to make sure the results made sense with the theoretical results found in the pre-lab report. In doing so, the percent difference was calculated between the experimental and

calculated results (using Eq.6). Then, the team members gauged the validity of the results to see if they fell within uncertainties. Seeing as there was room for error throughout this experiment, there most definitely could have been fluctuations in the true experimental values.

Table 4. Comparison of source 1 & 2 calculated vs. measured results.

	V _{Th} (V)		%	$\mathbf{R}_{\mathrm{Th}}\left(\Omega ight)$		%
	Calculated	Measured	Difference	Calculated	Measured	Difference
Source 1	5.00	5.017	0.34	150.00	149.35	0.43
Source 2	5.01	4.992	0.36	149.86	147.13	1.82

For ease of understanding, a sample calculation of the percent difference has been provided below (Eq. 11):

$$\% Diff_{R_{Th_1}} = \left| \frac{measured-calculated}{calculated} \right| \times 100 = \left| \frac{(149.35\Omega) - (150.00\Omega)}{(150.00\Omega)} \right| \times 100 = 0.43\%$$
 (11)

Nonetheless, as seen in Table 4, all values fell within 2% difference: further proving Theorem of equivalence holds true. The actual values and their meaning will be discussed further in the following section of the report.

Discussion

An examination of Thevenin's Theorem allowed us to compare the calculated and measured values of the two primary equivalent components in each source: voltage, and resistance.

Comparisons were conducted by measuring the equivalent and source voltages using multimeters. These provided experimental values and calculated values through given parameters (i.e. the source voltages). After conducting the experiment, various values were found experimentally and theoretically; some of which were given, like resistance values and voltage values.

Percent error between theoretical and experimental values were calculated using Eq.7 as well. While it is not particularly useful to discuss all %errors, the team has found it worthwhile to elaborate on the %error of the measured voltages (Table 2) using the formula for %Difference

and the Thevenin voltages as an example, it was found that (through Eq.9) they differed by a margin of approximately 0.50% (0.4998%).

While values remained within reasonable parameters (% difference \leq 5%), it still raises the question of why this value is higher than the rest. This could be due to many variables, but, for the purpose of this report, the following paragraphs elaborate on the most common errors that the team could think of.

Potential human errors encountered in the lab which could have altered our readings range from misreading the value on the multimeter or not waiting for the value displaying on the multimeter to stabilize. Mathematically, not using appropriate significant digits during the calculations could have equally caused a varying percent difference between the theoretical values and the experimental values.

Experimental errors consisted of some being unavoidable. Environmental factors such as a change in temperature would evidently alter the readings on the multimeter (e.g. an increase in resistance of the circuit that results from an increase in temperature). Faulty calibration of the devices used in the lab such as multimeter and voltage source may have resulted in a variation of the readings as well. As the name states, unavoidable errors cannot be altered directly, but being aware of these variables could equally reduce the margin of error while also creating a better understanding of the phenomena.

Conclusion

Experiment 2 provided a wide range of results that all revolved around the same thing: proving Thevenin equivalence between two sources as well as supporting the validity of the law itself. After a thorough analysis of the obtained results throughout the experiment, it was determined that both sources did, in fact, carry the same equivalent voltage (Source $1 = 5.017 \text{ V} \approx 4.992 \text{ V} = \text{Source 2}$) and resistance (Source $1 = 149.35 \Omega \approx 147.13 \Omega = \text{Source 2}$).

Although there were discrepancies in the values that made the source voltages/resistances differ, Thevenin's law was still proven valid since the percent difference of the values were not too large. In other words, the percent difference was no greater than 2% (the largest difference was R_{Th} Source 2 at 1.82%) which, in this case, is not large enough to impact the experiment greatly.

However, it is very important to reiterate the human and random errors that were encountered during the experiment that may have altered the actual results. Firstly, when calculating the Thevenin resistance, the team used the wrong value (100.0 Ω instead of 101.0 Ω) which would have obviously changed the true value, even if only slightly. Additionally, rounding errors during calculations could have ensued slight deviations as well.

Plus, during the experiment, due to an instrument failure, the team was forced to relocate to another station. This meant the power supply used was different half way through the experiment, which could have slightly changed the voltage source in the process. It is also worthy to note that not all resistances were measured. Because of this, measured values might not have been the same if the resistances did not hold their indicated value.

In conclusion, as with any experiment, the results could have been more accurate if it was performed multiple times. Given more time, the team could have meticulously gone through every circuit to assure a strong and reliable connection through each conductor as well. Through it all, however, the team successfully completed what they were set out to do: prove Thevenin's Theorem.

Summary of Roles

Dominic Geneau contributed an equitable amount during the lab, and more so during the writing stage of this report. He aided his partner in constructing the circuits and measuring the requested data during the experiment. He then researched and wrote all parts of this report except for the abstract as well as for the discussion. Finally, he proof-read the document in order to provide an adequate, presentable lab report.

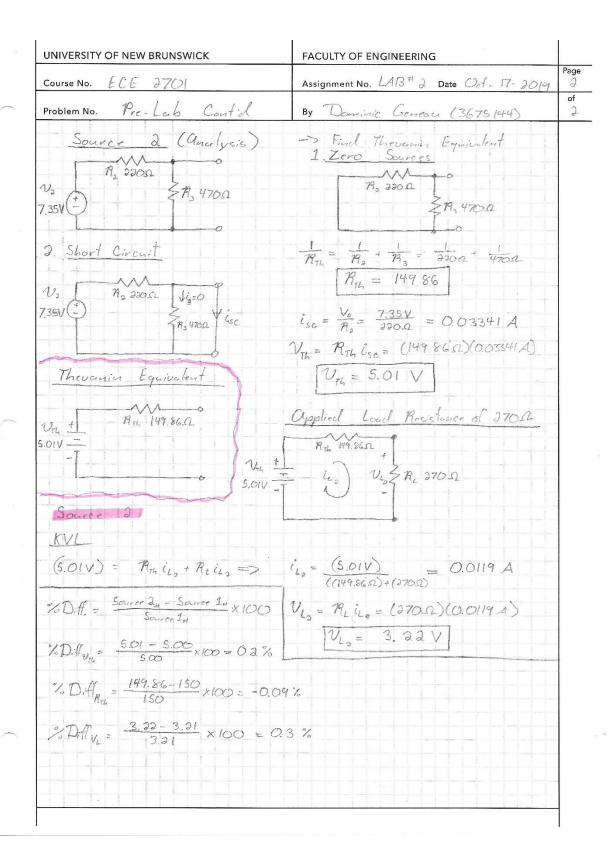
Syed Abdul Wasey Naqvi contributed an equitable amount during the lab, and during the writing stage of this report. He aided his partner in building of the circuits and measuring the data required during the experiment. He has written the abstract and discussion of this report and has done the calculations required for this lab. To end, he prof-read the lab report to try and ensure the lab report was presentable.

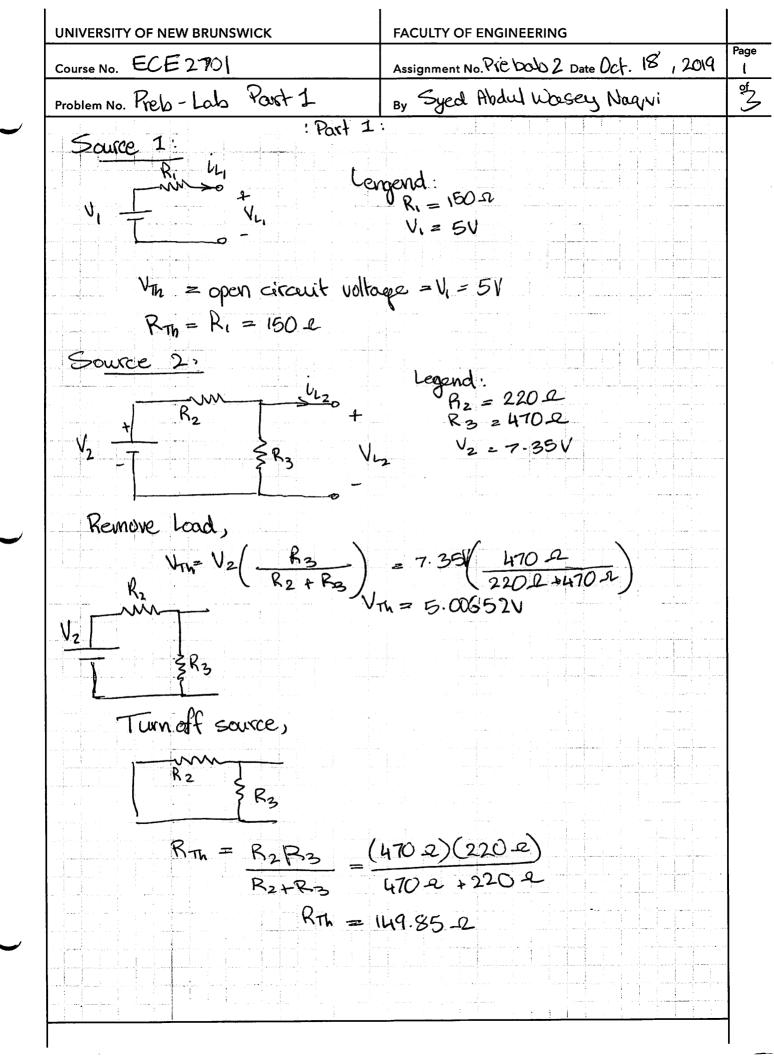
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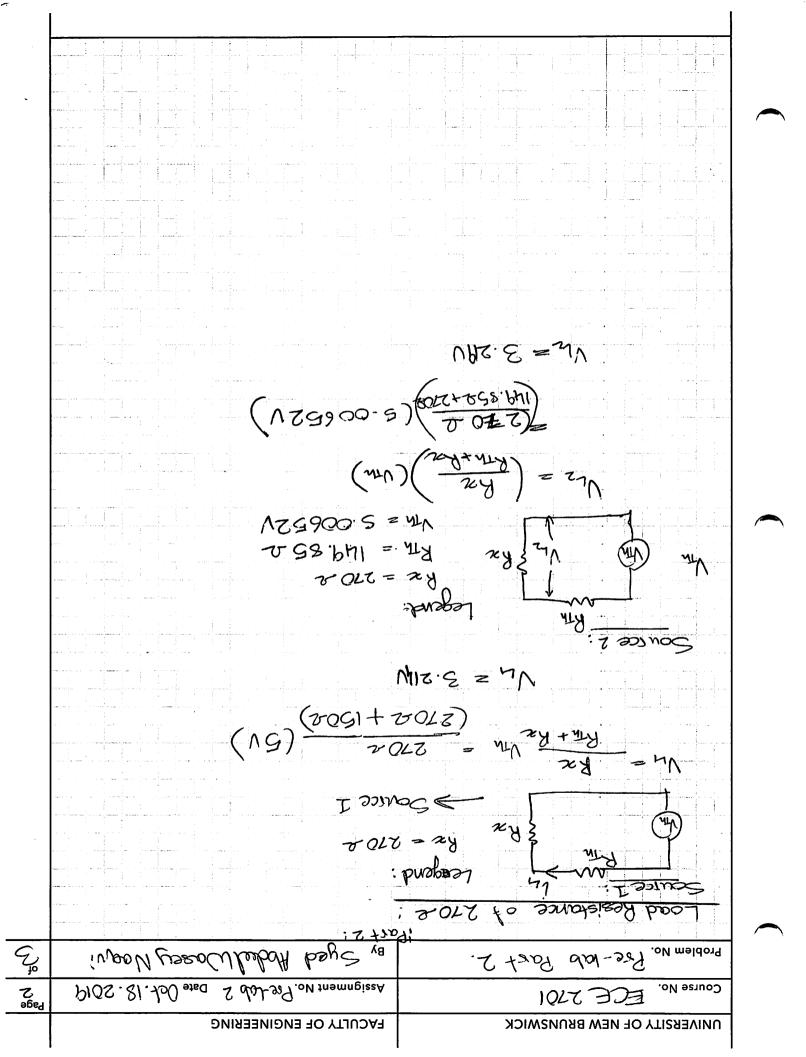
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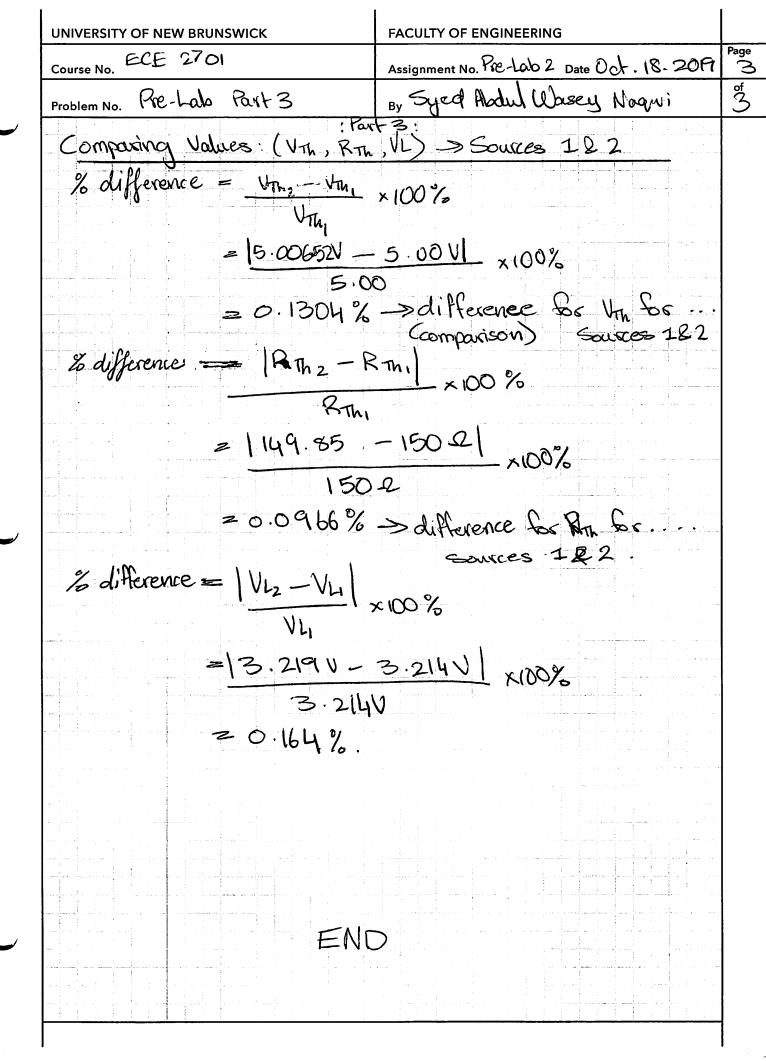
Appendix A – Pre-Lab Calculations

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Course No. ECE 2701	Assignment No. LAB* 2 Date Oct - 17 - 2019	
Problem No. Pre-Lab	By Dominic Geneau (3673144)	of 2
Source 1 (analysis) R, 1500. $\tilde{\epsilon}_{i}$ Source 1 Short Considering the second seco	Time! The varies equivalent? Zero Sources R, 150 \(\text{R} \) R_H = \(\text{R} \) = 150 \(\text{L} \) (5.00) (5.00) The varies Equivalent 1	
$V_{Th} = (1500)$ $V_{Th} = 5.0V$ Applied load resistance of	2)(0.03A)	
v_{th} R_{th} 150 Ω + V_{L} , $\geq R_{L}$ 270 Ω	$((150\Omega) + (270\Omega))$ $V_{L} = R_{L} i_{L} = (270\Omega)(0.0019 A)$	
	VL = 3.21 V Second Source and comparison on ment page	









${\bf Appendix}\; {\bf B}-{\bf Mathematical}\; {\bf Calculations}$

(Can be found on the following page)

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Course No. ECE 2701	Course No. ECE 2701 Assignment No. Lab 2 Date Och 18-201	
Problem No. Problem 5	By Syed Abdul Wasey Nagvi/Dominic	of S
Sources % Difference (V	Th) Geneau	-
% difference = VThz - VTh,	×100 ×	
which was a second of the seco		
= 14.992 V-	5.017V X100%	
= 0.4983%		
RTh for Source 1:		
$RTh = \left(\frac{Vo_1}{Vo_2} - I\right) R_X$		
(Vo ₂)		
= 15.017V \\\	00-20	
$=\left(\frac{5.017V}{2.012V}-1\right)\left(10^{-1}\right)$		
= 149.35-2		
RTH for Source 2:		
$RTh = \left(\frac{V_{01}}{V_{02}} - 1\right) R_{24}$		-
= / 4.992 V \\/		
$=\left(\frac{4.992 \text{ V}}{2.020 \text{ V}}-1\right)$	(100 -2)	
= 147.13 -2		
Rm % Difference:		
% difference = RTh2-RTh1 ×100%		
RTI		
= /149.135	2-149.35-21	
	7.35-2	
= 1.49 %		
		-

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Course No. ECE 2701	Assignment No. Lab 2 Date Oct. 18.2019	
Problem No. Problem 7	By Syed Abdul Wosey Nogrij Dominic Geneau	
VL % Difference:		
% Difference = VL2	-V4) ×100%	
I I I I I I I I I I I I I I I I I I I		
	121-3.160V × 100%	
= 1.64%	3.160v	
- marin brain	ra di kanangan di katangan di kanangan	
Calculated and Measured	Compassion:	
% Différence =	ulated 1	
colc	ulated	
Source 1/diff= 5.0V - 5.017V x100%		
20.34% 20.34%	504 10004	
Source 2 % diff = 5.006	52V-4.44ZV X100%	
(Vin)	.0060 20	
20.36%	for the control of th	
Source 2 % diff = 150.	02-149.352 X100%	
(RTh)	150.0-1	
20.47	3%	
	010 1471201 -0,	
Source 2% diff = [149.9] (Rm) = 1.82	149.86Q X100%	
(RTM)		
= 1.82		

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