### FACULTY OF COMPUTER SCIENCE

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### **Tutorial 0**

- 1) What is the personal data?
  - Answer: Any information that can be used to identify a living person directly and indirectly, or that relates to them.
    - This could be: name, an identification number, or location data, like an IP address.
    - It could also include other information that leads to an individual being identified (which could be: physical, genetic or cultural).
    - More care needs to be taken with sensitive personal data, e.g., health data, religious beliefs.
- 2) Why data privacy matters to us?
  - Answer:
    - We care we are responsible for handling people's most personal information.
    - This is an opportunity to make privacy central to what we do.
    - By not handling personal data properly we could put individuals at risk and the entitys reputation at stake.
    - Getting it wrong could result in significant fines.
    - We need robust systems and processes in place to make sure we use personal information properly and comply.
- 3) Briefly discuss the General Data Protection Regulation (GDPR), which has come in to force on 25 May 2018.
  - Answer: GDPR is a European law that will replace the current Data Protection Act and the UK government will still implement the rules after Brexit. The aim is to strengthen and unify personal data protection for all individuals living in the European Union. In addition, the Information Commissioners Office (ICO) will lead on GDPR in the UK and will hand out penalties for organisations who are in breach of the new law.



- 1) There are four key security issues that we need to consider in database, including availability, authenticity, integrity, and confidentiality. Please give a brief definition for each of them.
  - Answer:
    - Availability: Data should be available at all necessary times and only the appropriate users. In addition, data should be able to track who has access to and he/she has accessed what data.
    - Authenticity: Data should be edited by an authorized source and the database system should be accessed by the users who they say they are. Authenticity should verify that all report requests are for authorized users and any outbound data are going to the expected receiver.
    - Integrity: Integrity is to verify that any external data has the correct formatting and other metadata, and all input data
      is accurate and verifiable. At the same time, data is following the correct work flow rules for the institution/company.
      All data changes and who authored them to ensure compliance with corporate rules and privacy laws should be able
      to report.
    - Confidentiality: Confidential data is only available to correct people, and entire database is secure from external and internal system breaches. Confidentiality should provide for reporting on who has accessed what data and what they have done with it. Mission critical and legal sensitive data must be highly secure at the potential risk of lost business and litigation.
- 2) One of privacy threats to database privacy is knowledge discovery data mining (KDDM). Please give a brief description of KDDM.
  - Answer: KDMM is to extract information from database, and suggest a pattern regarding the data stored in the database. It is used to discover patterns that classify individuals into categories, revealing in the way confidential personal information with certain probability.
- 3) There are three main types of authentication techniques, including what you know, what you have, and what you are. Please give a brief description for each of them.
  - Answer:
    - What you know? We know the most well known form of authentication is user/password verification, which is used to gain access. There are two problems and security issues considerations: (i) potential for password to be written down on paper; (ii) password can be guessed or broken easily.
    - What you have? Physical object is needed in order to gain access, such as key card, ID card, token etc. At the same time, the most common uses includes gaining access to buildings or restricted areas. There are two problems or security issues: loss of object and replication.
    - What you are? We have biometric information and the unique biological characteristics can be used for identification. There are two problems or security issues: (i) the biological information may change with aging, injury and some other biological or environmental conditions; (ii) the identification system is not fully accurate, i.e., there are false-acceptance rate and false-rejection rate.
- 4) What are the models of anonymity?
  - Answer:
    - Interactive model: This model is similar to statistical database. Data owner acts as "gatekeeper" to data. Users submit queries in some agreed language and gatekeeper gives an (anonymized) answer, or refuses to answer.
    - "Send me your code" model: In this model, data owner executes code on their system and reports result, but it cannot be sure the code is not malicious.
    - Offline model or "publish and be damned" model: Data owner somehow anonymizes data set and publishes the results to the world and retires.
- 5) What are the goals of anonymity?
  - Answer: The goals are as follows.
    - Prevent (high confidence) inference of associations. In other words, it aims to prevent linking sensitive information to an individual.
    - Prevent inference of presence of an individual in the data set.
    - Have to model what knowledge might be known to attacker, including the background knowledge and domain knowledge.
- 6) What are identifiers, quasi-identifiers, and sensitive attributes in database?
  - Answer:
    - Identifiers: uniquely identity, e.g., Social Security Number
    - Quasi-Identifiers: combine several attributes together to be a new identity, which is enough to partially identify an individual in a database. For example, DOB+Sex+ZIP are unique for 87% of US residents.

- Sensitive attributes: the associations we want to hide, e.g., the salary in the "census".
- 7) Use an example to explain the De-Identification, and Linking Attack for Tabular Data.
  - Answer:
    - De-Identification:

We take the census data as an example to explain the de-identification. The following table contains some census data recording incomes and demographics, where SSN is an identifier and salary is a sensitive attribute. At the same time, salary association violates individual's privacy.

SSN	DOB	Sex	ZIP	Salary
11-1-111	1/21/76	М	53715	50,000
22-2-222	4/13/86	F	53715	55,000
33-3-333	2/28/76	М	53703	60,000
44-4-444	1/21/76	М	53703	65,000
55-5-555	4/13/86	F	53706	70,000
66-6-666	2/28/76	F	53706	75,000

We can remove SSN to create a de-identified table.

DOB	Sex	ZIP	Salary
1/21/76	М	53715	50,000
4/13/86	F	53715	55,000
2/28/76	М	53703	60,000
1/21/76	М	53703	65,000
4/13/86	F	53706	70,000
2/28/76	F	53706	75,000

- Linking Attack: linking de-identified private data and public available data is likely to achieve identification and access some sensitive data. For example, in the following table, the de-identified census data table and public available data table can be combined to identify users and their salary information.

53703

DOB	Sex	ZIP	Salary	SSN	DOB	Sex
1/21/76	М	53715	50,000	11-1-111	1/21/76	М
4/13/86	F	53715	55,000	33-3-333	2/28/76	M
2/28/76	М	53703	60,000			
1/21/76	М	53703	65,000			
4/13/86	F	53706	70,000			
2/28/76	F	53706	75,000			

- 8) What is k-Anonymization in in database privacy?
  - Answer: k-anonymity: Table T satisfies k-anonymity with regards to quasi-identifier QI if each tuple in (the multiset) T[QI] appears at least k times.
- 9) What is homogeneity attack in k-anonymity?
  - Answer: k-anonymity requires each tuple in (the multiset) T[QI] to appear at least k times. In this case, if (almost) all sensitive attribute values in a QI group are equal, privacy is lost. This is homogeneity attack.
- 10) What are l-Diversity and t-Closeness?
  - Answer:
    - *l*-diversity: Table T satisfies *l*-diversity with regards to quasi-identifier QI if each of its QI groups contains at least *l* well-represented values for the sensitive attributes.
    - t-closeness: Table T satisfies t-closeness with regards to quasi-identifier QI if the distance between the distribution of sensitive attribute values in the group and in the whole table is no more than threshold t.

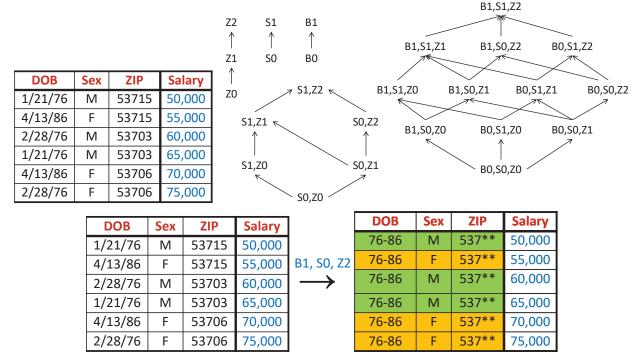
- 11) What is Differential Privacy (DP) technique?
  - Answer: The DP can guarantee that the privacy risk should not substantially increase as a result of participating in a statistical database. For example, there are two datasets X and X', and X is a neighbor of X' because they differ in one row. However, from the released statistics, it is hard to distinguish X and X'.
- 12) Discuss why the sensitivity of the function F = HISTOGRAM is 2, i.e., S(F) = 2.
  - Answer: The sensitivity of a function F is the maximum (absolute) change over all possible adjacent inputs

$$S(F) = Max_{D_1,D_2:|D_1-D_2|=1}|F(D_1) - F(D_2)|.$$

For the HISTOGRAM, removing any record affects one bin and changing any record affects two bins in the worst case. Therefore, S(F) = 2.

- 13) Incognito is one of approaches to implement the k-anonymity. Given a table below, the full-domain generalizations described by "domain vectors" are represented as follows.
  - $B_0 = \{1/21/76, 2/28/76, 4/13/86\} \rightarrow B_1 = \{76 86\}$
  - $S_0 = \{M, F\} \to S_1 = \{*\}$
  - $Z_0 = \{53715, 53710, 3706, 53703\} \rightarrow Z_1 = \{5371*, 5370*\} \rightarrow Z_2 = \{537**\}$

Based on the Lattice of Domain Vectors, we can achieve the k-anonymity for the original table. For example, with  $(B_1, S_0, Z_2)$  generalization, we can achieve k = 3-anonymity.



Please follow the example to apply  $(B_1, S_1, Z_1)$ ,  $(B_1, S_1, Z_2)$  to generalize the original table, and discuss what is the value of k in each generalized case?

• Answer: The generalized tales are as follows.

DOB	Sex	ZIP	Salary		DOB	Sex	ZIP	Salary
1/21/76	М	53715	50,000		76-86	*	5371*	50,000
4/13/86	F	53715	55,000	B1, S1, Z1	76-86	*	5371*	55,000
2/28/76	М	53703	60,000	$\rightarrow$	76-86	*	5370*	60,000
1/21/76	М	53703	65,000		76-86	*	5370*	65,000
4/13/86	F	53706	70,000		76-86	*	5370*	70,000
2/28/76	F	53706	75,000		76-86	*	5370*	75,000

DOB	Sex	ZIP	Salary		DOB	Sex	ZIP	Salary
1/21/76	М	53715	50,000		76-86	*	537**	50,000
4/13/86	F	53715	55,000	B1, S1, Z2	76-86	*	537**	55,000
2/28/76	М	53703	60,000	$\rightarrow$	76-86	*	537**	60,000
1/21/76	М	53703	65,000		76-86	*	537**	65,000
4/13/86	F	53706	70,000		76-86	*	537**	70,000
2/28/76	F	53706	75,000		76-86	*	537**	75,000

When applying  $(B_1, S_1, Z_1)$  to generalize the original table, k = 2. Similarly, when applying  $(B_1, S_1, Z_2)$  to generalize the original table, k = 6.

14) Let F(x) be the true answer on input x, and  $Lap(\lambda)$  be the noise sampled from Laplace distribution with parameter  $\lambda = S(F)/\epsilon$ . Please prove that the release of  $F(x) + Lap(\lambda) = F(x) + Lap(S(F)/\epsilon)$  can obtain  $\epsilon$ -DP guarantee. Proof: Suppose that  $A = F(x) + Lap(\lambda)$ , and  $D_1$  and  $D_2$  are any two adjacent DBs. Thus,  $A(D_1) = F(D_1) + x_1$  and  $A(D_2) = F(D_2) + x_2$ , where  $x_1$  and  $x_2$  are  $Lap(\lambda)$  distributed. Since  $\lambda = S(F)/\epsilon$ , the probability density for  $x_1$  is proportional to  $e^{-||x_2||_1(\frac{\epsilon}{S(F)})}$  and . Similarly, the probability density for  $x_1$  is proportional to  $e^{-||x_2||_1(\frac{\epsilon}{S(F)})}$ . Therefore, for any  $T \in range(A)$ 

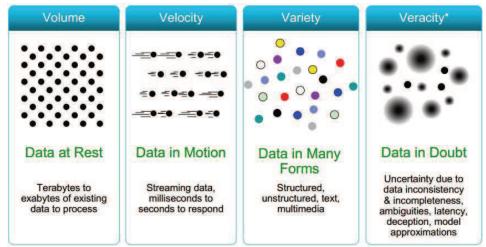
$$\frac{Pr[A(D_1) = T]}{Pr[A(D_2) = T]} = \frac{Pr[F(D_1) + x_1 = T]}{Pr[F(D_2) + x_2 = T]} = \frac{Pr[x_1 = T - F(D_1)]}{Pr[x_2 = T - F(D_2)]} = \frac{e^{-||T - F(D_1)||_1(\frac{\epsilon}{S(F)})}}{e^{-||T - F(D_2)||_1(\frac{\epsilon}{S(F)})}}$$
$$= e^{(-(||T - F(D_1)||_1 - ||T - F(D_2)||_1)(\frac{\epsilon}{S(F)})} < e^{-||F(D_2) - F(D_1)||_1(\frac{\epsilon}{S(F)})}$$

where the inequality follows form the triangle inequality. By the definition of sensitivity,

$$S(F) = Max_{D_1, D_2: |D_1 - D_2| = 1} |F(D_1) - F(D_2)|$$

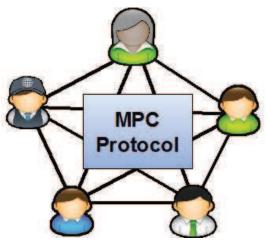
Thus,  $e^{-||F(D_2)-F(D_1)||_1(\frac{\epsilon}{S(F)})} \le e^{-\epsilon}$ . The ration is bounded by  $e^{-\epsilon}$ , yields  $\epsilon$ -Differential Privacy.

- 1) What are big data, its challenges and opportunities?
  - Answer:
    - Big data are high-volume, high-velocity, and/or high-variety information assets that require new forms of processing to enable enhanced decision making, insight discovery and process optimization. (Gartner 2012) In addition, Big data is the term for a collection of data sets so large and complex that it becomes difficult to process using on-hand database management tools or traditional data processing applications. (Wikipedia)
    - Challenges include capture, curation, storage, search, sharing, transfer, analysis, and visualization.
    - Opportunities: The trend to larger data sets is due to the additional information derivable from analysis of a single large set of related data. Compared to separate smaller sets with the same total amount of data, allowing correlations can find "spot business trends, determine quality of research, prevent diseases, link legal citations, combat crime, and determine real-time roadway traffic conditions."
- 2) Briefly explain 4V characteristics in big data.
  - Answer:
    - Volume: Data volume is increasing exponentially, e.g., 44x increase from 2009 to 2020 and from 0.8 zettabytes to 35zb.
    - Velocity: Data are generated fast and need to be processed fast.
    - Variety: Different types of data are involved, including relational data, text data, graph data, etc., which become more complex. All these types of data need to be linked together.
    - Veracity: The data inconsistency and incompleteness, ambiguities etc. will bring some uncertainty in big data. Veracity will consider some security issues in big data in order to protect data veracity.



- 3) Briefly describe some typical security issues often discussed in big data.
  - Answer:
    - Secure data collection: It aims to make the collection of data private as well as authenticated.
    - Secure data filtration: In order to reduce useless data, set the data filtering criteria for being useful and keep the filtering criteria secret even if it is executed at the source.
    - Data integrity and poisoning concerns: Only conduct data computing on authenticated data.
    - Proof of data storage: Check the file or data is actually stored in the cloud.
    - Secure outsourcing of computation: A weak client outsources a computation to the provider. After the provider returns the result and a "proof" that the computation was carried out correctly, the client can use less computation cost to verify the proof is true or not.
- 4) What are data aggregation, suppression, swapping, synthesis techniques?
  - Answer:
    - Aggregation: With aggregation, privacy is protected by aggregating individual records within a report-based and summarized format before release. Aggregation reduces disclosure risks by turning records at risk into less-risky records.
    - Suppression: In suppression, not all the data values are released. Some values are removed, withheld, or disclosed. Typically, data agencies remove sensitive values from the released dataset. They may select to suppress entire variables

- or just at-risk data values. However, suppression may lead to inaccurate data mining and analysis as important data values are suppressed and missing.
- Swapping: In swapping, data values of selected records are swapped to hide the true owner of the records, thereby making the matching inaccurate. Agencies may choose to select to have high rate of data swapping in which a large percentage of records are selected for swapping, or low rate in which only a small percentage of records are selected for swapping.
- Synthesis techniques: In this way, the values of sensitive variables are replaced with synthetic values generated by simulation. In addition, the synthetic values are basically random values generated by a probability distribution function simulator.
- 5) What is multi-party secure computation in big data?



### • Answer:

Generally, MPSC allows the players to perform an arbitrary on-going computation during which new inputs can be provided and security in MPSC means that the players' inputs remain secret (except for what is revealed by the intended results of the computation) and that the results of the computation are guaranteed to be correct.

- 1) What are ciphertext-only attack and known-plaintext attack on symmetric encryption?
  - Answer:
    - Ciphertext-only attack: An adversary only has several ciphertexts and he/she tried to recover the decryption key or plaintext of a given ciphertext.
    - Known-plaintext attack: An adversary has several pairs of (plaintext, ciphertext) and he/she tried to recover the decryption key or plaintext of a given ciphertext.
- 2) What is the Kerckhoffs principles?
  - Answer: The security of algorithms should depend on the confidentiality of the key. All other things except the key can be public.
- 3) What is the block cipher? What is the goal of padding in block cipher?
  - Answer:
    - Block ciphers break messages into fixed length blocks, and encrypt each block using the same key.
    - The goal of padding in block cipher: when the plaintext message is broken into blocks, the last block may be short of a whole block and needs padding.
- 4) What are advantages and disadvantages of symmetric encryption in terms of efficiency and key management?
  - Answer:
    - Efficiency: Symmetric encryptions are generally very fast.
    - Key management is a problem. When there are N users, the total number of required keys are  $\frac{N(N-1)}{2}$ .
- 5) What is the birthday attack?
  - Answer: Birthday attack is to find two people with the same birthday, which is the same thing as finding a collision for a particular hash function. For example, in a group of 23 randomly chosen people, at least two will share a birthday with probability at least 50%. If there are 30, the probability is around 70%.
- 6) Let P(N,q) denote the probability of at least one collision when we throw  $q \ge 1$  balls at random into  $N \ge q$  buckets. Then, we have

$$P(N,q) \le \frac{q(q-1)}{2N}, \quad P(N,q) \ge 1 - e^{\frac{-q(q-1)}{2N}}$$

when  $1 \le q \le \sqrt{2N}$ ,

$$P(N,q) \ge 0.3 \cdot \frac{q(q-1)}{N}$$

Please prove these birthday bounds.

Proof: Let  $C_i$  be the event that the *i*-th ball collides with one of the previous ones. Then,  $Pr[C_i]$  is at most  $\frac{i-1}{N}$ , since when the *i*-th ball is thrown in, there are at most i-1 different occupied slots and the *i*-th ball is equally likely to land in any of them. Thus,

$$P(N,q) = Pr[C_1 \lor C_2 \lor \cdots \lor C_q] \le Pr[C_1] \lor Pr[C_2] \lor \cdots \lor Pr[C_q] \le \frac{0}{N} + \frac{1}{N} + \cdots + \frac{q-1}{N} = \frac{q(q-1)}{2N}$$

Thus, this proves the upper bound.

Then, let  $D_i$  be the event that there is no collision after having thrown in the i-th ball. If there is no collision after throwing in i balls then they must all be occupying different slots, so the probability of no collision upon throwing in the (i+1)-th ball is exactly  $\frac{N-i}{N}$ . That is,  $Pr[D_{i+1}|D_i] = \frac{N-i}{N} = 1 - \frac{i}{N}$ .

Noth that  $Pr[D_1] = 1$ . The probability of no collision at the end of the game can now be computed via

$$1 - P(N, q) = Pr[D_q] = Pr[D_q | D_{q-1}] \cdot Pr[D_{q-1}] = \dots = \prod_{i=1}^{q-1} Pr[D_{i+1} | D_i] = \prod_{i=1}^{q-1} (1 - \frac{i}{N})$$

Note that  $\frac{i}{N} < 1$ , we can use the inequality  $1 - x \le e^{-x}$  for each term of the above expression. This means the above is not more than

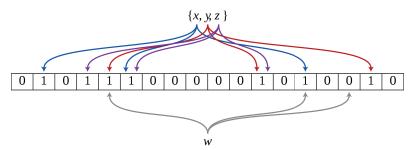
$$\prod_{i=1}^{q-1} (e^{-\frac{i}{N}}) = e^{-\frac{1}{N} - \frac{2}{N} - \dots - \frac{q-1}{N}} = e^{-\frac{q(q-1)}{2N}}$$

Therefore,  $P(N,q) \ge 1 - e^{-\frac{q(q-1)}{2N}}$ . Thus, this proves the lower bound.

When  $1 \le q \le \sqrt{2N}$ , we know  $\frac{q(q-1)}{2N} \le 1$ . Then, we can use the inequality  $(1-\frac{1}{e}) \cdot x \le 1-e^x$  to get  $P(N,q) \ge 1$  $(1-\frac{1}{e})\frac{q(q-1)}{2N}.$ 

Thus,  $P(N,q) \ge 0.3 \cdot \frac{q(q-1)}{N}$ .

7) Alice wants to send m=10 files to Bob, how to use the optimal Bloom Filter technique to assure the data integrity of



• Answer: Alice has m=10 files and the number of hash functions is k=3. In order to optimize the Bloom Filter technique and reduce the probability of false positive, it should satisfy that

$$k = \ln 2 \cdot \frac{n}{m}.$$

Then  $n = k * m / \ln 2 = 3 * 10 / \ln 2 = 44$ , so Alice can use a Bloom Filter array with size n to store these data.

8) Use the extended Euclidean algorithm to compute the value of  $x = 37^{-1} \mod 60$ .

	A
•	Answer

Dividend	Divisor	Quotient	Reminder
60	37	1	23
37	23	1	14
23	14	1	9
14	9	1	5
9	5	1	4
5	4	1	1
1) 1 5 (4 1	<u> </u>		L <del>*</del>

1) 
$$1 = 5 - (4 * 1) = 5 - 4 * 1$$

2) 
$$1 = 5 - (9 - 5 * 1) * 1 = 5 * 2 - 9$$

3) 
$$1 = (14 - 9 * 1) * 2 - 9 = 14 * 2 - 9 * 3$$

4) 
$$1 = 14 * 2 - (23 - 14 * 1) * 3 = 14 * 5 - 23 * 3$$

2) 
$$1 = 5 - (4 \times 1) = 3 - 4 \times 1$$
  
2)  $1 = 5 - (9 - 5 \times 1) \times 1 = 5 \times 2 - 9$   
3)  $1 = (14 - 9 \times 1) \times 2 - 9 = 14 \times 2 - 9 \times 3$   
4)  $1 = 14 \times 2 - (23 - 14 \times 1) \times 3 = 14 \times 5 - 23 \times 3$   
5)  $1 = (37 - 23 \times 1) \times 5 - 23 \times 3 = 37 \times 5 - 23 \times 8$ 

6) 
$$1 = 37 * 5 - (60 - 37 * 1) * 8 = 37 * 13 - 60 * 8$$
  
 $\rightarrow$  437 \* 13 - 60 \* 8 = 1 mod 60

$$\rightarrow$$
 437 \* 13 - 60 \* 8 = 1 mod 60

$$\rightarrow$$
 (37)<sup>-1</sup> mod 60 = 13

# 9) Prove the following results.

a) If p|10a - b, p|10c - d, then p|ad - bc.

Proof: From p|10a - b, we know  $p \cdot k_1 = 10a - b$  for some  $k_1$ . Then,

$$p \cdot k_1 \cdot c = (10a - b) \cdot c$$

Let  $k_2 = k_1 \cdot c$ , we have  $p \cdot k_2 = (10a - b) \cdot c = 10ac - bc$ .

Similarly, we have  $p \cdot k_4 = (10c - d) \cdot a = 10ca - ad$  for some  $k_4$ . Then,

$$p \cdot k_2 - p \cdot k_4 = 10ac - bc - 10ac + ad \Longrightarrow p(k_2 - k_4) = ad - bc$$

Therefore,

$$p|ad-bc$$

b) if n is odd, then  $3|2^n + 1$ .

Proof: Since  $2 + 1 \equiv 0 \mod 3$ , we have  $2 \equiv -1 \mod 3$ . Then,

$$2^n \equiv (-1)^n \mod 3$$

Because, n is odd, we have

$$2^n - (-1)^n \equiv 0 \mod 3 \Longrightarrow 2^n + 1 \equiv 0 \mod 3 \Longrightarrow 3|2^n + 1$$

c)  $k = 0, 1, 2, \cdots$  for  $n \in \mathbb{Z}$ , we have  $2n + 1|1^{2k+1} + 2^{2k+1} + \cdots + (2n)^{2k+1}$ . Proof: For each  $i = 1, 2, \cdots, n$ , we have

$$i + (2n+1) - i \equiv 2n+1 \equiv 0 \mod 2n+1$$

$$i \equiv -(2n+1-i) \mod 2n+1 \Longrightarrow i^{2k+1} \equiv [-(2n+1-i)^{2k+1}] \mod 2n+1$$

Since, 2k + 1 is odd, we have

$$i^{2k+1} + (2n+1-i)^{2k+1} \equiv 0 \mod 2n + 1$$

$$\sum_{i=1}^{n} [i^{2k+1} + (2n+1-i)^{2k+1}] \equiv 0 \mod 2n + 1$$

Therefore,

$$2n + 1|1^{2k+1} + 2^{2k+1} + \dots + (2n-1)^{2k+1} + (2n)^{2k+1}$$

d) if m - p|mn + pq, then m - p|mq + np.

Proof: Since

$$(m-p)|(m-p)(n-q) \Longrightarrow (m-p)|mn+pq-(mq+np)$$

Because m - p|mn + pq, we have m - p|mq + np.

e) if  $x \equiv 1 \mod m^k$ , then  $x^m \equiv 1 \mod m^{k+1}$ .

Proof: Since  $x \equiv 1 \mod m^k$ , we have  $x = 1 + k \cdot m^k = 1 + (k \cdot m^{k-1}) \cdot m$ . Thus,

$$m^k|x-1, \qquad x \equiv 1 \mod m$$

From  $x \equiv 1 \mod m$ , we have  $x^i \equiv 1^i \mod m$  for  $i = 0, 1, \dots, m-1$ . Then,

$$\sum_{i=1}^{m-1} x^i \equiv \sum_{i=1}^{m-1} 1^i \mod m$$

$$1 + x + x^2 + \dots + x^{m-1} \equiv m \mod m \Longrightarrow 1 + x + x^2 + \dots + x^{m-1} \equiv 0 \mod m$$

Then,

$$m|(1+x+x^2+\cdots+x^{m-1})$$

Finally, we have

$$m^k \cdot m | (x-1)(1+x+x^2+\cdots+x^{m-1}) \Longrightarrow m^{k+1} | x^m-1 \Longrightarrow x^m \equiv 1 \mod m^{k+1}.$$

10) Prove the Fermat's Little Theorem, i.e.,  $a^{p-1} \equiv 1 \mod p$ , where p is a prime, and a < p.

Proof: Suppose that  $Z_p^* = \{1, 2, 3 \cdots, p-1\}$  and  $B = \{a*1, a*2, a*3 \cdots, a*(p-1)\}$  for any  $a \in Z_p^*$ . We need to prove  $|Z_p^*| = |B|$  or there is not redundant element in B, so the p-1 multiples of a in B are distinct and nonzero. By contradiction, if  $i \neq j$ ,  $a*i \equiv a*j \mod p$ ,  $a*(i-j) \equiv 0 \mod p$ . Then,  $a \equiv 0 \mod p$  or  $i-j \equiv 0 \mod p$ . As we know,  $\gcd(a,p)=1$ , thus  $a \neq 0 \mod p$  and  $i-j \equiv 0 \mod p$ . Thus, i=j.

Now we know that  $|Z_P^*|$  and B have the same number of elements, and try to calculate the multiplication of element in  $Z_p^*$  and B.

$$\prod_{x_i \in Z_p^*} x_i \equiv \prod_{x_i \in Z_p^*} a * x_i \mod p$$

$$\prod_{x_i \in Z^*} x_i \equiv 1 * 2 * 3 * \dots * (p-1) \mod p$$

$$\prod_{x_i \in Z_p^*} a * x_i \equiv (a * 1) * (a * 2) * \dots * (a * (p - 1)) \equiv a^{p-1} * (1 * 2 * 3 * \dots * (p - 1)) \mod p$$

Assume,  $1 * 2 * 3 * \cdots * (p-1) \mod p = \beta$ . Then,

Then,  $\beta \equiv (a^{p-1}-1)*\beta \mod p$ , so  $(a^{p-1}-1)*\beta \equiv 0 \mod p$ . Thus,  $a^{p-1} \equiv 1 \mod p$ .

- 11) What's the difference between the public key encryption and the symmetric encryption?
  - Answer: In public key encryption, there are two different keys, i.e., public key and private key. In the encryption algorithm, public key is published and used for encryption. Private key is kept secret and used for decryption. However, in symmetric key, one same key is used for both encryption and decryption.
- 12) In a public-key system using RSA, you intercept the ciphertext C=8 sent to a user whose public key is e=5, n=35. What is the plaintext M?
  - Answer: Since n = 35, we have p = 5 and q = 7.

$$\phi(n) = (p-1)(q-1) = 24$$

As we know  $ed \equiv 1 \mod \phi(n)$ , so

$$d \equiv e^{-1} \mod \phi(n)$$

$$d \equiv 5^{-1} \equiv 5 \mod 24$$

According to the RSA decryption algorithm,  $M \equiv C^d \equiv \mod n$ . Therefore,

$$M \equiv 8^5 \equiv 8 \mod 35$$

13) Use the Chinese Remainder Theorem (CRT) to solve x, where

$$\begin{cases} x \equiv 2 \mod 3 \\ x \equiv 3 \mod 5 \\ x \equiv 4 \mod 7 \end{cases}$$

• Answer: Let  $m_1 = 3, m_2 = 5, m_3 = 7, a_1 = 2, a_2 = 3, a_3 = 4$ . We have  $M = m_1 \cdot m_2 \cdot m_3 = 3 \times 5 \times 7 = 105$ .  $\begin{array}{l} M_1 = M/m_1 = 35, M_2 = M/m_2 = 21, M_3 = M/m_3 = 15. \\ \alpha_1 = M_1^{-1} \mod m_1 = 35^{-1} \mod 3 = 2, \alpha_2 = M_2^{-1} \mod m_2 = 21^{-1} \mod 5 = 1, \alpha_3 = M_3^{-1} \mod m_3 = 15^{-1} \end{array}$ mod 7 = 1.Therefore,

$$x = a_1 \cdot \alpha_1 \cdot M_1 + a_2 \cdot \alpha_2 \cdot M_2 + a_3 \cdot \alpha_3 \cdot M_3 = 2 \times 2 \times 35 + 3 \times 1 \times 21 + 4 \times 1 \times 15 \mod M = 53$$

- 14) Consider a Diffie-Hellman scheme with a common prime q=11 and a primitive root  $\alpha=2$ .
  - If user A has public key  $Y_A = 3$ , what is A's private key  $X_A$ ?
    - Answer:  $X_A = 8$ , because  $2^8 \mod 11 = 3$ .
  - If user B has public key  $Y_B=8$ , what is the secret key K shared with A? Answer: Since  $K=Y_B^{X_A} \mod q$ , we have  $K=8^8 \mod 11=5$ .
- 15) Consider an Elgamal encryption scheme with a common prime q=11 and a primitive root  $\alpha=2$ . If B has public key  $Y_B = 3$  and A choose the random integer k = 2, what is the ciphertext of M = 9?
  - Answer: (4,4)

Since,

$$C_1 = \alpha^k = 2^2 = 4 \mod 11 = 4$$
  
 $C_2 = M \cdot Y_B^k = 9 \cdot 3^2 = 4 \mod 11 = 4$ 

16) What is the Man-In-The-Middle attack? How to deal with the Man-In-The-Middle attack in the Diffie-Hellman key exchange protocol?

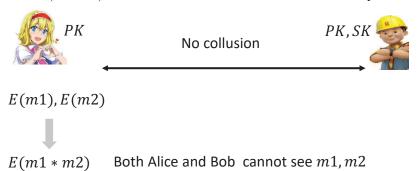
Answer:

- Man-In-The-Middle attack: When Alice and Bob run the Diffie-Hellman key exchange protocol. An attacker Darth can attack the protocol as follows.
  - Darth prepares for the attack by generating random numbers  $X_D, X_D'$  and then computing the corresponding public key  $Y_D, Y_D'$ .
  - Alice transmits  $Y_A$  to Bob.

- Darth intercepts  $Y_A$  and transmits  $Y_D$  to Bob.
- Bob receives  $Y_D$  and calculates  $K_2 = (Y_D)^{X_B} \mod q$ .
- Bob transmits  $Y_B$  to Alice.

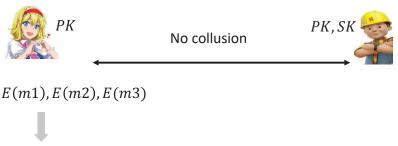
- Darth intercepts  $Y_B$  and calculates  $K_2 = (Y_B)^{X_D} \mod q$ . Darth transmits  $Y_D'$  to Alice. Darth also calculates  $K_1 = (Y_A)^{X_D'} \mod q$ . Alice receives  $Y_D'$  and calculates  $K_1 = (Y_D')^{X_A} \mod q$ . Then, Darth and Alice share the key  $K_1$ . At the same time, Darth and Bob share the key  $K_2$ .
- Solution: When running the key exchange protocol, each party signs its own DH value to prevent man-in-the-middle attack. The specific protocol is as follows.
  - Alice transmits  $ID_A, Y_A$  to Bob.
  - Bob receives  $ID_A, Y_A$  and calculates  $K = (Y_A)^{X_B}$ . Then, he/she returns  $ID_B, Y_B, SIG_B(ID_A, ID_B, Y_A, Y_B)$
  - Alice verifies the received message and calculates  $K = (Y_B)^{X_A}$ . Then, he/she sends  $SIG_A(ID_A, ID_B, Y_A, Y_B)$
  - Bob verifies the identity of Alice. Then, Alice and Bob share the key K.

- 1) What is the homomorphic encryption technique?
  - Answer: Homomorphic encryption is a form of encryption that allows computation on ciphertexts, generating an encrypted result which, when decrypted, matches the result of the operations as if they had been performed on the plaintexts.
- 2) Let E() be a Paillier homomorphic encryption scheme. Assume Bob has the public/private key pair (pk, sk) of E(), and Alice only has the public key pk of E(). Consider there is no collusion between Alice and Bob. When Alice has two ciphertexts  $E(m_1)$  and  $E(m_2)$ , please design a protocol run between Alice and Bob. With the protocol, Alice can finally obtain the ciphertext  $E(m_1 \cdot m_2)$  while both Alice and Bob have no idea on the plaintexts  $m_1$  and  $m_2$ .



### • Answer:

- Alice chooses two random numbers  $r_1$  and  $r_2$ . Then, it can compute  $E(m_1 + r_1) = E(m_1) * E(r_1)$ ,  $E(m_2 + r_2) = E(m_2) * E(r_2)$ . At the same time, it sends  $E(m_1 + r_1)$  and  $E(m_2 + r_2)$  to Bob.
- On receiving  $E(m_1 + r_1)$  and  $E(m_2 + r_2)$ , Bob recovers the plaintext  $m_2 + r_2$  with private key sk. Then, Bob computes  $E((m_1 + r_1) * (m_2 + r_2)) = E(m_1 + r_1)^{m_2 + r_2}$  and sends it to Alice.
- Alice receives  $E((m_1+r_1)*(m_2+r_2))$ . Because  $(m_1+r_1)(m_2+r_2)=m_1*m_2+m_1*r_2+m_2*r_1+r_1*r_2$ , we have  $m_1*m_2=(m_1+r_1)(m_2+r_2)-m_1*r_2-m_2*r_1-r_1*r_2$ . Then,  $E(m_1*m_2)=E((m_1+r_1)*(m_2+r_2))*(E(m_1)^{r_2})^{-1}*(E(m_2)^{r_1})^{-1}*E(r_1*r_2)^{-1}$ . Thus, Alice obtain ciphertext  $E(m_1*m_2)$ .
- 3) Let E() be a BGN homomorphic encryption scheme. Assume Bob has the public/private key pair (pk, sk) of E(), and Alice only has the public key pk of E(). Consider there is no collusion between Alice and Bob. When Alice has three ciphertexts  $E(m_1)$ ,  $E(m_2)$  and  $E(m_3)$ , please design a protocol run between Alice and Bob. With the protocol, Alice can finally obtain the ciphertext  $E(m_1 \cdot m_2 \cdot m_3)$  while both Alice and Bob have no idea on the plaintexts  $m_1$ ,  $m_2$ , and  $m_3$ .



• Answer:

- Alice chooses two random numbers  $r_1$  and  $r_2$ . Then, it can compute  $E(m_1 + r_1) = E(m_1) * E(r_1)$ ,  $E(m_2 + r_2) = E(m_2) * E(r_2)$ . At the same time, it sends  $E(m_1 + r_1)$  and  $E(m_2 + r_2)$  to Bob.

E(m1\*m2\*m3) Both Alice and Bob cannot see m1, m2, m3

- On receiving  $E(m_1 + r_1)$  and  $E(m_2 + r_2)$ , Bob recovers the plaintext  $m_2 + r_2$  with private key sk. Then, Bob computes  $E((m_1 + r_1) * (m_2 + r_2)) = E(m_1 + r_1)^{m_2 + r_2}$  and sends it to Alice.
- Alice receives  $E((m_1+r_1)*(m_2+r_2))$ . Because  $(m_1+r_1)(m_2+r_2)=m_1*m_2+m_1*r_2+m_2*r_1+r_1*r_2$ , we have  $m_1*m_2=(m_1+r_1)(m_2+r_2)-m_1*r_2-m_2*r_1-r_1*r_2$ . Then,  $E(m_1*m_2)=E((m_1+r_1)*(m_2+r_2))*(E(m_1)^{r_2})^{-1}*(E(m_2)^{r_1})^{-1}*E(r_1*r_2)^{-1}$ . Thus, Alice obtain ciphertext  $E(m_1*m_2)$ .
- Then, Alice calculates  $e(E(m_1 * m_2), E(m_3)) = E'(m_1 * m_2 * m_3)$ . Thus, Alice can finally obtain the ciphertext  $E'(m_1 \cdot m_2 \cdot m_3)$ .

- 1) What is the Tor protocol? How can it enable the anonymous communication?
  - Answer: Tor is an Internet networking protocol designed to anonymize the data relayed across it. Using Tor makes it more difficult to trace Internet activity to the user. Tor's intended use is to protect the personal privacy of its users, as well as their freedom and ability to conduct confidential communication by keeping their Internet activities from being monitored.

The Tor network runs through the computer servers of thousands of volunteers spread throughout the world. Your data is bundled into an encrypted packet when it enters the Tor network. Then, unlike the case with normal Internet connections, Tor strips away part of the packet's header, which is a part of the addressing information that could be used to learn things about the sender such as the operating system from which the message was sent. Finally, Tor encrypts the rest of the addressing information, called the packet wrapper. Regular Internet connections don't do this.

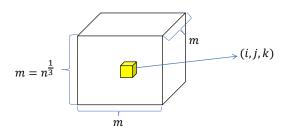
- 2) What is the Onion routing?
  - Answer: Onion routing is like an advanced form of proxy routing. Instead of routing through a single unprotected server, it uses a network of nodes that constantly encrypt your data packets at every step. Only at the end of this chain of onion nodes, your data become decrypted and sent to the final destination. In fact, only this exit node has the power to decrypt your message, so no other node can even see what you're sending.
- 3) What is the message frequency attack in anonymous communication system?
  - Answer: These types of attacks are similar to the other contextual attacks in that they exploit the fact that some communication are easy to distinguish from others. If a participant sends a non-standard (i.e., unusual) number of messages, a passive external attacker can spot these messages coming out of the mix-networks. In fact, unless all users send the same number of messages, this type of attack allows the adversary to gain non-trivial information.
- 4) What is the k-Anonymous message transmission?
  - Answer: k-Anonymous message transmission means that the adversary is able to learn something about the origin or destination of a particular message, but cannot narrow down its search to a set of less than k participants.
- 5) What is the Dining cryptographers network?
  - Answer: A Dining cryptographers network is an anonymous communication network. It consists of a set of participants that have mutually agreed on a secret and send messages to each other in every round. All round messages are summed together and the result is the message content sent by one of the participants. The sender is not identifiable in the set of participants (sender anonymity). Since the message content is broadcasted, all participants are receivers (receiver anonymity)
- 6) What is the "randomized" attack in k-Anonymous message transmission?
  - Answer: The attacker does not follow the protocol. After receiving shares  $S_{1,i}, \dots, S_{k,i}$ , instead of broadcasting  $S_i$ , it generates a random value r and broadcasts that instead. This will randomize the result of the DC-Net protocol.
- 7) What is the Pedersen Commitment Scheme? How to apply it to implement an electronic soccer lottery?



- Answer: The Pedersen commitment scheme runs between a committer C, holding a secret message  $m \in \mathbb{Z}_q$  to commit to, and a receiver R. It consists of three algorithms, i.e., Setup, Commit and Reveal.
  - Setup: Two large primes p, q are selected such that q|p-1.  $Z_q^*$  is a group and g is a generator of  $Z_p^*$ . Choose a random value a from  $Z_q$  and let  $h=g^a \mod p$ . Then, publish p,q,g,h and keep a as a secret.
  - Commit: To commit to some  $x \in Z_q$ , sender chooses random  $r \in Z_q$  and sends  $c = g^x h^r \mod p$  to receiver.
  - Reveal: To open the commitment, sender reveals x and r, and receiver verifies that  $c = q^x h^r \mod p$ .

The electronic soccer lottery can use Pedersen commitment scheme to implement. When a user intends to buy the lottery x, he/she selects a random number and makes a commitment c by running Commit algorithm. Then, he/she sends the commitment c to agency. When the user wins the prize, he/she has to provide x and r with the agency. Agency verifies whether the user actually wins the prize by comparing the commitment c and  $g^xh^r \mod p$ . If it is same, verification succeeds. Otherwise, verification fails.

- 1) What is private information retrieval (PIR)?
  - Answer: In cryptography, a private information retrieval (PIR) protocol is a protocol that allows a user to retrieve an item from a server in possession of a database without revealing which item is retrieved.
- 2) How to design an Information-Theoretic 2-Server PIR protocol with communication costs  $O(n^{1/3})$ ?
  - Answer: Suppose that each server holds n bit strings. Let  $m=n^{\frac{1}{3}}$  and the string can be represented as follows. Each bit in the string has 3 coordinates, i.e., (i, j, k). If a user wants to retrieve item (i, j, k) with privacy, it can do as follows.

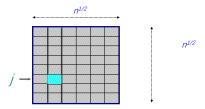


- First, user randomly chooses  $Q_{1x} \in \{0,1\}^m, Q_{1y} \in \{0,1\}^m, Q_{1z} \in \{0,1\}^m$ . Second, user sends  $(Q_{1x},Q_{1y},Q_{1z})$  to server  $S_1$  and sends  $(Q_{2x}=Q_{1x}+2^i,Q_{2y}=Q_{1y}+2^j,Q_{2z}=Q_{1z}+2^k)$ to server  $S_2$ .
- For each  $x=1,2,\cdots,m$ , server  $S_1$  computes sums all nodes where  $y\in Q_{1y},z\in Q_{1z}$ . Similarly,  $S_1$  sums all nodes for each  $y=1,2,\cdots,m$  where  $x\in Q_{1x},z\in Q_{1z}$  and sums all nodes for each  $z=1,2,\cdots,m$  where  $x \in Q_{1x}, y \in Q_{1y}$ . Then,  $S_1$  returns the result to user and let  $S_{xyz}$  denote the result.
- $S_2$  does the same computation as  $S_1$ . At the same time,  $S_2$  returns the computation result to user and let  $S'_{xyz}$  denote the result.
- When receiving the results from  $S_1$  and  $S_2$ , user can compute the item (i, j, k) as follows.
  - Choose x=i and compute  $S_{iyz}\oplus S'_{iyz}.$
  - Choose y = j and compute  $S_{xjz} \oplus S'_{xjz}$ .

  - Choose z=k and compute  $S_{xyk} \oplus S'_{xyk}$ . Compute  $X(i,j,k) = (S_{iyz} \oplus S'_{iyz}) \oplus (S_{xjz} \oplus S'_{xjz}) \oplus (S_{xyk} \oplus S'_{xyk})$

The communication cost from user to two servers is 3m + 3m = 6m. At the same time, the communication cost from two servers to user is also 3m + 3m = 6m. Thus, the communication cost in total is O(m), i.e.,  $O(n^{\frac{1}{3}})$ .

- 3) How to design a computational 1-server PIR protocol with communication costs  $O(n^{1/2})$ ?
  - Answer: The 1-server PIR protocol with communication costs  $O(n^{1/2})$  can be achieved by Paillier homomorphic encryption technique. Suppose that the server holds n bit strings. Let  $m=n^{\frac{1}{2}}$  and the string can be represented as follows. Each bit in the string has 2 coordinates, i.e., (i, j). If a user wants to retrieve item (i, j) with privacy, it can do as follows.

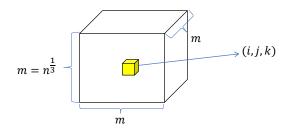


- User sends  $\{C_k = E(0), C_i = E(1) | k = 1, 2, \dots, m, (k \neq i) \}$  to server.
- Sever computes a "bit" for each row and returns to user. Suppose that data in the r-th row is  $\{x_1, x_2, \cdots, x_m\}$ . Then, server will compute one bit for the r-th row as  $b_r = \prod_{k=1}^m C_k^{x_k}$ . Finally, server returns  $\{b_r | r = 1, 2, \cdots, m\}$ to user.
- User recovers the item (i, j) from  $b_i$  by decryption.

The communication cost from user to two servers is m. At the same time, the communication cost from two servers to user is also m. Thus, the communication cost in total is O(m), i.e.,  $O(n^{\frac{1}{2}})$ .

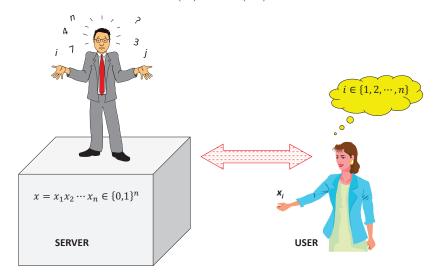
- 4) How to design a computational 1-server PIR protocol with communication costs  $O(n^{1/3})$ ?
  - Answer: The 1-server PIR protocol with communication costs  $O(n^{1/3})$  can be achieved by BGN homomorphic encryption technique, which satisfies  $e(E(a), E(b)) = E_T(a \cdot b)$ . Suppose that a server holds n bit strings. Let  $m = n^{\frac{1}{3}}$ and the string can be represented as follows. Each bit in the string has 3 coordinates, i.e., (i, j, k). If a user wants to

retrieve item (i, j, k) with communication costs  $O(n^{1/3})$ , it can do as follows.

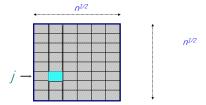


- User sends  $Str_1 = \{C_k = E(0), C_i = E(1) | k = 1, 2, \cdots, m, (k \neq i) \}$  and  $Str_2 = \{C_k = E(0), C_j = E(1) | k = 1, 2, \cdots, m, (k \neq j) \}$  to server.
- For  $z=1,2,\cdots,m$ , sever computes a "bit" value. Let  $V_{xyz}$  denote the value in (x,y,z). Then, for each  $z=1,2,\cdots,m$ , server will compute one bit  $b_z=\prod_{x,y\in 1,2,\cdots,m}e(C_x,C_y)^{V_{xyz}}$ . Finally, server returns  $\{b_z|z=1,2,\cdots,m\}$  to user.
- User recovers the item (i, j, k) from  $b_k$  by decryption.

The communication cost from user to server is 2m. At the same time, the communication cost from server to user is also m. Thus, the communication cost in total is O(m), i.e.,  $O(n^{\frac{1}{3}})$ .



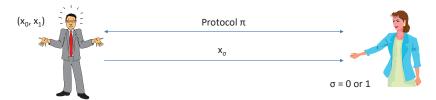
- 1) What is Oblivious Transfer (OT) protocol?
  - Answer: Oblivious Transfer (OT) protocol is a protocol by which sender sends information to receiver, but remains oblivious as to what is received.
- 2) What is the difference between OT protocol and PIR protocol?
  - Answer: In both OT and PIR, server cannot know  $x_i$ . For the user, it can only  $x_i$  in OT, but it can know  $x_i$  and others in PIR.
- 3) How to use the Paillier encryption to design 1-out-n OT with  $O(n^{1/2})$  Communication costs?
  - Answer: Suppose that Alice holds n messages and has Paillier's public key pk and private key sk, and Bob only has public key pk. Let  $l=n^{\frac{1}{2}}$  and these messages can be represented as follows. Each message is denoted by  $x_{i,j}$ . The OT protocol can be run as follows.



- Bob selects  $i, j \in \{1, 2, \dots, l\}$ . Then, he/she computes  $S = \{C_k = E(0), C_j = E(1) | k = 1, 2, \dots, m, (k \neq j)\}$  and returns S to Alice.
- After receiving message from Bob, Alice first computes a value for each row. In specific, for the k-th row, the value  $E(x_{k,j})$  can be computed as  $E(x_{k,j}) = \prod_{t=1}^l C_t^{x_{k,t}}$ . Then, Alice will have l values  $\{E(x_{1,j}), E(x_{2,j}), \cdots, E(x_{l,j})\}$ , and return them to Bob.
- Bob chooses a random number w and computes  $E(x_{i,j} + w) = E(x_{i,j}) \cdot E(w)$  and returns it to the Alice.
- Alice recovers  $x_{i,j} + w$  by decryption and returns it to the Bob.
- Bob obtains  $x_{i,j} = x_{i,j} + w w$ .

Thus, the communication cost in total is O(l), i.e.,  $O(n^{\frac{1}{2}})$ .

- 4) What is the weakness in the above Paillier based OT protocol? How to design a stronger 1-out-n OT with  $O(n^{1/2})$  Communication costs?
  - Answer:



**Weakness**: In the above Paillier based OT protocol, the weakness is that Bob must honestly follow the protocol. Otherwise, he/she can obtain other values except  $x_{i,j}$ . For example, when Bob returns  $E(x_{i,j} + w)$  to Alice, he/she may return  $E(x_{i,j} + 10000x_{k,j} + w)$ . In this case, Bob can compute  $x_{i,j} + 10000x_{k,j}$  on receiving the decryption result from Alice, and then he/she can obtain  $x_{i,j} = x_{i,j} + 10000x_{k,j}$  mod 10000 and  $x_{k,j} = \frac{x_{i,j} + 10000x_{k,j} - x_{i,j}}{10000}$ .

**Stronger scheme**: We can use BGN homomorphic encryption scheme to design a stronger 1-out-n OT with  $O(n^{1/2})$  communication cost. The protocol is as follows.

- Bob computes  $S_i = \{(C_{i,1}, W_{i,1}), (C_{i,2}, W_{i,2}), \cdots, (C_{i,l}, W_{i,l})\}$ , where  $C_{i,k} = E(0) (1 \le k \ne i \le l)$  and  $C_{i,i} = E(1)$ . At the same time,  $W_{i,k}$  is used to proof that  $C_{i,k}$  is either E(0) or E(1), where  $k = 1, 2, \cdots, l$ . In specific, suppose that if  $C_{i,k} = g^m h^r$ , the proof of  $W_{i,k}$  is  $g^{1-2m}h^{-r}$ . Then, the statement of whether  $C_{i,k}$  is either E(0) or E(1) can be checked by verifying whether  $e(C_{i,k}, gC_{i,k}^{-1})$  is equal to  $e(h, W_{i,k})$ . In addition, Bob needs to compute a proof  $W_{i,l+1}$  for  $\prod_{k=1}^l C_{i,k}$ , which is used to proof that there is only one E(1) in  $S_i$ . With the same method, Bob can compute  $S_j = \{(C_{j,1}, W_{j,1}), (C_{j,2}, W_{j,2}), \cdots, (C_{j,l}, W_{j,l})\}$  and  $W_{j,l+1}$  as a proof for  $\prod_{k=1}^l C_{j,k}$ . Then, he/she will send  $S_i$ ,  $W_{i,l+1}$ ,  $S_j$  and  $W_{j,l+1}$  to the Alice.
- After receiving message from Bob, Alice first verifies each value in  $S_i$  and  $S_j$  is either E(0) or E(1), and there is only one E(1) in both  $S_i$  and  $S_j$ . Then, Alice computes  $E'(x_{i,j}) = \prod_{k=1}^l \prod_{t=1}^l e(C_{i,k}, C_{j,t})^{x_{i,j}}$  and returns  $E'(x_{i,j})$  to Bob.
- Bob recovers  $x_{i,j}$  by decryption.

- 1) What is zero knowledge (ZK) proof?
  - Answer: A zero-knowledge proof protocol allows one party, usually called PROVER, to convince another party, called VERIFIER, that PROVER knows some facts (a secret, a proof of a theorem,...) without revealing to the VERIFIER any information about his knowledge (secret, proof,...).
- 2) Please formally describe the Fiat-Shamir identification scheme, and show why it is ZK?
  - $\bullet$  Answer: In the identification protocol, the verifier wishes to authenticate the identity of the prover, which is claimed to have a public key I. Thus, he requests the prover to convince him that he knows the secret key S corresponding to I. The scheme is as follows.
    - The prover chooses a random value 1 < R < n and computes  $X = R^2 \mod n$ . Then, the prover sends X to the verifier.
    - The verifier requests from the prover one of the following requests at random: R or  $RS \mod n$ .
    - The prover sends the requested information to the verifier.
    - The verifier verifies that he received the correct answer by checking whether:  $R^2 \equiv ?X \mod n$  or  $(RS)^2 \equiv ?IX \mod n$ .
    - If the verification fails, the verifier concludes that the prover does not know S, and thus he is not the claimed party.
    - This protocol is repeated t times, and if in all of them the verification succeeds, the verifier concludes that the prover is the claimed party.

In the following, we prove that the Fiat-Shamir scheme is zero knowledge by giving a simulator for the problem. The input for the simulator are numbers I, N, which the prover claims to know the square root of  $I \mod N$ . The output of the simulator is a forged transcript of a proof. A transcript for the problem is of the form:

$$(I, N)(X_1, i_1, M_1), \cdots (X_n, i_n, M_n)$$

where  $M_k$  is either the square root of  $X_k$  in case  $i_k = 1$ , or the square root of  $IX_i$  in case  $i_k = 2$ .

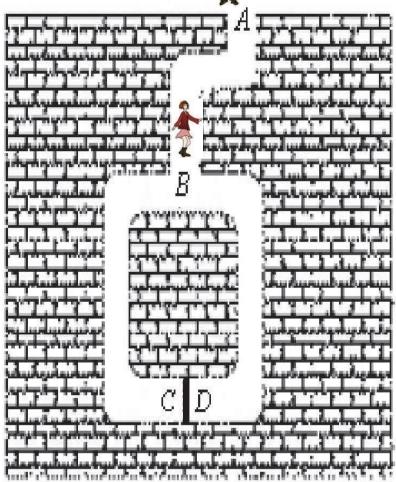
The transcript generated by the simulator is from the same probability distribution as the protocol runs. Thus, it is ZK.

- 3) Why is Parallel Fiat-Shamir identification not ZK?
  - Answer: In parallel Fiat-Shamir identification, all rounds can be run in parallel.
    - The prover chooses random values  $R_1, R_2, \dots, R_t$  and computes  $X_1 = R_1^2 \mod n, \dots, X_t = R_t^2 \mod n$ . Then, the prover sends  $X_1, \dots, X_t$  to the verifier.
    - The verifier sends t random bits  $i_1, i_2, \dots, i_t$ , where  $i_k = 0, 1$  mean the verifier requests  $R_k$  and  $R_kS$  respectively.
    - The prover sends the requested information  $Z_1, Z_2, \cdots, Z_t$  to the verifier.
    - The verifier accepts if  $Z_k^2 \equiv ?X_k I^{i_k} \mod n$ .

However, this protocol is no longer ZK. Consider the  $V^*$  such that  $V^*$  chooses  $i_1, i_2, \dots, i_t$  to be the first t bits of  $H(X_1, X_2, \dots, X_t)$ , where H is a cryptographic hash function. One can no longer generate an indistinguishable transcript.

- 4) How to design non-interactive zero-knowledge proof?
  - Answer: For the non-interactive zero-knowledge proof, the prover can publish the proof and anyone can verify the proof. In a normal ZK proof, the prover first issues a bunch of commitments, then the verifier issues challenges that the prover complies with. This proves anything only as long as the verifier is assumed to issue challenges normally without any prior understanding with the prover. In a non-interactive ZK proof, the verifier can be replaced by a hash function (or something similar) which is computed over the whole set of commitments: the hash function result is the challenge. If the hash function is really a random oracle then the prover cannot guess its output before trying it, i.e. before having produced his commitments, and that's where the security comes from.





- 1) What is the secret handshake?
  - Answer: The secret handshake considers a CIA agent who wants to authenticate herself to a server, but does not want to reveal her CIA credentials unless the server is a genuine CIA outlet. It also considers that the CIA server does not want to reveal its CIA credentials to anyone but CIA agents C not even to other CIA servers.
- 2) How to use the Paillier encryption to implement one privacy-preserving friend match protocol?
  - Answer: Suppose Alice has an interests list  $A = \{a_1, a_2, \dots, a_N\}$  and Bob has an interests list  $B = \{b_1, b_2, \dots, b_N\}$ . If they have more than or equal to t common interests, they can run the friend match protocol to make friends as follows.
    - Alice computes  $c_i=E(a_i)$  for  $i=1,2,\cdots,N$ , and sends  $\{c_i|i=1,2,\cdots,N\}$  to Bob. Bob calculates  $D=\prod_{i=1}^N c_i^{b_i}$  and returns it to Alice.

    - Alice decrypts D. If the result is more than or equal to t, make friend. Otherwise, stop.
- 3) How to design a non OT-based Private Equality Test protocol?
  - Answer: The homomorphic encryption techniques can be used to achieve private equality test protocol. Suppose that Alice and Bob have values x and y respectively. Alice can test the equality of x and y as follows.
    - Suppose that there is a homomorphic public encryption algorithm. Alice has public key pk and private key sk. Bob only has the public key pk.
    - Alice sends E(x) and E(y) to Bob.
    - Bob computes  $E(r(x-y)) = (\frac{E(x)}{E(y)})^r$  using the homomorphic property, where r is a random number. Then, Bob returns E(r(x-y)) to Alice.
    - Alice recovers r(x-y). If it is equal to 0, x is equal to y. Otherwise, x is not equal to y.

Note that in this protocol, except the comparison result, Alice and Bob do not obtain any other information.



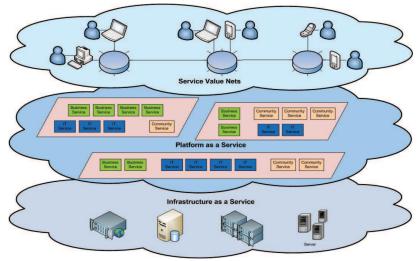




- 1) What is cloud computing?
  - Answer: Cloud computing is a general term used to describe a new class of network based computing that takes place over the Internet, and it has the following features.
    - basically a step on from Utility Computing.
    - a collection/group of integrated and networked hardware, software and Internet infrastructure (called a platform).
    - using the Internet for communication and transport provides hardware, software and networking services to clients.
- 2) What are the characteristics of cloud computing?
  - Answer:
    - On-demand self-service: A consumer can unilaterally provision computing capabilities, such as server time and network storage, as needed automatically without requiring human interaction with each services provider.
    - Broad network access: Capabilities are available over the network and accessed through standard mechanisms that promote use by heterogeneous thin or thick client platforms (e.g., mobile phones, laptops, and PDAs).
    - Resource pooling and multi-tenant model: There is a sense of location independence in that the customer generally has no control or knowledge over the exact location of the provided resources but may be able to specify location at a higher level of abstraction (e.g., country, state, or datacenter). Examples of resources include storage, processing, memory, network bandwidth, and virtual machines.
    - Rapid elasticity: Capabilities can be rapidly and elastically provisioned, in some cases automatically, to quickly scale out and rapidly released to quickly scale in. To the consumer, the capabilities available for provisioning often appear to be unlimited and can be purchased in any quantity at any time.
    - Measured service: Cloud systems automatically control and optimize resource use by leveraging a metering capability at some level of abstraction appropriate to the type of service (e.g., storage, processing, bandwidth, and active user accounts).
- 3) What are the security challenges in cloud computing?

Answer: Most security challenges stem from consumer's loss of control, lack of trust in the cloud and multi-tenancy issues in the cloud.

- Consumer's loss of control
  - Data, applications, resources are located with provider.
  - User identity management is handled by the cloud.
  - User access control rules, security policies and enforcement are managed by the cloud provider.
  - Consumer relies on provider to ensure data security, privacy, resource availability and monitoring and repairing of services/resources.
- Lack of trust in the cloud: trusting a third party requires taking risks.
- Multi-tenancy issues in the cloud: Multiple independent users share the same physical infrastructure. Thus, an attacker can legitimately be in the same physical machine as the target.
- 4) What are the key privacy concerns in cloud computing?



Answer: The key privacy concerns in cloud computing include storage, retention, destruction, auditing, monitoring and risk management.

- Storage: Information from different organizations may be commingled together when they use the same CSP. At the same time, the aggregation of data raises new privacy issues. Some governments may decide to search through data without necessarily notifying the data owner, depending on where the data resides. It is also important to control the storage access of the cloud provider, i.e., whether the cloud provider itself has any right to see and access customer data or not. In addition, some services today track user behaviour for a range of purposes, from sending targeted advertising to improving services.
- Retention: It concerns how long the personal information (that is transferred to the cloud) retained and the related
  policy that governs the data. At the same time, there are other challenges, e.g., whether the organization or the CSP
  own the data, who enforces the retention policy in the cloud, how exceptions to this policy are managed, etc.
- Destruction: This concerns several problems. The first one is how the cloud server destroys PII at the end of the retention period. The second one is how organizations ensure that their PII is destroyed by the CSP at the right point and is not available to other cloud users. The third one is how the organizations know the CSP really destroyed the data or just makes it inaccessible to the organizations, how the organizations know the CSP does not retain additional copies and whether the CSP keeps the information longer than necessary so that it can min the data for its own use.
- Auditing, monitoring and risk management: This concerns several problems. The first is how organizations monitor
  their CSP and provide assurance to relevant stakeholders that privacy requirements are met when their PII is in
  the cloud. The second one is whether they are regularly audited. The third one is if business-critical processes are
  migrated to a cloud computing model, internal security processes need to evolve to allow multiple cloud providers
  to participate in those processes, as needed. These include processes such as security monitoring, auditing, forensics,
  incident response, and business continuity.

- 1) What is smart grid?
  - Answer: Smart grid is an intelligent power system, which involves advanced technology, digital communications, sensing, measurement and control technologies etc. In addition, it integrates renewable sources and electric vehicles to achieve above goals.
- 2) What are the characteristics of smart grid?
  - Answer: Compared with the electrical grid, smart grid has the following characteristics.

Electrical Grid	Smart Grid
One way communications	Two way communications
Built for Centralized Generation	Accommodated Distributed Generation
Few Sensors	Monitors and Sensor Throughout
"Blind"	Self-monitoring
Manual Restoration	Semi-automated restoration and, eventually, self-healing
Prone to failures and blackouts	Adaptive protection and islanding
Check equipment manually	Monitor equipment remotely
Emergency decision by committee and phone	Decision support systems, predictive reliability
Limited control over power flows	Pervasive control systems
Limited price information	Full price information
Few customer choices	Many customer choices

- 3) What are the security and privacy challenges in smart grid? Answer:
  - Security challenges:
    - The information flow is vulnerable to the Cyber attacks.
    - Former CIA Director James Woolsey said the federal government's oversight of grid security is inadequate and attacks on the grid are entirely possible.
    - Smart grid is a double-edged sword.
  - Privacy challenges: Personal information can be inferred:
    - When you are at home.
    - Which appliances you use.
    - When you eat.
    - Whether you arrive late to work.

