# Assignment 2 Solution

ECE 2412 - Simulation and Engineering Analysis

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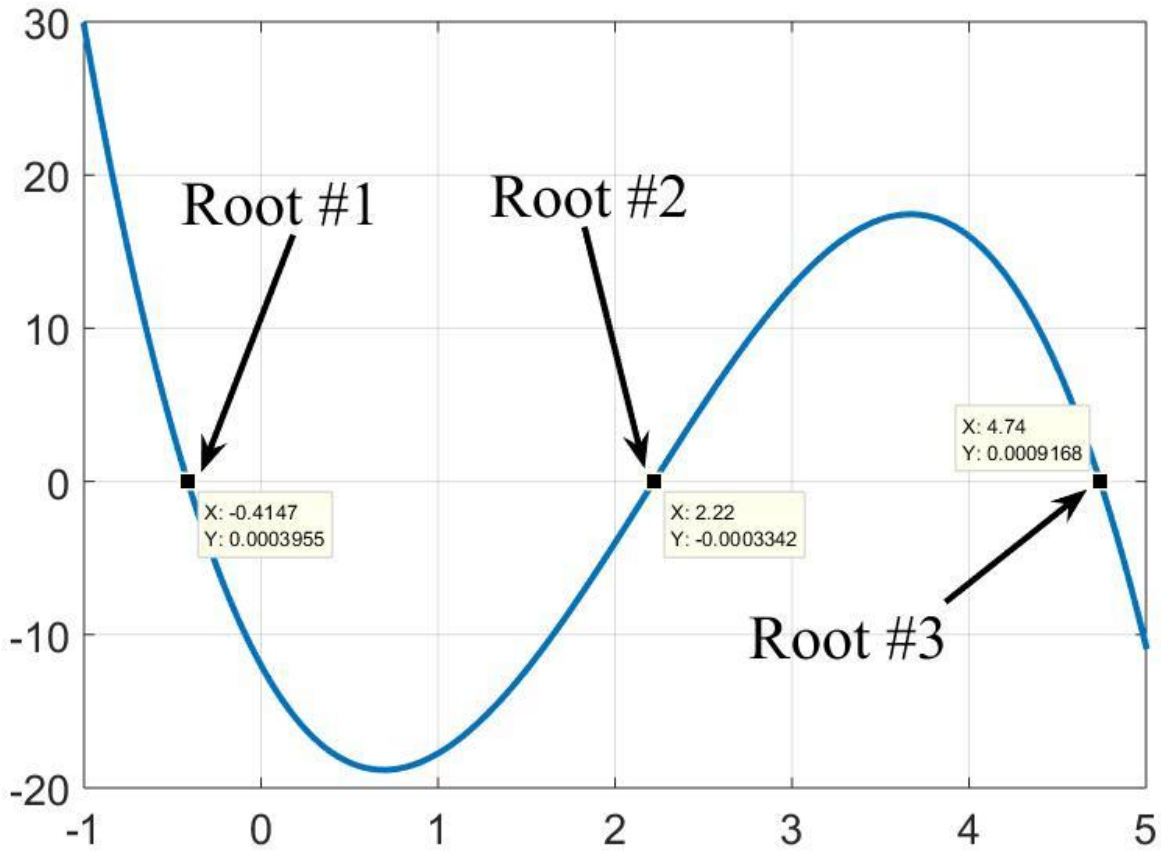
Problem 5.5:

a) Graphical method

```
%% Problem 5.5
clear;clc;close all

% Part (a)
x=-1:0.0001:5;
f_x = -12 - 21*x + 18*x.^2 - 2.75*x.^3;
plot(x,f_x)
grid on
```



(b) Bisection Method

```
%% Part (b) Bisection Method
clear;clc;close all
a=-1;
b=0;
E_s = 1/100; % The tolerance
E_a = 1 + E_s; % initial value of the approximated error must be greater
than E_s
x_r_old = a; % initial value of x_r_old  = a or b
count=0; %count will be used to count the number of iterations (optional)

f_a = -12 - 21*a + 18*a.^2 - 2.75*a.^3;
f_b = -12 - 21*b + 18*b.^2 - 2.75*b.^3;
% More efficent way is to replace f_a and f_b by MATALB function that takes
% the variable x as input and return f(x) as output.

% To check for change in sign in the interval (a,b)
if ( f_a*f_b <0) % There is change in sign

while(E_a>E_s)
    x_r = (a + b) / 2;     % new estimate of the root
    f_x_r = -12 - 21*x_r + 18*x_r.^2 - 2.75*x_r.^3; % f(xr)

    if(f_a*f_x_r < 0) % the root is located between a and x_r
        b=x_r;
        f_b=f_x_r;

    else % the root is located between x_r and b
        a=x_r;
        f_a=f_x_r;

    end
    %calculating the approximated error
    E_a = abs((x_r - x_r_old) / x_r) *100;
    % updating x_r_old
    x_r_old=x_r;
    count=count+1;

end

else % no change in sign between (a,b)
     error('F(a) and f(b) must have different sign')

end

fprintf(' The estimated root is %0.8f with relative approximated error of
%0.8f%% using the bisection method (%0.0f iterations)\n',x_r,E_a,count)
```

>> The estimated root is -0.41470337 with relative approximated error of 0.00735889% using the bisection method (15 iterations)

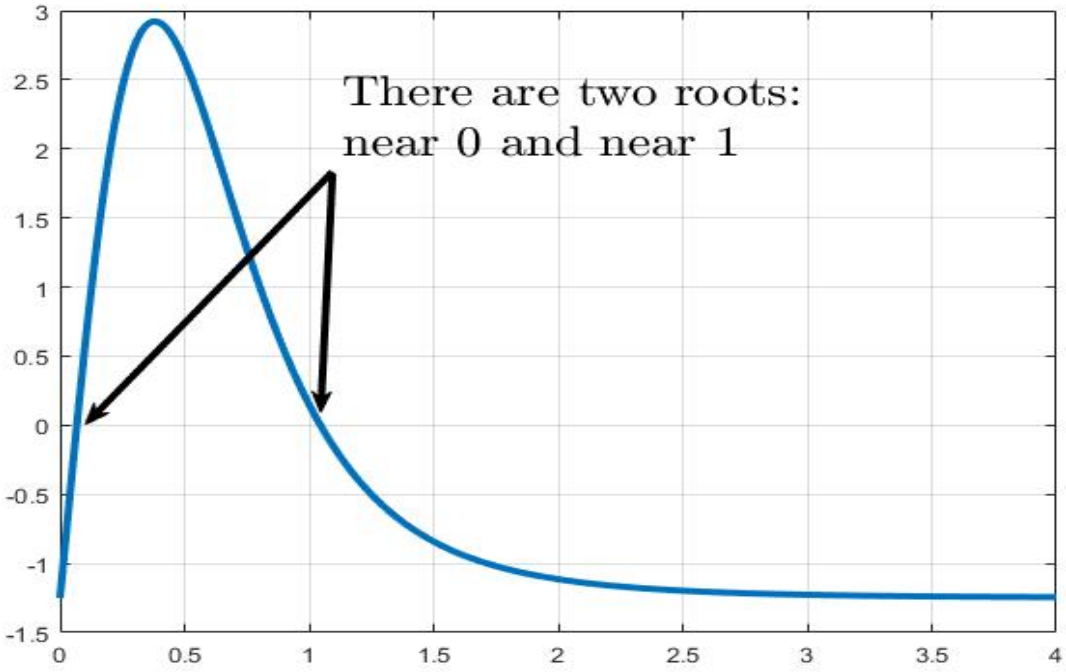
(b) False Position Method

```
%% (c) False Position
clear;clc;close all;
a=-1;
b=0;
E_s = 1/100; % The tolerance
E_a = 1 + E_s; % initial value of the approximated error must be greater
than E_s
x_r_old = a; % initial value of x_r_old  = a or b
count=0; %count will be used to count the number of iterations (optional)

f_a = -12 - 21*a + 18*a.^2 - 2.75*a.^3;
f_b = -12 - 21*b + 18*b.^2 - 2.75*b.^3;
% More efficent way is to replace f_a and f_b by MATALB function that takes
% the variable x as input and return f(x) as output.

% To check for change in sign in the interval (a,b)
if ( f_a*f_b <0) % There is change in sign

while(E_a>E_s)
    x_r = b - ( (f_b*(a-b))/(f_a-f_b));    % new estimate of the root
    f_x_r = -12 - 21*x_r + 18*x_r.^2 - 2.75*x_r.^3; % f(xr)

    if(f_a*f_x_r < 0) % the root is located between a and x_r
        b=x_r;
        f_b=f_x_r;

    else % the root is located between x_r and b
        a=x_r;
        f_a=f_x_r;

    end

    E_a = abs((x_r - x_r_old) / x_r) *100; %calculating the approximated
error
    x_r_old=x_r; % updating x_r_old
    count=count+1;
end

else % no change in sign between (a,b)
     error('F(a) and f(b) must have different sign')

end

fprintf(' The estimated root is %0.8f with relative approximated error of
%0.8f%% using the false position method (%0.0f iterations)\n',x_r,E_a,count)
```

>> The estimated root is -0.41467695 with relative approximated error of 0.00832633% using the false position method (8 iterations)

Problem 5.17

```
%% Problem 5_17
clear;clc;close all

% values of the constants
e0=8.9e-12;
Q=2e-5;
q=2e-5;
F=1.25;
a=0.85;

% f_x = (1/(4*pi*e0)*((q*Q*x)/((x^2+a^2)^3/2))-F

% Graphical method
x=0:0.01:4

% calculate f_x as a vector of values to be plotted
f_x = (1/(4*pi*e0)).*((q.*Q.*x)./((x.^2+a.^2).^(3/2)))-F;
plot(x,f_x)
grid on

% You can use any method to solve for the root, I will use the MATLAB
% function fzero()

% defined the function to be used in the fzero function
fx = @(x) (1/(4*pi*e0))*((q*Q*x)./((x^2+a^2)^3/2))-F;

% solve for the root xr and its value fval using initial guess of xi
% NOTE that the initial values of xi were determined from the graph
xi=0;
[xr1,fval1]=fzero(fx,xi);

xi=1;
[xr2,fval2]=fzero(fx,xi);

fprintf('The distance where the force is 1.25N = %0.8f m and %0.8f m
\n',xr1,xr2)
```




>> The distance where the force is 1.25N = 0.24104274 m and 1.29127956 m

Problem 6.1

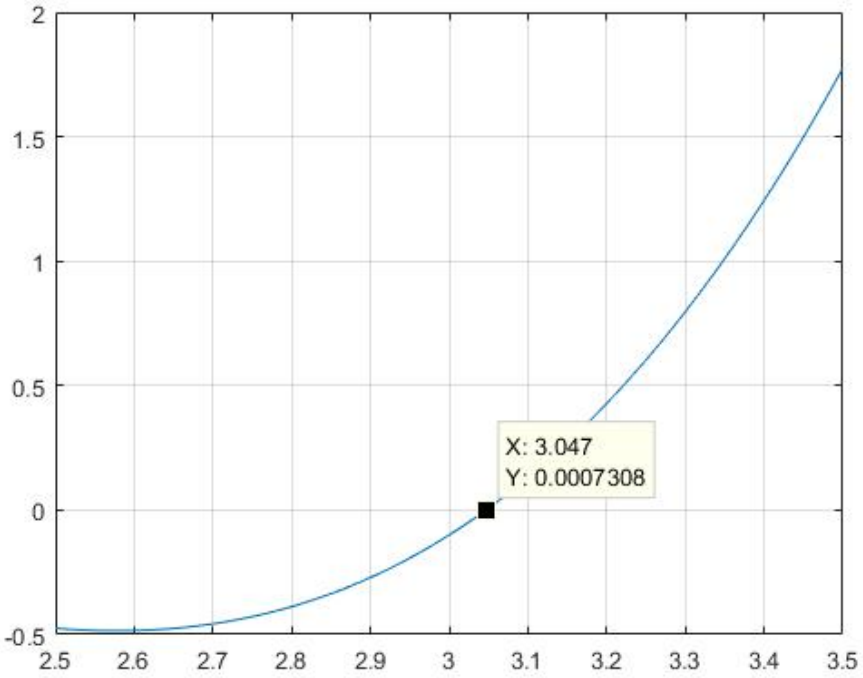
```
%% Problem 6.1
clear;clc;close all;

x_old=0.5; % initial guess
E_s = 0.01; % The tolerance
E_a = 1 + E_s; % initial value of the approximated error must be greater
than E_s
count=0; %count will be used to count the number of iterations (optional)

% Modified the function f(x) ------> x = f(x)+ x ----> x = sin (sqrt(x))


while(E_a>E_s)

    x_new = sin ( sqrt(x_old)); % estimate the new root

    E_a = abs((x_new - x_old) / x_new) *100; %calculating the approximated
error
    x_old=x_new; % updating x_old
    count=count+1;
end


fprintf(' The estimated root is %0.8f with relative approximated error of
%0.8f%% using the fixed point method (%0.0f iterations)\n',x_new,E_a,count)
```

>> The estimated root is 0.76860623 with relative approximated error of 0.00965506% using the fixed point method (9 iterations)

Problem 6.3

(a) Graphical Method

```
%(a) Graphically
clear;clc;close all;
x = 2.5:0.001:3.5;
f_x = (x.^3) - (6.*x.^2) + (11.*x) - 6.1;
plot(x,f_x)
grid on
```




(b) Newton-Raphson method

```
%% (b)  Newton-Raphson method
clear;clc;close all;
xi=3.5;              % initial guess
n = 3;  % 3 iterations

% For Newton-Raphson method, the derivative is required to calculate
the
% new estimated root:
% f(x) = x^3 - 6x^2 + 11x - 6.1
% df_dx(x) = 3x^2 - 12x + 11


for i=1:1:n %start the iteration until i=n

    f_xi = xi^3 - 6*xi^2 + 11*xi - 6.1;  % calculating f(xi)
    df_dx = 3*xi^2 - 12*xi + 11;        %calculating f'(xi)
    x_i_new = xi - ((f_xi)/(df_dx));      % estimating the new root
    xi= x_i_new; %updating xi;

end

fprintf(' The estimated root is %0.8f after %d iterations using the
Newton-Raphson method. \n',x_i_new,n)
```

>> The estimated root is 3.04731674 after 3 iterations using the Newton-Raphson method.

(c) Secant method

```
%% (c) Secant method
clear;clc;close all;
xi=3.5;   % initial guess at x_i
xi_1 = 2.5;  % initial guess at x_i-1
n = 3;  % 3 iterations

for i=1:1:n %start the iteration until i=n

    f_xi = xi^3 - 6*xi^2 + 11*xi - 6.1;  % calculating f(xi)
    f_xi_1 = xi_1^3 - 6*xi_1^2 + 11*xi_1 - 6.1;  % calculating f(xi-1)
    x_i_new = xi - ((f_xi)*(xi_1-xi)/(f_xi_1-f_xi));      % estimating the
new root
    xi_1=xi; %updating xi-1
    xi= x_i_new; %updating xi;

end

fprintf(' The estimated root is %0.8f after %d iterations using the Secant
method. \n',x_i_new,n)
```

>> The estimated root is 3.22192345 after 3 iterations using the Secant method.

(d) Modified Secant method

```
%% (d) Modified Secant method
clear;clc;close all;
xi=3.5;   % initial guess at x_i
delta = 0.01;  % initial guess at x_i-1
n = 3;  % 3 iterations

for i=1:1:n %start the iteration until i=n

    f_xi = xi^3 - 6*xi^2 + 11*xi - 6.1;  % calculating f(xi)
    f_delta_xi = (xi+delta*xi)^3 - 6*(xi+delta*xi)^2 + 11*(xi+delta*xi) -
6.1;  % calculating f(xi)
    x_i_new = xi - ((f_xi)*(delta*xi)/(f_delta_xi-f_xi));      % estimating
the new root
    xi= x_i_new; %updating xi;

end

fprintf(' The estimated root is %0.8f after %d iterations using the Modified
Secant method. \n',x_i_new,n)
```

>> The estimated root is 3.04881823 after 3 iterations using the Modified Secant method.

(e) All roots using MATLAB

```
%% (e) All roots using matlab
clear;clc;close all;
% represents f(x) as a vector of the coefficients of polynomial
f_x = [ 1 -6 11 -6.1];
% use matlab function roots()
f_x_roots = roots(f_x)
```

>> f_x_roots =

3.0467
1.8990
1.0544

Problem 6.24

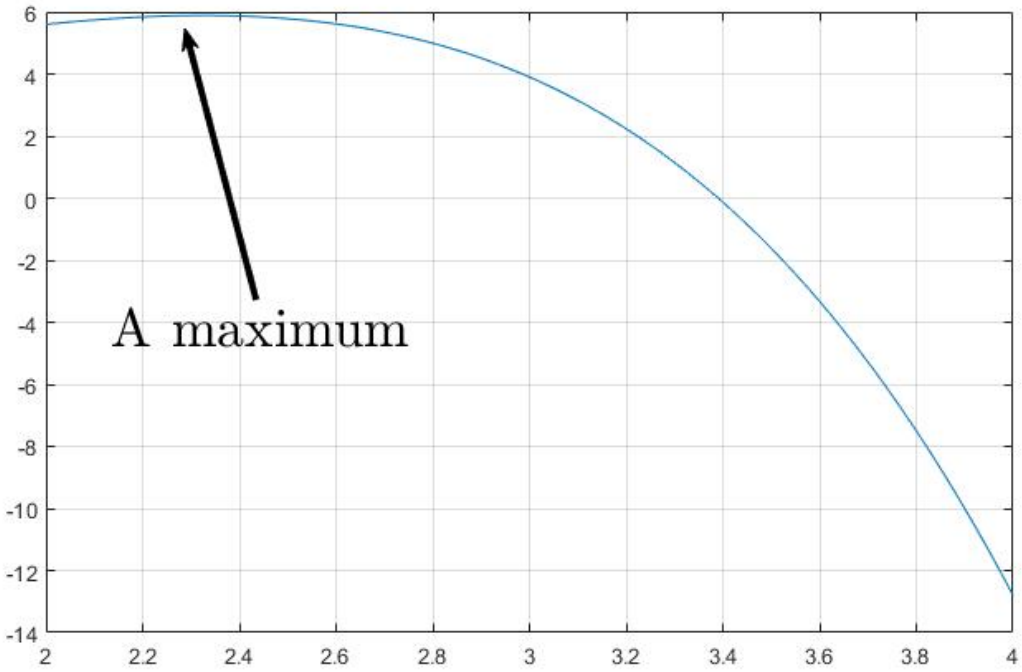
```
%% Problem 6.24
clear; clc; close all

% Since C(s) and N(s) are polynomials, they can be represented in MATLAB as
% a vector of coefficients.

C = [1 9 26 24];
N = [1 15 77 153 90];

% finding the roots of the numerator C(s)
roots_num = roots(C);

% finding the roots of the denominator N(s)
roots_den = roots(N);

fprintf('The numerator C(s) factors are (s%0.3f)(s%0.3f)(s%0.3f)
\n',roots_num(1),roots_num(2),roots_num(3))
fprintf('The denominator N(s) factors are (s%0.3f)(s%0.3f)(s%0.3f)(s%0.3f)
\n',roots_den(1),roots_den(2),roots_den(3),roots_den(4))
```

>> The numerator C(s) factors are (s-4.000)(s-3.000)(s-2.000)
The denominator N(s) factors are (s-6.000)(s-5.000)(s-3.000)(s-1.000)

Problem 7.7

Start with graphical solution in order to have an in-sight of the curve

```
%% Problem 7.7
clear;clc;close all;
x=2:0.01:4;
fx = 4*x - 1.8*x.^2 + 1.2*x.^3 - 0.3*x.^4;
plot(x,fx)
grid on
```




From the graph, the expected maximum location is near x=2.3 and its value f(x) ≈ 6

(a) The golden-section method

```
clear;clc;
a=2; b=4;
E_s = 1/100; % The tolerance
E_a = 1 + E_s; % initial value of the approximated error > E_s
count=0; %count will be used to count the number of iterations (optional)
phai = 1.6180339887; % The golden ratio (constant)
phai_1 = (phai-1); % calculate the constant outside the loop

% calculate d
d = phai_1*(b-a);
% calculate the two points x1 and x2
x1 = b - d;
x2 = a + d;
while(E_a>E_s)
    %calculate f(x1) and f(x2)
    fx1 = -(4*x1 - 1.8*x1^2 + 1.2*x1^3 - 0.3*x1^4);
    fx2 = -(4*x2 - 1.8*x2^2 + 1.2*x2^3 - 0.3*x2^4);

    % select the Xopt based on the comparison of f(x1) and f(x2)
    if(fx1 < fx2) % the true root is located between a and x2
        xopt=x1; % estimate of the maximum
        fxopt=fx1;
        b=x2;   % update the interval side
        x2=x1;  %update x2
        d= phai_1*(b-a);  %update d
        x1 = b - d; %calculate the new x1

    else % the root is located between x_r and b
        xopt=x2; % estimate of the maximum
        fxopt=fx2;
        a=x1;   % update the interval side
        x1=x2;  %update x1
        d= phai_1*(b-a);  %update d
       x2 = a + d; %calculate the new x2
    end

    E_a = (2-phai)*abs((b-a) / xopt) *100; %calculating the approximated
error
    count=count+1;

end
fprintf(' Using the golden-section method (%d iterations)\n',count)
fprintf(' The estimated maximum location is %0.8f \n',xopt)
fprintf(' The estimated maximum value is %0.8f \n',-fxopt)
fprintf(' The relative approximated error is %0.8f \n',E_a)
```

>> Using the golden-section method (17 iterations)
The estimated maximum location is 2.32634488
The estimated maximum value is 5.88534005
The relative approximated error is 0.00919583

(b) Parabolic interpolation method

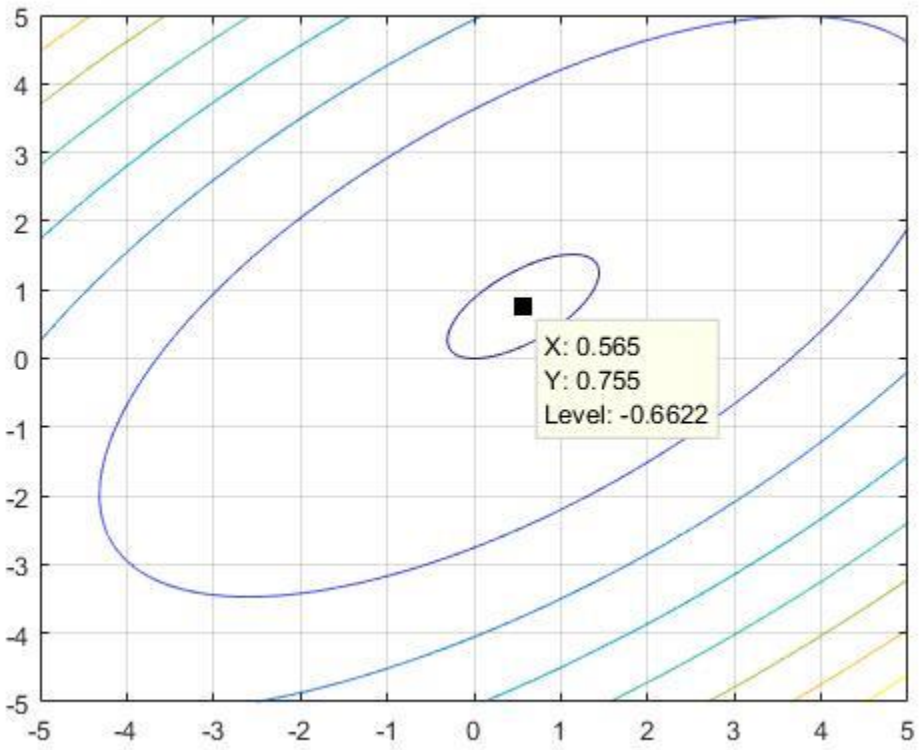
```
%% (b) Parabolic interpolation
clear;clc;
x1=1.75;  %initial value of x1
x2=2;   %initial value of x2
x3=2.5; %initial value of x3
n=5; % number of iterations

%evaluating f(x) at x1, x2, x3
fx1 = -(4*x1 - 1.8*x1^2 + 1.2*x1^3 - 0.3*x1^4);
fx2 = -(4*x2 - 1.8*x2^2 + 1.2*x2^3 - 0.3*x2^4);
fx3 = -(4*x3 - 1.8*x3^2 + 1.2*x3^3 - 0.3*x3^4);

for i=1:1:n

    % calculate the forth point
    num =  (x2-x1)^2*(fx2-fx3)-( (x2-x3)^2*(fx2-fx1)  );
    den =  (x2-x1)*(fx2-fx3)-( (x2-x3)*(fx2-fx1)  );
    x4 = x2 - 0.5 * (num) / (den);

    %calculate f(x) at x4
    fx4 = -(4*x4 - 1.8*x4^2 + 1.2*x4^3 - 0.3*x4^4);


    % select the Xopt based on the comparision of f(x2) and f(x4)

    if(fx2 < fx4) % the true root is located between a and x2
        xopt=x2; % estimate of the maximum
        fxopt=fx2;
        x3=x4;   % update the interval side

    else % the root is located between x_r and b
        xopt=x4; % estimate of the maximum
        fxopt=fx4;
        x1=x2;   % update the interval side
        x2=x4;
    end
end
fprintf(' Using the parabolic interpolation method (%d
iterations)\n',n)
fprintf(' The estimated maximum location is %0.8f \n',xopt)
fprintf(' The estimated maximum value is %0.8f \n',-fxopt)
```

>> Using the parabolic interpolation method (5 iterations)
The estimated maximum location is 2.33405797
The estimated maximum value is 5.60000000

Problem 7.23

```
%% Problem 7.23
clear; clc;close all

x=-5:0.005:5;
y=-5:0.005:5;
[X,Y]=meshgrid(x,y);
Z=2.*Y.^2 - 2.25.*X.*Y - 1.75.*Y + 1.5.*X.^2;
contour(X,Y,Z);
grid on

% from the contour, we can estimate the location of the minimum to be
% around the point (0.5, 0.5) ---> we will use it as initial guess
f=@(x) 2*x(2)^2 - 2.25*x(1)*x(2) - 1.75*x(2) + 1.5*x(1)^2;
[x,fval]=fminsearch(f,[0.5 0.5]);

fprintf('The location of the minimum is (%0.4f, %0.4f) with value of
%0.8f \n',x(1),x(2),fval)
```

>> The location of the minimum is (0.5676, 0.7568) with value of -0.66216216



Problem 7.23

```
%% Problem 7.24
clear; clc;close all

x=-3:0.01:3;
y=-3:0.01:3;
[X,Y]=meshgrid(x,y);
Z=-1*(4*X + 2*Y + X.^2 - 2*X.^4 + 2*X.*Y - 3*Y.^2);
contour(X,Y,Z);
grid on

f=@(x) -1*(4*x(1) + 2*x(2) + x(1)^2 - 2*x(1)^4 + 2*x(1)*x(2) -
3*x(2)^2);
[x,fval]=fminsearch(f,[1 2]);

fprintf('The location of the maximum is (%0.4f, %0.4f) with value of
%0.8f \n',x(1),x(2),-fval)
```

>> The location of the maximum is (0.9676, 0.6558) with value of 4.34400579