ECE 4643: Power Electronics

Assignment 2: Solution

Q1:

Q2

A 1ϕ full-wave rectifier supplies an inductive load ($R_L = 4\,\Omega$ and $L_L = 9$ mH) from a 60 Hz AC supply. Design an input LC filter that will limit the RF to $RF \leq 6\%$, and an output LC filter that will allow a voltage ripple as $\Delta v_o \leq 5\%$.

The input LC Filter

The values of the input filter parameters (L_S and C_S) can be determined based on the the value of the THD$_I$ or the RF as:

$$\frac{X_{Ls}}{X_{Cs}} = \frac{1}{3^2}\left(\frac{I_{an}(n\omega)}{(I_{an}(n\omega))_{Ls}} + 1\right) = \frac{1}{9}(RF + 1) = 0.111 \times (0.06 + 1) = 0.118$$

Let $C_S = 30\mu$F. This value produces L_S as:

$$2\pi \times 60 L_S = 0.118 X_{Cs} = \frac{0.118}{(2\pi)^2 \times 60^2 \times 30 \times 10^{-6}} = 10.4332 \Longrightarrow L_S = 27.7 \quad mH$$

The ouput LC Filter

The value of the output side capacitor C_O can be determined as:

$$\sqrt{R_L^2 + (2\pi 2 f_s L_L)^2} >> \frac{1}{2\pi 2 f_s C_O}$$

$$\sqrt{16 + 46.0476} = \frac{8}{240\pi C_O} \Longrightarrow C_O = 1300 \;\; \mu F$$

The value of L_O can be determined for the 2^{nd} harmonic on the output as:

$$\Delta v_o = \left|\frac{-1}{(2\pi 2 f_s)^2 L_O C_O - 1}\right| \Longrightarrow (240\pi)^2 L_O C_O - 1 = \frac{1}{\Delta v_o}$$

$$L_O = \frac{\frac{1}{\Delta v_o} + 1}{(240\pi)^2 C_O} = \frac{1/0.05 + 1}{(240\pi)^2 \times 0.0013}$$

get $L_O = 28.4$ mH.

Q3:

A 3ϕ full-wave rectifier supplies a 25 kW, 300 Vdc load. If this rectifier is supplied from a 3ϕ 60 Hz feeder determine:

a) The input AC line and phase rms and peak voltages

$$V_{dc} = \frac{3\sqrt{3}}{\pi} V_m \Longrightarrow V_m = \frac{300 \times \pi}{3\sqrt{3}} = 181.38 \ \ V$$

The rms value for the input voltage phase voltage is $181.38/\sqrt{2} = 128.26$ V. The line voltage peak and rms values are:

$$V_L = \sqrt{3} \times 128.26 = 222.15 \ \ V, \ \ (V_L)_{peak} = \sqrt{3} \times 181.38 = 314.16$$

b) The values of $(I_O)_{rms}$, I_{dc}, and $(I_s)_{rms}$.

The value of $(I_O)_{rms}$ can be determined as:

$$(I_O)_{rms} = \frac{V_{Orms}}{R_L}; \ \ R_L = \frac{V_{dc}^2}{P_O} = \frac{300^2}{25000} = 3.6\,\Omega$$

The value of V_{Orms} can be determined as:

$$V_{rmso} = 1.654 V_m = 1.654 \times 181.38 = 300.0 \ \ \ V$$

$$(I_O)_{rms} = \frac{300.0}{3.6} = 83.33 \quad A$$

The dc current demand of the load I_{dc} is determined as:

$$I_{dc} = \frac{P_L}{V_{dc}} = \frac{25 \times 10^3}{300} = 83.33\ A$$

The rms value of the input current $(I_s)_{rms}$ can be determined as:

$$(I_s)_{rms} = \sqrt{\frac{2}{3}}\,(I_O)_{rms} = 68.04 \quad A$$

c) The values for average and rms currents per diode. The average current in each diode:

The dc current demand of the load I_{dc} is determined as:

$$I_{dc} = \frac{P_L}{V_{dc}} = \frac{25 \times 10^3}{300} = 83.333\ A$$

The average current per-diode is determined as:

$$(I_D)_{avg} = \frac{I_{dc}}{3}\frac{83.333}{3} = 27.78\ A$$

The rms value of the current through each diode is:

$$(I_D)_{rms} = \frac{(I_{rms})_o}{\sqrt{3}} = \frac{83.38}{\sqrt{3}} = 48.14 \quad A$$

d) PIV for each diode.

The peak inverse voltage for each diode (PIV) is:

$$PIV = \sqrt{2} \times (V_s)_{LL} = \sqrt{2} \times 222.15 = 314.16 \quad V$$

e) peak-to-peak ripple in $v_O(t)$ and its frequency.

The output voltage fluctuates between $1.225\,(V_s)_{LL}$ and $\sqrt{2}\,(V_s)_{LL}$ as:

$$\left((v_{ripple}(t))_{out}\right)_{min} = 1.225\times 222.15 = 272.13\,V,\ and\ \left((v_{ripple}(t))_{out}\right)_{max} = \sqrt{2}\times 222.15 = 314.16$$

The peak value of the output voltage ripple is:

$$\left((v_{ripple}(t))_{out}\right)_{max} - \left((v_{ripple}(t))_{out}\right)_{min} = 314.16 - 272.13 = 42.03 \quad V$$

This ripple will have a fundamental frequency of $(f_{ripple})_1$ as:

$$(f_{ripple})_1 = 6 \times f_s = 6 \times 60 = 360 \quad Hz$$

Q4

A 3ϕ full-wave rectifier supplies an inductive load $R_L = 9\,\Omega$ and $L_L = 3$ mH. The input AC voltage is supplied from a feeder at 660 V, 60 Hz. Design an input LC filter that will ensure an input side RF of $RF \leq 6\%$, and an output LC filter for a ripple voltage $\Delta v_o \leq 8\%$.

The output DC voltage V_{dc} can be determined for the load as:

$$V_{dc} = \frac{3\sqrt{3}}{\pi} V_m$$

The feeder voltage is an rms line-to-line voltage (default specifications). The required the peak line-to-neutral value V_m:

$$V_m = \sqrt{2}\left(\frac{V_s}{\sqrt{3}}\right) = 538.8877 \quad V$$

V_{dc} becomes:

$$V_{dc} = \frac{3\sqrt{3}}{\pi} \times 538.8877 = 891.31 \quad V$$

The output DC current I_{dc} is determined as:

$$I_{dc} = \frac{V_{dc}}{R_L} = \frac{891.31}{9} = 99.03 \quad A$$

The input LC Filter

The values of the input filter parameters (L_S and C_S) can be determined based on the the value of the THD$_I$ or the RF as:

$$\frac{X_{Ls}}{X_{Cs}} = \frac{1}{5^2}\left(\frac{I_{an}(n\omega)}{(I_{an}(n\omega))_{Ls}} + 1\right) = \frac{1}{25}(RF + 1) = 0.04 \times (0.06 + 1) = 0.0424$$

Let $C_S = 70\mu$F. This value produces L_S as:

$$2\pi \times 60 L_S = 0.0424 X_{Cs} = \frac{0.0424}{2\pi \times 60 \times 70 \times 10^{-6}} = 1.6067 \Longrightarrow L_S = 4.26 \quad mH$$

The ouput LC Filter

The value of the output side capacitor C_O can be determined using the ripple voltage conditions as:

$$C_O = \frac{100\,(I_o(n=6))_{peak}}{\sqrt{2} \times (\Delta v_O\%) \times V_{dc} \times 12\pi \times f_s}$$

The peak value of the 6^{th} harmonic voltage is:

$$(v_o(n=6))_{peak} = 0.9549 V_m \times \frac{2}{35} = 0.9549 \times 538.8877 \times 0.0571 = 29.38 \quad V$$

The peak value of $I_o(n=6)$ can be determined as:

$$(I_o(n=6))_{peak} = \frac{(v_o(n=6))_{peak}}{Z_L} = \frac{29.38}{\sqrt{9^2 + (2\pi \times 6 \times 60 \times 0.003)^2}} = 2.6066 \quad A$$

The value of C_O can be determined as:

$$C_O = \frac{100 \times 2.6066}{\sqrt{2} \times 8 \times 891.31 \times 2 \times 6 \times \pi \times 60} = 11.43 \quad \mu F$$

The value of L_O can be determined for the 6^{th} harmonic on the output as:

$$\Delta v_o = \left| \frac{-1}{(2\pi 6 f_s)^2 L_O C_O - 1} \right| \Longrightarrow (720\pi)^2 L_O C_O - 1 = \frac{1}{\Delta v_o}$$

$$L_O = \frac{\frac{1}{\Delta v_o} + 1}{(720\pi)^2 C_O} = \frac{1/0.08 + 1}{(720\pi)^2 \times 23.89 \times 10^{-6}}$$

get $L_O = 231$ mH.

Q5

A 3ϕ full-wave controlled rectifier supplies a resistive load with 12 kW at 80% of the maximum possible output DC voltage. The input AC voltage is supplied from a Δ-connected feeder at 340 V, 60 Hz. For this controlled rectifier, determine:

a) The firing angle α.

The Δ-connected feeder will include a $\Delta - Y$ transformer, where the Y-connected side will feed the AC-DC converter. The phase voltage is $V_{sP} = 340/\sqrt{3} = 196.3$ V, $(V_m)_{peak} = \sqrt{2} \times V_{sP} = 277.61$ V. The maximum possible output DC voltage is obtained for $\alpha = 0$ as:

$$(V_{dc})_{max} = \frac{3\sqrt{3}V_m}{\pi} \cos(\alpha = 0) = 1.654 \times 277.61 = 459.17\, V$$

The firing angle α can be determined from the required DC voltage, which is $0.8\,(V_{dc})_{max} = 367.33$ V. For the output DC voltage:

$$\alpha = \cos^{-1}\left(\frac{367.33}{459.61}\right) = 36.94°$$

b) $(I_O)_{rms}$ and I_{dc}.

The rms output voltage is determined as:

$$(V_O)_{rms} = \sqrt{3}\,(V_m)_{Ph}\sqrt{\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos(2\alpha)} = \sqrt{3}{\times}277.61\sqrt{0.5 + 0.4135\cos(2\times 34.86°)} = 377.0\ V$$

$$(I_O)_{rms} = \frac{(V_O)_{rms}}{R_L};\ R_L = \frac{V_{dc}^2}{P_o} = \frac{367.33^2}{12000} = 11.24\Omega$$

$$(I_O)_{rms} = \frac{(V_O)_{rms}}{R_L} = \frac{377}{11.24} = 33.53\ \ A$$

The output DC current I_{dc} is:

$$I_{dc} = \frac{V_{dc}}{R_L} = \frac{367.33}{11.24} = 32.68\quad A$$

c) The current in each thyristor.

The average current in each thyristor I_T is determined as:

$$I_T = \frac{I_{dc}}{3} = \frac{32.68}{3} = 10.89\quad A$$

d) the efficiency of this controlled rectifier.

The efficiency η is determined as:

$$\eta = \frac{V_{dc} \times I_{dc}}{(V_O)_{rms} \times (I_O)_{rms}} = \frac{367.33 \times 32.68}{377 \times 33.53} = 94.96\%$$

e) The input power factor.

The input current $(I_s)_{rms}$ can be determined as:

$$(I_s)_{rms} = (I_O)_{rms} \times \sqrt{\frac{2}{3}} = 33.53 \times 0.8165 = 27.35 \quad A$$

The output power $P_{out} = 12$ kW. The input power factor is determined as:

$$PF = \frac{P_{out}}{\sqrt{3}\,(V_s)_{rms} \times (I_s)_{rms}} = \frac{12000}{\sqrt{3} \times 340 \times 27.35} = \frac{12}{22.613} = 0.745 \quad Lag$$

f) The commutation angle u.

From the output DC voltage:

$$V_{dc} = \frac{3\,(V_{LL})_{Peak}}{2\pi}(\cos(\alpha) + \cos(\alpha + u));\ 367.33 = \frac{3 \times \sqrt{2} \times 340}{2\pi}(\cos(36.94) + \cos(36.94 + u))$$

$$\cos(36.94 + u) = 0.8008 \Longrightarrow u = -0.15^{\circ} \approx 0$$