Review of Some Forecasting Methods

1. The Naive Approach

Naive forecasts are the most cost-effective forecasting model, and it provides a benchmark against which more sophisticated models can be compared [1]. This forecasting method is only suitable for time series data. In the naive approach, forecasts are produced that are equal to the last observed value. If the time series is believed to have seasonality, the seasonal naive approach may be more appropriate where the forecasts are equal to the value from last season. The seasonal naive method accounts for seasonality by setting each prediction to be equal to the last observed value of the same season. The seasonal naive method is particularly useful for data that has a very high level of seasonality.

It is important to 'baseline' the forecasts from other methods by comparing them to the naive forecast, because it allows us to understand how much value is being added to the current forecasting process. It also helps provide an impression of how difficult the products are to forecast; and when a better forecasting method is applied, it can be understood how much value the method is adding to the forecast. The reason for this because though it's a simple method, it performs really well in many cases where there is high similarity in the time series data [2]. The formula for the naive approach, and the seasonal naive approach are shown below respectively;

$$\hat{y}_{T+h|T} = y_T.$$

$$\hat{y}_{T+h|T} = y_{T+h-m(k+1)},$$

Where; y is the time series, h is the forecast horizon, m is the seasonal period, and k is the integer part of (h-1)/m [3]. The naive formula takes the last observed value as the future value, while the seasonal naive formula takes the value from the previous season.

2. Holt–Winters Method

Holt-Winters forecasting is a way to model and predict the behavior of a sequence of values over time in a time series. Holt-Winters is a model of time series behavior; it models three aspects of the time series: a typical value (average), a slope (trend) over time, and a cyclical repeating pattern (seasonality). Holt-Winters uses exponential smoothing to encode lots of values from the past and use them to predict "typical" values for the present and future. Holt-Winters is normally called triple exponential smoothing (value, trend, seasonality); the model requires several parameters: one for each smoothing (α, β, γ), the length of a season, and the number of periods in a season.

A season is a fixed length of time that contains the full repetition. Within the season, there are periods; if we wanted to model a value for every hour of every day within a week, the season will be 168 hours long and our period will be 1 hour. The idea behind this method is to apply exponential smoothing to the seasonal components in addition to level and trend. The Holt-Winters seasonal method comprises the forecast equation and three smoothing equations; one for the level L_t, one for trend b_t and one for the seasonal component denoted by S_t, with smoothing parameters α, β and γ. The formulas for this method can be seen below, where s is the length of the seasonal cycle, for $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and $0 \leq \gamma \leq 1$ [4].

$$\begin{aligned}
\text{level} \quad L_t &= \alpha(y_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}); \\
\text{trend} \quad b_t &= \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1}, \\
\text{seasonal} \quad S_t &= \gamma(y_t - L_t) + (1-\gamma)S_{t-s} \\
\text{forecast} \; F_{t+k} &= L_t + kb_t + S_{t+k-s},
\end{aligned}$$

Holt-Winters forecasting is very powerful despite its simplicity. It can handle lots of complicated seasonal patterns by simply finding the central value, then adding in the effects of slope and seasonality. The most challenging part of Holt-Winters forecasting is the selection of good parameters. Numerical optimization has been found to be a good method for picking the parameters for the model [5].

3. Autoregressive Integrated Moving Average (ARIMA)

ARIMA is a class of models that explains a given time series based on its own past values, that is, its own lags and the lagged forecast errors, so that equation can be used to forecast future values. An ARIMA model is characterized by p, d, q; where p is the order of the AR term, q is the order of the MA term, and d is the number of differencing required in order to make the time series stationary. An ARIMA model is one where the time series was differenced at least once to make it stationary and you combine the AR and the MA terms [6].

The first step to build an ARIMA model is to make the time series stationary; because, term 'Auto Regressive' in ARIMA means it is a linear regression model that uses its own lags as predictors. Linear regression models work best when the predictors are not correlated and are independent of each other. The integration of the time series is a common approach for making the data stationary; the d value is the minimum rounds of integration needed. The 'p' is the order of the 'Auto Regressive' (AR) term, it refers to the number of lags of the data to be used as predictors; and the 'q' is the order of the 'Moving Average' (MA) term, it refers to the number of lagged forecast errors that should go into the ARIMA Model.

The Autoregressive (AR) model is one where Yt depends only on its own lags. That is, Yt is a function of the 'lags of Yt'. The formula can be seen below.

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + .. + \beta_p Y_{t-p} + \epsilon_1$$

Where, Y_{t-1} is the lag1 of the series, ß is the coefficient of lag1 that the model estimates and α is the intercept term which is also estimated by the model. The Moving Average (MA) model is one where Yt depends only on the lagged forecast errors. The formula can be seen below.

$$Y_t = \alpha + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + .. + \phi_q \epsilon_{t-q}$$

Where, the error terms are the errors of the autoregressive models of the respective lags, e.g. the error E_t is the error gotten from this equation.

$$Y_t = \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + .. + \beta_0 Y_0 + \epsilon_t$$

The ARIMA model is one where the time series was differenced at least once to make it stationary and you combine the AR and the MA terms. So the equation becomes:

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + .. + \beta_p Y_{t-p} \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + .. + \phi_q \epsilon_{t-q}$$

ARIMA in the basic form is: the prediction Y_t equals the addition of; a constant, the linear combination lags of Y (up to p lags), and the linear combination of lagged forecast errors (up to q lags).

The SARIMA takes into account the seasonality of the dataset; seasonal differencing is used in this case. Seasonal differencing is similar to regular differencing, but instead it subtracts the value from the previous season. The model will be represented as SARIMA(p,d,q)x(P,D,Q), where, P, D and Q are SAR, order of seasonal differencing and SMA terms respectively and 'x' is the frequency of the time series. If we would like to consider exogenous variables, which considers more than one variable for the prediction; models like ARIMAX or SARIMAX should be considered.

4 References

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