

Complete Time and Space Complexity Guide (With Examples)

O(1) Constant Time

Accessing an element by index:

```
arr = [10, 20, 30]
print(arr[1]) # Output: 20
```

O(log n) Logarithmic Time

Binary search on sorted list:

```
def binary_search(arr, target):
    low, high = 0, len(arr) - 1
    while low <= high:
        mid = (low + high) // 2
        if arr[mid] == target:
            return mid
        elif arr[mid] < target:
            low = mid + 1
        else:
            high = mid - 1
    return -1
```

O(n) Linear Time

Traversing a list:

```
for item in [1, 2, 3, 4]:
    print(item)
```

O(n log n) Linearithmic Time

Merge sort (simplified structure):

```
def merge_sort(arr):
    if len(arr) <= 1:
        return arr
    mid = len(arr) // 2
    left = merge_sort(arr[:mid])
    right = merge_sort(arr[mid:])
    return merge(left, right)
```

O(n²) Quadratic Time

Nested loop:

```
for i in range(n):
    for j in range(n):
        print(i, j)
```

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$O(2^n)$ Exponential Time

```
Fibonacci (naive recursion):
def fib(n):
    if n <= 1:
        return n
    return fib(n-1) + fib(n-2)
```

$O(n!)$ Factorial Time

```
All permutations:
def permute(nums):
    if len(nums) <= 1:
        return [nums]
    res = []
    for i in range(len(nums)):
        for p in permute(nums[:i] + nums[i+1:]):
            res.append([nums[i]] + p)
    return res
```

Amortized Time Complexity

```
Append in dynamic arrays:
arr = []
for i in range(1000):
    arr.append(i) #  $O(1)$  amortized,  $O(n)$  when resized
```

```
Hash table insert:
my_map = {}
my_map["key"] = "value" #  $O(1)$  avg,  $O(n)$  worst-case
```

Rare Complexities

```
 $O(n)$ : Sieve of Eratosthenes
is_prime = [True] * (n+1)
for i in range(2, int(n**0.5) + 1):
    if is_prime[i]:
        for j in range(i*i, n+1, i):
            is_prime[j] = False
```

```
 $O(b^d)$ : BFS
Use BFS in a tree/graph with branching factor b and depth d
```

Data Structures Time & Space

```
Array:  $O(1)$  access,  $O(n)$  insert/delete
```

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Stack/Queue: $O(1)$ push/pop
HashMap: $O(1)$ avg, $O(n)$ worst
Set: Same as HashMap
Linked List: $O(n)$ access, $O(1)$ insert/delete
BST: $O(\log n)$ avg, $O(n)$ worst
Heap: $O(\log n)$ insert/delete
Trie: $O(L)$, Space $O(\text{ALPHABET_SIZE} * L)$
Segment Tree: $O(\log n)$ query/update
Union Find (with path compression): $O((n))$ constant

Space Complexity Examples

$O(1)$: Swap in place
 $a, b = 1, 2$
 $a, b = b, a$

$O(n)$: Copy list
 $\text{copy} = \text{arr}[:]$

$O(\log n)$: Recursive binary search call stack

Space Optimization Techniques

Sliding window ($O(1)$ space):
 $\text{def max_subarray}(\text{arr}, k):$
 $\text{max_sum} = \text{window} = \text{sum}(\text{arr}[:k])$
 for i in $\text{range}(k, \text{len}(\text{arr}))$:
 $\text{window} += \text{arr}[i] - \text{arr}[i - k]$
 $\text{max_sum} = \text{max}(\text{max_sum}, \text{window})$
 return max_sum

Bit manipulation:
 $x = x \oplus y$
 $y = x \oplus y$
 $x = x \oplus y$

Tips and Pitfalls

- Avoid ignoring recursion stack space
- Hashmaps are not always $O(1)$ in worst-case
- Consider amortized cost for dynamic arrays
- Understand worst vs. average vs. best case
- Input size growth can destroy performance