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Website CoSTAR

The MATLAB Toolbox CoSTAR

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INSTITUTE FOR MECHANICS

ENGINEERING DYNAMICS

Website Engineering Dynamics Group

Content

- Overview and Features
- Flow Chart and Code Structure
- Basic Use and Where To Start
- Outlook, Feedback and Download

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CoSTAR

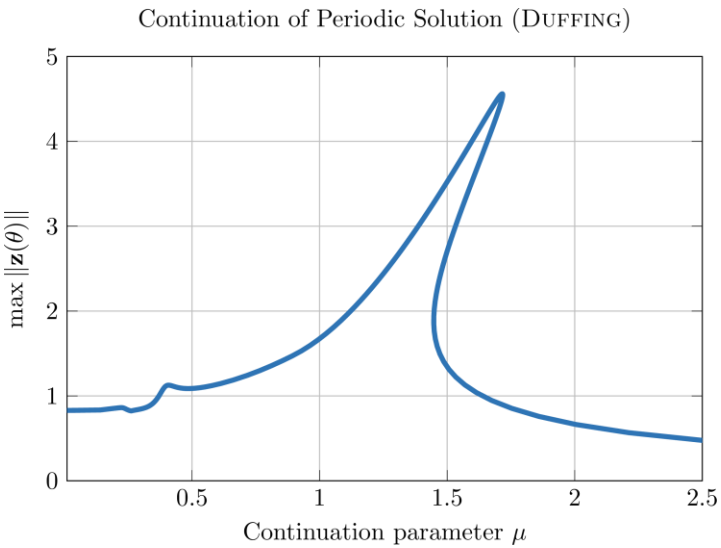
Continuation of Solution Torus AppRoximations

- **Computation of *stationary* solutions of dynamical systems**
  (*stationary* solution: solution type persists for infinite time interval)
  - Equilibrium solutions (**EQ**)
  - Periodic solutions (**PS**)
  - Quasi-periodic solutions (**QPS**)
    (2 base frequencies)

$$\dot{\mathbf{z}} = \mathbf{f}(t, \mathbf{z}, \mu)$$
$$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mu)$$

- **Approximation methods for (quasi-)periodic solutions**
  - Finite Difference Method (**FDM**)
  - Fourier-Galerkin Method (**FGM**)
  - (Multiple) Shooting Method (**SHM**)
    - **PS:** Multiple Shooting Method
    - **QPS:** Single Shooting Method

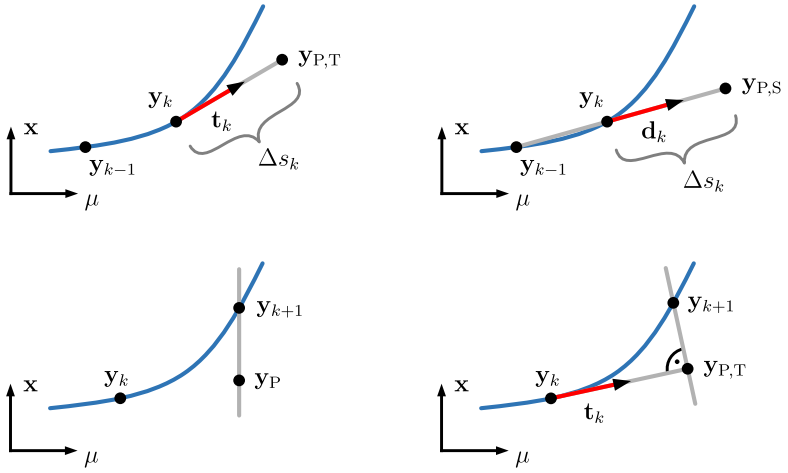
- **Continuation of solution branches**

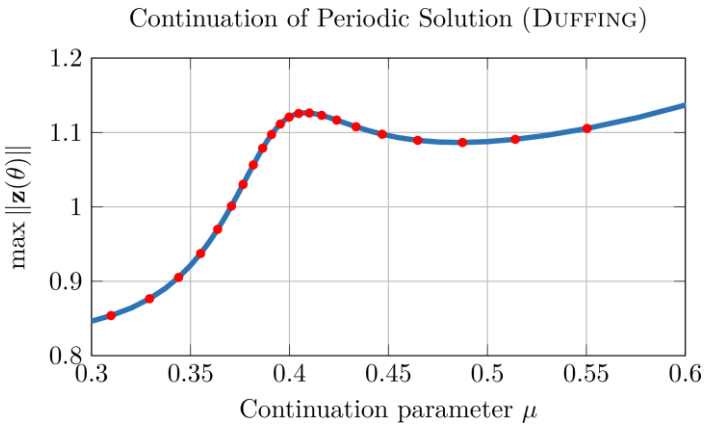


ENGINEERING DYNAMICS

All features can be used as required

- **Continuation: Predictor-corrector algorithm**
  - Predictors
    - Tangent
    - Polynomials of order 1, 2 and 3
  - Parametrisations
    - Natural
    - Arclength and pseudo-arclength
    - 1-norm
  - Step control
    - ➢ Various algorithms based on geometrical information and solver iterations
  - Live plot
    - ➢ Creating continuation plot during computation

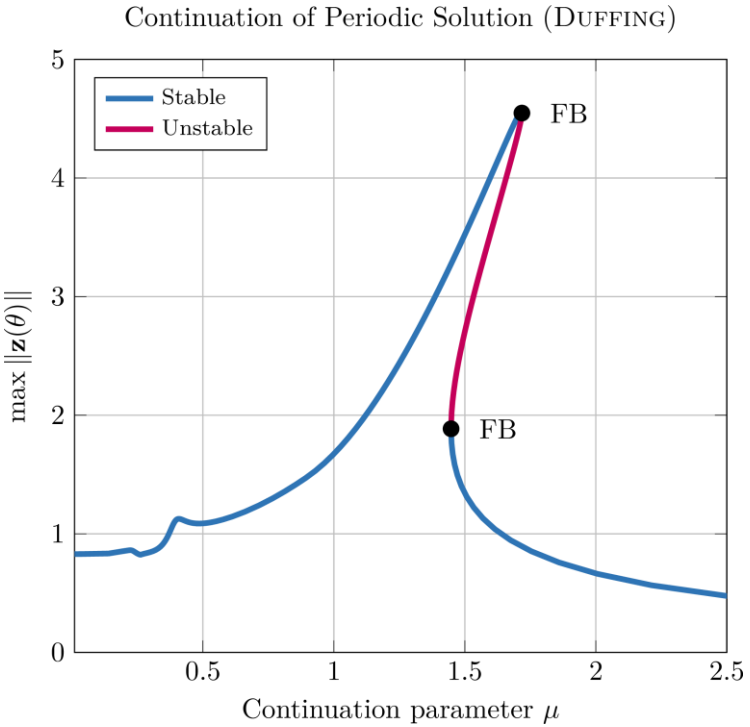




Features

All features can be used as required

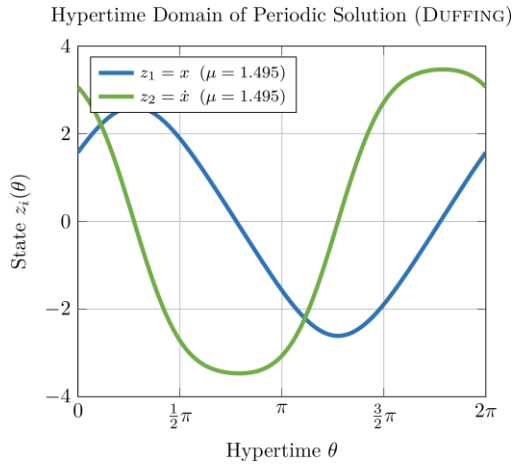
- **Stability computation of solutions**
  - Equilibrium solutions
  - Periodic solutions
    - **SHM**
    - **FDM** & **FGM** (internally resort to **SHM**)
  - Quasi-periodic solutions
    - **SHM**
- **Detection of bifurcation points**
  - Fold / Pitchfork / Transcritical (**FB**)
  - Period Doubling (**PDB**)
  - HOPF (**HB**)
  - NEIMARK-SACKER (**NSB**)

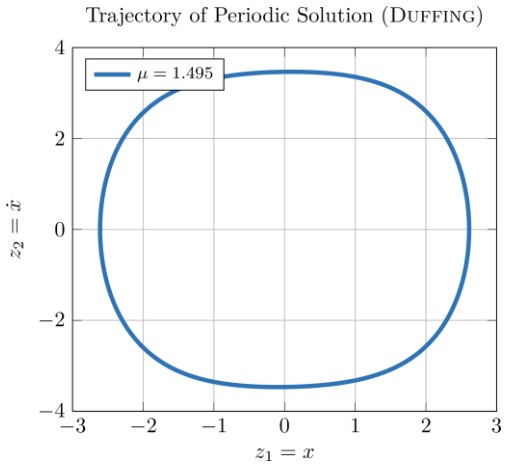


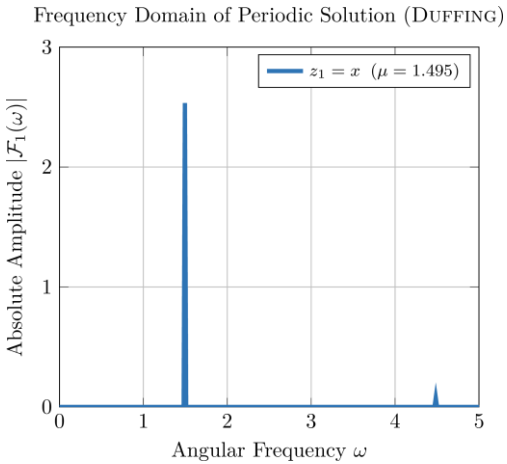
- **Error Control**
  - **FGM:** (PS & QPS)
    Automatic adaption of number of harmonics based on residuum

All features can be used as required

- **Postprocessing methods**
  - `contplot`
    - Creates continuation / bifurcation diagrams (plots solution branches with respect to μ)
  - `solplot`
    - Plots individual solutions in different *solution spaces*
      (Available *solution spaces*: time, hypertime, trajectory and frequency domain)





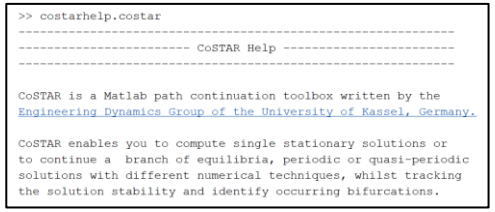


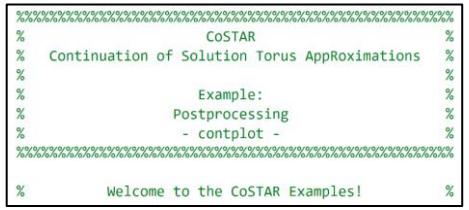
  - `solget`
    - Returns solution data in different solution spaces

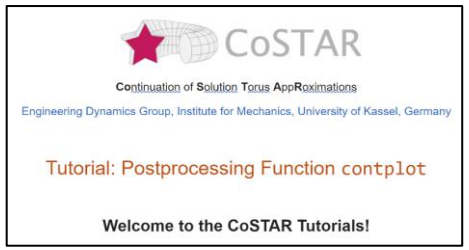
Help Features

- **Gatekeeper** *cannot be bypassed*
  - Checks the input (the defined options) from the user
  - Reports errors in case of illogical or invalid input

- **Help**
  - `costarhelp` function
    - Quick help in the command window
    - Overview of the available options with a short description
    - Type **`costarhelp.costar`** in the command window to start
  - Examples
    - Short MATLAB scripts
    - Sample code showing usage of a certain **CoSTAR** module
  - Tutorials
    - MATLAB live scripts
    - There is one tutorial for each example (identical code)
    - Comprehensive explanations of a certain **CoSTAR** module

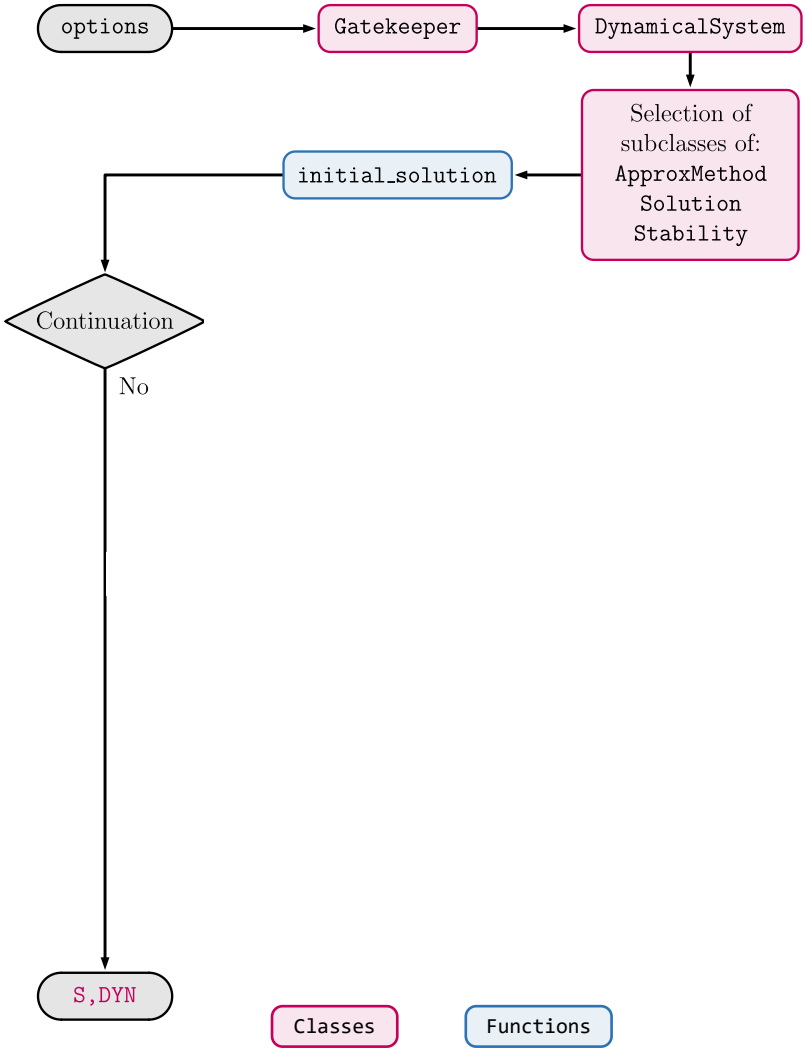






Source „*You shall not pass!*“ picture: https://knowyourmeme.com/photos/42374-you-shall-not-pass--2 (2024-11-25)

Flow Chart

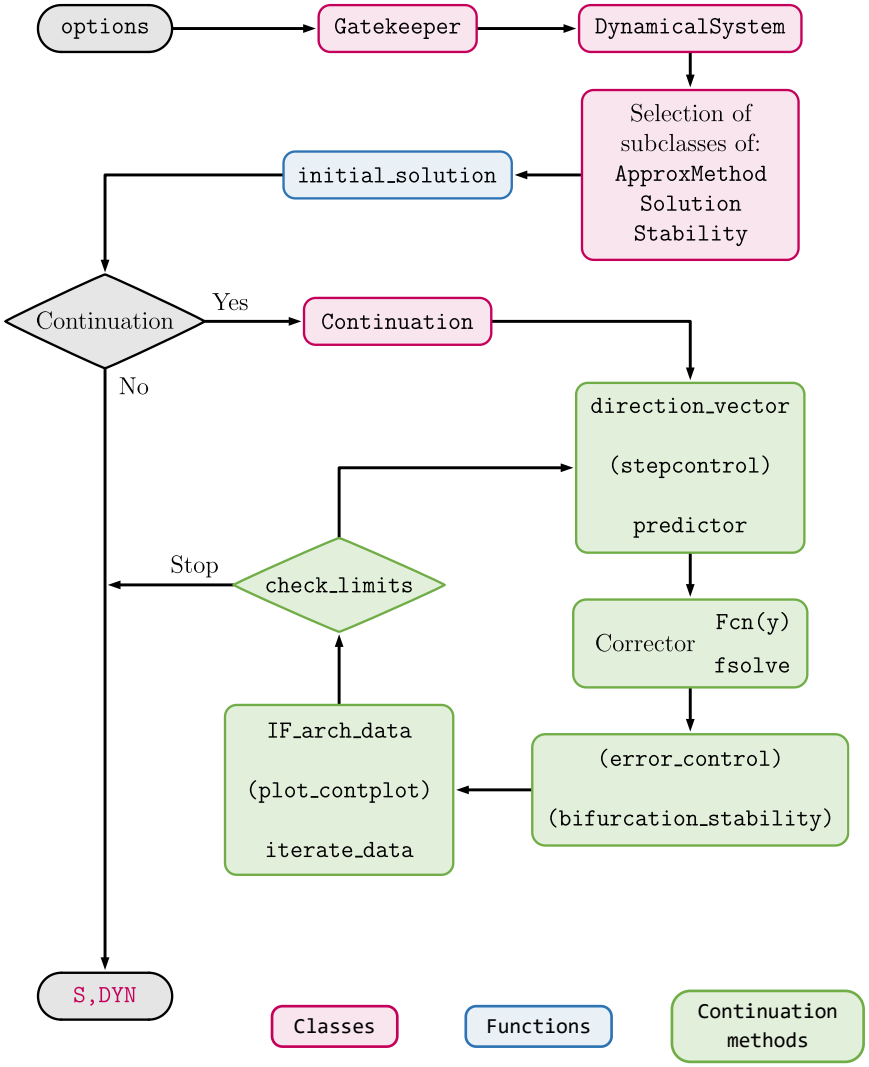


1. `options`
   - Structure array
   - Contains user-defined options for computation
2. `Gatekeeper`
   - Checks `options`
3. `DynamicalSystem`
   - Stores all options in object `DYN`
   - Can be used to restart the computation
4. Selection of subclasses
   - `ApproxMethod`
     - Applies approximation method
   - `Solution`
     - Stores solution data in object `S`
   - `Stability`
     - Used for stability computation
5. `initial_solution`
   - Computes the first *(initial)* solution

In case of no continuation:

6. Return `S` and `DYN`

Flow Chart



6. Continuation loop
   - 6.1. direction_vector
   - 6.2. (stepcontrol) (can be skipped)
   - 6.3. predictor
     - Computes direction vector, new step width and predictor point
   - 6.4. Corrector
     - fsolve solving Fcn(y) = 0
   - 6.5. (error_control) (can be skipped)
   - 6.6. (bifurcation_stability) (can be skipped)
     - Performs error control and computes stability as well as bifurcation point
   - 6.7. IF_arch_data
   - 6.8. (plot_contplot) (can be skipped)
   - 6.9. iterate_data
     - Stores data, updates live plot and performs iterations for next loop
   - 6.10. check_limits
     - Checks exit conditions

When exit condition is met:

7. Return S and DYN

Code Structure

Classes	All classes and associated methods
Functions	Functions not belonging to any class
RHS	Functions defining right-hand side of $\dot{\mathbf{z}} = \mathbf{f}(t, \mathbf{z}, \mu)$ or $\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mu)$
test	Scripts to test the code
Tutorials	Tutorial & example scripts
Version_Log	Version log files documenting code development
costar	Main **CoSTAR** function (to be called by the user)

Code Structure

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- Classes

  All classes and associated methods

  - @Continuation
    - Class and methods to perform the continuation
  - @costarhelp
    - Class and methods for the **`costarhelp`** feature
  - @DynamicalSystem
    - Class for storing the **`options`** structure
  - @Gatekeeper
    - Class and methods for the **`Gatekeeper`** feature
  - ApproxMethod_SC
    - Classes and methods to construct the residuum function
    - @AM_EQ
    - @AM_PS_FDM
    - @AM_PS_FGM
    - @AM_PS_SHM
    - @AM_QPS_FDM
    - @AM_QPS_FGM
    - @AM_QPS_SHM

      Subclasses and methods

    - @ApproxMethod
      - Superclass
  - Solution_SC
  - Stability_SC

Code Structure

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- Classes

  All classes and associated methods

  - @Continuation
    - Class and methods to perform the continuation
  - @costarhelp
    - Class and methods for the **`costarhelp`** feature
  - @DynamicalSystem
    - Class for storing the **`options`** structure
  - @Gatekeeper
    - Class and methods for the **`Gatekeeper`** feature
  - ApproxMethod_SC
    - Classes and methods to construct the residuum function
  - Solution_SC
    - Classes and methods to save computed data
    - @SOL_EQ
    - …
    - @Solution

    } Subclasses and methods (analogous to ApproxMethod_SC)

    - Superclass
  - Stability_SC
    - Classes and methods to compute stability
    - @ST_EQ
      - Subclass and methods for equililibrium solutions
    - @ST_PS_SHM
      - Subclass and methods for periodic shooting method
    - @ST_QPS_SHM
      - Subclass and methods for quasi-periodic shooting method
    - @Stability
      - Superclass and methods

costar

```
function [S,DYN] = costar(options)
```

- **Input:** `options` structure
  Output: `Solution` object `S`, `DynamicalSystem` object `DYN`

```
    %% Gatekeeper
    GC = Gatekeeper();
    options = GC.m_gatekeeper(options);
    clear GC;
```

- **Gatekeeper** checks all input `options`

```
    %% Dynamical System class
    DYN = DynamicalSystem(options);
```

- **Save** all `options` in `DynamicalSystem` object `DYN`

```
    %% Approximation Method class
    AM = ApproxMethod.s_method_selection(DYN);
```

- **Create** `ApproxMethod` object `AM`
  (methods construct the residuum function)

```
    %% Solution class
    S = Solution.s_solution_selection(DYN,AM);
```

- **Create** `Solution` object `S`
  (stores all solution data)

```
    %% Stability class
    ST = Stability.s_stability_selection(DYN,AM);
```

- **Create** `Stability` object `ST`
  (methods compute the stability of a solution)

```
    %% Calculate initial solution
    [S,AM,DYN] = initial_solution(DYN,S,AM,ST);
```

- **Compute** the initial (first) solution

```
    %% Continuation
    if strcmpi(DYN.cont,'on')
        CON = Continuation(options.opt_cont);
        S = CON.m_continuation(DYN,S,AM,ST);
    end
```

- **Create** `Continuation` object `CON`
- **Do** the continuation

1. Define important parameters and functions

(not necessarily needed, but it helps to keep the overview)

```
%% 1. Define important parameters and functions (not necessarily needed, but it helps to keep the overview)
D = 0.05;     kappa = 0.3;     g = 1;              % Parameters needed for the Duffing differential equation
mu_limit = [0.01, 2.5];                            % Limits of the continuation
eta0 = mu_limit(1);                                % Value of continuation parameter at start of continuation
param = {kappa, D, eta0, g};                       % Parameter array
active_parameter = 3;                              % Location of continuation parameter within the array
IC = [1; 0];                                       % Initial condition (point in state space) for fsolve

% Functions
non_auto_freq = @(mu) mu;                          % Non-autonomous excitation frequency
Fcn =  @(t,z,param) duffing_ap(t,z,param);          % Right-hand side of dz/dtau = f(tau,z,kappa,D,eta,g)
```

2. Define the options structure

(it comprises all information that CoSTAR needs)

```
%% 2. Define the options structure (it comprises all information that CoSTAR needs)
options.system = costaropts('order',1,'dim',2,'rhs',Fcn,'param',param,'info','continuation of Duffing equation');   % Properties of the system
options.opt_sol = costaropts('sol_type','periodic','approx_method','shooting','cont','on','stability','on', ...   % Properties of the solution
                             'non_auto_freq',non_auto_freq,'act_param',active_parameter);                          % Properties of the solution
options.opt_init = costaropts('ic',IC);                                                                            % Property for initial solution
options.opt_approx_method = costaropts('solver','ode45');                                                          % Properties of approximation method
options.opt_cont = costaropts('mu_limit',mu_limit);                                                                % Properties for continuation
```

3. Call CoSTAR (and do the continuation)

```
%% 3. Call CoSTAR and do the continuation
[S,DYN] = costar(options);                         % CoSTAR is called by costar(options)
```

4. Individual postprocessing

```
%% 4. Individual postprocessing
% ...
```

Where To Start

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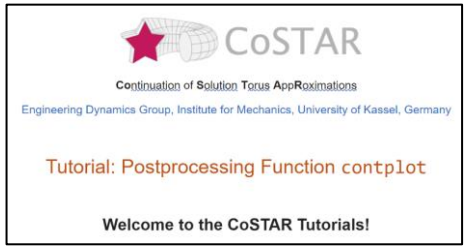
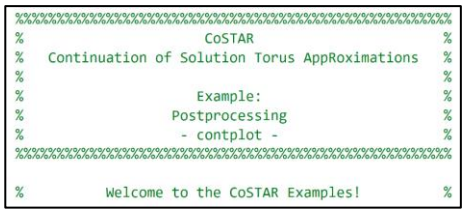
- **If you are new to CoSTAR or certain modules**
  - Tutorials
    - Comprehensive explanations of a certain **CoSTAR** module
    - Currently available:

Start with one of these if you have not used CoSTAR yet

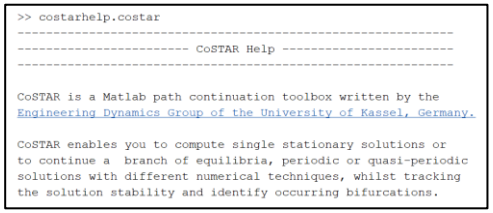
- ✓ Equilibrium solutions (`Tutorial_EQ`)
- ✓ Periodic and quasi-periodic solutions approximated by FDM, FGM and SHM (`Tutorial_PS_FDM`, `Tutorial_PS_FGM`, …, `Tutorial_QPS_FDM`, …)
- ✓ Postprocessing methods contplot, solplot and solget (`Tutorial_Postprocessing_contplot`, …)



- **If you already used CoSTAR**
  - Examples
    - Sample code showing usage of a certain **CoSTAR** module
    - Good rescue point to restart working with **CoSTAR**
    - There is one example for each tutorial (examples are labelled `Example_[…]`)



  - `costarhelp` feature
    - Overview of the available options with a short description
    - Quick help in the command window while using **CoSTAR**
    - Type **`costarhelp.costar`** in the command window to start



Outlook

Note: The following list may be incomplete and only lists ideas for future improvements to the toolbox. There is no guarantee for actual implementation.

- **Approximation methods**
  - Multiple Shooting Method for quasi-periodic solutions
- **Features**
  - **Stability computation**
    - **PS:** Directly from solution data when using FDM or FGM without JACOBIAN of SHM
    - **QPS:** For Finite Difference Method & FOURIER-GALERKIN Method (completely missing so far)
  - **Error control**
    - Finite Difference Method (PS & QPS)
    - Handle different exit flags from `fsolve` when computing new solution with updated discretization
  - **Step control**
    - Algorithm(s) based on convergence of solver (when self-written solver is available)
  - **Postprocessing**
    - **FGM & SHM** (QPS hypertime plots): `[1x2]` array for `options` structure field `'resolution'`
  - **Tutorials & Examples**
    - Update default-script tutorials to live scripts
    - Tutorials and examples for continuation options, step control and stability computation

Outlook

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Note: The following list may be incomplete and only lists ideas for future improvements to the toolbox. There is no guarantee for actual implementation.

- **Initial value** (for the solver to compute the initial solution)
  - Standardise the parameters, which create an initial value, for all approximation methods
  - Use of a solution of a different approximation method as initial value
  - Homotopy methods
- **Continuation**
  - Predictor: Polynomials of order > 3
  - Additionally compute the solution at specified (desired) μ-values
- **Computational effort**
  - Make parallel computing available to enhance performance
- **Solver**
  - Self-written solver to remove the need of MATLAB's Optimization Toolbox
- **Dynamic System**
  - Computation of non-hyperbolic manifolds (solutions of Hamiltonian systems)

Feedback and Download

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- **Download**
  - CoSTAR is available for free as GitHub repository
  - CoSTAR is licenced under the *Apache 2.0* licence

- **Report of bugs**
  - Please create a GitHub issue, labelled as bug, if you experience a new bug
  - If a GitHub issue already exists for your bug, no action is required

- **Suggestions for improvement, wishes and ideas for future releases**
  - Please create a GitHub issue for any wishes, improvements and ideas for future releases and label it accordingly

Website CoSTAR

Website Engineering Dynamics Group