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Website CoSTAR

The MATLAB Toolbox CoSTAR

Institute of Mechanics, Engineering Dynamics Group
Department of Mechanical Engineering (FB15)
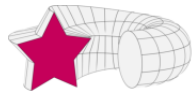University of Kassel (Germany)

INSTITUTE FOR
MECHANICS

ENGINEERING
DYNAMICS

Content

- Overview and Features
- Flow Chart and Code Structure
- Basic Use and Where To Start
- Outlook, Feedback and Download

Website Engineering
Dynamics Group

CoSTAR — **C**ontinuation of **S**olution **T**orus App**R**oximations

- **Computation of *stationary* solutions of dynamical systems**
  (*stationary* solution: solution type persists for infinite time interval)

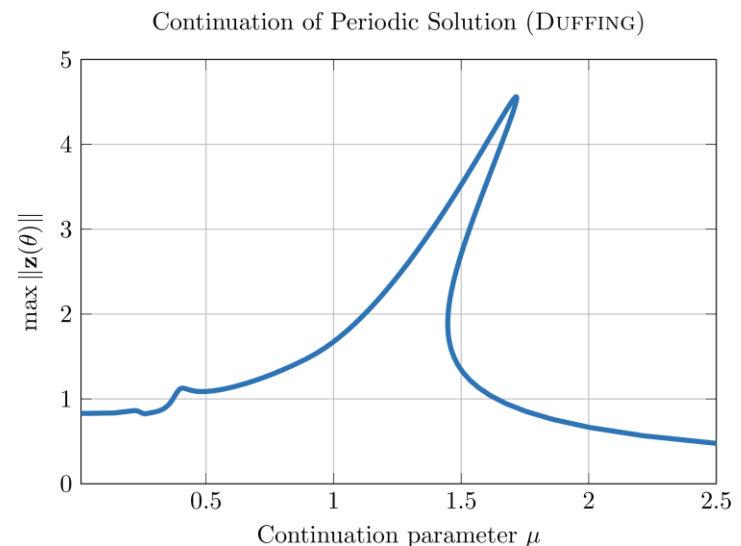 $$\dot{\mathbf{z}} = \mathbf{f}(t, \mathbf{z}, \mu)$$
  $$\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mu)$$

  – Equilibrium solutions (**EQ**)

  – Periodic solutions (**PS**)

  – Quasi-periodic solutions (**QPS**)
    (2 base frequencies)

- **Approximation methods for (quasi-)periodic solutions**

  – Finite Difference Method (**FDM**)

  – FOURIER-GALERKIN Method (**FGM**)

  – (Multiple) Shooting Method (**SHM**)

    o **PS:** Multiple Shooting Method

    o **QPS:** Single Shooting Method

- **Continuation of solution branches**

Continuation of Periodic Solution (DUFFING)

x-axis: Continuation parameter μ
y-axis: $\max \|\mathbf{z}(\theta)\|$

ENGINEERING DYNAMICS

All features can be used as required

- **Continuation: Predictor-corrector algorithm**

  – Predictors

    o Tangent

    o Polynomials of order 1, 2 and 3

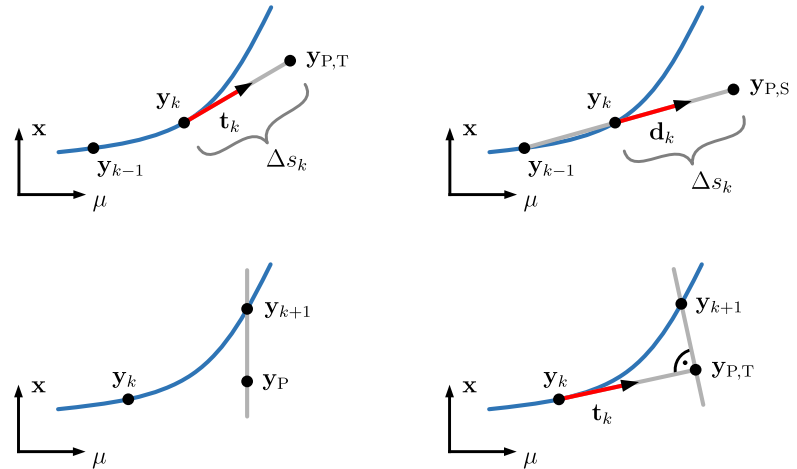 – Parametrisations

    o Natural

    o Arclength and pseudo-arclength
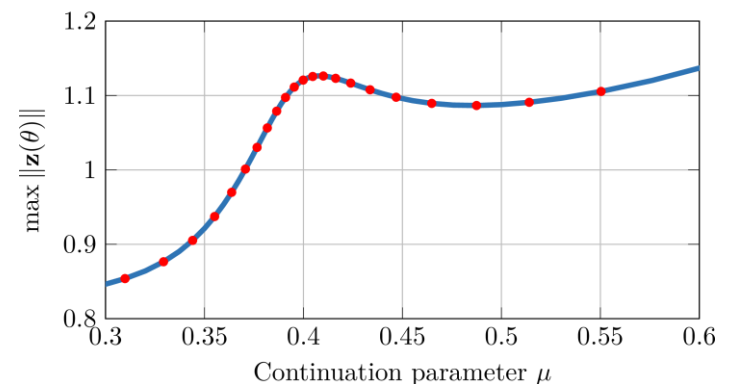
    o 1-norm

  – Step control

    ➢ Various algorithms based on geometrical information and solver iterations

  – Live plot

    ➢ Creating continuation plot during computation

Continuation of Periodic Solution (DUFFING)

All features can be used as required

- **Stability computation of solutions**
  - Equilibrium solutions
  - Periodic solutions
    - **SHM**
    - **FDM** & **FGM** (internally resort to **SHM**)
  - Quasi-periodic solutions
    - **SHM**

- **Detection of bifurcation points**
  - Fold / Pitchfork / Transcritical (**FB**)
  - Period Doubling (**PDB**)
  - HOPF (**HB**)
  - NEIMARK-SACKER (**NSB**)

- **Error Control**
  - **FGM:** (PS & QPS)
    Automatic adaption of number of harmonics based on residuum

Continuation of Periodic Solution (DUFFING)

UNI KASSEL
VERSITÄT
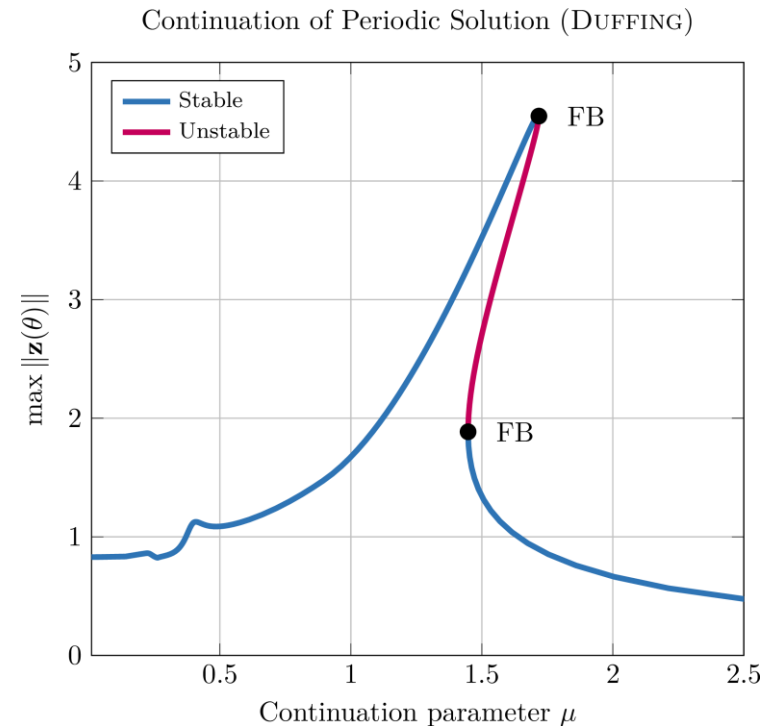
> ## All features can be used as required
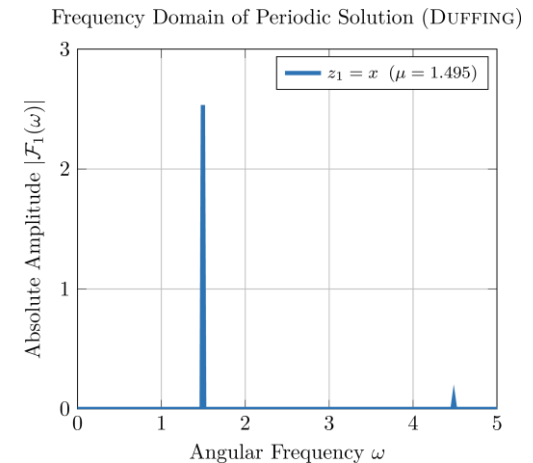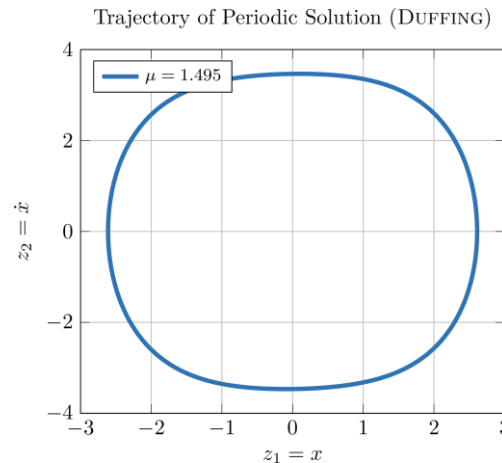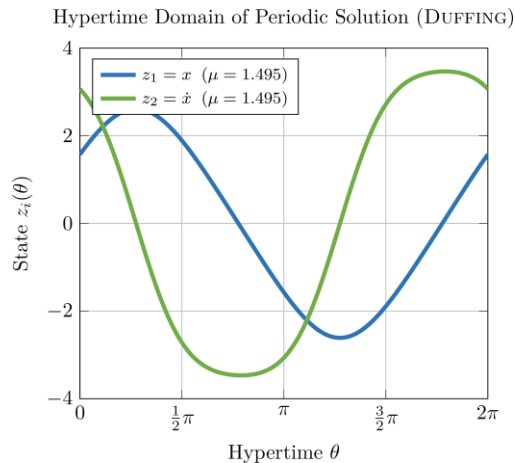
- **Postprocessing methods**
  - `contplot`
    - ➢ Creates continuation / bifurcation diagrams (plots solution branches with respect to μ)
  - `solplot`
    - ➢ Plots individual solutions in different *solution spaces*
      (Available *solution spaces*: time, hypertime, trajectory and frequency domain)

  - `solget`
    - ➢ Returns solution data in different solution spaces

ENGINEERING
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Help Features

- **Gatekeeper** *cannot be bypassed*

  - ➢ Checks the input (the defined options) from the user

  - ➢ Reports errors in case of illogical or invalid input ➡

  You shall not pass!

- **Help**

  - – `costarhelp` function
    - ➢ Quick help in the command window
    - ➢ Overview of the available options with a short description
    - ➢ Type `costarhelp.costar` in the command window to start

  - – Examples
    - ➢ Short MATLAB scripts
    - ➢ Sample code showing usage of a certain **CoSTAR** module

  - – Tutorials
    - ➢ MATLAB live scripts
    - ➢ There is one tutorial for each example (identical code)
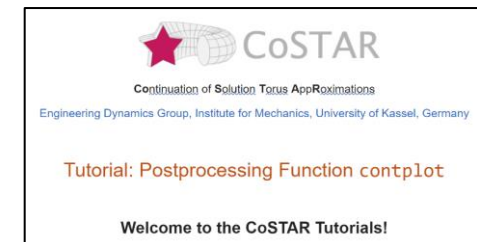   - ➢ Comprehensive explanations of a certain **CoSTAR** module

1. **options**
   - Structure array
   - Contains user-defined options for computation

2. **Gatekeeper**
   - Checks options

3. **DynamicalSystem**
   - Stores all options in object `DYN`
   - Can be used to restart the computation

4. **Selection of subclasses**
   - ApproxMethod
     - Applies approximation method
   - Solution
     - Stores solution data in object `S`
   - Stability
     - Used for stability computation

5. **initial_solution**
   - Computes the first *(initial)* solution

In case of no continuation:

6. Return `S` and `DYN`
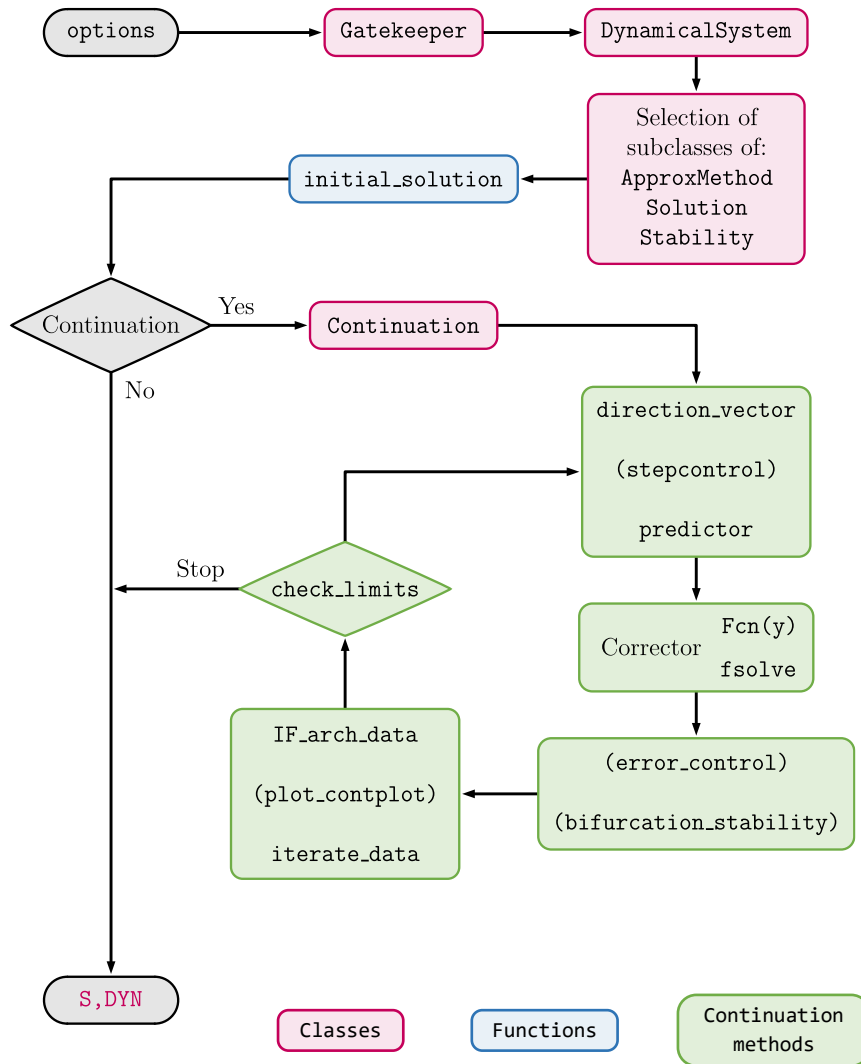
6. **Continuation loop**

6.1. `direction_vector`

6.2. `(stepcontrol)`   (can be skipped)

6.3. `predictor`
   ➢ Computes direction vector, new step width and predictor point

6.4. Corrector
   ➢ `fsolve` solving `Fcn(y) = 0`

6.5. `(error_control)`   (can be skipped)

6.6. `(bifurcation_stability)`   (can be skipped)
   ➢ Performs error control and computes stability as well as bifurcation point

6.7. `IF_arch_data`

6.8. `(plot_contplot)`   (can be skipped)

6.9. `iterate_data`
   ➢ Stores data, updates live plot and performs iterations for next loop

6.10. `check_limits`
   ➢ Checks exit conditions

When exit condition is met:

7. Return S and DYN

Code Structure

📁 **Classes** ➤ All classes and associated methods

📁 **Functions** ➤ Functions not belonging to any class

📁 **RHS** ➤ Functions defining right-hand side of $\dot{\mathbf{z}} = \mathbf{f}(t, \mathbf{z}, \mu)$ or $\dot{\mathbf{z}} = \mathbf{f}(\mathbf{z}, \mu)$

📁 **test** ➤ Scripts to test the code

📁 **Tutorials** ➤ Tutorial & example scripts

📁 **Version_Log** ➤ Version log files documenting code development

📄 **costar** ➤ Main **CoSTAR** function (to be called by the user)

ENGINEERING **DYNAMICS**

Code Structure

📁 **Classes**

> All classes and associated methods

    📁 **@Continuation**   ➢ Class and methods to perform the continuation

    📁 **@costarhelp**   ➢ Class and methods for the `costarhelp` feature

    📁 **@DynamicalSystem**   ➢ Class for storing the `options` structure

    📁 **@Gatekeeper**   ➢ Class and methods for the `Gatekeeper` feature

    📁 **ApproxMethod_SC**   ➢ Classes and methods to construct the residuum function

        📁 **@AM_EQ**

        📁 **@AM_PS_FDM**

        📁 **@AM_PS_FGM**

        📁 **@AM_PS_SHM**   ➢ Subclasses and methods

        📁 **@AM_QPS_FDM**

        📁 **@AM_QPS_FGM**

        📁 **@AM_QPS_SHM**

        📁 **@ApproxMethod**   ➢ Superclass

    📁 **Solution_SC**

    📁 **Stability_SC**

Code Structure

📁 **Classes**

> All classes and associated methods

- 📁 @Continuation — ➤ Class and methods to perform the continuation
- 📁 @costarhelp — ➤ Class and methods for the **costarhelp** feature
- 📁 @DynamicalSystem — ➤ Class for storing the **options** structure
- 📁 @Gatekeeper — ➤ Class and methods for the **Gatekeeper** feature
- 📁 ApproxMethod_SC — ➤ Classes and methods to construct the residuum function
- 📁 Solution_SC — ➤ Classes and methods to save computed data
  - 📁 @SOL_EQ
  - 📁 ...   ➤ Subclasses and methods (analogous to ApproxMethod_SC)
  - 📁 @Solution   ➤ Superclass
- 📁 Stability_SC — ➤ Classes and methods to compute stability
  - 📁 @ST_EQ   ➤ Subclass and methods for equililibrium solutions
  - 📁 @ST_PS_SHM   ➤ Subclass and methods for periodic shooting method
  - 📁 @ST_QPS_SHM   ➤ Subclass and methods for quasi-periodic shooting method
  - 📁 @Stability   ➤ Superclass and methods

costar

```matlab
function [S,DYN] = costar(options)

    %% Gatekeeper
    GC = Gatekeeper();
    options = GC.m_gatekeeper(options);
    clear GC;

    %% Dynamical System class
    DYN = DynamicalSystem(options);

    %% Approximation Method class
    AM = ApproxMethod.s_method_selection(DYN);

    %% Solution class
    S = Solution.s_solution_selection(DYN,AM);

    %% Stability class
    ST = Stability.s_stability_selection(DYN,AM);

    %% Calculate initial solution
    [S,AM,DYN] = initial_solution(DYN,S,AM,ST);

    %% Continuation
    if strcmpi(DYN.cont,'on')
        CON = Continuation(options.opt_cont);
        S = CON.m_continuation(DYN,S,AM,ST);
    end
```

➢ **Input:** options structure
Output: Solution object S, DynamicalSystem object DYN

➢ **Gatekeeper** checks all input options

➢ **Save** all options in DynamicalSystem object DYN

➢ **Create** ApproxMethod object AM
(methods construct the residuum function)

➢ **Create** Solution object S
(stores all solution data)

➢ **Create** Stability object ST
(methods compute the stability of a solution)

➢ **Compute** the initial (first) solution

➢ **Create** Continuation object CON
➢ **Do** the continuation

1. Define important parameters and functions
(not necessarily needed, but it helps to keep the overview)

```matlab
%% 1. Define important parameters and functions (not necessarily needed, but it helps to keep the overview)
D = 0.05;      kappa = 0.3;     g = 1;                  % Parameters needed for the Duffing differential equation
mu_limit = [0.01, 2.5];                                 % Limits of the continuation
eta0 = mu_limit(1);                                     % Value of continuation parameter at start of continuation
param = {kappa, D, eta0, g};                            % Parameter array
active_parameter = 3;                                   % Location of continuation parameter within the array
IC = [1; 0];                                            % Initial condition (point in state space) for fsolve

% Functions
non_auto_freq = @(mu) mu;                               % Non-autonomous excitation frequency
Fcn = @(t,z,param) duffing_ap(t,z,param);               % Right-hand side of dz/dtau = f(tau,z,kappa,D,eta,g)
```

2. Define the options structure
(it comprises all information that CoSTAR needs)

```matlab
%% 2. Define the options structure (it comprises all information that CoSTAR needs)
options.system = costaropts('order',1,'dim',2,'rhs',Fcn,'param',param,'info','continuation of Duffing equation');  % Properties of the system
options.opt_sol = costaropts('sol_type','periodic','approx_method','shooting','cont','on','stability','on', ...    % Properties of the solution
                             'non_auto_freq',non_auto_freq,'act_param',active_parameter);                          % Properties of the solution
options.opt_init = costaropts('ic',IC);                                                                            % Property for initial solution
options.opt_approx_method = costaropts('solver','ode45');                                                          % Properties of approximation method
options.opt_cont = costaropts('mu_limit',mu_limit);                                                               % Properties for continuation
```

3. Call CoSTAR (and do the continuation)

```matlab
%% 3. Call CoSTAR and do the continuation
[S,DYN] = costar(options);                              % CoSTAR is called by costar(options)
```

4. Individual postprocessing

```matlab
%% 4. Individual postprocessing
% ...
```

- **If you are new to CoSTAR or certain modules**
  - Tutorials
    - ➢ Comprehensive explanations of a certain **CoSTAR** module
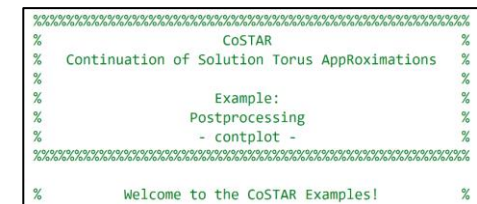   - ➢ Currently available:
      - ✓ Equilibrium solutions (Tutorial_EQ)
      - ✓ Periodic and quasi-periodic solutions approximated by FDM, FGM and SHM (Tutorial_PS_FDM, Tutorial_PS_FGM, …, Tutorial_QPS_FDM, …)
      - ✓ Postprocessing methods contplot, solplot and solget (Tutorial_Postprocessing_contplot, …)

Start with one of these if you have not used CoSTAR yet

- **If you already used CoSTAR**
  - Examples
    - ➢ Sample code showing usage of a certain **CoSTAR** module
    - ➢ Good rescue point to restart working with **CoSTAR**
    - ➢ There is one example for each tutorial (examples are labelled Example_[…])
  - costarhelp feature
    - ➢ Overview of the available options with a short description
    - ➢ Quick help in the command window while using **CoSTAR**
    - ➢ Type **costarhelp.costar** in the command window to start
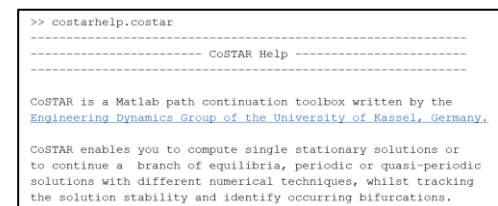
> **Note:** The following list may be incomplete and only lists <u>ideas</u> for future improvements to the toolbox. There is no guarantee for actual implementation.

- **Approximation methods**
  - Multiple Shooting Method for quasi-periodic solutions

- **Features**
  - **Stability computation**
    - **PS:** Directly from solution data when using FDM or FGM without JACOBIAN of SHM
    - **QPS:** For Finite Difference Method & FOURIER-GALERKIN Method (completely missing so far)
  - **Error control**
    - Finite Difference Method (PS & QPS)
    - Handle different exit flags from `fsolve` when computing new solution with updated discretization
  - **Step control**
    - Algorithm(s) based on convergence of solver (when self-written solver is available)
  - **Postprocessing**
    - **FGM & SHM** (QPS hypertime plots): [1x2] array for `options` structure field 'resolution'
  - **Tutorials & Examples**
    - Update default-script tutorials to live scripts
    - Tutorials and examples for continuation options, step control and stability computation

ENGINEERING DYNAMICS

> **Note:** The following list may be incomplete and only lists <u>ideas</u> for future improvements to the toolbox. There is no guarantee for actual implementation.

- **Initial value** (for the solver to compute the initial solution)
  - Standardise the parameters, which create an initial value, for all approximation methods
  - Use of a solution of a different approximation method as initial value
  - Homotopy methods

- **Continuation**
  - Predictor: Polynomials of order > 3
  - Additionally compute the solution at specified (desired) μ-values

- **Computational effort**
  - Make parallel computing available to enhance performance

- **Solver**
  - Self-written solver to remove the need of MATLAB'S Optimization Toolbox

- **Dynamic System**
  - Computation of non-hyperbolic manifolds (solutions of Hamiltonian systems)

Feedback and Download

- **Download**

  - CoSTAR is available for free as GitHub repository

  - Research, teaching and personal use only
    (commercial use is <u>not</u> allowed)

- **Report of bugs**

  - Please create a GitHub issue if you experience a new bug

  - If a GitHub issue already exists for your bug, no action is required

- **Suggestions for improvement, wishes and ideas for future releases**

  - Please contact us via engineering-dynamics@uni-kassel.de

Website CoSTAR

Website Engineering Dynamics Group