MAE 4300 Heat Transfer
Design Project

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Table of Contents

Introduction

The purpose of this project is to find six fin configurations that maximize the heat transfer rate over a CPU under the same operating conditions. Once six independent designs were developed, they were then analyzed to find the best design that removes the largest amount of heat from the CPU. After testing the different designs, a recommended configuration was obtained. The performance curve in terms of the total rate of heat transfer from the heat sink versus the angular speed of the micro-fan justifies the best optimal design.

This design consists of 0.2 kg of aluminum alloy that is to be extruded into square fins of length L and integrated on a heat sink (53 mm x 57 mm) for CPU cooling. For optimizing the thermal performance of the fin array, the volume was constrained to a constant. Furthermore, smaller fins in greater quantity were compared with larger fins of lesser quantity for removing the largest possible amount of heat from the CPU. In addition to the fin geometry, aligned and staggered fin configurations are also within the design considerations. To satisfy geometrical constraints, length of the fin is limited to 40 mm ($L \leq 40$ mm). To ensure smooth air flow through the fin arrays, the base area of each fin cannot exceed 9 mm^2. The air flow is driven by a micro-fan, which has a radius of 25 mm and spins at a constant angular speed. The CPU is mounted on a motherboard in a computer case, where local temperature is maintained at 23°C. The base temperature is fixed at 85°. This report will explain the design process and recommendations of different fin configurations that are the most efficient at removing large amounts of heat from the CPU. The final recommended design is detailed and shown explicitly for its effectiveness and robustness for having the greatest heat transfer rate.

Nomenclature

C_1 = constant
C_2 = constant
ω = rotations per minute
c_p = specific heat
t = time
μ = dynamic viscosity
Nu = ratio of convective to conductive heat transfer
α = thermal diffusivity
A = area
ṁ = mass flow rate
V = velocity
Nu_L = nusselt number
ρ = density
L = length
h = height
k = thermal conductivity
W = width
T_b = base temperature
T_∞ = ambient temperature
$\overline{h}$ = heat transfer coefficient
N = fin total
dT = temperature change
S_t = ratio of heat transferred into a fluid to the thermal capacity
q = heat transfer
R_e = Reynolds number
r = number of fins in a single row

Design Process

In order to figure out the best design that would dissipate the most heat from the CPU for square fins, a series of equations had to be used and manipulated to find total heat transfer q. These equations were solved using MATLAB and arithmetic to test different fin configurations. These values were obtained from the given and from interpolating the data for ambient temperature of 296 K in Table A.4 Thermophysical Properties of Gasses at Atmospheric Pressure.

After all the design constants were found and tabulated, the independent variables could be determined. To find the best fin array design, three variables could be changed or manipulated. These three variables are the fin width W, the number of fins N, and the length of the fin L. One given constraint that had to be factored into the design process was that all fins had to have the same operating conditions. This meant that the volume for the design had to be held constant for all configurations while being equal to or less than 0.2 kg of aluminum alloy regardless of the change in the number of fins, width, or the length of the fins. The volume was calculated by using the density of aluminum alloy. A volume of 73800 mm^3 was found and set constant for all of the designs. Another constraint that had to be applied was the fact that the area of the base/top of the fin could not exceed 9 mm^2. This meant the width of the fin could not be greater than 3 mm since it is a square. Equation 1 was used to vary N, W, and L to meet the constraints. N is equal to the square of the total row of fins.

$$N(W^2L) = 0.0000738 \tag{1}$$

Table 1 Design Constraints and Parameters

C_1	1.616E-6	P_R	.70804
C_2	2.4E-5	c_p	1.00692E3 kJ/kg•K
d	0.05	μ	182.6E-7 N•s/m²
ρ	1.1801 kg/m³	Nu	15.534E-6
k	25.98E-3 W/m•K	α	21.97199999999999E-6 m²/s
C	0.102	m	0.675
P_{RS}	0.70539	v	m²/s

After determining a N, W, and L value that meets the design constraints listed above, further calculations can be made to find the total heat transfer of the configuration. The mass flow rate is found using equation 2.

$$\dot{m} = C_1(w) + C_2 \tag{2}$$

This value was found using the constants in table 1. With mdot, the velocity could be calculated using equation 3.

$$V = \frac{\dot{m}}{\rho A} \tag{3}$$

With mdot and velocity, the Reynold's number can be calculated. Reynold's number was calculated using equation 4.

$$R_e = \frac{VW}{Nu} \tag{4}$$

Based on the value obtained in equation 4, a C and m could be picked from table 2 and inserted into the hbar equation to find the heat transfer coefficient that is very important in finding the total heat transfer. The Prandtl numbers, from table 1, are also used in finding hbar along with the thermal conductivity k. The hbar equation is given below as equation 5.

$$h_{bar} = \frac{k}{w}(C)(R_e^{\ m})(P_r^{\ 0.36})(\frac{P_r}{P_{RS}})^{1/4} \tag{5}$$

Table 2 C & m Values for

Configuration	$Re_{D,max}$	C	m
Aligned	$10–10^2$	0.80	0.40
Staggered	$10–10^2$	0.90	0.40
Aligned	$10^2–10^3$	Approximate as a single (isolated) cylinder	
Staggered	$10^2–10^3$		
Aligned $(S_T/S_L > 0.7)$[a]	$10^3–2 \times 10^5$	0.27	0.63
Staggered $(S_T/S_L < 2)$	$10^3–2 \times 10^5$	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered $(S_T/S_L > 2)$	$10^3–2 \times 10^5$	0.40	0.60
Aligned	$2 \times 10^5–2 \times 10^6$	0.021	0.84
Staggered	$2 \times 10^5–2 \times 10^6$	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

Table 3 Reynolds Number for Circular Cylinder

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

Table 4 Reynold's Number for Non-Circular Cylinders

Geometry	Re_D	C	m
Square			
$V \rightarrow$ D	5×10^3–10^5	0.246	0.588
$V \rightarrow$ D	5×10^3–10^5	0.102	0.675
Hexagon			
$V \rightarrow$ D	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
$V \rightarrow$ D	5×10^3–10^5	0.153	0.638
Vertical plate			
$V \rightarrow$ D	4×10^3–1.5×10^4	0.228	0.731

The equation for finding the heat transfer coefficient also contains the spacing between the center of each fin S_t. To find St, the length of the left side of the chip had to be used. Equation 6 represents how S_t is calculated based on the width of the fins.

$$S_T = \frac{(0.053 - r \bullet (W))}{(r-1)} \tag{6}$$

With the previous values obtained from equations 1-6, the change of temperature of Ts - To can be calculated with equation 7 seen below.

$$T_S - T_O = e^{\left(\frac{-\Delta T (h_{bar})(4)(W)(N)}{\rho (S_t)(V)(r-1)(C_p)}\right)} \tag{7}$$

With Ts - To, the log mean temperature difference can be calculated for the fin design. The log mean difference deltaTm is calculated using Ts - To subtracted from change in surface and ambient temperature divided by the log of surface and ambient temperature divided by Ts - To.

$$\Delta T_m = \frac{((85-23)-(T_s - T_o))}{\frac{log(85-23)}{(T_s - T_o)}} \tag{8}$$

Finally, with all the variables calculated from the above equations, the rate of heat transfer q can be calculated using equation 9 for all the different fin designs.

$$q = N(h_{bar})(4)(W)(L)(\Delta T_m) \tag{9}$$

The testing of different varying input variables resulted in each design obtaining a relatively effective heat transfer value. Equation 9 was used explicitly as the reference for comparing and adopting the best overall design in this report.

Design 1: Asa Hamilton

The first design is a rectangular 35 by 33 array giving a total of 1155 fins. The width was set at 1.5 mm which is smaller than the given constraint of 3 mm, and the volume is set at the constant value of 73800 mm^3. To ensure the design will have a length (L) no greater than 40 mm, the number of fins (N) and width (W) were plugged into equation 1 where L was found to be 28.39 mm. Once the design values for N, W, and L had met all of the requirements and constraints a value for the value for the total rate of heat transfer (q) could be calculated. To ensure accuracy and expedite the process of finding q, a MATLAB function was written using equation 8, and the q value of the design was found to be 348.0118 W.

Design 2: Alex Trout

This design took a different approach than the others. With the three dynamic variables regarding the fins, the length, the width, and the array. When considering the parameters, the goal for this configuration was to try and optimize the surface area of the fins. To do this, the configuration needed maximum length but needed a smaller width to allow for more fins to create in total more surface area. With these two established, then equation 1 can be used to find the total number of fins for the design. The number of fins needed would be 1845. Factors of 1845 are 41 and 45 which work correctly for the design. This is because the row of fins, when multiplied by the width of each fin, will not exceed the 53mm width or the 57mm length of the chip. Since the following values are all compatible with each other the total net of heat transfer can be solved by using MATLAB. This yields a high total heat transfer of 511.7329 W.

Design 3: Aleksandr Buechter

The goal of this design was to maximize the total heat transfer rate while having the highest total number of arrays. This meant that this configuration would have optimal fin surface area for squared fins. The width of 1 mm was selected because it did not affect the output of the heat transfer rate as drastically as the length did. The volume of 73800 mm^3 was held constant as with all the other designs while the width of 1 mm and the length 32.8 mm met the preliminary constraints for the specifications and parameters outlined in this report. Equations 1-9 were used for the thermal calculations that ultimately solved for the net heat transfer. MATLAB was employed to make the configurations and necessary calculations of this design. The heat transfer of 429.415 W was the result of this design based on its configuration. This design used a higher number of fins to achieve a greater heat transfer over the specific surface area that would be used under a CPU.

Design 4: Will Forsythe

The fourth design was developed by using a square fin array of 23 by 23 fins. This configuration started off with a set number of fins and a mid-range width of 2 mm. With the number of fins and width defined, the length was able to be calculated. With the help of equation 1, the length was calculated to be 34.877 mm. In order to get a reliable and usable design, the volume was held constant at 73800 mm^3 to ensure none of the fin dimensions exceeded this value. Some other constraints that needed to be followed for the design were that the top area of a single fin could not be greater than 9 mm^2 and the length must be less than or equal to 40 mm. After all of the values were verified for the design, MATLAB was then used to calculate the total rate of heat transfer for the square fin configuration. For the square fin configuration, the total rate of heat transfer came out to be 350.3389 W.

Design 5: Nick Holbert

The fifth design was created using a set number of fins and the maximum allowed width. This design used a 17 by 17 square-fin array, for a total of 289 fins. The width for this design was set at the maximum allowed, which was 3 mm. This made sure the maximum area requirement of 9 mm^2 was met. The length was calculated from those values in order to maintain the constant volume of 73800 mm^3. It was calculated to be 23.87 mm, which is under the 40 mm maximum. The values are plugged into the square-fin array MATLAB code, as shown in the appendix, in order to calculate the rate of heat transfer. The heat transfer rate was found to be 238.7634 Watts for this array.

Design 6: Pavel Buechter

The last design that was tested for square fins was an array of 15 by 15. This array was tested as it is a logical square design that is seen on a lot of computer ships. This design equated to a total of 225 fins. To maximize heat transfer, different widths were tested. One challenge with testing the different widths was that the total volume of the fins could not exceed 73800 mm^3. Also the area of the top of the fin could not be greater than 9 mm^2. To start, the fin width and length were set at maximum values of 3 mm and 40 mm respectively. Through running the code in MATLAB seen in the appendix, this configuration wouldn't work as it exceeds the volume limit. So from there the length or width had to be decreased to meet the volume requirement. From there the length was held constant at 40 mm and the width was tested with values from 1 mm to 3 mm in increments of 1 mm. It was found that the higher the width of the fins up to the maximum width gave a lower rate of heat transfer as compared to the other designs. This was true even with increasing the length of the fins. By fixing the number of arrays and fixing the width of each fin to 3 mm, the exact length of the fins could be calculated. Using equation 1, the calculated length was 36.4 mm. After finding all of the important metrics for the 15 by 15 array design, MATLAB was used to calculate the total rate of heat transfer for the design. The total heat transfer q was calculated to be 263.3598 W for the 15 by 15 array.

Design Analysis

Table 5 displays the six different designs for fin arrays and their respective parameters that were previously mentioned above. The heat transfer rate q is also tabulated for each design.

Table 5 Design Data for Rectangular Arrays

Design	Number of Fins (N)	Width (W) (mm)	Length (L) (mm)	Heat Transfer Rate (q) (W)
1	33x35=1155	1.5	28.39	348.0118
2	41x45=1845	1	40	511.7329
3	45x50=2250	1	32.8	429.415
4	23x23=529	2	34.877	350.3389
5	17x17=289	3	28.37	238.7634
6	15x15=225	3	36.4	263.3598

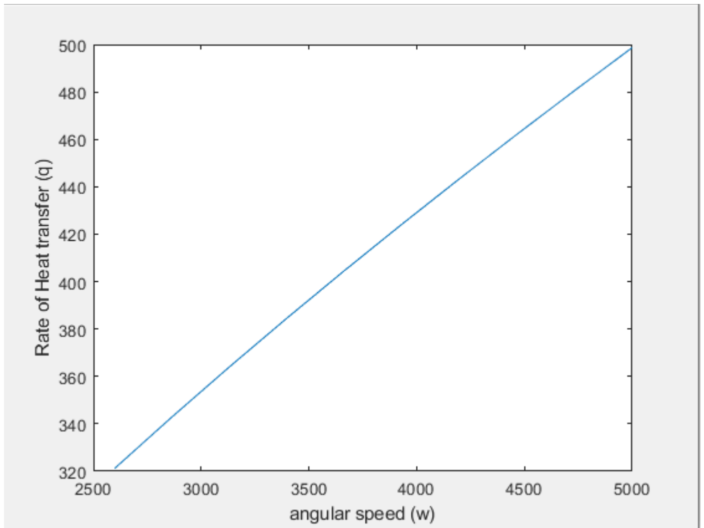

Figure 1 Performance Curve for Design Two

Six completely different designs were created in order to compare what makes the rate of heat transfer rise or fall based on the array. Each design needed to maintain three parameters. First, the area of the fin could not exceed 9 mm^2. Second, the length of the fins could not exceed 40 mm. Lastly, the volume needs to remain constant for each array while also being within the other two parameters. Once all parameters are met, the values can be put into a MATLAB code to solve for q, using equation 10. Once all six designs were created, their q values were calculated. From table 5, it can be seen that design two gave the highest q value while design five gave the lowest value. Design two maximized the length of the fins, while setting the width to 1 mm. The number of fins was then calculated in order to maintain constant volume, while also leaving enough room for gaps. Design five was designed with a maximum width of 3 mm and a set array of 17 by 17 for a total of 289 fins. The length was calculated to meet the constant volume and was under 40 mm. One of the biggest reasons design five has a very low rate of heat transfer is because it has the shortest length along with the maximum allowed width. From table 5, the arrays with longer and thinner fins paired together have the highest rate of heat transfer. That is why the rate of heat transfer is highest with design two. It has the longest length possible paired with the smallest width of all six designs. With a maximum length and small base area, there can be more fins. This leads to an increase in surface area, which correlates to a larger convective surface. That is why the rate of heat transfer is higher for design two and lower for design five.

Figure 1 shows the performance curve of the rate of heat transfer versus various angular speeds. With design two being the best design, it was compared to the same array with circular fins using data from table 3. The area of the fin was kept constant so the length and number of fins could remain the same for an accurate comparison. The rate of heat transfer for a circular fin array of design two comes out to be 532.6 W. This value is higher than that of the rectangular fin array. The difference is surprising because typically in CPU chips circular fins tend to be more efficient but not have a higher rate of heat transfer. To further optimize the best design, the orientation is then rotated at a ninety degree angle to have air flow hitting the corner of the square. The only values that change are the C and M constant values found in table 4, which are

0.246 and 0.588 respectively. This configuration yields 530.884 W. Lastly, a staggered fin array is tested with design 2. In order to get a correct Reynold's number for equation 6, a number between 500-5000 is needed. When comparing it to a staggered arrangement there is an improvement of 4.7% with 354.338 W. We then test it as a staggered circular array and get an improvement of just 4.4% with 532.5936 W. Finally, when we stagger the square array that has a different orientation the heat transfer rate improves by 4.73% with a value of 535.815 W. With all of this data, the most optimal design is a configuration of a staggered square array orientated by ninety degrees.

Conclusion

The purpose of this project was to develop six fin array designs that maximize heat transfer under given specifications and constraints. In order to meet these constraint requirements, conditional equations were created to ensure the constant volume was 73800 mm^3, the maximum width was 3 mm, and the maximum length was 40 mm for each design. Each of the six design parameters were then plugged into the equations for finding the total heat transfer. The value given by q determines the total rate of heat transfer from the CPU. From there, it was determined that design two, having a q value of 511.7329 W, was the most effective of the six aligned square designs. This is because design two used the maximum length and a small base area which allowed for more fins to dissipate the heat away from the CPU. The designs were then compared with circular arrays and square arrays that were oriented by ninety degrees using the same parameters to better optimize the design. The circular and oriented square fins resulted in a slightly higher q value in comparison to the aligned square arrays. The oriented square fins were the most optimal design as it produced the greatest heat transfer rate of 535.815 W. This small change in q is due to the fact that each of these designs are contained within a small area. A fin array of larger area would show a larger change in the effectiveness of q. The results of this project show that to have the best fin array design, the surface area needs to be maximized by having a smaller fin area. Furthermore, when changing the orientation of the fin shapes and the layout of the array ultimately generates a higher rate of heat transfer. Thus concluding design two with square fins, an orientation of ninety degrees, and with a staggered configuration, is the best design.

Appendix

```
%For Rectangular/Square fin array
%Constants:
C1=1.616E-6; C2=2.4E-5; w=5000; diameter=.05; density=1.1801;
Pr=.70804; Cp=1.00692E3; mu=182.6E-7;nu=15.534E-6; k=25.98E-3;
alpha=21.97199999999999E-6; C=0.102;m=0.675; Prs=0.70539;

%Fin array of 15* x 15* with Diameter of 0.003* m
```

```
%Asterisk* represents values that can change
%Fin width/Diameter*
W=0.001;
%W=width of the fins
%row size*
row1=41;
row2=45;
N=row1*row2;
%length of fin*
L=0.04; %has to be less than 0.04
if N*W^2*L⩽0.0000738 && W^2⩽0.000009
     disp('This fin configuration works!')
A=.025^2*pi
% area = m^2
mdot=C1*w+C2;
%  mdot =
V=mdot/(density*A);
%
Re=V*W/nu;
h_bar=k/W*C*Re^m*Pr^0.36*(Pr/Prs)^(1/4)
dT=62;
%Atotal=W*L*N
St=((0.053-row1*W)/(row1-1))+W
TsTo=exp(-(h_bar*4*W*N)/(density*St*V*(row1-1)*Cp))*62
deltaTm=((85-23)-TsTo)/(log((85-23)/TsTo))
q=N*h_bar*4*W*L*deltaTm
else
     disp('This fin configuration does not work =(')
end


%For Circular fin array
%Constants:
C1=1.616E-6; C2=2.4E-5; omega=5000; density=1.1801;
Pr=.70804; Cp=1.00692E3; mu=182.6E-7;nu=15.534E-6; k=25.98E-3;
```

```
alpha=21.97199999999999E-6 ;m=0.466; Prs=0.7;

%Asterisk* represents values that can change
%Fin width/Diameter*

%row size*
row1=41;
row2=45;
N=row1*row2;
%length of fin*
L=.04; %has to be less than 0.04
w=0.001; %width of square configuration
Area=w^2;
D=sqrt((Area*4)/pi) % calculation for new diameter (D)
if N*pi*D^2/4*L<0.0000738 && pi*D^2/4<0.000009
     disp('This fin configuration works!')
A=.025^2*pi
% area = m^2
mdot=C1*omega+C2;
%  mdot = .002448
V=mdot/(density*A);
St=((0.053-row1*D)/(row1-1))+D;
Sl=((0.053-row2*D)/(row2-1))+D;
Sd=sqrt((St/2)^2+Sl^2);
if 2*Sd-St > D
     Vmax=(St*V)/(St-D);
elseif 2*Sd-St < D
     Vmax=(St*V)/(2*(Sd-D));
end
%
Re=Vmax*D/nu
C=0.35*(St/Sl).^(1/5)
h_bar=k/D*C*Re^m*Pr^0.36*(Pr/Prs)^(1/4);
dT=62;
Atotal=(D^2*pi/4)*L*N
```

```
St=(0.053-row1*D)/(row1-1)+D;
TsTo=exp(-(h_bar*pi*D*N)/(density*St*V*(row1-1)*Cp))*62;
deltaTm=((85-23)-TsTo)/(log((85-23)/TsTo))
q=N*h_bar*pi*D*L*deltaTm
else
     disp('This fin configuration does not work =(')
end
```

Staggered

```
w=linspace(2600,5000,10)
C1=1.616E-6; C2=2.4E-5; density=1.1801;
Pr=.70804; Cp=1.00692E3; mu=182.6E-7;nu=15.534E-6; k=25.98E-3;
alpha=21.97199999999999E-6; C=0.102;m=0.675; Prs=0.70539;

%Fin array of 15* x 15* with Diameter of 0.003* m
%Asterisk* represents values that can change
%Fin width/Diameter*
W=0.001;
%W=width of the fins
%row size*
row1=41;
row2=45;
N=row1*row2;
%length of fin*
L=.04; %has to be less than 0.04
if N*W^2*L⩽0.0000738 && W^2⩽0.000009
     disp('This fin configuration works!')
A=.025^2*pi
% area = m^2
mdot=C1*w+C2;
%  mdot =
V=mdot/(density*A);
St=((0.053-row1*W)/(row1-1))+W;
Sl=((0.053-row2*W)/(row2-1))+W;
```

```
Sd=sqrt((St/2)^2+Sl^2);
if 2*Sd-St > W
     Vmax=(St*V)/(St-W);
elseif 2*Sd-St < W
     Vmax=(St*V)/(2*(Sd-W));
end
%
Re=Vmax*W/nu
h_bar=k/W*C*Re.^m*Pr.^0.36*(Pr/Prs).^(1/4)
dT=62;
%Atotal=W*L*N
TsTo=exp(-(h_bar*4*W*N)/(density*St*V*(row1-1)*Cp))*62
deltaTm=((85-23)-TsTo)/(log((85-23)/TsTo))
q=N*h_bar*4*W*L*deltaTm
else
     disp('This fin configuration does not work =(')
end
plot(w,q)
```

Supervised & mentored 2 Flt/CCs, oversaw/enhanced trng of 43 GMC cdts--accomplished 28 AFROTC objs
- 3.97 MAE GPA! Top in Det 440, created semester FTP trng plan, 20+ extra trng hrs--set stu ldrshp example
- Exceptional ldr, top 1/3 FT rank, RA of 67 residents, coord move in of 3000+ stu--push for SQ/CC or OG/CC

Evaluated & trnd 28+ GMC, directed 2 Flt/CCs, Honor Flight--cultured positive trng environment for GMC/POC

- Increases intensity/morale, great role model among peers/GMC, 60+ hrs of effective trng–exceeds expectations