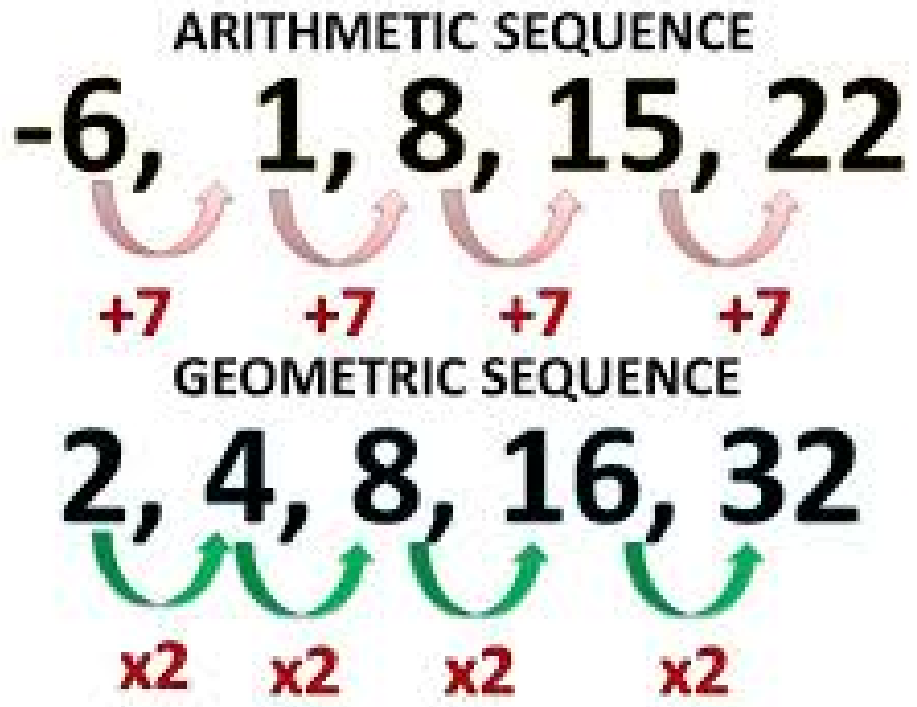
Arithmetic and Geometric Progression

Sequences & Series

- A sequence is a list of terms separated by commas
  - 2, 4, 6, 8, 10, 12
  - It has the notation n terms for the location in the list n=1, n=2, n=3 ect.
  - It has the notation a terms for the value in that list a_1=2, a_2=4, a_3=6
- A series is a sum, you are adding up all the terms in the list
  - 2+4+6+8+10+12

Arithmetic vs Geometric

- Arithmetic sequences add the same thing each time to get to the next term.
- Geometric sequences multiply by the same thing each time to get to the next term.



Are These Sequences Arithmetic or Geometric

- 1, 5, 9, 13, 17…
- 10, 40, 160, 640…
- 1024, 962, 900, 838…
- x, 8x, 64x, 512x…
- 2x, 2x-3, 2x-6, 2x-9…
- $a_n = 2+4(n-1)$
- $a_n = 2(4)^{n-1}$

Finding a Value in an Arithmetic Sequence

- We want to find the 6^{th} value in this sequence
- 8, 12, 16, 20, 24, ?
- Common difference between values (d) = 12-8 = 4
- By the 6^{th} value how many times will we have added 4? (5 times)
- We can take our first value and then add on all the times we would have added 4
- 8 + 4 + 4 + 4 + 4 + 4 = 8+4(5) = 28
- We can reverse engineer our equation for the value in an arithmetic sequence as $a_n = a_1+d(n-1)$

Finding a Value in a Geometric Sequence

- We want to find the 9^{th} value in this sequence
- 12, 36, 108, 324, …, 26244, ?
- Common ratio between values (r) = 36/12 = 3
- By the 9^{th} value how many times will we have multiplied by 3? (8 times)
- We can take our first value and then multiply by the common ratio for the number of times.
- $8 * 3 * 3 * 3 * 3 * 3 * 3 * 3 * 3 = 8(3)^8$
- We can reverse engineer our equation for the value in an arithmetic sequence as $a_n = a_1(r)^{n-1}$

Your Turn

Can you find the nth term a,b,c,d,e,f of each sequence:

n values:

a) n = 6
b) n = 12
c) n = 25
d) n = 100
e) n = 2.5
f) n = -10

Sequences:

1) 3, 6, 9, 12, 15
2) 6, 18, 54, 162
3) 10, 8, 6.4, 5.12, 4.096
4) 5.6, 11.7, 17.8, 23.9, 30
5) 8, -10, 12.5, -15.625, 19.53125

Arithmetic:

$a_n = a_1+d(n-1)$

Geometric:

$a_n = a_1(r)^{n-1}$

Bonus Challenge:

If we have a geometric sequence with a common ratio of 1.01, an initial value of 8.2 and $a_n = 31.13948635$ what is n?

Answers

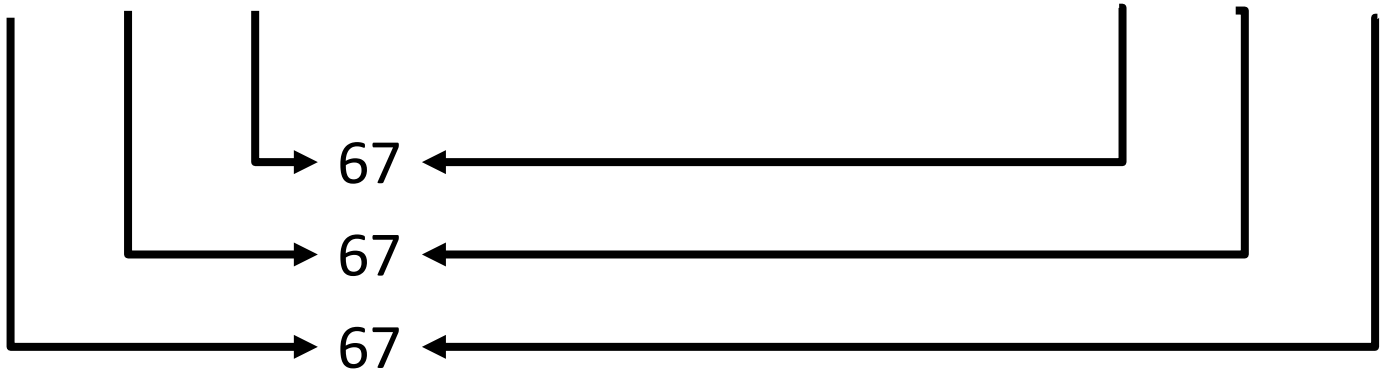
	a) n = 6	b) n = 12	c) n = 25	d) n = 100	e) n = 2.5	f) n = -10
1	18	36	75	300	7.5	-30
2	1458	1062882	1.694×10^{12}	1.030×10^{48}	31.176	3.387×10^{-05}
3	3.2768	0.859	0.047	2.546×10^{-09}	7.155	116.415
4	36.1	72.7	152	609.5	14.75	-55.4
5	-24.414	-93.132	1694.066	-3.141×10^{10}	UNDEFINED	-0.687

Bonus Challenge:

135.1

Finding the Sum of an Arithmetic Series

- 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 + 56 + 61



- When finding the value of a series we could add up each individual value but for long series that could take ages
- We know that pairs of numbers around the median add up to a number but how pairs are there?
- n/2

Finding the Sum of an Arithmetic Series

- 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 + 56 + 61

- How do we find the difference between pairs?
- We can work out the second number by adding n-1(d) to the first number so in this example 6 + 12-1(5) = 61
- To work out the sum of that pair we can do 2x the initial number plus the gap between the two numbers for example (2*6)+12-1(5) = (2*6)+55 = 67
- This times by the n/2 gives us our sum value $\frac{n}{2}\left(2a_1 + (n-1)d\right)$
- So in this example: $\frac{12}{2}\left(2 \cdot 6 + (12-1)5\right) = 402$

Finding the Sum of a Geometric Series

- To find the sum of a geometric sequence we use this formula:

$$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

- So for the sequence 12, 24, 48, 96, 192
- We do $s_n = \frac{12\left(1 - 2^5\right)}{1 - 2}$ as n = 5, r = 2, a_1 = 12
- This gives us 372

Your Turn

Can you find the sum @ the nth term a,b,c,d,e,f of each sequence:

n values:

a) n = 6
b) n = 12
c) n = 25
d) n = 100
e) n = 2.5
f) n = -10

Series:

1) 3 + 6 + 9 + 12 + 15…
2) 6 + 18 + 54 + 162…
3) 10 + 8 + 6.4 + 5.12 + 4.096…
4) 5.6 + 11.7 + 17.8 + 23.9 + 30…
5) 8 + -10 + 12.5 + -15.625 + 19.53125…

Bonus Challenge:

If we have a geometric series with a common ratio of 1.01, an initial value of 8.2 and s_n = 67.94249862 what is n?

Answers

	a) n = 6	b) n = 12	c) n = 25	d) n = 100	e) n = 2.5	f) n = -10
1	81	270	1050	15450	20.625	105
2	2184	1594320	2.542×10^{12}	1.546×10^{48}	43.765	-2.999
3	36.89	46.564	49.811	49.999	21.378	-415.661
4	125.1	469.8	1970	30755	25.4375	279.5
5	17.119	55.296	944.703	1.745×10^{10}	9.767	3.937

Bonus Challenge:

8

Equations

	Arithmetic	Geometric
Sequence	$a_n = a_1 + d(n-1)$	$a_n = a_1(r)^{n-1}$
Series	$s_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$	$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$