

Integration



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What is integration

- To solve an integration, you just do the inverse of what you do when finding the derivative
- This means integration follows the same rules that differentiation does but in reverse
- You will often hear it defined as the “antiderivative”

Integration representation

- When we solve an integration, we must use a set of symbols
- The first is the integral symbol \int which we put before our expression to make it clear it's an integration
- The next is dx , dy , dt , etc. which we put after the expression to tell us the variable of integration
- So, a full integration might look like

Definite vs Indefinite Integration

- **Indefinite Integration:**

- Finds a general formula for the antiderivative.
- This means we use value c to represent any potential constant that we don't know
- We do this when we don't have limits
- For indefinite we use an empty integration symbol \int

- **Definite Integration:**

- Calculates the numerical area under a curve between two limits.
- This means we don't have to include value c as we are just finding a number not an expression
- We do this when we have limits from the question
- For definite we use a filled in integration symbol \int_b^a

Power rule (indefinite integration)

- The integration power rule is as such:

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

- So, if we have x^4

- $\int x^4 dx = \frac{x^{4+1}}{4+1} + c = \frac{x^5}{5} + c$

Constant multiple rule (indefinite integration)

- The integration multiple rule is as such:


- $\int ax^n dx = a * \frac{x^{n+1}}{n+1} + c$

- So, if we have $20x^4$

- $\int x^4 dx = 20 * \frac{x^{4+1}}{4+1} + c = 20 * \frac{x^5}{5} + c = 4x^5 + c$

Sum/difference rule (indefinite integration)

- The integration sum/difference rule is as such:
- $\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
- So, if we have $f(x) = 30x^2$ & $g(x) = 4x^3$ & $f(x) + g(x)$
- $\int [f(x) + g(x)] = 10x^3 + x^4 + c$



Note we only have one c which we add in at the end, not 2

Integral of a constant(indefinite integration)

- The rule when integrating a constant is as such:
- $\int a \, dx = ax + c$
- So, if we have $f(x) = 5$
- $\int f(x) = 5x + c$

Exponential Rule (indefinite integration)

- The rule when integrating an exponential (e^x) is as such:
- $\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
- So, if we have $f(x) = 5$
- $\int f(x) = 5x + c$

Trigonometric Integration (indefinite integration)

- The rules of trigonometric integration are:

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + C$$

$$\int \csc^2(ax) dx = -\frac{1}{a} \cot(ax) + C$$

$$\int \sec(ax) \tan(ax) dx = \frac{1}{a} \sec(ax) + C$$

$$\int \csc(ax) \cot(ax) dx = -\frac{1}{a} \csc(ax) + C$$

Definitive integration

- The primary rule in definitive integration is
- $\int_a^b f(x)dx = F(b) - F(a)$
- Where $F(x)$ is $\int f(x)dx$ (of course without the constant)

Example of Definitive integration

- $\int_0^5 15x^2 dx = F(5) - F(0)$
- $\int 15x^2 = 5x^3$
- $[5x^3]_0^5 = 5(5^3) - 5(0^3) = 625 - 0 = 625$