ADVANCED MATHEMATICS

Engineering Informatics

Calculus Reminder: Integration and differentiation of real functions.

A) Find derivatives of the following functions:

1) $f(x) = 2x^3 - x^2 - 5x + 21$
Ans: $f'(x) = 6x^2 - 2x - 5$

2) $g(x) = (\log x)^7$.
Ans: $g'(x) = \frac{7(\log x)^6}{x}$.

3) $h(x) = e^x \sin x$
Ans: $h'(x) = e^x(\sin x + \cos x)$.

4) $f(x) = \frac{x^2+1}{x^2-1}$
Ans: $f'(x) = -\frac{4x}{(x^2-1)}$

5) $g(x) = xe^x \cos x$
Ans: $g'(x) = e^x(\cos x + x\cos x - x\text{sin}x)$

6) $f(x) = \sqrt{\cos x - e^{2x}}$
Ans: $f'(x) = \frac{-\text{sin}x - 2e^{2x}}{2\sqrt{\cos x - e^{2x}}}$

7) $g(x) = \sqrt{\frac{x-1}{x+1}}$.
Ans: $g'(x) = \frac{1}{(x-1)^{1/2}(x+1)^{3/2}}$.

8) $g(x) = \frac{\sqrt{x}-1}{\sqrt{x}+1}$
Ans: $g'(x) = \frac{1}{\left(\sqrt{x}+1\right)^2\sqrt{x}}$.

9) $h(x) = \sec\sqrt{x}$ ($\sec u := 1/\cos u$).
Ans: $h'(x) = \frac{1}{2\sqrt{x}}\sec(\sqrt{x})\tan(\sqrt{x})$.

10) $u(x) = \log(\sin x)$
Ans: $u'(x) = \frac{\cos x}{\text{sen}x} := \cot x$

11) $v(x) = \frac{\text{arcsin}x}{\log x}$.
Ans: $v'(x) = \frac{x\log x - \sqrt{1-x^2}\text{arcsin}x}{x\sqrt{1-x^2}(\log x)^2}$.

12) Compute the derivative of $f(x) = x^{\log x}$, finding first the derivative of $g(x) = \log f(x)$.
¿What is the value of $\lim_{x\to 0^+} x^{\log x}$?

B) Compute the primitive of the following functions. Review of: integration by parts, integration of rational functions, change of variable and primitives of elementary trigonometric functions.

1) i)$\int \frac{dx}{x}$, ii) $\int \tan x\, dx$ (use that $\cos' x = -\text{sen}x$).
Ans: i) $\log|x| + c$, ii) $-\log|\cos x| + c$.

2) $\int \frac{2x^2-5x+6}{(x-1)^3}dx$ (use that $\frac{2x^2-5x+6}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$).
Ans: $2\log|x-1| + (x-1)^{-1} - \frac{3}{2}(x-1)^{-2} + c$.

3) $\int \frac{-x-2}{x(x-1)(x-2)}dx$ (use that $\frac{-x-2}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$).
Ans: $-\log|x| + 3\log|x-1| - 2\log|x-2| + c$.

4) $\int \frac{x^2+1}{x^2(x-1)(x+1)}dx$ (use $\frac{x^2+1}{x^2(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{x+1}$).
Ans: $\frac{1}{x} + \log|x-1| - \log|x+1| + c$.

5) $\int \frac{5x^2-x+3}{x(x^2+1)}dx$ (use $\frac{5x^2-x+3}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$).
Ans: $3\log|x| + \log(x^2+1) - \arctan x + c$

6) $\int x\, \sin x\, dx$ (use $u = x$, $dv = \text{sin}x\, dx$).
Ans: $-x\cos x + \text{sin}x + c$.

7) $\int 3xe^{-2x}dx$
Ans: $-\frac{3}{2}xe^{-2x} - \frac{3}{4}e^{-2x} + c$.

8) $\int x^\alpha \log x\, dx$, if $\alpha \neq -1$.(use $u = \log x$, $dv = x^\alpha dx$).
Ans: $\frac{x^{\alpha+1}}{\alpha+1}\log x - \frac{x^{\alpha+1}}{(\alpha+1)^2} + c$.

9) $\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$ (use $x = t^6$).
Ans: $2x^{1/2} + 3x^{1/3} + 6x^{1/6} + 6\log\left|x^{1/6} - 1\right| + c$.

10) $\int \frac{e^{6x}dx}{e^{2x}+1}$ (use $t = e^{2x}$).
Ans: $\frac{e^{4x}}{4} - \frac{e^{2x}}{2} + \frac{1}{2}\log(e^{2x}+1) + c$

11) $\int e^{\sqrt{x}}dx$ (use $t = \sqrt{x}$ and combine with integration by parts).
Ans: $2e^{\sqrt{x}}(\sqrt{x} - 1) + c$