Differential Equations, Dynamical Systems and an Introduction to Chaos

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2 Planar Linear Systems

2.(b)

Solution. The eigenvalues of A are 0 and 7, and the corresponding eigenvectors are $(2, -1)$ and $(1, 3)$. Hence, the general solution is

$$X(t) = \alpha \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \beta e^{7t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$\square$

5.

Solution. Given $X(0) = X_0$, clear that $X(t) = X_0$ is a solution and since $X' = 0$, it is the only solution. $\square$

7.

Proof. When $a = 1$, A has repeated eigenvalues 1. If $a \neq 1$, then the eigenvectors associated with a and 1 are $(1, 1 - a)$ and $(0, 1)$ respectively. As a approaches 1, the eigenvectors of A tends to be orthogonal. $\square$