Real Analysis

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Contents

3 Lebesgue Measure

3.1 Introduction

1.

Proof. Since $\mathfrak{M}$ is an σ-algebra, $B \backslash A \in \mathfrak{M}$ as long as $A, B \in \mathfrak{M}$. Since $B \backslash A$ and A are disjoint, $mB = mA + m(B \backslash A) \geq mA$ since m is nonnegative. $\square$

2.

Proof. Let $A_0 = E_0$ and $E_k = A_k \backslash A_{k-1}$ for $k \geq 1$. Clear that E_i and E_j are disjoint for distinct i and j, $\bigcup A_n = \bigcup E_n$ and $A_i \subset E_i$ for every i. Hence,

$$m\left(\bigcup E_n\right) = m\left(\bigcup A_n\right) = \sum mA_n \leq \sum mE_n,$$

where the last inequality comes from Exercise 1. $\square$

3.

Proof. Suppose that $mA < \infty$. Then $mA = m(A \cup \varnothing) = mA + m\varnothing$, implying that $m\varnothing = 0$. $\square$