Differential Equations, Dynamical Systems and Linear Algebra

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2.

Solution.
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(b) $x_1(t) = k_1 e^t$, $x_2(t) = k_2 e^{-2t}$ and $x_3(t) = k_3$.
(b) $x_1(t) = k_1 e^t$, $x_2(t) = k_2 e^{-2t}$ and $x_3(t) = k_3 e^{2t}$. $\square$

4.

Solution.
$$x'(t) = \begin{bmatrix} e^t \\ 4e^{2t} \\ 8e^{2t} \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 2 & \\ & & 2 \end{bmatrix} \begin{bmatrix} e^t \\ 2e^{2t} \\ 4e^{2t} \end{bmatrix} = \begin{bmatrix} 1 & & \\ & 2 & \\ & 2 & 1 \end{bmatrix} \begin{bmatrix} e^t \\ 2e^{2t} \\ 4e^{2t} \end{bmatrix}.$$
$\square$

6.

Solution. Suppose that $A = \operatorname{diag}(a_1, \dots, a_n)$, then $x_i(t) = K_i \exp(a_i t)$. Hence, $x(t) \to 0$ as $t \to \infty$ for all solutions iff $a_i < 0$ for each $i = 1, 2, \dots, n$. $\square$

8.

Proof.
(a) It follows immediately that both the differential operator and A are linear operators.
(b) Since every solution is of form $x(t) = (K_1 e^t, K_2 e^{-2t})$, $u(t) = (e^t, 0)$ and $v(t) = (0, e^{-2t})$ are two suitable solutions. $\square$