Inner Product Spaces. Hilbert Spaces Inner Product Spaces. Hilbert Spaces 2. proof $\|x+y\|^2 = \langle x + y, x+y\rangle = \|x\|^2+\|y\|^2+2\langle x, y\rangle = \|x\|^2+\|y\|^2, wherethelastequalitycomesfromthehypothesisoforthogonality.Nowwesho$

$\left\|\sum_{i=1}^{m} x_i\right\|^2 = \sum_{i=1}^{m} \|x_i\|^2, byinductionon$m$.Thecasewhere$m=2$hasalreadybeenshowedandweassumethattheequationhol$

$1.Since$x$_m$ is orthogonal with each $i = 1, \ldots, m-1$, x_m is orthogonal to $x_1+\cdots+x_{m-1}$. Hence, $\left\|\sum_{i=1}^{m} x_i\right\|^2 = \left\|\sum_{i=1}^{m-1} x_i\right\|^2 + \|x_m\|^2 = \sum_{i=1}^{m} \|x_i\|^2, completingtheproof.$