

7.15 3, 1, 4, 1, 5, 9, 2, 6
 ↓
 (3, 1, 4, 1) (5, 9, 2, 6)
 ↓
 (3, 1), (4, 1) (5, 9), (2, 6)
 ↓
 3, 1, 4, 1, 5, 9, 2, 6
 ↓
 (1, 3), (1, 4), (5, 9), (2, 6)
 ↓
 (1, 1, 3, 4) (2, 5, 6, 9)
 ↓

1 < 2
 1,
 1 < 2
 1, 1,
 3 > 2
 1, 1, 2
 3 < 5
 1, 1, 2, 3,
 4 < 5
 1, 1, 2, 3, 4

↓
 [1, 1, 2, 3, 4, 5, 6, 9] Final sorted array.

7.17

- a. sorted input : $O(n \log n)$ * Note if its Natural merge sort, it will be $O(n)$
 b. reversed-order input : $O(n \log n)$
 c. Random input : $O(n \log n)$

7.19 3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5
 ① find median from beginning, middle and end. 3, 9, 5; median = 5
 move 9 and 5
 Index: 0 1 2 3 4 5 6 7 8 9 10
 3, 1, 4, 1, 5, 9, 5, 2, 6, 5, 3, 9 Pivot = 5
 ↑
 p

② swap Pivot with second-last
 Index: 0 1 2 3 4 5 6 7 8 9 10
 3, 1, 4, 1, 5, 3, 2, 6, 5, 9, 5, 9
 ↑ ↑
 6 > 5 j = 8
 i = 6

set $i=0$, $j=9$ where pivot is.
 when $i=6$, $j=8$, swap the values

↓
 3, 1, 4, 1, 5, 3, 2, 5, 6, 9, 5, 9
 ↑ ↑
 i j
 when $i=8$, $j=7$, crossed, swap i with Pivot.

↓
 Index 0 1 2 3 4 5 6 7 8 9 10
 3, 1, 4, 1, 5, 3, 2, 5, 5, 6, 9, 9
 ↑
 previous pivot
 Take left side's subarray.

0 1 2 3 4 5 6 7
 3, 1, 4, 1, 5, 3, 2, 5
 median = 1, 3, 5
 Pivot = 3
 swap with 2nd last element

↓
 1, 1, 4, 3, 5, 3, 2, 5
 ↓
 1, 1, 4, 2, 5, 3, 3, 5
 ↑ ↑
 i j
 when $i=2$, $j=5$

↓
 1, 1, 3, 2, 5, 4, 3, 5
 ↑ ↑
 i j

0 1 2 3 4 5 6 7

1, 1, 3, 2, 5, 4, 3, 5

\uparrow
 j

\downarrow
1, 1, 3, 2, 3, 4, 5, 5

1, 1, 3, 2 \rightarrow 1, 3, 1, 2

\uparrow
 j

1, 1, 3, 2

③ 1, 1, 3, 2, ~~3~~, 3, 4, 5, 5, 5, 6, 9

Use insertion sort index 2 and 3, "3, 2" \rightarrow "2, 3"
since cutoff = 3

④ final array: 1, 1, 2, 3, 3, 4, 5, 5, 5, 6, 9

5, 6, 9 4

when $i = 4$, $j = 3$,
swap i with pivot

new subarray = 1, 1, 3, 2
median = 1

swap 1 with 2nd last

when $i = 1$, $j = 0$

swap i with pivot

7.20

a. sorted input: $O(N \log N)$

b. reversed input: $O(N \log N)$

c. random input: $O(N \log N)$

7.21

a. the first element: $O(N^2)$

b. larger of the first two distinct elements:

c. a random element: $O(N^2)$

d. the average: $O(N \log N)$

7.22

- The worst case will be $O(N \log N)$ as the implementation uses median-3 partitioning.
- the two while loops need to be changed to

line 19 ① while ($a[i] \leq \text{pivot} \ \&\& \ i < j$)
 line 20 ② while ($\text{pivot} \leq a[j] \ \&\& \ j > i$)

The run time will be $O(N^2)$ since the median selected will be the same always, so it is worst-case scenario.

c.

~~① while ($a[i] \leq \text{pivot}$)~~
 line 20 ① while ($\text{pivot} \leq a[j] \ \&\& \ j > i$)

The run time will be $O(N^2)$ when keys are equal

7.23.

Unless the array is sorted, picking the middle can still be the smallest/largest value causing it to have $O(N^2)$ time. But if it is sorted, it will greatly reduce the chance that quicksort will require quadratic time.

7.24

Let the array be [20, 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2]

This median-3 will always be 20, which will be worst case scenario.