

7.15    3, 1, 4, 1, 5, 9, 2, 6  
           ↓  
       (3, 1, 4, 1)    (5, 9, 2, 6)  
           ↓  
       (3, 1), (4, 1)    (5, 9), (2, 6)  
           ↓  
       3, 1, 4, 1, 5, 9, 2, 6  
           ↓  
       (1, 3), (1, 4), (5, 9), (2, 6)  
           ↓  
       (1, 1, 3, 4)    (2, 5, 6, 9)  
           ↓

1 < 2  
 1,  
 1 < 2  
 1, 1,  
 3 > 2  
 1, 1, 2  
 3 < 5  
 1, 1, 2, 3,  
 4 < 5  
 1, 1, 2, 3, 4

↓  
 [1, 1, 2, 3, 4, 5, 6, 9] Final sorted array.

7.17

- a. sorted input :  $O(n \log n)$  \* Note if its Natural merge sort, it will be  $O(n)$   
 b. reversed-order input :  $O(n \log n)$   
 c. Random input :  $O(n \log n)$



7.19 3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5  
 ① find median from beginning, middle and end. 3, 9, 5; median = 5  
 move 9 and 5  
 Index: 0 1 2 3 4 5 6 7 8 9 10  
 3, 1, 4, 1, 5, 9, 5, 2, 6, 5, 3, 9 Pivot = 5  
 ↑  
 p

② swap Pivot with second-last  
 Index: 0 1 2 3 4 5 6 7 8 9 10  
 3, 1, 4, 1, 5, 3, 2, 6, 5, 9, 5, 9  
 ↑ ↑  
 6 > 5 j = 8  
 i = 6

set  $i=0$ ,  $j=9$  where pivot is.  
 when  $i=6$ ,  $j=8$ , swap the values

↓  
 3, 1, 4, 1, 5, 3, 2, 5, 6, 9, 5, 9  
 ↑ ↑  
 i j  
 when  $i=8$ ,  $j=7$ , crossed, swap  $i$  with Pivot.

↓  
 Index 0 1 2 3 4 5 6 7 8 9 10  
 3, 1, 4, 1, 5, 3, 2, 5, 5, 6, 9, 9  
 ↑  
 previous pivot  
 Take left side's subarray.

0 1 2 3 4 5 6 7  
 3, 1, 4, 1, 5, 3, 2, 5  
 median = 1, 3, 5  
 Pivot = 3  
 swap with 2nd last element

↓  
 1, 1, 4, 3, 5, 3, 2, 5  
 ↓  
 1, 1, 4, 2, 5, 3, 3, 5  
 ↑ ↑  
 i j  
 when  $i=2$ ,  $j=5$

↓  
 1, 1, 3, 2, 5, 4, 3, 5  
 ↑ ↑  
 i j



0 1 2 3 4 5 6 7

1, 1, 3, 2, 5, 4, 3, 5

i  
j

1, 1, 3, 2, 3, 4, 5, 5

1, 1, 3, 2 → 1, 3, 1, 2

i j

1, 1, 3, 2

③ 1, 1, 3, 2, ~~3~~, 3, 4, 5, 5, 5, 6, 9

Use insertion sort index 2 and 3, "3, 2" → "2, 3"  
since cutoff = 3

④ final array: 1, 1, 2, 3, 3, 4, 5, 5, 5, 6, 9

7.20

a. sorted input:  $O(N \log N)$

b. reversed input:  $O(N \log N)$

c. random input:  $O(N \log N)$

7.21

a. the first element:  $O(N^2)$

b. larger of the first two distinct elements:  $O(N^2)$

c. a random element:  $O(N^2)$

d. the average:  $O(N \log N)$

5, 6, 9 4  
when  $i = 4$ ,  $j = 3$ ,  
swap  $i$  with pivot.

new subarray = 1, 1, 3, 2  
median = 1.

swap 1 with 2nd last  
when  $i = 1$ ,  $j = 0$   
swap  $i$  with pivot



7.22

- The worst case will be  $O(N \log N)$  as the implementation uses median-3 partitioning.
- the two while loops need to be changed to

line 19 ① while ( $a[i] \leq \text{pivot} \ \&\& \ i < j$ )  
 line 20 ② while ( $\text{pivot} \leq a[j] \ \&\& \ j > i$ )

The run time will be  $O(N^2)$  since the median selected will be the same always, so it is worst-case scenario.

c.

~~① while ( $a[i] \leq \text{pivot}$ )~~  
 line 20 ① while ( $\text{pivot} \leq a[j] \ \&\& \ j > i$ )

The run time will be  $O(N^2)$  when keys are equal

7.23.

Unless the array is sorted, picking the middle can still be the smallest/largest value causing it to have  $O(N^2)$  time. But if it is sorted, it will greatly reduce the chance that quicksort will require quadratic time.

7.24

Let the array be  $[20, 19, 18, 17, 16, 15, 14, 13, 12, 10, 9, 8, 7, 6, 5, 4, 3, 2]$

This median-3 will always be 20, which will be worst case scenario.