CA6 Q2©

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$y' = -y \ln y, \quad y(0) = 3$

Exact formula :-

$y(x) = e^{(\ln 3)e^{-t}}$

Formula :-

$y_{i+1} = y_i + h f(x_i, y_i)$

When, $h = \frac{1}{2}$, $n = 2$:-

$i = 0, \ y_0 = 3, \ x_0 = 0$

$y_1 = y_0 + h f(x_0, y_0)$

$= 3 + \frac{1}{2}(-3 \ln(3))$

$= 1.35208$

$i = 1, \ y_1 = 1.35208, \ x_1 = 1/2$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.148157038$

$i = 2, \ y_2 = 1.148157038, \ x_2 = 1$

$y_3 = y_2 + h f(x_2, y_2)$

$= 1.068843451$

$exact(1) = 1.49803$

$Error = |exact(1) - y_3|$

$= |1.49803 - 1.068843451|$

$= 0.429187$

when $h = \frac{1}{4}$, $n = 4$:-

$i = 0, \ y_0 = 3, \ x_0 = 0$

$y_1 = y_0 + h f(x_0, y_0)$

$= 3 + \frac{1}{4}(-3 \ln(3))$

$= 2.17604$

$i = 1, \ y_1 = 2.17604, \ x_1 = 1/4$

$y_2 = y_1 + h f(x_1, y_1) = 1.753069$

THE ERROR BECOMES SMALLER AS "N" INCREASES

When H=1/2

i	x[i]	y[i]	y[i+1]	Error
0	0	3	1.352082	1.647918
1	0.5	1.352082	1.148157	0.203925
2	1	1.148157	1.068843	0.079314

WHEN H=1/4

i	x[i]	y[i]	y[i+1]	Error
0	0	3	2.176041	0.823959
1	0.25	2.176041	1.753069	0.422972
2	0.5	1.753069	1.50704	0.246029
3	0.75	1.50704	1.352513	0.154527
4	1	1.352513	1.25041	0.102103

$i=2,\ y_2 = 1.753069,\ x_2 = 1/2$

$y_3 = y_2 + hf(x_2, y_2) = 1.507039811$

$i=3,\ y_3 = 1.507039811,\ x_3 = 3/4$

$y_4 = y_3 + hf(x_3, y_3) = 1.355251772$

$i=4,\ y_5 = 1.355251772,\ x_4 = 1$

$y_5 = 1.250411036$ ✗

$error = |exact(1) - y_5|$

$= |1.49803 - 1.250411036|$

$= 0.247618964$

when $h = \frac{1}{8}$

$i=0,\ y_0 = 3,\ x_0 = 0$

$y_1 = y_0 + hf(x_0, y_0)$

$= 3 + \frac{1}{8}(-3\ln(3))$

$= 2.588020392$

$i=1,\ y_1 = 2.588020392,\ x_1 = 1/8$

$y_2 = y_1 + hf(x_1, y_1) = 2.280404$

$i=2,\ y_2 = 2.280404,\ x_2 = 2/8$

$y_3 = y_2 + hf(x_2, y_2) = 2.045421873$

$i=3,\ y_3 = 2.045421873,\ x_3 = 3/8$

$y_4 = y_3 + hf(x_3, y_3) = 1.862457848$

$i=4,\ y_4 = 1.862457848,\ x_4 = 4/8$

$y_5 = y_4 + hf(x_4, y_4) = 1.71767572$

$i=5,\ y_5 = 1.71767572,\ x_5 = 5/8$

$y_6 = y_5 + hf(x_5, y_5) = 1.6015239$

$i=6,\ y_6 = 1.6015239,\ x_6 = 6/8$

$y_7 = y_6 + hf(x_6, y_6) = 1.507243066$

$i=7,\ y_7 = 1.507243066,\ x_7 = 7/8$

i	x[i]	y[i]	y[i+1]	error
0	0	3	2.588020392	0.411979608
1	0.125	2.58802039	2.280404	0.307616392
2	0.25	2.280404	2.045421873	0.234982126
3	0.375	2.04542187	1.862457848	0.182964025
4	0.5	1.86245785	1.71767572	0.144782128
5	0.625	1.71767572	1.6015239	0.11615182
6	0.75	1.6015239	1.507243066	0.094280834
7	0.875	1.50724307	1.429943692	0.077299375
8	1	1.42994369	1.366018941	0.063924751

$y_8 = y_7 + hf(x_7, y_7) = 1.429943692$

$i=8 \quad y_8 = 1.429943692, \quad x_8 = 1$

$y_9 = y_8 + hf(x_8, y_8) = 1.366018941$

$error = |exact(1) - y_9|$

$= 0.132011$