Simpson's Rule (Composite)

$$S_2(f) = \frac{h}{3}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$S_2(f) = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Composite:

$$S_n(f) = \sum_{i=1}^{n/2} \frac{h_i}{3}\left[f(x_{2i-2}) + 4f(x_{2i-1}) + f(x_{2i})\right]$$

$$h_i = \frac{x_{2i} - x_{2i-2}}{2}$$

Example

$$\int_0^4 e^x \, dx$$

By Basic Way :-

$$h = \frac{4-0}{2} = 2 \qquad \int_0^4 e^x dx \approx \frac{2}{3}\left(e^0 + 4e^2 + e^4\right) = 56.76258$$

Composite way :-

$$\int_0^4 e^x dx \approx \int_0^1 e^x dx + \int_1^2 e^x dx + \int_2^3 e^x dx + \int_3^4 e^x dx$$

$$\approx \frac{0.5}{3}\left(e^0 + 4e^{0.5} + e^1\right)$$

$$\frac{0.5}{3}\left(e^1 + 4e^{1.5} + e^2\right)$$

$$\frac{0.5}{3}\left(e^{2} + 4e^{2.5} + e^{3}\right)$$

$$0.5\left(e^{3} + 4e^{3.5} + e^{4}\right)$$

$= 53.616222.\#$