

Question 1 Euler

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$$y(0) = \frac{1}{2}, h = \frac{1}{4}$$

$$y' = -y \ln y = f(x, y)$$

$$y_{i+1} = y_i + hf(x_i, y_i) = y_i + h(-y_i \ln y_i)$$

$$h = \frac{1}{4}, x_0 = 0, y_0 = \frac{1}{2} \Rightarrow \frac{x_1 - x_0}{h} = \frac{1 - 0}{\frac{1}{4}} = \frac{1}{4}$$

$$\begin{aligned} i=0, \quad y_1 &= y_0 + hf(x_0, y_0) & y_1 &= y_0 + hf(x_0, y_0) \\ &= \frac{1}{2} + \left(\frac{1}{4}\right)\left(-\frac{1}{2} \ln \frac{1}{2}\right) & &= 0.5866 + \frac{1}{4}(-0.3466 \ln(0.5)) \\ &= 0.5866 & &= 0.6609 \end{aligned}$$

$$\begin{aligned} i=1, \quad y_2 &= y_1 + hf(x_1, y_1) & i=2, \quad y_2 &= y_1 + hf(x_1, y_1) \\ &= 0.5866 + \left(\frac{1}{4}\right)(-0.6609 \ln 0.5866) & &= 0.7327 + \frac{1}{4}(-0.7327 \ln 0.7327) \\ &= 0.7327 & &= 0.7976 \end{aligned}$$

$$\begin{aligned} i=4, \quad y_5 &= y_4 + hf(x_4, y_4) \\ &= 0.7976 + \frac{1}{4}(-0.7976 \ln 0.7976) \\ y_5 &= 0.8362 \end{aligned}$$

i	x _i	y _i	y _{i+1} = y _i + hf(x _i , y _i)
0	0	$\frac{1}{2}$	0.5866
1	$\frac{1}{4}$	0.5866	0.6609
2	$\frac{1}{2}$	0.6609	0.7327
3	$\frac{3}{4}$	0.7327	0.7976
4	1	0.7976	0.8362

Error, |exact value - Euler's value|

$$i=0, \Delta y = y(x_0) - y_0 = 0.718 -$$

Exact Solution:-

$$y' = -y \ln y$$

$$\frac{dy}{dx} = -y \ln y$$

$$\int \frac{dy}{-y \ln y} = \int dx$$

$$-\int \frac{dy}{y \ln y} = \int dx \quad \text{use } u = \ln y$$

$$-\int \frac{du}{u} = x + C$$

$$-\ln |\ln y| = x + C$$

$$\ln y = e^{-x} \Rightarrow y = e^{-e^{-x}}$$

$$\begin{aligned} \ln(1-y) &= x \\ 1-y &= e^x \\ -y &= e^x - 1 \\ y &= 1 - e^x \end{aligned}$$