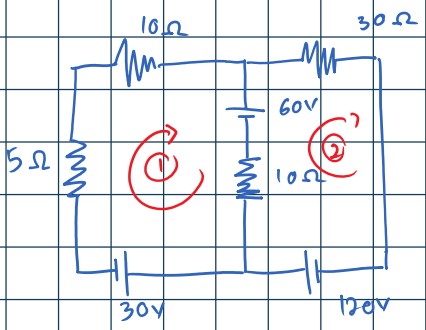
CA5 Q3

Wednesday, 9 June, 2021 8:49 PM



Loop ① $10i_1 + 5i_1 + 10(i_1 - i_2) = -60 - 30$

$10i_1 + 5i_1 + 10(i_1 - i_2) = -90$

Loop ② $30i_1 + 10(i_2 - i_1) = 120 + 60$

① $25i_1 - 10i_2 = -90$

② $-10i_1 + 40i_2 = 180$

Change ① and ② into Matrix form

$$\begin{bmatrix} 25 & -10 \\ -10 & 40 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -90 \\ 180 \end{bmatrix}$$

Using Doolittle method:-

$$L = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \quad U = \begin{bmatrix} b & c \\ 0 & d \end{bmatrix} \quad A = \begin{bmatrix} 25 & -10 \\ -10 & 40 \end{bmatrix} \quad B = \begin{bmatrix} -90 \\ 180 \end{bmatrix}$$

$$LU = \begin{bmatrix} b & c \\ ba & ac + d \end{bmatrix}$$ compare with A

$b = 25 \quad c = -10$

$ba = -10$

$a = \frac{-10}{b}$

$a = \frac{-10}{25} = -\frac{2}{5}$

$ac + d = 40$

$d = 40 - ac$

$d = 40 + \frac{2}{5}(-10)$

$d = 36$

$$L = \begin{bmatrix} 1 & 0 \\ -2/5 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 25 & -10 \\ 0 & 36 \end{bmatrix}$$

Solving Linear Equation

$$LY = B \Rightarrow \begin{bmatrix} 1 & 0 \\ -2/5 & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} -90 \\ 180 \end{bmatrix}$$

$$Y_1 = -90$$

$$-2/5\, Y_1 + Y_2 = 180$$

$$Y_2 = 180 + 2/5\, Y_1$$

$$Y_2 = 144$$

$$Y = \begin{bmatrix} -90 \\ 144 \end{bmatrix}$$

$$UX = Y$$

$$\begin{bmatrix} 25 & -10 \\ 0 & 36 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} -90 \\ 144 \end{bmatrix}$$

$$25x_1 - 10x_2 = -90$$

$$36x_2 = 144$$

$$x_2 = 4$$

$$25x_1 - 10(3.333) = -90$$

$$x_1 = \frac{-90 + 10(4)}{25}$$

$$x_1 = -2$$

$$i_1 = -2.0 \text{ A}$$

$$i_2 = 4.0 \text{ A}$$