

$$y(x, t) = 0.03 \cos(2.4x - 12t + 0.1) \phi_0$$

x is measured in cm
 y in s

a) Frequency

$$v = f\lambda \quad \text{or} \quad T = \frac{2\pi}{\omega} \quad f = \frac{1}{T}$$

$$= \frac{2\pi}{12} \quad f = \frac{1}{1/6\pi} = 1.90985 \text{ Hz}$$

$$T = 1/6\pi$$

b) Wave speed, v

$$v = f\lambda \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{2.4} = 5/6\pi \text{ m}$$

$$v = 1.90985 (5/6\pi)$$

$$v = 4.990$$

$$v \approx 5 \text{ m s}^{-1}$$

c) Particle velocity :-

Differentiate will be $-\sin(C)$

$$y(x, t) = A \cos(kx - \omega t + \phi_0)$$

$$\dot{y}(x, t) = \omega A \sin(kx - \omega t + \phi_0) \quad \leftarrow \text{This formula}$$

$$\ddot{y}(x, t) = -\omega^2 A \cos(kx - \omega t + \phi_0)$$

$$\begin{aligned}
 y(x,t) &= (12)(0.03) \sin(2.4(15 \times 10^{-2}) - 12(0.2) + 0.1) \\
 &= -0.335741405 \\
 &\approx -0.3357 \text{ m s}^{-1} \quad \text{✓}
 \end{aligned}$$

↓) Maximum particle acceleration:-

$$\omega^2 A = (12)^2 (0.03) = 4.32 \text{ m s}^{-2} \quad \text{✓}$$

Travelling Harmonic wave ,

$$A = 0.05 \text{ m}$$

$$\text{Wave number} = 0.1 \text{ m}^{-1}$$

$$\text{phase velocity} = 50 \text{ ms}^{-1} \text{ in } -x \text{ dire}$$

$$\text{at } x=0, t=2 \text{ :-}$$

$$y = 0.0125 \text{ and } \ddot{y}(x,t) \text{ is negative.}$$

Write the wave function, $y(x,t)$

k is a wave number,

$$k = 0.1 \text{ m}^{-1}$$

To find ω , angular frequency.

v is the phase velocity,

$$v = 50 \text{ ms}^{-1}$$

$$\omega = vk$$

$$\omega = (50)(0.1)$$

$$\omega = 5 \text{ s}^{-1}$$

$$y(x,t) = A \sin(kx - \omega t + \phi_0)$$

$$\ddot{y}(x,t) = -\omega^2 A \cos(kx - \omega t + \phi_0)$$

$$\ddot{y}(x,t) = -\omega^2 A \sin(kx - \omega t + \phi_0)$$

$$\text{at } x=0, t=2 \text{ :-}$$

$$y = 0.0125 \text{ and } \ddot{y}(x,t) \text{ is negative.}$$

So,

$$0.0125 = 0.05 \sin(0.1x - 5t + \phi_0)$$

$$0.0125 = 0.05 \sin(0.1(0) - 5(2) + \phi_0)$$

$$0.0125 = 0.05 \sin(-10 + \phi_0)$$

$$\sin^{-1}\left(\frac{0.0125}{0.05}\right) = -10 + \phi_0$$

$$0.2527 = -10 + \phi_0$$

$$\phi = 10.2527$$

$$y(x,t) = -\omega A \cos(kx - \omega t + \phi_0)$$

$$= -5(0.05) \cos(0.1(0) - 5(2) + 10.2527)$$

$$= -0.24786$$

Verified \checkmark

Therefore,

$$y(x,t) = 0.05 \sin(0.1x - 5t + 10.2527) \quad \checkmark$$

Q3

Sunday, 13 June, 2021 6:36 PM

Tension $F = 0.16$

$$y(x, t) = 0.0024 \sin(5\pi x - 40\pi t) \quad \phi_0 = 0$$

 x in metres y in seconds

a) wavelength and frequency

$$f = \frac{\omega}{2\pi} = \frac{40\pi}{2\pi} = 20 \text{ Hz}$$

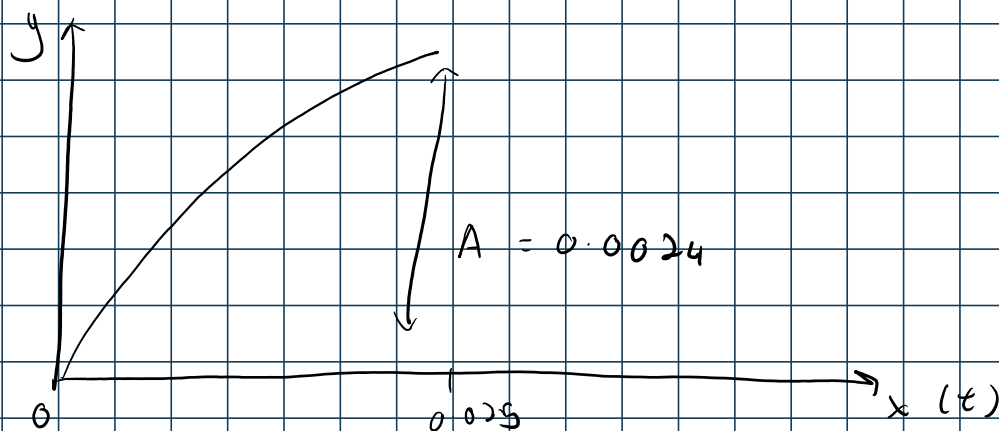
$$\lambda = \frac{\omega}{k} \times \frac{1}{f} = \frac{40\pi}{5\pi} \times \frac{1}{20} = \frac{8}{20} = \frac{2}{5} \text{ m}$$

b) sketch the shape of the string

$$20 \text{ Hz} = 20 \text{ cycles/sec}$$

$$\text{so } 20 \times 0.025 = 0.5$$

Only from half a wave



c) maximum particle velocity,

$$\max v = \omega A = 5\pi \cdot 10^{-2} \cdot 24 = 0.037699 \text{ m s}^{-1}$$

d) linear density, μ

$$v = \sqrt{\frac{F}{\mu}}$$

$$v^2 = \frac{F}{\mu}$$

$$\mu = \frac{F}{v^2}$$

$$v = f\lambda$$

$$= 20 \left(\frac{2}{5} \right) = 8$$

$$\mu = \frac{600}{8^2} = 2.5 \text{ kg m}^{-1}$$

Q4

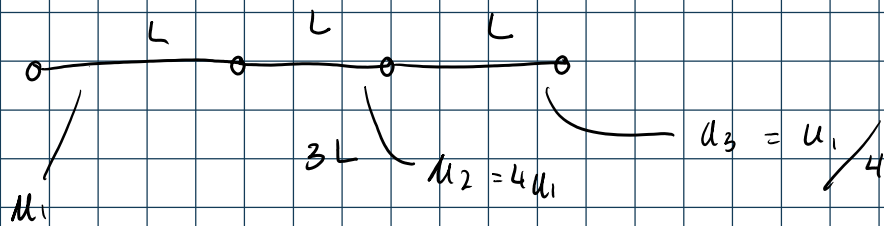
Sunday, 13 June, 2021

6:54 PM

YEAH I'M SKIPPING THIS FOR NOW

Q5

Sunday, 13 June, 2021 6:54 PM



if the combined string is under tension F ,
How much time does it take to traverse its full length.

Give in terms of L , F , μ_1

Does this answer depend on the order in which the pieces are joined

BABA ~ ~ ~ ~ ~

Q6

Sunday, 13 June, 2021 6:57 PM

$$y(x, 0) = \frac{A^3}{A^2 + x^2}$$

$$A = 1 \text{ cm} \\ = 1 \times 10^{-2} \text{ m}$$

$$y(x, 0) = \frac{(1 \times 10^{-2})^3}{(1 \times 10^{-2})^2 + x^2}$$

$$v = 20 \text{ m s}^{-1}$$

a) write $y(x, t)$

GAS BABY GAS !!!

Not in notes !p, will come back later.

Q7

Sunday, 13 June, 2021 7:00 PM

STANDING WAVES ~ ~ ~ ~ ~

$$y(x, t) = 0.04 \sin(0.5x) \cos(30t)$$

$$y(x, t) = 0.02 [2 \sin 0.5x \cos 30t]$$

$$= 0.02 [\sin 0.5x \cos 30t + \cos 0.5x \sin 30t + \sin 0.5x \cos 30t - \cos 0.5x \sin 30t]$$

$$= 0.02 [\sin(0.5x + 30t) + \sin(0.5x - 30t)]$$

$$= 0.02 \sin(0.5x + 30t) + 0.02 \sin(0.5x - 30t)$$

$$\uparrow$$

 $y_1(x, t)$

$$\uparrow$$

 $y_2(x, t)$

$$\text{For } y_1(x, t) = 0.02 \sin(0.5x + 30t)$$

$$k = 0.5, \quad \omega = -30, \quad A = 0.02$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{-30}{2\pi}$$

cannot be in negative

$$f = -4.477 \text{ Hz}$$

$$= 4.477 \text{ Hz}$$

$$\lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{0.5} = 4\pi$$

$$\text{Wave speed} = v = \frac{\omega}{k}$$

$$v = \frac{-30}{0.5}$$

$$v = -60 \text{ ms}^{-1}$$

$$y_2(x, t) = 0.02 \sin(0.5x - 30t)$$

$$k = 0.5, \quad \omega = 30, \quad A = 0.02$$

$$f = 4.477 \text{ Hz}$$

$$\lambda = 4\pi$$

$$v = 60 \text{ ms}^{-1}$$

b) particle velocity at $x = 2.4 \text{ m}$ and $t = 0.8 \text{ s}$

$$y_1(x, t) = -\omega A \sin(kx - \omega t + \phi_0)$$

$$y_1(2.4, 0.8) = 30(0.02) \cos(0.5(2.4) + 30(0.8) + 0)$$

$$= 0.60 \text{ ms}^{-1}$$

$$y_2(2.4, 0.8) = -30(0.02) \cos(0.5(2.4) - 30(0.8))$$

$$= 0.4142 \text{ ms}^{-1}$$

c) length of the string

Fundamental mode, $n = 1$

$$i) \quad \frac{\omega L}{v} = n\pi$$

$$v = 60, w = -30, n = 1$$

$$\frac{-30L}{60} = 1\pi$$

$$L = 1\pi \times \frac{60}{-30}$$

$$L = -2\pi m$$

$$L = 2\pi m$$

for both y' and y. ✓

$$L = 60 \text{ cm}, \quad m = 1.2 \text{ g}, \quad n = 3, \quad y(x, t) = 2 \text{ mm} = x$$

$$V = 420 \text{ ms}^{-1}$$

a) Wave length, λ_n frequency, ω_n :-

$$L = n \frac{\lambda_n}{2}$$

$$L = 3 \frac{\lambda_3}{2}$$

$$\frac{2L}{3} = \lambda_3$$

$$\lambda_3 = \frac{2(0.6)}{3}$$

$$\lambda_3 = 0.4 \text{ m}$$

$$\omega_n = \frac{n \pi v}{L}$$

$$\omega_3 = \frac{3 \pi (420)}{0.60}$$

$$\omega_3 = 2100\pi$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{2100\pi}{2\pi} \overset{f_3}{=} 1050 \text{ Hz}$$

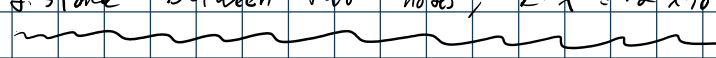
b) equation of the standing wave

$$y(x, t) = 2A \sin(kx) \cos(\omega t)$$

a) $A = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

$D = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$

distance between two nodes, $\Delta x = 12 \times 10^{-2} \text{ m}$



$\mu = 2.7 \text{ g m}^{-1} = \frac{2.7 \text{ g}}{1 \text{ m}} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 2.7 \times 10^{-3} \text{ kg m}^{-1}$

$F = 14 \text{ N}$,

Write the wave function of the standing wave

$y(x,t) = 2A \sin kx \cos \omega t$

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{14}{2.7 \times 10^{-3}}} = 72 \text{ ms}^{-1}$

$v = f \lambda$

$D = \frac{\lambda^2}{\Delta \lambda} = \frac{\lambda^2}{1} = 12 \times 10^{-3}$
 (assume as 1) $\lambda = \frac{\sqrt{120}}{50} \text{ m}$

$k = 2\pi \times \frac{50}{\sqrt{30}} = 57.357$

$v = \frac{\omega}{k} \Rightarrow \omega = kv = 72(57.357) = 4129.704$

$y(x,t) = 2A \sin(kx) \cos(\omega t)$
 $= 2(2 \times 10^{-3}) \sin(57.35x) \cos(4129.704t)$

b) $L = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$

$\mu = 1.5 \text{ g m}^{-1} = 1.5 \times 10^{-3} \text{ kg m}^{-1}$

$f_2 = 450 \text{ Hz}$

i) speed of the transverse wave

ii) Tension in the string, \nearrow

$f_n = \frac{nv}{2L}$

$v = \sqrt{\frac{F}{\mu}}$

$$f = \frac{nv}{2L}$$

$$f_2 = \frac{2v}{2L}$$

$$f_2 = \frac{v}{L}$$

$$450(L) = v$$

$$v = 450(60 \times 10^{-3}) = 270 \text{ m/s} \quad \times$$

$$v = \sqrt{\frac{F}{\mu}}$$

$$v^2 = \frac{F}{\mu}$$

$$F = v^2 \mu$$

$$= (270)^2 (1.5 \times 10^{-3})$$

$$F = 109.35 \text{ N} \quad \times$$