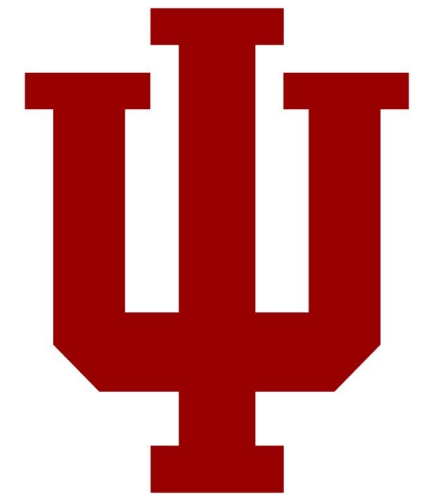


20: Parallelism III

ENGR 315: Hardware/Software CoDesign

Andrew Lukefahr

Indiana University



Announcements

- P8: Due Friday
- P9: Parallelism
- Report: Coming up

Input
[[0.1 0.2 0.3]]

Weights
[1. 2. 3. 4.]
[5. 6. 7. 8.]
[9. 10. 11. 12.]

Output
= [3.8000002 4.4 5. 5.6000004]
[0.1 0.2 0.3 0.4]

Dependencies

$$0.1 \cdot 1 + 0$$

$$0.1 \cdot 2 + 0$$

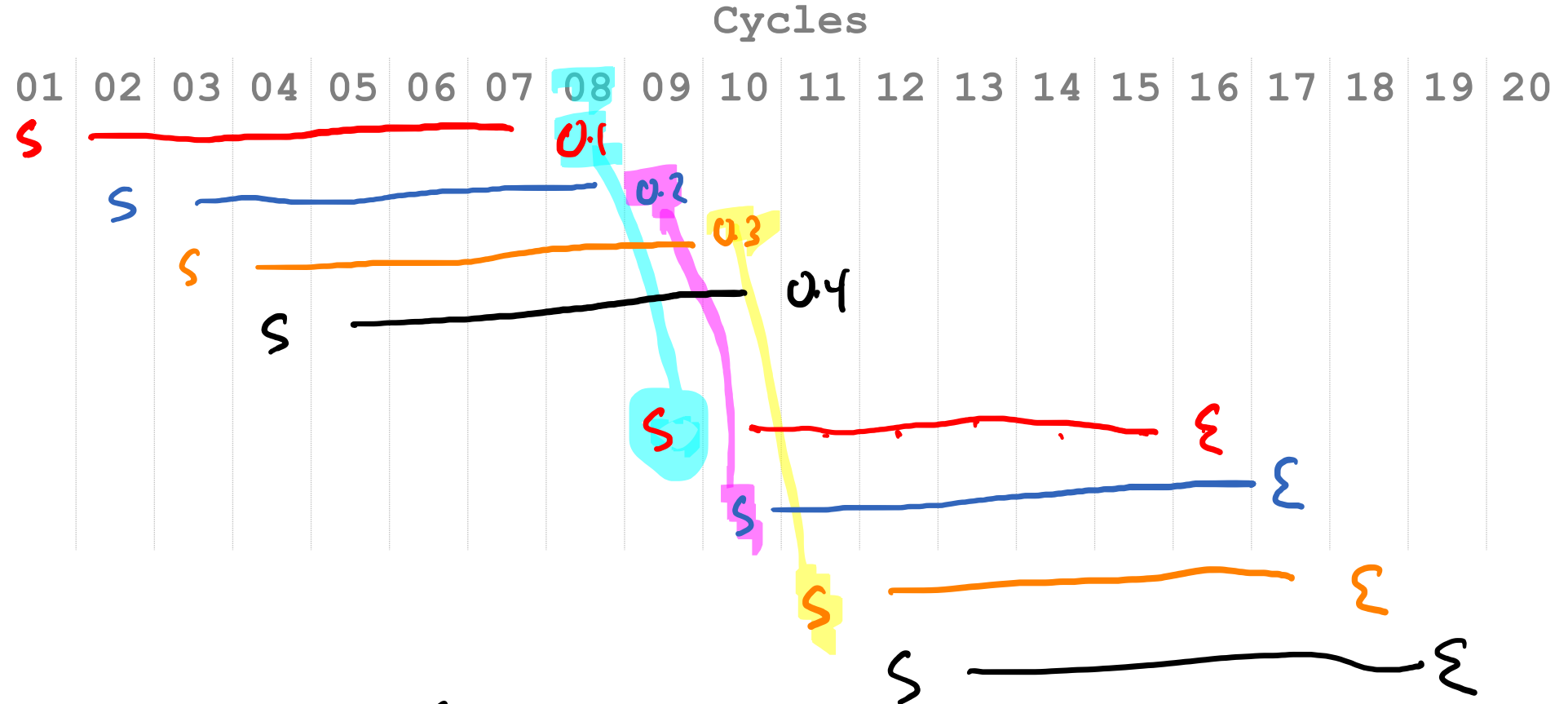
$$0.1 \cdot 3 + 0$$

$$0.1 \cdot 4 + 0$$

$$0.2 \cdot 5 + 0.1$$

$$0.2 \cdot 6 + 0.2$$

$$0.2 \cdot 7 + 0.3$$



↑
"bubbles"

Verilog Parameters

```
module adder #(parameter BITS = 2) (  
    input [BITS-1:0] a,  
    input [BITS-1:0] b,  
    output [BITS-1:0] c) );  
    assign c = a + b;  
endmodule
```

```
// 2-bit adder
```

```
adder add2 (a2, b2, c2);
```

```
//another 2-bit adder
```

```
adder #(BITS=2) add2b (a2b, b2b, c2b);
```

```
//a 3-bit adder
```

```
adder #(BITS=3) add3 (a3, b3, c3);
```

```
Adder #(BITS=32) add32 (a32, b32,  
c32);
```


data-dependence graph
(data-flow graph)

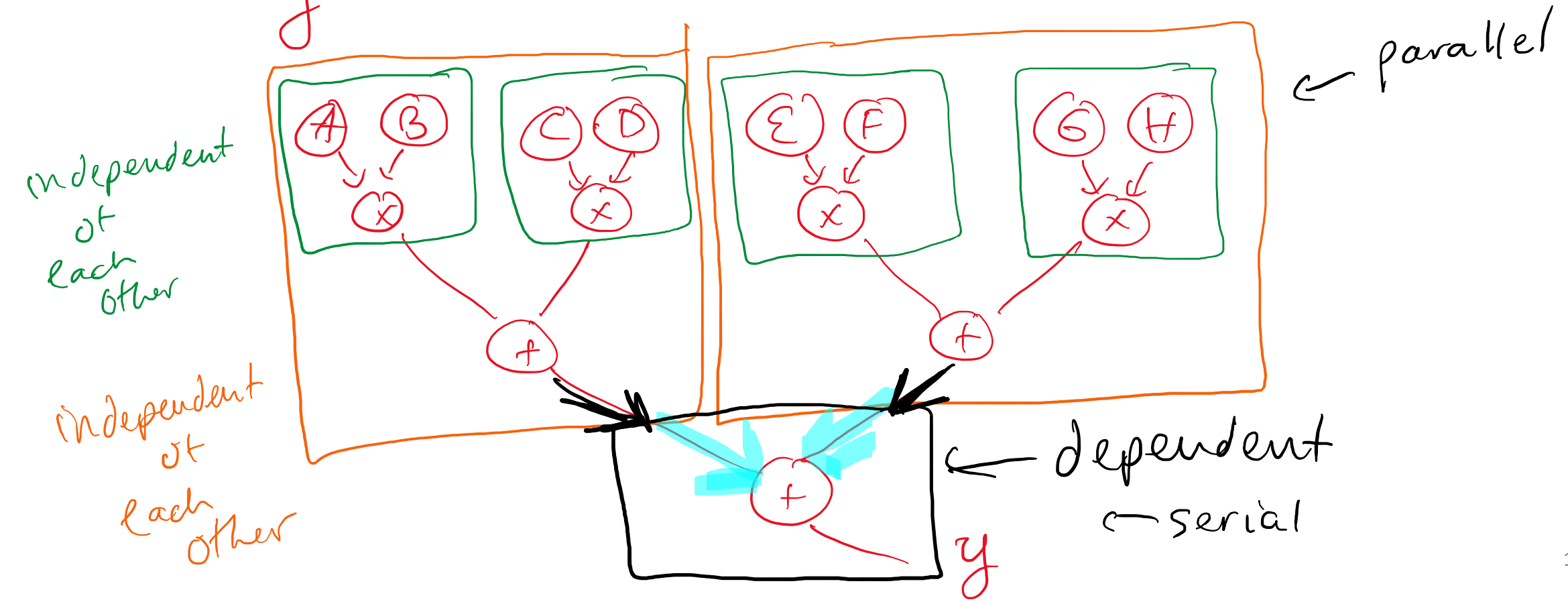
$$Z = C(A+B) - D \cdot E$$

data-dependence graph
(data-flow graph)

$$y = A \cdot B + C \cdot D + E \cdot F + G \cdot H$$

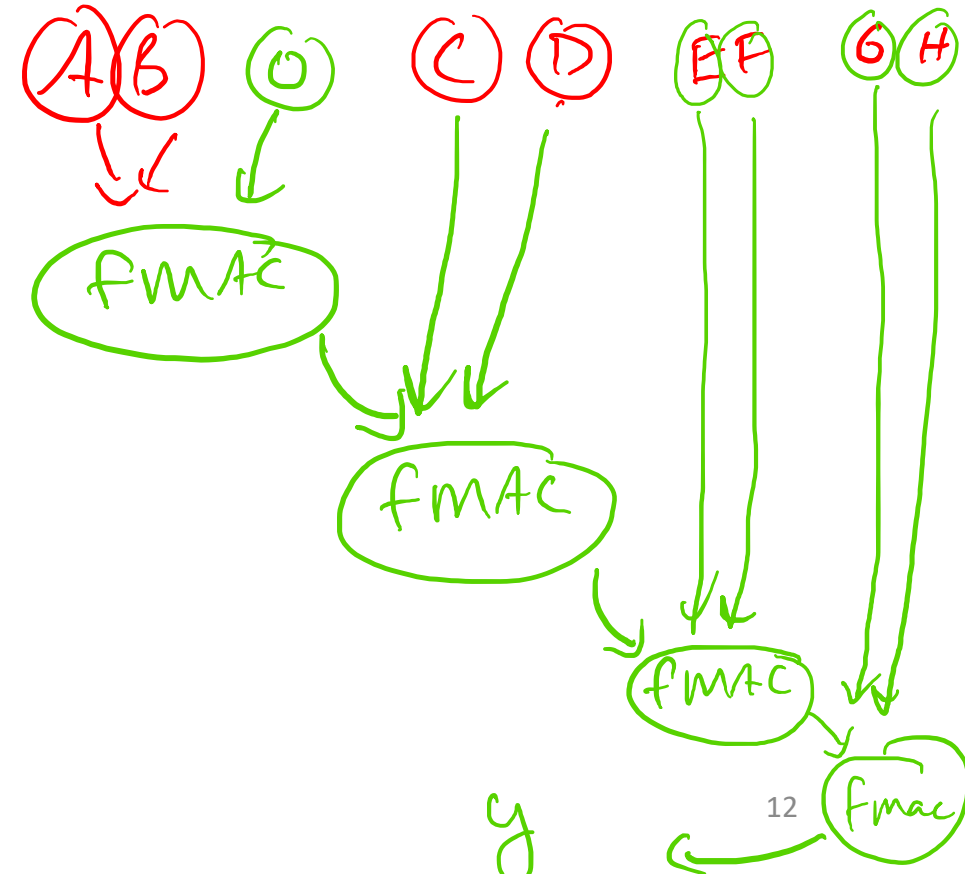
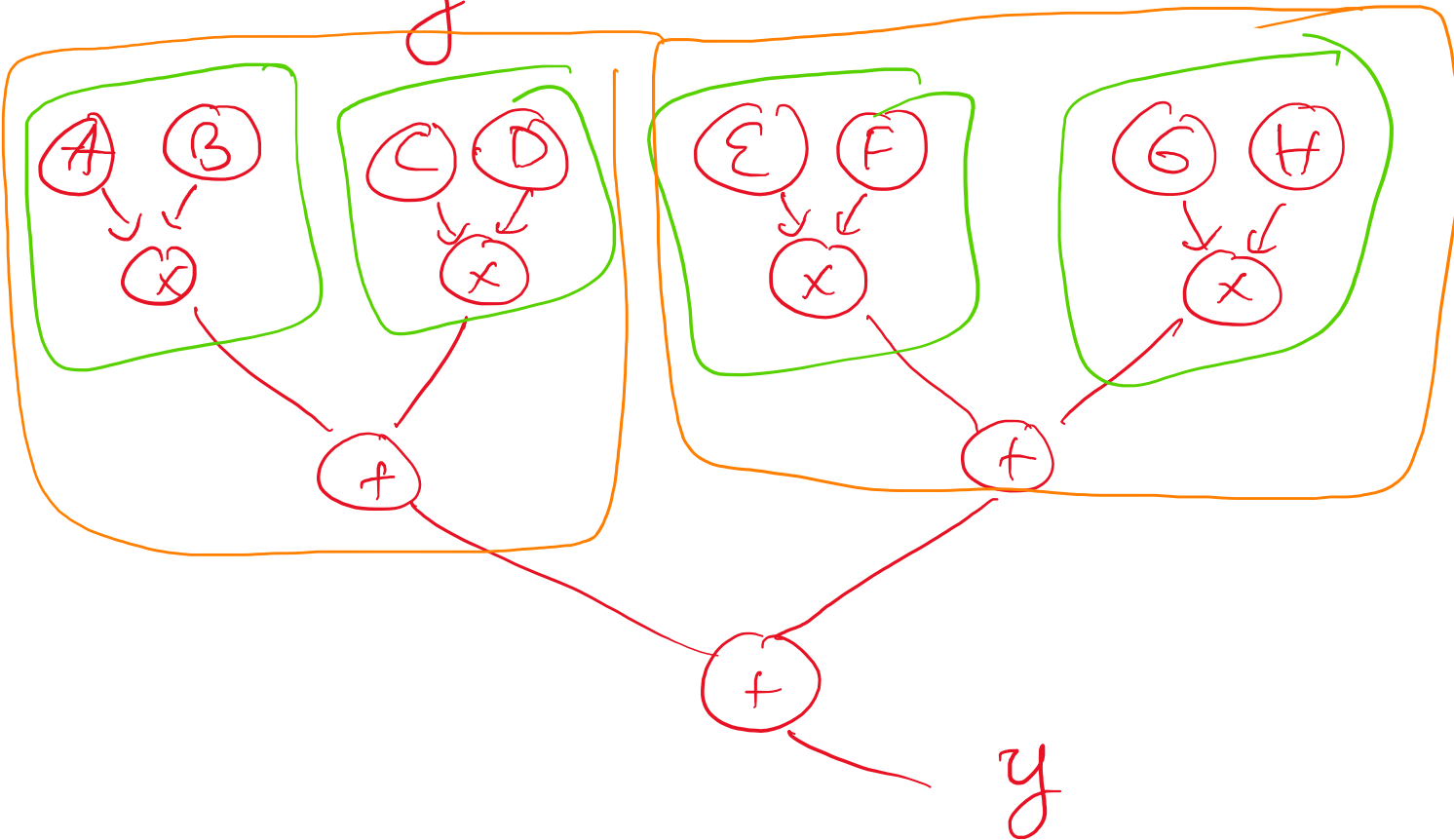
data-dependence graph (data-flow graph)

$$y = A \cdot B + C \cdot D + E \cdot F + G \cdot H$$



data-dependence graph (data-flow graph)

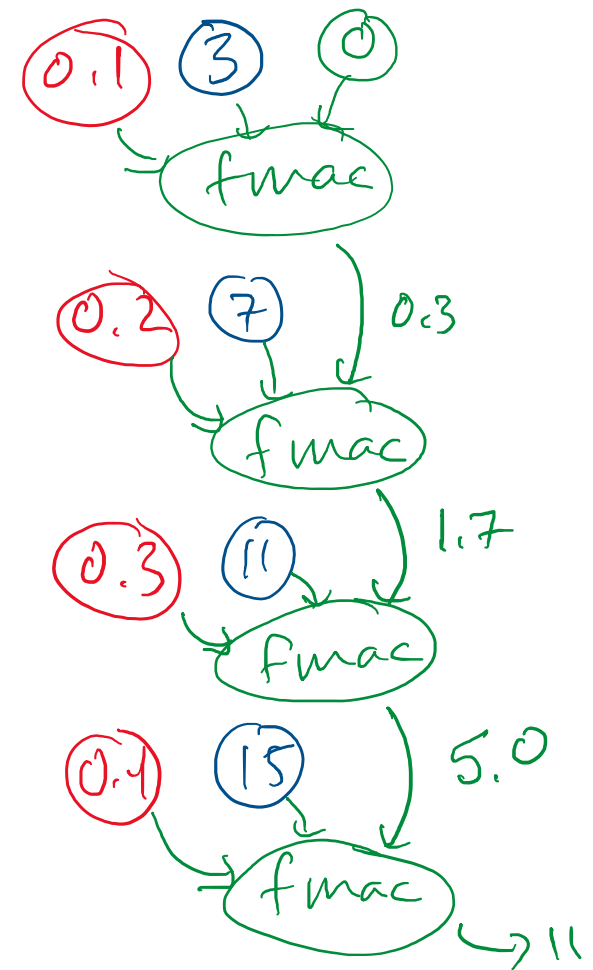
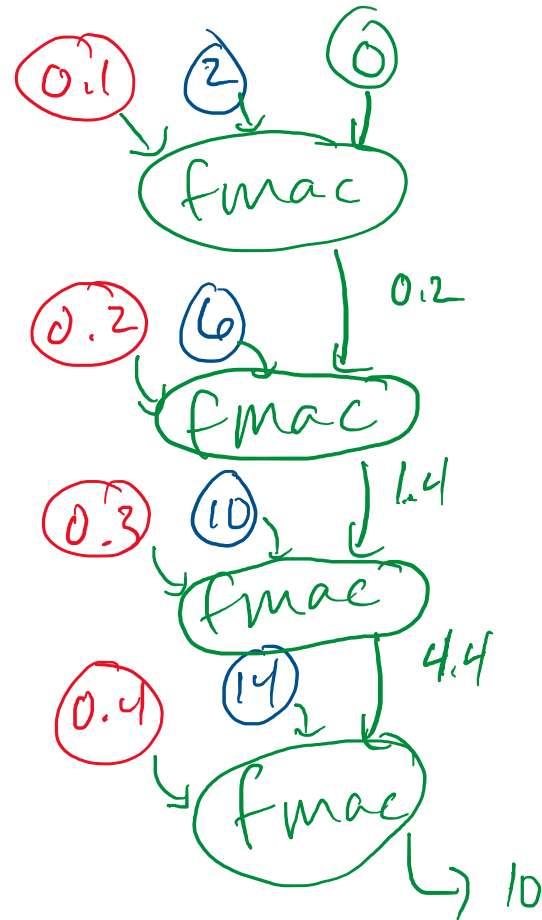
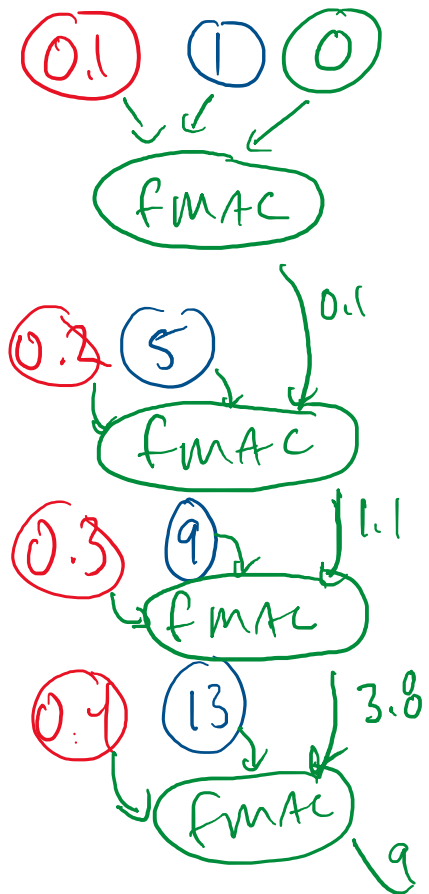
$$y = A \cdot B + C \cdot D + E \cdot F + G \cdot H$$



$$y = A \cdot B + C \cdot D + E \cdot F + G \cdot H$$

$$[0.1 \quad 0.2 \quad 0.3 \quad 0.4] \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = [9 \quad 10 \quad 11 \quad 12]$$

↑ outputs are independent



4th output not shown

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = \begin{bmatrix} 9 & 10 & 11 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = \begin{bmatrix} 9 & 10 & 11 & 12 \end{bmatrix}$$

$$[0.1 \ 0.2 \ 0.3 \ 0.4] \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = [9 \ 10 \ 11 \ 12]$$

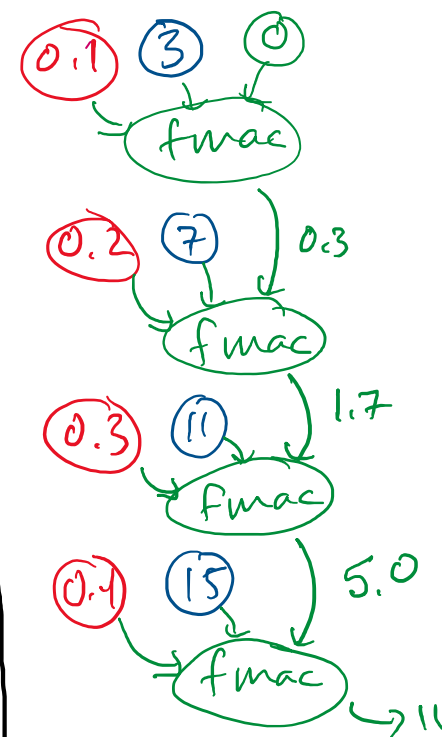
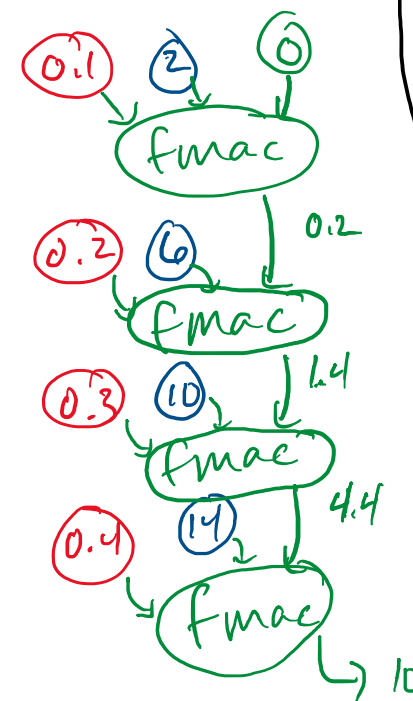
$$[0.1 \ 0.2 \ 0.3 \ 0.4] \begin{bmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \\ 13 & 14 \end{bmatrix} = [9 \ 10]$$

$$[0.1 \ 0.2 \ 0.3 \ 0.4] \begin{bmatrix} 3 & 4 \\ 7 & 8 \\ 11 & 12 \\ 15 & 16 \end{bmatrix} = [11 \ 12]$$

$$[0.1 \ 0.2 \ 0.3 \ 0.4] \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = [9 \ 10 \ 11 \ 12]$$

$$[0.1 \ 0.2 \ 0.3 \ 0.4] \begin{bmatrix} 1 & 2 \\ 5 & 6 \\ 9 & 10 \\ 13 & 14 \end{bmatrix} = [9 \ 10]$$

$$[0.1 \ 0.2 \ 0.3 \ 0.4] \begin{bmatrix} 3 & 4 \\ 7 & 8 \\ 11 & 12 \\ 15 & 16 \end{bmatrix} = [11 \ 12]$$

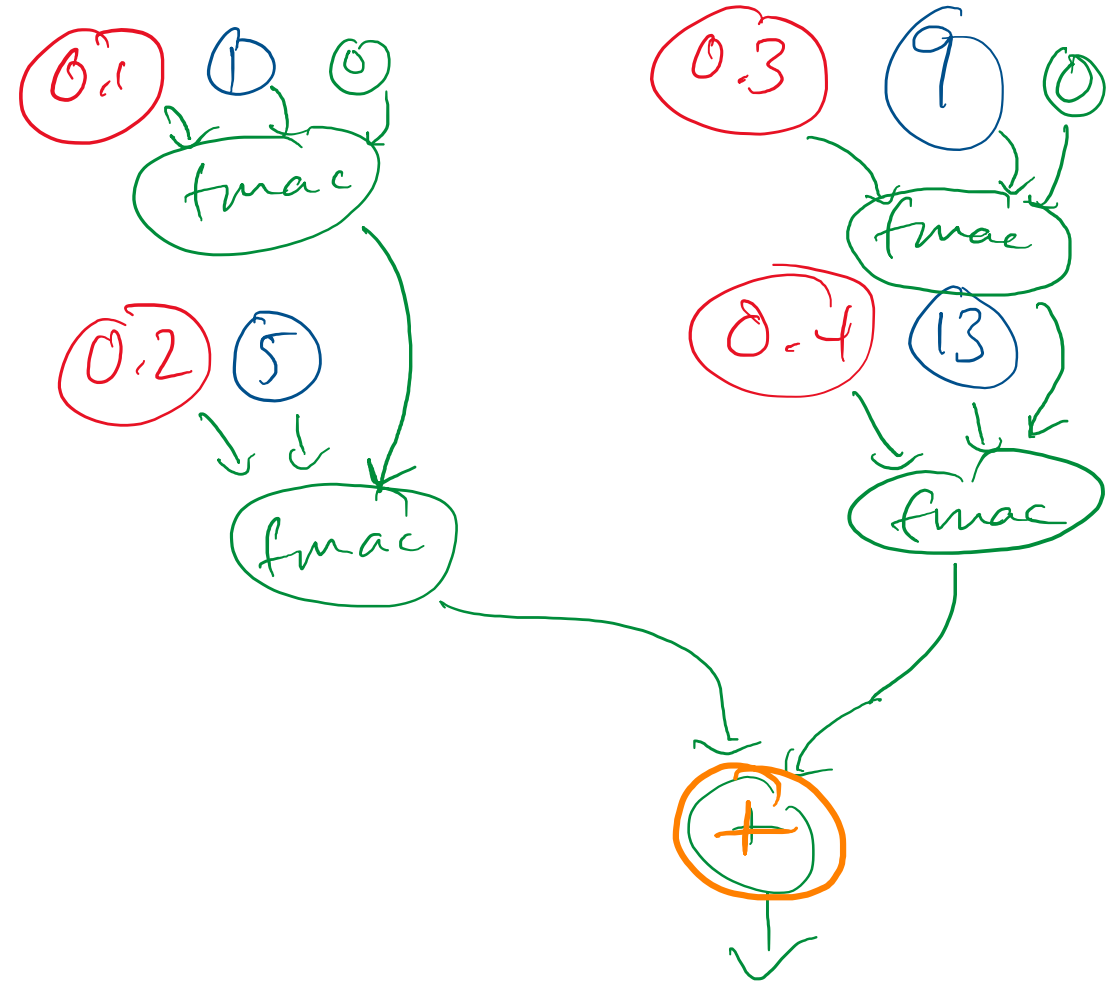


$$[0.1 \ 0.2 \ 0.3 \ 0.4] \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} = [9 \ 10 \ 11 \ 12]$$

$$[0.1 \ 0.2] \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

$$+ [0.3 \ 0.4] \begin{bmatrix} 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$[9 \ 10 \ 11 \ 12]$$



inputs: 0.1 0.2 0.3 0.4

$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

Can we parallelize Dot?

```
# how its done in dot.py
def pydot(inputs, weights):
    inputs = inputs[0] # remove outer nesting
    outs = np.zeros(weights.shape[1], dtype=np.float32)
    for i in range(weights.shape[0]): # input length
        for j in range(weights.shape[1]): # output length
            outs[j] = outs[j] + weights[i][j] * inputs[i]
    return outs
```

0.1 0.2 0.3 0.4

```
weights = np.array( [[1,2,3,4],[5,6,7,8],[9,10,11,12],[13,14,15,16]], dtype=np.float32)
inputs = np.array([[0.1,0.2,0.3,0.4]], dtype=np.float32)
outputs = np.dot(inputs, weights)
```

| Input | | Weights | | Output |
|-------------------|---|-------------------|---|-------------------|
| [0.1 0.2 0.3 0.4] | . | [1. 2. 3. 4.] | = | [9. 10. 11. 12.] |
| | | [5. 6. 7. 8.] | | |
| | | [9. 10. 11. 12.] | | |
| | | [13. 14. 15. 16.] | | |

```
outputs1 = np.dot([[0.1,0.2,0.3, 0.4]], [[1,2],[5,6],[9,10],[13,14]])
outputs2 = np.dot([[0.1,0.2,0.3, 0.4]], [[3,4],[7,8],[11,12],[15,16]])
outputs = np.concatenate((outputs1, outputs2), axis=1)
```

```
[[ 9. 10.]]
[[11. 12.]]
[[ 9. 10. 11. 12.]]
```

```
weights = np.array( [[1,2,3,4],[5,6,7,8],[9,10,11,12],[13,14,15,16]], dtype=np.float32)
inputs = np.array([[0.1,0.2,0.3,0.4]], dtype=np.float32)
outputs = np.dot(inputs, weights)
```

Input Weights Output

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 1. & 2. & 3. & 4. \\ 5. & 6. & 7. & 8. \\ 9. & 10. & 11. & 12. \\ 13. & 14. & 15. & 16. \end{bmatrix} = \begin{bmatrix} 9. & 10. & 11. & 12. \end{bmatrix}$$

```
outputs1 = np.dot([[0.1,0.2,0.3, 0.4]], [[1,2],[5,6],[9,10],[13,14]])
outputs2 = np.dot([[0.1,0.2,0.3, 0.4]], [[3,4],[7,8],[11,12],[15,16]])
outputs = np.concatenate((outputs1, outputs2), axis=1)
```

```
[[ 9. 10.]]
[[11. 12.]]
[[ 9. 10. 11. 12.]]
```

```
outputs1 = np.dot([[0.1, 0.3]], [[1,2,3,4],[9,10,11,12]])
outputs2 = np.dot([[0.2, 0.4]], [[5,6,7,8],[13,14,15,16]])
outputs = outputs1 + outputs2
```

```
[[2.8 3.2 3.6 4. ]]
[[6.2 6.8 7.4 8. ]]
→ [[ 9. 10. 11. 12.]]
```

Can we parallelize Dot?

Can we parallelize Dot?

21: Hardware Acceleration V

Engr 315: Hardware / Software Codesign
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