17: Pipelining II

ENGR 315: Hardware/Software CoDesign

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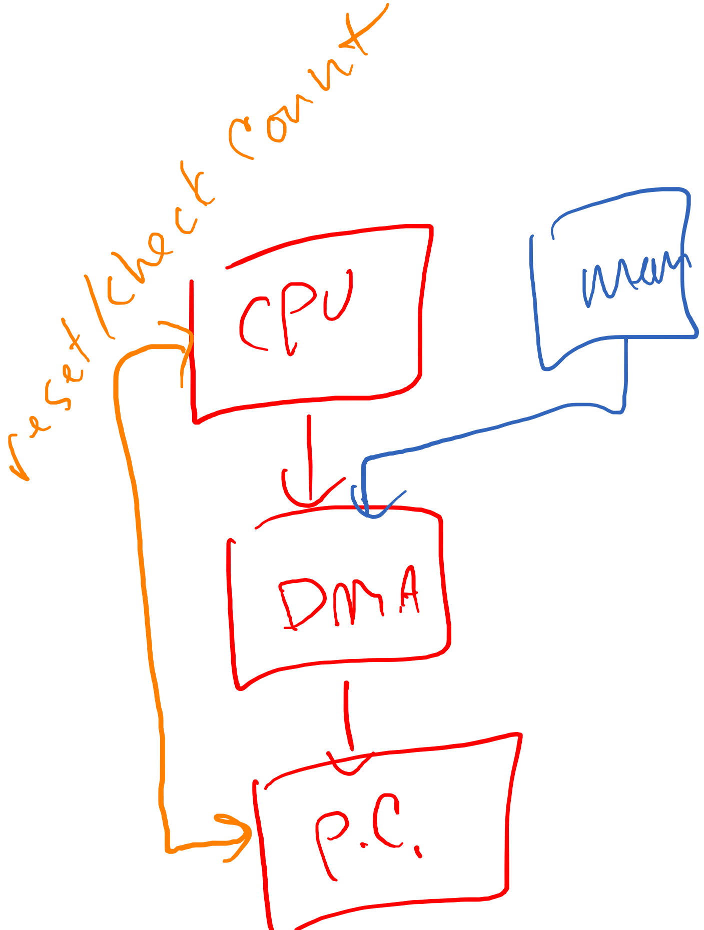
Some material taken from:
https://github.com/trekhleb/homemade-machine-learning/tree/master/homemade/neural_network
http://cs231n.github.io/neural-networks-1/

Announcements

- P6 out
- No Class on Wednesday (Oct 26th)
- New SD Cards coming for P7…
- Exam Nov 7th

P6: Adds DMA + AXI-Stream to Popcount



- DMA
  - Add DMA engine to move data via AXI4-Full to AXI-Stream interface
- Popcount.sv:
  - Add AXI-Stream Interface
  - Keep AXI4-Lite Interface to read result

P7 – DMA from C

1. Start the MM2S channel running by setting the run/stop bit to 1 (MM2S_DMACR.RS = 1). The halted bit (DMASR.Halted) should deassert indicating the MM2S channel is running.

2. Skip

3. Write a valid source address to the MM2S_SA register.

4. Write the number of bytes to transfer in the MM2S_LENGTH register.

   The MM2S_LENGTH register must be written last.

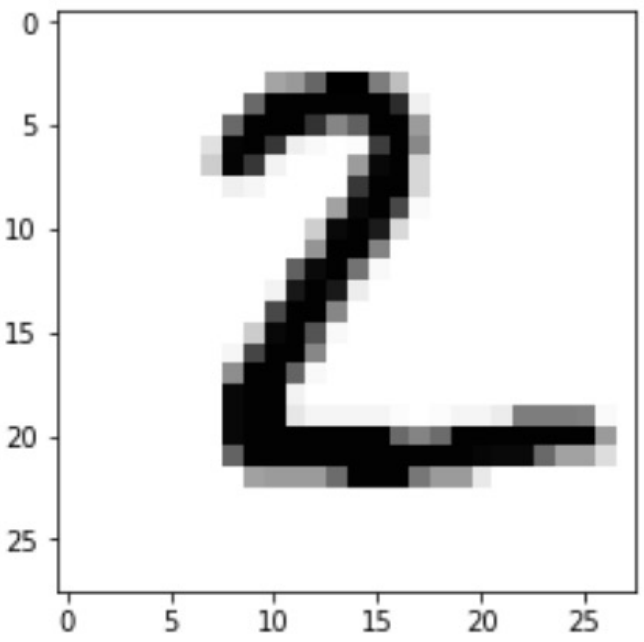
5. Wait until MM2S_DMASR.Idle==1 for completion

P8+ Accelerate Machine Learning

- Goal: Accelerate reference neural network
- Harder, more open-ended projects

Simple Neural Network

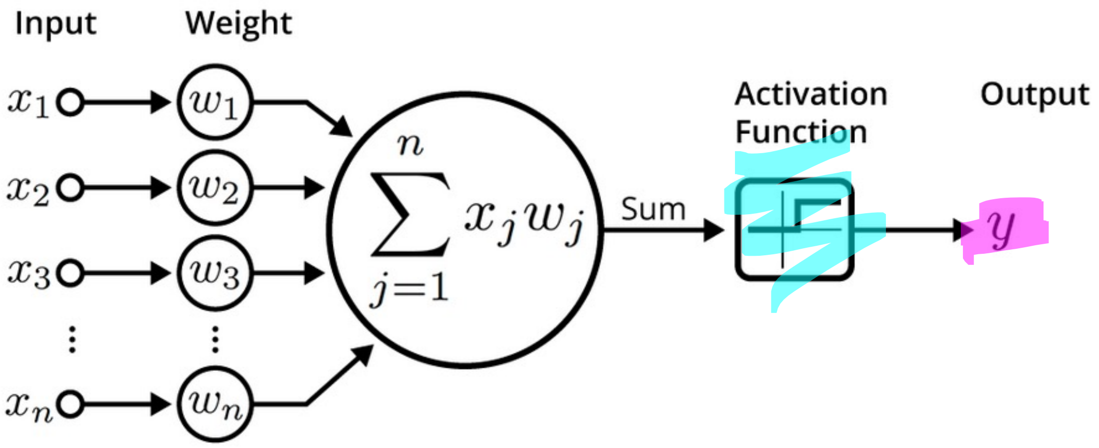
```
=================================
Index: 0
Image:
```




```
ML Classification Result: 2
Real Value: 2
Correct Result: True
=================================
```

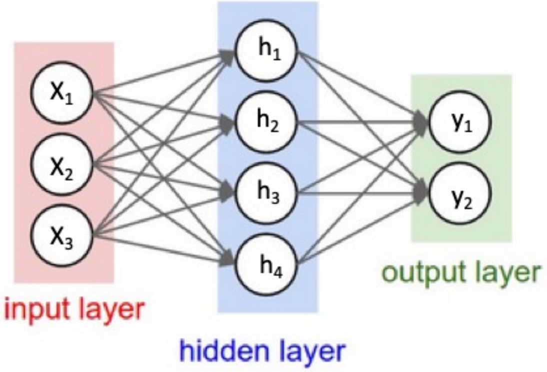
- Takes in image of number
- Returns integer value
- How? artificial neural network

Python Neuron




```
class Neuron(object):
  # ...
  def forward(self, inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

Why Dot Product?




```
# forward-pass of a 3-layer neural network:
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Matrix Multiplication (Dot Product)

$$\begin{bmatrix} i_0 & i_1 \end{bmatrix} \times \begin{bmatrix} w_{00} & w_{10} & w_{20} \\ w_{01} & w_{11} & w_{21} \end{bmatrix} = \begin{bmatrix} o_0 & o_1 & o_2 \end{bmatrix}$$

$$o_0 = i_0 \cdot w_{00} + i_1 \cdot w_{01}$$

$$o_1 = i_0 \cdot w_{10} + i_1 \cdot w_{11}$$

$$o_2 = i_0 \cdot w_{20} + i_1 \cdot w_{21}$$

Matrix Multiplication (Dot Product)

$$\underbrace{\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}}_{\text{inputs}} \times \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{\text{weights}} =$$

$$= \begin{bmatrix} (0.1 \times 1 + 0.2 \times 4) & (0.1 \times 2 + 0.2 \times 5) & (0.1 \times 3 + 0.2 \times 6) \end{bmatrix}$$

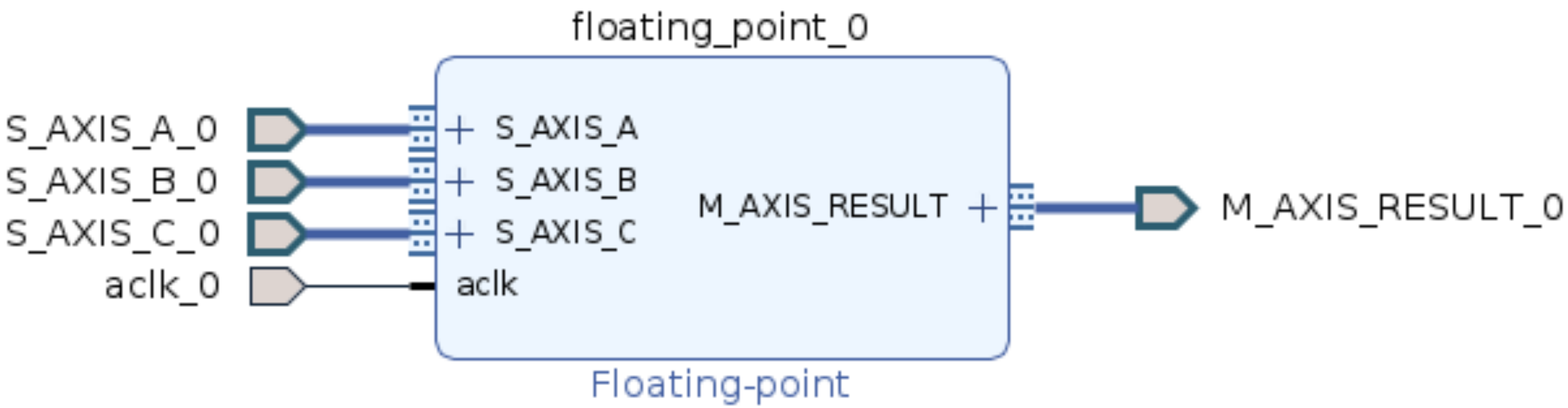
(Answer)

$$= \begin{bmatrix} 0.9 & 1.2 & 1.5 \end{bmatrix}$$

Floating-Point Multiply-Accumulate (FMAC)

- Math: a * b + c

Floating-Point Multiply in Hardware

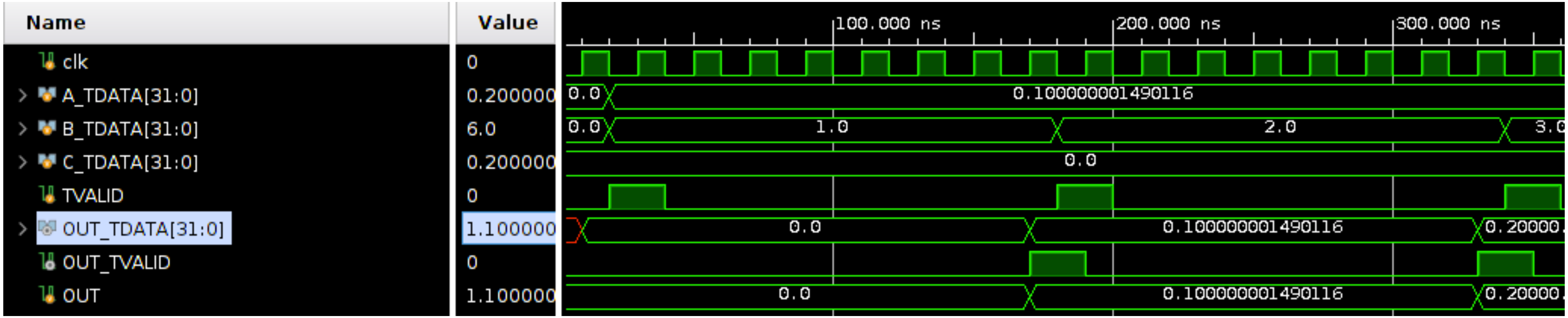


- result = a * b + c

Floating-Point math takes 8 cycles.

- Floating-Point is complicated.
- 8 cycles of complicated.

Demo Time



Name
Value
clk
0
A_TDATA[31:0]
0.200000
B_TDATA[31:0]
6.0
C_TDATA[31:0]
0.200000
TVALID
0
OUT_TDATA[31:0]
1.100000
OUT_TVALID
0
OUT
1.100000
100.000 ns
200.000 ns
300.000 ns
0.0
0.100000001490116
0.0
1.0
2.0
0.0
0.0
0.100000001490116
0.0
0.100000001490116

Floating-Point math takes 8 cycles.

- Floating-Point is complicated.
- 8 cycles of complicated.
- How do we work around an 8 cycle latency?
- Pipelining!

Floating-Point math takes 8 cycles.

$$\underbrace{A \cdot B + C}_{MAC}$$

- Floating-Point is complicated.
- How do we work around an 8 cycle latency?
- Pipelining!

Pipelining in hardware

```
   2        10        10        10
 x 3  =>  x 11  =>   x 1   +  x 10
 ---      ----      ----      ----
   6       110      10     +   100  = 110
```

Pipelining in hardware

```
  2          10           10           10
x 3  =>  x  11  =>    x  1     +  x  10
  6         110       10       +   100  = 110
```

FMAC Pipelining

	Cycles																			
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20
$0.1 \cdot 1 + 0$																				
$0.1 \cdot 2 + 0$																				
$0.1 \cdot 3 + 0$																				

FMAC Pipelining

Cycles

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20

Latency vs. Throughput

- **Latency**: How long does an individual operation take to complete?

- **Throughput**: How many operations can you complete per second (or per cycle)?

Pipelining

- FMAC takes 8 cycles for 1 value
- But can accept a new value every cycle.

- What is Latency:
- What is Throughput:

Recall: Matrix Multiplication (Dot Product)

$$\underset{\text{inputs}}{\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}} \times \overset{\text{weights}}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} =$$

$$= \begin{bmatrix} (0.1 \times 1 + 0.2 \times 4) & (0.1 \times 2 + 0.2 \times 5) & (0.1 \times 3 + 0.2 \times 6) \end{bmatrix}$$

(Answer)

$$= \begin{bmatrix} 0.9 & 1.2 & 1.5 \end{bmatrix}$$

Alternative Dot Computations

$$[0.1 \quad 0.2] \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = [0.9 \quad 1.2 \quad 1.5]$$

partial result

$$[0.1\cdot 1 \quad 0.1\cdot 2 \quad 0.1\cdot 3] = [0.1 \quad 0.2 \quad 0.3]$$

$$[0.2\cdot 4 \quad 0.2\cdot 5 \quad 0.2\cdot 6] = + [0.8 \quad 1.0 \quad 1.2]$$

$$[0.9 \quad 1.2 \quad 1.5]$$

Multiply-Accumulate Dot Computations

$$\begin{bmatrix} 0.1 & 0.2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 0.9 & 1.2 & 1.5 \end{bmatrix}$$

Python Time

$$\underset{\text{inputs}}{\begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix}} \times \overset{\text{weights}}{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}} =$$

```
weights = np.array( [[1,2,3,4],[5,6,7,8],[9,10,11,12]], dtype=np.float32)
inputs = np.array([[0.1,0.2,0.3]], dtype=np.float32)
outputs = np.dot(inputs, weights)
```

```
Input                    Weights                  Output
[0.1 0.2 0.3]     .      [1. 2. 3. 4.]      = [3.8000002 4.4      5.       5.6000004]
                         [5. 6. 7. 8.]
                         [ 9. 10. 11. 12.]
```

```
Input                Weights                Output
[[0.1 0.2 0.3]]   .   [1. 2. 3. 4.]      =  [3.8000002 4.4      5.       5.6000004]
                      [5. 6. 7. 8.]
                      [ 9. 10. 11. 12.]
```

Mult-Accum Dot

```
Inputs (Shape):
 (1, 3)
Output (Shape):
 (1, 4)
Weights (Shape):
 (3, 4)
```

```python
# how its done in dot.sv
def pydot(inputs,weights):
    inputs = inputs[0] # remove outer nesting
    outs = np.zeros(weights.shape[1], dtype=np.float32)
    for i in range(weights.shape[0]): # input length
        for j in range(weights.shape[1]): # output length
            outs[j] = outs[j] + weights[i][j] * inputs[i]
    return outs
```

```
Input                    Weights               Output
[[0.1 0.2 0.3]]    .    [1. 2. 3. 4.]      =  [3.8000002 4.4       5.        5.6000004]
                        [5. 6. 7. 8.]
                        [ 9. 10. 11. 12.]
```

Dependencies

Cycles

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20

Next Time: More on Dependencies

Latency on Pipelined FMAC

- Solution: Stall at the end of a row.
- Drain the pipeline.

Hardware Parallelism

- CPU: 1 Floating-Point Unit
- FPGA? 10 Floating-Point Units?

  20 ?

  100 ?

Finding Parallelism

- Some some computation that doesn't depend on other computation's results

- Shared Inputs are OK.

Next Time: Can we use 2+ FMACs?

$$[0.1 \quad 0.2 \quad 0.3] \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} = [3.8 \quad 4.4 \quad 5 \quad 5.6]$$

Stopped here!

cycle	FMAC comp
0	$0.1 \cdot 1 + 0$
1	$0.1 \cdot 2 + 0$
2	$0.1 \cdot 3 + 0$
3	$0.1 \cdot 4 + 0$

Option 1

Cycle	FMAC 1	FMAC 2
0	$0.1 \cdot 1 + 0$	$0.1 \cdot 2 + 0$
1	$0.1 \cdot 3 + 0$	$0.1 \cdot 3 + 0$

Option 2

→ $0.1 * 1 + 0$	$0.2 * 5 + 0$
$0.1 * 2 + 0$	$0.2 * 6 + 0$
$0.1 * 3 + 0$	$0.2 * 7 + 0$
$0.1 * 4 + 0$	$0.2 * 8 + 0$

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} = \begin{bmatrix} 3.8 & 4.4 & 5 & 5.6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

$$0.1 \cdot \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix}$$

$$0.2 \cdot \begin{bmatrix} 5 & 6 & 7 & 8 \end{bmatrix} + \begin{bmatrix} 0.1 & 0.2 & 0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} 1.1 & 1.4 & 1.7 & 2.0 \end{bmatrix}$$

$$0.3 \cdot \begin{bmatrix} 9 & 10 & 11 & 12 \end{bmatrix} + \begin{bmatrix} 1.1 & 1.4 & 1.7 & 2.0 \end{bmatrix} = \underline{\begin{bmatrix} 3.8 & 4.4 & 5 & 5.6 \end{bmatrix}}$$

Parallize Alternative Dot Computations?

$$[0.1 \quad 0.2] \times \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = [0.9 \quad 1.2 \quad 1.5]$$

$[0, \; 0, \; 0] \leftarrow$ result

$0.1 \cdot [1 \quad 2 \quad 3] + [0 \quad 0 \quad 0] =$

$[0.1 \quad 0.2 \quad 0.3] + [0 \quad 0 \quad 0] = [0.1 \quad 0.2 \quad 0.3] \leftarrow$ temp result

$0.2 \cdot [4 \quad 5 \quad 6] + [0.1 \quad 0.2 \quad 0.3] =$

$[0.8 \quad 1.0 \quad 1.2] + [0.1 \quad 0.2 \quad 0.3] = [0.9 \quad 1.2 \quad 1.5]$

Can we parallelize Dot?

```
# how its done in dot.sv
def pydot(inputs,weights):
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    for i in range(weights.shape[0]): # input length
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            outs[j] = outs[j] + weights[i][j] * inputs[i]
    return outs
```

Can we parallelize Dot?

```python
# how its done in dot.sv
def pydot(inputs,weights):
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    for i in range(weights.shape[0]): # input length
        for j in range(weights.shape[1]): # output length
            outs[j] = outs[j] + weights[i][j] * inputs[i]
    return outs
```

```python
def par_pydot(inputs, weights):
    par_inputs = [inputs[:,::2], inputs[:,1::2]]
    par_weights = [weights[::2,:], weights[1::2,:]]

    par_outputs = [pydot(par_inputs[0], par_weights[0]),
                   pydot(par_inputs[1], par_weights[1])]

    outputs = par_outputs[0] + par_outputs[1]
    return outputs
```

Can we parallelize Dot?

```
# how its done in dot.sv
def pydot(inputs,weights):
    inputs = inputs[0] # remove outer nesting
    outs = np.zeros(weights.shape[1], dtype=np.float32)
    for i in range(weights.shape[0]): # input length
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```
def par_pydot(inputs, weights):
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    par_weights = [weights[::2,:], weights[1::2,:]]

    par_outputs = [pydot(par_inputs[0], par_weights[0]),
                   pydot(par_inputs[1], par_weights[1])]

    outputs = par_outputs[0] + par_outputs[1]
    return outputs
```

19: Hardware Acceleration III

Engr 315: Hardware / Software Codesign
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