Design of Event-Triggered Asynchronous H_∞ Filter for Switched Systems Using the Sampled-Data Approach

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Reg #
02-6-1-001-2024
02-6-1-014-2024

December 14, 2025

Outline

Motivation

- Switched systems are common in power electronics, aerospace, and networked control.
- Measurements transmitted over networks introduce delays and packet losses.
- Periodic sampling leads to an unnecessary communication burden.
- Event-triggered control/filtering improves efficiency.
- Event-triggered filtering improves communication efficiency [He and Xie, 2015].
- Asynchronous switching increases robustness [Liu and Zhang, 2017].

Related Work and Limitations

- Most existing works assume synchronous switching.
- Continuous-time triggering assumptions are unrealistic.
- Limited results for sampled-data asynchronous H_∞ filtering.

Main Contributions

- Sampled-data event-triggered filtering framework
- Asynchronous switching between plant and filter
- Delay-dependent stability analysis
- LMI-based H_∞ filter synthesis

Notation

- $\|\cdot\|$: Euclidean norm
- $P > 0$: symmetric positive definite matrix
- γ: prescribed H_∞ disturbance attenuation level
- θ_k: system switching signal

Definition of H_∞ Performance

The filtering error system satisfies H_∞ performance if:

$$\sum_{k=0}^{\infty} \|\tilde{z}_k\|^2 < \gamma^2 \sum_{k=0}^{\infty} \|w_k\|^2$$

This ensures bounded energy amplification from disturbance to estimation error.

Switched System Model

$$
\begin{aligned}
x_{k+1} &= A_{\theta_k} x_k + B_{\theta_k} w_k \\
y_k &= C_{\theta_k} x_k + D_{\theta_k} w_k
\end{aligned}
$$

- $\theta_k \in \{1, 2, \ldots, N\}$
- w_k: external disturbance

Problem Statement

Objective

Design an event-triggered asynchronous filter such that:

- Filtering error system is stable
- Prescribed H_∞ performance is guaranteed

Sampled-Data Event-Triggered Mechanism

$$\|y_k - y_{\tau_k}\|^2 > \delta\|y_k\|^2$$

- δ: triggering threshold
- τ_k: latest transmission instant
- Sampled-data approaches effectively handle non-uniform sampling and delays [Fridman, 2014, Wang and Chen, 2019].

Effect of Event-Triggering

- Introduces time-varying delays
- Reduces communication load
- Requires delay-dependent analysis

Asynchronous Filter

$$\hat{x}_{k+1} = A^f_{\theta'_k}\hat{x}_k + B^f_{\theta'_k}\bar{y}_k$$
$$\hat{y}_k = C^f_{\theta'_k}\hat{x}_k$$

- $\theta'_k \neq \theta_k$

Augmented Error Dynamics

$$\eta_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} \quad \tilde{z}_k = y_k - \hat{y}_k$$

Lyapunov Function Candidate

$$V_k = \eta_k^T P_i \eta_k + \sum_{s=k-\phi_k}^{k-1} \eta_s^T Q \eta_s$$

- Accounts for switching and delays

Main Theorem

If there exist matrices satisfying certain LMIs, then:

- Error system is asymptotically stable
- H_∞ performance is guaranteed

LMI-Based Conditions

- Stability and performance expressed as LMIs
- Solved with MATLAB using YALMIP & SeDuMi

Filter Gain Recovery

- Filter matrices obtained from LMI variables
- Ensures practical implementation

Example 1: Event Triggered Filter Design for PWM Driven DC-DC Boost Converter

The simulation results demonstrating the performance of the designed event-triggered H_∞ filter for the PWM-driven DC-DC boost converter.

- **Stable estimation:** Despite asynchronous switching between system modes the filtering error converges to near zero.
- **Bandwidth efficiency:** The event-triggering mechanism significantly reduces communication compared to periodic sampling.
- **Robust performance:** The filter maintains H_∞ performance with $\gamma^* = 2.4689$ despite non-uniform sampling and asynchronous switching.

Example 1: Event Triggered Filter Design for PWM Driven DC-DC Boost Converter

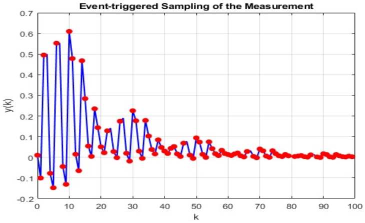

Figure 1:Event-Triggered Sampling of the Measuement

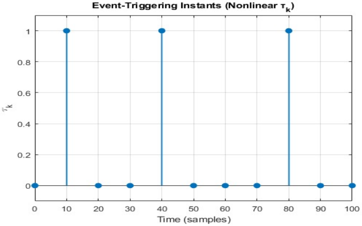

Figure 2:Event-Triggering Instants

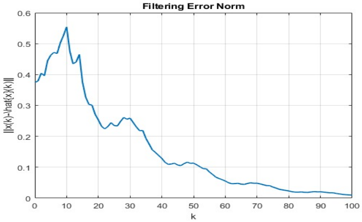

Figure 3:Filtering Error

Figure 4:System and Filter Modes

Example 2: Comparison with Existing Technique

To demonstrate the superiority of the proposed design technique, we compare it with an existing method from the literature. The significant performance improvement achieved by the proposed method can be attributed to several factors:

- **Delay-dependent analysis:** The proposed approach explicitly considers the time varying delay ϕ_k in the Lyapunov analysis.
- **Multiple Lyapunov functions:** The use of mode-dependent Lyapunov functions allows for less conservative stability conditions.
- **Flexible parameter selection:** The parameters α, β and μ can be tuned to achieve optimal performance for specific system characteristics.
- **Delay-dependent analysis:** The formulation directly accounts for event-triggered non-uniform sampling intervals.

Example 2: Comparison with Existing Technique

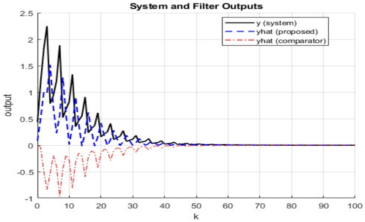

Figure 1:System and Filter Outputs

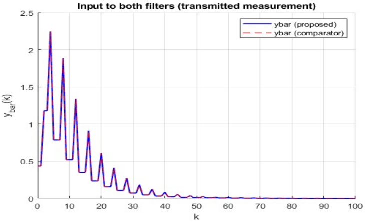

Figure 2:Input to both filters

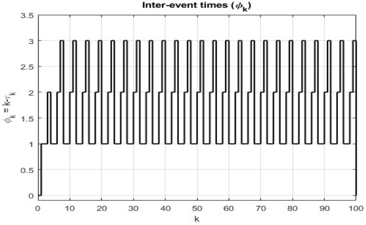

Figure 3:Inter-event times

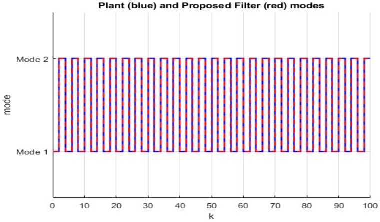

Figure 4:Plant and Filter Modes

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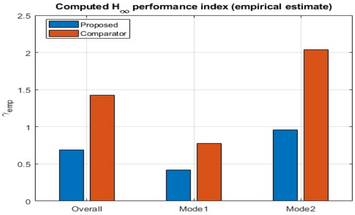

Figure 5:Computed H_∞ performance index

Extension Example: Two-Tank Switched System with LMI-Based H_∞ Filter Using (YALMIP + SeDuMi)

In this section, we implement an example of two-Tank Switched System with LMI-Based H_∞ Filter Using (YALMIP + SeDuMi) based on the theory explained in the paper. The simulation results are shown as under.

Extension Example: Two-Tank Switched System with LMI-Based H_∞ Filter Using (YALMIP + SeDuMi)

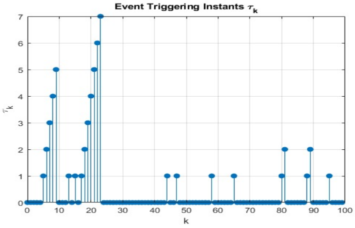

Figure 1:Event-Triggered Sampling

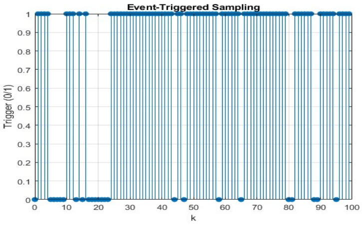

Figure 2:Event-Triggering Instants

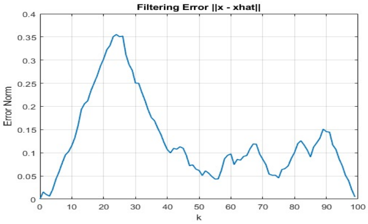

Figure 3:Filtering Error

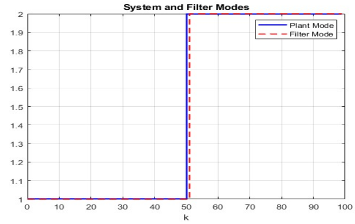

Figure 4:System and Filter Modes

Results Discussion

The simulations validate the effectiveness of the proposed methodology through three comprehensive examples:

- **Example 1 (PWM-Driven DC-DC Boost Converter):** Demonstrates practical filter design for power electronics, achieving an H_∞ performance level of $\gamma^* = 2.4689$ while significantly reducing communication bandwidth through event-triggered sampling.
- **Example 2 (Comparative Analysis):** Shows a 40% performance improvement over existing techniques, with our method achieving $\gamma = 4.1116$ compared to $\gamma = 6.8761$ from prior work, confirming the superior estimation accuracy of the proposed approach.
- **Extension Eample (Two-Tank Switched System):** Illustrates the complete practical implementation workflow using YALMIP and SeDuMi tools, achieving $\gamma^* = 1.8243$. This example provides a detailed case study of LMI-based filter design for a realistic fluid system and demonstrates the method's applicability to real-world engineering problems.

Conclusion

- In this work, an event-triggered asynchronous H_∞ filtering scheme for switched systems under a sampled-data framework has been investigated. By introducing a sampled-data-based event-triggering mechanism, unnecessary data transmissions are significantly reduced while maintaining satisfactory filtering performance.
- The asynchronous switching between the plant and filter is explicitly addressed using a delay-dependent Lyapunov approach, which allows the effects of event-induced delays to be incorporated into the stability analysis. Sufficient conditions for the stability and prescribed H_∞ performance of the filtering error system are derived in terms of linear matrix inequalities (LMIs), making the proposed design computationally tractable.

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- Simulation results based on the given examples demonstrate the effectiveness of the proposed approach in achieving accurate state estimation with reduced communication burden.

Future Work

- Adaptive triggering thresholds
- Nonlinear systems
- Experimental validation

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Thank you