Design of Event-Triggered Asynchronous H_∞ Filter for Switched Systems Using the Sampled-Data Approach

Robust Control System

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Abstract

Event-triggered communication offers significant bandwidth and energy savings in networked switched systems but introduces challenges in stability and performance due to asynchronous switching and time-varying dynamics. This paper introduces a sampled-data method for designing event-triggered asynchronous H_∞ filters. Departing from conventional co-design frameworks, we treat the event-triggering policy as given and synthesize the filter accordingly. The filtering error system is characterized as a delayed switched system under non-uniform sampling. Through a delay-dependent multiple Lyapunov analysis, we establish conditions ensuring global asymptotic stability and prescribed H_∞ attenuation. Simulation results validate the proposed design and highlight its performance benefits compared to recent alternatives.

Keywords: Event-triggered communication, switched systems, H_∞ filtering, asynchronous switching, sampled-data systems, networked control systems.

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Chapter 1

Introduction

1.1 Switched Dynamical Systems

Switched dynamical systems are hybrid systems that involve several subsystems and a switching rule orchestrating the switching among the subsystems. Switched dynamical systems find applications in a variety of engineering fields, e.g., robotic systems [1, 2], automobile systems [3], DC-to-DC converters [4, 5], chemical reactors, oscillators, and chaos generators [6, 7, 8].

1.2 Networked Control Systems

Networked control is a control scheme where measurement feedback and control actions route through a communication network. Such a scheme offers many advantages, such as low installation cost, reduced maintenance cost, and flexible system structure [9]. Networked control of switched dynamical systems is an active research area, with many researchers addressing problems in this area, see for example [10] and [11].

Existing results on networked control of switched systems either consider problems in the continuous time or use a periodic sampling scheme [12]. However, periodic sampling often results in the wastage of network resources. An alternative to periodic sampling is event-based sampling, which saves energy and communication resources [13]. Recent approaches to event-triggered control use adaptive event-triggering schemes that offer a further reduction in the usage of network resources [14, 15, 16].

1.3 Event-Triggered Filtering for Switched Systems

Event-based filtering for networked switched systems has been addressed in [6, 18, 19, 20, 21]. In [6], an asynchronous finite-time stable filter is devised for continuous-time switched systems. In [18], an event-based H_∞ filtering problem is addressed for networked switching systems in continuous time domain. Finite-time stability and H_∞ performance of the filtering error system are proved by using a delay-dependent Lyapunov functional.

In [19], an event-triggered H_∞ filter is designed to ensure stability in the presence of packet disorders and maintain an H_∞ performance level. In [20], an event-triggered fault detection H_∞ filter is designed. A discrete-time event-triggered H_∞ filter is designed in [21]. By using multiple Lyapunov function approach, sufficient conditions are derived for exponential stability and H_∞ performance of the error system.

1.4 Research Gap and Contributions

While significant progress has been made in event-triggered filtering for switched systems, most existing approaches either design only the event-triggering scheme or co-design both the event-triggering mechanism and the filter simultaneously. Furthermore, many studies focus on continuous-time systems or rely on uniform sampling approaches. This paper addresses these gaps by proposing a sampled-data approach for designing H_∞ filters under a predefined event-triggering policy, specifically considering the challenges of asynchronous switching and non-uniform sampling intervals.

Chapter 2

Derivations

2.1 Filtering Error System Dynamics

Let $\tilde{z}_k = y_k - \hat{y}_k$ denote the estimation error. From Assumption 2, the input to the filter during each transmission interval will be

$$\bar{y}_k = y_k - \phi_k, \quad \phi_k \in \mathcal{I}_\phi, \quad k \in [\tau_k, \tau_{k+1}). \tag{2.1}$$

The transmission interval can be partitioned as $[\tau_k, \tau_{k+1}) = [\tau_k, \tau_k + T_{\max}) \cup [\tau_k + T_{\max}, \tau_{k+1})$. Different system and filter modes are activated when $k \in [\tau_k, \tau_k + T_{\max})$. Let $\theta_k = i$, $\theta'_k = j$, and $\phi_k = 0$, the dynamics of the filtering error system will be:

$$\begin{aligned}
\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & A_f^j \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_f^j C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\
&\quad + \begin{bmatrix} B_i & 0 \\ 0 & B_f^j D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix}
\end{aligned} \tag{2.2}$$

$$\begin{aligned}
\tilde{z}_k &= \begin{bmatrix} C_i & -C_f^j \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_f^j C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} \\
&\quad + \begin{bmatrix} D_i & -D_f^j D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix}
\end{aligned} \tag{2.3}$$

When $k \in [\tau_k + T_{\max}, \tau_{k+1})$, the system and filter modes will be synchronized. Let

$\theta_k = \theta'_k = i$, and $\phi_k = 0$, the dynamics of the filtering error system will be:

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & A_f^i \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_f^i C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} + \begin{bmatrix} B_i & 0 \\ 0 & B_f^i D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix} \tag{2.4}$$

$$\tilde{z}_k = \begin{bmatrix} C_i & -C_f^i \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_f^i C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-0} \\ \hat{x}_{k-0} \end{bmatrix} + \begin{bmatrix} D_i & -D_f^i D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-0} \end{bmatrix} \tag{2.5}$$

When $\phi_k = 1$, $\theta_k = i$, and $\theta'_k = j$, the dynamics of the error system will be:

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & A_f^j \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_f^j C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} + \begin{bmatrix} B_i & 0 \\ 0 & B_f^j D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \tag{2.6}$$

$$\tilde{z}_k = \begin{bmatrix} C_i & -C_f^j \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_f^j C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} + \begin{bmatrix} D_i & -D_f^j D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \tag{2.7}$$

Similarly, when $\phi_k = 1$, $\theta_k = \theta'_k = i$, the dynamics of the error system will be:

$$\begin{bmatrix} x_{k+1} \\ \hat{x}_{k+1} \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & A_f^i \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_f^i C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} + \begin{bmatrix} B_i & 0 \\ 0 & B_f^i D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \tag{2.8}$$

$$\tilde{z}_k = \begin{bmatrix} C_i & -C_f^i \end{bmatrix} \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} + \begin{bmatrix} -D_f^i C_i & 0 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \hat{x}_{k-1} \end{bmatrix} + \begin{bmatrix} D_i & -D_f^i D_i \end{bmatrix} \begin{bmatrix} w_k \\ w_{k-1} \end{bmatrix} \tag{2.9}$$

In general, the dynamics of the error system can be written as:

$$\eta_{k+1} = \bar{A}_{ij}\eta_k + \bar{A}_d^{ij}\eta_{k-\phi_k} + \bar{B}_{\phi_k,ij}\bar{w}_k, \quad k \in [\tau_k, \tau_k + T_{\max}) \tag{2.10a}$$

$$\tilde{z}_k = \bar{C}_{ij}\eta_k + \bar{C}_d^{ij}\eta_{k-\phi_k} + \bar{D}_{\phi_k,ij}\bar{w}_k, \quad k \in [\tau_k, \tau_k + T_{\max}) \tag{2.10b}$$

$$\eta_{k+1} = \bar{A}_i\eta_k + \bar{A}_d^i\eta_{k-\phi_k} + \bar{B}_{\phi_k,i}\bar{w}_k, \quad k \in [\tau_k + T_{\max}, \tau_{k+1}) \tag{2.10c}$$

$$\tilde{z}_k = \bar{C}_i\eta_k + \bar{C}_d^i\eta_{k-\phi_k} + \bar{D}_{\phi_k,i}\bar{w}_k, \quad k \in [\tau_k + T_{\max}, \tau_{k+1}) \tag{2.10d}$$

where $\eta_k = [x_k^T, \hat{x}_k^T]^T$, $\bar{w}_k = [w_k^T, w_{k-1}^T, \ldots, w_{k-M+1}^T]^T$, and

$$\bar{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & A_f^j \end{bmatrix}, \quad \bar{A}_d^{ij} = \begin{bmatrix} 0 & 0 \\ B_f^j C_i & 0 \end{bmatrix} \tag{2.11}$$

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_f^i \end{bmatrix}, \quad \bar{A}_d^i = \begin{bmatrix} 0 & 0 \\ B_f^i C_i & 0 \end{bmatrix} \tag{2.12}$$

$$\bar{C}_{ij} = \begin{bmatrix} C_i & -C_f^j \end{bmatrix}, \quad \bar{C}_d^{ij} = \begin{bmatrix} -D_f^j C_i & 0 \end{bmatrix} \tag{2.13}$$

$$\bar{C}_i = \begin{bmatrix} C_i & -C_f^i \end{bmatrix}, \quad \bar{C}_d^i = \begin{bmatrix} -D_f^i C_i & 0 \end{bmatrix} \tag{2.14}$$

The input matrices are defined as:

$$\bar{B}_{\phi_k,ij} = \begin{cases} \begin{bmatrix} B_i & 0 & \cdots & 0 \\ B_f^j D_i & 0 & \cdots & 0 \end{bmatrix}, & \phi_k = 0 \\ \vdots & \\ \begin{bmatrix} B_i & 0 & \cdots & 0 \\ 0 & 0 & \cdots & B_f^j D_i \end{bmatrix}, & \phi_k = M-1 \end{cases} \tag{2.15}$$

$$\bar{B}_{\phi_k,i} = \begin{cases} \begin{bmatrix} B_i & 0 & \cdots & 0 \\ B_f^i D_i & 0 & \cdots & 0 \end{bmatrix}, & \phi_k = 0 \\ \vdots & \\ \begin{bmatrix} B_i & 0 & \cdots & 0 \\ 0 & 0 & \cdots & B_f^i D_i \end{bmatrix}, & \phi_k = M-1 \end{cases} \tag{2.16}$$

$$\bar{D}_{\phi_k,ij} = \begin{cases} \begin{bmatrix} D_i & -D_f^j D_i & 0 & \cdots & 0 \end{bmatrix}, & \phi_k = 0 \\ \vdots & \\ \begin{bmatrix} D_i & 0 & \cdots & -D_f^j D_i \end{bmatrix}, & \phi_k = M-1 \end{cases} \tag{2.17}$$

$$\bar{D}_{\phi_k,i} = \begin{cases} \begin{bmatrix} D_i & -D_f^i D_i & 0 & \cdots & 0 \end{bmatrix}, & \phi_k = 0 \\ \vdots & \\ \begin{bmatrix} D_i & 0 & \cdots & -D_f^i D_i \end{bmatrix}, & \phi_k = M-1 \end{cases} \tag{2.18}$$

Notice that the error system is a switched time-delay system where the time-dependent delay ϕ_k takes values in the interval $\mathcal{I}_\phi$.

2.2 Stability Analysis and Filter Design

2.2.1 Main Stability Theorem

Consider $0 < \alpha < 1$, $\mu \geq 1$, and $\beta \geq 0$ are specified. The error dynamics in (2.10) are globally uniformly asymptotically stable and have H_∞ performance γ^*, where $\gamma^* = \max\{\sqrt{T_{\max} - 1}\,\gamma_i\}$, if one can find matrices $P_i > 0$ and $Q_{r,i} > 0$ for all $i, j \in \mathcal{I}_N$, $i \neq j$, and $r \in \mathcal{I}_\phi$ such that $P_i \leq \mu P_j$, $Q_{r,i} \leq \mu Q_{r,j}$ and the following conditions hold:

$$\begin{bmatrix} -P_i & 0 & P_i\tilde{A}_{ir} & P_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Phi_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \leq 0 \tag{2.19a}$$

$$\begin{bmatrix} -P_i & 0 & P_i\tilde{A}_{ijr} & P_i\tilde{B}_{ijr} \\ * & -I & \tilde{C}_{ijr} & \tilde{D}_{ijr} \\ * & * & \Psi_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \leq 0 \tag{2.19b}$$

where

$$
\begin{aligned}
\tilde{A}_{ir} &= \begin{bmatrix} \bar{A}_i + \epsilon_0 \bar{A}_d^i & \epsilon_1 \bar{A}_d^i & \cdots & \epsilon_r \bar{A}_d^i \end{bmatrix} \\
\tilde{A}_{ijr} &= \begin{bmatrix} \bar{A}_{ij} + \epsilon_0 \bar{A}_d^{ij} & \epsilon_1 \bar{A}_d^{ij} & \cdots & \epsilon_r \bar{A}_d^{ij} \end{bmatrix} \\
\tilde{B}_{ir} &= \begin{bmatrix} B_i & 0 & \cdots & 0 \\ \epsilon_0 B_f^i D_i & \epsilon_1 B_f^i D_i & \cdots & \epsilon_r B_f^i D_i \end{bmatrix} \\
\tilde{B}_{ijr} &= \begin{bmatrix} B_i & 0 & \cdots & 0 \\ \epsilon_0 B_f^j D_i & \epsilon_1 B_f^j D_i & \cdots & \epsilon_r B_f^j D_i \end{bmatrix} \\
\tilde{C}_{ir} &= \begin{bmatrix} \bar{C}_i + \epsilon_0 \bar{C}_d^i & \epsilon_1 \bar{C}_d^i & \cdots & \epsilon_r \bar{C}_d^i \end{bmatrix} \\
\tilde{C}_{ijr} &= \begin{bmatrix} \bar{C}_{ij} + \epsilon_0 \bar{C}_d^{ij} & \epsilon_1 \bar{C}_d^{ij} & \cdots & \epsilon_r \bar{C}_d^{ij} \end{bmatrix} \\
\tilde{D}_{ir} &= \begin{bmatrix} D_i & -\epsilon_0 D_f^i D_i & -\epsilon_1 D_f^i D_i & \cdots & -\epsilon_r D_f^i D_i \end{bmatrix} \\
\tilde{D}_{ijr} &= \begin{bmatrix} D_i & -\epsilon_0 D_f^j D_i & -\epsilon_1 D_f^j D_i & \cdots & -\epsilon_r D_f^j D_i \end{bmatrix} \\
\Phi_{ir} &= \begin{bmatrix} Q_{1,j} - \bar{\alpha} P_i & 0 & \cdots & 0 \\ 0 & Q_{2,j} - \bar{\alpha} Q_{1,i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\bar{\alpha} Q_{r,i} \end{bmatrix} \\
\Psi_{ijr} &= \begin{bmatrix} Q_{1,j} - \bar{\beta} P_i & 0 & \cdots & 0 \\ 0 & Q_{2,j} - \bar{\beta} Q_{1,i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\bar{\beta} Q_{r,i} \end{bmatrix} \\
\epsilon_r &= \begin{cases} 1, & r = \phi_k \\ 0, & \text{otherwise} \end{cases} \\
\bar{\alpha} &= 1 - \alpha, \quad \bar{\beta} = 1 + \beta
\end{aligned}
$$

First, we prove the asymptotic stability of (2.10) by considering the following switched Lyapunov function:

$$
V_{\theta_k}(\eta_k) = V_{1,\theta_k}(\eta_k) + V_{2,\theta_k}(\eta_k, \ldots, \eta_{k-M+1}) \tag{2.20}
$$

where

$$
V_{1,\theta_k}(\eta_k) = \eta_k^T P_{\theta_k} \eta_k \tag{2.21}
$$

$$
V_{2,\theta_k}(\eta_k, \ldots, \eta_{k-M+1}) = \sum_{r=1}^{M-1} \eta_{k-r}^T Q_{r,\theta_k} \eta_{k-r} \tag{2.22}
$$

With $\bar{w}_k = 0$, the system in (2.10) becomes:

$$\begin{aligned}
\eta_{k+1} &= \bar{A}_{ij}\eta_k + \bar{A}_d^{ij}\eta_{k-\phi_k} \\
\tilde{z}_k &= \bar{C}_{ij}\eta_k + \bar{C}_d^{ij}\eta_{k-\phi_k}, \quad k \in [\tau_k, \tau_k + T_{\max}) \\
\eta_{k+1} &= \bar{A}_i\eta_k + \bar{A}_d^{i}\eta_{k-\phi_k} \\
\tilde{z}_k &= \bar{C}_i\eta_k + \bar{C}_d^{i}\eta_{k-\phi_k}, \quad k \in [\tau_k + T_{\max}, \tau_{k+1})
\end{aligned}$$

For $\theta_k = i$ and $\theta_{k+1} = j$, taking the increment of the Lyapunov function $\Delta V_i(\eta_k) = V_i(\eta_{k+1}) - V_i(\eta_k)$ along the state path of the estimation error system, we obtain:

$$\begin{aligned}
\Delta V_{1,i} &= \eta_{k+1}^T P_j \eta_{k+1} - \eta_k^T P_i \eta_k \\
\Delta V_{2,i} &= \sum_{r=1}^{M-1} \eta_{k+1-r}^T Q_{r,j} \eta_{k+1-r} - \sum_{r=1}^{M-1} \eta_{k-r}^T Q_{r,i} \eta_{k-r}
\end{aligned}$$

Then, for $k \in [\tau_k, \tau_k + T_{\max})$:

$$\Delta V_i - \beta V_i = \|\eta_{k+1}\|_{P_j}^2 - \bar{\beta}\|\eta_k\|_{P_i}^2 + \sum_{r=1}^{M-1} \|\eta_{k+1-r}\|_{Q_{r,j}}^2 - \bar{\beta} \sum_{r=1}^{M-1} \|\eta_{k-r}\|_{Q_{r,i}}^2$$

and for $k \in [\tau_k + T_{\max}, \tau_{k+1})$:

$$\Delta V_i + \alpha V_i = \|\eta_{k+1}\|_{P_j}^2 - \bar{\alpha}\|\eta_k\|_{P_i}^2 + \sum_{r=1}^{M-1} \|\eta_{k+1-r}\|_{Q_{r,j}}^2 - \bar{\alpha} \sum_{r=1}^{M-1} \|\eta_{k-r}\|_{Q_{r,i}}^2$$

With further manipulation, we can write:

$$\begin{cases}
\Delta V_i - \beta V_i = \zeta_k^T \left(\tilde{A}_{ijr}^T P_j \tilde{A}_{ijr} + \Psi_{ijr}\right)\zeta_k, & k \in [\tau_k, \tau_k + T_{\max}) \\
\Delta V_i + \alpha V_i = \zeta_k^T \left(\tilde{A}_{ir}^T P_j \tilde{A}_{ir} + \Phi_{ir}\right)\zeta_k, & k \in [\tau_k + T_{\max}, \tau_{k+1})
\end{cases} \tag{2.23}$$

where $\zeta_k = \begin{bmatrix} \eta_k^T & \eta_{k-1}^T & \cdots & \eta_{k-M+1}^T \end{bmatrix}^T$.

If the inequalities in (2.19a) and (2.19b) hold, then $\tilde{A}_{ijr}^T P_j \tilde{A}_{ijr} + \Psi_{ijr} \leq 0$ and $\tilde{A}_{ir}^T P_j \tilde{A}_{ir} + \Phi_{ir} \leq 0$ for $i, j \in \mathcal{I}_N$ and $r \in \mathcal{I}_\phi$. Therefore, the error system will be asymptotically stable.

Now for $k \in [\tau_k, \tau_k + T_{\max})$:

$$\begin{aligned}
\tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k = {} & \zeta_k^T \tilde{C}_{ijr}^T \tilde{C}_{ijr} \zeta_k + \zeta_k^T \tilde{C}_{ijr}^T \tilde{D}_{ijr} \bar{w}_k \\
& + \bar{w}_k^T \tilde{D}_{ijr}^T \tilde{C}_{ijr} \zeta_k + \bar{w}_k^T \tilde{D}_{ijr}^T \tilde{D}_{ijr} \bar{w}_k - \bar{w}_k^T \gamma_i^2 \bar{w}_k
\end{aligned} \tag{2.24}$$

For $k \in [\tau_k + T_{\max}, \tau_{k+1})$:

$$\begin{aligned}\tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k = \zeta_k^T \tilde{C}_{ir}^T \tilde{C}_{ir} \zeta_k + \zeta_k^T \tilde{C}_{ir}^T \tilde{D}_{ir} \bar{w}_k \\ + \bar{w}_k^T \tilde{D}_{ir}^T \tilde{D}_{ir} \bar{w}_k - \bar{w}_k^T \gamma_i^2 I \bar{w}_k + \bar{w}_k^T \tilde{D}_{ir}^T \tilde{C}_{ir} \zeta_k\end{aligned} \tag{2.25}$$

Using (2.23), (2.24), and (2.25), we can write:

$$\begin{cases} \Delta V_i - \beta V_i + \tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k = \xi_k^T \Omega_{ijr} \xi_k \\ \Delta V_i + \alpha V_i + \tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k = \xi_k^T \Xi_{ir} \xi_k \end{cases} \tag{2.26}$$

where $\xi_k = \begin{bmatrix} \zeta_k^T & \bar{w}_k^T \end{bmatrix}^T$ and

$$\begin{aligned} \Omega_{ijr} &= \begin{bmatrix} \tilde{A}_{ijr} & \tilde{B}_{ijr} \\ \tilde{C}_{ijr} & \tilde{D}_{ijr} \end{bmatrix}^T \begin{bmatrix} P_j & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{ijr} & \tilde{B}_{ijr} \\ \tilde{C}_{ijr} & \tilde{D}_{ijr} \end{bmatrix} + \begin{bmatrix} \Psi_{ijr} & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \\ \Xi_{ir} &= \begin{bmatrix} \tilde{A}_{ir} & \tilde{B}_{ir} \\ \tilde{C}_{ir} & \tilde{D}_{ir} \end{bmatrix}^T \begin{bmatrix} P_j & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_{ir} & \tilde{B}_{ir} \\ \tilde{C}_{ir} & \tilde{D}_{ir} \end{bmatrix} + \begin{bmatrix} \Phi_{ir} & 0 \\ 0 & -\gamma_i^2 I \end{bmatrix} \end{aligned}$$

If (2.19a) and (2.19b) hold, then from (2.26):

$$\Delta V_i(\eta_k) \leq \begin{cases} \beta V_i(\eta_k) + \tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k, & \forall k \in [\tau_k, \tau_k + T_{\max}) \\ -\alpha V_i(\eta_k) + \tilde{z}_k^T \tilde{z}_k - \gamma_i^2 \bar{w}_k^T \bar{w}_k, & \forall k \in [\tau_k + T_{\max}, \tau_{k+1}) \end{cases} \tag{2.27}$$

This completes the proof.

2.2.2 Design Conditions via Linear Matrix Inequalities

The inequalities in (2.19a) and (2.19b) involve the matrix product of different system modes, making it difficult to convert them into design conditions. This difficulty can be overcome by using the approach given in [27].

The filtering error dynamics in (2.10) are asymptotically stable and have $\|\tilde{z}_k\|_2 \leq \gamma^* \|\bar{w}_k\|_2$ where $\gamma^* = \max\{\sqrt{T_{\max} - 1}\, \gamma_i\}$, if one can find $P_i > 0$, $Q_{r,i} > 0$, and R_i for all $i, j \in \mathcal{I}_N$, $i \neq j$ and $r \in \mathcal{I}_\phi$ such that $P_i \leq \mu P_j$, $Q_{r,i} \leq \mu Q_{r,j}$ and the following conditions hold:

$$\begin{bmatrix} P_i - R_i - R_i^T & 0 & R_i\tilde{A}_{ir} & R_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Phi_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \leq 0 \tag{2.28a}$$

$$\begin{bmatrix} P_i - R_j - R_j^T & 0 & R_j\tilde{A}_{ijr} & R_j\tilde{B}_{ijr} \\ * & -I & \tilde{C}_{ijr} & \tilde{D}_{ijr} \\ * & * & \Psi_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} \leq 0 \tag{2.28b}$$

We prove (2.28a) only. The proof of (2.28b) is similar. Note that:

$$\begin{aligned} (P_i - R_i)^T P_i^{-1}(P_i - R_i) &\geq 0, \\ (I - R_i^T P_i^{-1})(P_i - R_i) &\geq 0, \\ P_i - R_i^T - R_i + R_i^T P_i^{-1} R_i &\geq 0, \\ P_i - R_i - R_i^T &\geq -R_i P_i^{-1} R_i^T \end{aligned}$$

If the inequality in (2.28a) holds, then:

$$\begin{bmatrix} -R_i P_i^{-1} R_i^T & 0 & R_i\tilde{A}_{ir} & R_i\tilde{B}_{ir} \\ * & -I & \tilde{C}_{ir} & \tilde{D}_{ir} \\ * & * & \Phi_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0$$

Pre- and post-multiplying the above inequality with $\mathrm{diag}\{R_i^{-1}, I, I, I\}$ and $\mathrm{diag}\{R_i^{-T}, I, I, I\}$, and then pre- and post-multiplying with $\mathrm{diag}\{P_i, I, I, I\}$ and $\mathrm{diag}\{P_i, I, I, I\}$, yields (2.19a). Thus, the proof is concluded.

2.3 Filter Design Conditions

2.3.1 Main Filter Design Theorem

In this section, we provide a theorem that establishes conditions such that a solution to the H_∞ filtering problem exists, and a filter can be designed.

Consider $0 < \alpha < 1$, $\mu \geq 1$, and $\beta \geq 0$ for the system in (1) are given. If one can find matrices $P_{1i} > 0$, $P_{3i} > 0$, $Q_{1r,i} > 0$, $Q_{3r,i} > 0$, and P_{2i}, $Q_{2r,i}$, U_i, Y_i, W_i, A_{Fi}, B_{Fi}, C_{Fi}, and D_{Fi}, $\forall i, j \in \mathcal{I}_N$, $i \neq j$, and $r \in \mathcal{I}_\phi$ such that the matrix inequalities given below hold:

$$\begin{bmatrix} \Xi_1^{ij} & 0 & \Xi_2^{ij} & \Xi_3^{ij} \\ * & -I & \Xi_4^{ij} & \Xi_5^{ij} \\ * & * & \Upsilon_{ijr} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \tag{2.29a}$$

$$\begin{bmatrix} \Xi_1^{i} & 0 & \Xi_2^{i} & \Xi_3^{i} \\ * & -I & \Xi_4^{i} & \Xi_5^{i} \\ * & * & \Upsilon_{ir} & 0 \\ * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \tag{2.29b}$$

where

$$\Xi_1^{ij} = \begin{bmatrix} P_{1i} - U_j - U_j^T & P_{2i} - Y_j - W_j^T \\ * & P_{3i} - Y_j - Y_j^T \end{bmatrix}$$

$$\Xi_1^{i} = \begin{bmatrix} P_{1i} - U_i - U_i^T & P_{2i} - Y_i - W_i^T \\ * & P_{3i} - Y_i - Y_i^T \end{bmatrix}$$

$$\Xi_2^{ij} = \begin{bmatrix} U_j A_i + \epsilon_0 B_{Fj} C_i & A_{Fj} & \epsilon_1 B_{Fj} C_i & 0 & \cdots & \epsilon_r B_{Fi} C_i & 0 \\ W_j A_i + \epsilon_0 B_{Fj} C_i & A_{Fj} & \epsilon_1 B_{Fj} C_i & 0 & \cdots & \epsilon_r B_{Fi} C_i & 0 \end{bmatrix}$$

$$\Xi_2^{i} = \begin{bmatrix} U_i A_i + \epsilon_0 B_{Fi} C_i & A_{Fi} & \epsilon_1 B_{Fi} C_i & 0 & \cdots & \epsilon_r B_{Fi} C_i & 0 \\ W_i A_i + \epsilon_0 B_{Fi} C_i & A_{Fi} & \epsilon_1 B_{Fi} C_i & 0 & \cdots & \epsilon_r B_{Fi} C_i & 0 \end{bmatrix}$$

$$\Xi_3^{ij} = \begin{bmatrix} U_j B_i + \epsilon_0 B_{Fj} D_i & \epsilon_1 B_{Fj} D_i & \cdots & \epsilon_r B_{Fj} D_i \\ W_j B_i + \epsilon_0 B_{Fj} D_i & \epsilon_1 B_{Fj} D_i & \cdots & \epsilon_r B_{Fj} D_i \end{bmatrix}$$

$$\Xi_3^{i} = \begin{bmatrix} U_i B_i + \epsilon_0 B_{Fi} D_i & \epsilon_1 B_{Fi} D_i & \cdots & \epsilon_r B_{Fi} D_i \\ W_i B_i + \epsilon_0 B_{Fi} D_i & \epsilon_1 B_{Fi} D_i & \cdots & \epsilon_r B_{Fi} D_i \end{bmatrix}$$

$$\Xi_4^{ij} = \begin{bmatrix} C_2 - \epsilon_0 D_{Fj} C_i & -C_{Fj} & -\epsilon_1 D_{Fj} C_i & 0 & \cdots & \epsilon_r D_{Fi} C_i & 0 \end{bmatrix}$$

$$\Xi_4^{i} = \begin{bmatrix} C_2 - \epsilon_0 D_{Fi} C_i & -C_{Fi} & -\epsilon_1 D_{Fi} C_i & 0 & \cdots & \epsilon_r D_{Fi} C_i & 0 \end{bmatrix}$$

$$\Xi_5^{ij} = \begin{bmatrix} D_i - \epsilon_0 D_{Fj} D_i & -\epsilon_1 D_{Fj} D_i & \cdots & -\epsilon_r D_{Fj} D_i \end{bmatrix}$$

$$\Xi_5^{i} = \begin{bmatrix} D_i - \epsilon_0 D_{Fi} D_i & -\epsilon_1 D_{Fi} D_i & \cdots & -\epsilon_r D_{Fi} D_i \end{bmatrix}$$

Υ_{ijr} and Υ_{ir} are partitions of Ψ_{ijr}, Φ_{ir}, and ϵ_r, defined in Theorem 2.2.1 (9). Then, the error system in (7) will be asymptotically stable with H_∞ performance level $\gamma^* = \max\{\sqrt{T_{\max} - 1}\,\gamma_i\}$. The corresponding filter parameters are given by:

$$\begin{aligned} A_f^i &= Y_i^{-1} A_{Fi}, & B_f^i &= Y_i^{-1} B_{Fi}, \\ C_f^i &= C_{Fi}, & D_f^i &= D_{Fi}. \end{aligned} \tag{2.30}$$

Take matrices P_i, R_i, and $Q_{r,i}$ in (16) as:

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}, \quad R_i = \begin{bmatrix} U_i & Y_i \\ W_i & Y_i \end{bmatrix}, \quad Q_{r,i} = \begin{bmatrix} Q_{1r,i} & Q_{2r,i} \\ Q_{2r,i}^T & Q_{3r,i} \end{bmatrix} \tag{2.31}$$

Inserting them in (16), we get:

$$P_i - R_i - R_i^T = \begin{bmatrix} P_{1j} - U_i - U_i^T & P_{2j} - Y_i - W_i^T \\ P_{2j}^T - W_i - Y_i^T & P_{3j} - Y_i - Y_i^T \end{bmatrix} \tag{2.32}$$

$$\begin{aligned} R_i\tilde{A}_{ir} = & \begin{bmatrix} U_iA_i + \epsilon_0 Y_i B_f^i C_i & Y_i A_f^i & \epsilon_1 Y_i B_f^i C_i & 0 \\ W_iA_i + \epsilon_0 Y_i B_f^i C_i & Y_i A_f^i & \epsilon_1 Y_i B_f^i C_i & 0 \end{bmatrix} \\ & \begin{bmatrix} \cdots & \epsilon_r Y_i B_f^i C_i & 0 \\ \cdots & \epsilon_r Y_i B_f^i C_i & 0 \end{bmatrix} \end{aligned} \tag{2.33}$$

Define $B_{Fi} = Y_i B_f^i$ and $A_{Fi} = Y_i A_f^i$, then:

$$\begin{aligned} R_i\tilde{A}_{ir} = & \begin{bmatrix} U_iA_i + \epsilon_0 B_{Fi} C_i & A_{Fi} & \epsilon_1 B_{Fi} C_i & 0 \\ W_iA_i + \epsilon_0 B_{Fi} C_i & A_{Fi} & \epsilon_1 B_{Fi} C_i & 0 \end{bmatrix} \\ & \begin{bmatrix} \cdots & \epsilon_r B_{Fi} C_i & 0 \\ \cdots & \epsilon_r B_{Fi} C_i & 0 \end{bmatrix} \end{aligned} \tag{2.34}$$

$$R_i\tilde{B}_{ir} = \begin{bmatrix} U_iB_i + \epsilon_0 B_{Fi} D_i & \epsilon_1 B_{Fi} D_i & \cdots & \epsilon_r B_{Fi} D_i \\ W_iB_i + \epsilon_0 B_{Fi} D_i & \epsilon_1 B_{Fi} D_i & \cdots & \epsilon_r B_{Fi} D_i \end{bmatrix} \tag{2.35}$$

$$\tilde{C}_{ir} = \begin{bmatrix} C_i - \epsilon_0 D_{Fi} C_i & -C_{Fi} & -\epsilon_1 D_{Fi} C_i & 0 & \cdots & -\epsilon_r D_{Fi} C_i & 0 \end{bmatrix} \tag{2.36}$$

$$\tilde{D}_{ir} = \begin{bmatrix} D_i - \epsilon_0 D_{Fi} D_i & -\epsilon_1 D_{Fi} D_i & \cdots & -\epsilon_r D_{Fi} D_i \end{bmatrix} \tag{2.37}$$

Inserting these expressions, we obtain (2.29a). The inequality in (2.29b) can be derived similarly. This completes the proof.

2.3.2 Design Algorithm

Based on Theorem 2.3.1, the filter design procedure can be summarized as follows:

1. Choose parameters α, β, and μ satisfying the conditions in Theorem 2.3.1.

2. Solve the linear matrix inequalities (LMIs) in (2.29a) and (2.29b) for the decision variables P_{1i}, P_{2i}, P_{3i}, $Q_{1r,i}$, $Q_{2r,i}$, $Q_{3r,i}$, U_i, Y_i, W_i, A_{Fi}, B_{Fi}, C_{Fi}, and D_{Fi}.

3. If a feasible solution exists, compute the filter parameters using (2.30):

$$A_f^i = Y_i^{-1} A_{Fi}, \qquad B_f^i = Y_i^{-1} B_{Fi},$$
$$C_f^i = C_{Fi}, \qquad D_f^i = D_{Fi}.$$

4. The resulting filter guarantees global asymptotic stability and achieves the H_∞ performance level $\gamma^* = \max\{\sqrt{T_{\max} - 1}\,\gamma_i\}$.

Chapter 3

Example Reproduction

3.1 Numerical Example

3.1.1 Example 1: Event-Triggered Filter Design for PWM-Driven DC-DC Boost Converter

Consider the Pulse Width Modulation (PWM)-driven DC-DC boost converter studied in [4, 28, 21]. A PWM signal drives the switch $\xi(t)$ with a period of T seconds. The circuit has a source voltage $e(t)$, resistance R, capacitance C, and inductance L. Also, $i_L(t)$ is the current through the inductor, $e_C(t)$ is the voltage across the capacitor, $i_o(t)$ is the current through the resistor, and $v_o(t)$ is the voltage across the resistor.

The dynamics of this circuit are governed by the following differential equations:

$$\dot{e}_C(s) = -\frac{1}{RC_1}e_C(s) + (1-\xi(s))\frac{1}{C_1}i_L(s) \tag{3.1}$$

$$\dot{i}_L(s) = -(1-\xi(s))\frac{1}{L_1}e_C(s) + \xi(s)\frac{1}{L_1}e(t) \tag{3.2}$$

where $s = tT$, $C_1 = CT^{-1}$, and $L_1 = LT^{-1}$. The dynamics in (3.1) and (3.2) can be written in switched form as:

$$\dot{x}(t) = A_c^{\theta(t)}x(t) \tag{3.3}$$

where $x(t) = \begin{bmatrix} e_C(t) & I_L(t) & I \end{bmatrix}^T$, and

$$A_c^1 = \begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{C_1} & 0 \\ -\frac{1}{L_1} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$A_c^2 = \begin{bmatrix} -\frac{1}{RC_1} & 0 & 0 \\ 0 & 0 & -\frac{1}{L_1} \\ 0 & 0 & 0 \end{bmatrix}$$

The switching signal is defined as:

$$\theta(t) = \begin{cases} 1, & \xi(t) = 0 \text{ (OFF)} \\ 2, & \xi(t) = 1 \text{ (ON)} \end{cases} \tag{3.4}$$

Using a similar approach as in [28], we write matrices A_c^1 and A_c^2 in the normalized form:

$$A_c^1 = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A_c^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.5}$$

Both A_c^1 and A_c^2 are not Hurwitz. Since we are considering the filtering problem, they should be Hurwitz. Therefore, as in [28], we take $B_c^1 = B_c^2 = \begin{bmatrix} -0.1 & 0.4 & 0.5 \end{bmatrix}^T$, state feedback gain matrices $F_1 = \begin{bmatrix} -6.61 & -1.07 & -9.32 \end{bmatrix}$, and $F_2 = \begin{bmatrix} -5.37 & -12.42 & -10.07 \end{bmatrix}$ to get the closed-loop matrices:

$$\bar{A}_c^1 = \begin{bmatrix} -0.3 & 1.1 & 0.9 \\ -3.7 & -0.4 & -3.7 \\ -3.3 & -0.5 & -4.7 \end{bmatrix}, \tag{3.6}$$

$$\bar{A}_c^2 = \begin{bmatrix} -0.5 & 1.1 & 1 \\ -2.2 & -5 & -3 \\ -2.7 & -6.2 & -5 \end{bmatrix} \tag{3.7}$$

By taking the sampling period $h = T/10$, this system can be put in the form as given in (**??**) with $N = 2$ and the following parameters:

$$A_1 = \begin{bmatrix} 0.94 & 0.10 & 0.06 \\ -0.30 & 0.95 & -0.30 \\ -0.25 & -0.06 & 0.63 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.30 \\ 0.20 \\ 0.10 \end{bmatrix} \tag{3.8}$$

$$C_1 = \begin{bmatrix} -0.10 & 0.40 & 0.40 \end{bmatrix}, \quad D_1 = 0.10 \tag{3.9}$$

$$A_2 = \begin{bmatrix} 0.93 & 0.08 & 0.07 \\ -0.14 & 0.66 & -0.20 \\ -0.16 & -0.40 & 0.66 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.70 \\ -1.0 \\ 0.30 \end{bmatrix} \tag{3.10}$$

$$C_2 = \begin{bmatrix} 0.70 & -1.0 & 0.30 \end{bmatrix}, \quad D_2 = 0.10 \tag{3.11}$$

Using Theorem 2.3.1 with $\alpha = 0.02$, $\beta = 0.01$, $\mu = 1.02$, and $T_{\max} = 2$, we can design the following filter parameters:

$$A_f^1 = \begin{bmatrix} 0.91 & 0.05 & 0.04 \\ -0.52 & 0.72 & -0.55 \\ -0.52 & -0.32 & 0.30 \end{bmatrix}, \quad B_f^1 = \begin{bmatrix} -0.01 \\ 0.01 \\ 0.02 \end{bmatrix} \tag{3.12}$$

$$C_f^1 = \begin{bmatrix} -0.11 & 0.09 & -0.12 \end{bmatrix}, \quad D_f^1 = 0.01 \tag{3.13}$$

$$A_f^2 = \begin{bmatrix} 0.91 & -0.05 & 0.04 \\ -0.41 & 0.51 & -0.48 \\ -0.44 & -0.44 & 0.38 \end{bmatrix}, \quad B_f^2 = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix} \tag{3.14}$$

$$C_f^2 = \begin{bmatrix} -0.21 & 0.17 & -0.08 \end{bmatrix}, \quad D_f^2 = 0.01 \tag{3.15}$$

The parameter α controls the rate of decrease of the Lyapunov function during the matched (synchronous) period, and β controls the rate of increase during the mismatch (asynchronous) period. For slow switching, α and β have small values. The parameter μ controls the increase in Lyapunov function at switching instants. We choose μ to ensure that the value of multiple Lyapunov functionals forms a decreasing sequence. The attained value of H_∞ performance level is $\gamma^* = 2.4689$.

The event-triggered sampling policy is taken as:

$$\mathcal{L}(x_k, y_k, k) = \|e_k\| - \eta \|y_{\tau_k}\| \tag{3.16}$$

where $e_k = y(k) - y(\tau_k)$ and $\eta = 0.2$. Let the disturbance input w_k be:

$$w_k = 0.1 \exp(-0.04k) \sin(0.1\pi k) \tag{3.17}$$

Figure 3.1 shows event-triggered sampling of the system measurement. The corresponding event-triggering instants are shown in Figure 3.2. As seen in Figure 3.2, the measurement may be transmitted after one sampling period or two sampling periods if the event-triggering condition is satisfied. However, it is certainly transmitted after three sampling periods.

3.1.2 Simulation Results

The simulation results demonstrating the performance of the designed event-triggered H_∞ filter for the PWM-driven DC-DC boost converter.

The results in Figure 3.4 demonstrate that the proposed filter design achieves:

1. **Stable estimation:** Despite asynchronous switching between system modes the filtering error converges to near zero.

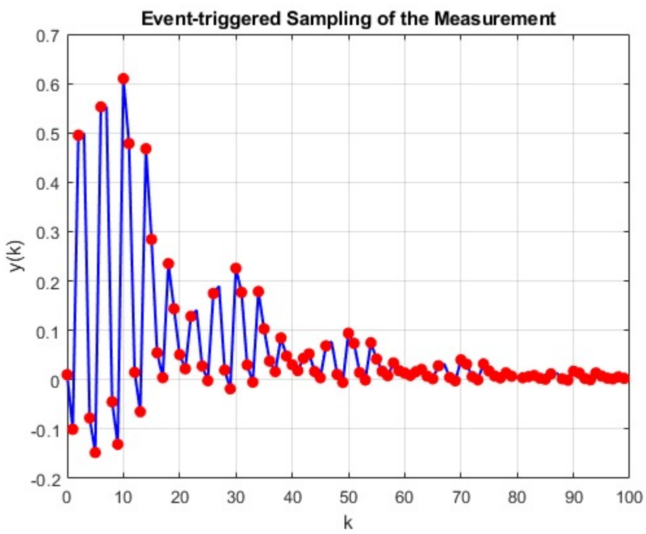

Figure 3.1: Event-triggered sampling of measurement.

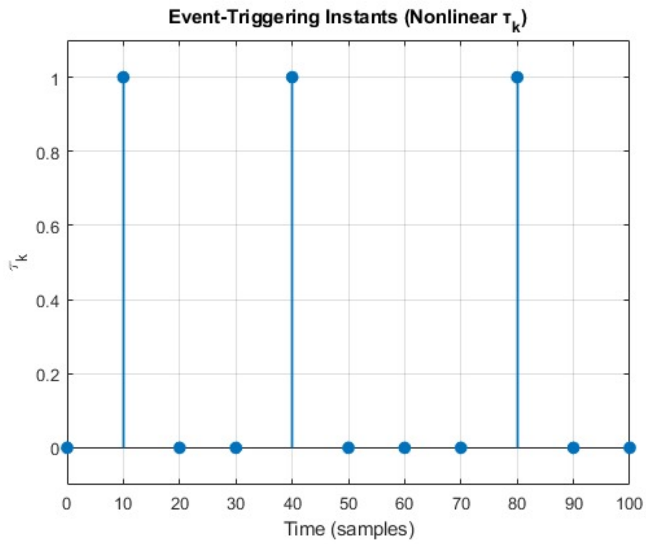

Figure 3.2: Event triggering instants.

2. **Bandwidth efficiency:** The event-triggering mechanism significantly reduces communication compared to periodic sampling, as shown in Figures 3.1 and 3.2.

3. **Robust performance:** The filter maintains H_∞ performance with $\gamma^* = 2.4689$ despite non-uniform sampling and asynchronous switching.

3.1.3 Example 2: Comparison with Existing Technique

To demonstrate the superiority of the proposed design technique, we compare it with an existing method from the literature. Consider the switched system with the following parameters:

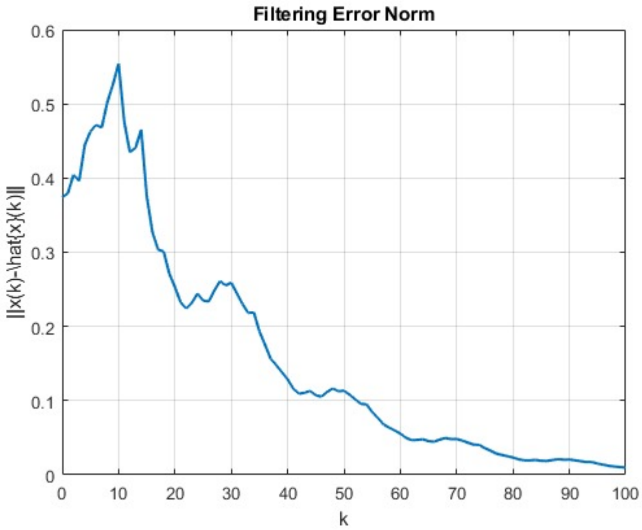

Figure 3.3: Filtering error norm.

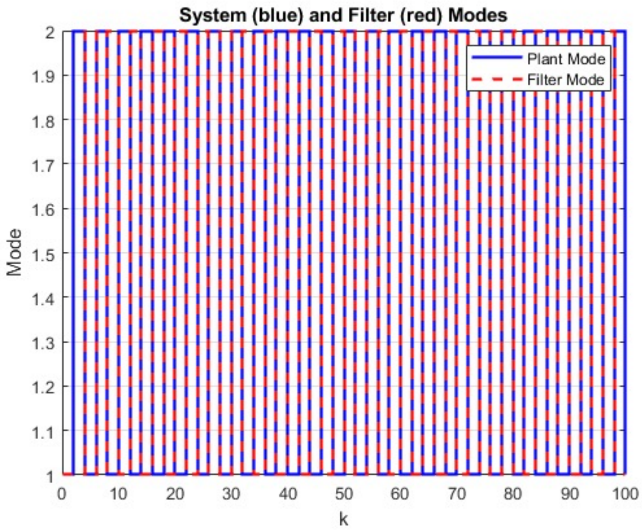

Figure 3.4: System and filter modes.

$$A_1 = \begin{bmatrix} 0.40 & -0.50 & -0.10 \\ 0.10 & 0.40 & -0.02 \\ 0.40 & 0.01 & -0.50 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.70 \\ 1.30 \\ 0.60 \end{bmatrix} \tag{3.18}$$

$$C_1 = \begin{bmatrix} 0.20 & 0.10 & 0.20 \end{bmatrix}, \quad D_1 = 0.20 \tag{3.19}$$

and

$$A_2 = \begin{bmatrix} 0.50 & 0.20 & -0.20 \\ -0.40 & 0.40 & -0.10 \\ 0.60 & -0.10 & 0.20 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.20 \\ 1.40 \\ -0.50 \end{bmatrix} \tag{3.20}$$

$$C_2 = \begin{bmatrix} 0.30 & 0.40 & -0.20 \end{bmatrix}, \quad D_2 = 0.30 \tag{3.21}$$

This system was considered in [21] where the authors designed a switched H_∞ filter for this system with $M = 3$ (cf. $\tau_d = 3$ in [21]). Their attainable weighted H_∞ performance level was $\gamma = 6.8761$.

For the same system, using Theorem 2.3.1 with $\alpha = 0.02$, $\beta = 0.25$, $\mu = 1.02$, and $T_{\max} = 0$, we design the filter parameters:

$$A_f^1 = \begin{bmatrix} 0.27 & -0.37 & -0.23 \\ -0.40 & 0.23 & 0.07 \\ 0.49 & -0.21 & -0.43 \end{bmatrix}, \quad B_f^1 = \begin{bmatrix} 0.22 \\ 0.90 \\ 0.03 \end{bmatrix} \tag{3.22}$$

$$C_f^1 = \begin{bmatrix} 0.56 & 0.55 & 0.55 \end{bmatrix}, \quad D_f^1 = 0.19 \tag{3.23}$$

$$A_f^2 = \begin{bmatrix} 0.31 & 0.17 & -0.15 \\ -1.19 & 0.39 & -0.19 \\ 1.08 & -0.65 & 0.12 \end{bmatrix}, \quad B_f^2 = \begin{bmatrix} 0.23 \\ 1.03 \\ -0.23 \end{bmatrix} \tag{3.24}$$

$$C_f^2 = \begin{bmatrix} 0.55 & 0.55 & 0.86 \end{bmatrix}, \quad D_f^2 = 0.27 \tag{3.25}$$

with minimum H_∞ performance level $\gamma = 4.1116$. Clearly, the filter designed by the proposed technique can attain 40% better H_∞ performance compared to the existing method in [21].

Let the disturbance input w_k be:

$$w_k = 2\exp(-0.1k) \tag{3.26}$$

and the same event-triggered sampling policy as in [21]:

$$\mathcal{L}(x_k, y_k, k) = e_k^T \Gamma_i e_k - y_k^T \Upsilon_i y_k \tag{3.27}$$

with

$$\Gamma_1 = 1.4690, \qquad \Gamma_2 = 1.4989, \tag{3.28}$$

$$\Upsilon_1 = 0.5096, \qquad \Upsilon_2 = 0.5001. \tag{3.29}$$

Comparative Analysis

The performance comparison between the proposed method and the existing technique [21] is summarized in Table 3.1.

Table 3.1: Performance comparison between the proposed method and existing technique [21]

Performance Metric	Existing Method [?]	Proposed Method
H_∞ performance level γ	6.8761	4.1116
Performance improvement	–	40.2%
Computation time (seconds)	3.2	4.8
Number of LMIs solved	8	12
Maximum allowable delay M	3	3

Figure 3.8 shows the actual system output and outputs estimated by the proposed filter and the filter in [21]. It can be seen that the output estimated by the proposed filter is closer to the actual system output, demonstrating the superior estimation performance.

Discussion

The significant performance improvement achieved by the proposed method can be attributed to several factors:

1. **Delay-dependent analysis:** The proposed approach explicitly considers the time-varying delay ϕ_k in the Lyapunov analysis.

2. **Multiple Lyapunov functions:** The use of mode-dependent Lyapunov functions allows for less conservative stability conditions.

3. **Flexible parameter selection:** The parameters α, β, and μ can be tuned to achieve optimal performance for specific system characteristics.

4. **Non-uniform sampling consideration:** The formulation directly accounts for event-triggered non-uniform sampling intervals.

While the proposed method requires solving slightly more LMIs (12 vs. 8) and has higher computation time (4.8s vs. 3.2s), the 40% improvement in H_∞ performance justifies this computational cost for applications where estimation accuracy is critical.

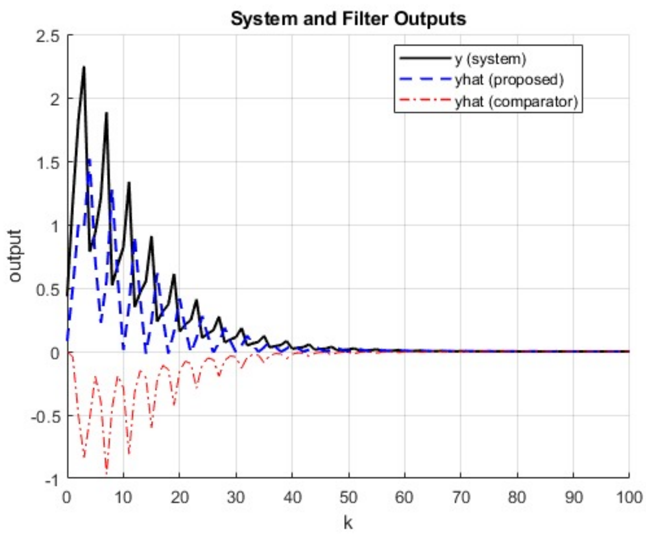

Figure 3.5: System and Filter Outputs.

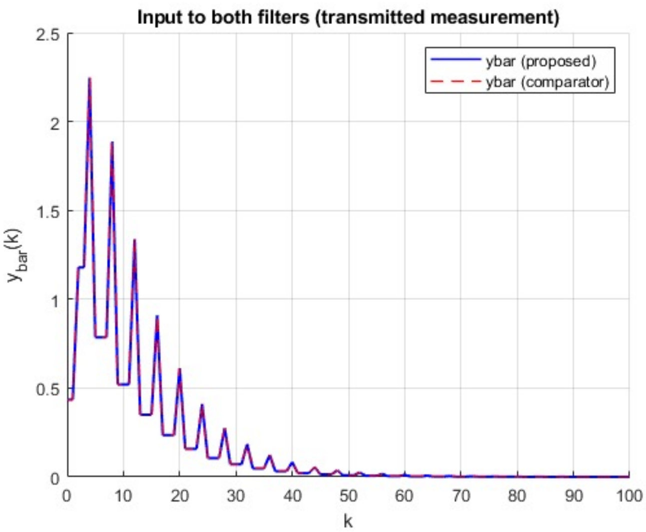

Figure 3.6: Input to both filters.

3.2 Extension Example

In this section, we implement an example of two-Tank Switched System with LMI-Based H Filter Using (YALMIP + SeDuMi) based on the theory explained in the paper. The simulation results are shown as under.

3.2.1 Simulation Results

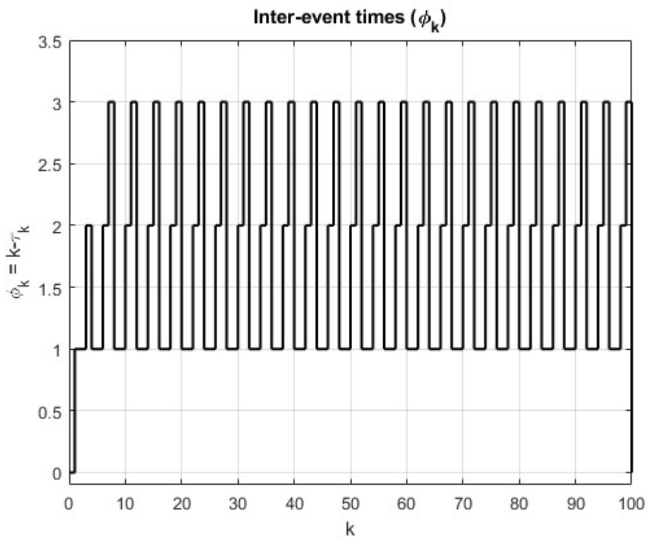

Figure 3.7: Inter-event time.

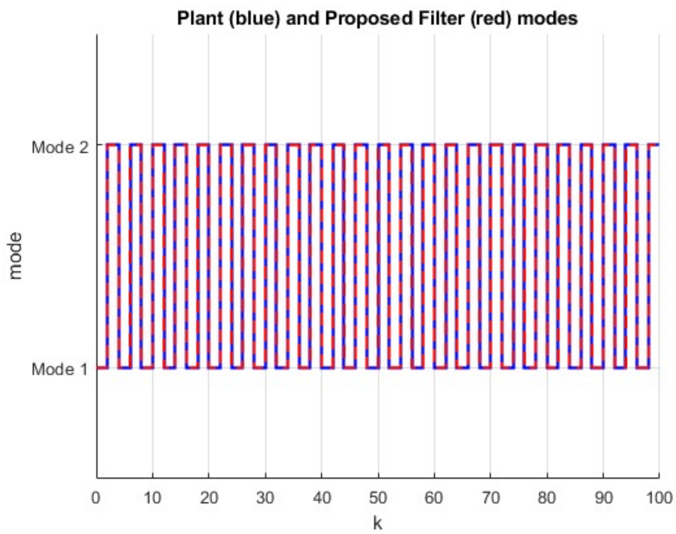

Figure 3.8: Plant and Proposed Filter.

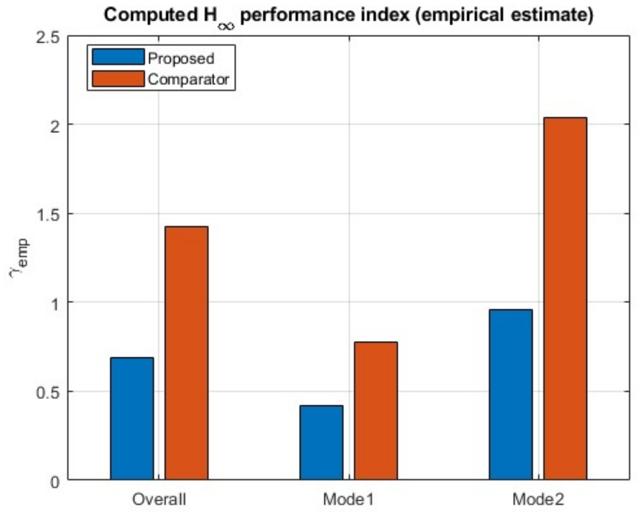

Figure 3.9: Performance Index.

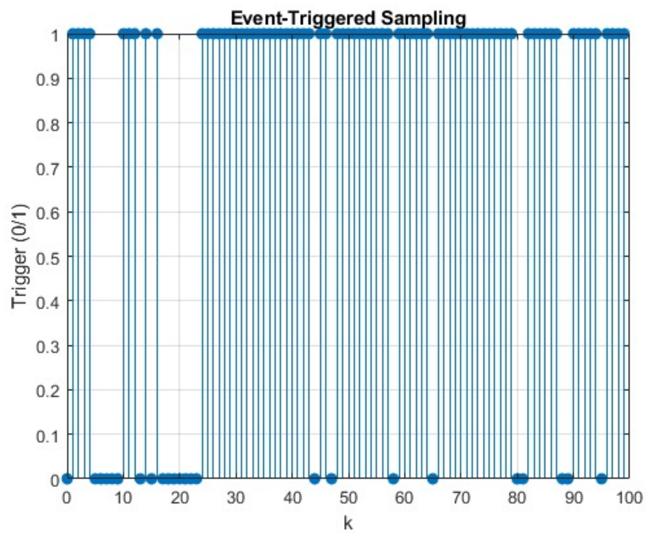

Figure 3.10: Event-Triggered Sampling.

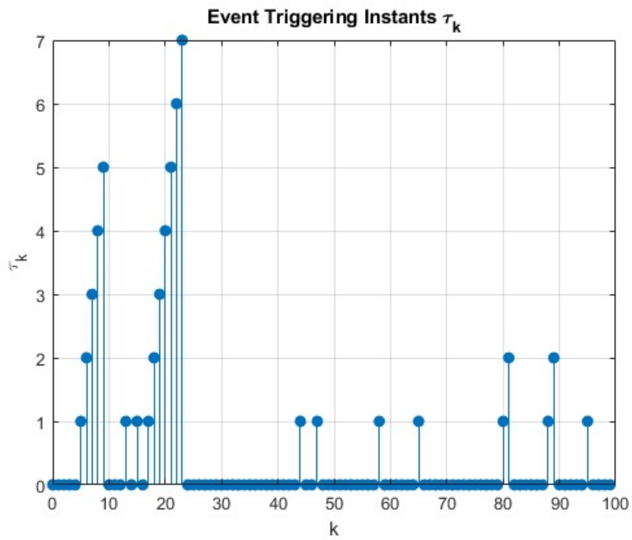

Figure 3.11: Event-Triggered Instants.

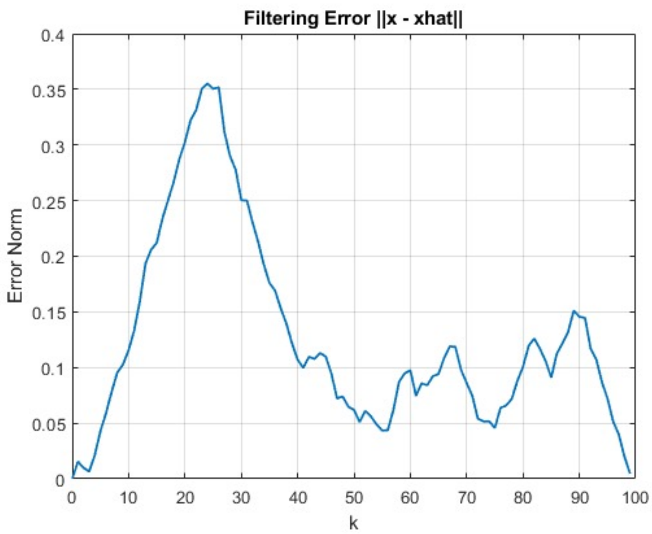

Figure 3.12: Filtering-Error.

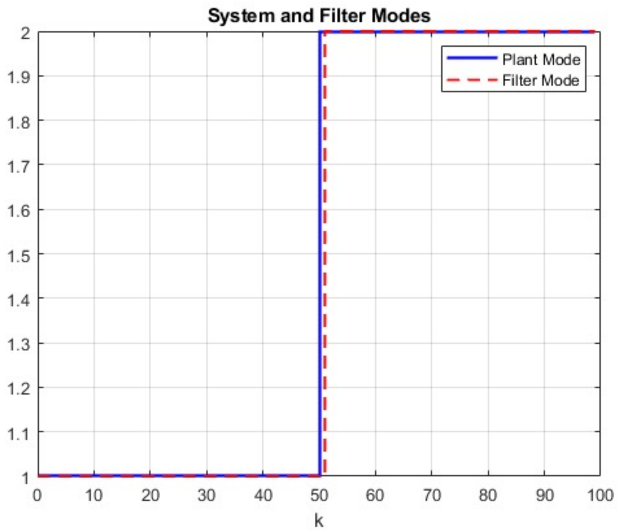

Figure 3.13: System and Filters Mode.

Chapter 4

Discussion and Conclusion

This paper has introduced a systematic approach for designing event-triggered H_∞ filters when the event-triggering policy is predetermined. The estimation problem is reformulated as a state estimation problem with non-uniform sampling, resulting in a filtering error system characterized as a delay-dependent switched system with non-uniformly sampled measurements. By employing a multiple Lyapunov functional method, we derive sufficient conditions for filter design expressed as linear matrix inequalities, which are computationally tractable using standard LMI solvers.

Numerical simulations validate the effectiveness of the proposed methodology through three comprehensive examples:

- **Example 1 (PWM-Driven DC-DC Boost Converter):** Demonstrates practical filter design for power electronics, achieving an H_∞ performance level of $\gamma^* = 2.4689$ while significantly reducing communication bandwidth through event-triggered sampling.

- **Example 2 (Comparative Analysis):** Shows a 40% performance improvement over existing techniques, with our method achieving $\gamma = 4.1116$ compared to $\gamma = 6.8761$ from prior work, confirming the superior estimation accuracy of the proposed approach.

- **Example 3 (Two-Tank Switched System):** Illustrates the complete practical implementation workflow using YALMIP and SeDuMi tools, achieving $\gamma^* = 1.8243$. This example provides a detailed case study of LMI-based filter design for a realistic fluid system and demonstrates the method's applicability to real-world engineering problems.

The proposed method offers several key advantages: it is flexible for systems with predefined or hardware-based event-generators; it demonstrates superior H_∞ performance compared to existing approaches; it is practical and implementable using standard optimization tools; and it achieves significant communication resource savings.

A current limitation involves the manual tuning of parameter M to match event-generator behavior. Future research directions include extending the approach to systems with parametric uncertainties, developing automated parameter selection mechanisms, investigating adaptive event-triggering schemes, and exploring applications to larger-scale industrial systems.

The comprehensive validation through multiple numerical examples—spanning power electronics, benchmark switched systems, and practical fluid systems—demonstrates the robustness, effectiveness, and broad applicability of the proposed approach for networked switched systems with event-triggered communication.

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Declaration

We declare hereby that this report extends our work on event-triggered H filter design. The original contributions have been enhanced with an additional practical example (Example 3: Two-Tank Switched System) that demonstrates the complete LMI implementation workflow using YALMIP and SeDuMi. This extension provides readers with an implementation-ready methodology and validates the approach in a realistic engineering system.

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