Lecture 4:

Image Enhancement in Spatial Domain

Basic pixel operations

Histogram Equalization

Common pixel operations

- Image negatives
- Log transformations
- Power-law transformations

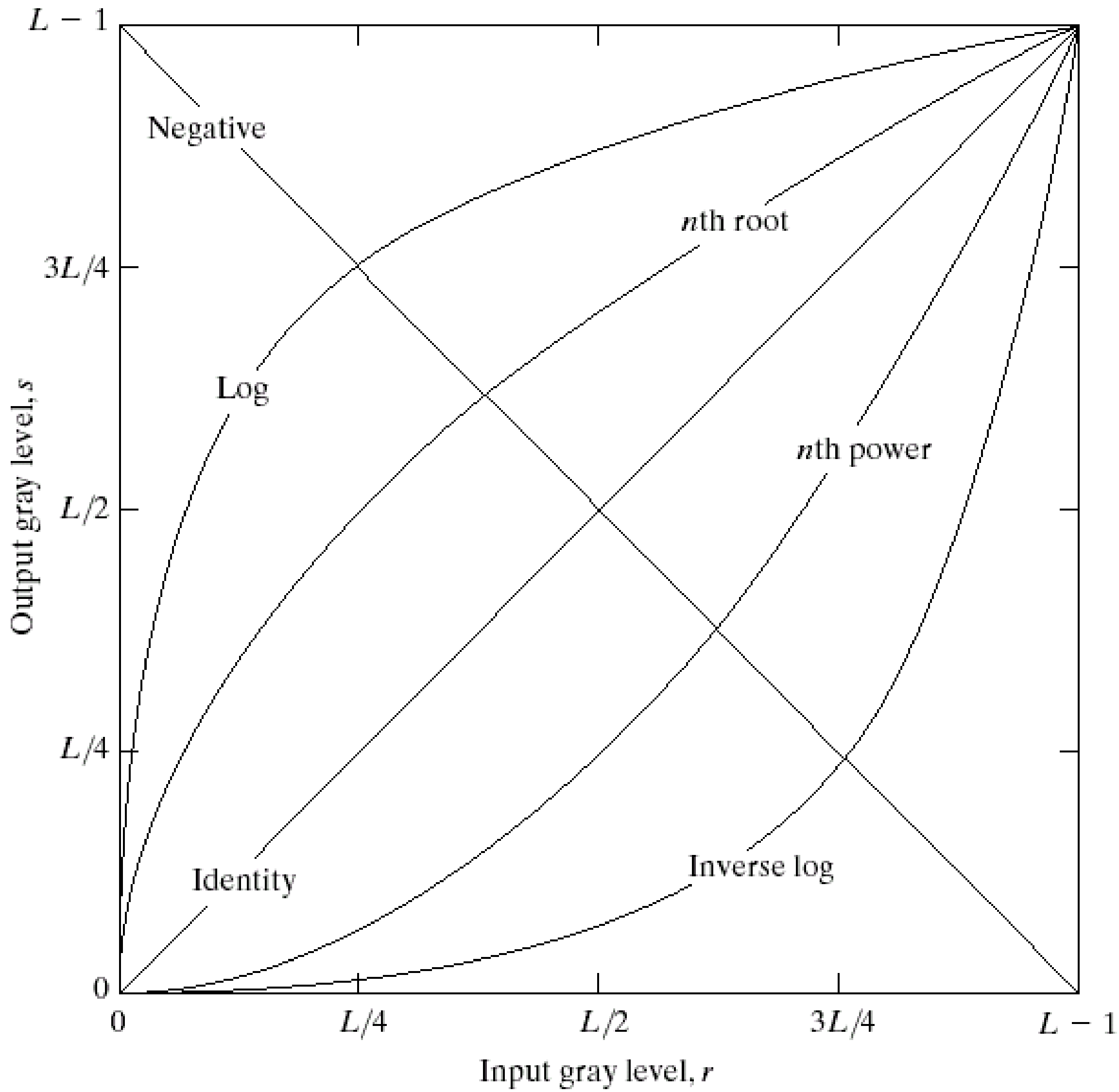
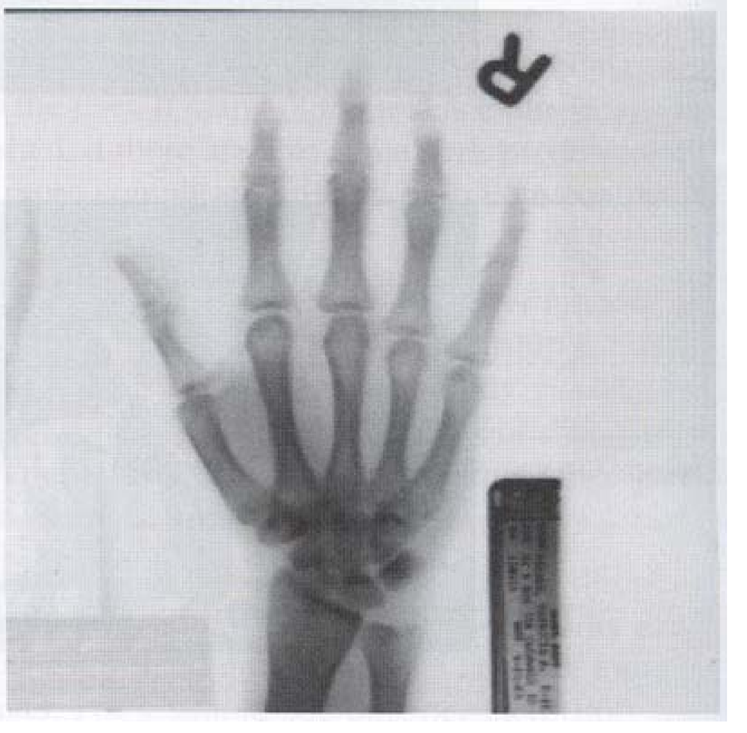


Image negatives

- Reverses the gray level order
- For L gray levels the transformation function is

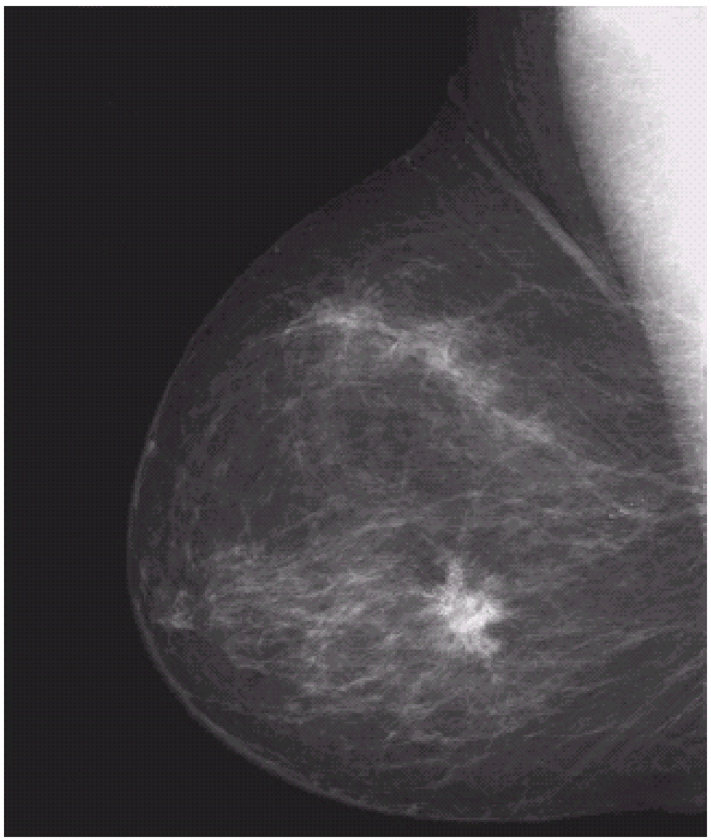
$$s = T(r) = (L-1)-r$$



Input image (X-ray image) **Output image (negative)**

Image negatives

Application: To enhance the visibility for images with more dark portion



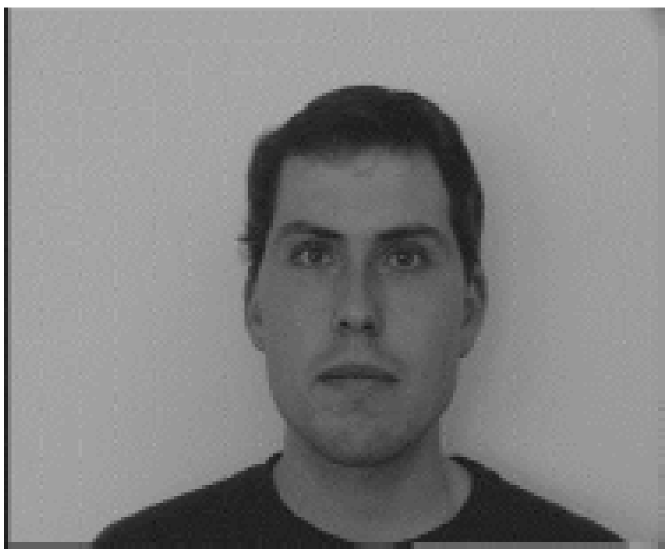
Original digital mammogram

Output image

Image scaling

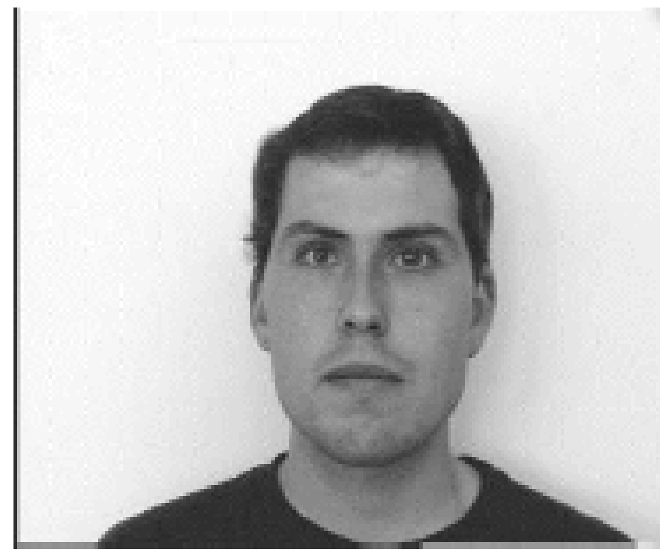
$s = T(r) = a.r$ (a is a constant)

Original image



$f(x,y)$

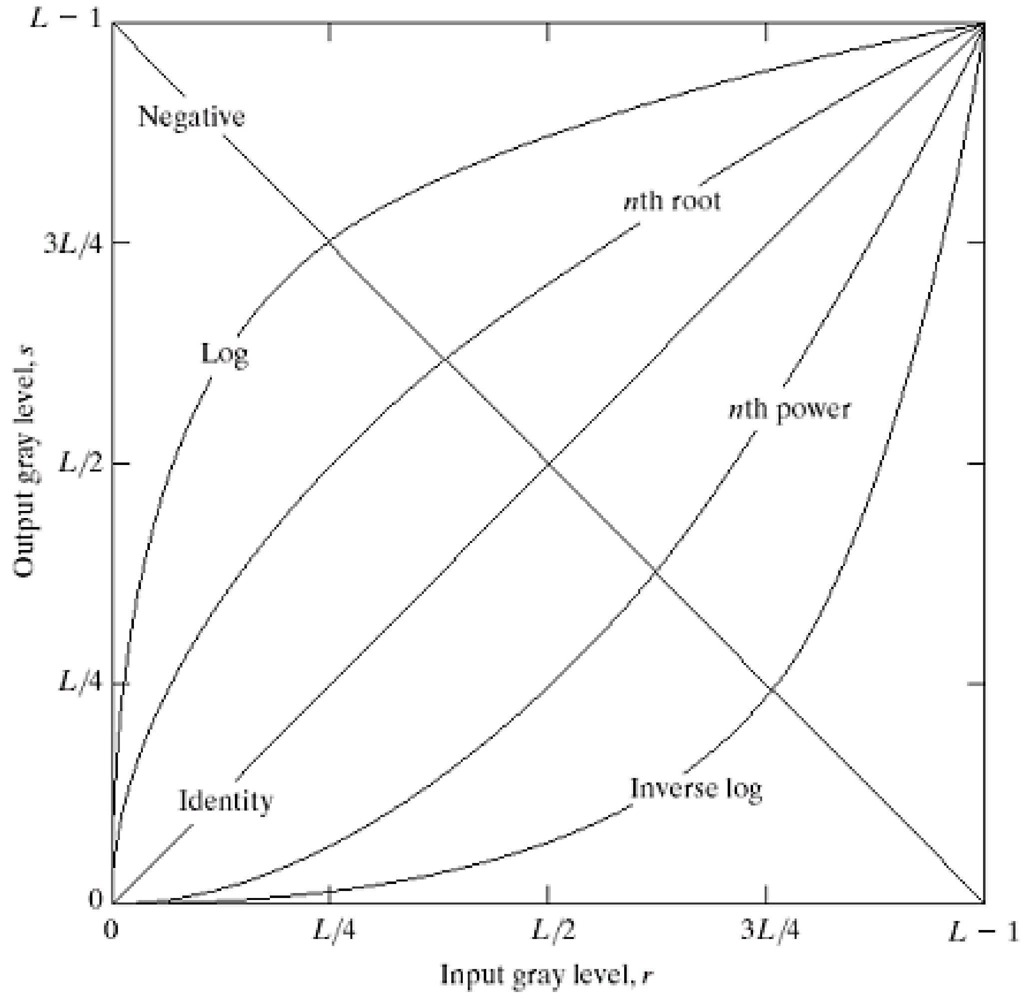
Scaled image



$a \cdot f(x,y)$

Log transformations

Function of $s = c \, \mathrm{Log}(1+r)$



Log transformations

Properties of log transformations

– *For lower amplitudes of input image the range of gray levels is expanded*

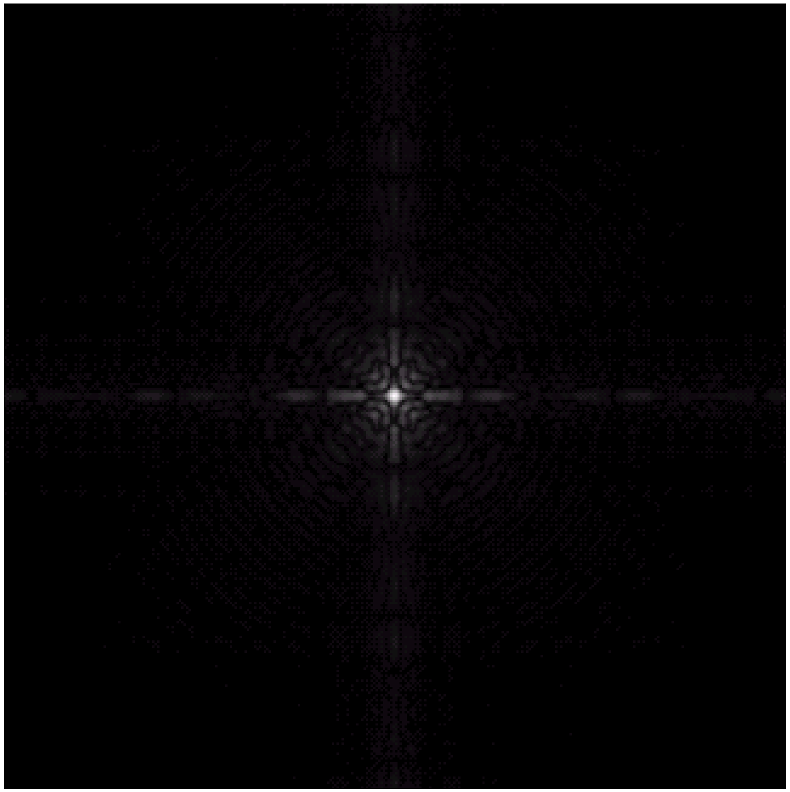
– *For higher amplitudes of input image the range of gray levels is compressed*

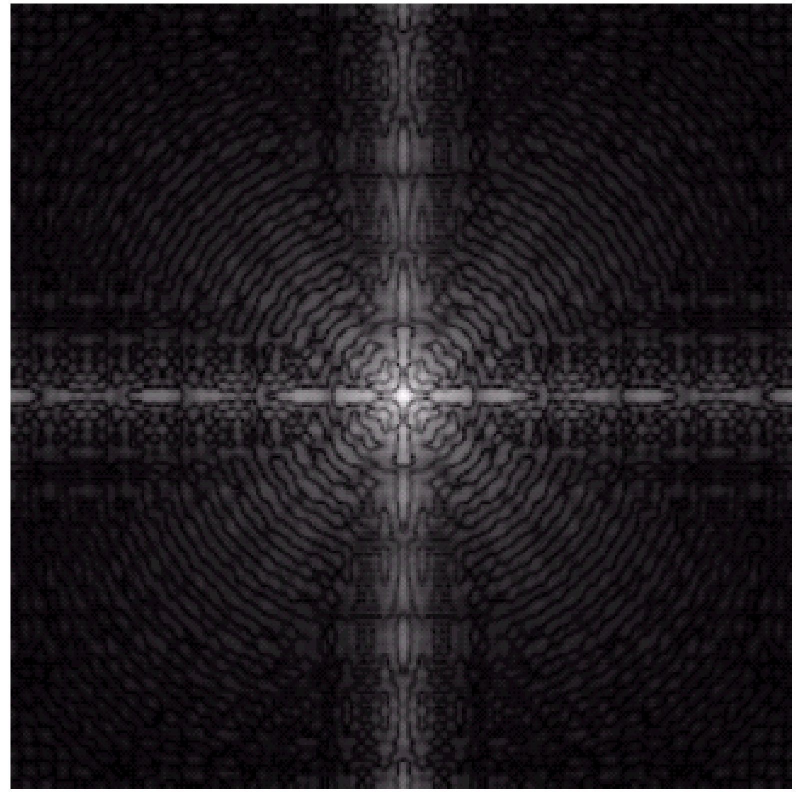
Application:

– *This transformation is suitable for the case when the dynamic range of a processed image far exceeds the capability of the display device (e.g. display of the Fourier spectrum of an image)*

– *Also called "dynamic-range compression / expansion"*

Log transformations





Fourier spectrum with values of range 0 to 1.5×10^6 scaled linearly

The result applying log transformation, $c = 1$

Power-law Transformation

Basic form:

$$s = cr^{\gamma} ,$$

where c & γ

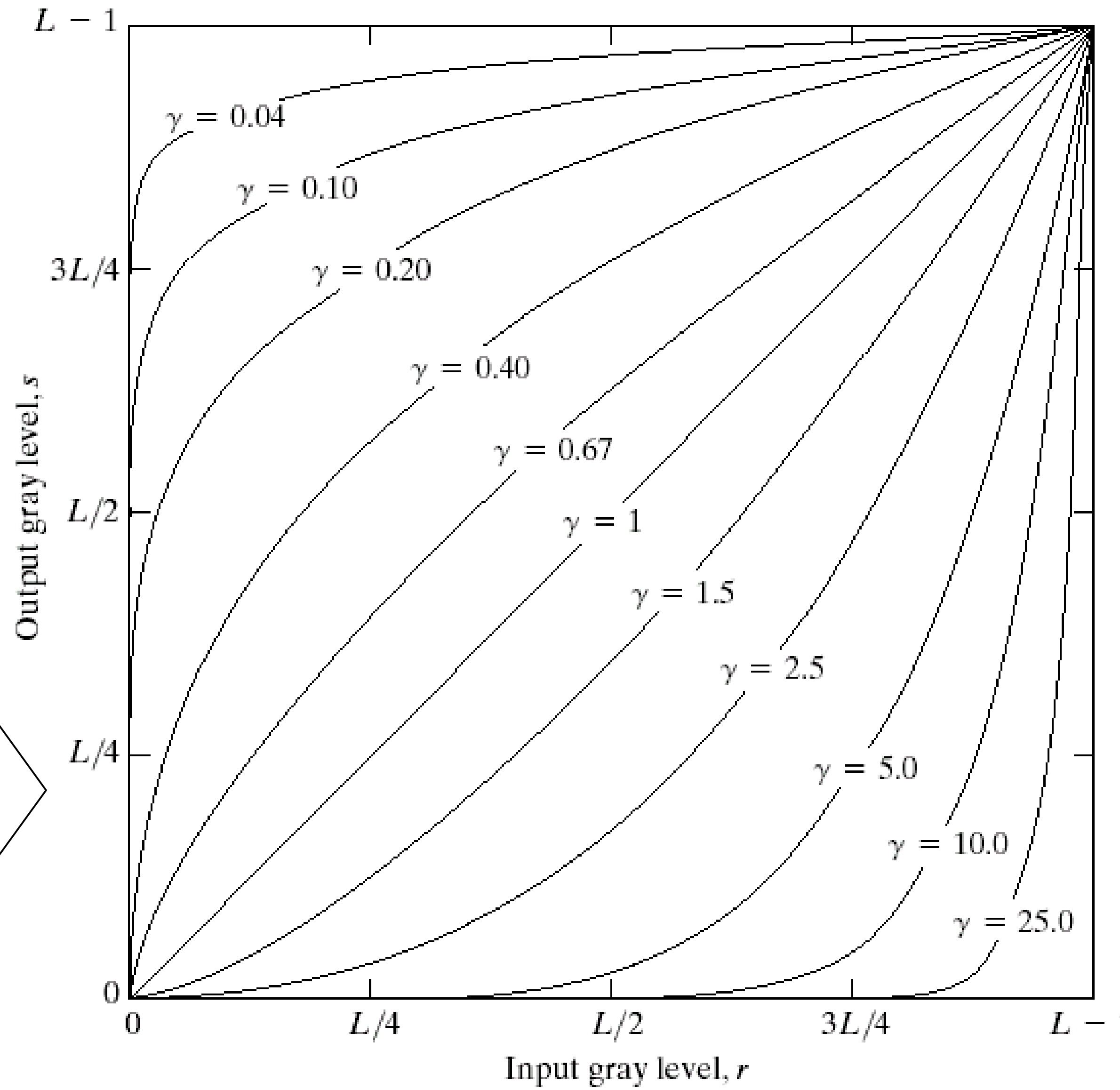
are positive

Plots of equation

$s = cr^{\gamma}$,

For various values of γ

(c = 1)



Power-law Transformation

For $\gamma < 1$: Expands values of dark pixels, compress values of brighter pixels

For $\gamma > 1$: Compresses values of dark pixels, expand values of brighter pixels

If γ=1 & c=1: Identity transformation ($s = r$)

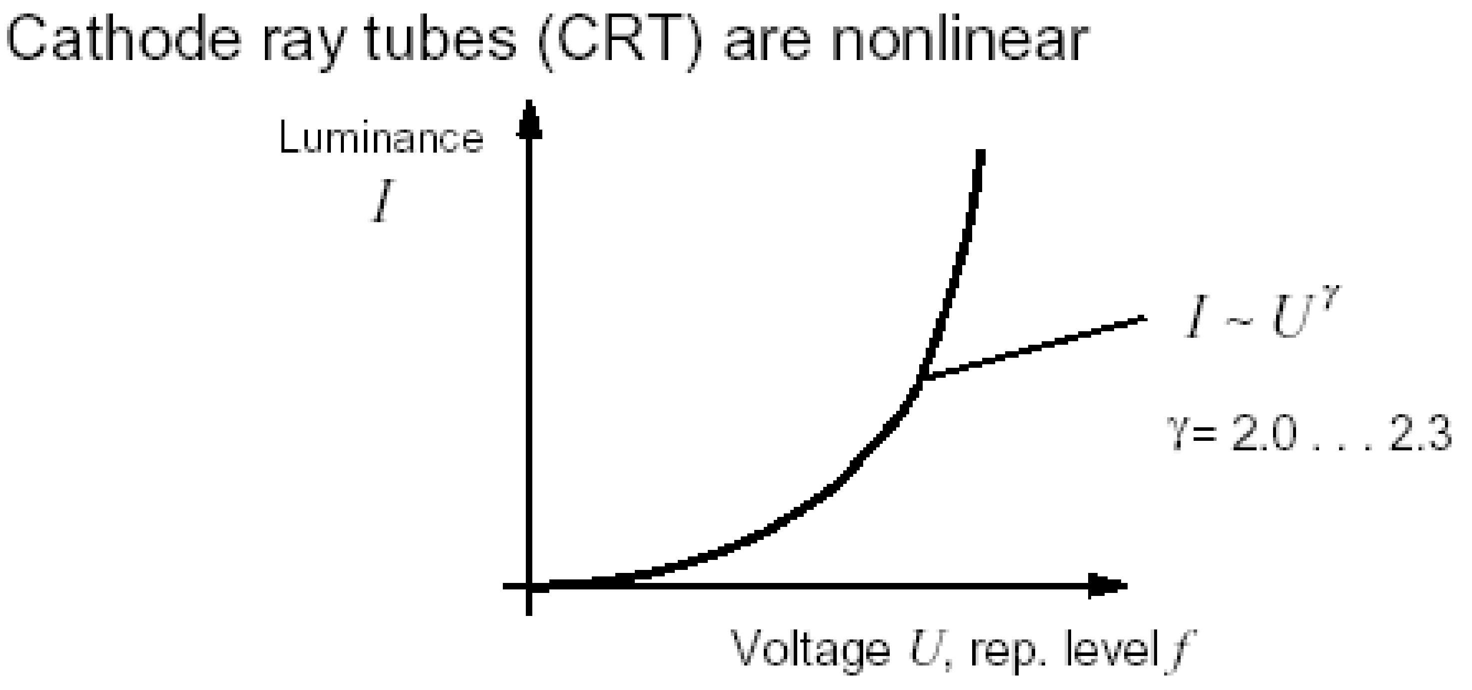
A variety of devices (image capture, printing, display) respond according to a power law and need to be corrected;

Gamma (γ) correction

The process used to correct the power-law response phenomena

Power-law Transformation

- Example of gamma correction

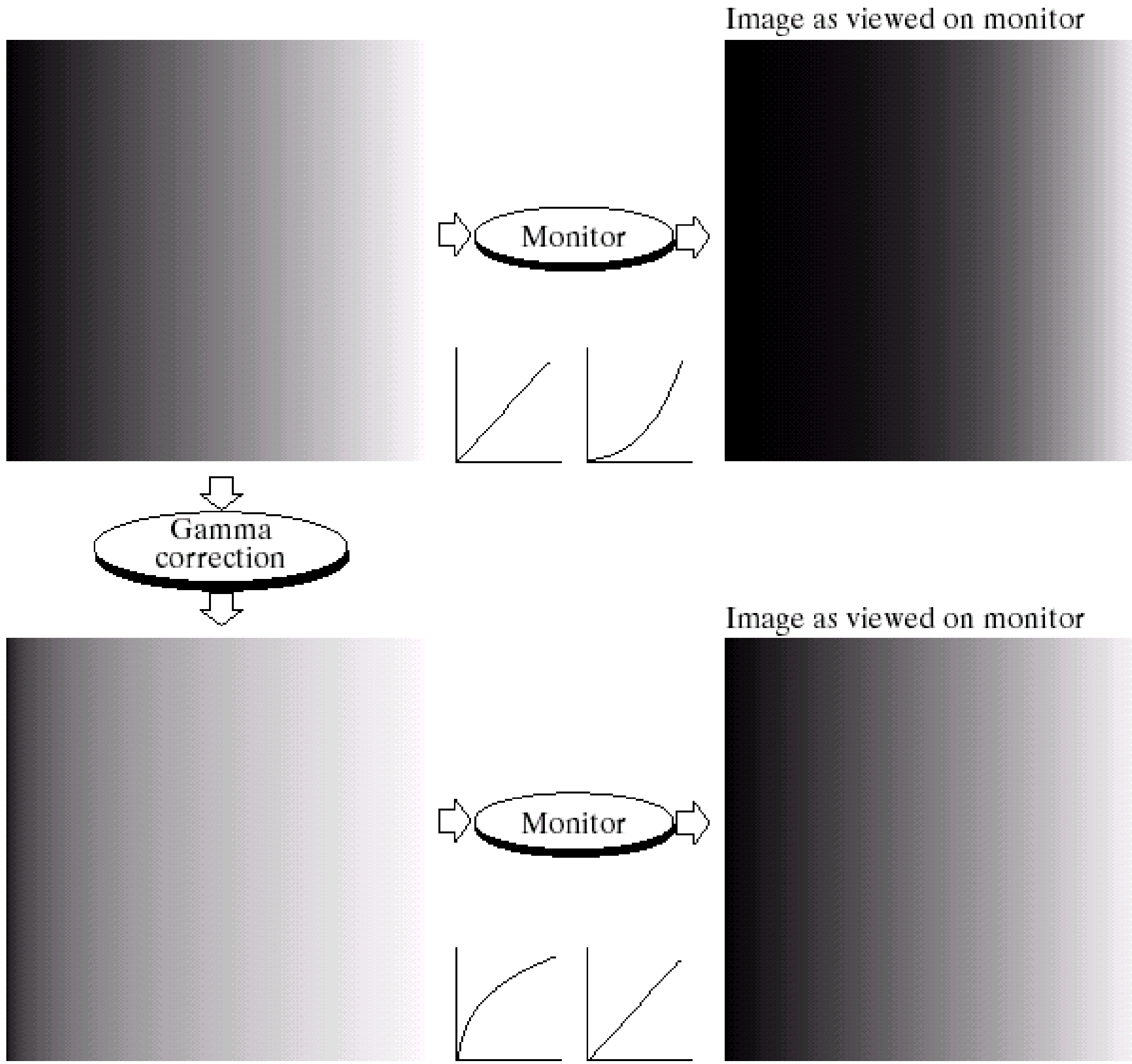


- To linearize the CRT response a pre-distortion circuit is needed $s = cr^{1/\gamma}$

Gamma correction

Linear wedge gray scale image

Response of CRT to Linear wedge



Gamma corrected wedge

Output of monitor

Power-law Transformation: Example

MRI image of fractured human spine

Result of applying power-law transformation

c = 1, $\gamma = 0.6$

Result of applying power-law transformation

c = 1, $\gamma = 0.4$

Result of applying power-law transformation

c = 1, $\gamma = 0.3$

Power-law Transformation: Example

Original satellite image

Result of applying power-law transformation

$c = 1, \gamma = 3.0$

Result of applying power-law transformation

$c = 1, \gamma = 4.0$

Result of applying power-law transformation

$c = 1, \gamma = 5.0$

Piecewise-linear transformation

Contrast stretching

Goal:

Increase the dynamic range of the gray levels for low contrast images

Low-contrast images can result from

- *poor illumination*
- *lack of dynamic range in the imaging sensor*
- *wrong setting of a lens aperture during image acquisition*

Piecewise-linear transformation: contrast stretching

Method

$$s = T(r) = \begin{cases} a_1 r, \ 0 \le r < r_1 & s_1 = T(r_1) \\ a_2(r - r_1) + s_1, \ r_1 \le r < r_2 & s_2 = T(r_2) \\ a_3(r - r_2) + s_2, \ r_2 \le r \le (L-1) & \end{cases}$$

where a_1, a_2, and a_3 control the result of contrast stretching

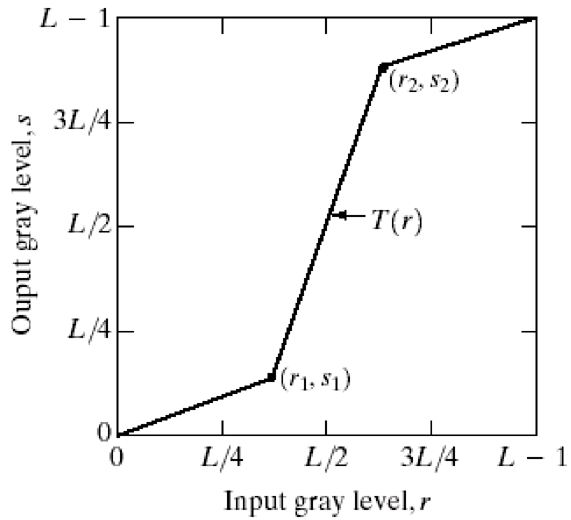
if $a_1 = a_2 = a_3 = 1$ no change in gray levels

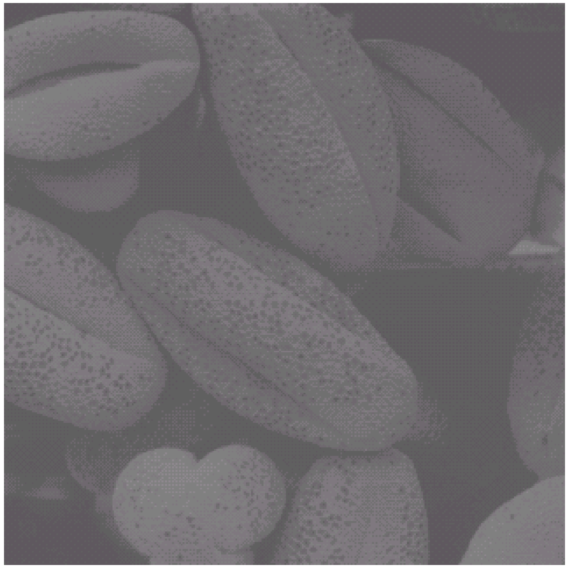
if $a_1 = a_3 = 0$ and $r_1 = r_2$, $T(*)$ is a thresholding function,

the result is a binary image

Contrast Stretching Example

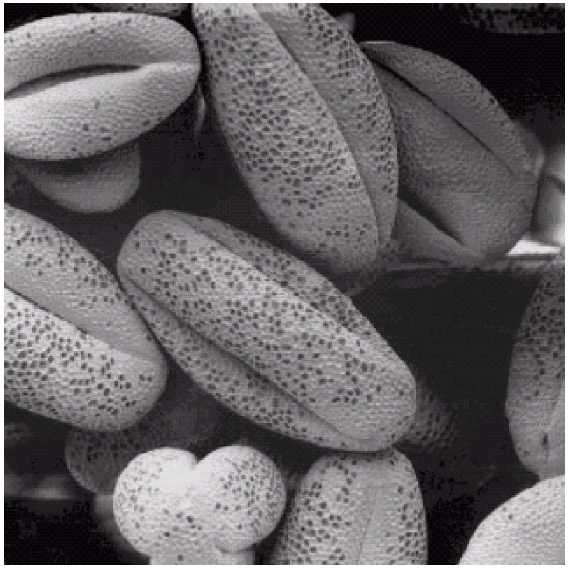
Form of Transformation function





Original low-contrast image

Result of contrast stretching



Result of thresholding

Histograms

Histogram of an image with gray level (0 to L-1):

A discrete function $h(r_k) = n_k$, where r_k is the k^{th} gray level and n_k is the number of pixels in the image having gray level r_k .

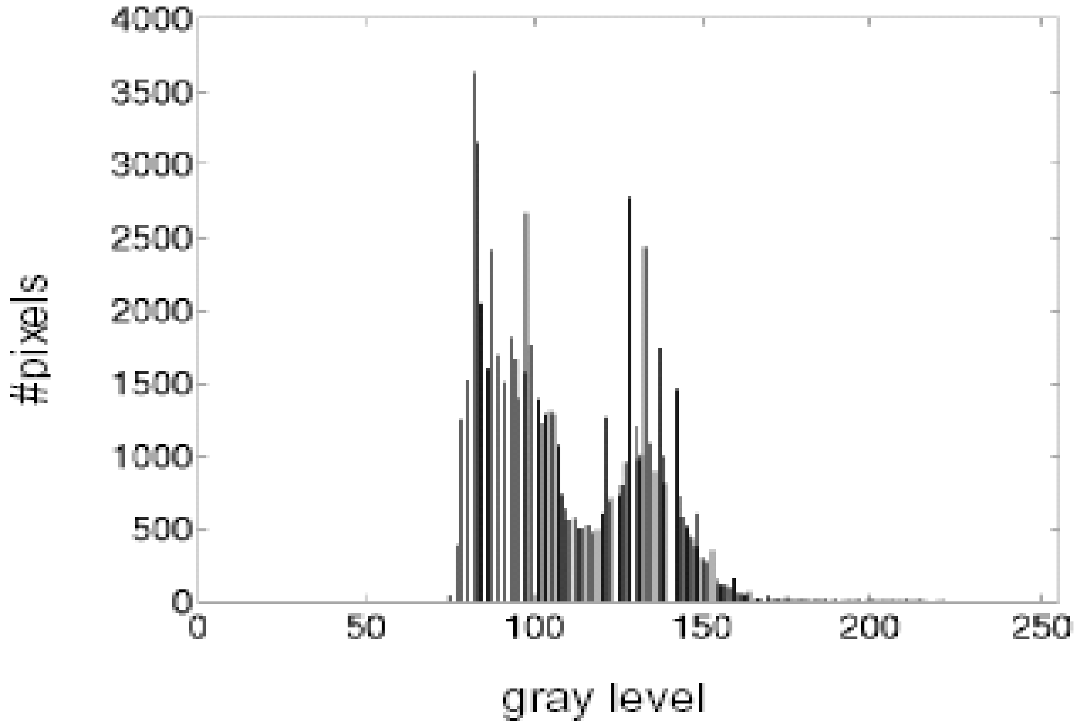
How a histogram is obtained?

- *For B-bit image, initialize 2^B counters with 0*
- *Loop over all pixels x,y*
- *When encountering gray level f(x,y)=i, increment counter # i*

Normalized histogram: A discrete function $p(r_k) = n_k/n$, where n is the total number of pixels in the image. $p(r_k)$ estimates probability of occurrence of gray-level r_k

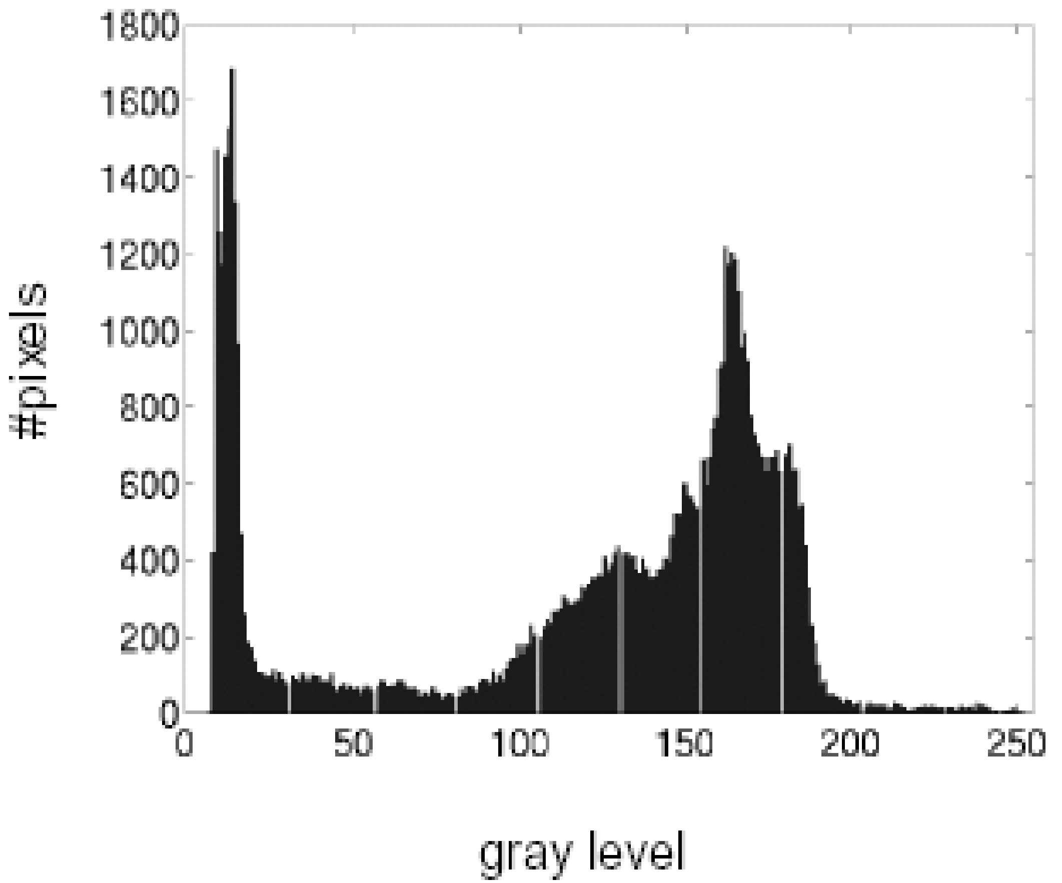
- Distribution of gray-levels can be judged by measuring a histogram
- Histogram provides global descriptions of the image (no local details)
- Fewer, larger bins can be used to trade off amplitude resolution against sample size.

Example Histogram



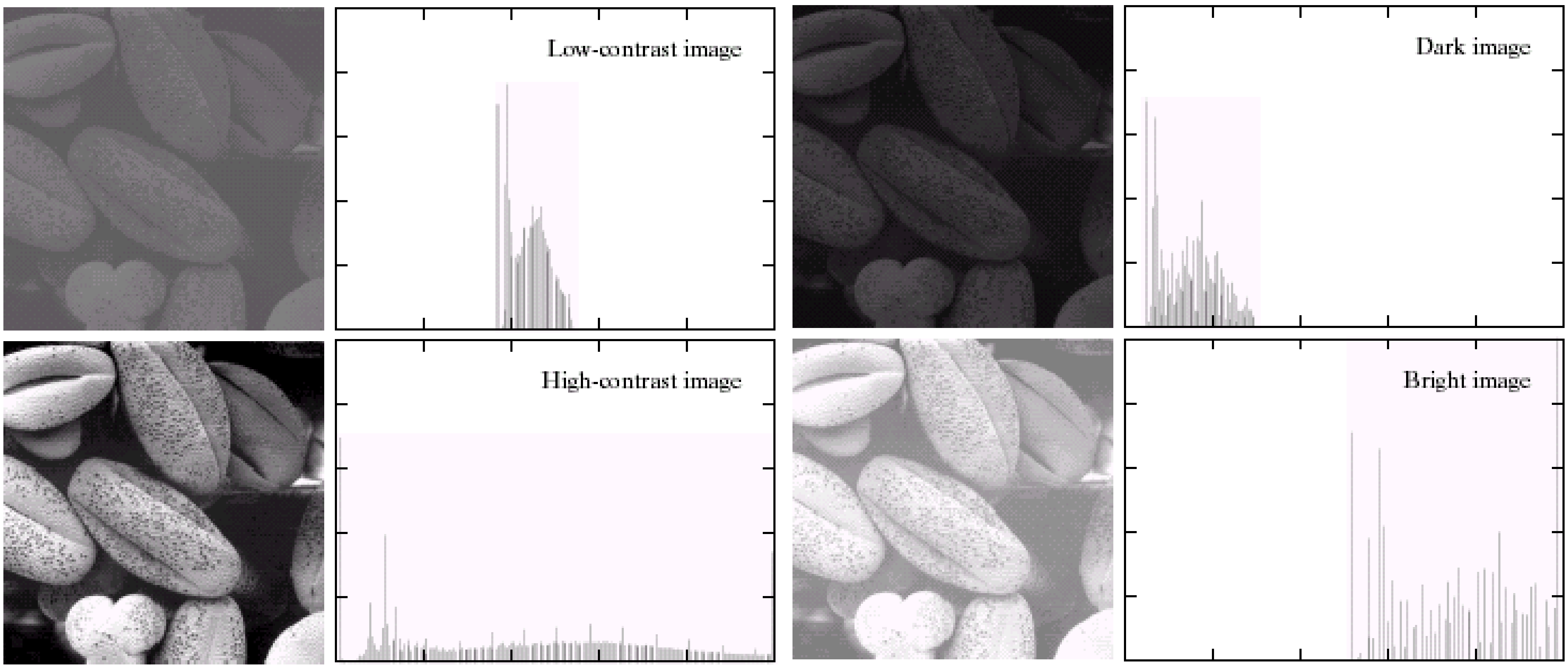
Pout
image

Example Histogram



Cameraman image

Histogram Examples

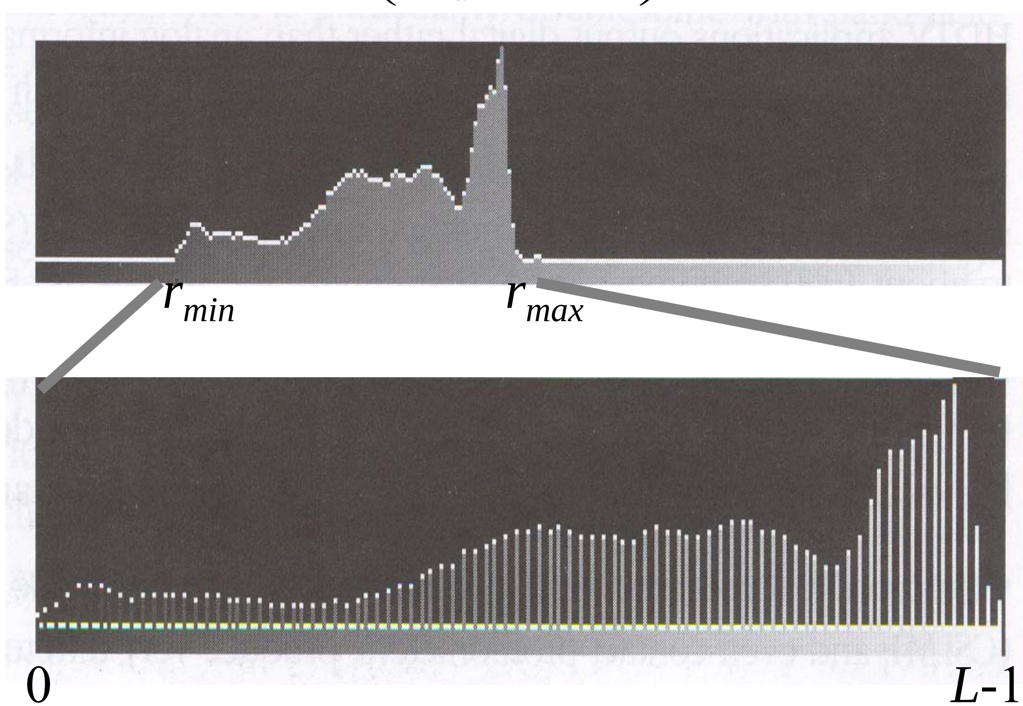


Contrast stretching through histogram

If r_{max} and r_{min} are the maximum and minimum gray level of the input image and L is the total gray levels of output image

The transformation function for contrast stretching will be

$$s = T(r) = (r - r_{\min})\left(\frac{L}{r_{\max} - r_{\min}}\right)$$



Histogram equalization

- Idea: To find a non-linear transformation

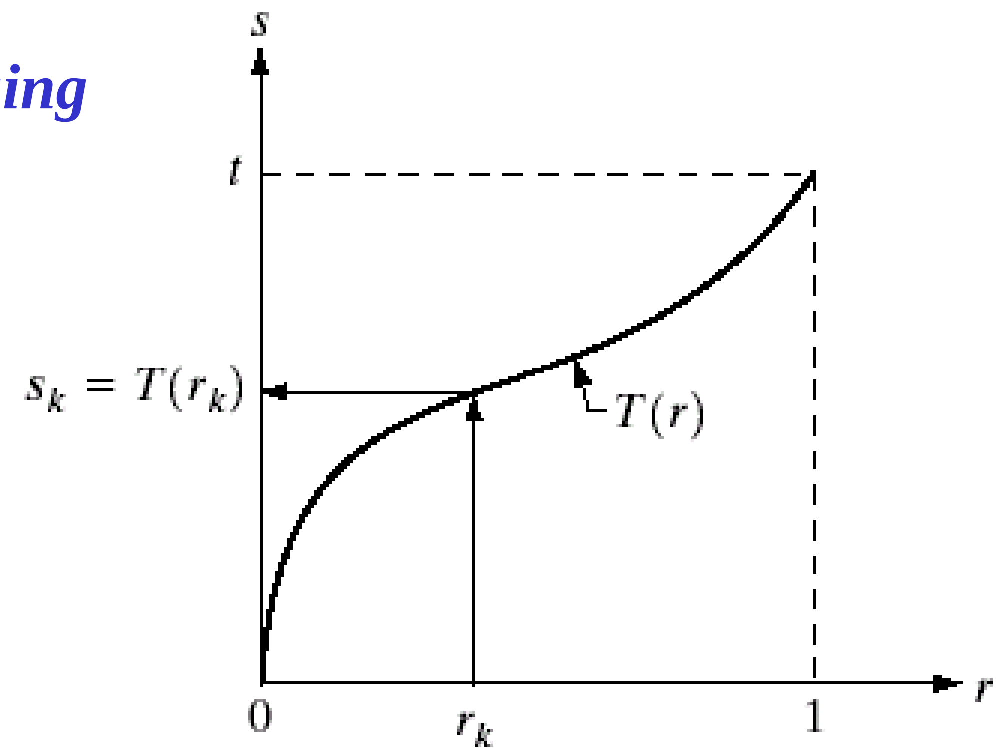
$$s = T(r)$$

to be applied to each pixel of the input image $f(x,y)$, such that a uniform distribution of gray levels in the entire range results for the output image $g(x,y)$.

- Assuming ideal, continuous case, with normalized histograms
  - *that* $0 \le r \le 1$ *and* $0 \le s \le 1$
  - *T(r) is single valued i.e., there exists* $r = T^{-1}(r)$
  - *T(r) is monotonically increasing*

Histogram equalization

A function $T(r)$ is ***monotonically increasing***
if $T(r_1) < T(r_2)$ for $r_1 < r_2$,
and ***monotonically decreasing***
if $T(r_1) > T(r_2)$ for $r_1 < r_2$.



Example of a transformation function which is both single valued and monotonically increasing

Background (probability distribution)

Assume ***continuous random variables***

The ***cumulative probability distribution function*** or ***cumulative distribution function* (cdf)**

The probability that the random variable is less than or equal to a specified constant a. We write this as

$$\boxed{F(a) = P(x \leq a).}$$

for all values of a (i.e., $-\infty < a < \infty$),

The ***probability density function* (pdf)** or ***density function*** of random variable x is defined as the derivative of the cdf:

$$\boxed{p(x) = \frac{dF(x)}{dx}.}$$

Histogram equalization

- $F_r(r)$ and $F_s(s)$: cdfs of original and transformed gray levels r and s.
- $p_r(r)$ and $p_s(s)$: pdfs of original and transformed gray levels r and s.

For strictly monotonically increasing transformation function

$$F_s(s) = F_r(r) \quad \text{or} \quad p_s(s)\,ds = p_r(r)\,dr$$

- Goal of histogram equalization:

  Gray levels are uniformly distributed

  i.e. pdf $p_s(s) = 1$ over the range $0 \le s \le 1$

$$p_s(s) = p_r(r)\left(\frac{dr}{ds}\right) = 1 \quad \text{or} \quad p_r(r) = \frac{ds}{dr} = \frac{dT(r)}{dr}$$

$$\boxed{\Rightarrow s = T(r) = \int_0^r p_r(\omega)\,d\omega}$$

Histogram equalization

If the following transformation function is used

$$s = T(r) = \int_0^r p_r(\omega)d\omega \quad \text{for} \qquad 0 \le r \le 1$$

Then the pdf $p_s(s) = 1$ over the range $0 \le s \le 1$

In words

If we select $T(r)$ as the cumulative distribution of r

Then the output image will have a uniform pdf of gray levels

Now Consider

1. a digital (gray level) case
2. the gray levels $0 \le r \le L - 1$

Histogram equalization

The discrete approximation of the transformation function for histogram equalization is:

$$\boxed{s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) \quad \text{for} \quad 0 \le k \le L-1}$$

where $p_r(r_j) = \dfrac{n_j}{n}, \quad j = 0,\ldots\ldots, L-1 \quad and \quad n = \sum_{j=0}^{L-1} n_j$

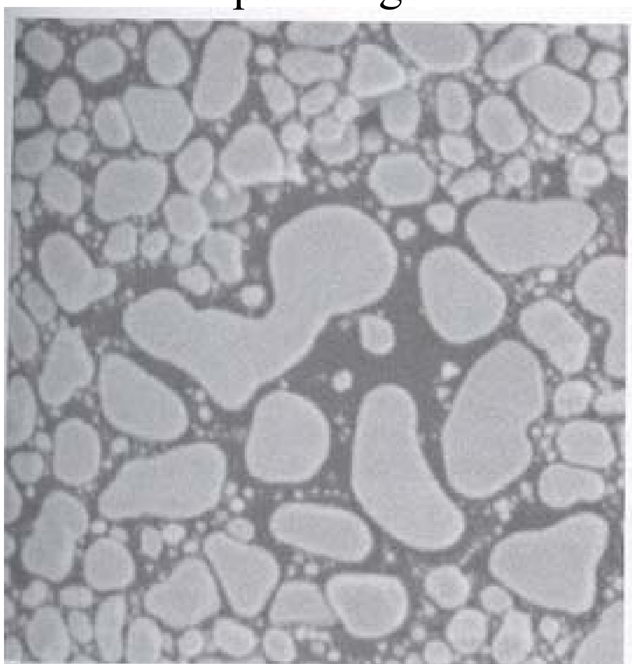
$\boldsymbol{n_j}$: number of pixels with gray level $\boldsymbol{r_j}$

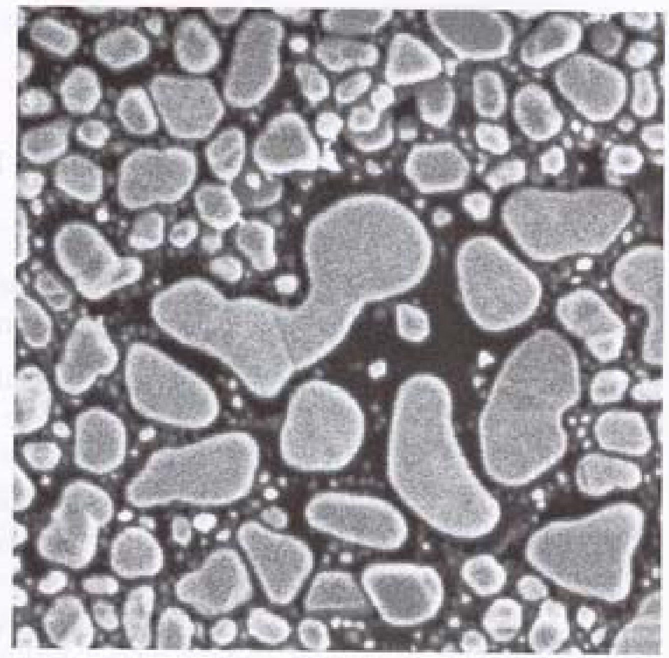
$\boldsymbol{n}$: total number of pixels

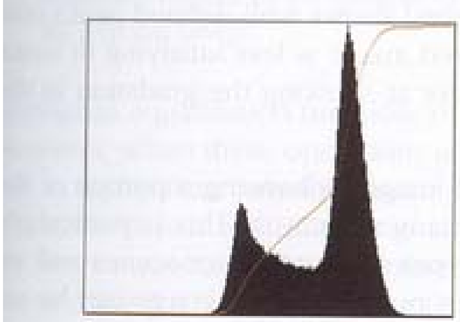
Note: For digital images, gray-level pdf cannot be exactly uniform after histogram equalization

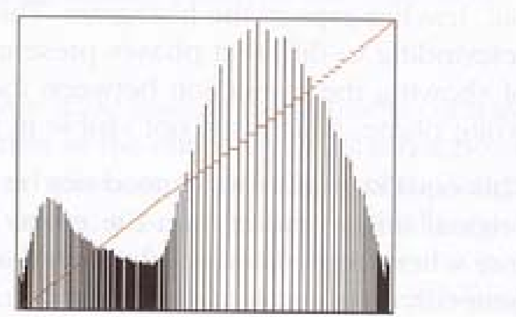
Histogram equalization examples

Input image | Output image





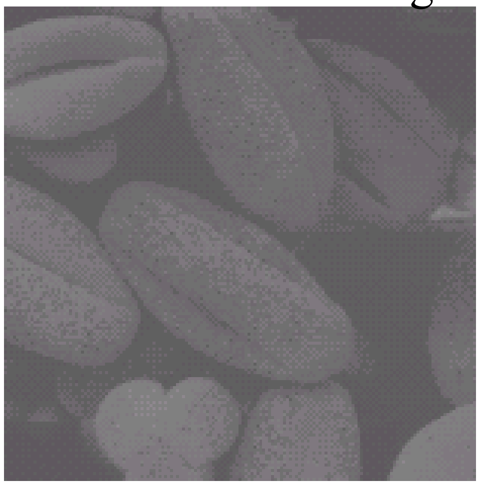




Input histogram and cdf | Output histogram and cdf

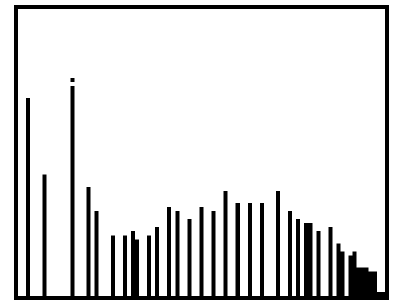
Histogram equalization examples

Low contrast image



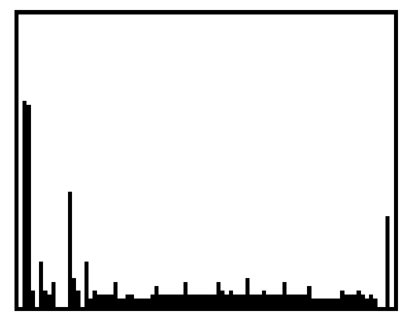
Output image

Equalized histogram



high contrast image

Output image

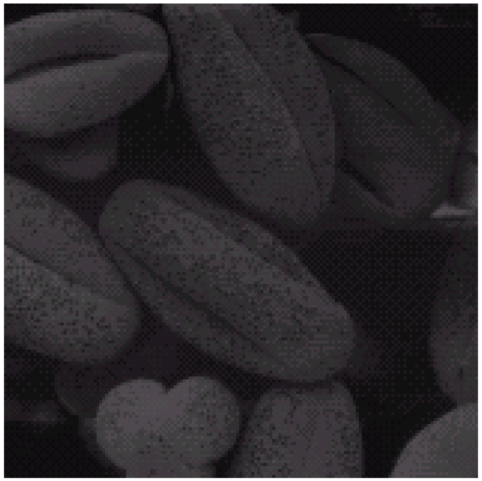


Equalized histogram

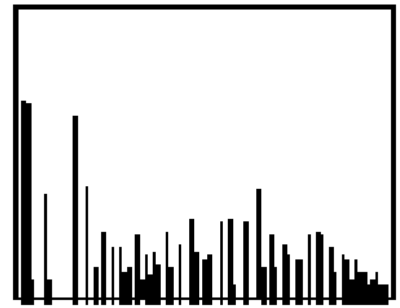
Histogram equalization examples

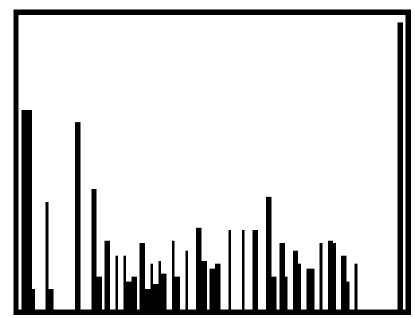
Dark input image

Output image



Equalized histogram



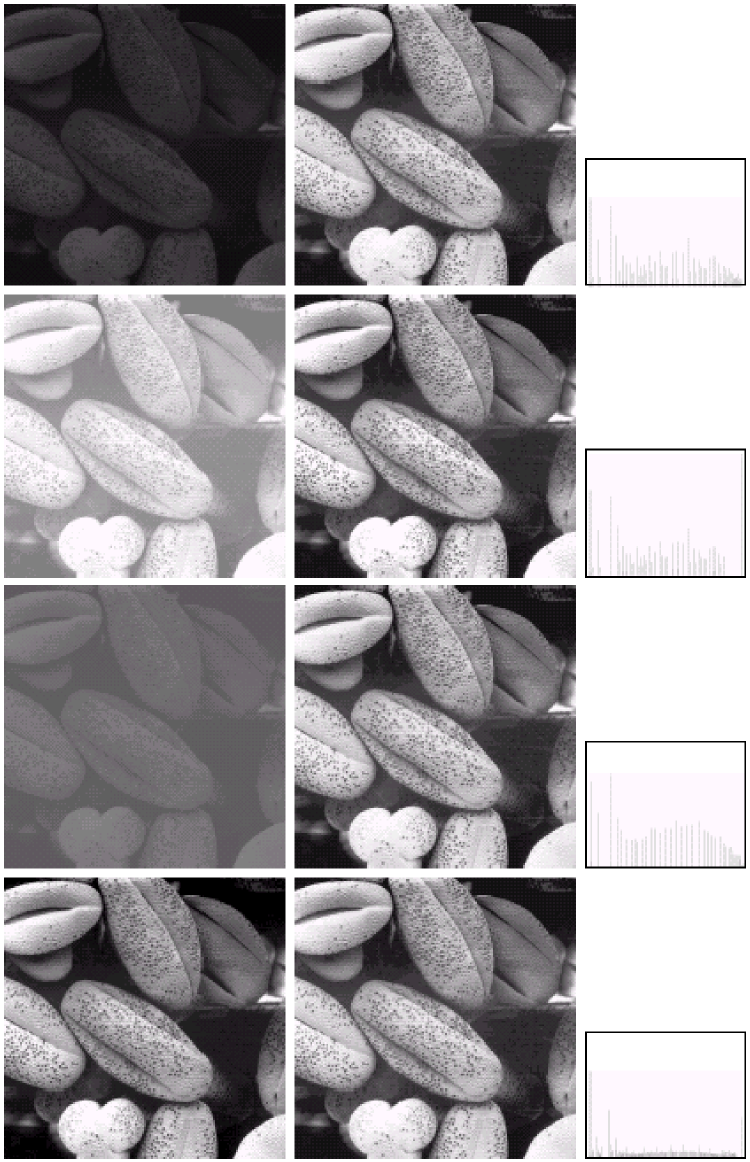


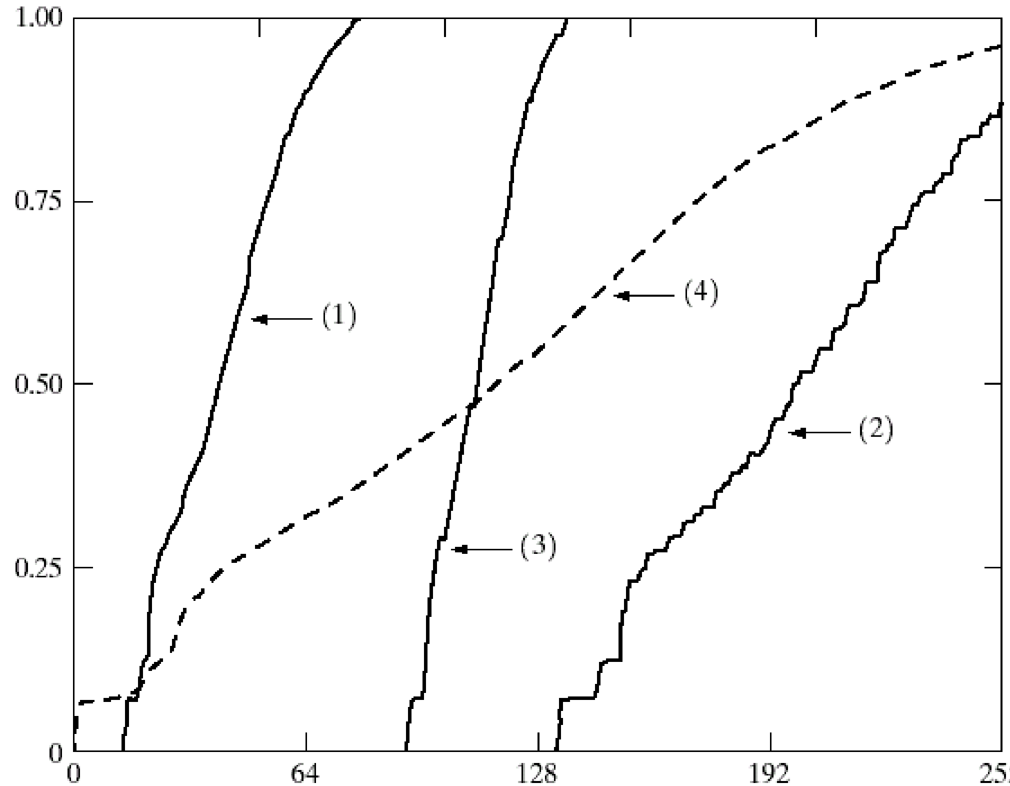
Bright input image

Output image

Equalized histogram

Histogram equalization examples





Transformation functions for histogram equalization

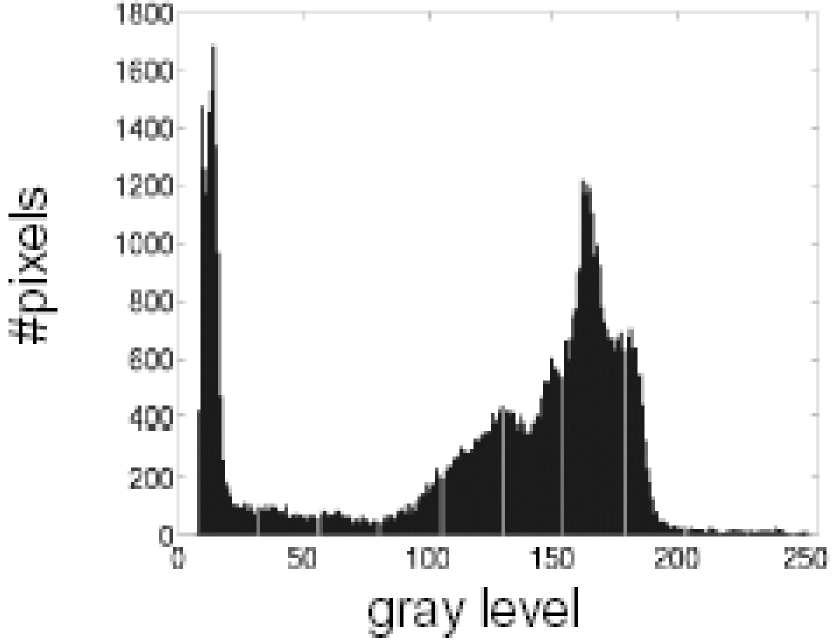
Histogram equalization examples

Original image
Cameraman

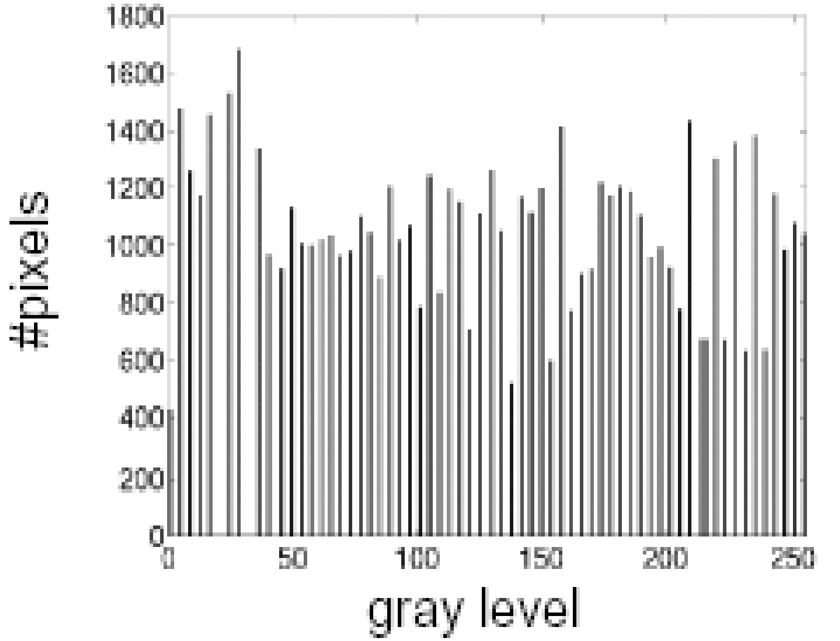
Cameraman
after histogram equalization

Histogram equalization examples

Original image *Cameraman*



. . . after histogram equalization



Histogram equalization examples

Original image *Moon*

Moon
after histogram equalization

Histogram equalization examples

Original image *Cameraman*

. . . after histogram equalization